Math 271 Winter 2019

Assignment 4

Due on Thursday, April 4, 2019. Please **hand in your assignment to your lab instructor at the beginning of the lab on April 4, 2019**. Assignments must be understandable by the marker (i.e., logically correct as well as legible), and must be done by the student in his/her own words. Answer all questions, but only one question will be marked for credit. Please make sure that (i) the cover page has only your UCID number and your instructor's name (you might also want to draw some picture on the cover page so it is easily recognized), (ii) your name and ID numbers are on the top right corner of each of the remaining pages, and (iii) your assignment is **STAPLED**.

Please make sure that you hand in your assignment to the lab instructor of the lab that you enrolled in.

1. Let n be a positive integer. The relation congruence modulo n on the set $\mathbb{Z}$ is defined by:

for all $x,\ y \in \mathbb{Z}$, $x \equiv y \pmod{n}$ if and only if $n \mid (x - y)$.

(a) Use the Euclidean Algorithm to compute $\gcd(271, 98)$ and use that to find integers x and y so that $\gcd(271, 98) = 271x + 98y$.

(b) Use part (a) to find an inverse a of 98 modulo 271 so that $0 < a < 271$; that is, to find the integer a so that $271a \equiv 1 \pmod{271}$ and $0 < a < 271$. Make sure to verify that a is an inverse of 98 modulo 271 using the definition of congruence modulo n.

(c) Find an integer b so that $98b \equiv 99 \pmod{271}$ and $0 < b < 271$. Make sure to verify that b satisfies the condition $98b \equiv 99 \pmod{271}$ using the definition of congruence modulo n.

2. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let $T = \{2, 4, 6, 8\}$. Let R be the relation on $\mathcal{P}(A)$ defined by

for all $X,\ Y \in \mathcal{P}(A)$, $(X, Y) \in R$ if and only if $|X - T| = |Y - T|$.

(a) Prove that R is an equivalence relation.

(b) How many equivalence classes are there? Explain.

(c) How many elements of $[\varnothing]$, the equivalence class of $\varnothing$, are there? Explain.

(d) How many elements of $[\{1, 2, 3, 4\}]$, the equivalence class of $\{1, 2, 3, 4\}$, are there? Explain.

3. Let $A = \{1, 2, 3, 4\}$. For any function $f : A \to A$ and any relation R on A, we define the relation S on A by:

for any $a, b \in A$, $\quad aSb$ if and only if $(f(a), f(b)) \in R$.

For each of the following statements, prove or disprove the statement.

(a) For all functions $f : A \to A$ and all relations R on A, if S is reflexive then R is reflexive.

(b) For all functions $f : A \to A$ and all relations R on A, if R is reflexive then S is reflexive.

(c) For all functions $f : A \to A$ and all relations R on A, if S is symmetric then R is symmetric.

(d) For all functions $f : A \to A$ and all relations R on A, if R is symmetric then S is symmetric.