Chapter 10

10.1

$ENR = 22\,dB\ (T_1)$, $T_2 = 77K$, $Y = 15.83dB$

$$ENR = 10 \log \frac{T_1 - T_0}{T_0} \Rightarrow \frac{T_1 - T_0}{T_0} = 10^{22/10} = 158.5$$

$$T_1 = T_0(1 + 158.5) = 4.63 \times 10^4\ K$$

$$Y = 10^{15.83/10} = 38.28$$

$$T_e = \frac{T_1 - Y T_2}{Y - 1} = 1.163 \times 10^3\ K$$

$$F = 1 + \frac{T_e}{T_0} = 5.01 = \underline{7.0\ dB}\ \checkmark$$

10.2

$$T_e = \frac{T_1 - \gamma T_2}{\gamma - 1}$$

$$T_e + \Delta T_e = \frac{T_1 - (\gamma + \Delta\gamma)T_2}{(\gamma + \Delta\gamma) - 1}$$

$$\Delta T_e = \frac{T_1 - (\gamma + \Delta\gamma)T_2}{(\gamma + \Delta\gamma) - 1} - \frac{T_1 - \gamma T_2}{\gamma - 1}$$

$$= \frac{T_1 - (\gamma + \Delta\gamma)T_2}{(\gamma - 1)\left(1 + \frac{\Delta\gamma}{\gamma - 1}\right)} - \frac{T_1 - \gamma T_2}{\gamma - 1}$$

$$\simeq \frac{[T_1 - (\gamma + \Delta\gamma)T_2]\left[1 - \frac{\Delta\gamma}{\gamma - 1}\right] - (T_1 - \gamma T_2)}{\gamma - 1}$$

$$\simeq \frac{-\frac{T_1}{\gamma - 1} + \frac{\gamma T_2}{\gamma - 1} - T_2}{\gamma - 1}\Delta\gamma = \frac{(T_2 - T_1)}{(\gamma - 1)^2}\Delta\gamma$$

$$\frac{\Delta T_e}{T_e} = \frac{(T_2 - T_1)\gamma}{(\gamma - 1)^2 T_e}\frac{\Delta\gamma}{\gamma} = \frac{(T_1 + T_e)(T_2 + T_e)}{T_e(T_2 - T_1)}\frac{\Delta\gamma}{\gamma}$$

minimize with respect to T_e:

$$\frac{d}{dT_e}\left[\frac{\Delta T_e}{T_e}\right] = \frac{\left(\frac{T_1}{T_e} + 1\right)\left(\frac{T_2}{T_e} + 1\right) + T_e\left(\frac{-T_1}{T_e^2}\right)\left(\frac{T_2}{T_e} + 1\right) + T_e\left(\frac{T_1}{T_e} + 1\right)\left(\frac{-T_2}{T_e^2}\right)}{T_2 - T_1} = 0$$

Thus,

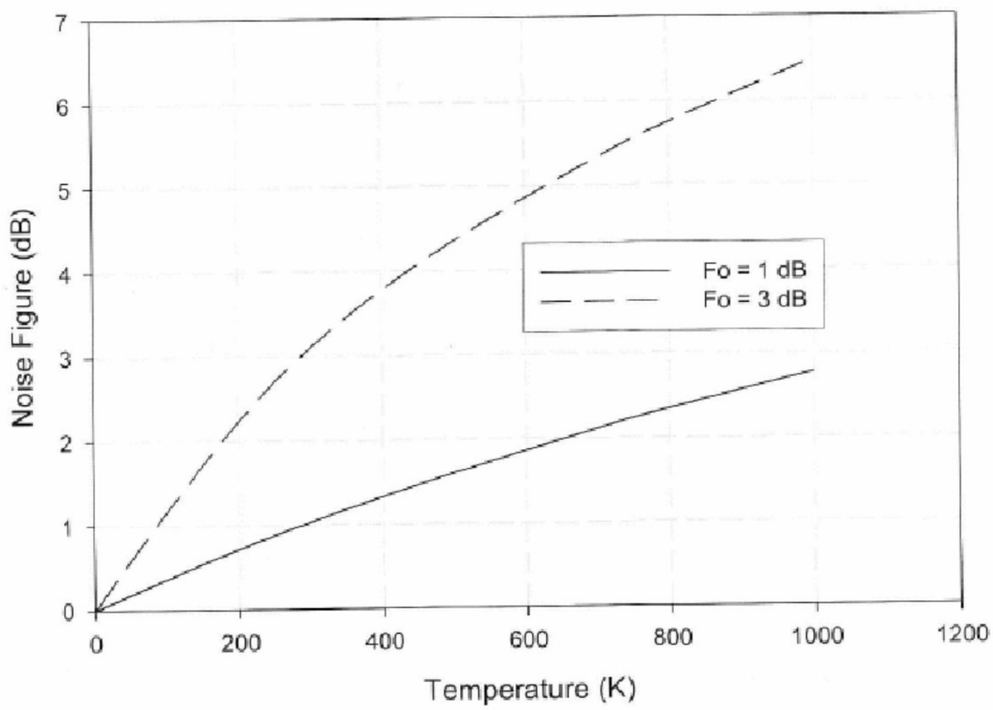
$$T_e = \sqrt{T_1 T_2} \checkmark$$

10.3 $T = 0$ to $1000\,K$, $F_0 = 1\,dB$, $3\,dB$.

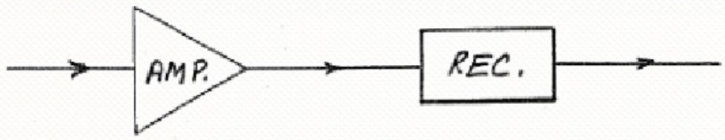
From (10.16) $F = 1 + (L-1)\,T/T_0$

When $F_0 = 1\,dB$, $L = 1\,dB = 1.259$
$F_0 = 3\,dB$, $L = 3\,dB = 2.00$

T (K)	F ($F_0 = 1\,dB$)	F ($F_0 = 3\,dB$)
0	0	0
250	0.88	2.70
500	1.60	4.35
750	2.23	5.55
1000	2.77	6.48



10.4



$G = 12dB = 15.8$
$BW = 150MHz$
$F = 4dB = 2.51$

$T_e = 900K$

The noise figure of the receiver is, from (10.11),

$$F_2 = 1 + \frac{T_e}{T_0} = 1 + \frac{900}{290} = 4.10$$

Then the noise figure of the cascade is, from (10.21),

$$F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1) = 2.51 + \frac{4.10-1}{15.8} = 2.71 = 4.3dB \checkmark$$

10.5

a) $$T_e = \frac{P}{kB} = \frac{(.001)\times 10^{-9.5}}{(1.38\times10^{-23})(75\times10^6)} = 305.5K \checkmark$$

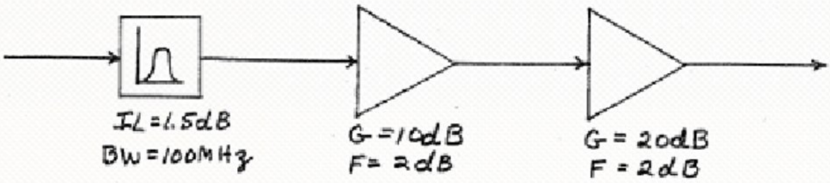
b) $$F_\ell = 1 + (L-1)\frac{T}{T_0} = 1 + (1.413-1)\frac{300}{290} = 1.43, \quad F_a = 1 + \frac{T_e}{T_0} = 1.62 = 2.1dB$$

c) $$F_c = F_\ell + \frac{F_a - 1}{G_\ell} = 1.43 + \frac{1.62-1}{1/1.413} = 2.30 = \underline{3.6dB} \checkmark$$

$$T_c = (F_c - 1)T_0 = (2.30-1)(290) = \underline{378K} \checkmark$$

d) $$N_o = k(T_c + T_i)BG = (1.38\times10^{-23})(378 + 305.5)(75\times10^6)\left(\frac{15.8}{1.413}\right)$$
$$= 7.9\times10^{-12}\,W = 7.9\times10^{-9}\,mW = \underline{-81.0\,dBm} \checkmark$$

10.6



From (10.23) the noise figure of the cascade is ($F_1 = IL = 1.5dB$)

$$F_{CAS} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} = 1.41 + (1.41)(1.58-1) + \frac{1.41}{10}(1.58-1)$$

$$= 2.31 = 3.64dB$$

If $P_{in} = -90$ dBm, then $P_{out} = -90dBm - 1.5dB + 10dB + 20dB = -61.5dBm$

The noise power output is then,

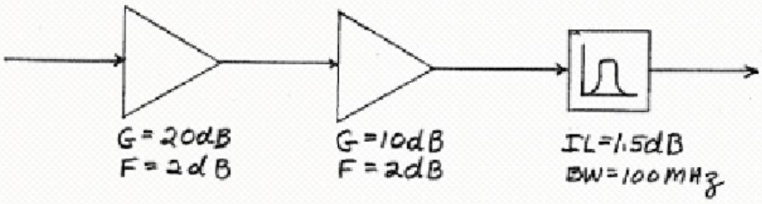
$$P_n = G_{cas} k T_{e_{cas}} B = k(F_{cas} - 1) T_o B G_{cas}$$

$$= (1.38 \times 10^{-23})(2.31-1)(290)(10^8)(10^{28.5/10}) = 3.71 \times 10^{-10} \text{ W}$$

$$= -64.3 \text{ dBm}$$

Thus, $\quad \frac{S_o}{N_o} = -61.5 + 64.3 = 2.8dB$

The best noise figure would be achieved with the arrangement shown below:



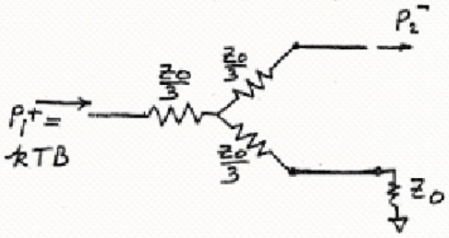
Then,

$$F_{CAS} = 1.58 + \frac{(1.58-1)}{100} + \frac{(1.41-1)}{1000} = 1.586 = 2.0dB$$

(In practice, however, the initial filter may serve to prevent overload of the amplifier, and may not be allowed to be moved.)

10.7

a) RESISTIVE DIVIDER



$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

When the input noise power at port 1 is kTB, and the divider is at temperature T, the system is in thermodynamic equilibrium. Thus the output noise power at port 2 must be kTB. We can also express this as due to the attenuated input noise power and noise power added by the network (ref. at input). Thus,

$$P_2^- = kTB = \frac{kTB}{4} + \frac{N_{added}}{4}$$

$$\therefore N_{added} = 3\,kTB$$

The equivalent noise temperature is then,

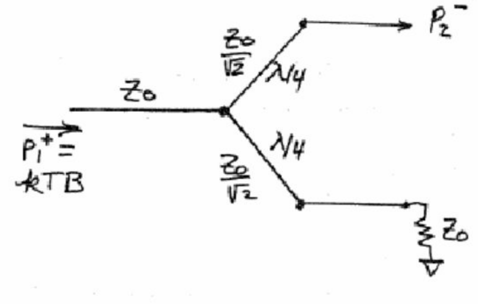
$$T_e = \frac{N_{added}}{kB} = 3T$$

and the noise figure is,

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{3T}{T_0}$$

at room temperature, $T = T_0$, so $F = 4 = 6\,dB$.
(this result checks with that obtained using the available gain method)

b) WILKINSON DIVIDER



$$[S] = \begin{bmatrix} 0 & -j/\sqrt{2} & -j/\sqrt{2} \\ -j/\sqrt{2} & 1/2 & -1/2 \\ -j/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$$

In this case, if the input noise power is kTB, and the system is in thermodynamic equilibrium, the net output power at port 2 is $\frac{3}{4}kTB$, because of the mismatch of the output ports (1/4 of output power is reflected). Then we have,

$$P_2^- = \frac{3}{4}kTB = \frac{kTB}{2} + \frac{N_{added}}{2} \qquad (N_{added} \text{ ref. at input})$$

$$\therefore N_{added} = \frac{1}{2}kTB$$

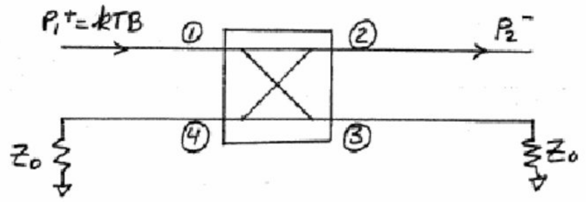
$$T_e = \frac{N_{added}}{kB} = \frac{T}{2}$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T}{2T_0}$$

If $T = T_0$, $F = \frac{3}{2} = 1.76$ dB.

(Result verified with HP-MDS, calculations using available gain, and direct measurement)

c) QUADRATURE HYBRID



$$[S] = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

Using the same thermodynamic arguments as above, the output noise power is kTB (outputs are matched). Thus,

$$P_2^- = kTB = \frac{kTB}{2} + \frac{N_{added}}{2}$$

$$\therefore N_{added} = kTB$$

$$T_e = \frac{N_{added}}{kB} = T$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T}{T_0}$$

If $T = T_0$, we have $F = 2 = 3dB$

10.8 From (10.33), $T_e = \dfrac{(L-1)(L+|\Gamma_s|^2)}{L(1-|\Gamma_s|^2)}T$

let $x^2 = |\Gamma_s|^2$; $C = (L-1)T/L$. Then $T_e = C\dfrac{L+x^2}{1-x^2}$

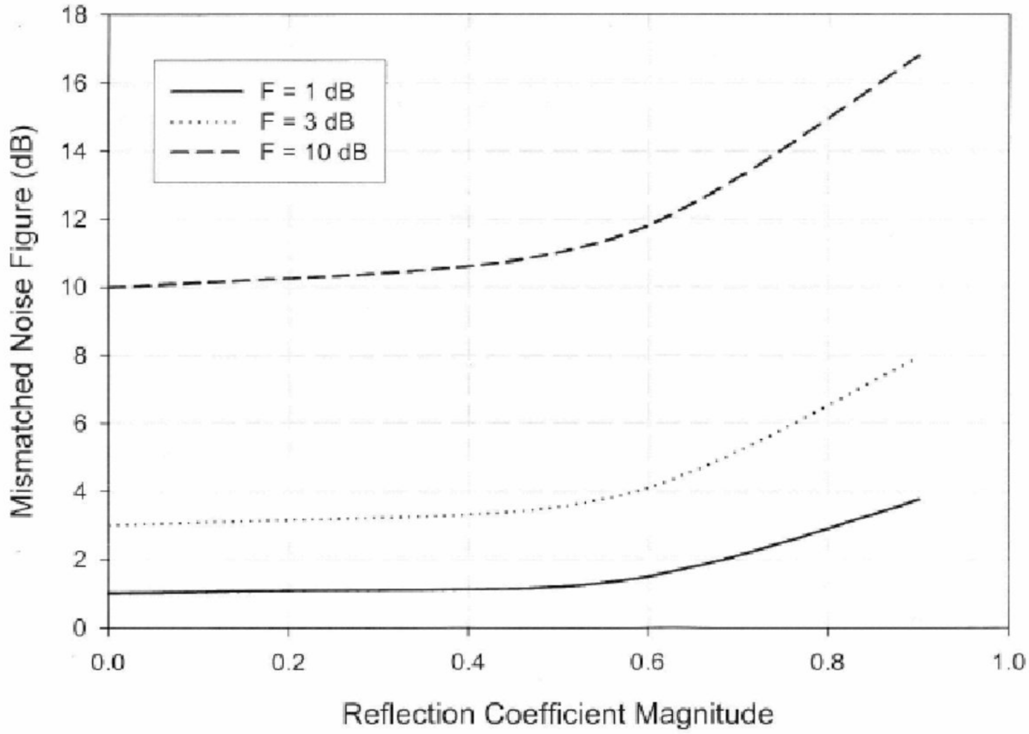
$$\frac{dT_e}{dx} = C\frac{(1-x^2)(2x)+(2x)(L+x^2)}{(1-x^2)^2} = \frac{2x(1+L)}{(1-x^2)^2} = 0$$

Thus $x=0$, so $|\Gamma_s| = 0$ minimizes T_e ✓

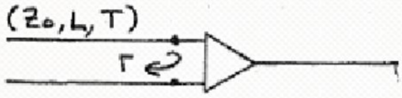
10.9 $|\Gamma| = 0$ to 0.9, $F = 1\,dB, 3\,dB, 10\,dB.$

From (10.36) $\qquad F_M = 1 + \dfrac{F-1}{1-|\Gamma|^2}$

$\lvert\Gamma\rvert$	F=1dB	F=3dB	F=10dB
0	1.00	3.00	10.0
0.3	1.09	3.22	10.4
0.6	1.48	4.09	11.8
0.9	3.74	7.97	16.8



10.10



Solution using noise temperature:

Let $N_i = kT_0B$

Then $N_0 = \underbrace{\frac{kT_0BG}{L}(1-|\Gamma|^2)}_{\text{INPUT NOISE}} + \underbrace{\frac{(L-1)}{L}kTB(1-|\Gamma|^2)G}_{\text{NOISE ADDED BY LINE}} + \underbrace{kT_0(F-1)GB}_{\text{NOISE ADDED BY AMP}}$

also,
$$S_o = \frac{G(1-|\Gamma|^2)}{L}S_i$$

So,
$$F_{CAS} = \frac{S_iN_o}{S_oN_i} = \frac{L}{\cancel{G}(1-|\Gamma|^2)} \cdot \frac{\frac{\cancel{kT_0BG}}{L}(1-|\Gamma|^2)+\left(\frac{L-1}{L}\right)\cancel{G}\cancel{k}T\cancel{B}(1-|\Gamma|^2)+\cancel{k}T_0(F-1)\cancel{G}\cancel{B}}{\cancel{kT_0B}}$$

$$= 1 + (L-1)\frac{T}{T_0} + \frac{L(F-1)}{1-|\Gamma|^2} \checkmark$$

Solution using cascade formula:

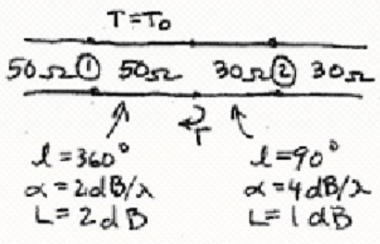
$$T_{e(LINE)} = (L-1)T$$
$$F_{(LINE)} = 1 + (L-1)T/T_0$$
$$G_{(LINE)} = \frac{1}{L}(1-|\Gamma|^2)$$

$$F_{CAS} = F_{(LINE)} + \frac{F_{AMP}-1}{G_{(LINE)}} = 1 + (L-1)\frac{T}{T_0} + \frac{L}{1-|\Gamma|^2}(F-1) \checkmark$$

CHECK: IF $\Gamma = 0$, $F_{CAS} = 1 + (L-1)\frac{T}{T_0} + L(F-1) \checkmark$

IF $\Gamma = 0$ AND $T = T_0$, $F_{CAS} = 1 + (L-1) + L(F-1) = LF \checkmark$

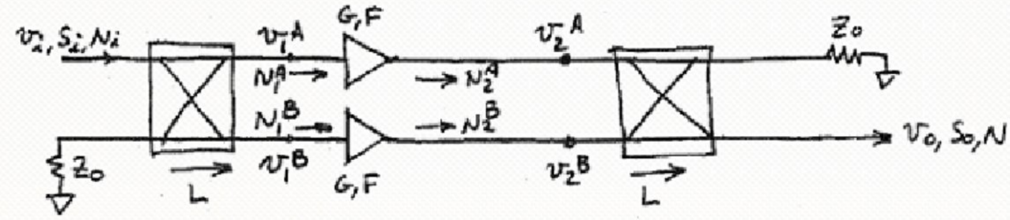
NUMERICAL CHECK:



$$\Gamma = \frac{30-50}{30+50} = \frac{-1}{4}$$

$F_{CAS} = 3.06\,dB$ – AGREES WITH SERENADE.

10.11



$$S_i = v_i^2/2$$

$$v_1^A = \frac{v_i}{\sqrt{2L}} \qquad v_1^B = -j\frac{v_i}{\sqrt{2L}}$$

$$v_2^A = \frac{v_i\sqrt{G}}{\sqrt{2L}} \qquad v_2^B = -j\frac{v_i\sqrt{G}}{\sqrt{2L}}$$

$$v_o = -j\frac{v_2^A}{\sqrt{2L}} + \frac{v_2^B}{\sqrt{2L}} = -j\frac{v_i\sqrt{G}}{2L} - j\frac{v_i\sqrt{G}}{2L} = -j\frac{v_i\sqrt{G}}{L}$$

$$S_o = \frac{v_o^2}{2} = \frac{v_i^2 G}{2L^2} = \frac{G S_i}{L^2} \quad ✓$$

$$N_1^A = N_1^B = kT_0B$$

$$N_2^A = N_2^B = kT_0BG + kT_eBG = kT_0BG(1+F-1) = kT_0BGF$$

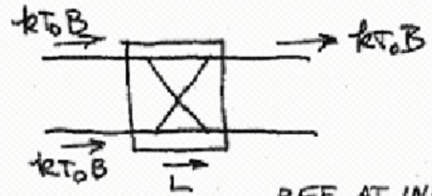
$$N_o = \frac{N_2^A}{2L} + \frac{N_2^B}{2L} + \frac{N_{ADDED}}{2L} = \frac{kT_0BG}{L}F + \underbrace{\frac{kT_0B}{2L}(2L-2)}_{\text{SEE BELOW}}$$

$$= \frac{kT_0BG}{L}F + kT_0B\left(1-\frac{1}{L}\right)$$

$$F_{TOT} = \frac{S_iN_o}{S_oN_i} = \frac{L^2}{G}\left[\frac{GF}{L} + \left(1-\frac{1}{L}\right)\right] = LF + \frac{L}{G}(L-1) \quad ✓$$

CHECK: IF $L=1$, $F_{TOT} = F$ ✓

N_{ADDED} FOR HYBRID:



REF. AT INPUT

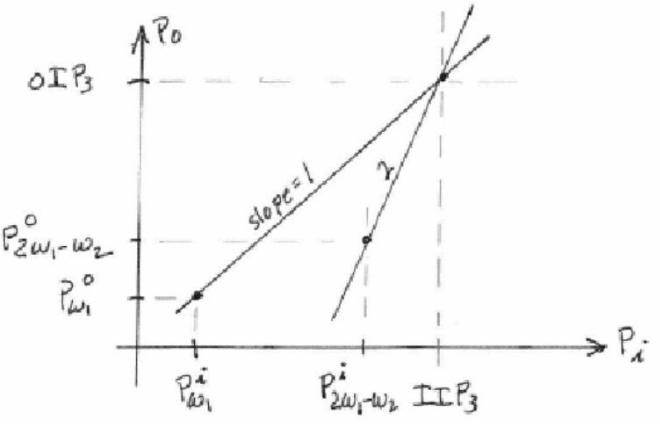
$$N_o = \frac{kT_0B}{2L} + \frac{kT_0B}{2L} + \frac{N_{ADDED}}{2L} = kT_0B$$

$$\therefore\ N_{ADDED} = 2kT_0B(L-1) \quad \text{(REF. AT INPUT)}$$

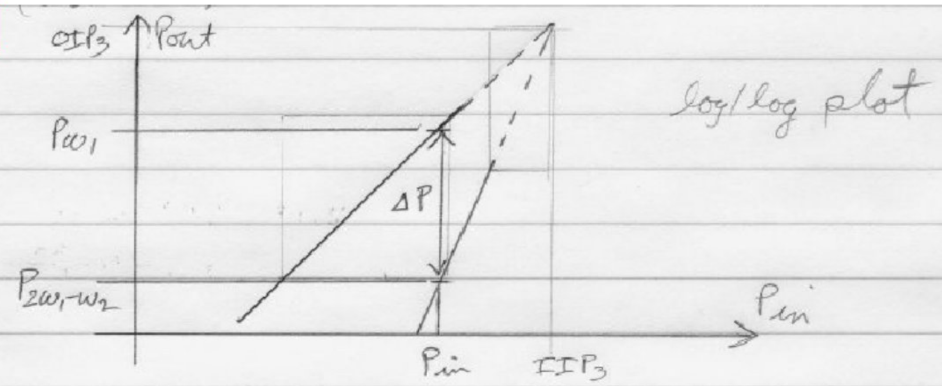
10.12

The ratio $\dfrac{OIP_3 - P^o_{\omega_1}}{IIP_3 - P^i_{\omega_1}} = 1$ defines the slope of the fundamental (1st order response)

The ratio $\dfrac{OIP_3 - P^o_{2\omega_1-\omega_2}}{IIP_3 - P^i_{2\omega_1-\omega_2}} = 3$ defines the slope of the third order response.



10.13



$P_{\omega_1} = P_{in} + b_1$ (eq. of line, slope = 1)

$P_{2\omega_1-\omega_2} = 3P_{in} + b_2$ (eq. of line, slope = 3)

subtract:

$$\Delta P = P_{\omega_1} - P_{2\omega_1-\omega_2} = -2P_{in} + b_1 - b_2$$

Now,

$IIP_3 = P_{in}$ when $\Delta P = 0$, so

$$\Delta P = -2P_{in} + b_1 - b_2$$
$$0 = -2(IIP_3) + b_1 - b_2$$

so $\Delta P = -2P_{in} + 2(IIP_3)$

or,

in dB $\begin{cases} IIP_3 = P_{in} + \Delta P/2 & \text{(ref. at input)} \\ OIP_3 = P_{\omega_1} + \Delta P/2 & \text{(ref at output)} \end{cases}$

Example: $P_{\omega_1} = 5\,dBm$, $P_{2\omega_1-\omega_2} = -27\,dBm$, $P_{in} = -4\,dBm$ (Egan, p. 100)

Then $\Delta P = 32\,dB$

$IIP_3 = -4 + 32/2 = 12\,dBm$

$OIP_3 = 5 + 32/2 = 21\,dBm$

check: $G = P_{\omega_1}/P_{in} = 5 + 4 = 9\,dB$

$OIP_3 = G(IIP_3) = 9 + 12 = 21\,dBm$ ✓

10.14 Retaining only the terms that give rise to the third order intermodulation products:

$$v_0 \sim k\,(v_1 \cos\omega_1 t + v_2 \cos\omega_2 t)^3$$

$$\sim k\,(v_1^2 v_2 \cos^2\omega_1 t \cos\omega_2 t + v_1 v_2^2 \cos\omega_1 t \cos^2\omega_2 t)$$

$$\sim k\,(v_1^2 v_2 \cos 2\omega_1 t \cos\omega_2 t + v_1 v_2^2 \cos\omega_1 t \cos 2\omega_2 t)$$

$$\sim \frac{k}{2}\left[v_1^2 v_2 \cos(2\omega_1 - \omega_2)t + v_1 v_2^2 \cos(2\omega_2 - \omega_1)t\right]$$

So the ratio of the powers in the two outputs is,

$$\left(\frac{v_1^2 v_2}{v_1 v_2^2}\right)^2 = \left(\frac{v_1}{v_2}\right)^2 = 6\,dB$$

Note that the individual output powers vary as P_{in}^3.

10.15 Moving the reference for IP_3' to the output of the mixer gives

$$OIP_3' = 13\ dBm - 6\ dB = 7\ dBm$$

numerical values:

$$OIP_3'' = 22\ dBm = 158\ mW \qquad (AMP)$$
$$OIP_3' = 7\ dBm = 5\ mW \qquad (MIXER)$$
$$G_2 = 20\ dB = 100 \qquad (AMP)$$

Assuming coherent products, using (10.53),

$$OIP_3 = \left(\frac{1}{G_2(OIP_3')} + \frac{1}{OIP_3''}\right)^{-1} = \left(\frac{1}{(100)(5)} + \frac{1}{158}\right)^{-1} = 120\ mW$$

$$= \underline{20.8\ dBm}\ \checkmark$$

Assuming non-coherent products, using (10.54),

$$OIP_3 = \left[\frac{1}{G_2^2(OIP_3')^2} + \frac{1}{(OIP_3'')^2}\right]^{-1/2} = \left[\frac{1}{(100)^2(5)^2} + \frac{1}{(158)^2}\right]^{-1/2}$$

$$= 150.7\ mW = \underline{21.8\ dBm}$$

10.16

For $v_i = V_o \cos \omega_o t$

$$V_{\omega_o} = a_1 V_o + \frac{3}{4} a_3 V_o^3 \qquad (10.40)$$

$$V_{3\omega_o} = \frac{1}{4} a_3 V_o^3$$

For $v_i = V_o(\cos \omega_1 t + \cos \omega_2 t)$

$$V_{2\omega_1 - \omega_2} = \frac{3}{4} a_3 V_o^3 \qquad (10.43)$$

assume a_3 is of opposite sign to a_1 (for compression).
Now let V_o be input voltage where 3rd order term reduces 1st order term by 1 dB:

$$\frac{|a_1|V_o - \frac{3}{4}|a_3|V_o^3}{|a_1|V_o} = 1 - \frac{3}{4}\left|\frac{a_3}{a_1}\right| V_o^2 = 10^{-1/20} = 0.8913$$

$$\frac{3}{4}\left|\frac{a_3}{a_1}\right| V_o^2 = 0.10875$$

From (10.44) the input voltage at IP_3 is

$$V_{IP}^2 = \frac{4a_1}{3a_3}, \text{ so } \frac{V_o^2}{V_{IP}^2} = 0.10875 = \frac{IP_{1dB}}{IIP_3} = -9.64\,dB$$

Then, $OP_{1dB} = G + IP_{1dB} - 1\,dB = G + IIP_3 - 1\,dB - 9.64\,dB$

$$= OIP_3 - 10.64\,dB \quad ✓ \quad \text{(see Egan, P. 103)}$$

10.17 $OP_{1dB} = 5\ dBm, \quad G = 15\ dB, \quad B = 1\ GHz, \quad T_e = 250K$

$$N_i = kT_eB = (1.38\times10^{-23})(250)(10^9) = 3.5\times10^{-12}\ W = -84.5\ dBm$$
$$N_o = GN_i = 15dB + (-84.5dBm) = -69.5\ dBm$$

From (10.55), $LDR = OP_{1dB} - N_o = 5-(-69.5) = \underline{74.5\ dB}$
(Note: using input levels will result in a value 1dB higher, but LDR is usually defined as output levels)

10.18 $F = 6dB = 4, \quad G = 30dB = 10^3, \quad B = 20MHz,$
$OIP_3 = 33dBm, \quad OP_{1dB} = 21\ dBm, \quad SNR = 8\ dB.$

$$N_i = -105\ dBm = 3.16\times10^{-14}\ W$$
$$T_e = (F-1)T_o = 3(290) = 870K \ ✓$$
$$N_o = G(N_i + kT_eB)$$
$$= 10^3[3.16\times10^{-14} + (1.38\times10^{-23})(870)(20\times10^6)]$$
$$= 2.72\times10^{-10}\ W = -65.7\ dBm \ ✓$$

From (10.55) $LDR = OP_{1dB} - N_o = 21-(-65.7) = \underline{86.7\ dB}$ ✓

From (10.60) $SFDR = \frac{2}{3}(OIP_3 - N_o) - SNR$
$$= \frac{2}{3}(33+65.7) - 8 = \underline{57.8\ dB}$$

Chapter 11

11.1

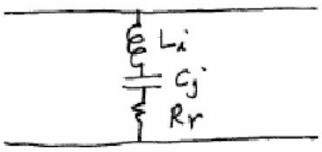
$C_j = 0.38\,pF$, $R_S = 4\,\Omega$, $I_S = 0.3\,\mu A$

$I_o\,(\mu A)$	$R_j\,(\Omega)$	$\beta_V\,(V/mW)$
0	8.3×10^4	8.7
20	1.23×10^3	6.4
50	4.97×10^2	4.6

11.2 INFINEON BA592 PIN : $R_f = 0.36\,\Omega$, $C_j = 1.4\,pF$

assume $R_r = 0.5\,\Omega$, $L_i = 0.5\,nH$

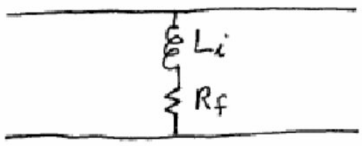
SWITCH ON (diode OFF)



$Z_d = 0.5 - j\,15.8\,\Omega$

$Y_d = 1.98 + j\,63.01\,mS$

SWITCH OFF (diode ON)



$Z_d = 0.36 + j\,12.6\,\Omega$

$Y_d = 2.28 - j\,79.5\,mS$

Use stub with $Y_s = -j\,0.06301 = -j\,3.15/Z_0 \Rightarrow \ell_s = 0.30\lambda$.

ON: $Z = 1/.00198 = 505.1\,\Omega$

$$IL(ON) = -20\log\left|\frac{2Z}{2Z+Z_0}\right| = 0.42\,dB \checkmark$$

OFF: $Z = 0.112 + j\,7.01$

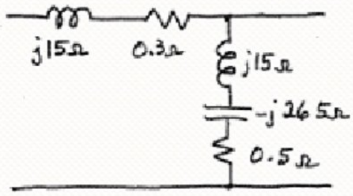
$$IL(OFF) = -20\log\left|\frac{2Z}{2Z+Z_0}\right| = 11.4\,dB \checkmark$$

11.3

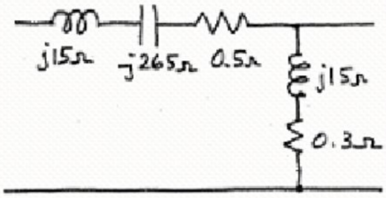
$\omega L_i = 15\,\Omega$

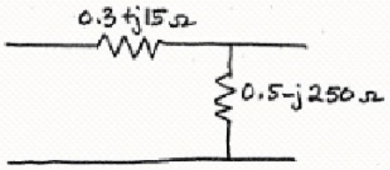
$1/\omega C_j = 265\,\Omega$

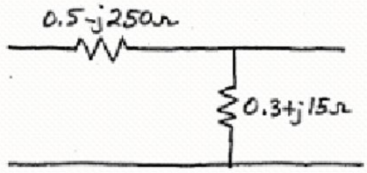
SWITCH ON:



SWITCH OFF:

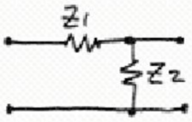






ABCD matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/Z_2 & 1 \end{bmatrix} = \begin{bmatrix} 1 + Z_1/Z_2 & Z_1 \\ 1/Z_2 & 1 \end{bmatrix}$$



Convert to S_{21}:

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{1 + Z_1/Z_2 + Z_1/Z_0 + Z_0/Z_2 + 1} = \frac{2}{2 + \frac{Z_1}{Z_2} + \frac{Z_1}{Z_0} + \frac{Z_0}{Z_2}}$$

ON STATE: $Z_1 = 0.3 + j15\,\Omega$, $Z_2 = 0.5 - j250\,\Omega$

$S_{21} = 0.995\angle -14°$

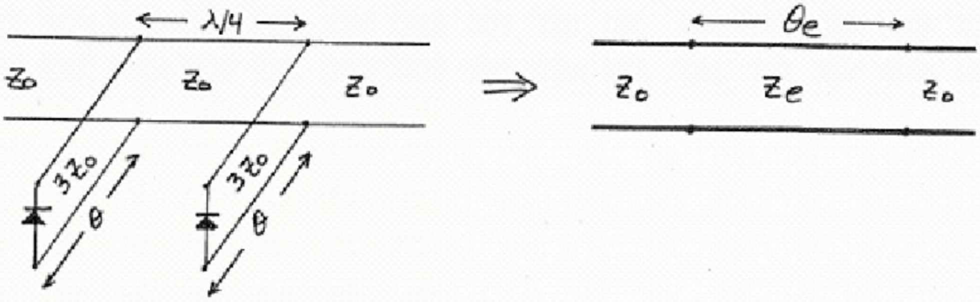
$IL = 0.044\,dB$

OFF STATE: $Z_1 = 0.5 - j250\,\Omega$, $Z_2 = 0.3 + j15\,\Omega$

$S_{21} = 0.118\angle 149°$

$IL = 18.6\,dB$

11.4



From (10.74), $\cos\theta_e = -b$

$Z_e = Z_0/\sqrt{1-b^2}$

where b is the normalized stub susceptance.

For diodes ON, $b = -\frac{1}{3}\cot\theta$

$\cos\theta_e = \frac{1}{3}\cot\theta$

For diodes OFF, $b = 1/3\tan\theta$

$\cos\theta_e = \frac{-1}{3}\tan\theta$

So $\Delta\phi = 45° = \cos^{-1}(\frac{1}{3}\cot\theta) - \cos^{-1}(\frac{-1}{3}\tan\theta)$ (ON-OFF)

Solving this equation numerically:

θ	$\Delta\phi$
110°	73°
120°	46°
130°	40°
122°	44.3°
121°	45.2°

So we choose $\theta = 121°$. (Using $\theta = 31°$ gives $\Delta\phi = -45°$)

Insertion loss for $\theta = 121°$:

Using (10.73), $b = BZ_0$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{-BZ_0 + j(1-B^2Z_0^2) - BZ_0} = \frac{2}{-2b + j(2-b^2)}$$

$$|S_{21}|^2 = \frac{4}{4b^2 + (2-b^2)^2}$$

DIODES ON: $b = \frac{-1}{3}\cot\theta = 0.20$

$|S_{21}|^2 = 0.9996$

$IL = 0.0017\,dB \sim 0\,dB$ ✓

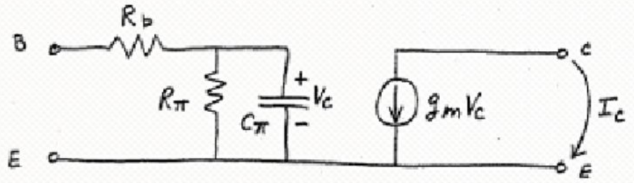
DIODES OFF: $b = \frac{1}{3}\tan\theta = -0.555$

$|S_{21}|^2 = 0.977$

$IL = 0.102\,dB$ ✓

(SuperCompact analysis gives $IL_{ON} = 0\,dB$, $\phi_{ON} = -101.5°$, $IL_{OFF} = 0.10\,dB$, $\phi_{OFF} = -56.7°$, thus $\Delta\phi = 44.8°$)

11.5 Unilateral bipolar transistor model:



From (10.77) the short-circuit current gain is,

$$G_i^{sc} = \left|\frac{I_c}{I_b}\right|_{V_{ce}=0} = \frac{g_m V_c}{I_b} = \frac{g_m I_b \left|\frac{R_\pi / j\omega C_\pi}{R_\pi + 1/j\omega C_\pi}\right|}{I_b}$$

$$= g_m \frac{R_\pi}{|1 + j\omega R_\pi C_\pi|} = \frac{g_m}{\left|\frac{1}{R_\pi} + j\omega C_\pi\right|} \simeq \frac{g_m}{\omega C_\pi} \quad \text{since } R_\pi \gg 1/\omega C_\pi$$

(e.g., if $R_\pi = 110\Omega$, $C_\pi = 18pF$, $f = 1GHz$, then $1/\omega C_\pi = 9\Omega$)

11.6 First find z_{ij}, then convert to S_{ij}:

$$Z_{11} = R_i - j/\omega C_{gs}$$
$$Z_{22} = \left(\frac{1}{R_{ds}} + j\omega C_{ds}\right)^{-1}$$
$$Z_{12} = 0$$
$$Z_{21} = -g_m Z_{22}/j\omega C_{gs} = j g_m Z_{22}/\omega C_{gs}$$

$$S_{11} = \frac{(Z_{11}-Z_0)(Z_{22}+Z_0)}{\Delta Z}, \quad \Delta Z = (Z_{11}+Z_0)(Z_{22}+Z_0)$$

$$= \frac{Z_{11}-Z_0}{Z_{11}+Z_0} \checkmark$$

$$S_{12} = 0$$

$$S_{21} = \frac{2Z_{12}Z_0}{\Delta Z} = \frac{2jZ_0 g_m Z_{22}/\omega C_{gs}}{(Z_{11}+Z_0)(Z_{22}+Z_0)}$$

$$S_{22} = \frac{(Z_{11}+Z_0)(Z_{22}-Z_0)}{\Delta Z} = \frac{Z_{22}-Z_0}{Z_{22}+Z_0} \checkmark$$

11.7 From 11.6,

$$Z_{11} = Z_0 \frac{1+S_{11}}{1-S_{11}} = R_i - j/\omega C_{gs}$$

$$R_i = Re\{Z_{11}\} \quad , \quad C_{gs} = -1/\omega\, Im\{Z_{11}\}$$

$$Z_{22} = Z_0 \frac{1+S_{22}}{1-S_{22}} = \left(\frac{1}{R_{ds}} + j\omega C_{ds}\right)^{-1}$$

$$R_{ds} = 1/Re\{Z_{22}\} \quad , \quad C_{ds} = Im\{1/Z_{22}\}/\omega$$

$$g_m = -j\omega C_{gs} Z_{21}/Z_{22}, \quad Z_{21} = \frac{S_{21}\Delta Z}{2Z_0} = \frac{S_{21}(Z_{11}+Z_0)(Z_{22}+Z_0)}{2Z_0}$$

From Table 11.7, MESFET @ 2GHz :

$$S_{11} = 0.9 \angle -55^\circ$$
$$S_{21} = 3.56 \angle 129^\circ$$
$$S_{12} = 0.08 \angle 54^\circ \sim 0$$
$$S_{22} = 0.65 \angle -37^\circ$$

Then,

$$R_i = 12.2\ \Omega \quad , \quad C_{gs} = 0.84\ pF$$
$$R_{ds} = 213\ \Omega \quad , \quad C_{ds} = 0.51\ pF$$
$$g_m = 54\ mS.$$

Chapter 12

12.1 The [S] matrix for a 3-dB matched attenuator is,

$$[S] = \begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}$$

For $Z_L = 50\Omega$: $\Gamma_L = \Gamma_{in} = 0$, $\Gamma_S = 0$, $\Gamma_{out} = 0$

Then from (11.12), (11.13), and (11.8) we have

$$G_A = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)} = |S_{21}|^2 = 0.5 \checkmark$$

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_S\Gamma_{in}|^2|1-S_{22}\Gamma_L|^2} = |S_{21}|^2 = 0.5 \checkmark$$

$$G = \frac{|S_{21}|^2\ (1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)|1-S_{22}\Gamma_L|^2} = |S_{21}|^2 = 0.5 \checkmark$$

For $Z_L = 25\Omega$: $\Gamma_L = -1/3$, $\Gamma_S = \Gamma_{out} = 0$, $\Gamma_{in} = S_{11} + \dfrac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} = -1/6$

$$G_A = |S_{21}|^2 = 0.5 \checkmark$$

$$G_T = |S_{21}|^2(1-|\Gamma_L|^2) = 0.444 \checkmark$$

$$G = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)} = 0.457 \checkmark$$

For $Z_S = 25\Omega$, $Z_L = 50\Omega$: $\Gamma_L = \Gamma_{in} = 0$, $\Gamma_S = -1/3$, $\Gamma_{out} = S_{22} + \dfrac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S} = -1/6$

$$G_A = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{(1-|\Gamma_{out}|^2)} = 0.457$$

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{1} = 0.444$$

$$G = |S_{21}|^2 = 0.5$$

12.2 Infineon BFP SiGe HBT at 1 GHz

(a) $\Gamma_S = \Gamma_L = 0$, so $\Gamma_{in} = S_{11}$, $\Gamma_{out} = S_{22}$

(12.8) $$G = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)|1-S_{22}\Gamma_L|^2} = \frac{|S_{21}|^2}{(1-|S_{11}|^2)} = \frac{(3.92)^2}{1-(.91)^2} = 89.4 \checkmark$$

(12.12) $$G_A = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{(1-|\Gamma_{out}|^2)|1-S_{11}\Gamma_S|^2} = \frac{|S_{21}|^2}{1-|S_{22}|^2} = \frac{(3.92)^2}{1-(.93)^2} = 113.7 \checkmark$$

(12.13) $$G_T = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_S\Gamma_{in}|^2|1-S_{22}\Gamma_L|^2} = |S_{21}|^2 = (3.92)^2 = 15.4 \checkmark$$

(b) for G, let $\Gamma_L = S_{22}^*$, $\Gamma_S = 0$ (G does not depend on Γ_S)
Then, for $S_{12}=0$, $\Gamma_{in} = S_{11}$

$$G = \frac{|S_{21}|^2}{(1-|S_{11}|^2)(1-|S_{22}|^2)} = 661.7 \checkmark$$

for G_A, let $\Gamma_S = S_{11}^*$, $\Gamma_L = 0$ (G_A does not depend on Γ_L)
Then, with $S_{12}=0$, $\Gamma_{out} = S_{22}$

$$G_A = \frac{|S_{21}|^2}{(1-|S_{22}|^2)(1-|S_{11}|^2)} = 661.7 \checkmark$$

For G_T, let $\Gamma_S = S_{11}^*$, $\Gamma_L = S_{22}^*$.
Then, with $S_{12}=0$, $\Gamma_{in} = S_{11}$, $\Gamma_{out} = S_{22}$.

$$G_T = \frac{|S_{21}|^2}{(1-|S_{11}|^2)(1-|S_{22}|^2)} = 661.7 \checkmark$$

12.3 $S_{11} = 0.61\angle{-170^\circ}$, $S_{12} = 0.06\angle{70^\circ}$, $S_{21} = 2.3\angle{80^\circ}$, $S_{22} = 0.72\angle{-25^\circ}$

$Z_S = 25\,\Omega$, $Z_L = 100\,\Omega$, $V_S = 2V$.

a)

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.641\angle{-174^\circ} \checkmark$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} = 0.777\angle{-25^\circ} \checkmark$$

$G = 12.8 = \underline{11.1\,dB}$ ✓

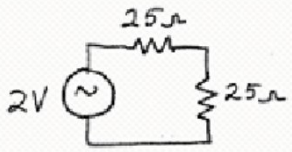
$G_A = 18.4 = \underline{12.6\,dB}$ ✓

$G_T = 10.8 = \underline{10.3\,dB}$ ✓

$$G_{TU} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-S_{11}\Gamma_S|^2\,|1-S_{22}\Gamma_L|^2} = 10.4 = \underline{10.2\,dB} \checkmark$$

(verified with Serenade)

b)



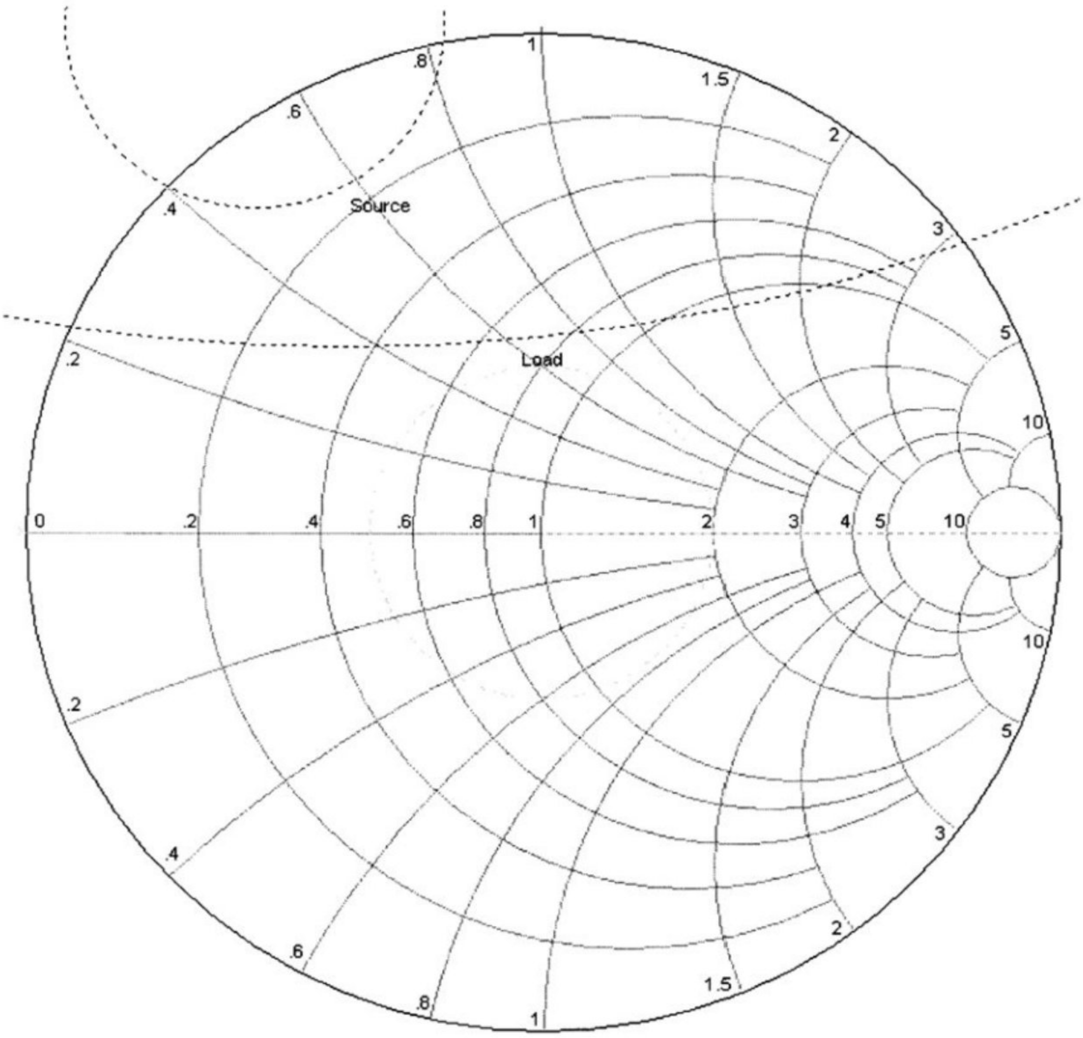
$P_{AVS} = \frac{1}{2}\left(\frac{2}{2}\right)^2\frac{1}{25} = \underline{0.02\,W}$

$P_L = G_T\,P_{AVS} = (10.8)(.02) = \underline{0.216\,W}$

12.4 $S_{11} = 0.880\angle{-115}$, $S_{12} = 0.029\angle{31}$, $S_{21} = 9.4\angle{110}$, $S_{22} = 0.328\angle{-67}$

$C_L = 4.0\angle{96^\circ}$, $R_L = 3.60$, $C_S = 1.16\angle{119^\circ}$, $R_S = 0.367$

$K = 0.275$



12.5

GaN HEMT

S-parameters from Table 11.8

f(GHz)	K	Δ	STABILITY
0.5	0.604	0.686	pot. unstable
1.0	0.452	0.725	pot. unstable
2.0	1.030	0.705	uncond. stable
4.0	3.262	0.770	uncond. stable

12.6

Using (12.30) to compute μ:

DEVICE	S_{11}	S_{12}	S_{21}	S_{22}	μ	
A	$0.34\angle{-170°}$	$0.06\angle{70°}$	$4.3\angle{80°}$	$0.45\angle{-25°}$	1.193	UNC. STABLE
B	$0.75\angle{-60°}$	$0.2\angle{70°}$	$5.0\angle{90°}$	$0.5\angle{60°}$	0.283	POT. UNSTABLE
C	$0.65\angle{-140°}$	$0.04\angle{60°}$	$2.4\angle{50°}$	$0.7\angle{-65°}$	1.057	UNC. STABLE

Device A has the best stability.

12.7 From (11.30) the μ-parameter test is,

$$\mu = \frac{1-|S_{11}|^2}{|S_{22}-S_{11}^*\Delta| + |S_{21}S_{12}|} > 1$$

If $S_{12}=0$ (unilateral) then we have,

$$\Delta = S_{11}S_{22}$$

So,

$$\mu = \frac{1-|S_{11}|^2}{|S_{22}-|S_{11}|^2 S_{22}|} = \frac{1-|S_{11}|^2}{|S_{22}||1-|S_{11}|^2|} > 1$$

Since the denominator is positive and μ is positive, the numerator must also be positive, thus $|S_{11}|<1$. Then the above reduces to,

$$\mu = \frac{1}{|S_{22}|} > 1 \ ,$$

so,

$$|S_{22}| < 1 .$$

12.8 From the definitions of (11.41),

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \quad , \quad C_1 = S_{11} - \Delta S_{22}^* .$$

Similar to the expansion used after (11.35), it can be verified by direct expansion that,

$$|C_1|^2 = |S_{11}-\Delta S_{22}^*|^2 = |S_{12}S_{21}|^2 + (1-|S_{22}|^2)(|S_{11}|^2-|\Delta|^2)$$

So the condition that $B_1^2 - 4|C_1|^2 > 0$ implies that,

$$(1+|S_{11}|^2-|S_{22}|^2-|\Delta|^2)^2 > 4|S_{12}S_{21}|^2 + 4(1-|S_{22}|^2)(|S_{11}|^2-|\Delta|^2)$$

$$1+2|S_{11}|^2-2|S_{22}|^2-2|\Delta|^2+|S_{11}|^4-2|S_{11}|^2|S_{22}|^2-2|S_{11}|^2|\Delta|^2+|S_{22}|^4$$
$$+2|\Delta|^2|S_{22}|^2+|\Delta|^4 > 4|S_{12}S_{21}|^2+4(|S_{11}|^2-|\Delta|^2-|S_{11}|^2|S_{22}|^2+|\Delta|^2|S_{22}|^2)$$

$$1-2|S_{11}|^2-2|S_{22}|^2+2|\Delta|^2+|S_{11}|^4+2|S_{11}|^2|S_{22}|^2-2|S_{11}|^2|\Delta|^2+|S_{22}|^4$$
$$-2|\Delta|^2|S_{22}|^2+|\Delta|^4 > 4|S_{12}S_{21}|^2$$

$$(1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2)^2 > 4|S_{12}S_{21}|^2$$

or,

$$K^2 = \frac{(1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2)^2}{4|S_{12}S_{21}|^2} > 1 \quad ✓$$

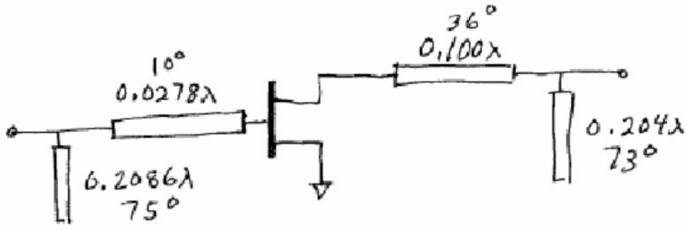
12.9 Data from Table 11.7 GaAs MESFET

conjugate matching for maximum gain (at 8 GHz):

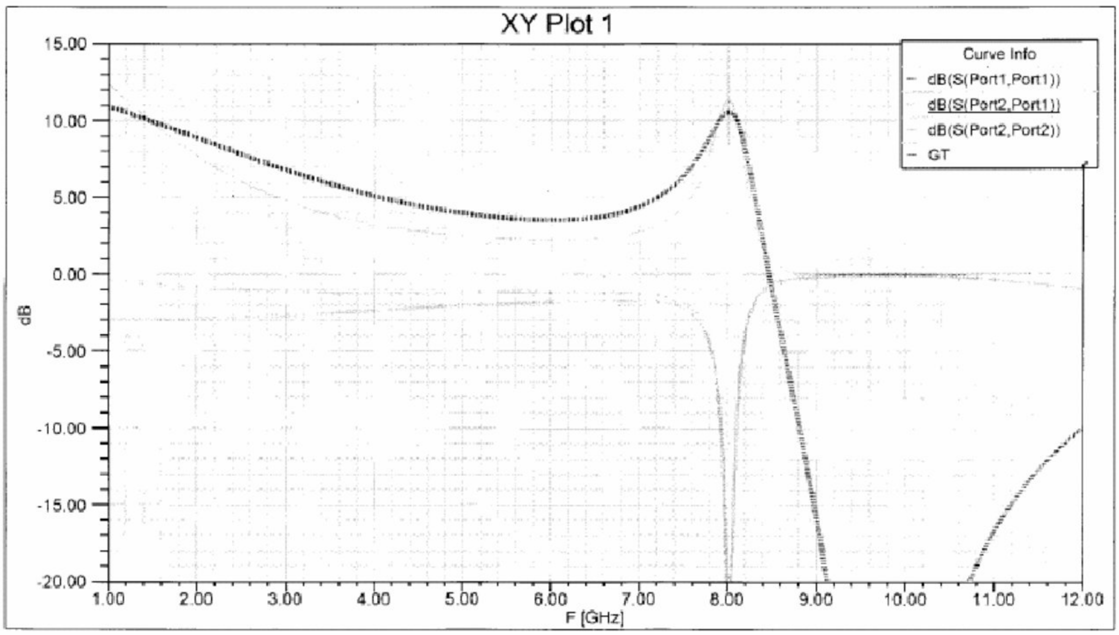
$$\Gamma_S = \Gamma_{in}^* = 0.883\angle{-172^\circ}$$

$$\Gamma_L = \Gamma_{out}^* = 0.859\angle{139^\circ}$$

$G_S = 4.53,\ G_o = 4.00,\ G_L = 0.623 \Rightarrow G_T = 11.29 = 10.53\,dB$



CAD modeling gives gain = 10.5 dB; no stability problems from 1-12 GHz. Gain is plotted below.



12.10

Assuming a lossless reciprocal network,

$$S_{12} = S_{21}, \quad |S_{11}|^2 + |S_{12}|^2 = |S_{22}|^2 + |S_{12}|^2 = 1$$

$$S_{11} S_{12}^* + S_{12} S_{22}^* = 0$$

For the second circuit, $\Gamma = S_{22} + \dfrac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S} = S_{22}$ since $\Gamma_S = 0$.

For the first circuit, at port 1, $\Gamma_L = \Gamma^* = S_{22}^*$, so

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} = S_{11} + \frac{S_{12}^2 S_{22}^*}{1 - |S_{22}|^2}$$

$$= S_{11} + \frac{S_{12}^2 S_{22}^*}{|S_{12}|^2} = \frac{S_{12}(S_{11} S_{12}^* + S_{12} S_{22}^*)}{|S_{12}|^2} = 0 \quad ✓$$

12.11 $S_{11} = 0.61\angle{-170°}$, $S_{21} = 2.24\angle{32°}$, $S_{12} = 0$, $S_{22} = 0.72\angle{-83°}$

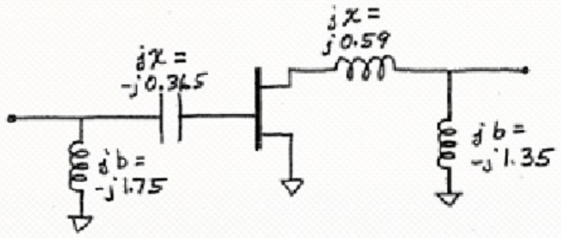
The transistor is unconditionally stable since $K = \infty$ and $|\Delta| < 1$. Since the transistor is unilateral,

$\Gamma_S = S_{11}^* = 0.61\angle{170°}$ ✓ , $\Gamma_L = S_{22}^* = 0.72\angle{83°}$ ✓.

and the maximum gain is, from (11.45),

$$G_{TU_{MAX}} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2} = 16.6 = 12.2\,dB$$

Matching was done with a Smith chart. The final circuit is:



The matching element values are, at 6 GHz,

$$C = \frac{-1}{\omega Z_0 x_c} = 1.45\,pF \checkmark \qquad L = \frac{Z_0 x_L}{\omega} = 0.78\,nH \checkmark$$

$$L = \frac{-Z_0}{\omega b_L} = 0.76\,nH \checkmark \qquad L = \frac{-Z_0}{\omega b_L} = 0.98\,nH \checkmark$$

SuperCompact analysis gives $|S_{11}| = 0.035$, $|S_{22}| = 0.008$, and $G = 12.2\,dB$ ✓

12.12 $S_{11} = 0.61\angle{-170°}$, $S_{21} = 2.24\angle{32°}$; $S_{12} = 0$, $S_{22} = 0.72\angle{-83°}$

$G = 10dB$, $G_S = 1dB$, $G_L = 2dB$

Since $K = \infty$ and $|\Delta| < 1$, the transistor is unconditionally stable. From (11.45), we have

$$G_{S_{MAX}} = \frac{1}{1-|S_{11}|^2} = 1.59 \checkmark \quad , \quad G_{L_{MAX}} = \frac{1}{1-|S_{22}|^2} = 2.08 \checkmark$$

So for $G_S = 1dB = 1.26$, and $G_L = 2dB = 1.58$, we have from (11.46),

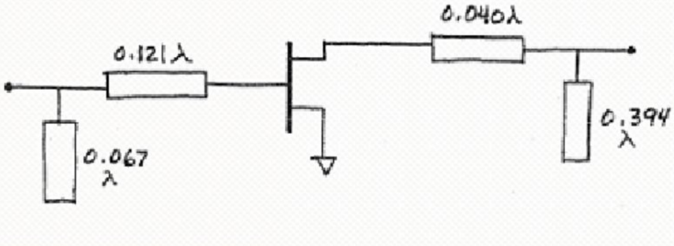
$$g_S = \frac{G_S}{G_{S_{MAX}}} = 0.792 \quad , \quad g_L = \frac{G_L}{G_{L_{MAX}}} = 0.760$$

Then the centers and radii of the constant gain circles can be found from (11.49)-(11.50) :

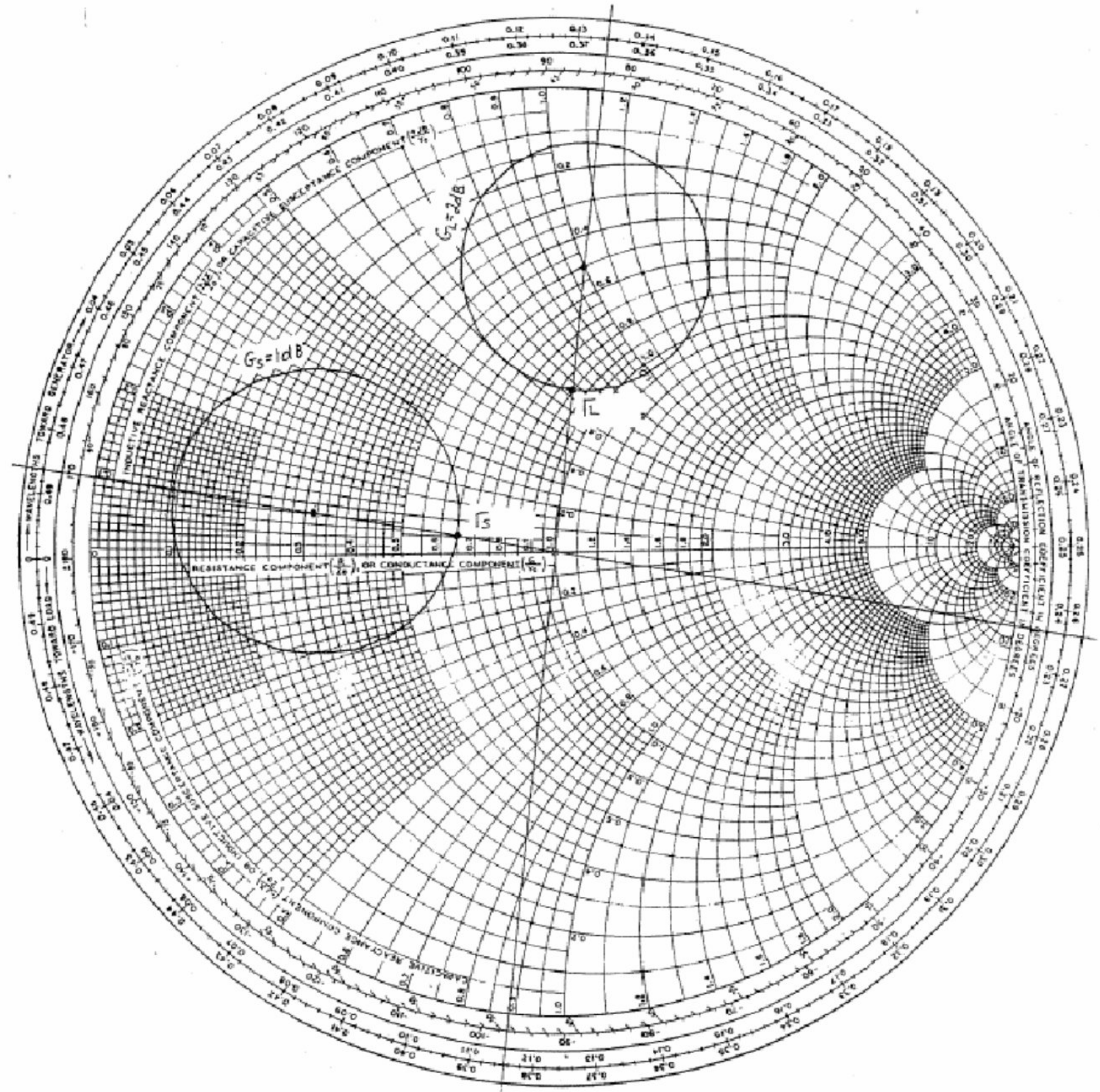
$C_S = 0.524\angle{170°}$ ✓ $\qquad C_L = 0.625\angle{83°}$ ✓

$R_S = 0.310$ ✓ $\qquad R_L = 0.269$ ✓

Since $G_0 = 10 \log |S_{21}|^2 = 7.0 dB$, using the $G_S = 1dB$ and the $G_L = 2dB$ gain circles will give an overall gain of 10dB. We plot these circles on the Smith chart, and choose $\Gamma_S = 0.215\angle{170°}$ ✓ and $\Gamma_L = 0.361\angle{83°}$ ✓ to minimize the magnitude of these values. after matching, we have the following amplifier circuit:



SuperCompact analysis gives $|S_{11}| = 0.45$, $|S_{22}| = 0.48$, $G = 10.05dB$ ✓ (reflections at input and output serve to reduce the gain to 10dB). Smith chart shown on following page.



$G_S = 1dB$
$G_L = 2dB$
Γ_L
Γ_S

12.13

$S_{11} = .88\angle{-115^\circ}$, $S_{12} = .029\angle{31^\circ}$, $9.40\angle{110^\circ}$, $S_{22} = .328\angle{-67^\circ}$

From (12.46) the unilateral figure of merit is

$$U = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)} = \frac{(.029)(9.4)(.88)(.328)}{[1-(.88)^2][1-(.328)^2]} = 0.391$$

From (12.45) this bounds the error in G_T/G_{TU} by

$$.517 = \frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2} = 2.70$$

in dB, $\quad -2.9\,dB < G_T(dB) - G_{TU}(dB) < 4.3\,dB$

An assumption that the device is unilateral is not justified in this case.

12.14 From (11.48) and (11.47), when $G_S = 1$ we have,

$$g_s = \frac{1}{G_{S_{MAX}}} = 1 - |S_{11}|^2 \quad , \quad 1 - g_s = |S_{11}|^2$$

so (11.51) reduces to,

$$C_S = \frac{(1-|S_{11}|^2)\, S_{11}^*}{1-|S_{11}|^4} = \frac{S_{11}^*}{1+|S_{11}|^2}$$

$$R_S = \frac{|S_{11}|(1-|S_{11}|^2)}{1-|S_{11}|^4} = \frac{|S_{11}|}{1+|S_{11}|^2}$$

So the equation of the constant gain circle becomes,

$$\left| \Gamma_S - \frac{S_{11}^*}{1+|S_{11}|^2} \right| = \frac{|S_{11}|}{1+|S_{11}|^2}$$

One solution to this equation occurs for $\Gamma_S = 0$, so the circle must pass through the center of the Smith chart.

12.15 $S_{11} = 0.7\angle{-110°}$, $S_{12} = 0.02\angle{60°}$, $S_{21} = 3.5\angle{60°}$, $S_{22} = 0.8\angle{-70°}$

$F_{MIN} = 2.5dB$, $\Gamma_{OPT} = 0.7\angle{120°}$, $R_N = 15\Omega$

First check stability: $K = 1.07$, $|\Delta| = 0.53$

Since $K>1$ and $|\Delta|<1$ the device is unconditionally stable.
Minimum noise figure occurs for $\Gamma_S = \Gamma_{OPT} = 0.7\angle{120°}$. Then we maximize gain by conjugate matching the output.
From (11.41b),

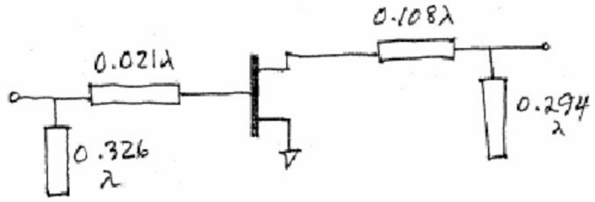
$$\Gamma_L = \left(S_{22} + \frac{S_{12}S_{21}\Gamma_S^*}{1-S_{11}\Gamma_S}\right)^* = 0.873\angle{74°} \checkmark$$

So the noise figure will be $F = F_{min} = 2.5dB$, and the gain will be,

$$G_T = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|S_{22}|^2}{|1-S_{22}\Gamma_L|^2}$$

$$= (1.85)(12.25)(3.81) = 86.3 = 19.4 dB$$

Impedance matching is done with a Smith chart; the final amplifier circuit is shown below.



Serenade analysis of this amplifier gives

$|S_{11}| = 0.33$, $|S_{22}| = 0.13$,

$G = 19.7dB$, $F = 2.5dB$ ✓.

The solution is simpler if S_{12} is set to zero, resulting in $G = 18 dB$.

12.16 $S_{11} = 0.6\angle{-60°}$, $S_{21} = 2.\angle{181°}$, $S_{12} = 0$, $S_{22} = 0.7\angle{-60°}$

$F_{MIN} = 2.0dB$, $\Gamma_{OPT} = 0.62\angle{100°}$, $R_N = 20\Omega$

Since $S_{12} = 0$ and $|S_{11}||S_{22}| < 1$, the device is unconditionally stable. The overall gain is, $G_{TU} = G_S G_0 G_L$, where $G_0 = |S_{21}|^2 = 4 = 6dB$. ✓ So $G_S + G_L = 0\,dB$.

Plot noise figure circles for $F = 2.0, 2.05, 2.2,$ and $3.0\,dB$:

F(dB)	N	C_F	R_F
2.05	0.0134	$0.61\angle{100°}$	0.09
2.20	0.055	$0.59\angle{100°}$	0.18
3.00	0.30	$0.48\angle{100°}$	0.40
2.00	0.	$0.62\angle{100°}$	0

Now plot constant gain circles for $G_S = G_L = 0dB$:

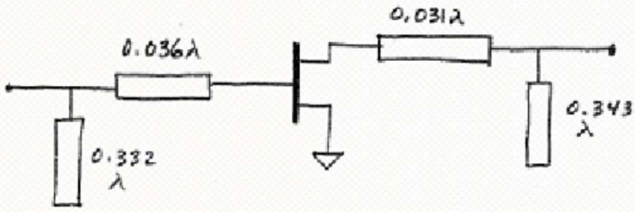
$G_{S_{MAX}} = 1.56$ ✓ $\quad$ $G_{L_{MAX}} = 1.96$ ✓

$g_S = 0.641$ $\quad$ $g_L = 0.510$

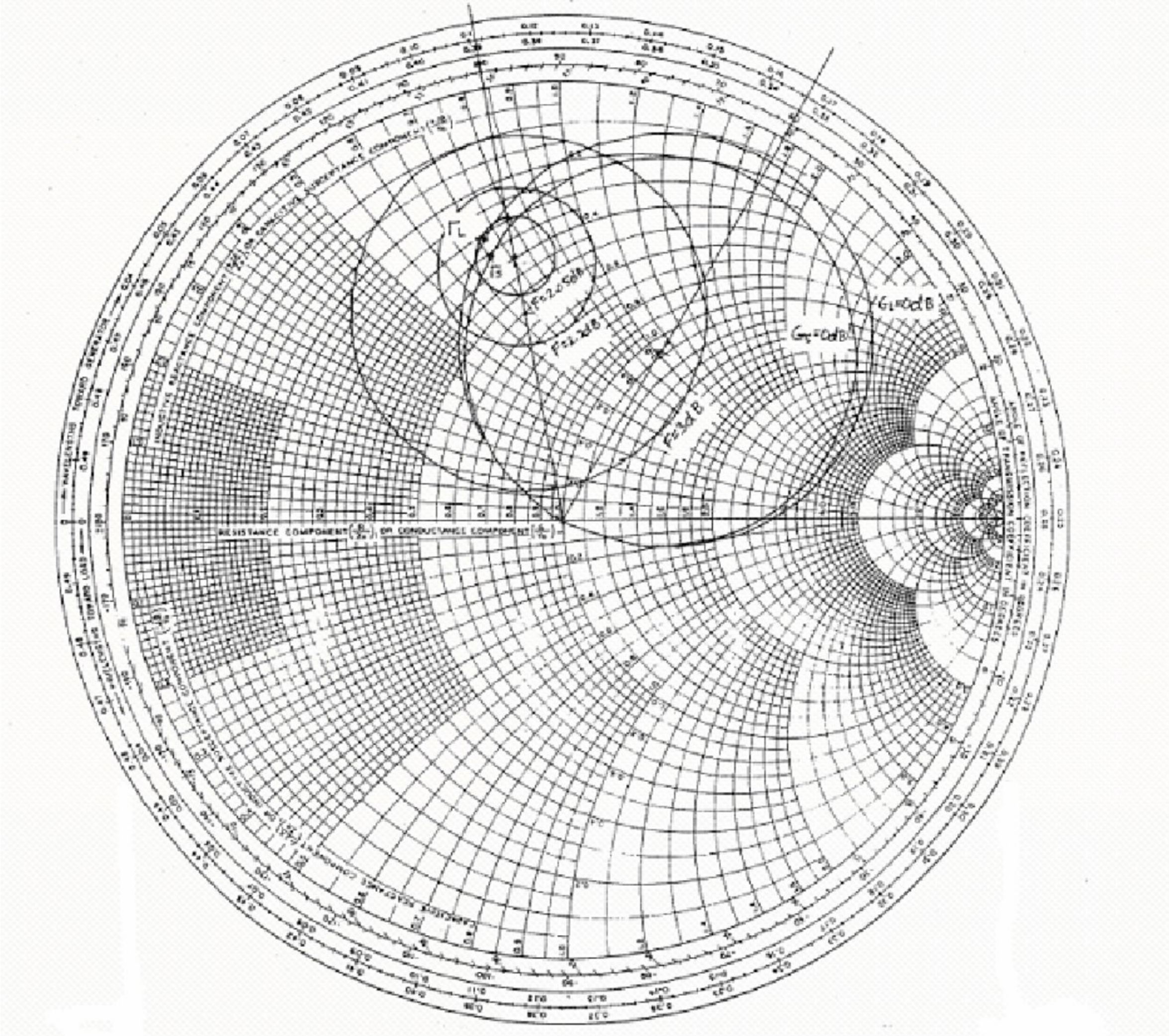
$C_S = 0.44\angle{60°}$ $\quad$ $C_L = 0.47\angle{60°}$

$R_S = 0.44$ $\quad$ $R_L = 0.47$

These two circles are close together near the $F = 2dB$ point. We choose $\Gamma_L = 0.66\angle{105°}$, $\Gamma_S = 0.62\angle{105°}$. Then we should obtain $F \simeq 2.04dB$. The final AC amplifier circuit is:



Super Compact analysis gives $|S_{11}| = 0.62$, $|S_{22}| = 0.67$, $G = 6.1dB$, and $F = 2.04dB$ ✓ The gain and noise circles are shown below.



Γ_L
Γ_S
F=2.25dB
F=2dB
F=3dB
G_L=0dB
G_S=0dB
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)

12.17 S-parameters and noise parameters of Problem 11.14

Plot the F = 2.5dB constant noise figure circle:

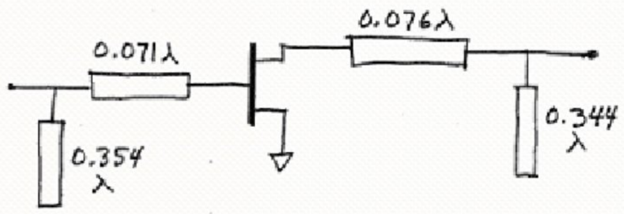
$N = 0.141$, $C_F = 0.543\angle 100°$, $R_F = 0.286$

Now, $G_{S_{MAX}} = 1.56 = 1.93\,dB$, $G_{L_{MAX}} = 1.96 = 2.92\,dB$

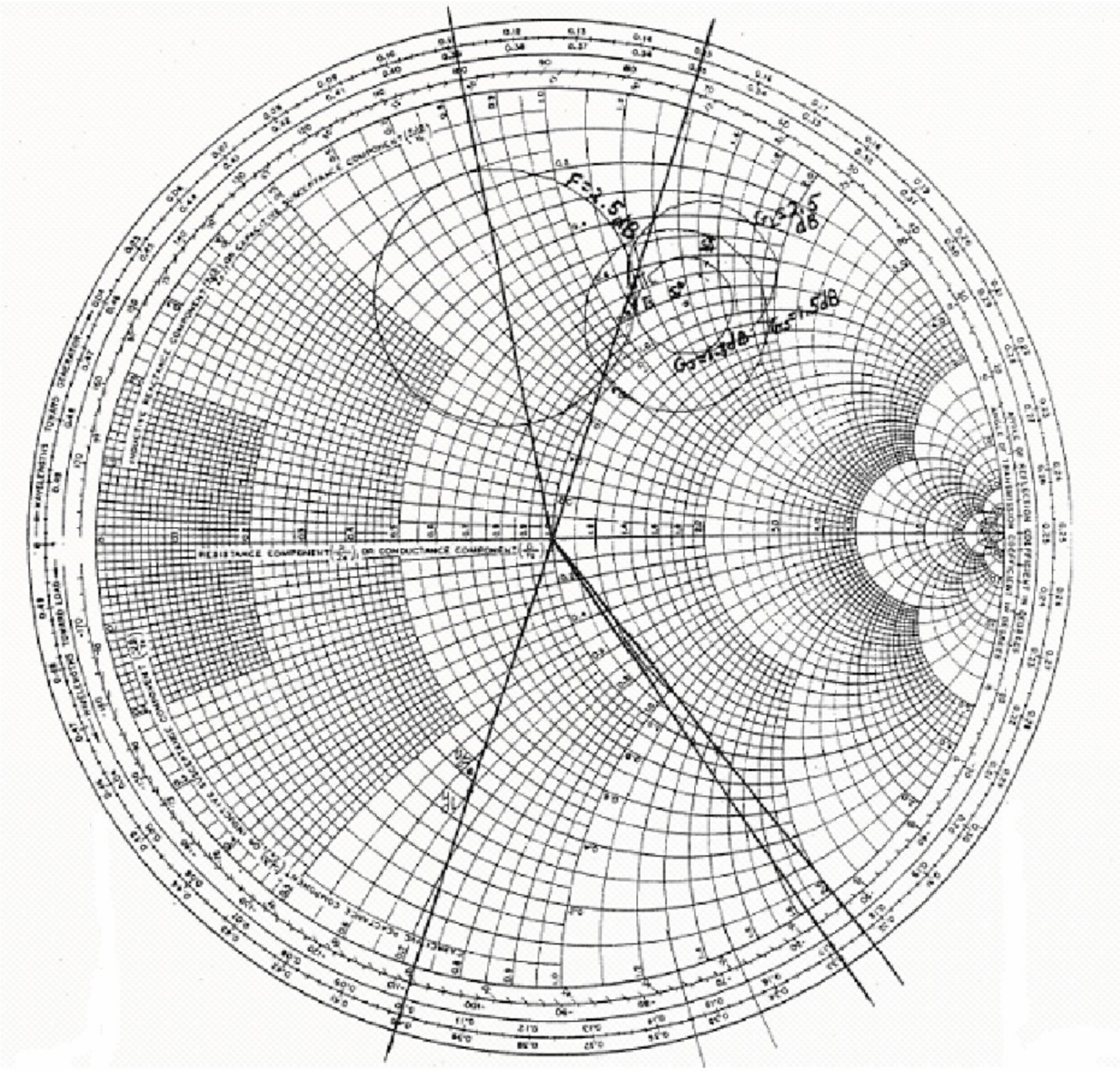
But these points (S_{11}^*, S_{22}^*) do not lie on the F = 2.5dB circle. We can plot some gain circles to just give intersections with the F = 2.5dB noise circle:

$G_S = 1.5\,dB$	$g_S = 0.905$	$C_S = 0.56\angle 60°$	$R_S = 0.204$
$G_L = 2.5\,dB$	$g_L = 0.907$	$C_L = 0.67\angle 60°$	$R_L = 0.163$
$G_S = 1.7\,dB$	$g_S = 0.948$	$C_S = 0.58\angle 60°$	$R_S = 0.149$
$G_S = 1.8\,dB$	$g_S = 0.970$	$C_S = 0.59\angle 60°$	$R_S = 0.112$

The $G_S = 1.8\,dB$ and $G_L = 2.5\,dB$ circles are close to optimum (the F = 2.5dB noise circle). Thus we have $\Gamma_S = 0.545\angle 70°$, $\Gamma_L = 0.59\angle 72°$, which will yield a gain of $G_T = 1.8 + 2.5 + 6 = 10.3\,dB$. The final AC amplifier circuit is shown below:



SuperCompact analysis of this circuit gives $|S_{11}| = 0.20$, $|S_{22}| = 0.28$, $G = 10.3\,dB$ ✓, and $F = 2.4\,dB$ ✓. The noise and gain circles are shown on the following page.


F=2.5dB
RESISTANCE COMPONENT

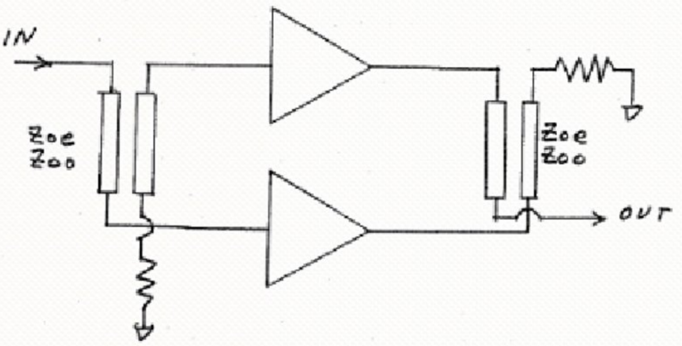
12.18

FOR THE COUPLED LINE COUPLERS:

$$C = 10^{-3/20} = 0.708$$

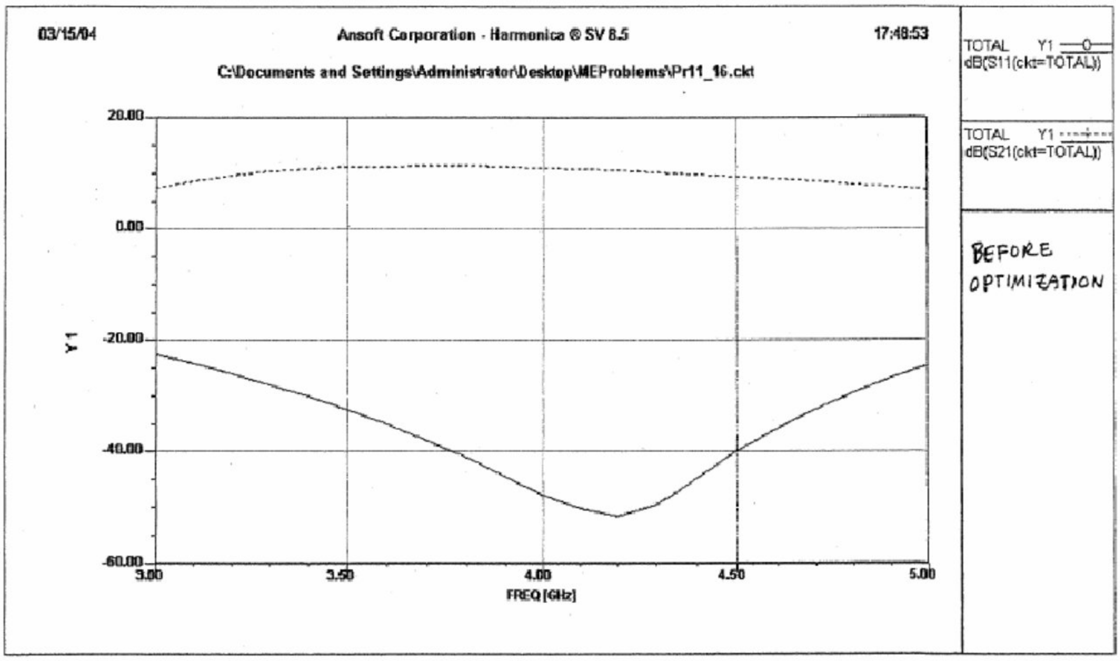
$$Z_{oe} = 50\sqrt{\frac{1+c}{1-c}} = 121\,\Omega$$

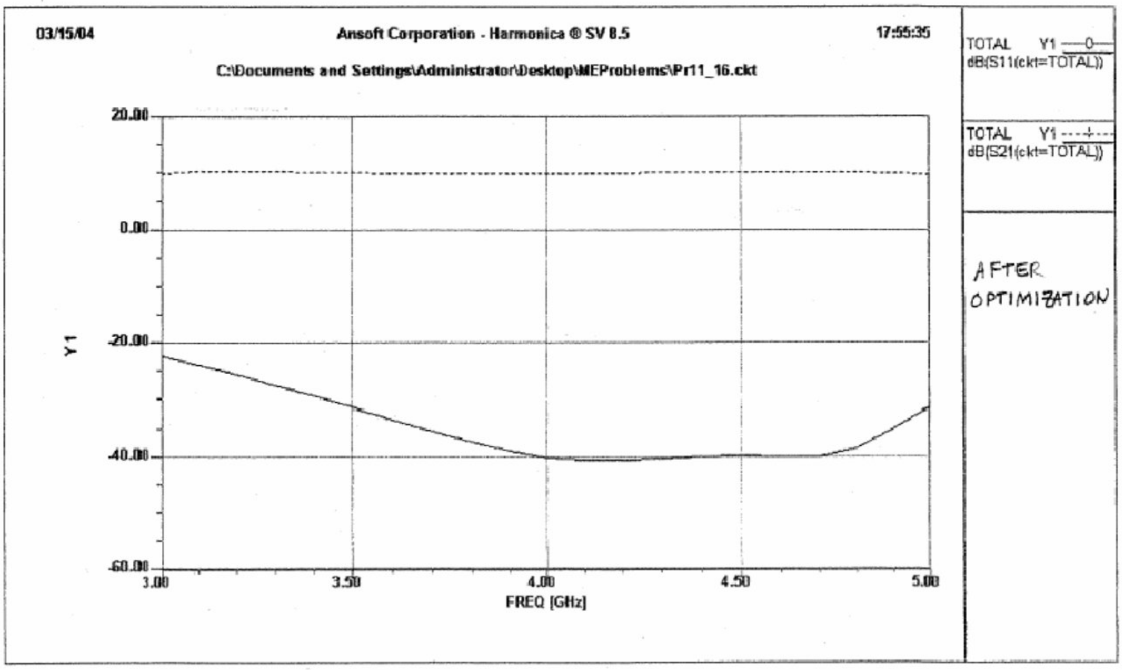
$$Z_{oo} = 50\sqrt{\frac{1-c}{1+c}} = 21\,\Omega$$



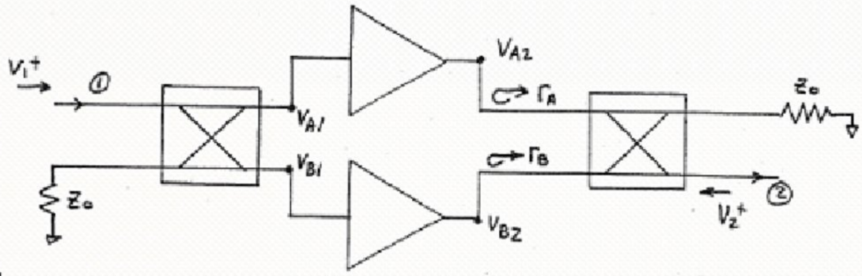
The amplifier circuit of Example 11.4 was used for both amplifiers here. As in Example 11.6, the amplifier matching networks were optimized using SuperCompact to give a flat 10dB gain response, with good input matching. Results of the optimization are given below, including the line and stub lengths before and after optimization, the SuperCompact data file, and the calculated gain and input return loss of the balanced amplifier before and after optimization. Results seem to be a bit better than those of Example 11.6.

PARAMETER	BEFORE OPT.	AFTER OPT.
INPUT SECTION STUB LENGTH	0.100λ	0.125λ
INPUT SECTION LINE LENGTH	0.179λ	0.119λ
OUTPUT SECTION LINE LENGTH	0.045λ	0.089λ
OUTPUT SECTION LINE LENGTH	0.432λ	0.458λ


03/15/04
Ansoft Corporation - Harmonica ® SV 8.5
17:48:53
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr11_16.ckt
TOTAL Y1
dB(S11(ckt=TOTAL))
TOTAL Y1
dB(S21(ckt=TOTAL))
BEFORE OPTIMIZATION
Y1
20.00
0.00
-20.00
-40.00
-60.00
3.00
3.50
4.00
4.50
5.00
FREQ [GHz]


03/15/04
Ansoft Corporation - Harmonica ® SV 8.5
17:55:35
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr11_16.ckt
TOTAL Y1
dB(S11(ckt=TOTAL))
TOTAL Y1
dB(S21(ckt=TOTAL))
AFTER OPTIMIZATION
Y1
20.00
0.00
-20.00
-40.00
-60.00
3.00
3.50
4.00
4.50
5.00
FREQ [GHz]

12.19



The analysis for S_{22} is identical to that for S_{11} in eqs (11.61)–(11.65), but with input V_2^+ at port 2.

Thus, if the input at port 2 is V_2^+, then the voltages incident at the amplifiers are,

$$V_{A2}^- = \frac{1}{\sqrt{2}} V_2^+$$

$$V_{B2}^- = \frac{-j}{\sqrt{2}} V_2^+$$

Then the reflected output voltage at port 2 is,

$$V_2^- = \frac{1}{\sqrt{2}} V_{A2}^+ + \frac{-j}{\sqrt{2}} V_{B2}^+ = \frac{1}{\sqrt{2}} \Gamma_A V_{A2}^- + \frac{-j}{\sqrt{2}} \Gamma_B V_{B2}^-$$

$$= \frac{1}{2} V_2^+ (\Gamma_A - \Gamma_B)$$

Thus, $$S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+=0} = \frac{1}{2}(\Gamma_A - \Gamma_B) \checkmark$$

12.20 From (11.77), $G = \dfrac{g_m^2 Z_d Z_g}{4} \dfrac{(e^{-N\alpha_g l_g} - e^{-N\alpha_d l_d})^2}{(e^{-\alpha_g l_g} - e^{-\alpha_d l_d})^2}$

differentiating with respect to N and setting to zero gives,

$$\alpha_g l_g e^{-N\alpha_g l_g} - \alpha_d l_d e^{-N\alpha_d l_d} = 0$$

$$\ln \alpha_g l_g - N\alpha_g l_g = \ln \alpha_d l_d - N\alpha_d l_d$$

$$\ln \frac{\alpha_g l_g}{\alpha_d l_d} = N(\alpha_g l_g - \alpha_d l_d)$$

$$N = \frac{\ln(\alpha_g l_g / \alpha_d l_d)}{\alpha_g l_g - \alpha_d l_d} \checkmark$$

12.21 $R_i = 5\Omega$, $R_{ds} = 200\Omega$, $C_{gs} = 0.3\,pF$, $g_m = 40\,mS$

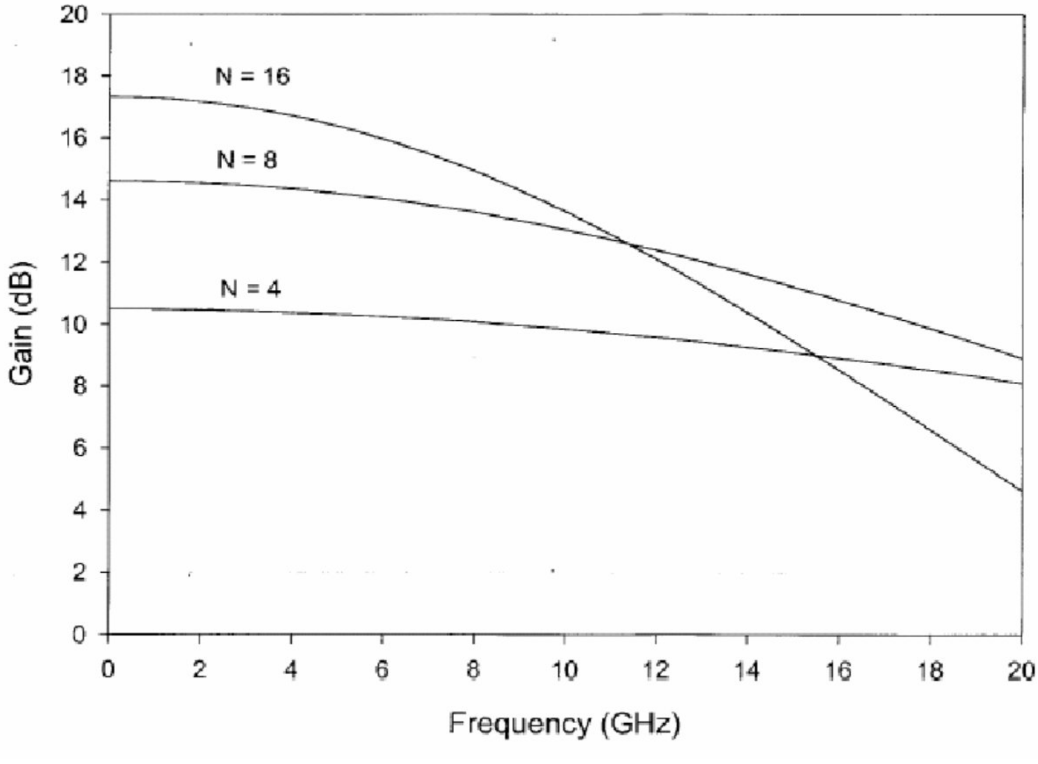
$f = 0-20\,GHz$, opt G at $16\,GHz$. $N = 4, 8, 16$.

at $f = 16\,GHz$, $\alpha_g l_g = 0.1137$

$\alpha_d l_d = 0.125$

$$N_{opt} = \frac{\ln \alpha_g l_g / \alpha_d l_d}{(\alpha_g l_g - \alpha_d l_d)} = 8.4$$

gain vs. frequency is plotted below for $N = 4, 8, 16$.
graph shows $N_{opt} \sim 8$ at $16\,GHz$ ✓



12.22 $S_{11} = 0.76\angle 169°$, $S_{12} = 3.08\angle 69°$, $S_{21} = 0.079\angle 53°$, $S_{22} = 0.36\angle -169°$

$\Gamma_{SP} = 0.797\angle -187°$, $\Gamma_{LP} = 0.491\angle 185°$, $G_P = 10\,dB$. $f = 1\,GHz$.

Check stability using small-signal S-parameters:

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.452\angle -27°$$

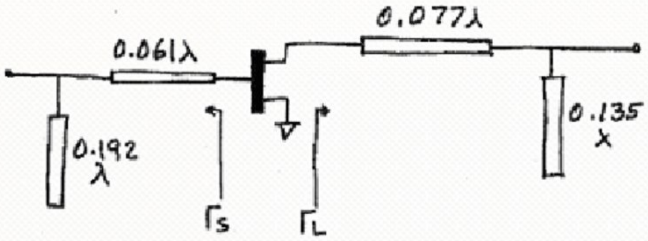
$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 1.02$$

Since $|\Delta| < 1$ and $K > 1$, the device is unconditionally stable at this frequency.

Using the given large-signal source and load reflection coefficients gives,

$$\Gamma_S = 0.797\angle -187°, \quad \Gamma_{LP} = 0.491\angle 185°$$

Then the matching circuits can be designed, resulting in the following AC circuit:

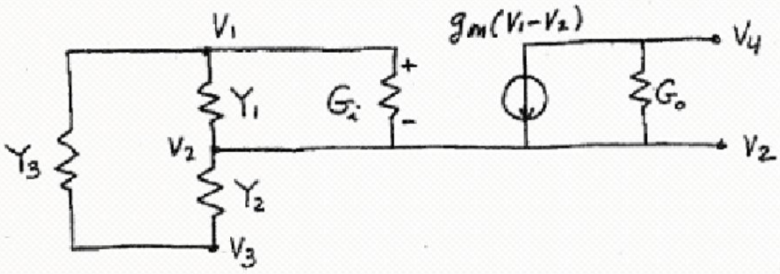


Since the gain with this Γ_S, Γ_P is 10 dB, the input power for a 1 W output is,

$$P_{in} = P_{out} - G_P = 30\,dBm - 10 = 20\,dBm = 100\,mW.$$

Chapter 13

13.1



Writing KCL for nodes V_1, V_2, V_3, V_4:

V_1: $(V_3-V_1)Y_3 + (V_2-V_1)Y_1 + (V_2-V_1)G_i = 0$

V_2: $(V_1-V_2)Y_1 + (V_3-V_2)Y_2 + (V_1-V_2)G_i + g_m(V_1-V_2) + (V_4-V_2)G_o = 0$

V_3: $(V_1-V_3)Y_3 + (V_2-V_3)Y_2 = 0$

V_4: $(V_2-V_4)G_o - g_m(V_1-V_2) = 0$

Rearranging:

$$V_1(Y_1+Y_3+G_i) + V_2(-Y_1-G_i) + V_3(-Y_3) + V_4(0) = 0$$
$$V_1(-Y_1-G_i-g_m) + V_2(Y_1+Y_2+G_i+G_o+g_m) + V_3(-Y_2) + V_4(-G_o) = 0$$
$$V_1(-Y_3) + V_2(-Y_2) + V_3(Y_2+Y_3) + V_4(0) = 0$$
$$V_1(g_m) + V_2(-G_o-g_m) + V_3(0) + V_4 G_o = 0$$

which agrees with the matrix of (12.3).

13.2 From (13.4),

$$\det\begin{bmatrix}(Y_1+Y_3+G_i) & -Y_3 \\ (g_m-Y_3) & (Y_2+Y_3)\end{bmatrix}=0 \quad \text{for oscillation.}$$

For a Colpitts oscillator, let $Y_1=j\omega C_1$, $Y_2=j\omega C_2$, $Z_3=R+j\omega L_3$.

Then,

$$\det[\cdot]=\left(j\omega C_1+\frac{1}{R+j\omega L_3}+G_i\right)\left(j\omega C_2+\frac{1}{R+j\omega L_3}\right)+\left(\frac{1}{R+j\omega L_3}\right)\left(g_m-\frac{1}{R+j\omega L_3}\right)=0$$

$$\left[1+(G_i+j\omega C_1)(R+j\omega L_3)\right]\left[1+j\omega C_2(R+j\omega L_3)\right]+g_m(R+j\omega L_3)-1=0$$

$$\cancel{1}+j\omega C_2\,\cancel{(R+j\omega L_3)}+(G_i+j\omega C_1)\cancel{(R+j\omega L_3)}+j\omega C_2(G_i+j\omega C_1)(R+j\omega L_3)^{\cancel{2}}+$$
$$+\,g_m\cancel{(R+j\omega L_3)}-\cancel{1}=0$$

$$j\omega C_2+G_i+j\omega C_1+j\omega C_2(G_i+j\omega C_1)(R+j\omega L_3)+g_m=0$$

Re: $\quad G_i+g_m-\omega^2L_3G_iC_2-\omega^2C_1C_2R=0$

Im: $\quad \omega C_2+\omega C_1+\omega C_2G_iR-\omega^3C_1C_2L_3=0$

$$C_1+C_2+C_2G_iR-\omega^2C_1C_2L_3=0$$

$$\omega=\sqrt{\frac{C_1+C_2+C_2G_iR}{C_1C_2L_3}}=\sqrt{\frac{1}{L_3}\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{G_iR}{C_1}\right)} \quad ✓$$

13.3

From (13.4) :

$$\begin{vmatrix} (Y_1+Y_2+g_m+G_0) & -(Y_2+G_0) \\ -(G_0+g_m+Y_2) & (Y_2+Y_3+G_0) \end{vmatrix} = 0$$

$$(Y_1+Y_2+g_m+G_0)(Y_2+Y_3+G_0) - (Y_2+G_0)(G_0+g_m+Y_2) = 0$$

Simplifying gives

$$Y_1Y_2 + Y_1Y_3 + Y_1G_0 + Y_2Y_3 + g_mY_3 + G_0Y_3 = 0$$

$$\frac{1}{Y_3} + \frac{1}{Y_2} + \frac{G_0}{Y_2Y_3} + \frac{g_m}{Y_1Y_2} + \frac{G_0}{Y_1Y_2} = 0$$

For Colpitts, let $\frac{1}{Y_3} = Z_3 = R + j\omega L$; $\frac{1}{Y_1} = 1/j\omega C_1$; $\frac{1}{Y_2} = 1/j\omega C_2$.

$$R + j\omega L_3 - \frac{j}{\omega C_2} + G_0(R + j\omega L_3)\left(\frac{1}{j\omega C_2}\right) - \frac{j}{\omega C_1} - \frac{(g_m+G_0)}{\omega^2 C_1 C_2} = 0$$

Re: $$R + G_0 L_3/C_2 - \frac{(g_m+G_0)}{\omega^2 C_1 C_2} = 0 \quad ✓$$

Im: $$\omega L_3 - \frac{1}{\omega C_2} - G_0 R/\omega C_2 - 1/\omega C_1 = 0$$

$$\omega L_3 = \frac{1}{\omega}\left(\frac{1}{C_2} + \frac{G_0 R}{C_2} + \frac{1}{C_1}\right)$$

$$\omega = \sqrt{L_3\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{G_0 R}{C_2}\right)} \quad ✓$$

Design: $f = 200$ MHz common gate FET, $g_m = 20$ mS,
$R_0 = 1/G_0 = 200\Omega$, $L_3 = 15$ nH, $Q = 50$.

Then, $\frac{1}{\omega_0^2 L_3} = \frac{C_1 C_2'}{C_1 + C_2'} = 42.2$ pF , $C_2' = \frac{C_2}{1 + G_0 R}$

$$R = \frac{\omega_0 L_3}{Q} = 0.38\,\Omega$$

Assume $C_1 = C_2'$, then $C_1 = 84.4\text{ pF} = C_2'$

$$C_2 = C_2'(1+G_0R) = 84.4\text{ pF}$$

$$R < \frac{g_m + G_0}{\omega^2 C_1 C_2} - \frac{G_0 L_3}{C_2} = 1.33\,\Omega$$

$$Q_{MIN} = \frac{\omega_0 L_3}{R_{MAX}} = \underline{14.}$$

13.4

Let $Z = R + jX$, $R > 0$ (POSITIVE RESISTANCE)

Then $z = Z/Z_0 = r + jx$, $r > 0$

$$\Gamma = \frac{z-1}{z+1} = \frac{(r-1)+jx}{(r+1)+jx}$$

Now let $Z = -R + jX$, $R > 0$ (NEGATIVE RESISTANCE)

$z = -r + jx$, $r > 0$

Then,
$$\Gamma = \frac{z-1}{z+1} = \frac{-(r+1)+jx}{(-r+1)+jx} = \frac{(r+1)-jx}{(r-1)-jx}$$

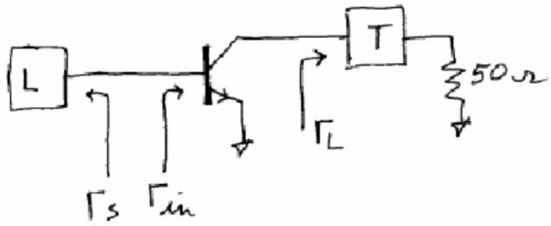
So,
$$\frac{1}{\Gamma^*} = \frac{(r-1)+jx}{(r+1)+jx}$$

which has the same form as Γ for positive resistance. So we can read the resistance circles as negative, and interpret the "reflection coefficient" read from the Smith chart as $1/\Gamma^*$.

13.5

$f = 1.9$ GHz, Si BJT

$S_{11} = .72\angle 157°$, $S_{12} = .15\angle 56°$, $S_{21} = 1.9\angle 52°$, $S_{22} = .63\angle -63°$

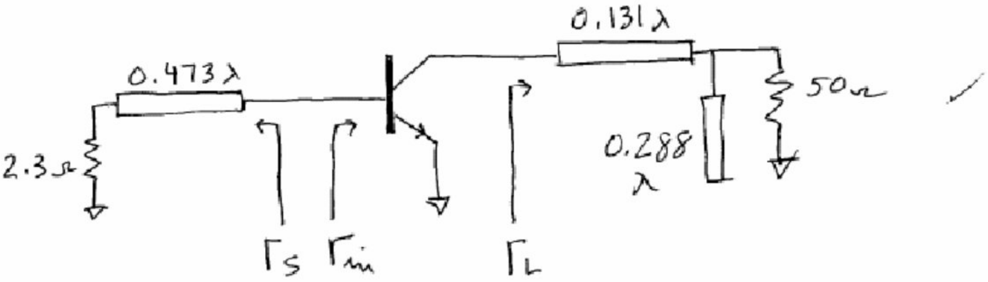


From 13.3a, $\Gamma_{in} = S_{11} + \dfrac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$

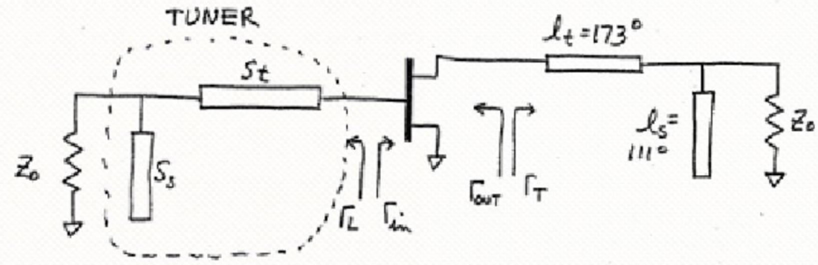
Choose Γ_L so that $|\Gamma_L| < 1$ and $|\Gamma_{in}|$ is large.
By trial and error,

Γ_L	Γ_{in}	Z_{in}
$.6\angle 160$	$.99\angle 160$	$.2 + j.9$
$.8\angle 60$	$1.17\angle 160$	$-4.2 + j8.7$
$.9\angle 50$	$1.26\angle 150$	$-6.2 + j12.9$
$.9\angle 60$	$1.31\angle 160$	$-6.8 + j8.6$

Select $\Gamma_L = 0.9\angle 60°$. Then $\Gamma_{in} = 1.31\angle 160°$, $Z_{in} = -6.8 + j8.6\,\Omega$
Let $Z_s = -R_{in}/3 - jX_{in} = 2.3 - j8.6\,\Omega$. Stub matching circuits were designed as shown below.



13.6

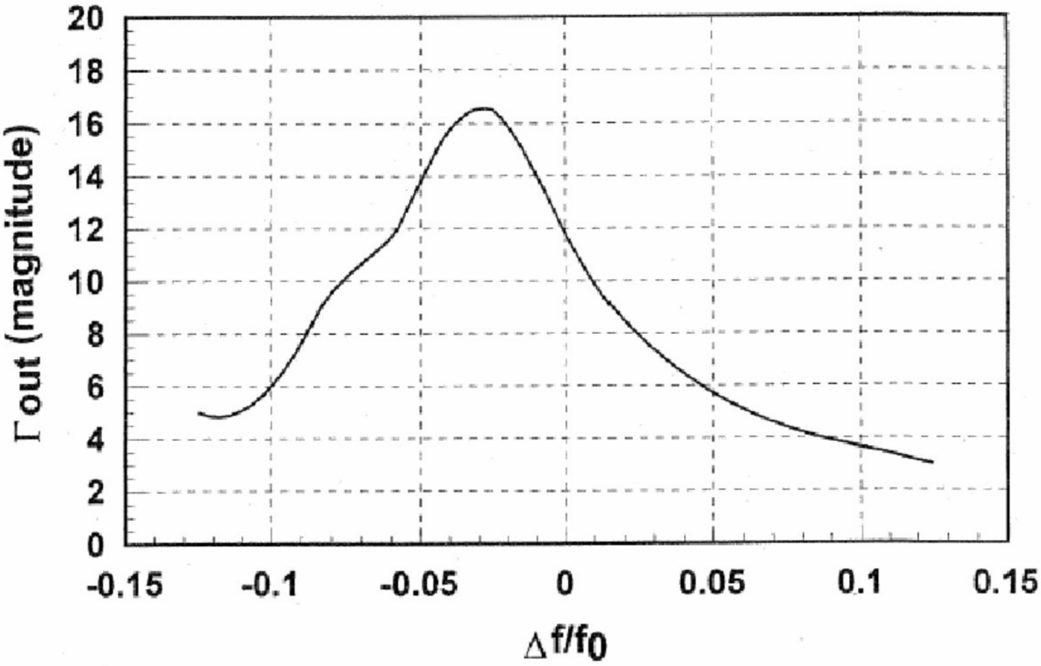


As in Example 12.4, choose $\Gamma_L = 0.6 \angle{-130°}$. The Γ_{out}, Z_{out}, Z_T, l_t, and l_s are unchanged. Then we have the simple matching problem of using the stub tuner to match 50Ω to Γ_L. The stub susceptance is $jb_s = +j1.56$, or a stub length of $S_s = 0.158\lambda$. The line length is $S_t = 0.18 - 0.176 = 0.004\lambda$.

We then analyze the above circuit to compute Γ_{out} versus frequency:

f (GHz)	$\Delta f / f_0$	$\lvert\Gamma_{out}\rvert$
2.10	−0.125	5.0
2.18	−0.092	7.2
2.20	−0.083	9.1
2.26	−0.058	11.9
2.30	−0.042	15.4
2.34	−0.025	16.5
2.38	−0.008	13.6
2.40	0	11.8
2.42	0.008	10.2
2.46	0.025	7.9
2.50	0.042	6.3
2.60	0.083	4.1
2.66	0.110	3.4
2.70	0.125	3.0

The maximum of $\lvert\Gamma_{out}\rvert$ does not occur at $\Delta f = 0$ because the tuner is not resonant at f_0. The "Q" is much lower than in Example 12.4. This problem shows the advantage of using a high-Q resonator for the oscillator. $\lvert\Gamma_{out}\rvert$ vs f is plotted on the following page.



13.7

$S_{11} = 1.2\angle 150°$, $S_{12} = 0.2\angle 120°$, $S_{21} = 3.7\angle -72°$, $S_{22} = 1.3\angle -67°$

As in Example 12.4, maximize $|\Gamma_{out}|$ by choosing $S_{11}\Gamma_L \simeq 1$, since

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{11}\Gamma_L}$$

Thus let $\Gamma_L = 0.8\angle -150°$ ✓. Then $\Gamma_{out} = 15.88\angle -99.3°$, and

$$Z_{out} = Z_0 \frac{1+\Gamma_{out}}{1-\Gamma_{out}} = -7.6 + j1.9\,\Omega$$

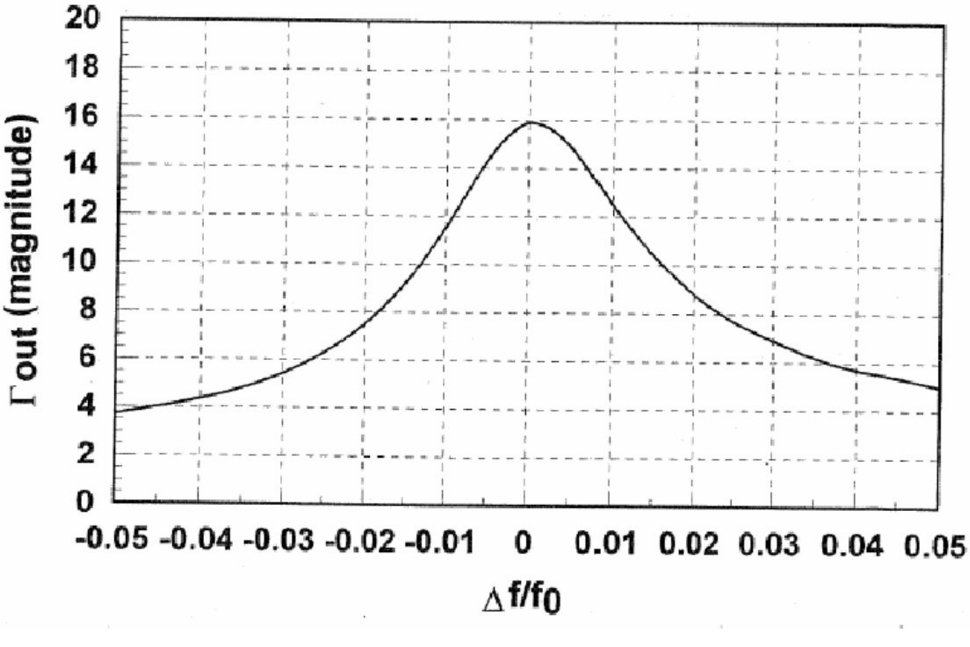
$$Z_T = \frac{-R_{out}}{3} - jX_{out} = 2.53 - j1.9\,\Omega \quad (z_T = 0.0506 - j0.038)$$

Matching Z_T to the load impedance gives $l_t = 0.031\lambda$ with a required stub susceptance of $+j4$. Thus $l_s = 0.21\lambda$.

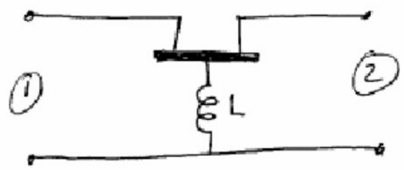
At the dielectric resonator, $\Gamma_L' = \Gamma_L e^{2j\beta l_r} = (0.8\angle -150°)e^{2j\beta l_r} =$
$= 0.8\angle 180°$. Thus $l_r = 0.4583\lambda$.

$$Z_L' = Z_0 \frac{1+\Gamma_L'}{1-\Gamma_L'} = 5.55\,\Omega = N^2 R$$

$|\Gamma_{out}|$ vs f was calculated with SuperCompact, and is plotted below:



13.8 common gate: $S_{11} = .46\angle 178$, $S_{12} = .045\angle 73$,
$S_{21} = 1.41\angle -19$, $S_{22} = 1.02\angle -12$



Find L to minimize μ, using CAD.

L (nH)	μ
0	.922
1	.088
2	−.928
3	−.927
5	−.89
1.5	−.88
2.5	−.931

13.9 From (12.49),

$$S_\phi = \frac{kT_0F}{P_0}\left(\frac{K\omega_\alpha\omega_h^2}{\Delta\omega^3} + \frac{\omega_h^2}{\Delta\omega^2} + \frac{K\omega_\alpha}{\Delta\omega} + 1\right)$$

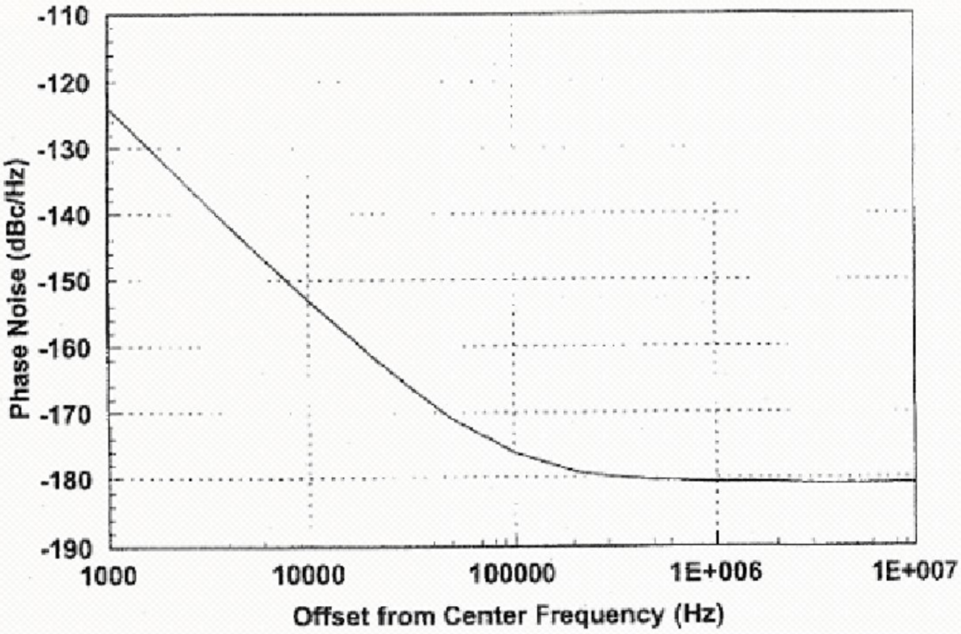
$$\mathcal{L}(f) = S_\phi/2.$$

For $F = 6\text{dB} = 4$, $f_0 = 100\text{MHz}$, $Q = 500$, $P_0 = 10\text{dBm} = 10\text{mW}$, $K = 1$,
$f_\alpha = 50\text{kHz}$, $f_h = f_0/2Q = 100\text{kHz}$, $\Delta f = f - f_0$

a short computer program was written to compute data for the plot shown below.

(a) $\Delta f = 1\text{MHz}$, $S_\phi = -178\text{dBm}$, $\mathcal{L}(1\text{MHz}) = -181\text{ dBc/Hz}$

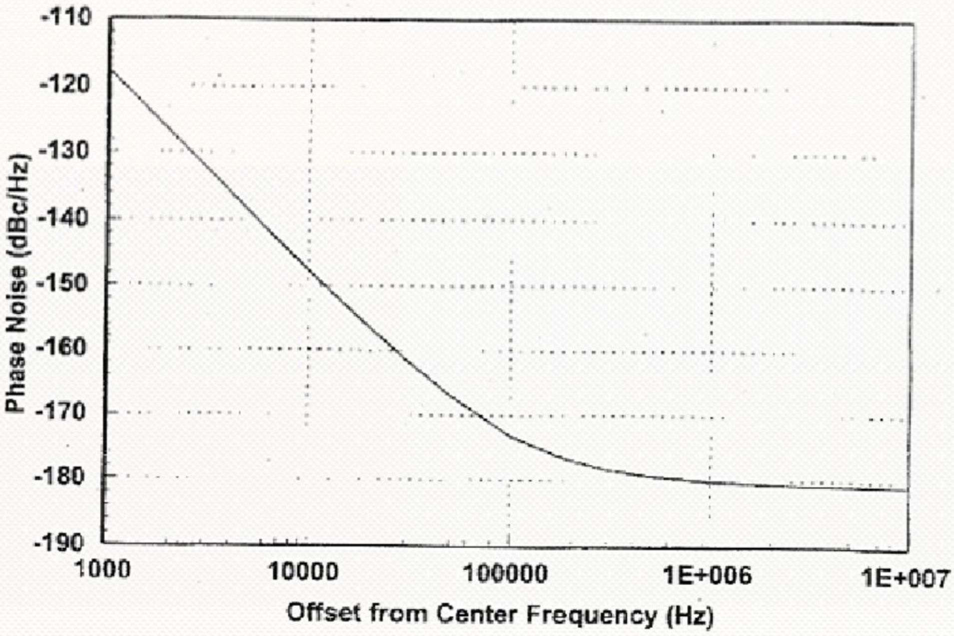
(b) $\Delta f = 10\text{kHz}$, $S_\phi = -150\text{dBm}$, $\mathcal{L}(10\text{kHz}) = -153\text{ dBc/Hz}$



13.10 This calculation is similar to that of Problem 13.9, but with $f_\alpha = 200\,kHz$. Plot shown below.

(a) $\Delta f = 1\,MHz$, $S_\phi = -177\,dBm$, $\mathcal{L}(1\,MHz) = -180\,dBc/Hz$

(b) $\Delta f = 10\,kHz$, $S_\phi = -144\,dBm$, $\mathcal{L}(10\,kHz) = -147\,dBc/Hz$.



13.11

If C is the desired signal level, I is the undesired signal level, S is the desired rejection ratio, $\mathcal{L}(f)$ the phase noise, and B the filter bandwidth, then

$$S = \frac{C}{I B \mathcal{L}(f)}$$

In dB,

$$\mathcal{L}(f) = C(dBm) - I(dBm) - S(dB) - 10\log(B) . \checkmark$$

13.12 $B = 12\text{ kHz}, \quad S = 80\text{ dB}, \quad C = I$

From (13.50),

$$\mathcal{L}(30\text{kHz}) = C(dBm) - I(dBm) - S(dB) - 10\log(B)$$
$$= -80dB - 10\log(12\times10^3)$$
$$= -121 \text{ dBc/Hz}.$$

13.13 Assume excitation at f_1, f_2; o.c. at all other frequencies except $f_3 = f_1 + f_2$. Then all power terms are zero except for $n=\pm1$, $m=0$; $n=0$, $m=\pm1$; and $n=m=\pm1$. So the Manley-Rowe relations give,

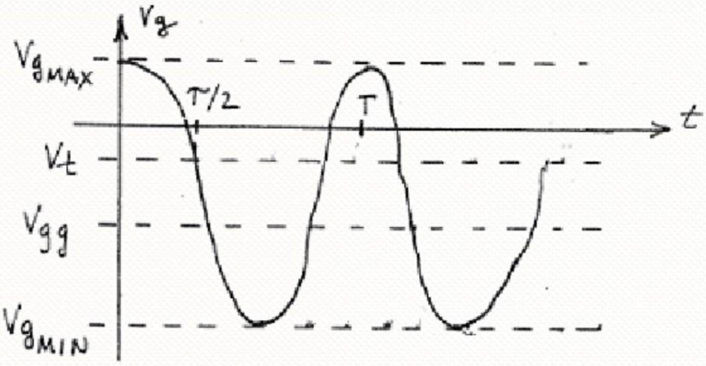
$$\frac{P_{10}}{\omega_1} + \frac{P_{11}}{\omega_1+\omega_2} = 0$$

$$\frac{P_{01}}{\omega_2} + \frac{P_{11}}{\omega_1+\omega_2} = 0$$

For sources at f_1, f_2, we have $P_{10} > 0$ and $P_{01} > 0$. Then $P_{11} < 0$, representing power at $f_3 = f_1 + f_2$ ($m=n=1$). Conversion gain is then,

$$G_C = \frac{-P_{11}}{P_{10}} = \frac{\omega_1+\omega_2}{\omega_1} = 1 + \omega_2/\omega_1 . \checkmark$$

13.14



$$V_g = V_{gg} + V_g \cos\frac{2\pi t}{T}$$

$$V_g = (V_{g_{MAX}} - V_{g_{MIN}})/2 \qquad \text{(PEAK)}$$

$$V_{gg} = (V_{g_{MAX}} + V_{g_{MIN}})/2 \qquad \text{(AVG)}$$

$$V_t = V_{gg} + V_g \cos\frac{\pi\tau}{T}$$ solve for $\cos\frac{\pi\tau}{T}$:

$$\cos\frac{\pi\tau}{T} = \frac{V_t - V_{gg}}{V_g} = \frac{2V_t - V_{g_{MAX}} - V_{g_{MIN}}}{V_{g_{MAX}} - V_{g_{MIN}}} \quad ✓$$

13.15

$$v_{RF}(t) = V_{RF}\left[\cos(\omega_{LO} - \omega_{IF})t + \cos(\omega_{LO} + \omega_{IF})t\right]$$

$$v_{LO}(t) = V_{LO} \cos\omega_{LO} t$$

after mixing and LPF:

$$v_{OUT}(t) = \frac{K V_{RF} V_{LO}}{2}\left[\cos\omega_{IF} t + \cos\omega_{IF} t\right] = K V_{RF} V_{LO} \cos\omega_{IF} t$$

(both sidebands convert to same IF)

13.16 $i(t) = e^{3v(t)} - 1$, $v(t) = 0.1\cos\omega_1 t + 0.1\cos\omega_2 t$

$$i(t) = i\Big|_{v=0} + \frac{di}{dv}\Big|_{v=0} v + \frac{d^2 i}{dv^2}\Big|_{v=0}\frac{v^2}{2} + \frac{d^3 i}{dv^3}\Big|_{v=0}\frac{v^3}{6} + \cdots$$

$$i\Big|_{v=0} = 0 \; ; \; \frac{di}{dv}\Big|_{v=0} = 3 \; ; \; \frac{d^2 i}{dv^2}\Big|_{v=0} = 9 \; ; \; \frac{d^3 i}{dv^3}\Big|_{v=0} = 27.$$

So,

$$i(t) = 3v + 4.5v^2 + 4.5v^3 + \cdots$$

$$v^2 = .01\left[\cos^2\omega_1 t + 2\cos\omega_1 t\cos\omega_2 t + \cos^2\omega_2 t\right]$$

$$= .01\left[1 + \tfrac{1}{2}\cos^2\omega_1 t + \tfrac{1}{2}\cos 2\omega_2 t + \cos(\omega_1+\omega_2)t + \cos(\omega_1-\omega_2)t\right]$$

$$v^3 = .001\left[\cos^3\omega_1 t + 3\cos^2\omega_1 t\cos\omega_2 t + 3\cos\omega_1 t\cos^2\omega_2 t + \cos^3\omega_2 t\right]$$

$$= .001\left[\tfrac{1}{4}\cos 3\omega_1 t + \tfrac{3}{4}\cos\omega_1 t + \tfrac{3}{2}\cos\omega_2 t + \tfrac{3}{4}\cos(2\omega_1 - \omega_2)t\right.$$

$$+ \tfrac{3}{4}\cos(2\omega_1+\omega_2)t + \tfrac{3}{2}\cos\omega_1 t + \tfrac{3}{4}\cos(\omega_1 - 2\omega_2)t$$

$$\left. + \tfrac{3}{4}\cos(\omega_1+2\omega_2)t + \tfrac{3}{4}\cos\omega_2 t + \tfrac{1}{4}\cos 3\omega_2 t\right]$$

ω	I	
0	$(4.5)(.01)$	$=0.045$ ✓
ω_1, ω_2	$(3)(.1)+(4.5)(.001)(\frac{3}{2}+\frac{3}{4})$	$=0.3101$ ✓
$2\omega_1, 2\omega_2$	$(4.5)(.01)(\frac{1}{2})$	$=0.0225$ ✓
$3\omega_1, 3\omega_2$	$(4.5)(.001)(1/4)$	$=0.00113$ ✓
$\omega_1+\omega_2$	$(4.5)(.01)(1)$	$=0.045$ ✓
$\omega_1-\omega_2$	$(4.5)(.01)(1)$	$=0.045$ ✓
$2\omega_1-\omega_2$	$(4.5)(.001)(3/4)$	$=0.003375$ ✓
$2\omega_1+\omega_2$	$(4.5)(.001)(3/4)$	$=0.003375$ ✓
$\omega_1-2\omega_2$	$(4.5)(.001)(3/4)$	$=0.003375$ ✓
$\omega_1+2\omega_2$	$(4.5)(.001)(3/4)$	$=0.003375$ ✓

13.17 $f_{RF} = 1800$ MHz, $f_{IF} = 87$ MHz

possible LO frequencies are

$$f_{LO} = f_{RF} \pm f_{IF} = 1887 \text{ MHz}, \ 1713 \text{ MHz}.$$

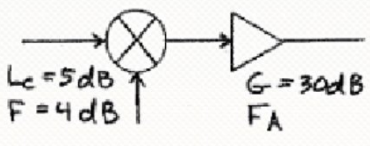
image frequency for $f_{LO} = 1887$ MHz is

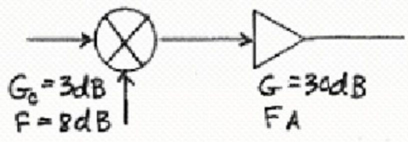
$$f_{IM} = f_{LO} + f_{IF} = 1974 \text{ MHz}$$

image frequency for $f_{LO} = 1713$ MHz is

$$f_{IM} = f_{LO} - f_{IF} = 1626 \text{ MHz}$$

13.18





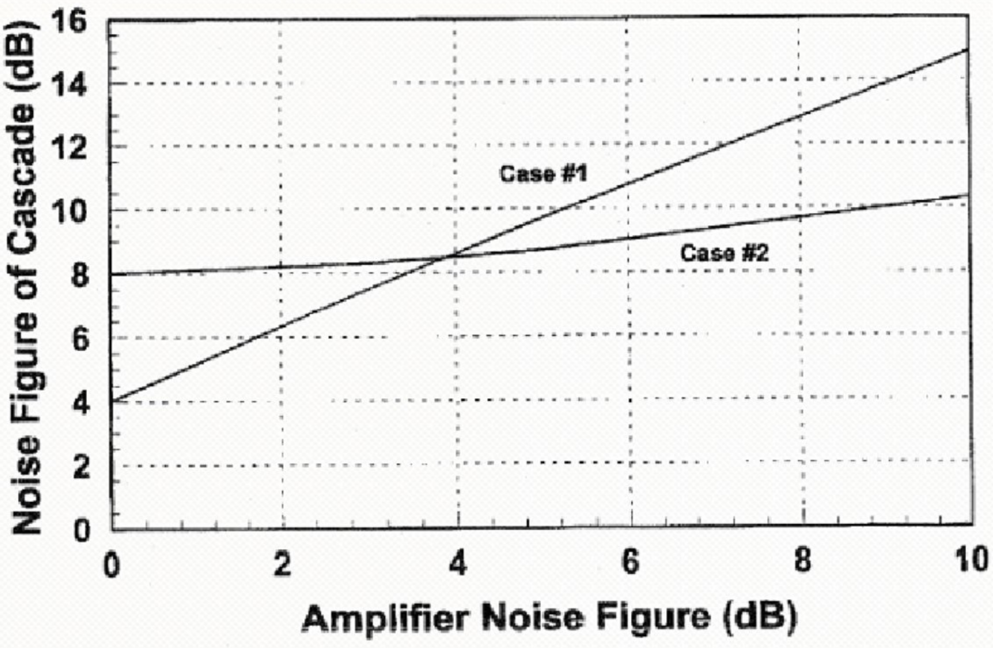
$F_C = 2.51 + \frac{F_A - 1}{1/3.16}$

$F_C = 6.31 + \frac{F_A - 1}{2.0}$

F_A (dB)	F_C(#1) dB	F_C(#2) dB
0	4.0	8.0
3	7.5	8.3
5	9.7	8.7
10	14.9	10.3

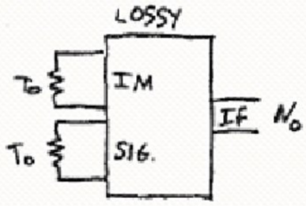
3dB = 2.0
4dB = 2.51
5dB = 3.16
8dB = 6.31

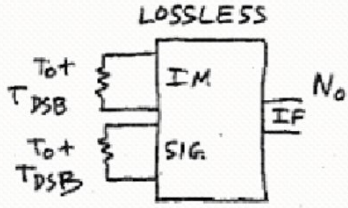
RESULTS ARE PLOTTED BELOW:



13.19

DSB





$$N_o = N_{ADDED} + \frac{kT_oB}{L} + \frac{kT_oB}{L} \qquad N_o = \frac{2kB}{L}\left(T_o + T_{DSB}\right)$$

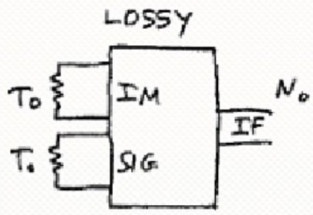
(B is SSB)

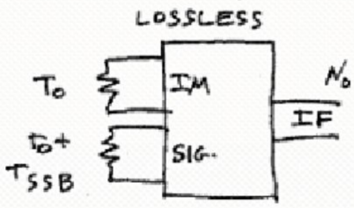
$$\therefore \quad N_{ADDED} = \frac{2kB}{L}\, T_{DSB}$$

$$F_{DSB} = \frac{S_i N_o}{S_o N_i} = \frac{N_o L}{2kT_oB} = 1 + \frac{T_{DSB}}{T_o}$$

(INPUT NOISE – $N_i = 2kT_oB$)

SSB





$$N_o = N_{ADDED} + \frac{2kT_oB}{L} \qquad N_o = \frac{2kBT_o}{L} + \frac{kBT_{SSB}}{L}$$

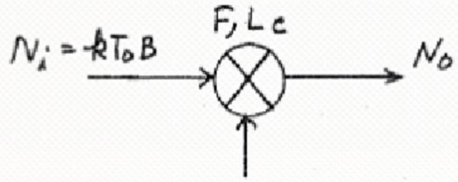
$$\therefore \quad T_{SSB} = \frac{L\, N_{ADDED}}{kB}$$

$$\underline{T_{SSB} = 2\,T_{DSB}} \; \checkmark$$

$$F_{SSB} = \frac{S_i N_o}{S_o N_i} = \frac{N_o L}{kT_oB} = 2 + \frac{T_{SSB}}{T_o} = 2\left(1 + \frac{T_{DSB}}{T_o}\right) = 2\,F_{DSB} \; \checkmark$$

(INPUT NOISE – $N_i = kT_oB$)

13.20



$$N_0 = \underbrace{\frac{kT_0B}{L_c}}_{\text{INPUT NOISE}} + \underbrace{\frac{kT_0B(F-1)}{L_c}}_{\text{MIXER NOISE}} = \frac{kT_0BF}{L_c} \checkmark$$

13.21 $v_1 = v_0 \cos\omega t$; $v_2 = v_0 \cos(\omega t + \theta)$

as in (12.112)-(12.113), the diode currents in a mixer using a quadrature hybrid will be,

$$i_1 = k v_0^2 [\cos(\omega t - \pi/2) + \cos(\omega t + \theta - \pi)]^2$$
$$= k v_0^2 [\sin\omega t - \cos(\omega t + \theta)]^2$$
$$i_2 = -k v_0^2 [\cos(\omega t - \pi) + \cos(\omega t + \theta - \pi/2)]^2$$
$$= -k v_0^2 [-\cos\omega t + \sin(\omega t + \theta)]^2$$

Low-pass filtering leaves the following DC components:

$$i_1 = k v_0^2 (1 + \tfrac{1}{2} \sin\theta)$$
$$i_2 = -k v_0^2 (1 - 1/2 \sin\theta)$$

So the output is $i_1 + i_2 = k v_0^2 \sin\theta$ ✓

If a mixer with a 180° hybrid is used, the diode currents become,

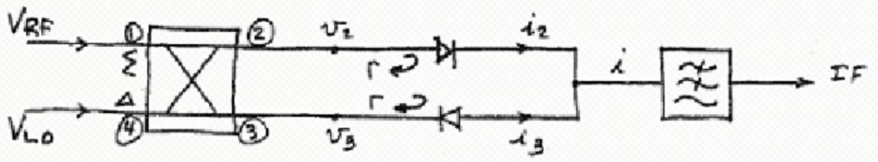
$$i_1 = k v_0^2 [\cos\omega t + \cos(\omega t + \theta)]^2$$
$$i_2 = -k v_0^2 [\cos\omega t - \cos(\omega t + \theta)]^2$$

Then low-pass filtering leaves the following DC components:

$$i_1 = k v_0^2 (1 + \tfrac{1}{2} \cos\theta)$$
$$i_2 = -k v_0^2 (1 - \tfrac{1}{2} \cos\theta)$$

So the output is $i_1 + i_2 = k v_0^2 \cos\theta$ ✓

13.22



$$[S] = \frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

let $v_{RF}(t) = V_{RF}\cos\omega_{RF}t = v_1(t)$

$v_{LO}(t) = V_{LO}\cos\omega_{LO}t = v_4(t)$

Then the diode voltages are,

$$v_2(t) = \frac{1}{\sqrt{2}}V_{RF}\cos(\omega_{RF}t - 90°) + \frac{1}{\sqrt{2}}V_{LO}\cos(\omega_{LO}t + 90°)$$

$$v_3(t) = \frac{1}{\sqrt{2}}V_{RF}\cos(\omega_{RF}t - 90°) + \frac{1}{\sqrt{2}}V_{LO}\cos(\omega_{LO}t - 90°)$$

assume $i_2 = k v_2^2$, $i_3 = -k v_2^2$. $\omega_{IF} = \omega_{RF} - \omega_{LO}$.

Then, after LP filtering, the diode currents are,

$$i_2 = \frac{k}{4}V_{RF}V_{LO}\cos(\omega_{RF}t - 90° - \omega_{LO}t - 90°) = \frac{k}{4}V_{RF}V_{LO}\cos\omega_{IF}t.$$

$$i_3 = -\frac{k}{4}V_{RF}V_{LO}\cos(\omega_{RF}t - 90° - \omega_{LO}t + 90°) = -\frac{k}{4}V_{RF}V_{LO}\cos\omega_{IF}t.$$

So the IF output current is $i(t) = -\frac{k}{2}V_{RF}V_{LO}\cos\omega_{IF}t$ ✓

AT RF INPUT:

$$V_2^+ = \Gamma V_2^- = \frac{-j}{\sqrt{2}}\Gamma V_{RF}^+ \;;\; V_3^+ = \Gamma V_3^- = \frac{-j}{\sqrt{2}}\Gamma V_{RF}^+$$

$$V_{RF}^{\Sigma} = V_1^- = V_2^+(-j/\sqrt{2}) + V_3^+(-j/\sqrt{2}) = \underline{-\Gamma V_{RF}^+} \quad ✓$$

$$V_{RF}^{\Delta} = V_4^- = V_2^+(j/\sqrt{2}) + V_3^+(-j/\sqrt{2}) = \underline{0} \quad ✓$$

AT LO INPUT:

$$V_2^+ = \Gamma V_2^- = \frac{j}{\sqrt{2}}\Gamma V_{LO}^+ \;;\; V_3^+ = \Gamma V_3^- = \frac{-j}{\sqrt{2}}\Gamma V_{LO}^+ .$$

$$V_{LO}^{\Sigma} = V_1^- = V_2^+(-j/\sqrt{2}) + V_3^+(-j/\sqrt{2}) = \underline{0} \quad ✓$$

$$V_{LO}^{\Delta} = V_4^- = V_2^+(j/\sqrt{2}) + V_3^+(-j/\sqrt{2}) = \underline{-\Gamma V_{LO}^+} \quad ✓$$

Assume now that

$$v_{LO}(t) = V_{LO}^{(2)}\cos 2\omega_{LO}t.$$

Then after LPF,

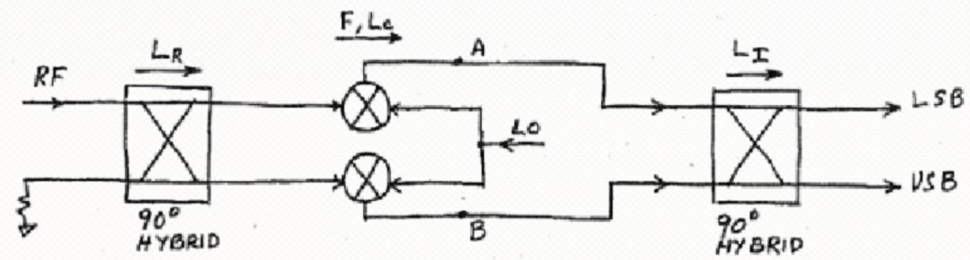
$$v_2^2(t) = \frac{1}{\sqrt{2}} V_{RF} V_{LO}^{(2)} \cos(\omega_{RF} t + 2\omega_{LO} t + 90°) + \frac{1}{\sqrt{2}} V_{RF} V_{LO}^{(2)} \cos(\omega_{RF} t - 2\omega_{LO} t + 90°)$$

$$v_3^2(t) = \frac{1}{\sqrt{2}} V_{RF} V_{LO}^{(2)} \cos(\omega_{RF} t + 2\omega_{LO} t + 90°) + \frac{1}{\sqrt{2}} V_{RF} V_{LO}^{(2)} \cos(\omega_{RF} t - 2\omega_{LO} t + 90°)$$

Then forming

$$i(t) = k(v_2^2 - v_3^2)\Big|_{LPF} = 0 \quad \text{for } \omega_{RF} \pm 2\omega_{LO} \text{ frequencies.}$$

13.23



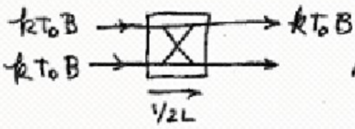
The noise power due to the RF hybrid and the mixer, ref. to IF output of mixer, is

$$N_A = N_B = \frac{kB}{L_c}\left[T_0 + (F-1)\,T_0\right] = \frac{kBT_0F}{L_c},$$

since the noise power output of the matched hybrid is kT_0B. The total noise power output is (at either LSB or USB),

$$N_o = \frac{N_A}{2L_I} + \frac{N_B}{2L_I} + \frac{N_{added}}{2L_I} = \frac{kBT_0F}{L_IL_c} + \frac{N_{added}}{2L_I}$$

N_{added} is the output noise power of the IF hybrid when not terminated at second input port:



$$N_o = \frac{kT_0B}{2L} + \frac{kT_0B}{2L} + \frac{N_{added}}{2L} = kT_0B$$

Thus $N_{added} = 2kT_0B(L-1)$

So, $N_o = \dfrac{kBT_0F}{L_IL_c} + kT_0B\left(1-\dfrac{1}{L_I}\right)$; $S_o = \dfrac{4S_i}{L_c}\dfrac{1}{4L_IL_R} = \dfrac{S_i}{L_cL_IL_R}$

$N_i = kT_0B$

and then,

$$F_{TOT} = \frac{S_iN_o}{S_oN_i} = \frac{L_cL_IL_R}{kT_0B}\left[\frac{kBFT_0}{L_IL_c} + kT_0B\left(1-\frac{1}{L_I}\right)\right] = FL_R + L_cL_RL_I - L_cL_R$$

CHECK: if $L_R = L_I = 1$, $F_{TOT} = F + 2L_c - 2L_c = F$ ✓ (mixer noise only)

CHECK: if $F = L_c$ (passive mixer loss only), $F_{TOT} = L_cL_IL_R$ ✓

(The cascade noise figure formula can be used to obtain the same result if we set $F_R = L_R$, $F_I = L_I$.)

Chapter 14

14.1

Data on satellite fading at L-band in various environments can be found in "Handbook of Propagation Effects for Vehicular and Personal Mobile Satellite Systems", by J. Goldhirsh and W. Vogel, and in "Satellite Systems for Personal and Broadband Communications", by E. Lutz, M. Werner, and A. Jahn, as well as from various other sources. Typically, one can expect fading levels of 15 to 20 dB for domestic and commercial buildings, for 95% link availability. For vehicles, the fading levels can be 20 dB or more. On the other hand, a line-of-sight system (as when the handset is used outdoors with little or no blockage to the satellite) would require a link margin of 0 dB in principal, although a few dB of margin would provide a more robust system. In view of this data, it is not clear why the Iridium system was designed with a link margin of 16 dB.

14.2 $F_\theta(\theta,\phi) = A \sin^2\theta \cos\phi$

Main beam at $\theta = 90°$, $\phi = 0°$ or $180°$

3 dB points at $\theta = 90°$ (az) plane:

$$\cos\phi = 0.707 \Rightarrow \phi = 45° \text{ or } 135°$$

$$HPBW_\phi = 135 - 45 = 90° \quad ✓$$

3 dB points in $\phi = 0°$ (el) plane:

$$\sin^2\theta = 0.707 \Rightarrow \theta = \sin^{-1}\sqrt{.707} = 57.2°, \ 122.8°$$

$$HPBW_\theta = 122.8 - 57.2 = 65.6°$$

$$\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} F^2(\theta,\phi)\sin\theta\, d\theta\, d\phi = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \sin^5\theta \cos^2\phi\, d\theta\, d\phi$$

$$= \left(\frac{16}{15}\right)\pi$$

$$D = \frac{4\pi F^2_{MAX}}{\iint} = \frac{15}{4} = 3.75 = \underline{5.74\, dB}$$

linearly polarized in vertical direction.

14.3 $F_\theta(\theta,\phi) = \begin{cases} A\sin\theta & \text{for } 0 \le \theta \le \pi/2 \\ 0 & \text{otherwise} \end{cases}$

$$D = \frac{4\pi F_{MAX}^2}{\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} F_\theta^2(\theta,\phi)\sin\theta\, d\theta\, d\phi} = \frac{4\pi}{2\pi\int_{\theta=0}^{\pi/2}\sin^3\theta\, d\theta} = \frac{4\pi}{2\pi\left(\frac{2}{3}\right)}$$

$= 3 = \underline{4.8\,dB}$ (verified with PCAAD)

14.4 $f = 12.4$ GHz, $D_{iam} = 18'' = 0.457$ m, $\eta_{ap} = 65\%$

$\lambda = \frac{c}{f} = 0.0242$ m ✓

$A = \pi R^2 = \pi\left(\frac{D_{iam}}{2}\right)^2 = 0.164\, m^2$

From (14.13)

$$D = \eta_{ap}\frac{4\pi A}{\lambda^2} = (0.65)\frac{4\pi(0.164)}{(0.0242)^2} = 2287 = \underline{33.6\,dB} ✓$$

14.5 $f = 38\,\text{GHz}$, $G = 39.0\,\text{dB}$, $D_{iam} = 12.0''$, $\eta_{rad} = 90\%$

a) From (14.11), $D = \dfrac{G}{\eta_{rad}} = \dfrac{10^{39/10}}{0.9} = 8{,}826.$

From (14.13), $D = \dfrac{4\pi A}{\lambda^2}\eta_{ap}$

$$\eta_{ap} = \frac{\lambda^2 D}{4\pi A} = \left(\frac{c}{\pi f D_{iam}}\right)^2 D = \underline{\underline{60\%}}$$

b) From (14.9), $D = \dfrac{32{,}400}{\theta_3^2}$

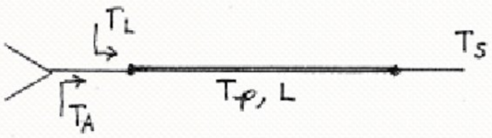
$$\theta_3 = \sqrt{\frac{32{,}400}{8826}} = \underline{\underline{1.9^\circ}}$$

14.6 From (14.18), $T_A = \eta_{rad} T_b + (1-\eta_{rad}) T_p$
$= (T_b - T_p)\eta_{rad} + T_p$

Thus,

$$\eta_{rad} = \frac{T_A - T_p}{T_b - T_p} = \frac{105-290}{5-290} = \underline{65\%}$$

14.7



$T_A = \eta_{rad} T_b + (1-\eta_{rad}) T_p$ (at antenna) (14.18)

$T_L = (L-1) T_p$ (at antenna) (10.15)

$T_S = \frac{1}{L}(T_A + T_L)$ (at output)

$= \frac{1}{L}\left[\eta_{rad} T_b + (1-\eta_{rad}) T_p\right] + \frac{(L-1)}{L} T_p$ (14.20) ✓

14.8

$$T = \eta_{rad} T_b + (1-\eta_{rad}) T_p + T_R$$

$T_b = 50K$
$T_p = 290K$
$F = 1.1 dB = 1.29$
$L = 2.5 dB = 1.78$
$G = 33.5 dB = 2240$

The noise temperature of the receiver is,

$$T_R = (F-1)T_o = (1.29-1)(290) = 84K$$

The efficiency of the array is,

$$\eta_{rad} = \frac{1}{L} = \frac{1}{1.78} = 0.56$$

Thus,

$$T = (0.56)(50) + (1-0.56)(290) + 84 = \underline{240K}$$

Then,

$$\frac{G}{T}(dB) = 10 \log \frac{2240}{240} = 9.7 \underline{dB/K}$$

This value is well below the desired minimum of 12 dB/K.

14.9 Solving (14.23) for G gives,

$$G = \frac{4\pi S R^2}{P_t} = \frac{4\pi(7.5\times10^{-3})(300)^2}{85} = 100 = \underline{\underline{20 dB}}$$

14.10

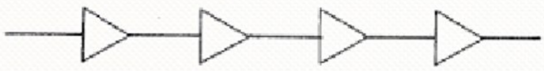
1) RADIO LINK: $f = 28\,GHz \Rightarrow \lambda = 0.0107\,m$; $G_t = G_r = 25\,dB = 316$.

Let $P_r = 1\,W$. Then,

$$P_t = \frac{(4\pi R)^2}{P_r G_t G_r \lambda^2} = \frac{(4\pi)^2 (5000)^2}{(1)(316)^2(0.0107)^2} = 3.45 \times 10^8\ W$$

$$\text{ATTENUATION} = 10 \log \frac{P_r}{P_t} = 10 \log \frac{1}{3.45 \times 10^8} = -\underline{85.4\,dB}\ ✓$$

2) WIRED LINK: $\alpha = 0.05\,dB/m = 0.0057$ neper/m; $4 \times 30\,dB$ REPEATERS.



$$\text{ATTENUATION OF LINE} = 10 \log e^{-2\alpha R}$$
$$= 10 \log e^{-2(0.0057)(5000)}$$
$$= -250\,dB$$

$$\text{TOTAL LOSS} = -250 + 4(30) = \underline{-130\,dB}\ ✓$$

The radio link has much less link loss than the wired link, and will thus require less transmit power.

14.11 GSM downlink, 935 - 960 MHz, EIRP = 20 W, $G_r = 0$ dBi, $T = 450$ K, SNR = 10 dB, LM = 30 dB, $B = 200$ kHz, $F_R = 8$ dB.

$f = 947.5$ MHz $\Rightarrow \lambda = 0.317$ m, $G_r = 0\,dB_i = 1$

$F_R = 8\,dB = 6.31$, $SNR = 10\,dB = 10$.

$$T_{SYS} = T_A + T_R = T_A + (F_R - 1)T_0 = 450 + (6.31-1)(290)$$
$$= 1990\,K$$
$$N_o = kT_{sys}B = (1.38\times10^{-23})(1990)(200\times10^{3})$$
$$= 5.5\times10^{-15}\,W = -112.6\,dBm \quad \text{(at rec. input)}$$

$$S_o(dB_m) = \left(\frac{S_o}{N_o}\right) + N_o + LM = 10 - 112.6 + 30 = -72.6\,dBm$$
$$= 5.5\times10^{-11}\,W$$

$$R = \sqrt{\frac{P_t G_t G_r \lambda^2}{(4\pi)^2 S_o}} = \sqrt{\frac{(20)(1)(.317)^2}{(4\pi)^2(5.5\times10^{-11})}} = \underline{15.2\ km}$$

(stated base station range for GSM 900 is 2-20km)

14.12 Carrier power at receiver:

$$C = S_i G_A G/L \qquad (S_i \text{ ref. to antenna w/ 0 dBi})$$

at input to amplifier:

$$T_e = T_A + (F-1)T_0 + (L-1)T_0/G$$

The noise power at input to receiver:

$$N = kT_e G/L = \frac{C}{(C/N)}$$

$L = 25\,dB = 316.2$

$S_i = 1 \times 10^{-16}$

$G_A = 5\,dB = 3.16$

$\frac{C}{N} = 32\,dB = 1580.$

$G = 10\,dB = 10$

So,

$$T_e = \frac{CL}{k\left(\frac{C}{N}\right)G} = \frac{S_i G_A}{k\left(\frac{C}{N}\right)}$$

$$F = 1 + \frac{T_e}{T_0} - \frac{T_A}{T_0} - \frac{(L-1)}{G} = 1 + \frac{S_i G_A}{kT_0\left(\frac{C}{N}\right)} - \frac{T_A}{T_0} - \frac{(L-1)}{G}$$

$$= 1 + \frac{(1\times10^{-16})(3.16)}{(1.38\times10^{-23})(290)(1.58\times10^{3})} - \frac{300}{290} - \frac{(316.2-1)}{10}$$

$$= 18.4 = \underline{\underline{12.6\,dB}}$$

14.13 $N_o = kT_bB = S_o$ for $S_o/N_o = 0\,dB$

$$kT_bB = P_r = \frac{P_t G^2 \lambda^2}{(4\pi R)^2}$$

$$R = \sqrt{\frac{P_t G^2 \lambda^2}{16\pi^2 k T_b B}} = \sqrt{\frac{(1000)(2.51)^2(4.48)^2}{16\pi^2(1.38\times10^{-23})(4)(4\times10^{6})}} = \underline{1.9\times10^{9}}\,m$$

If $SNR = 30\,dB = 1000$, $R = 6.0\times10^{7}\,m$

$R_{VENUS} = 4.2\times10^{10}\,m$, so the signal will not even reach the nearest planet.

14.14 Mariner 10, PSK, $P_b = 0.05$, $(E_b/\eta_o) = 1.4\,dB$,
$R = 1.6\times10^8\,km$, $f = 2.295\,GHz$. $G_t = 27.6\,dB$
$P_t = 16.8\,W$, $G_r = 61.3\,dB$, $T_{sys} = 13.5\,K$

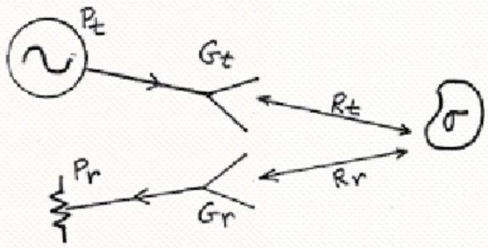
$\lambda = 0.1307\,m$, $P_t = 12.25\,dBW$, $E_b/\eta_o = 1.38$

$$L_o = 20\log(4\pi R/\lambda) = 263.7\,dB.$$
$$P_r = P_t + G_t - L_o + G_r = -162.6\,dBW = 5.56\times10^{-17}\,W \checkmark$$
$$\eta_o = kT_{sys}$$

From (14.36), $R_b = \left(\frac{P_r}{\eta_o}\right)\left(\frac{\eta_o}{E_b}\right) = \frac{5.56\times10^{-17}}{(1.38\times10^{-23})(13.5)}\left(\frac{1}{1.38}\right)$

$$= \underline{216\,kbps} \checkmark$$

14.15



From (14.23) the power density incident on the target is,

$$S = \frac{P_t G_t}{4\pi R_t^2}$$

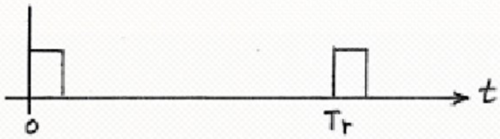
The scattered power density at the receiver is, from (14.40),

$$S_r = \frac{P_t G_t \sigma(\theta_t, \phi_t; \theta_r, \phi_r)}{(4\pi)^2 R_t^2 R_r^2},$$

where $\sigma(\theta_t, \phi_t; \theta_r, \phi_r)$ is the radar cross-section of the target seen at θ_r, ϕ_r with an incident wave at θ_t, ϕ_t. Then the received power can be found using (14.14):

$$P_r = P_t \frac{G_t G_r \lambda^2 \sigma(\theta_t, \phi_t; \theta_r, \phi_r)}{(4\pi)^3 R_t^2 R_r^2} \quad ✓$$

14.16



When a pulse is transmitted at $t=0$, the return pulse must come back before the next pulse is transmitted at $t=T_r$, to avoid an ambiguity in range. The round-trip time for a pulse return is,

$$T = 2R/c,$$

so the maximum unambiguous range is,

$$R_{MAX} = \frac{cT_r}{2} = \frac{c}{2f_r} \quad ✓$$

14.17 From (14.43) the doppler frequencies are,

$$f_{d(MIN)} = \frac{2 v_{MIN} f_o}{c} = \frac{2(1\,\frac{m}{sec})(12\,GHz)}{3\times10^{8}\,m/sec} = 80\ Hz$$

$$f_{d(MAX)} = \frac{2 v_{MAX} f_o}{c} = \frac{2(20\,m/sec)(12\,GHz)}{3\times10^{8}\,m/sec} = 1.6\ kHz$$

So the necessary passband is 80–1600 Hz.

14.18 From (14.41) the received power is,

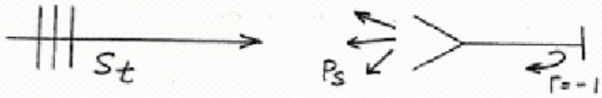
$$P_r = P_t \frac{G^2\lambda^2\sigma}{(4\pi)^3 R^4} = 1000\,\frac{(1000)^2(0.15)^2\,(20)}{(4\pi)^3\,(10^4)^4} = 2.27\times10^{-11}\ W$$

$$= -76\ dBm$$

The transmitter power is $10\log(10^4)(10^3) = 70\ dBm$. So the isolation between receiver and transmitter must be,

$$I = 70\ dBm - (-76\ dBm) + 10\ dB = 156\ dB$$

14.19



Assume an incident plane wave with power density S_t. Then the received power of the antenna is, from (13.14) and (14.15)

$$P_\ell = S_t A_e = S_t \frac{\lambda^2 G}{4\pi}$$

Because of the short-circuit termination, all of this power is re-transmitted (assuming a lossless antenna), giving a radiated power in the main beam direction of, $P_s = G P_\ell$. Then the RCS can be found from (14.39) :

$$\sigma = \frac{P_s}{S_t} = \frac{\lambda^2 G^2}{4\pi} \checkmark$$

14.20

$$\Delta T_{TRUE} = T_p - T_2$$

$$\Delta T_S = T_S\Big|_{T_B=T_1} - T_S\Big|_{T_B=T_2}$$

From (14.19), with $L=1$, $T_S = (1-\Gamma^2)\left[\eta_{rad} T_b + (1-\eta_{rad}) T_p\right]$

$$\Delta T_S = (1-\Gamma^2)\left[\eta_{rad} \cancel{T_p} + \cancel{(1-\eta_{rad})} T_p\right] - (1-\Gamma^2)\left[\eta_{rad} T_2 + \cancel{(1-\eta_{rad})} T_p\right]$$

$$\frac{\Delta T_S}{\Delta T_{TRUE}} = \frac{(1-\Gamma^2)\eta_{rad}(T_p - T_2)}{T_p - T_2} = (1-\Gamma^2)\eta_{rad} \quad ✓$$

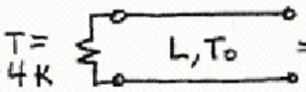
14.21

LOOKING TOWARD ZENITH, $\ell = 4000$ m. $= 4$ km.

LOOKING TOWARD HORIZON,

$$\ell = \sqrt{(6404)^2 - (6400)^2} = 226 \text{ km}.$$

$\alpha = 0.005$ dB/km



$$\Rightarrow T_e = \frac{4}{L} + (L-1)T_0$$

AT ZENITH: $L = (0.005 \text{ dB/km})(4 \text{ km}) = 0.02 \text{dB} = 1.0046$

$\underline{T_e = 5.3K}$

AT HORIZON: $L = (0.005 \text{dB/km})(226 \text{ km}) = 1.13 \text{dB} = 1.297$

$\underline{T_e = 89K}$

14.22 $f = 28\,GHz \Rightarrow \lambda_0 = 0.0107\,m, \; G = 32\,dB = 1585, \; P_t = 5\,W$

a) $R = \sqrt{\dfrac{G P_t}{4\pi S}} = \sqrt{\dfrac{(1585)(5)}{4\pi(.01\,W/cm^2)}} = 251\,cm = \underline{2.51\,m}$

b) $G = 32 - 10 = 22\,dB = 158.5$

$R = \sqrt{\dfrac{(158.5)(5)}{4\pi\;(.01\,W/cm^2)}} = 79.5\,cm = \underline{0.8\,m}$

c) $d = \sqrt{\dfrac{\lambda^2 D}{\pi^2}} = 0.175\,m$ (use $D = \dfrac{G}{\eta_{ap}} = 2641.$)

$R_{ff} = \dfrac{2d^2}{\lambda} = \underline{5.7\,m}$

(neither distance is in the far-field of the antenna)

14.23 $S = 1300\,W/m^2 = \frac{1}{2}|\bar{E}||\bar{H}| = \frac{1}{2\eta_0}|\bar{E}|^2 = \frac{\eta_0}{2}|\bar{H}|^2$

$|\bar{E}| = \sqrt{2\eta_0 S} = \sqrt{2(377)(1300)} = 990\,V/m$

$|\bar{H}| = \sqrt{\dfrac{2S}{\eta_0}} = \sqrt{\dfrac{2(1300)}{377}} = 2.6\,A/m$

check: $\frac{1}{2}|\bar{E}||\bar{H}| = \frac{1}{2}(990)(2.6) = 1300\,W/m^2$ ✓