Paper Summary: Stochastic Poisson Surface Reconstruction

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1 Contribution

A statistical extension of Poisson surface reconstruction where the constructed surface is regarded as a Gaussian Process. It provides a new understanding of PSR, and enables more versatile applications, such as:

- Query the possibility that a point in space is inside/outside the surface (would be useful in tasks like collision detection and automated driving).
- Provide feedback information quantifying scan quality so that we can know when to terminate scanning.
- Utilize priors to improve the reconstruction result (mean of Gaussian process) when the point cloud data is incomplete

2 Method

Given a point cloud equipped with normal vectors, traditional PSR is divided into two primary steps:

1. Build a grid where each vertex stores a normal vector interpolated/extrapolated from the original data
2. Build an implicit function on the grid by taking the above vector field as its gradient. This is equivalent to solving a PDE (Possion equation).

Firstly, SPSR modifies the formula of vector field interpolation by defining PSR semicovariance and then replacing it with a symmetric counterpart, Stochastic PSR covariance. The key observation is that the modified interpolation formula can be interpreted as the mean of a supervised learning task under the assumption of a Gaussian Process with $m = 0$ and Stochastic PSR covariance.

Gaussian process is a collection of random variables parameterized by some continuous parameter x. Any finite subset of it follows a multivariate Gaussian distribution. In the surface reconstruction context, we can view the surface normal field as a Gaussian process where the random variables are normal vectors and the continuous parameter is the position of any point on the surface. Any scanned point cloud data (with normal vectors) is a subset of this Gaussian process.

Now we just need to do some linear algebra to calculate the stochastic vector field. However, the K_3^{-1} is computationally expansive, especially when the point cloud data grows. #1 A simple solution is taking the assumption that samples are independent, but it introduces some unbalance in the resulting vector field influenced by the sampling density (but why?). #2 This paper proposes to use a technique similar to the mass matrix lumping step in finite element literature: making each sample's variance proportional to sampling density.

Secondly, using the linear operators used in the Poisson equation and the properties of mean and variance, we can transform the above formulas for the stochastic normal field to get formulas for the stochastic implicit function, by just adding some linear operations. The stochastic implicit function formulation is the core contribution of this paper.

The reconstructed stochastic implicit function field now has the meaning of probability, so we can make statistical queries/quantify uncertainties using CDF and PDF.

Then we can define total uncertainty by integrating the "absolute deviation between the probability and 0.5" in the bounding box. This quantity is useful to quantify the quality of scanning.

The traditional PSR can be viewed as taking zero-norm gradient ($m = 0$) as its prior. Fortunately, with SPSR we can easily use other priors to improve the reconstruction result when the data is incomplete.

I think one main difference between traditional PSR and SPSR is that PSR cares only about the values near the surface while SPSR builds a (meaningful) field in the whole space.

3 Question?

I have a tiny question in section 4.1, about "the probability density of being on the surface". Should the formula be $p(o_i \in \partial\Omega)$ instead of $p(o_i \in \Omega)$?