ARK 1 TDT4171 - METODER I KUNSTIG INTELLIGENS

VÅR 2021 01/02 - 2021 HÅKON GRÆSBERG

ØVING 1

Prob 1) Probability of a person has 0, 1, 2, 3, 4, 5 or more is;

$P(S=0) = 0.15$, S = siblings

$P(S=1) = 0.49$

$P(S=2) = 0.27$

$P(S=3) = 0.06$

$P(S=4) = 0.02$

$P(S=5) = 0.01$

a) Probability that child has ~~at~~ at most 2 siblings; $P(S \leq 2)$

$$P(S \leq 2) = P(S=0) + P(S=1) + P(S=2) = 0.15 + 0.49 + 0.27 = \underline{\underline{0.91}}$$

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b) Probability child has more than 2 siblings, given has at least 1?

$$P(S > 2 \mid S \geq 1) = \frac{\overset{\text{II}}{P(S \geq 1 \mid S > 2)} \cdot \overset{\text{I}}{P(S > 2)}}{P(S \geq 1)} = \frac{1 \cdot (1 - 0.91)}{(1 - 0.15)} = \underline{\underline{0.11}}$$

I $P(S > 2) = 1 - P(S \leq 2)$ (from a))

II $P(S \geq 1 \mid S > 2) = 1$, if know have more than 2 sibling, clearly more than 1.

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c) Three friends gathered; Probability they have three siblings combined

$$P(S_{TOT} = 3) = \overset{\text{I}}{3} \cdot P(S_x = 0 \wedge S_y = 0 \wedge S_z = 3) + \overset{\text{II}}{6} \cdot P(S_x = 0 \wedge S_y = 1 \wedge S_z = 2)$$
$$+ \text{III}\, P(S_x = 1 \wedge S_y = 1 \wedge S_z = 1)$$
$$= 3 \cdot (0.15 \cdot 0.15 \cdot 0.06) + 6 \cdot (0.15 \cdot 0.49 \cdot 0.27) + (0.49^3)$$
$$= \underline{\underline{0.24}}$$

I 3 combinations of 2 person 0 sibling and one three

II 6 combo of 1 with 0, 1 with 1 and 1 with 2 siblings

III 1 combo of all have 1 sibling each.

ARK 2 TDT4171 ØVING 1 01/2-21 HÅKON GRÆSBERG

d) Emma and Jacob are not siblings, but combined they have total of 3 siblings. What is probability that Emma has no siblings?

$$P(S_E = 0 \mid S_E + S_J = 3) =$$

$$= \frac{P(S_E = 0 \wedge (S_J + S_E = 3))}{P(S_J + S_E = 3)} = \frac{P(S_J + S_E = 3 \mid S_E = 0)\, P(S_E = 0)}{P(S_J + S_E = 3)}$$

$$P(S_E = 0) = \underline{0.15}$$

$$P(S_J + S_E = 3) = 2 \cdot P(S_x = 0 \wedge S_y = 3) + 2 \cdot P(S_x = 1 \wedge S_y = 2)$$

$$= 2 \cdot (0.15 \cdot 0.06) + 2 \cdot (0.49 \cdot 0.27)$$

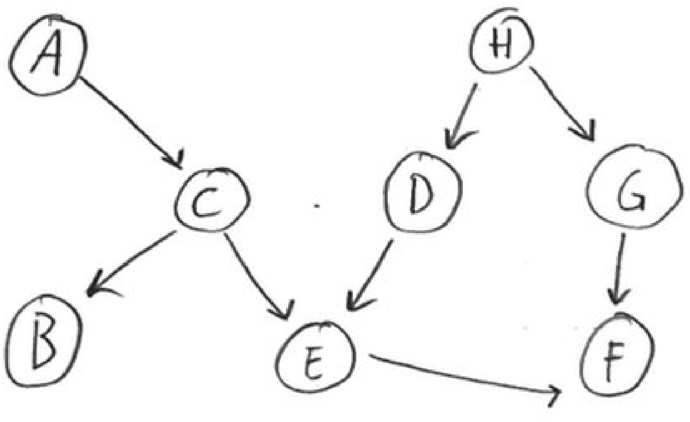
$$= \underline{0.28}$$

$$P(S_J + S_E = 3 \mid S_E = 0) = P(S_J + 0 = 3) = P(S_J = 3) = \underline{0.06}$$

$$P(S_E = 0 \mid S_E + S_J = 3) = \frac{P(S_J + S_E \mid S_E = 0)\, P(S_E = 0)}{P(S_E + S_J = 3)}$$

$$= \frac{0.06 \cdot 0.15}{0.28} = \underline{\underline{0.03}}.$$

Prob2) Bayesian Network Structure, decide statements and explain.



a) Every var has boolean state, Then Bayesian network can be represented by 18 numbers.

True, the numbers to represent a variable is defined by how many parents it has by formula $O(2^k)$ (worst case), k = parents.

A and H have no parents; $2 \cdot O(2^0) = 2$ numbers to represent these
C, D, G and B have one parent; $4 \cdot O(2^1) = 4 \cdot 2 = 8$ numbers.
E and F have 2 parents; $2 \cdot O(2^2) = 2 \cdot 4 = 8$ numbers.

By that, we need $2+8+8 = 18$ numbers to represent this network.

□.

ARK 4 TDT4171 Ø1 HÅKON GRÆSBERG

b) $G \perp\!\!\!\perp A$? (G independent of A?)

This means if $P(G|A) = P(G)$? (or $P(A|G) = P(A)$)

Yes, A and G are independent when we dont have any more information of the network. If we know A, it does not give anything in knowing only G.

c) $E \perp\!\!\!\perp H \mid \{D, G\}$?

$P(E \mid (H \mid D \lor H \mid G)) = P(E)$?

Yes, $P(E \mid H, D) = P(E \mid D)$, do not need (or give us anything) to know H if we know E given D. → Knowing Hs child D makes it independent from E.
When know D, H is not informative in knowing E.

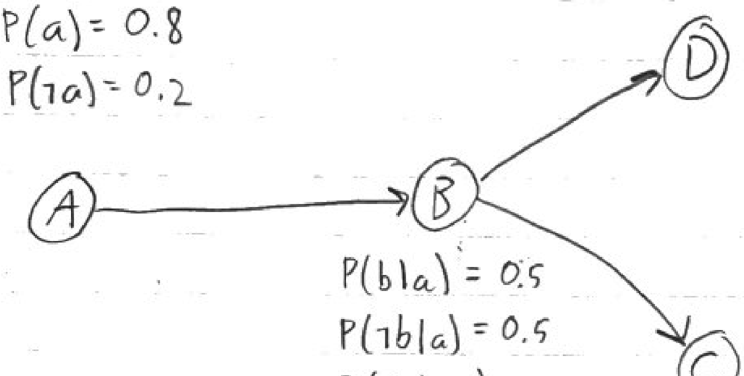
d) $E \perp\!\!\!\perp H \mid \{C, D, F\}$

No We know a common child, F. This affects the probability of both H an G, and they are not independent.
$E \not\perp\!\!\!\perp H \mid F$. If we also knew G, they would be independent again, but in knowing F propagetes up to H.

ARK5 TDT4171 Ø1 HÅKON GRÆSBERG

Prob 3) Bayesian network with binary states.
Conditional probability listed for each state. Calculate the probabilities;

$P(a) = 0.8$
$P(\neg a) = 0.2$



$P(d|b) = 0.6$
$P(\neg d|b) = 0.4$
$P(d|\neg b) = 0.8$
$P(\neg d|\neg b) = 0.2$

$P(b|a) = 0.5$
$P(\neg b|a) = 0.5$
$P(b|\neg a) = 0.2$
$P(\neg b|\neg a) = 0.8$

$P(c|b) = 0.1$
$P(\neg c|b) = 0.9$
$P(c|\neg b) = 0.3$
$P(\neg c|\neg b) = 0.7$

a) $P(b)$;

Represent the Network as Joint distribution: (by global semantics)

$$P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B)$$
$$= P(A)P(B|A)P(C|B)P(D|B)$$

$$P(b) = P(A,b,C,D) = P(A)P(b|A)P(C|b)P(D|b)$$

$$P(b) = \sum A \sum C \sum D \, P(A,b,C,D) \quad ; \text{ 8 summations}$$

$(A{=}a, C{=}c, D{=}d) = P(a)P(b|a)P(c|b)P(d|b)$
$(a, c, \neg d) \quad + P(a)P(b|a)P(c|b)P(\neg d|b)$
$(a, \neg c, d) \quad + P(a)P(b|a)P(\neg c|b)P(d|b)$
$(a, \neg c, \neg d) \quad + P(a)P(b|a)P(\neg c|b)P(\neg d|b)$
$(\neg a, c, d) \quad + P(\neg a)P(b|\neg a)P(c|b)P(d|b)$
$(\neg a, c, \neg d) \quad + P(\neg a)P(b|\neg a)P(c|b)P(\neg d|b)$
$(\neg a, \neg c, d) \quad + P(\neg a)P(b|\neg a)P(\neg c|b)P(d|b)$
$(\neg a, \neg c, \neg d) \quad + P(\neg a)P(b|\neg a)P(\neg c|b)P(\neg d|b)$

ARK6 TDT4171 Ø1 HÅKON GRÆSBERG

Prob3) P(b);

a)

Uses the process of avoiding calculating the prior prob by computing posterior and know the prior value equals the normalization-factor. Bayes Rule with normalization

$$P(A|b) = \frac{1}{P(b)} P(b|A)P(A) = \alpha P(b|A)P(A)$$

$$= \alpha \langle P(b|a)P(a),\ P(b|\neg a)P(\neg a) \rangle$$

$$= \alpha \langle 0.5 \cdot 0.8,\ 0.2 \cdot 0.2 \rangle$$

$$= \alpha \langle 0.4,\ 0.04 \rangle$$

$$= \frac{1}{0.4+0.04} \langle 0.4, 0.04 \rangle$$

Not necessary

$$= \frac{1}{0.44} \langle 0.4, 0.04 \rangle = \langle 0.91, 0.09 \rangle$$

$\rightarrow$ know that $\alpha = \frac{1}{P(b)}$ and $\alpha = \frac{1}{0.44}$

$\rightarrow \frac{1}{P(b)} = \frac{1}{0.44} \rightarrow \underline{P(b) = 0.44}$

b) $P(d)$:

Bayes Rule: $P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$

With normalization, $= \alpha P(Y|X)P(X) = \frac{1}{P(Y)} P(Y|X)P(X) = P(X|Y)$

Use this to find $P(d)$ by finding it as the normalization value.

$P(B|d) = \frac{P(d|B)P(B)}{P(d)} = \alpha P(d|B)P(B)$

$= \alpha \langle P(d|b)P(b), P(d|\neg b)P(\neg b) \rangle$

$P(d|b) = 0.6$
$P(d|\neg b) = 0.8$
$P(b) = 0.44$, from a)
$P(\neg b) = 1 - P(b) = \underline{0.56}$

$= \alpha \langle 0.6 \cdot 0.44, 0.8 \cdot 0.56 \rangle$

$= \alpha \langle 0.264, 0.448 \rangle$

↓ Normalize, multiply by $\alpha = \frac{1}{P(d)} = \frac{1}{0.264 + 0.448}$

$= \frac{1}{0.712} \langle 0.264, 0.448 \rangle$

$\underline{\underline{P(d) = 0.712}}$

$(= \langle 0.371, 0.629 \rangle)$

↳ Not necessary, only interested in $P(d)$ here

ARK 8 TDT4171 Ø1 HÅKON GRÆSBERG

c) $P(c \mid \neg d)$

Use the semantics to find the representation of the joint Distribution.

$$P(A,B,C,D) = \prod_{i=1}^{n} P(X \mid parents(X))$$

$$= P(A)\,P(B \mid A)\,P(C \mid B)\,P(D \mid B)$$

Find $P(c \mid \neg d) = \alpha \sum_a \sum_b P(a,b,c,\neg d) = (P(A,B,c,\neg d))$

$$= \alpha \sum_a \sum_b P(a)P(b \mid a)P(c \mid b)P(\neg d \mid b)$$

$$= \alpha \sum_a P(a) \sum_b P(b \mid a)P(c \mid b)P(\neg d \mid b)$$

c = true

$$= \alpha\Big(\big(P(a)\cdot(P(b \mid a)P(c \mid b)P(\neg d \mid b) + P(\neg b \mid a)P(c \mid \neg b)P(\neg d \mid \neg b)\big) + P(\neg a)\cdot(P(b \mid \neg a)P(c \mid b)P(\neg d \mid b) + P(\neg b \mid \neg a)P(c \mid \neg b)P(\neg d \mid \neg b))\Big)$$

$$= \alpha\big(0.8\cdot(0.5\cdot 0.1\cdot 0.4 + 0.5\cdot 0.3\cdot 0.2) + (0.2(0.2\cdot 0.1\cdot 0.4 + 0.8\cdot 0.3\cdot 0.2))\big) = \underline{0.0512}$$

$$= \alpha\langle 0.0512, ?\rangle \quad \left(\text{Dont need this now!} \rightarrow \frac{1}{P(\neg d)}\langle 0.0512, ?\rangle \rightarrow \langle 0.178, 0.822\rangle \quad (1-0.178)\right)$$

Know that $\alpha = \dfrac{1}{P(\neg d)} = \dfrac{1}{1-P(d)} = \dfrac{1}{1-0.712} = \dfrac{1}{\underline{0.288}}$

$\uparrow$ from b)

$$P(c \mid \neg d) = \alpha\, 0.0512 = \frac{0.0512}{0.288} = \underline{\underline{0.178}}$$

ARK9 TDT4171 Ø1 HÅKON GRÆSBERG

d) $P(a|\neg c, d)$

$$P(A,B,C,D) = P(A)P(B|A)P(C|B)P(D|B)$$

$$P(a|\neg c,d) = \alpha P(a,B,\neg c,d)$$

$$= \alpha \sum_b P(a,b,\neg c,d) = \alpha \sum_b P(a)P(b|a)P(\neg c|b)P(d|b)$$

$$= \alpha P(a) \sum_b P(b|a)P(\neg c|b)P(d|b)$$

$$= \alpha P(a)\left(P(b|a)\cdot P(\neg c|b)P(d|b) + P(\neg b|a)P(\neg c|\neg b)P(d|\neg b)\right)$$

$$= \alpha\left(0.8\cdot(0.5\cdot 0.9\, 0.6 + 0.5\cdot 0.7\cdot 0.8)\right)$$

$$\underline{= \alpha \cdot 0.44}$$

Finds for $P(\neg a|\neg c,d)$ to find normalizationvalue and by that find $P(a|\neg c,d)$.

ARK10 TDT4171 Ø1 HÅKON GRÆSBERG

$P(\neg a \mid \neg c, d) = \alpha P(\neg a, B, \neg c, d)$

$= \alpha \sum_{b} P(\neg a, b, \neg c, d) = \alpha \sum_{B} P(\neg a) P(b \mid \neg a) P(\neg c \mid b) P(d \mid b)$

$= \alpha P(\neg a) \sum_{b} P(b \mid \neg a) P(\neg c \mid b) P(d \mid b)$

$= \alpha P(\neg a) \cdot ((P(b \mid \neg a) P(\neg c \mid b) P(d \mid b) + P(\neg b \mid \neg a) P(\neg c \mid \neg b) P(d \mid \neg b)$

$= \alpha \cdot 0.2 (0.2 \; 0.9 \cdot 0.6 + 0.8 \cdot 0.7 \cdot 0.8)$

$= \alpha \cdot 0.111$

$\alpha \, 0.111 + \alpha \, 0.44 = 1$

$\downarrow$

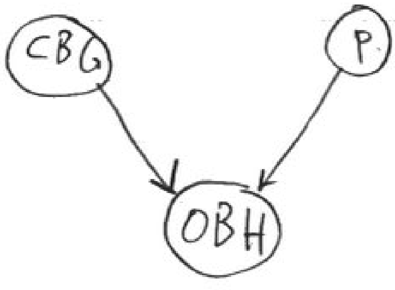
$\alpha = \frac{1}{0.111 + 0.44} = \frac{1}{0.551}$

$P(a \mid \neg c, d) = \alpha \, 0.44 = \frac{0.44}{0.551} = 0.799$

ARK 11 TDT4171 Ø1 HÅKON GRÆSBERG

4c) Chosen By Guest = CBG D_1 = door 1
Opened By Host = OBH D_2 = door 2
Prize = P D_3 = door 3

$P(CBG=D_1)=0.33$
$P(CBG=D_2)=0.33$
$P(CBG=D_3)=0.33$



$P(P=D_1)=0.33$
$P(P=D_2)=0.33$
$P(P\neq D_3)=0.33$

$P(OBH \mid CBG, P) = \langle D_1, D_2, D_3 \rangle$

$P(OBH \mid CBG=D_1 \wedge P=D_1) = \langle 0, 0.5, 0.5 \rangle$

$P(OBH \mid CBG=D_1 \wedge P=D_2) = \langle 0, 0, 1 \rangle$

$P(OBH \mid CBG=D_1 \wedge P=D_3) = \langle 0, 1, 0 \rangle$

$P(OBH \mid CBG=D_2 \wedge P=D_1) = \langle 0, 0, 1 \rangle$

$P(OBH \mid CBG=D_2 \wedge P=D_2) = \langle 0.5, 0, 0.5 \rangle$
$P(OBH \mid CBG=D_2 \wedge P=D_3) = \langle 1, 0, 0 \rangle$
$P(OBH \mid CBG=D_3 \wedge P=D_1) = \langle 0, 1, 0 \rangle$
$P(OBH \mid CBG=D_3 \wedge P=D_2) = \langle 1, 0, 0 \rangle$
$P(OBH \mid CBG=D_3 \wedge P=D_3) = \langle 0.5, 0.5, 0 \rangle$

Oppg.4

a)

You can see the topologically sorted nodes under resulting nodes and the object values for each variable. The sorting is topological.

```
The resulting nodes:
 [<__main__.Variable object at 0x00D91250>, <__main__.Variable object at 0x00D8B0B0>, <__main__.Variable object at 0x00D91170>, <__main__.Variable object at 0x00E239F0>]

 {'A': <__main__.Variable object at 0x00D91250>, 'B': <__main__.Variable object at 0x00D8B0B0>, 'C': <__main__.Variable object at 0x00D91170>, 'D': <__main__.Variable object at 0x00E239


[0.45294118 0.54705882]
```

b)

The result I got from b) were not right, but it calculated some values that are normalized. See the bottom of the snippet over.

```
Probability distribution, P(C | !D)
+----------+----------+
|   C(0)   |  0.4529  |
+----------+----------+
|   C(1)   |  0.5471  |
+----------+----------+
```

c)

I think I created the problem correctly. Here are snippets of the distributions and the answer. The code is in the bottom of the python-file.

```
Probability distribution, P(CBG)
+----------+----------+
|  CBG(0)  |  0.3333  |
+----------+----------+
|  CBG(1)  |  0.3333  |
+----------+----------+
|  CBG(2)  |  0.3333  |
+----------+----------+

Probability distribution, P(P)
+----------+----------+
|   P(0)   |  0.3333  |
+----------+----------+
|   P(1)   |  0.3333  |
+----------+----------+
|   P(2)   |  0.3333  |
+----------+----------+
```

```
Probability distribution, P(OBH | P, CBG)
+----------+----------+----------+----------+----------+----------+----------+----------+----------+----------+
|   CBG    |  CBG(0)  |  CBG(1)  |  CBG(2)  |  CBG(0)  |  CBG(1)  |  CBG(2)  |  CBG(0)  |  CBG(1)  |  CBG(2)  |
+----------+----------+----------+----------+----------+----------+----------+----------+----------+----------+
|    P     |   P(0)   |   P(0)   |   P(0)   |   P(1)   |   P(1)   |   P(1)   |   P(2)   |   P(2)   |   P(2)   |
+----------+----------+----------+----------+----------+----------+----------+----------+----------+----------+
|  OBH(0)  |  0.0000  |  0.0000  |  0.0000  |  0.0000  |  0.5000  |  1.0000  |  0.0000  |  1.0000  |  0.5000  |
+----------+----------+----------+----------+----------+----------+----------+----------+----------+----------+
|  OBH(1)  |  0.5000  |  0.0000  |  1.0000  |  0.0000  |  0.0000  |  0.0000  |  1.0000  |  0.0000  |  0.5000  |
+----------+----------+----------+----------+----------+----------+----------+----------+----------+----------+
```

```
The resulting nodes:
 [<__main__.Variable object at 0x01251290>, <__main__.Variable object at 0x0124B0F0>, <__main__.Variable object at 0x0124B0F0>, <__main__.Variable object at 0x012511B0>]

 {'CBG': <__main__.Variable object at 0x01251290>, 'P': <__main__.Variable object at 0x0124B0F0>, 'OBH': <__main__.Variable object at 0x012511B0>}


Probability distribution, P(P | CBG = 1 OBH = 3)
+----------+----------+
|   P(0)   |  0.3333  |
+----------+----------+
|   P(1)   |  0.6667  |
+----------+----------+
|   P(2)   |  0.0000  |
+----------+----------+
```

The Monty hall did return the correct answer, but the prize-object were added to resulted topological sort twice. Not sure why, and this did not work on the example problem 3c), but it did now. Probably something strange with the sorting as well.