Resistors and Resistive Networks

1.1 Objectives

In this lab, you will be examining the behavior of three simple resistive networks: a two-way resistive voltage divider, a two-way resistive current divider, and an R-$2R$ ladder network. Each of these simple networks find applications in more complicated circuits, some of which we will see in later labs. In the process, you should become familiar with the operation of the two-channel USB source-measure unit (SMU) that you will be using through out the course and with the use of MATLAB for analyzing and plotting experimental data.

1.2 Prelab

The following prelab questions have been constructed to help you prepare to do the lab efficiently. Please complete these questions *before* you come to lab. While you may discuss the prelab questions with your lab partner or with other students in the class, each student in a lab group should complete the prelab assignment individually, so that you each understand the circuit(s) that you will be testing and what you will be doing in the lab.

1. **Electrical Measurement Concepts**. Choose the phrases which correctly complete each of the following statements.

   (a) An ideal voltage meter has **(zero|infinite)** internal resistance and should be connected in **(series|parallel)** with the circuit or device being tested.

   (b) An ideal current meter has **(zero|infinite)** internal resistance and should be connected in **(series|parallel)** with the circuit or device being tested.

2. **Resistive Divder Accuracy**. Two-way resistive voltage and current divider ratios are both expressible as the ratio of one resistance to the sum of two resistances. Suppose that each resistance in the divider is subject to some finite tolerance (e.g., typically $\pm 5\%$ for a 0.25-W carbon resistor or $\pm 1\%$ for a metal-film resistor) about its nominal value. Determine how the tolerance on the divider ratio depends on the tolerances of the individual components.

3. **R-2R Ladder Network**. Consider the circuit shown in Fig. 1.1. It is called an R-$2R$ *ladder network*, because if you were to turn it sideways it would look like a ladder. This type of circuit is commonly used to build digital-to-analog (D/A) converters to interface computers to the real world, as you will be considering in the postlab section. As a function of the applied voltage, V, and the unit resistance, R, what are the currents, I_1 through I_N, that flow through each of the $2R$ branches?

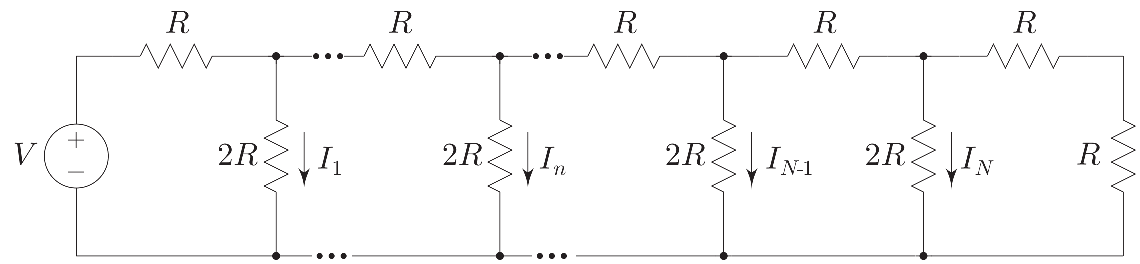

Figure 1.1: An R-$2R$ ladder network.

4. **Accurate 2:1 Resistor Ratios**. In order to obtain accurate current division at each branch in the R-$2R$ ladder network shown in Fig. 1.1, we need to have accurate 2:1 ratios between the various branches in the network. When designing integrated circuits, we obtain the most accurate ratios (for rational numbers) when we use an integral number of well-matched unit-sized devices. In this case, we would like to construct two effective resistances in a 2:1 ratio from three identical resistors, each with a resistance R. Come up with two different methods to accomplish this goal. In building an R-$2R$ ladder network, which method uses fewer unit resistors?

1.3 Experiments

You will be doing four experiments in this lab. In the first experiment, you will be measuring the resistance of a 0.25-W resistor in various ways and comparing the value that you obtain from each method. In the second experiment, you will be constructing a two-way resistive voltage divider, measure its voltage transfer characteristic, and extracting the actual divider ratio. In the third experiment, you will be constructing a two-way resistive current divider, measuring its current transfer characteristic, and extracting the actual divider ratio. In the fourth experiment, you will be constructing an R-$2R$ ladder network and measuring the currents in each stage as a function of positition in the ladder. For Experiments 2, 3, and 4, you will be using Bourns dual in-line thick-film resistor array chips. These sixteen-pin chips have eight independent resistors that are reasonably well matched integrated into a single package. The pinout for this chip is shown in Fig. 1.2. The tolerance on their absolute values is $\pm 2\%$.

In your lab report, you should include graphs of all theoretical and experimental curves. In general, you should plot the measurements in a point style so the individual points are distinguishable. Any theoretical fits to the data should be plotted on the same graph as the experimental data in a line style.

1.3.1 Experiment 1: Resistance Measurement

Obtain a 0.25-W resistor from the ECE stockroom whose resistance is $1\,\mathrm{k\Omega}$ or larger. Record the resistance as indicated by the color code printed on the resistor. Next, use a Keithley

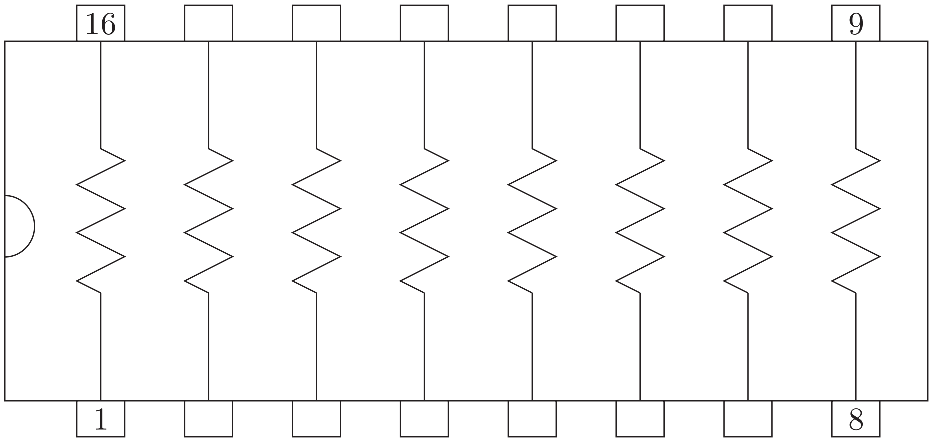

Figure 1.2: Pinout of the Bourns isolated thick-film resistor array.

2400 SourceMeter in resistance mode to measure the resistance of your resistor. Finally, use one channel of the SMU to measure the resistor's current–voltage characteristic. Take at least 50 points on the characteristic. Using MATLAB, fit a straight line to this curve and extract the value of the resistance from the slope of this characteristic. Make a plot showing both the measured data (as individual points) and the theoretical fit (as a solid line) along with the extracted resistance value. Compare the three values of resistance that you have obtained for this resistor. Comment on any discrepancies and suggest possible explanations for them. Which technique do you think is the most accurate? Why?

1.3.2 Experiment 2: Resistive Voltage Division

Obtain a two Bourns resistor array chips and measure all of the resistance values on both chips with a Keithley 2400 SourceMeter in resistance mode. Within a single chip, how do the resistances match each other compared to the $\pm 2\%$ tolerance specified for their absolute values? How do the resistance values on one chip compare to those on the second chip? In your report, include a table showing all of the resistor values from both chips. You will receive five bonus points on your lab grade if you send all 16 of your measured resistance values via electronic mail to `bradley.minch@olin.edu` as a two-line comma-separated-values (CSV) file in which each line corresponds to the eight resistance values from one of the resistor array chips. (Note that the CSV file is not a substitute for the table of values in your lab report.)

Construct a two-way resistive voltage divider in which the divider ratio is a ratio of two small integers from the resistors on one of your array chips by connecting these unit resistors in series or in parallel. With one channel of the SMU, apply an input voltage to the divider as you measure the output voltage with the other channel of the SMU. Measure the output voltage as a function of input voltage. Using MATLAB, fit a straight line to the divider's voltage transfer characteristic and extract the value of the divider ratio. Make a plot showing both the measured data and the theoretical fit along with the extracted value of the divider ratio. How does the actual divider ratio compare to the theoretical one? Is this discrepancy consistent with the level of resistance mismatch that you observed in your resistor array?

1.3.3 Experiment 3: Resistive Current Division

Next, construct a two-way resistive current divider in which the divider ratio is a ratio of two small integers from the resistors in your array by connecting these resistors in series or parallel. With one channel of the SMU, apply an input current to the divider as you measure the output current with the other channel of the SMU. Measure the output current as a function of input current. Using MATLAB, fit a straight line to the divider's current transfer characteristic and extract the value of the divider ratio. Make a plot showing both the measured data and the theoretical fit along with the extracted value of the divider ratio. How does the actual divider ratio compare to the theoretical one? Is this discrepancy consistent with the level of resistance mismatch that you observed in your resistor array?

1.3.4 Experiment 4: R-2R Ladder Network

From your two resistor array chips, construct an R-$2R$ ladder network with four $2R$ branches. You should be able to build an R-$2R$ ladder with two $2R$ branches from a single chip. Apply a voltage to the network with one channel of the SMU and measure the current flowing in each of the $2R$ branches in turn with the other SMU channel. Change the externally applied voltage and repeat these current measurements. Enter your measurements into MATLAB and make a semilog plot of current as a function of position for both measurements on a single graph along with appropriate theoretical expected values. Do these currents vary with position as you expect?

1.4 Postlab

We live in a physical world of analog signals, such as light intensity, sound pressure, force, position, acceleration, and temperature, whose values can vary continuously. As digital computers have become smaller, cheaper, and more powerful, we have tried to put these remarkable devices almost everywhere in order to monitor and control physical systems in ways that, a few years ago, would have been inconceivable. Information is usually represented inside of digital computers as binary numbers, a representation that is usually not directly compatible with the analog world. Consequently, if our computer systems are to monitor and control bits and pieces of the physical world, we need to build some kind of system that can convert both from the analog representation to the digital format and from the digital format back to the analog format. The former devices are called *analog-to-digital* (A/D) converters while the latter devices are called *digital-to-analog* (D/A) converters.

Binary numbers are just like decimal numbers, except that the value of each *b*inary dig*it* (bit) can only take on the values of 0 or 1 instead of any integral value between 0 and 9. Also, in the binary number system, the weight assigned to each position is a successive power of two instead of a power of ten, as it is in the decimal system; that is, for a binary number, we have a ones position (i.e., 2^0), a twos position (i.e., 2^1), and a fours position (i.e., 2^2), instead of a ones position (i.e., 10^0), a tens position (i.e., 10^1), and a hundreds position (i.e.,

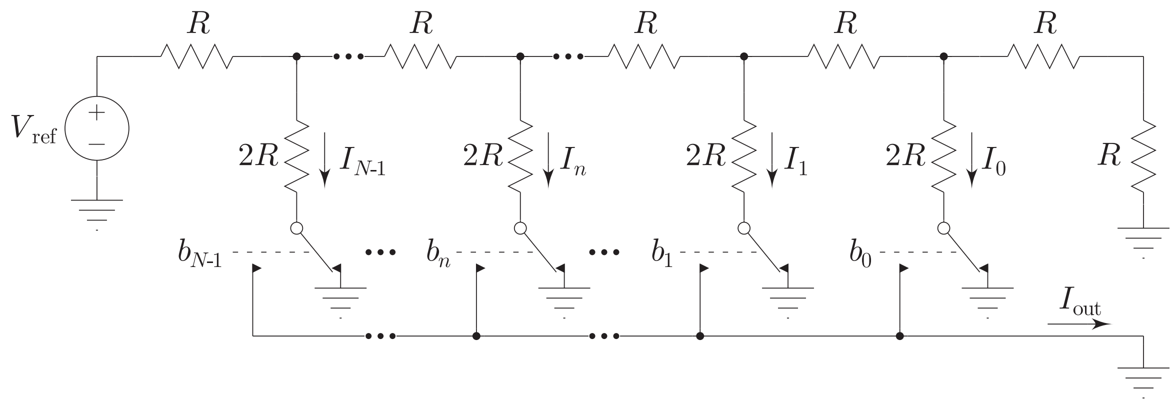

Figure 1.3: A conceptual N-bit R-$2R$ ladder D/A converter. The position of each switch is controlled by a different bit of the N bit binary number to be converted into an analog current value. In this case, all switches shown are in the 0 position.

10^2) that we would have for a decimal number. Accordingly, the value of a binary number that has been expressed as a string of N bits $b_{N-1} \dots b_1 b_0$ is given by

$$\sum_{n=0}^{N-1} b_n 2^n,$$

where b_n is the value of the nth bit, which can be equal either to 0 or to 1.

An N-bit D/A converter is an analog circuit that takes N bits and produces an analog signal (usually either a current or a voltage) whose value is proportional to the sum just given. In order to perform this operation, we need to produce voltages or currents that are in ratios of successive powers of two, and then to add them up selectively, depending on whether the value of a particular bit is zero or one. If a given bit is a zero, then the current or voltage at its position is not added into the sum. If a given bit is a one, then the current or voltage at its position is added to the sum.

Consider the circuit shown in Fig. 1.3, called an R-$2R$ *ladder network*, which is a commonly used structure in building D/A and A/D converters. Derive an expression relating the current flowing through each of the $2R$ resistors to the reference voltage, V_{ref} and to the *unit resistance*, R. In the circuit shown in Fig. 1.3, each switch selectively steers one of these currents to the left or to the right based on the value of one of the binary digits in an N-bit binary number. Those currents that are steered to the left are combined to form an output current, I_{out}, flowing into ground. Derive an expression relating I_{out} to the values of the binary digits (i.e., $b_{N-1}, \dots, b_0$), the reference voltage (i.e., V_{ref}), and the unit resistance (i.e., R). Please carefully note the differences between Fig. 1.1 and Fig. 1.3. Your answers must be expressed using the indexing scheme shown in Fig. 1.3 in order to receive full credit.