Math 271 Winter 2019

Assignment 3

Due on Thursday, March 21, 2019. Please **hand in your assignment to your lab instructor at the beginning of the lab on March 21, 2019**. Assignments must be understandable by the marker (i.e., logically correct as well as legible), and must be done by the student in his/her own words. Answer all questions, but only one question will be marked for credit. Please make sure that (i) the cover page has only your UCID number and your instructor's name (you might also want to draw some picture on the cover page so it is easily recognized), (ii) your name and ID numbers are on the top right corner of each of the remaining pages, and (iii) your assignment is **STAPLED**.

Please make sure that you hand in your assignment to the lab instructor of the lab that you enrolled in.

1. Let $f : A \to B$ and $g : B \to C$ be functions. Prove or disprove each of the following.
   (a) If $g \circ f$ is onto then f is onto.
   (b) If $g \circ f$ is onto then g is onto.
   (c) If $g \circ f$ is onto and g is one-to-one then f is onto.

2. Let $f : \mathbb{Z} \to \mathbb{Z}$ and $g : \mathbb{Z} \to \mathbb{Z}$ be functions defined by $f(x) = 3x - 1$ and $g(x) = \left\lfloor \frac{x+1}{2} \right\rfloor$. Prove or disprove each of the following:
   (a) $f \circ g$ is one-to-one.
   (b) $f \circ g$ is onto.
   (c) $g \circ f$ is one-to-one.
   (d) $g \circ f$ is onto.

3. Let $A = \{1, 2, 3, 4\}$. Let F be the set of all function from A to A. We note that the identity function $I_A : A \to A$ is defined by $I_A(x) = x$ for each $x \in A$.
   (a) Find a function $f \in F$ so that $f \neq I_A$ and $f \circ f = I_A$. Make sure to show that $f \circ f = I_A$.
   (b) Prove or disprove the statement: "For all $f \in F$, if $f \circ f = I_A$ then f is one-to-one and onto."
   (c) Prove or disprove the statement: "For all $f,\ g \in F$, if $f \circ f = g \circ g$ then $f = g$."
   (d) Prove or disprove the statement: "For all $f,\ g \in F$, if $f \circ g = g \circ f$ then $f = g$."