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FULL LENGTH ARTICLE

Robust pre-departure scheduling for a nation-wide air traffic flow management

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Abstract Air traffic flow management has been a major means for balancing air traffic demand and airport or airspace capacity to reduce congestion and flight delays. However, unpredictable factors, such as weather and equipment malfunctions, can cause dynamic changes in airport and sector capacity, resulting in significant alterations to optimized flight schedules and the calculated pre-departure slots. Therefore, taking into account capacity uncertainties is essential to create a more resilient flight schedule. This paper addresses the flight pre-departure sequencing issue and introduces a capacity uncertainty model for optimizing flight schedule at the airport network level. The goal of the model is to reduce the total cost of flight delays while increasing the robustness of the optimized schedule. A chance-constrained model is developed to address the capacity uncertainty of airports and sectors, and the significance of airports and sectors in the airport network is considered when setting the violation probability. The performance of the model is evaluated using real flight data by comparing them with the results of the deterministic model. The development of the model based on the characteristics of this special optimization mechanism can significantly enhance its performance in addressing the pre-departure flight scheduling problem at the airport network level.

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1. Introduction

The 2018 International Civil Aviation Organization (ICAO) statistics showed that 4.3 billion passengers were served by air transport, a 6.1% increase from the previous year.[1] Despite the construction of new infrastructure to enhance the capacity and efficiency of air traffic systems, flight delays remain an issue due to unpredictable elements such as extreme weather. In 2018, the flight departure delays within Europe rose to 14.7 min per flight, a 2.3 min increase from 2017.[2] The imbalance between the capacity of the air traffic system and the demand for traffic is still the primary cause of flight delays. Consequently, optimization of air traffic operations is still a major area of research.

A great deal of research has been conducted to improve the operational effectiveness of air traffic systems since the 1990s. From the investigation of the Ground Holding Problem (GHP) of single airports, multiairports and airport networks,[3,4] to the examination of the dynamic flow problem of the network and the formulation of the uncertainty problem,[5,6] researchers from all over the world have made remarkable advances in the air transportation field. Since 2010, researchers have been attempting to optimize the flight schedule in a way that is more beneficial to all parties, taking into account fairness.[7–11] Furthermore, to create a more reliable flight schedule, the uncertainty factor in the airport network has also received attention.[12–14] As research results continue to be enriched, systematized integrated models have also been featured in many studies.[15–17] Some researchers have shifted their focus from the supply side to the demand side of air transport service.[18,19] The use of mathematics and big data has also been explored to optimize flight schedules.[20–22] In recent years, a large body of literature has been developed around the topic of airport capacity and demand management.

Air Traffic Flow Management (ATFM) is a key technique used by traffic management operators to balance the capacity of airports/airspace and air traffic demand. This has been a major area of research for many years. ATFM can be broken down into three stages: strategic ATFM, pre-tactical ATFM and tactical ATFM. Strategic ATFM begins several months before the day of operation and involves slot allocation and airspace organization. Pre-tactical ATFM can start from a week or a day before the day of operation, during which flight schedules may be adjusted. The tactical phase is usually the day of flight operation and the main activities include executing the daily plan developed in the pre-tactical phase, and formulating and implementing tactical traffic flow management measures to address unexpected supply–demand imbalances. The implementation of ATFM involves airlines, airports, and air traffic control authority. For more than two decades, the concept of Collaborative Decision Making (CDM) has been explored around the world to enhance the effectiveness of ATFM. This concept has now been widely adopted in major airports and Airport CDM (A-CDM) has become a successful example of its application. A-CDM's operation process helps to reduce delays, improve punctuality, and optimize resource utilization. It provides an information sharing platform and decision-making mechanism for multiple parties in air transport, allowing them to optimize resource utilization while still meeting the interests of all parties. The specific process and the 16 milestones are illustrated in Fig. 1. [23] All parties involved share their latest information and plan to keep everyone maintaining the same situation awareness. Two important information provided by air traffic control for departure flights are the Target Take-off Time (TTOT) and the Target Start-up Approval Time (TSAT). More accurate and stable timing information is vital to the successful implementation of A-CDM.

However, insufficient focus has been given to examining the robust optimization of departure slots for all flights within the aviation system. Although current research and decision making tools can identify the best flight schedule for air traffic management authorities, changes in departure slots may occur due to unexpected events or operational constraints that affect capacity in downstream sectors or airports. Such occurrences are commonly observed in actual operational scenarios. If the actual capacity is lower than the expected capacity, and too many flights were scheduled during the period, then some flights will not be able to take off as planned, resulting in flight delays. On the other hand, if the actual capacity is higher than the predicted capacity, for instance, the capacity of the affected airport or airspace is quickly restored, the preassigned departure slots may have to be changed earlier to avoid wasting capacity and unnecessary flight delays. In such situations, airlines have to rearrange their resources to facilitate earlier departures, which could result in increased workload and cost. One of the primary obstacles is making departure decisions when faced with capacity uncertainty. There is still a deficiency in methods for accurately forecasting airport and airspace capacity. The second issue lies in the complexity of the air traffic management system as a networked system, where modifications in operations at a specific sector or airport can impact the operations of other airports and sectors within the network. In instances where a hub airport's operational capacity falls short of the anticipated level, traffic control restrictions will affect both incoming and outgoing flights at that airport. Over-scheduled flights have to be delayed, leading to delays that will propagate through the entire air transportation network. Conversely, the repercussions of over-scheduling are less significant at smaller airports with lower levels of traffic. No research has been conducted to investigate the impact of network effects on the optimized strategies in air traffic flow management problems when using optimization techniques that account for uncertainty in stochastic scenarios.

This paper investigates the pre-departure scheduling in the tactical phase of air traffic flow management. Real-time data are used to make optimal adjustments to the flight schedule to enhance airport efficiency and punctuality. The contributions of our work are outlined below. First, a pre-departure scheduling model is proposed which considers uncertain capacity of airport and airspace to minimize the total cost of flight delays. The chance constraint method is used to manage the uncertainty of airport and sector capacity. Second, our model takes the characteristics of air traffic network into account while the computational results are compared with the optimization results of the deterministic capacity model, as well as the superiority of the capacity uncertainty model considering the node importance over the traditional model that only considers capacity uncertainty. This study is valuable for improving the effectiveness of collaborative air traffic flow management, and could have significant implications for designing network control strategies in uncertain conditions.

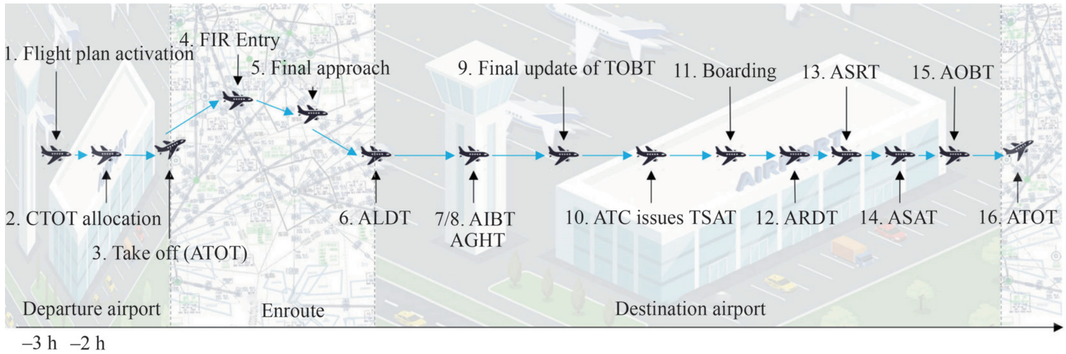

Fig. 1 Airport CDM milestones. A total of 16 key events during arrival and departure are marked as milestones.[23] Some critical time events are Calculated Take-off Time (CTOT), Actual Take-off Time (ATOT), Actual Landing Time (ALDT), Actual In-block Time (AIBT), Actual Commencement of Ground Handling Operations Time (ACGHOT), Target Off-block Time (TOBT), Aircraft Ready for Departure Time (ARDT), Aircraft Startup Requested Time (ASRT), Actual Start-up Approval Time (ASAT), Actual Off-block Time (AOBT), Actual Take-off Time (ATOT).

This paper is organized as follows. Section 2 reviews the existing literature on air traffic flow management, uncertainty problems, and the chance constraint method. Section 3 presents a model for allocating uncertainty departure and arrival slots flights at the airport network level and explains the transformation of chance constraints. Section 4 examines the structural characteristics of the national air traffic network and evaluates the importance of the airport and sector nodes. Section 5 introduces two methods to set the probability of violation of chance constraints and provides experimental results. Section 6 is the conclusion.

2. Literature review

The majority of the world's busiest airports experience significant issues with traffic congestion and flight delays. Expanding airport infrastructure to increase capacity is a potential solution, but it is a lengthy and costly process that makes it difficult to quickly solve the problem. An effective and practical alternative is air traffic flow management. In 1987, Odoni proposed a mathematical model for the first time to address the air traffic flow management problem. The model seeks to balance airport capacity and traffic demand, converting air delay into ground delay, thus reducing the cost of flight operations.[24] Significant attention has been focused on researching air traffic flow management since that time. Comprehensive reviews of air traffic flow management can be found in Ref. 25–27. This section provides an overview of research conducted on pre-tactical and network-level tactical air traffic flow management. The studies can be categorized into deterministic traffic management models and stochastic traffic management models, depending on whether they consider the future impact on airport or sector capacity as deterministic or uncertain.

2.1. Deterministic air traffic flow management models

The first research on ground holding strategies for air traffic flow management is discussed in Ref. 28, where a deterministic model is introduced for the holding of the ground in a single airport. The model takes into account the airport's capacity, which is determined by factors such as weather conditions (such as wind speed and visibility), to calculate flight departure times and delays. However, this model does not account for the ripple effect of delays spreading between successive flights of the same aircraft. In Ref. 3, a collaborative multi-airport ground delay model is proposed and a heuristic algorithm is developed to find a feasible solution in a reasonable time. This study examines the ground delay strategies for a network of airports where the impacts of delay propagation over time are carefully considered. To account for both airport capacity and sector capacity constraints, a 0–1 integer program model is developed.[29] Nevertheless, the model does not account for flight rerouting and cancellations. Subsequently, the authors proposed a dynamic network flow method to examine the reroute issue within air traffic flow management. [5]Lulli argued that the circumstances in Europe are distinct from those in the U.S. and proposed a deterministic dynamic model for European air traffic flow management.[30] A novel integer programming model for solving large-scale air traffic flow problems are presented in.[31] The model covers all the phases of flights, including departure, cruising, and landing, and considers various regulatory actions such as ground delays, route changes, speed adjustments, and airborne holding. It incorporates a multivariable framework with three categories of constraints to enhance relaxation conditions, enabling efficient resolution of air traffic management scenarios comparable in size to the entire United States. In a different research venue, a team of researchers investigates air traffic flow management through the use of Eulerian models. For example, a Eulerian model is proposed in Ref. 32, which is based on the Cell Transmission Model (CTM). This model operates under the assumption that flights between specific Origin-Destination (OD) pairs can be represented as paths, which consist of a sequence of links. Each link corresponds to a sector, and these links are further segmented into cells, where the average flight duration between OD pairs defines the path length and each cell's length is considered as 1 min. In the Cell Transmission Model (CTM), the state variable is the total count of aircraft in each cell during every time interval, and the total aircraft count in a sector is the aggregate of all aircraft across all cells within that sector. This modeling approach pertains to the airway level, and the model's dimensions are dictated by the network structure. Another interesting work is in Ref. 33, where an Eulerian-

Lagrangian model is developed by taking into account the departure and arrival fix information of the flights. A Linear Transmission Model (LTM) has been created, and a method for pairwise decomposition is introduced to discover the best global solution (Cao and Sun[34]). Similar to the CTM, this model can achieve the optimum outcome, yet it operates approximately six times faster than the CTM. The above mentioned studies can determine the optimal air traffic flow management strategies assuming that airport and sector capacities are fixed. However, these capacities are subject to change due to stochastic factors, such as weather, equipment malfunctions, and air traffic controller's personal capability. Several methods have been suggested to tackle the issue of stochastic air traffic flow management.

2.2. Stochastic air traffic flow management models

In Ref. 35, a dynamic multi-stage stochastic integer programming model is introduced for a single airport. This model employs a scenario tree to depict the uncertain capacity, and as time progresses, the decision tree branches out to create numerous scenarios. Hoffman argued that single airport flow management does not take into account the connectivity between flights, which can lead to impractical situation due to the impact of previous flight. To deal with the unpredictability of airport capacity in ground delay programs, Liu et al.[36] developed an airport capacity scenario tree using historical data on airport capacity distribution. Agustín et al.[37] developed a multistage hybrid 0–1 programming model to solve the air traffic management problem, taking into account the uncertainty of the capacity of airports and sectors simultaneously, as well as flight rerouting. Stochastic optimization has become a widely used method for dealing with optimization problems that involve uncertain parameters. The main idea behind is to minimize the risk of exceeding capacity constraints in cases where the probability distributions of the nodes are understood. Bertsimas and Sim[6] discussed the challenge of dealing with data uncertainty in the context of the network flow optimization problem. They introduced a robust optimization approach to tackle this uncertainty, allowing users to manage the level of constraint violation and consequently adjust the model's level of conservatism. However, when cost coefficients contain uncertainties, the 0–1 discrete optimization problem on n variables requires at most $n + 1$ instances of the original problem to solve the robust counterpart, which is computationally intensive. Janak et al.[38] proposed a novel robust optimization approach to address planning problems with bounded uncertainty. The model was implemented in real industrial cases to achieve dependable and resilient solutions. Gupta and Bertsimas[39] proposed a robust optimization approach to tackle the uncertainty in air traffic flow management. However, the model optimized the worst case of uncertain parameters, resulting in an overly conservative and resource-wasteful allocation scheme. Clare and Richards[40] proposed a robust optimization method to solve the planning problem with bounded uncertainty, assuming that the future capacity probability distribution information is known. They integrated a deterministic discrete decision mixed integer linear model with sector capacity violation probability constraints to create a chance-constrained model. However, The study employed a brute force algorithm to address the model, leading to computational constraints and challenges in solving large-scale practical issues. Chen et al.[41] developed a chance-constrained air traffic flow management model by adding probabilistic constraints to a deterministic integer programming model. The capacity information of the probabilistic sector is incorporated within the chance constraints. A polynomial approximation chance constraint optimization algorithm is developed, which is capable of providing optimal traffic management scheme with efficient computational speed. Scenarios generate method is used to reduce the number of possible operation scenarios in order to solve the chance constrained models.[42,43] These stochastic air traffic flow management models provide mathematical frameworks used to optimize the flow of air traffic in uncertain conditions, considering the random nature of factors such as weather, aircraft delays, and airspace congestion. By incorporating probability distributions and stochastic processes, they aim to find optimal solutions that minimize delays, fuel consumption, and overall system costs while ensuring safety and efficiency. However, most of work consider only one or a few sectors operating under uncertainty. Little attention is given to address the uncertainty of capacity in the whole network. Moreover, no research has been conducted on establishing the thresholds for capacity violations, leaving air traffic flow management staff to rely on their own expertise to make decisions.

3. A chance constrained model for air traffic flow management

3.1. Model framework and assumptions

The overall structure of the model is illustrated in Fig. 2. It takes into account the capacity of airports and sectors, flight plans, and the predetermined probability of violation at the airport/sector. Each flight plan includes the departure airport, destination airport, planned departure time, planned arrival time, list of sectors, and the associated flying time for traversing each sector, turnaround time at the airport, etc. The model optimizes the departure time and arrival time for each flight to ensure that the traffic demand and capacity are balanced. We use five-minute intervals as the time unit. Due to the data constraints, we assume that each flight can pass through the sector with the minimum sector flying time. In future work, we can consider variable flying times for different flights.

3.2. Description of notations

The sets, parameters and variables used in the model are given in Tables 1–4.

3.3. Objective function

The aim of the model is to reduce the total cost of flight delays. Flight delay is defined as the time difference between its scheduled time of operation and its actual time of operation. It should be noted that the departure displacement includes both ground delays and early departures. The objective function is represented in Eq. (1):

$$Q = \min\left(\sum_{f\in F_{\text{wait}},a=O_f} \beta\left|t - t_f^{\text{dep}}\right| X_{f,a,t} + \sum_{f\in F}\theta y_f Y_f\right) \tag{1}$$

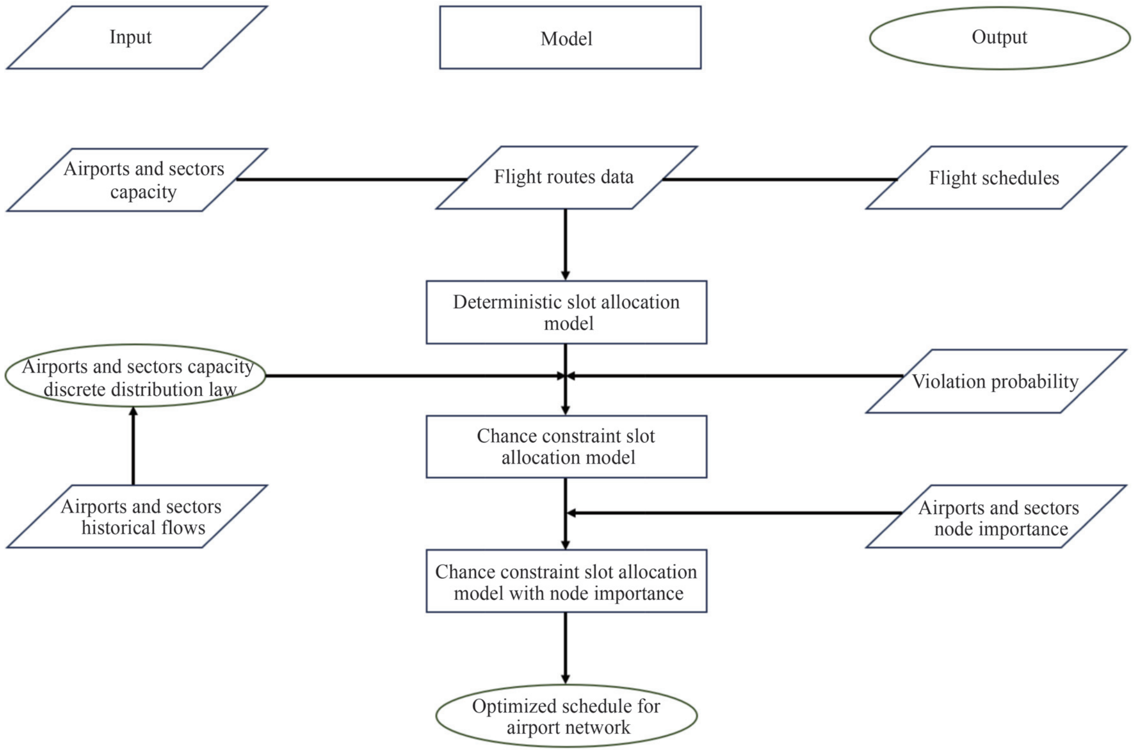

Fig. 2 Model framework.

Table 1 Sets of model.

Set	Description
T/T_{15}	Set of 5-minute/15-minute slots in a day
F_{wait}	Set of flights scheduled to depart at $T_{\min}$ and within 3 h
F_{fly}	Set of flights that have not yet arrived at $T_{\min}$
F	Set of all flights, $F = F_{\text{wait}} \cup F_{\text{fly}}$
$S^{\text{by}}_{f,T_{\min}}$	Set of sectors that flight f has passed through or is in at $T_{\min}, f \in F_{\text{fly}}$
S_f	Set of all sectors that flight f passes through(include departure and arrival airports), $f \in F$
S'_f	Set of sectors that flight f passes through(exclude arrival airport compared to S_f), $f \in F$
S	Set of sectors
F_s	Set of flights passing through sector $s, s \in S$
A_f/A'_f	Set of departure and arrival airports/pairs of departure and arrival airports (a, a') for flight $f, f \in F$
$A^{\text{dep}}/A^{\text{arr}}$	Set of departure/arrival airports
$F^{\text{dep}}_a/F^{\text{arr}}_a$	Set of flights with a as departure/arrival airport, $a \in A^{\text{dep}}/A^{\text{arr}}$
P	Set of pairs of connecting flights (f_1, f_2)
P'	Set of airport pairs (a, a') corresponding to the connecting flight pair (f_1, f_2), where a_1 is the arrival airport of f_1 and a_2 is the departure airport of f_2

Table 2 Parameters of model.

Parameter	Description
$T_{\min}$	The start time of the flight schedule to be optimized
$T_{\max}$	The end time of the flight schedule to be optimized
$t^{\text{dep}}_f/t^{\text{arr}}_f/t^{\text{fly}}_f$	Original departure/arrival/fly time of flight $f, f \in F$
O_f/D_f	Departure/Arrival airport for flight $f, f \in F$
β/θ	Cost of unit slot displacement of departure/air delay
$S_{f,T_{\min}}$	Sector in which flight f is located at $T_{\min}, f \in F_{\text{fly}}$
$S^{\text{next}}_{f,s}$	Next sector of sector s where flight f is located now, $f \in F, s \in S'_f$
$t^{\text{in}}_{f,s}$	The slot when flight f enters sector $s, f \in F_{\text{fly}}, s \in S^{\text{by}}_{f,T_{\min}}$
$t^{\text{fly}}_{f,s,\min}$	Minimum fly time of flight f in sector $s, f \in F, s \in S_f$
$\lambda^{\text{dep}}_{a,t}/\mu^{\text{arr}}_{a,t}$	Capacity of airport a at slot $t, a \in A^{\text{dep}}/A^{\text{arr}}, t \in T_{15}$
$\varphi_{s,t}$	Capacity of sector s at slot $t, s \in S, t \in T_{15}$
U	Maximum air delay for any flight which set to 1 h
L	Maximum delay for departure flight which set to 3 h
K	Maximum advanced arrival time for flight which set to 1 h
V	Minimum turnaround time for connecting flight which set to 30 min
M	A positive number of infinite size
α	Violation probability

The cost in the objective function consists of two parts. The first part is the cost of adjusting the departure slot for departure flights as expressed as $\sum_{f \in F_{\text{wait}}, a=O_f} \beta \left| t - t^{\text{dep}}_f \right| X_{f,a,t}$, and the second part, $\sum_{f \in F} \theta y_f Y_f$, is the cost of air delays for all flights.

Operators and managers prefer to keep flights on the ground rather than in the air, as this increases fuel consumption and the chances of unsafe events. Based on previous studies on the study of Ground Delay Program(GDP),[44,45] this paper sets the cost ratio of airborne delay to ground delay as

Table 3 Decision variables of model.

Decision variable	Description
$X_{f,a,t}$	Binary variables, where 1 indicates that flight f is assigned to slot t in airport a,otherwise 0,$f \in F$,$a \in A_f$,$t \in T$
$W_{f,s,t}$	Binary variables, where 1 indicates at slot t flight f has left or is in sector/airport s,otherwise 0,$f \in F$,$s \in S_f$,$t \in T$

Table 4 Indirect decision variables of model.

Indirect decision variable	Description
$b_{f,s,t}$	Binary variables, where 1 indicates at slot t flight f is in sector s,otherwise 0,$f \in F$,$s \in S_f$,$t \in T_{15}$
y_f	integer variables, where positive numbers indicate flight f occurred air delay, otherwise 0,$f \in F$
Y_f	Binary variables, where 1 indicates flight f occurred air delay, otherwise 0,$f \in F$

2:1. For simplicity, the cost of one unit of airborne delay is 100, and for ground delay it is 50.

3.4. Deterministic model constraints

The constraints of the model are capacity constraints, flight connection constraints, and flight time constraints. Eq. (2) to Eq. (21) are the constraints of the model.

(1) Constraints for the flights in the air

Eq. (2) ensures that the decision variable $X_{f,a,t}$ and for flights that are in the air is equal to its actual departure time. Eq. (3) ensures that the decision variable $W_{f,s,t}$ can for flights that are in the air is equal to its actual entry time of each sector it traverses. Eq. (4) ensures that the decision variable $W_{f,s,t}$ is not decreasing.

$$X_{f,a,t} = 1, \forall f \in F_{\text{fly}}, a = O_f, t = t_f^{\text{dep}} \tag{2}$$

$$W_{f,s,t} = 1, \forall f \in F_{\text{fly}}, s \in S_{f,T_{\min}}^{\text{by}}, t \in \left[t_{f,s}^{\text{in}}, 287\right] \tag{3}$$

$$W_{f,s,t-1} \leqslant W_{f,s,t}, \forall f \in F, s \in S_f, t \in [1, 287] \tag{4}$$

$$X_{f,a,t} = W_{f,s,t} - W_{f,s,t-1}, \forall f \in F, a \in A_f, s = a, t \in [1, 287] \tag{5}$$

(2) Flight operational constraints

Eq. (5) expresses the relation between the decision variables $X_{f,a,t}$ and $W_{f,s,t}$. When $X_{f,a,t} = 0$, this means that the flight f has not left the airport a in the slot t. In this case, $W_{f,s,t}$ can be either 0 or 1. If a is an arrival airport, Eq. (6) ensures that all flights have unique departure and arrival slots. Eq. (7) ensures that flights do not arrive earlier than $T_{\min}$. Eq. (8) guarantees that the maximum departure delay does not exceed 3 h. Eq. (9) ensures that flights arrive no more than 1 h ahead of schedule. Eq. (10) ensures that connecting flights must meet the minimum connection time requirement. Eq. (11) ensures that flights meet the sector minimum flying time. Eqs. (12)–(15) are the air delay of the flight and make sure that the maximum air delay does not exceed 1 h. Eq. (16) guarantees that transit time for connecting flights is not less than 30 min.

$$\sum_{t \in T} X_{f,a,t} = 1, \forall f \in F, a \in A_f \tag{6}$$

$$\sum_{t=T_{\min}}^{287} X_{f,a,t} = 1, \forall f \in F_{\text{fly}}, a \in D_f \tag{7}$$

$$\sum_{t=T_{\min}}^{t_f^{\text{dep}}+L} X_{f,a,t} = 1, \forall f \in F_{\text{wait}}, a \in O_f \tag{8}$$

$$\sum_{t=t_f^{\text{arr}}-K}^{287} X_{f,a,t} = 1, \forall f \in F, a \in D_f \tag{9}$$

$$\sum_{t \in T} t^* X_{f,a,t} \leqslant \sum_{t \in T} t^* X_{f,a',t}, \forall f \in F, (a, a') \in A'_f \tag{10}$$

$$W_{f,S_{f,s}^{\text{next}},t+t_{f,s,\min}^{\text{fly}}} \leqslant W_{f,s,t}, \forall f \in F, s \in S'_f, t \in \left[T_{\min}, 287 - t_{f,s,\min}^{\text{fly}}\right] \tag{11}$$

$$\sum_{t \in T} t^* X_{f,a',t} - \sum_{t \in T} X_{f,a,t} - t_f^{\text{fly}} \leqslant U, \forall f \in F, (a, a') \in A'_f \tag{12}$$

$$\sum_{t \in T} t^* X_{f,a',t} - \sum_{t \in T} X_{f,a,t} - t_f^{\text{fly}} = y_f, \forall f \in F, (a, a') \in A'_f \tag{13}$$

$$y_f \geqslant 0.5 - M^*\left(1 - Y_f\right), \forall f \in F \tag{14}$$

$$y_f \leqslant 0.5 + M^* Y_f, \forall f \in F \tag{15}$$

(3) Airport and sector capacity constraints

Eqs. (17)–(21) ensure that traffic at the airport or sector will not exceed its capacity.

$$W_{f_2,a_2,t+V} \leqslant W_{f_1,a_1,t}, \forall (f_1, f_2) \in P, (a_1, a_2) \in P', t \in [0, 287 - V] \tag{16}$$

$$\sum_{t=3t}^{3t+2} \sum_{f \in F_a^{\text{dep}}} X_{f,a,t} \leqslant \lambda_{a,t}^{\text{dep}}, \forall a \in A^{\text{dep}}, t \in \left[\frac{T_{\min}}{3}, \frac{T_{\max}}{3} + 12\right] \tag{17}$$

$$\sum_{t=3t}^{3t+2} \sum_{f \in F_a^{\text{arr}}} X_{f,a,t} \leqslant \mu_{a,t}^{\text{arr}}, \forall a \in A^{\text{arr}}, t \in \left[\frac{T_{\min}}{3}, \frac{T_{\max}}{3} + 12\right] \tag{18}$$

$$\sum_{t=3t}^{3t+2} \left(W_{f,s,t} - W_{f,S_{f,s}^{\text{next}},t}\right) \geqslant 0.5 - M^*\left(1 - b_{f,s,t}\right), \forall f \in F_s, s \in S, t \in \left[\frac{T_{\min}}{3}, \frac{T_{\max}}{3} + 12\right] \tag{19}$$

$$\sum_{t=3t}^{3t+2}\left(W_{f,s,t}-W_{f,S_{f,s}^{\text{next}},t}\right)\leqslant 0.5+M^{*}b_{f,s,t},\forall f\in F_s,s\in S,t\in\left[\frac{T_{\min}}{3},\frac{T_{\max}}{3}+12\right] \tag{20}$$

$$\sum_{f\in F_s}b_{f,s,t}\leqslant \varphi_{s,t},\forall s\in S,t\in\left[\frac{T_{\min}}{3},\frac{T_{\max}}{3}+12\right] \tag{21}$$

3.5. Chance constraints

3.5.1. Establishment of chance constraints

Chen et al.[41] proposed a chance constrained model to address the problem of Air Traffic Flow Management (ATFM) under uncertainty. The main idea is to replace sector capacity constraints in the Integer Programming optimization model with probabilistic constraints. That is the capacity of a sector can be exceeded with a given probability. Here, we extend their work to consider both stochastic capacity at airports and sectors. Therefore, Eq. (17), Eq. (18), and Eq. (21) are replaced by Eq. (22):

$$P\begin{pmatrix}\sum_{t=3t}^{3t+2}\sum_{f\in F_a^{\text{dep}}}X_{f,a,t}\leqslant\lambda_{a,t}^{\text{dep}}\\ \sum_{t=3t}^{3t+2}\sum_{f\in F_a^{\text{arr}}}X_{f,a,t}\leqslant\mu_{a,t}^{\text{arr}}\\ \sum_{f\in F_s}b_{f,s,t}\leqslant\varphi_{s,t}\end{pmatrix}\geqslant 1-\alpha \tag{22}$$

This model enumerates all potential combinations of airport and sector capacities, and chooses the capacity combination that meets the violation probability. It creates a feasible set of capacity combinations, which is also the feasible set of node traffic allocations. The optimal allocation scheme is the solution in the feasible set that has the best objective function value.

The most accurate solution to the chance constraint model can be determined by enumerating all possible combinations of capacities. However, as the number of airports and sectors increases, the computational complexity increases drastically. This type of constraint model has difficulty in finding the optimal solution for large-scale problems. Therefore, we transfer the joint constraints into individual constraints based on the Bonferroni conservative approximation.[46] Thus, this article improves the chance constraint method by setting the probability of violation for a single node instead of the joint distribution for a number of nodes. Doing so will enhance the computational speed of the model and allow for the customization of node violation probabilities according to the significance of nodes in the air traffic network. As a result, this will lead to more beneficial and logically optimized strategies. The capacity constraints in our chance constraint model are expressed in Eqs. (23)–(25):

$$P\left(\sum_{t=3t}^{3t+2}\sum_{f\in F_a^{\text{dep}}}X_{f,a,t}\leqslant\lambda_{a,t}^{\text{dep}}\right)\geqslant 1-\alpha_a,\forall a\in A^{\text{dep}},t\in\left[\frac{T_{\min}}{3},\frac{T_{\max}}{3}+12\right] \tag{23}$$

$$P\left(\sum_{t=3t}^{3t+2}\sum_{f\in F_a^{\text{arr}}}X_{f,a,t}\leqslant\mu_{a,t}^{\text{arr}}\right)\geqslant 1-\alpha_a,\forall a\in A^{\text{arr}},t\in\left[\frac{T_{\min}}{3},\frac{T_{\max}}{3}+12\right] \tag{24}$$

$$P\left(\sum_{f\in F_s}b_{f,s,t}\leqslant\varphi_{s,t}\right)\geqslant 1-\alpha_s,\forall s\in S,t\in\left[\frac{T_{\min}}{3},\frac{T_{\max}}{3}+12\right]. \tag{25}$$

3.5.2. Transformation of chance constraints into deterministic constraints

In this section, we show how to convert the chance constraints expressed in Eqs. (23)–(25) into deterministic constraints. As an example, the four capacity scenarios of a sector s are presented in Table 5.

The first row in Table 5 shows the capacity value ($\varphi_{s,t}$) of the sector s for each scenario; the second row displays the probability ($\text{PC}_{s,t}$) of the capacity in each scenario; the third row presents the cumulative probability ($\text{PC}_{s,t}^{\text{sum}}$) of the sector. Let n be the scenario number $n\in\{1,2,3,4\}$. The cumulative probability of scenario n indicates the proportion of scenarios with capacity values less than scenario n. It establishes the connection between the chance constraint and the deterministic capacity constraint by considering the probability of violation. The following example demonstrates the specific transformation process.

For example, if the probability of violation in sector s is 0.05, then it means that the maximum allowable flow is the quantile 5% of all possible capacity scenarios. Since the scenario with capacity of 5 is in the quantile 0% to 10%, the probability of violation of 0.05 is within the cumulative probability of Scenario 1 and Scenario 2. Therefore, the chance constraint can be changed into a deterministic constraint that traffic flow is less than or equal to 5. When the probability of violation is 0.95, the maximum allowed traffic is in the 95% quantile of all possible capacity scenarios. Since the capacity in Scenario 4 is in the 80% to 100% quantile, the chance constraint can be converted into a deterministic constraint with a flow less than or equal to 8. Given the empirical distribution of sector capacity, it is essential to ensure that converting a chance constraint into a deterministic one closely to the original situation. Thus, for sector s in the time slot t, a chance constraint with a probability of violation 95% can only be translated into a deterministic constraint where the flow is less than or equal to 8.

Table 5 Capacity distribution for sector s at slot t.

Parameter	Scenario 1	Scenario 2	Scenario 3	Scenario 4
$\varphi_{s,t}$	5	6	7	8
$\text{PC}_{s,t}$	0.1	0.3	0.4	0.2
$\text{PC}_{s,t}^{\text{sum}}$	0	0.1	0.4	0.8

$$P\left(\sum_{f\in F_s} b_{f,s,t} \leqslant \varphi_{s,t}\right) \geqslant 1-\alpha_s = \begin{cases} \sum_{f\in F_s} b_{f,s,t} \leqslant 5, & 0 \leqslant \alpha_s \leqslant 0.1 \\ \sum_{f\in F_s} b_{f,s,t} \leqslant 6, & 0.1 < \alpha_s \leqslant 0.4 \\ \sum_{f\in F_s} b_{f,s,t} \leqslant 7, & 0.4 < \alpha_s \leqslant 0.8 \\ \sum_{f\in F_s} b_{f,s,t} \leqslant 8, & 0.8 < \alpha_s \leqslant 1 \end{cases} \tag{26}$$

4. Characteristics of air traffic flow network

Due to its important role in moving passengers and cargo, the properties of the air transportation network have become a significant subject of study within the network science field. Early studies have investigated the structural features of air transport networks. For instance, Guimerà et al. examined the structure of worldwide air transportation network.[47] These networks, whether global or national, exhibit the characteristics of scale-free small-world networks. Subsequently, the emphasis of research shifted towards analyzing vital nodes and fundamental characteristics of the network. Given that connectivity patterns influence diffusion phenomena (such as flight delays and epidemics) across the network, understanding the function and essential elements of the network is crucial.[48] Earlier research has demonstrated that reductions in capacity at various airports affect the overall performance of air transport systems differently.[49,50] Motivated by these findings, it is hypothesized that the characteristics of the network play a role in determining probabilistic constraints and optimization approaches.

To illustrate how flights operate in the air transportation system, Fig. 3 presents a two-layer air traffic flow network. The bottom layer represents the airport network, where the nodes represent the airports, and the edges represent the scheduled direct flights connecting the airports. The upper layers are the sector network, where the nodes are the sectors, and the edges represent the flow of flight traffic between the two sectors. Arrows connect that airport and the sector in the two networks, indicating the traffic flow between the airport and the sectors. Several studies have investigated the structural characteristics of aviation networks. Most networks have airports as nodes and a link connects two airports if there is a flight operating between them.[51] In real operation, the airspace is divided into different sectors, each of which will be managed by one or two air traffic controllers.[51] To capture the operational characteristics of the air traffic network, the sector network is represented as a graph $\mathscr{G}\int = \left(\mathscr{N}\int, \mathscr{E}\int\right)$, where $\mathscr{N}\int$ is the set of nodes and $\mathscr{E}\int$ is the set of edges. Nodes n ($n \in \mathscr{N}\int$) of the network $\mathscr{G}\int$ are airports and sectors, while an edge e ($e \in \mathscr{E}$) is added between two nodes if there is traffic between them. In this paper, an edge e represents direct flight connections between a sector and a sector (or an airport). Let $\mathscr{A}\int$ be the adjacency matrix of the network $\mathscr{G}\int$, with each element a_{ij} representing the relationship between node i and j. For example, in an undirected, unweighted network, if there is an edge between i and j, then $a_{ij} = 1$; otherwise, $a_{ij} = 0$. To measure the importance of nodes in the network, we introduce the following metrics.

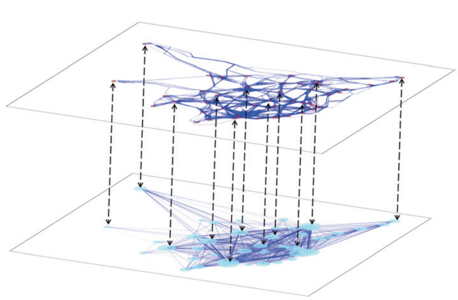

Fig. 3 Air traffic network. The bottom layer is the airport network with the size of node indicating the traffic volume at the airports. The upper layer in the sector network.

(1) Degree and degree distribution. The degree of a node v is denoted as k_v, which is defined as the number of edges connected to that node. In an undirected network $\mathscr{G}$, the degree of node v is calculated as

$$k_v = \sum_u a_{v,u} \tag{27}$$

The degree distribution is often represented as a histogram, showing the count of nodes for each degree.

(2) Degree centrality. The degree centrality is defined as

$$C_{\mathrm{D}}(v) = \frac{k_v}{\max\_\deg'} \tag{28}$$

where $\max_{\mathrm{d}}\mathrm{eg}$ is the maximum degree in the network.

(3) Betweenness centrality. Betweenness is to measure the influence of the node v over traffic flow between other node. Betweenness centrality is defined as

$$C_{\mathrm{B}}(v) = \sum_{i\neq v\neq j} \frac{\sigma_{ij}(v)}{\sigma'_{ij}} \tag{29}$$

where σ_{ij} is the total number of shortest path from node i to node j, and $\sigma_{ij}(v)$ is the number of those paths that pass through node v.

(4) Closeness centrality. Closeness centrality is a measure that quantifies how close a node is to all other nodes in a network. Mathmatically, closeness centrality is defined as

$$C_{\mathrm{C}}(v) = \frac{n-1}{\sum_u d(v,u)'} \tag{30}$$

where $d(v,u)$ is the shortest path length from node v to node u.

The importance of nodes in a network can be evaluated using metrics such as degree, betweenness, and closeness. The degree could reflect the potential for interaction between the node and other nodes in the network. However, we should note that the importance of a node is not solely determined by its degree value, but also by the degree values of its neighboring nodes. Betweenness was originally proposed to measure the social status of a node, as it is determined by the number of shortest paths that pass through it. Closeness is a measure of the proximity of a node to other nodes in the network, with a higher value indicating that the node is closer to other nodes and its information will spread more quickly. When it comes to air traffic, flight delays can spread throughout the system, leading to further delays. To reduce the amount of delays, it is essential to first minimize the initial delays, and then manage the propagation of delays in the network. Therefore, it is important to assess the importance of nodes in sector networks. Closeness could be a suitable measure for the sector nodes. Nodes with higher closeness should have a lower chance of exceeding capacity, which would control the initial delay generated by important nodes and thus limit the spread of delays in the network.

Airports, which are the beginning and end points of the air traffic management system, differ from sectors. Therefore, it is not appropriate to use the same network topology metric to assess their importance. To this end, the airport network $\mathcal{G}^{f}$ is created with all airports as nodes and a link between two airports if there is a scheduled traffic flow between them. The degree of the airport node reflects the number of cities served by the airport, but the traffic volume of the airport with high degree is not necessarily large. For example, the node degrees of ZBTJ(Tianjin) and ZLXY(Xi'an) are approximately equal, however, the flight volume of ZLXY is almost twice as much as that of ZBTJ. This means that ZLXY has a higher traffic volume, making it more prone to delays. Furthermore, due to the larger number of flights, ZLXY flight delays are more likely to spread to other airports through the network. Moreover, a prior research on the resilience of the airport network has indicated that the traffic volume of airports is the most reliable indicator of the effect of a disturbance on network efficiency.[50] Therefore, we use traffic volume as a metric to evaluate the importance of airport nodes.

To further calculate the importance of the nodes, we use the standard max–min normalization method to transfer the original node important metric data into the range (0, 1). Fig. 4 plots the importance of sector nodes and airport nodes. The names of sectors are shown in the upper X-axis, while the names of the airports are shown in the bottom X-axis. The sectors whose closeness are smaller than 0.209, are assumed to have enough capacity handling additional flights. These sectors are mostly located at the periphery of the network, and the probability of delay caused by these sectors is low. Thirty-eight of these airports have an average daily flights of more than 200, while the remaining airports have a smaller average daily flights and a low probability of traffic exceeding airport capacity. Thus, it is assumed that the capacity of these airports is always enough to meet traffic demand.

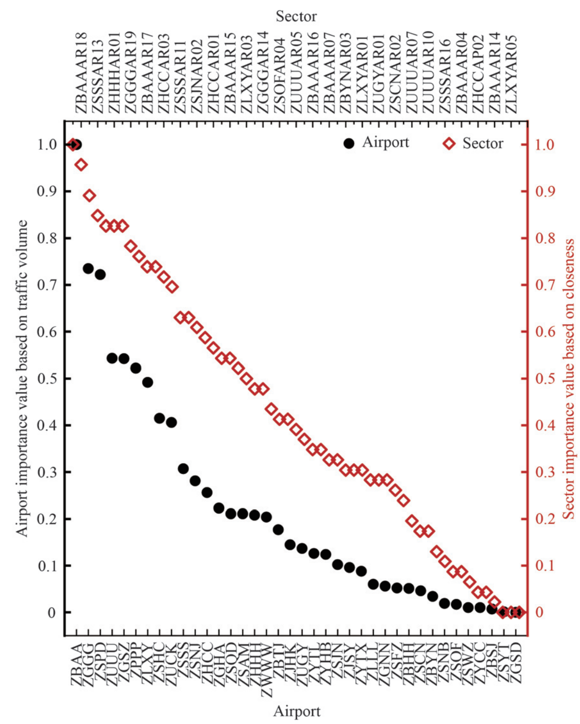

Fig. 4 Importance indicator of sectors and airports. The importance of airports is determined by assessing their significance through traffic volume. Similarly, the importance of sectors is determined by evaluating their significance through closeness centrality.

5. Case study

5.1. Experimental setup

5.1.1. Data

We examine the effectiveness and performance of our model by comparing the optimized results with those obtained from the deterministic model. The flight schedule from 9:00 am to 12:00 pm on August 1, 2018 is used as input data for the models. During this period, there were 3435 flights. At 9:00AM, 1156 flights were in the air, while 2279 were still on the ground. The air traffic network consists of 195 airport nodes and 188 sector nodes. The sector network is formulated based on airspace data from the Air Traffic Management Bureau. Estimation of the capacity of an airport or of a sector is a complex task, which is beyond the scope of this study. The airport capacity is provided by the air traffic control authority in the form of declared airport capacity. Statistical results indicate that hourly arrivals/departures of the airport vary between 40% and 70% of its declared capacity. Therefore, we use 60% of the airport declared capacity as departure capacity and arrival capacity. As there is no data available on the capacity of the sector, the sector capacity is determined as the 50th percentile of the past traffic volume moving through the sector. The primary goal of this study is to investigate the efficiency and benefits of using a chance constrained model in air traffic flow management when faced with uncertainty, taking into account the structural characteristics of the air traffic network. It is assumed that the precise determination of airport and sector capacities does not significantly influence this research. Further validation efforts can be conducted using more realistic capacity data.

5.1.2. Optimization schemes

We evaluate ten different optimization strategies in addition to the baseline (deterministic) model to compare the performance of our chance-constrained models. These ten optimization schemes are divided into two groups: (A) Method 1: Chance-constrained model with equal probability of violation for all airports and sectors; and (B) Method 2: Chance-constrained model with the probability of violation setting based on the importance of the node. In the base optimization scheme (S_0), all constraints in the worst-case scenario model must be satisfied. This means that optimized traffic must not exceed airport or sector capacity in all possible scenarios.

In fact, the operational capacity of airports or sectors is stochastic, which means that traffic flow may exceed a given capacity limit at some points. To account for this, we allow capacity constraints to be violated with a small probability. To set this probability, we used two different approaches. The first is to set all nodes with the same probability of violation. If this probability is set too high, it could lead to airport and airspace congestion and large flight delays in actual operations. However, if set too low, the optimized schedule may be more robust to various operating scenarios, but the limited airport capacity may be underused. To balance capacity utilization and robustness to optimization, we set five schemes $\alpha \in \{0.05, 0.08, 0.11, 0.14, 0.17\}$ which are referred to as S_1 to S_5, respectively. We suggest a second approach that takes into account the characteristics of the air traffic network. Small disturbances that occur at a critical airport or sector can cause long unforeseen delays throughout the network. Therefore, optimized traffic flow must be more resilient to uncertainty in these airports and sectors, and operations strategies should be more conservative. We propose a method to set the probability of violation for airports and sectors, which takes into account the importance of the airport or sector. For simplicity, we use the term "node n" to refer to an airport or a sector.

Let H_n be the importance value of node n, then $\mathrm{Max}_{ix} = \max\{H_n\}$ and $\mathrm{Min}_{ix} = \min\{H_n,\}$ are the maximum importance value and the minimum importance value of all nodes. The probability of violation $\alpha_{n,i}$ of the optimization scheme ith, $i \in \{6, 7, 8, 9, 10\}$ is calculated as follows:

$$\alpha_n^i = \alpha_{n,\min}^i + \left(\alpha_{n,\max}^i - \alpha_{n,\min}^i\right)\frac{\mathrm{Max}_{ix} - H_n}{\mathrm{Max}_{ix} - \mathrm{Min}_{ix}} \tag{31}$$

where $\alpha_{n,\max}^i$ and $\alpha_{n,\min}^i$ are the predetermined violation probabilities for the nodes with the maximum importance value and the minimum importance value. Here, $\alpha_{n,\max}^i$ are set to 0.20, while $\alpha_{n,\min}^i$ are set to $\{0.05, 0.08, 0.11, 0.14, 0.17\}$, corresponding to the schemes S_6 to S_{10} respectively. Table 6 presents data regarding the probablity of violation α for each optimization scheme. The algorithms are coded in Python and Groubi, and are executed on a computer equipped with a 32 GB RAM and a 12th Gen Intel(R) Core(TM) i7-12700 Cpu (2.10 GHz).

5.2. Results

5.2.1. Total cost for each optimization scheme

Table 7 shows the total cost of and the computation time for obtaining the optimal flight schedule under the optimization schemes from S_0 to S_{10}. The total cost of the optimal solution for the base model is 416450, with 13.2% of this cost being attributed to air delay and 86.8% to ground slot displacements. In the optimized schedule, 64.9% of the flights are assigned their scheduled departure time, and 35.1% are delayed (98.7%) or depart earlier (1.3%). In contrast, 81.1% of the arrival flights are not assigned their original scheduled arrival slot, with 42.8% being delayed and 57.2% assigned earlier slots. This is likely due to the fact that airlines adding buffer time to their scheduled blocked time to account for potential flight delays.[52]

In uncertainty scenarios, the total delay cost decreases as the probability of violation increases for both the random setting and the importance-based node setting. This is because an increase in the capacity chance constraint violation probability enlarges the feasible domain of the model solution, prompting the model to search for a flight schedule with a lower total cost. Since excessive air delay can cause airspace congestion and air traffic safety concerns, the unit cost of air delay is higher than the unit cost of ground delay in the objective function setting of the model. This leads to the ground delay being greater than the air delay. Generally, the optimization results based on Method 2 are better than those of Method 1. The variation is smaller, indicating greater stability. As the probability of violation increases, the percentage of flights experiencing flight delay decreases in all schemes in Method 1. Method 2, however, has a concentration of 6.9%-13.6% of departure flights with delays, and as capacity constraints become more relaxed, the percentage of departure flights delayed decreases. It is evi-

Table 6 Violation probability for eleven optimization schemes.

Scheme	Base method	Method 1					Method 2				
	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
α	0	0.05	0.08	0.11	0.14	0.17					
$\alpha_{\min}$							0.05	0.08	0.11	0.14	0.17
$\alpha_{\max}$							0.20	0.20	0.20	0.20	0.20

Table 7 Total cost and computation time (in second) for each optimization scheme.

Cost	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
Total	416 540	146 950	64 750	39 750	22 850	17 250	61 200	27 400	18 600	16 000	13 900
Air	54 900	14 625	6675	4350	3375	2550	8100	4050	3150	2700	2400
Gnd	364 550	132 325	58 075	35 400	19 475	14 700	53 100	23 350	15 450	13 300	11 500
Time(s)	125 624	34 520	24 867	14 684	5324	1387	10,170	5006	1131	1277	1086

dent that the percentage of departure flights that are delayed in Method 2 is much lower than that in Method 1. The results show that about 90% of the departure flights depart on time. In comparison, the proportion of flights with delayed arrival flight is higher. Method 1 and Method 2 generally do not have a clear increasing or decreasing pattern but with an overall downward fluctuate.

In Fig. 5, we plot the number of slot displacements for arrival and departure flights in each optimization scheme, while Fig. 6 further shows the ratio of slot forward and slot backward displacement. We observe that in the departure flight slot displacement, the slot forward displacement maintaining numerical stability and the slot backward displacement decrease with the increase in violation probability. The backward displacement of the departure slot for all scenarios is almost always greater than the forward displacement, and the numerical change in the backward displacement is also significantly greater than that of the forward displacement. In the flight arrival slot displacement, the slot backward displacement is much smaller than the slot forward displacement, and its proportion is continuously decreasing and approaching zero. The forward displacement of the slot determines the direction of the departure slot for the flight.

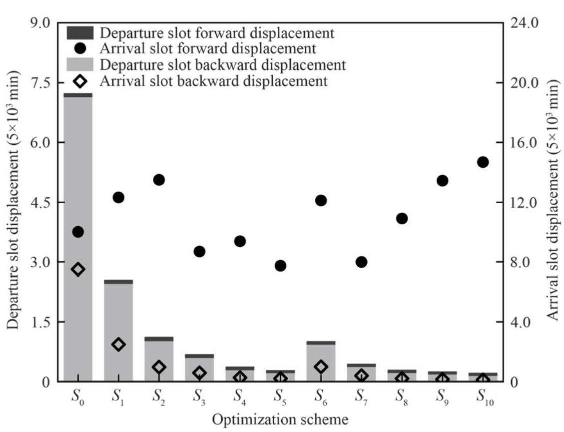

Fig. 5 Number of slot displacements for arrival and departure flights in each scenario.

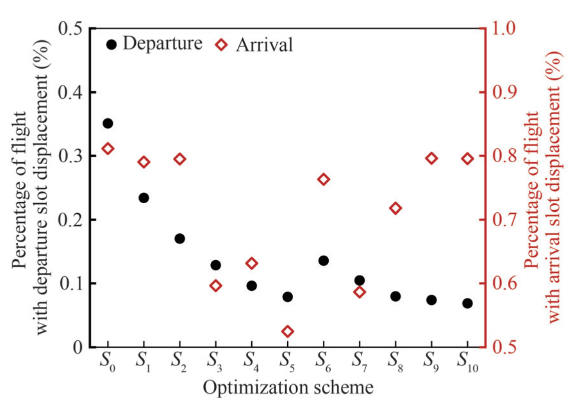

Fig. 6 Percentage of flights with arrival and departure slot displacement in each scenario.

The runtime of the optimization model using various schemes are displayed in Fig. 7. Our scenario involves 3435 flights, 195 airports, and 188 sectors. Traffic characteristics are comparable to those examined in,[41] although our network is substantially larger (up to 20 sectors in their work). Our model was executed using a desktop computer equipped with a single 12th Gen Intel(R) i7-12700 CPU running at 2.10 GHz and 32 GB of RAM, whereas the model presented in the study by Chen et al. was run on a Spark Cluster consisting of 6 nodes, with each node containing an 8-processor CPU. While it is not possible to directly compare the computational complexities of the two studies, our model can be successfully solved within a 20-minute timeframe for all optimization schemes, with the exception of S_6 and S_7.

5.3. Robustness tests

In Section 5.2, we compare and analyze the total cost, the proportion of flights that experience slot displacements, and the amount of slot displacement for arrivals and departures. As expected, the total expected cost would decrease as the probability of violation increases. The reason for this is the inclusion of additional ‘‘predicted‘‘ capacity resources to meet traffic demand. There is a potential risk when expanding capacity, as the actual capacity may be lower than anticipated. Consequently, overscheduled flights must be rescheduled for a later departure or arrival time. One advantage of the chance-constrained model is that it helps to ensure that constraints are not violated at a specified probability level. To evaluate the effectiveness of our approach, we use optimized flight schedule as input to the operational air traffic network. However, it is difficult to accurately predict what the future state of airspace will be; therefore, a robust and stable flight schedule in actual operation is more desirable. The term ‘‘robust’’ is commonly defined to characterize a software, a strategy, or an item that functions effectively and demonstrates minimal failures in various scenarios. In this research, we define the robustness of a flight schedule as the total amount of excess traffic scheduled for all sectors and airports. Here, we conducted a total of 20 experiments to compare the robustness of the optimal flight schedule under various schemes. Each experiment generated 50 sets of combinations of capacity based on the empirical distributions of the capacity of airports and sectors. We compare the amount of traffic with the capac-

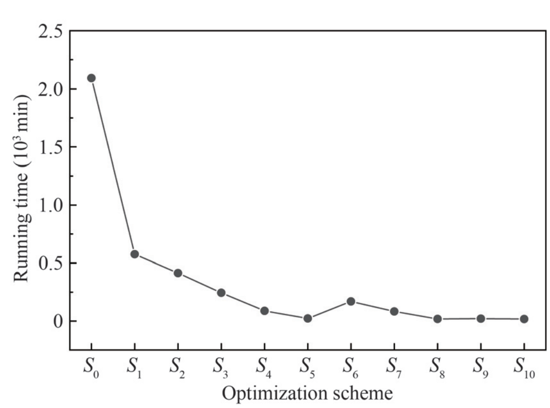

Fig. 7 Runtime of models under different setting.

ity for each flight schedule, and calculate the total amount traffic that exceeds the capacity. Let R_q be the robustness of optimization scheme q, then we have

$$R_q = \frac{1}{N}\sum_{j=1}^{N}\sum_{t\in T'}\left[\sum_{a\in A^{\text{dep}}\cup A^{\text{arr}}}\max\left(0,\left(F_{j,a,t}^{q}-C_{j,a,t}\right)\right)\right.$$
$$\left.+\sum_{s\in S}\max\left(0,\left(F_{j,s,t}^{q}-C_{j,s,t}\right)\right)\right] \tag{32}$$

where N represents the total number of simulated operation scenarios, $F\,j,a,tq$ and $F\,j,s,tq$ denote 500 the number of flights at airport a and sector s during time period t under optimization scheme q, while $C_{j,a,t}$ and $C_{j,s,t}$ are the simulated actual capacities at airport a and sector s during time period t under optimization scheme q.

5.3.1. Robustness test results for base model

The base model corresponds to the scenario in the uncertainty model where the violation probability is 0. Flight schedule data obtained from its optimization is then compared with the 50 sets of capacity combinations generated in each experiment. The robustness R_0 is then calculated and the statistical results are shown in Fig. 8. It is evident that the robust model-optimized flight schedule is more robust than the original flight schedule, as its R is substantially lower. This further confirms that flight schedule optimization is essential to make the air traffic system more stable and improve its resilience to various uncertainties.

5.3.2. Robustness test results for uncertainty model

The base model is too conservative. Airport and sector capacity may be unused due to low scheduled traffic. The more robust a flight schedule is, the higher the total cost. Based on the robustness test, the optimal flight schedule robustness metric for Schemes 1–10 has been calculated. We compare the robustness of the uncertainty models with that of the base model based on the robustness metric. Additionally, we also compare the robustness between Method 1 and Method 2, as well as the robustness of the optimal flight schedule for each scheme within Method 1 and Method 2, to analyze the effect of node importance and violation probability on the robustness of the models.

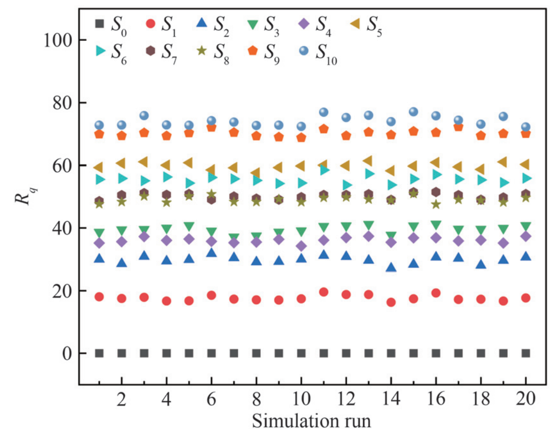

Fig. 9 Comparison of optimal flight schedule robustness metric by scheme.

We can see from Fig. 9 that the optimal flight schedule for all schemes of Method 1 is less robust than the base model. The capacity restriction becomes more stringent as the probability of violation decreases, leading to a decrease in overcapacity and a more robust flight schedule. The results of the robustness test for each scheme of Method 1 are in line with the initial hypothesis. This implies that Method 1, which takes into account capacity uncertainty, is less robust than the base model. However, the base model sacrifices the total cost of the flight schedule in order to achieve strong robustness. Fig. 10 shows that the total cost of the flight schedule for the robust scheme is approximately 35. 4%, 15. 6%, 9. 49%, 5. 38%, and 4. 05% of the base model, respectively. This indicates that the base model incurs a considerable cost in its pursuit of robustness, which may not be desirable to stakeholders. In comparison, Method 1 achieves a significant reduction in the total cost of the flight schedule by sacrificing a bit of robustness and appears to be more profitable. However, considering that the optimization objective of the model in this paper is the flight schedule at the airport network level, it is possible to

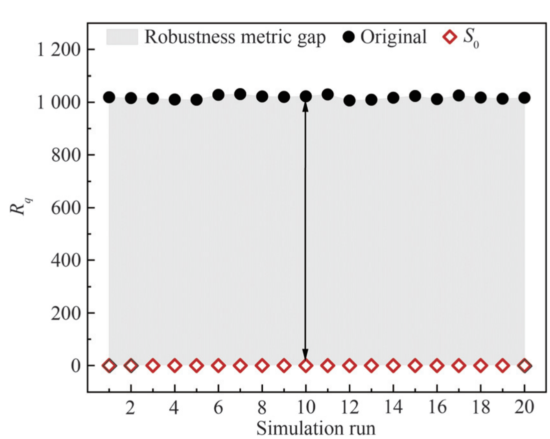

Fig. 8 Comparison of robustness metric between original flight schedule and optimal flight schedule of deterministic model.

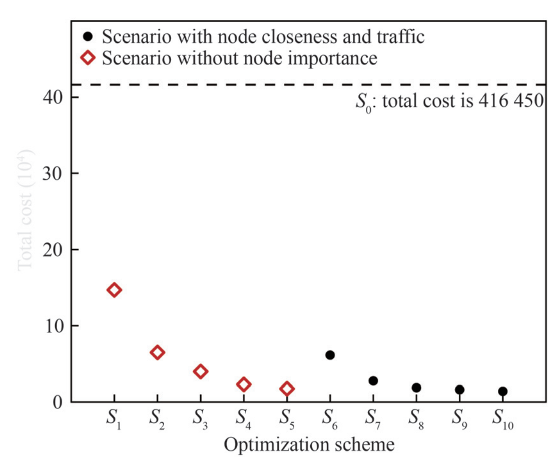

Fig. 10 Comparison of total cost of optimal flight schedule by scheme.

incorporate the structural characteristics of the network into Method 1. This has the potential to result in obtaining a robust flight schedule and reducing costs, thereby creating a model better suited for optimizing the flight schedule at the airport network level.

In an airport network, if there is delay or congestion at an airport or sector with high average daily flow or closeness, it can have a ripple effect on other downstream airports and sectors. To minimize the likelihood of this occurrence, Method 2 considers the importance of both the airport and the sector nodes. This results in a smaller probability of violations for important airports and sectors, leading to a robust flight schedule with a low total cost sacrifice. The total cost of the optimal flight schedule for all the schemes of Method 2 is significantly lower than that of the robust model and Method 1, as seen in Fig. 10. Additionally, the overcapacity of the optimal flight schedule for almost schemes of Method 2 is higher than that of Method 1, as seen in Fig. 9. Considering the importance of the node in the model, incorporating it into the optimization model can result in a flight schedule that demonstrates strong resilience and minimizes overall cost. This, in turn, improves the efficiency of optimizing the flight schedule at the airport network level.

Further examination of Fig. 9 reveals two interesting findings: (A) we observe that the optimal flight schedule of

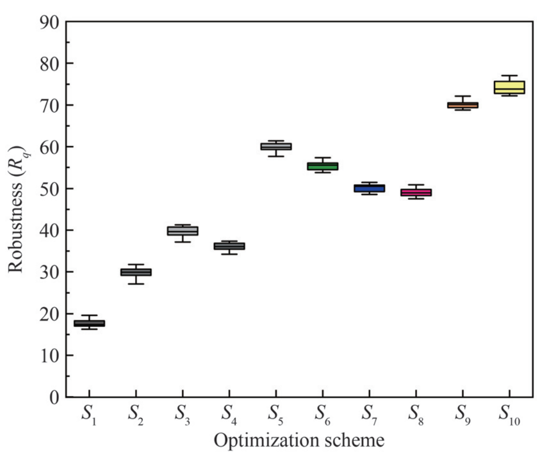

Fig. 11 Comparison of optimal flight schedule robustness metric.

Scheme 9 in Method 2 performs better in terms of robustness, even though the total cost is lower than that of Scheme 4 and 5 of Method 1 by 30.7% and 7.8%, respectively. This clearly shows the robustness and superiority of Method 2 compared to Method 1. Inclusion of the importance of the nodes in the model has advantages for the optimization performance of the model, making it more suitable for optimizing the flight schedule at the airport network level. The results of the robustness metric comparison for the above schemes are shown in Fig. 11. (B) The optimization performance of the model still has room to improve. Fig. 11 shows the robustness metric for each scheme of Method 2. Since Scheme 7 has lower violation probabilities, the optimized schedule should be more robust. However, its robustness metric is not as good as those of Schemes 8 and 9. We hypothesize that this is due to the combination of the probability of violation and the structural characteristics of the airport network. The importance value cannot fully capture the importance of the sector or airport in the network. To further investigate this, we performed additional experiments by adjusting the values of Eq. (27). The maximum violation probability in Method 2 is set to 0.2, the maximum violation probability ranged from 0.23 to 0.35 with a step change of 0.03. In total, five sets of optimization experiments similar to Method 2 were conducted, with the only difference being the combination of violation probability.

Table 8 presents the total cost and robustness of the flight schedule. The data in the tables show that as the probability of violation increases, the total cost of the optimal flight schedule from Scheme 6 to Scheme 10 decreases, but the robustness does not necessarily increase. The results for different values of α demonstrate that the robustness of schemes with a high probability of violation can be lower than that of schemes with a low probability of violation. Method 2 is more suitable for flight schedule optimization at the airport network level than Method 1, however, it does not take into account all the special points of the airport network. The combination of violation probability calculated by linear interpolation based on node importance is probably not the optimal combination for the corresponding schemes. This is evidenced by the lack of a clear trend in the average robustness in Table 8.

5.4. Comparison with other method of setting violation probability

The importance of the sector node was determined using closeness, while the importance of the airport node was assessed

Table 8 Total cost (robustness) for schemes in Method 2 with different maximum violation probability.

α_{max}	S_6	S_7	S_8	S_9	S_{10}
	α_{min} = 0.05	α_{min} = 0.08	α_{min} = 0.11	α_{min} = 0.14	α_{min} = 0.17
0.20	61 200 (55.5)	27 400 (50.2)	18 600 (49.1)	16 000 (70.2)	13 900 (74.2)
0.23	51 250 (64.1)	21 750 (54.3)	16 600 (65)	13 300 (74.3)	12 750 (82.8)
0.26	47 750 (69.5)	19 650 (67.1)	15 100 (68.2)	11 900 (75.6)	10 700 (74)
0.29	46 550 (73.8)	18 650 (56.9)	12 800 (85.6)	10 600 (78.1)	9650 (92.8)
0.32	43 950 (90.3)	16 050 (86.8)	10 850 (77.3)	9250 (98.2)	8900 (96.8)
0.35	41 450 (90.4)	14 750 (81.4)	10 250 (96.4)	8750 (98.1)	7350 (92.1)

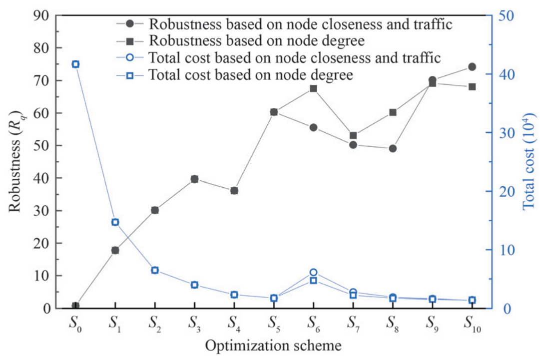

Fig. 12 Total cost and robustness of results under different optimization scheme.

based on the volume of traffic. In this section, we compare the optimal results with an alternative approach that relies on a different network metric to assess the importance of nodes. Specifically, we use the degree of the node as the measure of importance. The total cost and robustness of the results under different optimization schemes are plotted in Fig. 12. There is minimal variation in total cost resulting from two network metrics, with only a slight distinction observed in the schemes S_6 and S_7. However, there are clear discrepancies in the robustness of the optimized schedule when using the two different methods to set the probability of violations. Using traffic and closeness metrics to assess the importance of nodes can lead to a more resilient flight schedule compared to relying solely on the degree of the network. The excess capacity at each airport and sector is determined by employing Eq. (32) In Figs. 13(a) and (b), the average number of flights surpassing the capacity of airports or sectors is illustrated for various optimization approaches. Among the airports, only ZBAA, ZGSZ, and ZUCK exhibit more than 3 flights exceeding capacity, while the majority of airports and sectors experience fewer than 2 flights exceeding capacity. Sectors ZHHHAR01 and ZBAAAR15 show more than two flights surpassing sector capacity when applying violation probability settings based on closeness. Overall, the optimized flight schedules demonstrate resilience to operational uncertainties.

5.5. Discussion

We identified congested points in the air traffic network by using flight traffic data and capacity data. The level of congestion was measured with a number in the range of 0 to 1, with higher numerical values indicating more congestion. Similar to R_q, we define $R_{q,w,t}$ as value of the level of congestion for node w at 15-minutes slot t of optimization scheme q.$R_{q,w,t}$ can be calculated by Eq. (33). The results of Scheme 10 are shown in Fig. 14. We see the optimized flight schedule of Scheme 10 still has some congested airports and sectors. To achieve a flight schedule with fewer congested airports and airspace, increased robustness, and lower total cost, we can modify the probability of a violation of restrictions in airports and sectors. However, depending solely on static data such as node importance to set the probability of violation of airports and sectors may not be an optimal strategy. It is challenging to decide on the optimal combination of probability of violation, and congested nodes in the air traffic network should be taken into consideration to dynamically adjust the probability of violation of each chance constraint. To improve the probability of violation and the quality of the solution, it is possible to reduce the probability of violation at congested points and increase the probability of violation at points with a sufficient capacity margin. This can be done by "cutting the peak and filling the valley", which can be further studied by monitoring the changes in the probability of the chance of violation of the constraints of each node, thereby exploring the trigger conditions and intrinsic mechanisms of the optimization dynamic of the airport network.

$$R_{q,w,t} = \frac{1}{N}\sum_{j=1}^{N}\max\left(0,\left(F^{q}_{j,w,t} - C_{j,w,t}\right)\right), \quad \forall w \in A^{\mathrm{dep}} \cup A^{\mathrm{arr}} \cup S, t \in T'$$

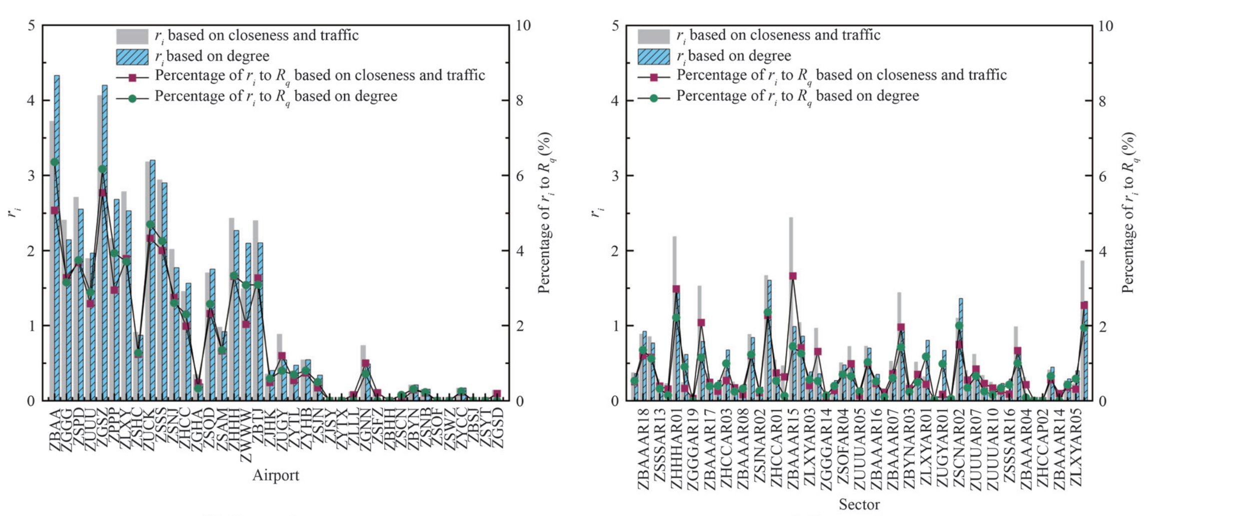

Fig. 13 Robustness of airports and sectors.

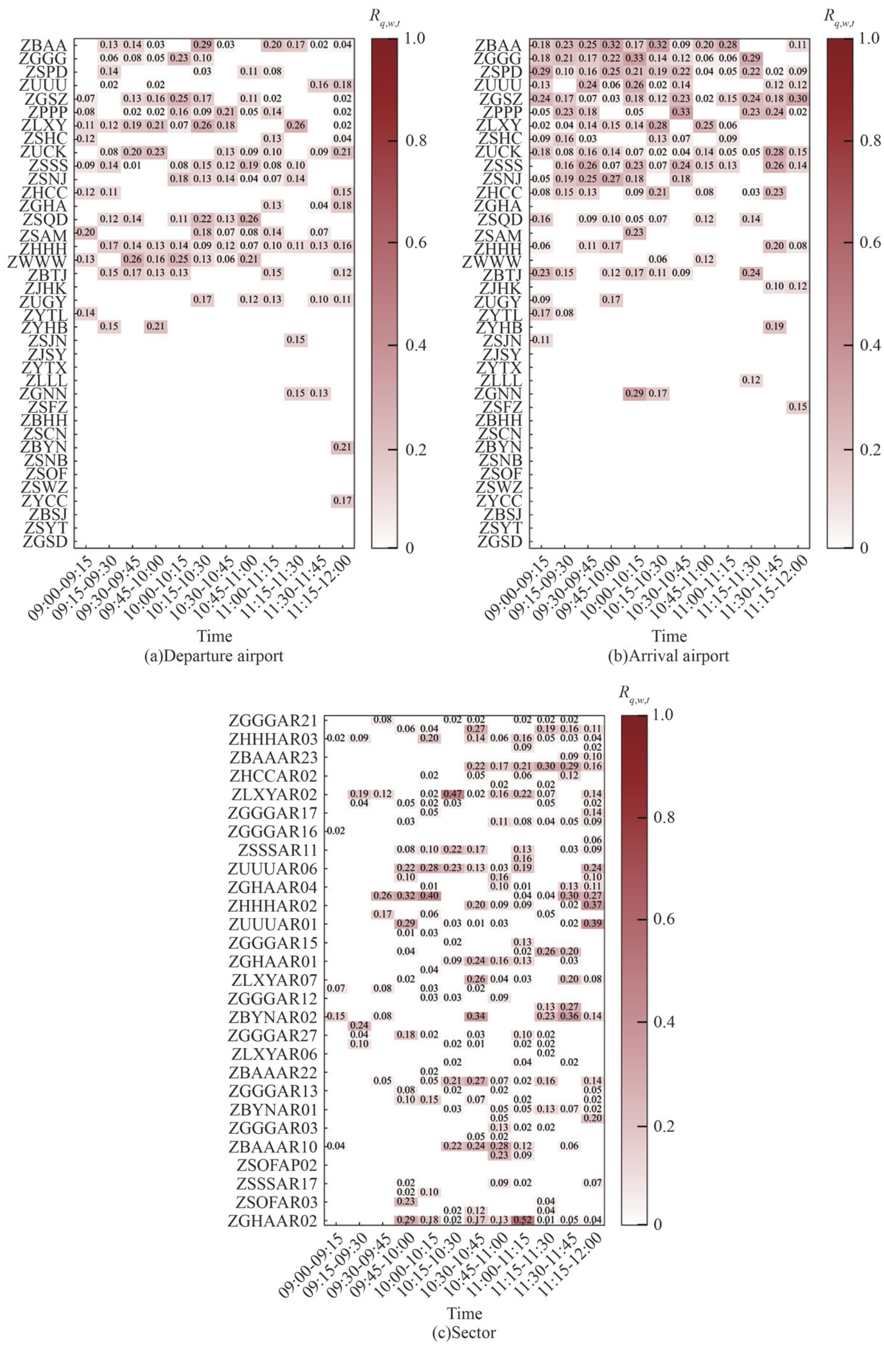

Fig. 14 Heatmap of congestion at airports and sectors in Scheme 10. (a) Congestion level of departure airports; (b) Congestion level of arrival airports; (c) The congestion level of sectors.

Our research has practical implications in real-world scenarios. Critical sectors and airports can be identified using existing and predicted air traffic conditions, along with meteorological data. Supported by air traffic flow management systems, the personnel involved in managing air traffic flow determines the permissible risk levels for key sectors and airports. Subsequently, our model is capable of assigning the best departure times for each flight. In the future, the efficacy of the model could be improved through the application of machine learning and artificial intelligence technologies.

6. Conclusions

The demand for air transport and the complexity of the air traffic system have both grown, leading to a number of uncertainties in the implementation of flight schedules. This paper presents a new capacity uncertainty chance constraint model to address the flight schedule optimization problem at the airport network level. This model takes into account the capacity uncertainty of airports and sectors in the air traffic system and also innovatively incorporates the importance of network nodes into the model. The experimental data show that the traditional capacity uncertainty model is not sufficient to handle the uncertainty problem. The cost of having a reliable flight schedule is high. To address this issue and create a model with better optimization results, this paper takes into account the importance of the nodes in the airport network in the model. Compared to the deterministic model and the traditional capacity uncertainty model, the results show that our model can produce a more robust and lower total cost flight schedule through the appropriate setting of the chance constraint violation probability.

This study has the following limitations. First, due to the limitations of the current dataset, it is difficult to find the optimal combination of probability of violation. This, in turn, reduces the efficiency of the model. To improve the optimization performance, the dynamic data of the congested points in the airport network should be taken into account when adjusting the probability of violation. Second, the length of the slot discussed in this paper is 5 min. Reducing the slot to 1 min could provide more accurate results. However, this would also significantly increase the computational complexity of the model. Third, the sector minimum flight time for each flight should be varied, as this would make the flight schedule more practical. This is because the performance of the aircraft can vary greatly depending on the model of the plane being used. Therefore, the sector minimum flight time should be adjusted accordingly for each flight. Future research efforts should be made in two areas: (A) to develop algorithms based on artificial intelligence to generate representative airspace operation scenarios; and (B) investigating the unique optimization process of the airport network.

CRediT authorship contribution statement

Jianzhong YAN: Writing – original draft, Validation, Investigation, Formal analysis, Conceptualization. **Haoran HU:** Writing – original draft, Software, Methodology, Investigation, Formal analysis. **Yanjun WANG:** Writing – review & editing, Writing – original draft, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Xiaozhen MA:** Methodology, Investigation, Formal analysis. **Minghua HU:** Investigation, Formal analysis. **Daniel DELAHAYE:** Writing – review & editing, Writing – original draft, Methodology, Formal analysis. **Sameer ALAM:** Writing – review & editing, Writing – original draft, Investigation, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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