Tutorial-4

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Section - 4

Subject - PAA (Design and Analysis of Algorithms)

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Q1.

$$T(n) = 3T(n/2) + n^2$$

$$T(n) = aT(n/b) + f(n^2)$$

$$a \geq 1, \quad b \geq 1.$$

On comparing

$$a = 3, \quad b = 2, \quad f(n) = n^2$$

Now $$c = \log_b a = \log_2 3 = 1.584$$

$$n^c = n^{1.584} < n^2$$

$$\therefore f(n) > n^c$$

$$\therefore T(n) = O(n^2)$$

Q2.

$$T(n) = 4T(n/2) + n^2$$

$$a \geq 1, \quad b > 1$$

$$a = 4, \quad b = 2$$

$$c = \log_2 4 = 2$$

$$n^c = n^2 = f(n) = n^2$$

$$\therefore T(n) = \theta(n^2 \log \tfrac{n}{2})$$

Q3. $T(n) = T(n/2) + 2^n$

$a = 1, \; b = 2$

$f(n) = 2^n$

$c = \log_b^a = \log_2^c = 0$

$n^c = n^b = 1$

$f(n) > n^c$

$T(n) = \theta(2^n)$

=

Q4. $T(n) = 2^n T(n/2) + n^n$

$a = 2^n, \; b = 2$

$f(n) = n^n$

$c = \log_b^a = \log_2^{2^n}$

$= n$

$n^c \to n^n$

$f(n) = n^c$

$f(n) = \theta(n \log_2 n)$

Q5. $T(n) = 16\,T(n/4) + n$

$a = 16, \; b = 4$

$f(n) = n$

$c = \log_4 16 = \log_2 (4)^2 = 2 \log_4 4 = 2$

$n^c = n^2$

$n^c > f(n^2)$

$\therefore T(n) = \theta(n^2)$

Q6. $T(n) = 2\,T(n/2) + n \log n$

$a = 2, \; b = 2$

$f(n) = n \log n$

$c = \log_2 2 = 1$

$n^c = n$

$n \log n > n$

$f(n) > n^c$

$\therefore T(n) = \theta(n \log n)$

Q7. $T(n) = 2T(n/2) + n/\log n$

$a = 2, \; b = 2, \; f(n) = n/\log n$

$L = \log_2 2 = 1$

$n^L = n^1 = n$

$\frac{n}{\log n} < n$

$f(n) < n^L$

$\therefore T(n) = \theta(n)$

Q8. $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, \; b = 4, \; f(n) = n^{0.51}$

$L = \log_b a = \log_4 2 = 0.5$

$n^L = n^{0.5}$

$n^{0.5} < n^{0.51}$

$f(n) > n^L$

$\therefore T(n) = \theta(n^{0.51})$

Q9. $T(n) = 0.5\,T(n/2) + 1/n$

$a = 0.5, \; b = 2$

$a \geq 1$ but a is 0.5

So we cannot apply Master's Theorem

Q10. $T(n) = 16\,T(n/4) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

$n! > n^2$

$\therefore T(n) = \theta(n!)$

Q11. $4\,T(n/2) + \log n$

$a = 4,\ b = 2,\ f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \theta(n^c) = \theta(n^2)$

Q12. $T(n) = \text{sqrt}(n) + T(n/2) + \log n$

$a = \sqrt{n},\ b = 2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log n$

$\therefore \frac{1}{2}\log n < \log n$

$f(n) > n^c$

$T(n) = \theta(\log(n))$

Q13. $T(n) = 3T(n/2) + n$

$a = 3, \; b = 2, \; f(n) = n$

$c = \log_b^a = \log_2^3 = 1.58$

$n^c = n^{1.54}$

$n < n^{1.58}$

$f(n) < n^c$

$T(n) = \theta(n^{1.58})$

Q14. $T(n) = 3T(n/3) + sqrt(n)$

$a = 3 \; , b = 3$

$c = \log_b^a = \log_3^3 = 1$

$n^c = n^1 = n$

$sqrt(n) < n$

$f(n) < n^c$

$\therefore T(n) = \theta(n)$

Q15. $T(n) = 4T(n/2) + n$

$a = 4 \; , b = 2$

$c = \log_b^a = \log_2^4 = 2$

$n^c = n^2$

$n < n^2$ (for any constant)

$f(n) < n^c$

$T(n) = \theta(n^2)$

Q16. $T(n) = 3T(n/4) + n \log n$

$a = 3$, $b = 4$, $f(n) = n \log n$

$$L = \log_b^a = \log_4^3 = 0.792$$

$$n^L = n^{0.792}$$

$$n^{0.792} < n \log n$$

$$T(n) = \theta(n \log n)$$

Q17. $T(n) = 3T(n/3) + n/2$

$a = 3$, $b = 3$

$$L = \log_b^a = \log_3^3 = 1$$

$$f(n) = n/2$$

$$\therefore \quad n^L = n^1 = n$$

As $n/2 < n$

$$f(n) < n^L$$

$$\therefore T(n) = \theta(n)$$

Q18. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6$, $b = 3$

$$L = \log_b^a = \log_3^6 = 1.6309$$

$$n^L = n^{1.6309}$$

As $n^{1.6309} < n^2 \log n$

$$\therefore T(n) = \theta(n^2 \log n)$$

Q19. $T(n) = 4T(n/2) + n\log n$

$a = 4, \quad b = 2, \quad f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\therefore \frac{n}{\log n} < n^2$

$\therefore T(n) = \theta(n^2)$

Q20. $T(n) = 64T(n/8) - n^2\log n$

$a = 64, \quad b = 8$

$c = \log_b a = \log_8 64 = \log_8 8^2$

$c = 2$

$n^c = n^2$

$\therefore n^2\log n > n^2$

$T(n) = \theta(n^2\log n).$

Q21. $T(n) = 7T(n/3) + n^2$

$a = 7, \quad b = 3, \quad f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$

$T(n) = \theta(n^2)$

Q22. $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1 \quad , \quad b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos n) > n^c$

$T(n) = \theta(n(2 - \cos n))$