Math 271 Winter 2019

Assignment 2

Due on Thursday, February 14, 2019. Please **hand in your assignment to your lab instructor at the beginning of the lab on February 14, 2019**. Assignments must be understandable by the marker (i.e., logically correct as well as legible), and must be done by the student in his/her own words. Answer all questions, but only one question will be marked for credit. Please make sure that (i) the cover page has only your UCID number and your instructor's name (you might also want to draw some picture on the cover page so it is easily recognized), (ii) your name and ID numbers are on the top right corner of each of the remaining pages, and (iii) your assignment is **STAPLED**.

Please make sure that you hand in your assignment to the lab instructor of the lab that you enrolled in.

1. Let $a,\ b,\ n$ be integers where $n \geq 1$ and $a \neq b$.
   (a) Show algebraically that $a^{n+1} - b^{n+1} = a\,(a^n - b^n) + b^n(a - b)$.
   (b) Use part (a) to prove by induction on n that $(a - b) \mid (a^n - b^n)$ for all integers $n \geq 1$.
   (c) Use part (b) to prove that $11 \mid \left(7^{271} + 4^{271}\right)$.

2. The sequence $a_1,\ a_2,\ a_3, ...$ is defined by: $a_1 = 0$ and for integers $n \geq 1$, $a_{n+1} = a_n + 2n + 1$.
   (a) Calculate $a_2,\ a_3,\ a_4$ and a_5.
   (b) Use part (a) to guess a formula for a_n for all positive integers n.
   (c) Prove by induction on n that your guess in part (b) is correct.
   (d) Prove that a_n is composite for all integers $n \geq 3$.

3. The sequence $b_0,\ b_1,\ b_2,\ ...$ is defined as follows: $b_0 = 0,\ b_1 = \frac{1}{2}$, and for integers $n \geq 2,\ b_n = \sqrt{b_{n-1}b_{n-2}} + \frac{3n}{2} - 1$.
   (a) Calculate $b_2,\ b_3,\ b_4$ and b_5.
   (b) Use part (a) to guess a formula for b_n for all integers $n \geq 0$.
   (c) Prove by induction on n that your guess in part (b) is correct.