Math 271 Winter 2019

Assignment 1

Due on Thursday, January 31, 2019. Please **hand in your assignment to your lab instructor at the beginning of the lab on January 31st, 2019**. Assignments must be understandable by the marker (i.e., logically correct as well as legible), and must be done by the student in his/her own words. Answer all questions, but only one question will be marked for credit. Please make sure that (i) the cover page has only your UCID number and your instructor's name (you might also want to draw some picture on the cover page so it is easily recognized), (ii) your name and ID numbers are on the top right corner of each of the remaining pages, and (iii) your assignment is **STAPLED**.

Please make sure that you hand in your assignment to the lab instructor of the lab that you enrolled in.

1. Let $\mathcal{P}$ be the statement: "For all real numbers x and y, if $x - \lfloor x \rfloor < \frac{1}{2}$ and $y - \lfloor y \rfloor < \frac{1}{2}$ then $\lfloor x+y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$.
   (a) Is $\mathcal{P}$ true? Prove your answer.
   (b) State the converse of $\mathcal{P}$. Is the converse of $\mathcal{P}$ true? Prove your answer.
   (c) State the contrapositive of $\mathcal{P}$. Is the contrapositive of $\mathcal{P}$ true? Explain.
   (d) State the negation of $\mathcal{P}$. Is the negation of $\mathcal{P}$ true? Explain.

2. Let $\mathbb{Z}^+$ be the set of all positive integers.
   (a) Use the Euclidean Algorithm to compute $\gcd(2019, 271)$ and use that to find integers x and y so that $\gcd(2019, 271) = 2019x + 271y$.
   (b) Find integers m and n so that $\gcd(2019, 271) = 2019m + 271n$, but $m \neq x$ and $n \neq y$. Note that x and y are the integers you found in part (a).
   (c) Is it true that $\forall a, b, c \in \mathbb{Z}^+$, $\gcd(a, c) + \gcd(b, c) = \gcd(a+b, c)$. Prove your answer.
   (d) Is it true that $\exists a, b, c \in \mathbb{Z}^+$ so that $\gcd(a, c) + \gcd(b, c) = \gcd(a+b, c)$. Prove your answer.

3. Prove for disprove each of the following statements.
   (a) For all integers x, $x^3 + x$ is even.
   (b) For all integers y, there is an integer x so that $x^3 + x = y$.
   (c) For all integers x and y, if $x^3 + x = y^3 + y$ then $x = y$.