CPSC 313 — Winter 2020
Assignment 2 — Context-Free Languages and Grammars

Due Monday, March 9, at 11:55 pm on Gradescope

Prior to submission, be sure to familiarize yourself with the **Policies and Guidelines** as well as the **Submission Procedure** as detailed on the assignments course webpage

http://people.ucalgary.ca/~rscheidl/313/assignments.html.

Assignments that don't follow these instructions will incur penalties, possibly even a score of zero.

1. **Non-regular languages and the Pumping Lemma**

   Let $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, =\}$ and consider the language L of all strings over Σ that constitute a valid and correct equation of the form $a + b = c$ where a, b, c are non-negative integers represented in base 10, without leading zeros. Some elements of L include $13 + 17 = 30$ and $99 + 0 = 99$, but not $13 + 17 = 29$ (wrong arithmetic) or $99 + 01 = 100$ (leading zero in the number 1). Use the Pumping Lemma to prove that L is not regular.

2. **Regular languages are context-free**

   Formally prove that every regular language is context-free. Use the following ingredients in your proof.

   - The recursive definition of regular expressions;
   - The fact that L is a regular language if and only if $L = L(e)$ for some regular expression e;
   - Strong induction on the length of a regular expression — recall that every regular expression is a string of length at least 1 consisting of elements in Σ as well as the symbols $\cup, *, (,), \varepsilon, \emptyset$;
   - The fact that context-free languages are closed under the regular operations; that is, if L_1 and L_2 are any context-free languages, then $L_1 \cup L_2$, $L_1 L_2$ and L_1^* are context-free. (You may use this result without proof; we will prove it in Week 7 of the course.)

3. **Designing context-free grammars and languages**

   (a) Design a context-free grammar for the language

   $$L = \left\{ a^{2i} b^j v c^j (ac)^i \mid i, j \geq 0, v \in \{a, b\}^* \right\}$$

   over the alphabet $\Sigma = \{a, b, c, \}$. Your grammar must have at most 3 variables and at most 7 rules. Clearly state the variables, the terminals, the rules, and the start variable for your grammar. You need *not* formally prove your grammar correct, but you should give a concise and convincing explanation of its correctness (in case of errors, such an explanation may also secure you partial credit).

(b) Consider the context-free grammar $G = (V, \Sigma, R, S)$ where $\Sigma = \{a, b, c\}$, $V = \{S, A, B, C\}$, S is the start variable and R consists of the rules

$$\begin{aligned} S &\to ASA \mid B \\ A &\to a \mid b \\ B &\to BC \mid \varepsilon \\ C &\to cc \end{aligned}$$

Give a formal description of $L(G)$, in the form $L(G) = \{\cdots \mid \cdots\}$. You need not formally prove your language correct, but you should again give a concise, coherent, convincing explanation of how you obtained your answer. (Again, in case of errors, such an explanation may help you gain partial credit for this problem).