

Our goal in this lab is to find the stiffness and natural length of the 6 rubber bands that will be used in our jungle bridge project. We accomplished this by measuring the length of the rubber bands under different weights, which gave us a series of data points that relates F (mass of the weights $\times g$) $= m * l + b$. where l is the length of the rubber band while being pulled by the weights. To find the line that best predicts the force applied on the rubber band, you optimize the cost function, $E(m, b) = \sum_{i=1}^n (m * x_i + b - y_i)^2$ (which represents the relationship between length and force) for the smallest difference in your line and the measured force of the data. In this case x is the measured lengths and y is the measured force. The way you optimize the cost function is by taking the derivative of that function and setting it equal to 0, and solving for m and b . This will give you the m and b which outputs a line that is as close to every point as possible. The derived formula for this is:

$$\begin{bmatrix} m \\ b \end{bmatrix} = (A^T A)^{-1} A^T Y$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \text{ and } A = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix}$$

Once you have minimized the cost function, you can compare $F = m * l + b$ to $F = k * l + k * l_0$. (The second equation is Hooke's Law), and because the two equations are of the same form we know this must be true: $k = m$, and $b = -l_0 * k$, so: $l_0 = -\frac{b}{k}$. This provides us with the k and l_0 values, or stiffness and natural length for any rubber band we have data for.