# Description of Diffusion-Limited Aggregation (DLA) Simulation

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## 1. Introduction

Diffusion-Limited Aggregation (DLA) is a stochastic process that models the formation of complex branching structures through random motion and aggregation. It was first introduced by Witten and Sander (1981) to describe cluster growth phenomena governed by diffusion. The process begins with a seed particle placed at the origin. Successive particles are released from a certain distance away and undergo a random walk. When a particle contacts the existing cluster, it becomes part of it. Over time, this process produces fractal patterns similar to those observed in dielectric breakdown, electrodeposition, and mineral formation.

The simulation presented here implements a two-dimensional DLA model using a discrete grid representation. The model follows the typical physical rules of diffusion and aggregation but is expressed computationally through a series of random walks and occupation updates within a bounded grid.

## 2. Algorithm Overview

The algorithm follows these main steps:

1. **Initialization:**
   - Define a square lattice of dimension $(-n, n)$ along both axes, initialized with zero values.
   - Set the origin $(0, 0)$ as the initial occupied site (seed) of the cluster.
   - Mark the four nearest neighbors of the seed as potential aggregation sites.
2. **Particle Release:**
   Each new particle starts its random walk from a point on a circular boundary with radius $r_{\max}$, where the initial radius is set slightly beyond the current cluster size. The launch angle $\theta$ is chosen uniformly from 0 to $2\pi$:

   $$\theta = 2\pi \times \text{rand}()$$

   The initial position is therefore:

   $$(x_{\text{walk}}, y_{\text{walk}}) = (\text{nint}(r_{\max} \cos\theta), \text{nint}(r_{\max} \sin\theta))$$

3. **Random Walk:**
   The particle undergoes a random walk, taking one step at a time in either the $x$ or $y$ direction. The direction is determined by random variables $a$, $dx$, and $dy$, where:

$$dx = \text{sign}(1, 2\text{rand}() - 1), \quad dy = \text{sign}(1, 2\text{rand}() - 1)$$

   If $a \geq 0$, the particle steps along the $y$-axis; otherwise, it moves along the $x$-axis.

   Boundary conditions are enforced so that any particle reaching the grid edge is discarded and reinitialized at a new random starting position.

4. **Aggregation Rule:**
   When the random walker reaches a site adjacent to an occupied point (marked as 1), it adheres to that site and becomes part of the cluster (value set to 2). The nearest four neighbors of that newly occupied site are then marked as potential attachment sites (value 1).

5. **Cluster Growth:**
   The effective cluster radius $r$ is continuously updated after each successful aggregation:

$$r_{\text{temp}} = \sqrt{x_{\text{walk}}^2 + y_{\text{walk}}^2}$$

   If $r_{\text{temp}} > r$, the radius is updated as $r = r_{\text{temp}}$, and the new release radius becomes $r_{\text{max}} = r + 5.0$.

6. **Output:**
   Each aggregated particle's coordinates and particle index are written to a data file for later visualization or analysis.

# 3. Simulation Characteristics

This implementation produces a classical fractal structure typical of DLA growth. The morphology exhibits self-similarity, with a fractal dimension $D$ that generally lies between 1.6 and 1.7 in two dimensions. The randomness in walker motion and aggregation direction leads to the emergence of branching and screened growth zones.

The simulation parameters can be adjusted as follows:

- $n$ — grid half-size (spatial domain)
- $r_{\text{max}}$ — maximum starting radius for new walkers
- $np$ — number of particles (cluster growth size)

# 4. Discussion

This DLA algorithm demonstrates the power of simple stochastic processes to generate complex structures. The diffusion and aggregation rules mimic real-world transport-limited phenomena such as crystal growth, bacterial colony formation, and viscous fingering. From a computational physics perspective, the model emphasizes randomness, self-organization, and emergent pattern formation, providing an intuitive framework for studying non-equilibrium growth systems.

# 5. Conclusion

The presented simulation successfully models the formation of fractal aggregates through diffusion-limited aggregation. Despite its simplicity, the algorithm captures the essential physics of diffusion-driven growth. The implementation allows flexibility in domain size, particle count, and visualization, making it suitable for educational purposes, computational physics experiments, or research exploring fractal growth dynamics.