$Assignment\ 1$

Haohu Shen UCID: 30063099
CPSC 359 - Computing Machinery II
Feb 6, 2019

Step 1

State the problem in words:

A nine-LED display consists of 9 individual LEDs that could be turned on and off in specific combinations to display particular patterns using the labelling of the LEDs are from A to I and are arranged in a 3x3 matrix. There are totally 16 display patterns mapping to corresponding 16 bit-patterns as below:

- When the display shows that only A, B, C are on, it is mapping to bits 0000
- When the display shows that only D, E, F are on, it is mapping to the bits 0001
- When the display shows that only G, H, I are on, it is mapping to the bits 0010
- When the display shows that only A, D, G are on, it is mapping to the bits 0011
- When the display shows that only B, E, H are on, it is mapping to the bits 0100
- When the display shows that only C, F, I are on, it is mapping to the bits 0101
- When the display shows that only C, E, G are on, it is mapping to the bits 0110
- When the display shows that only A, E, I are on, it is mapping to the bits 0111
- When the display shows that only A, B, C, G, H, I are on, it is mapping to the bits 1000
- When the display shows that only A, D, G, C, F, I are on, it is mapping to the bits 1001
- When the display shows that only B, D, E, F, H are on, it is mapping to the bits 1010
- When the display shows that only A, B, C, E, G, H, I are on, it is mapping to the bits 1011
- When the display shows that only A, C, D, E, F, G, I are on, it is mapping to the bits 1100
- When the display shows that only A, C, E, G, I are on, it is mapping to the bits 1101
- When the display shows that only A, B, C, D, F, G, H, I are on, it is mapping to the bits 1110
- When the display shows that only E are on, it is mapping to the bits 1111

Designs a combinational logic circuit that takes a bit number of length of 4 as an input and generate 9 outputs, that is, from A to I, suppose that 0 means off and 1 means on.

Step 2

Input a 4-bit number(a binary number ranging from 0000 to 1111)

Output 9 LEDs of the display

Step 3

- There are 4 input variables, from left to right of the 4-bit number:
  - W indicates the first bit
  - X indicates the second bit
  - Y indicates the third bit
  - Z indicates the fourth bit
- There are 9 output variables, each labels one of the 9 LEDs, they are A, B, C, D, E, F, G, H, I.

Step 4

We create the truth table that defines the relationships between inputs and outputs as below:

W	X	Y	Z		A	B	C	D	E	F	G	H	I
0	0	0	0		1	1	1	0	0	0	0	0	0
0	0	0	1		0	0	0	1	1	1	0	0	0
0	0	1	0		0	0	0	0	0	0	1	1	1
0	0	1	1		1	0	0	1	0	0	1	0	0
0	1	0	0		0	1	0	0	1	0	0	1	0
0	1	0	1		0	0	1	0	0	1	0	0	1
0	1	1	0		0	0	1	0	1	0	1	0	0
0	1	1	1		1	0	0	0	1	0	0	0	1
1	0	0	0		1	1	1	0	0	0	1	1	1
1	0	0	1		1	0	1	1	0	1	1	0	1
1	0	1	0		0	1	0	1	1	1	0	1	0
1	0	1	1		1	1	1	0	1	0	1	1	1
1	1	0	0		1	0	1	1	1	1	1	0	1
1	1	0	1		1	0	1	0	1	0	1	0	1
1	1	1	0		1	1	1	1	0	1	1	1	1
1	1	1	1		0	0	0	0	1	0	0	0	0

Step 5

We obtain the simplified function for each output using the map method.

For output A, according to the truth table, we have

$$A = \sum(0, 3, 7, 8, 9, 11, 12, 13, 14) = W'X'Y'Z' + W'X'YZ + W'XYZ + WX'Y'Z' + WX'Y'Z + WX'YZ + WXY'Z' + WXY'Z + WXYZ'$$

Thus, after placing the values on the corresponding positions of 4-variable K-Map, we have

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$	1	0	1	0
$W'X$	0	0	1	0
WX	1	1	0	1
WX'	1	1	1	0

And we can decompose it into 5 groups below:

- Group 1

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX	1	1		
WX'	1	1		

And its simplified expression is WY'

- Group 2

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$			1	
$W'X$				
WX				
WX'			1	

And its simplified expression is $X'YZ$

- Group 3

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$			1	
$W'X$			1	
WX				
WX'				

And its simplified expression is $W'YZ$

- Group 4

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX	1			1
WX'				

And its simplified expression is WXZ'

- Group 5

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$	1			
$W'X$				
WX				
WX'	1			

And its simplified expression is $X'Y'Z'$

- Thus, we combine all simplified expressions from all groups and we have

$$A = WY' + X'YZ + W'YZ + WXZ' + X'Y'Z'$$

For output B, according to the truth table, we have

$$B = \sum(0, 4, 8, 10, 11, 14) \ = W'X'Y'Z' + W'XY'Z' + WX'Y'Z' + WX'YZ' + WX'YZ + WXYZ'$$

Thus, after placing the values on the correspoding positions of 4-variable K-Map, we have

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$	1	0	0	0
$W'X$	1	0	0	0
WX	0	0	0	1
WX'	1	0	1	1

And we can decompose it into 4 groups below:

- Group 1

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$	1			
$W'X$				
WX				
WX'	1			

And its simplified expression is $X'Y'Z'$

- Group 2

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX				1
WX'				1

And its simplified expression is WYZ'

- Group 3

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$	1			
$W'X$	1			
WX				
WX'				

And its simplified expression is $W'Y'Z'$

- Group 4

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX				
WX'			1	1

And its simplified expression is $WX'Y$

- Thus, we combine all simplified expressions from all groups and have

$$B = X'Y'Z' + WYZ' + W'Y'Z' + WX'Y$$

For output C, according to the truth table, we have

$$C = \sum(0, 5, 6, 8, 9, 11, 12, 13, 14) \ = W'X'Y'Z' + W'XY'Z + W'XYZ' + WX'Y'Z' + WX'Y'Z + WX'YZ + WXY'Z' + WXY'Z + WXYZ'$$

Thus, after placing the values on the correspoding positions of 4-variable K-Map, we have

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$	1	0	0	0
$W'X$	0	1	0	1
WX	1	1	0	1
WX'	1	1	1	0

And we can decompose it into 5 groups below:

- Group 1

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX	1	1		
WX'	1	1		

And its simplified expression is WY'

- Group 2

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				1
WX				1
WX'				

And its simplified expression is XYZ'

- Group 3

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$	1			
$W'X$				
WX				
WX'	1			

And its simplified expression is $X'Y'Z'$

- Group 4

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$		1		
WX		1		
WX'				

And its simplified expression is $XY'Z$

- Group 5

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX				
WX'		1	1	

And its simplified expression is $WX'Z$

- Thus, we combine all simplified expressions from all groups and have

$$C = WY' + XYZ' + X'Y'Z' + XY'Z + WX'Z$$

For output D, according to the truth table, we have

$$D = \sum(1, 3, 9, 10, 12, 14) \ = W'X'Y'Z + W'X'YZ + WX'Y'Z + WX'YZ' + WXY'Z' + WXYZ'$$

Thus, after placing the values on the corresponding positions of 4-variable K-Map, we have

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$	0	1	1	0
$W'X$	0	0	0	0
WX	1	0	0	1
WX'	0	1	0	1

And we can decompose it into 4 groups below:

- Group 1

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$		1	1	
$W'X$				
WX				
WX'				

And its simplified expression is $W'X'Z$

- Group 2

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX	1			1
WX'				

And its simplified expression is WXZ'

- Group 3

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$		1		
$W'X$				
WX				
WX'		1		

And its simplified expression is $X'Y'Z$

- Group 4

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX				1
WX'				1

And its simplified expression is WYZ'

- Thus, we combine all simplified expressions from all groups and have

$$D = W'X'Z + WXZ' + X'Y'Z + WYZ'$$

For output E, according to the truth table, we have

$$E = \sum(1, 4, 6, 7, 10, 11, 12, 13, 15) \ = W'X'Y'Z + W'XY'Z' + W'XYZ' + W'XYZ + WX'YZ' + WX'YZ + WXY'Z' + WXY'Z + WXYZ$$

Thus, after placing the values on the corresponding positions of 4-variable K-Map, we have

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$	0	1	0	0
$W'X$	1	0	1	1
WX	1	1	1	0
WX'	0	0	1	1

And we can decompose it into 4 groups below:

- Group 1

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$	1			1
WX				
WX'				

And its simplified expression is $W'XZ'$

- Group 2

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$			1	
WX			1	
WX'				

And its simplified expression is XYZ

- Group 3

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX	1	1		
WX'				

And its simplified expression is WXY'

- Group 4

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX				
WX'			1	1

And its simplified expression is $WX'Y$

- And the residual bool expressions which cannot be mapped are:

$$W'X'Y'Z$$

- Thus, we combine all simplified expressions from all groups, with the residual unmapped variables, and we have

$$E = W'XZ' + XYZ + WXY' + WX'Y + W'X'Y'Z$$

For output F, according to the truth table, we have

$$F = \sum(1, 5, 9, 10, 12, 14) \ = W'X'Y'Z + W'XY'Z + WX'Y'Z + WX'YZ' + WXY'Z' + WXYZ'$$

Thus, after placing the values on the correspoding positions of 4-variable K-Map, we have

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$	0	1	0	0
$W'X$	0	1	0	0
WX	1	0	0	1
WX'	0	1	0	1

And we can decompose it into 4 groups below:

- Group 1

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$		1		
$W'X$				
WX				
WX'		1		

And its simplified expression is $X'Y'Z$

- Group 2

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX	1			1
WX'				

And its simplified expression is WXZ'

- Group 3

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$		1		
$W'X$		1		
WX				
WX'				

And its simplified expression is $W'Y'Z$

- Group 4

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX				1
WX'				1

And its simplified expression is WYZ'

- Thus, we combine all simplified expressions from all groups and we have

$$F = X'Y'Z + WXZ' + W'Y'Z + WYZ'$$

For output G, according to the truth table, we have

$$G = \sum(2,3,6,8,9,11,12,13,14) = W'X'YZ' + W'X'YZ + W'XYZ' + WX'Y'Z' + WX'Y'Z + WX'YZ + WXY'Z' + WXY'Z + WXYZ'$$

Thus, after placing the values on the corresponding positions of 4-variable K-Map, we have

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$	0	0	1	1
$W'X$	0	0	0	1
WX	1	1	0	1
WX'	1	1	1	0

And we can decompose it into 4 groups below:

- Group 1

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX	1	1		
WX'	1	1		

And its simplified expression is WY'

- Group 2

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$			1	1
$W'X$				
WX				
WX'				

And its simplified expression is $W'X'Y$

- Group 3

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				1
WX				1
WX'				

And its simplified expression is XYZ'

- Group 4

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$			1	
$W'X$				
WX				
WX'			1	

And its simplified expression is $X'YZ$

- Thus, we combine all simplified expressions from all groups and we have

$$G = WY' + W'X'Y + XYZ' + X'YZ$$

For output H, according to the truth table, we have

$$H = \sum(2, 4, 8, 10, 11, 14) \ = W'X'YZ' + W'XY'Z + WX'Y'Z' + WX'YZ' + WX'YZ + WXYZ'$$

Thus, after placing the values on the corresponding positions of 4-variable K-Map, we have

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$	0	0	0	1
$W'X$	1	0	0	0
WX	0	0	0	1
WX'	1	0	1	1

And we can decompose it into 4 groups below:

- Group 1

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				1
$W'X$				
WX				
WX'				1

And its simplified expression is $X'YZ'$

- Group 2

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX				1
WX'				1

And its simplified expression is WYZ'

- Group 3

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX				
WX'	1			1

And its simplified expression is $WX'Z'$

- Group 4

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX				
WX'			1	1

And its simplified expression is $WX'Y$

- And the residual bool expressions which cannot be mapped are:

$$W'XY'Z'$$

- Thus, we combine all simplified expressions from all groups, with the residual unmapped variables, and we have

$$H = X'YZ' + WYZ' + WX'Z' + WX'Y + W'XY'Z'$$

For output I, according to the truth table, we have

$$I = \sum(2, 5, 7, 8, 9, 11, 12, 13, 14) \ = W'X'YZ' + W'XY'Z + W'XYZ + WX'Y'Z' + WX'Y'Z + WX'YZ + WXY'Z' + WXY'Z + WXYZ'$$

Thus, after placing the values on the corresponding positions of 4-variable K-Map, we have

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$	0	0	0	1
$W'X$	0	1	1	0
WX	1	1	0	1
WX'	1	1	1	0

And we can decompose it into 4 groups below:

- Group 1

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX	1	1		
WX'	1	1		

And its simplified expression is WY'

- Group 2

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$		1	1	
WX				
WX'				

And its simplified expression is $W'XZ$

- Group 3

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX	1			1
WX'				

And its simplified expression is WXZ'

- Group 4

	$Y'Z'$	$Y'Z$	YZ	YZ'
$W'X'$				
$W'X$				
WX				
WX'		1	1	

And its simplified expression is $WX'Z$

- And the residual bool expressions which cannot be mapped are:

$$W'X'YZ'$$

- Thus, we combine all simplified expressions from all groups, with the residual unmapped variables, and we have

$$I = WY' + W'XZ + WXZ' + WX'Z + W'X'YZ'$$

Step 6

NOTE The file *assign1.circ* has been uploaded to D2L dropbox. And diagrams for output from A to I are hand-written.