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November 14, 2023

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1 Introduction

This lab aimed to measure charge distribution over the surface of various shapes in differing scenarios. This objective was accomplished by sampling a point on the object's surface using a proof plane and measuring the voltage using a Faraday Ice Pail. Since voltage is a measure of electric potential difference per unit charge, the changes in voltage were analogous to changes in charge, which is why no conversions between the two were made. Figure 1 depicts how charge density was measured.

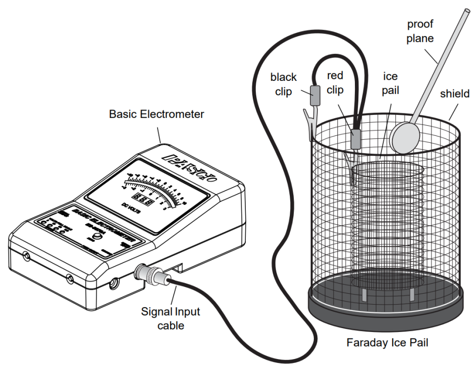

Fig. 1. Diagram of the equipment used in the experiments (Basic Electrometer Manual, n.d.)

The first experiment utilised two sampling spheres to explore a charged sphere's effect on a grounded sphere. This experiment aimed to observe if a non-uniform charge distribution arose on the sampling sphere due to the attraction of its free-moving electrons towards the positively charged sphere.

The second and third experiments aimed to analyse the effect a conductor's shape has on its charge distribution. Measurements for experiments 2 and 3 were taken at different points over the surface of a charged conical shape and hollow sphere, respectively.

2 Results

2.1 Addressing Experimental Error

Throughout all of the experiments, it was noted that there was a high amount of noise and variability in the results. Furthermore, taking measurements was difficult, as the voltage on the electrometer dropped from the moment the sample was placed in the ice pail. It was reasoned that this was due to a fault in the equipment. Possible reasons for this variability could be a grounding issue with the ice pail, a lack of electric insulation of the proof plane, a buildup of static charge on the person performing the experiment, or due to other external factors such as humidity.

Regardless of the source of error, five trials were performed to account for the randomness in results. It was ensured that each trial was entirely independent of the other to address any variables that may carry over. To achieve this, the proof plane was grounded between trials; in addition, the pail and shield of the ice pail were connected to discharge any residual charge still contained in the ice pail from previous experiments. Finally, the sampling sphere (in experiment 1) was set aside from the charged sphere before being grounded to not give the sampling sphere a net charge (this is further elaborated in the discussion for the first experiment).

2.2 Error Analysis Methods

Mathematical functions able to model the factors affecting charge distribution were beyond the scope of this experiment. Because of this, more analysis was done on the raw data and its relation to the expected theoretical value. Five trials are insufficient to calculate a standard deviation. Therefore, a maximum residual and maximum error was calculated for each point over multiple trials.

Mean:

$$\overline{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$

Maximum Absolute Residual:

$$\mathrm{MR} = \max\left(|x_i - \overline{x}|\right)$$

Maximum Error:

$$\mathrm{ME} = \frac{\mathrm{MR}}{\sqrt{N}}$$

An additional residual calculation was performed in cases with a uniform charge density. This residual was calculated for every point with respect to the average charge of the entire sampling sphere. This ”per-trial” residual gave an overall maximum residual comparing the charge at different points in a given trial. This residual calculation was done in conjunction with the established residual calculation that gave an overall maximum residual comparing the charge at the same points across the five trials.

Additionally, the digital display of the voltmeter (Model ES-9078A Electrometer) had an inherent uncertainty of $\pm$ 0.1V. While this does not take into account the experimental difficulties encountered with this instrument as discussed earlier, this was deemed a sufficient value of uncertainty.

2.3 Experiment 1 - 2 spheres

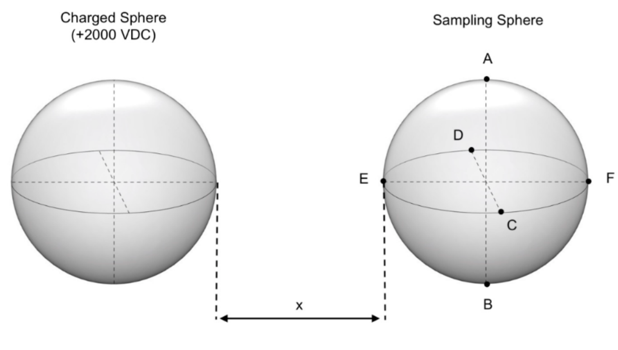

Fig. 2. Experiment 1: Model of the two conductive spheres, including the points measured

2.3.1 Part 1

Table 1: Results From Two Conductive Spheres 50 cm apart

	CHARGE (V)							
POINTS	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	**Average**	**MR**	**ME**
A (Top)	0.9	0.2	0	0	-0.1	0.2	0.7	0.313049517
B (Bottom)	-0.4	-0.1	-0.1	0.1	0	-0.1	0.3	0.134164079
C (Front)	0.9	0.3	0	0.4	0.1	0.34	0.56	0.250439613
D (Back)	0.4	0.4	0.4	0.4	0.1	0.34	0.24	0.107331263
E (Left)	0.1	0.1	0	0.1	0.1	0.08	0.08	0.035777088
F (Right)	0.8	0.9	0.3	0.3	0.1	0.48	0.42	0.18782971
Overall Average	0.223333333							
Overall MR	0.676666667							
Overall ME	0.123542							

The results from two conductive spheres 50 cm apart, as seen in Table 1, show an extremely low voltage with an overall average of 0.22V. The different points on the sphere are similarly charged with very little difference between points. Therefore, the charges on the sphere can be considered uniform.

2.3.2 Part 2

Table 2: Results From Two Conductive Spheres 1 cm apart

	CHARGE (V)							
POINTS	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	**Average**	**MR**	**ME**
A (Top)	0.4	0.5	0.6	0.3	0.4	0.44	0.16	0.071554175
B (Bottom)	-0.1	-0.7	-1.2	-1.5	-1.5	-1	0.9	0.402492236
C (Front)	0.4	0.3	0.3	0.3	0.1	0.28	0.18	0.080498447
D (Back)	0.1	0.1	-0.1	-0.1	0.1	0.02	0.12	0.053665631
E (Left)	-6.3	-6.2	-7.1	-5.3	-6.5	-6.28	0.98	0.438269324
F (Right)	0.7	0.9	0.9	0.6	0.7	0.76	0.16	0.071554175

The results from two conductive spheres 1 cm apart, as seen in Table 2, show a significant disparity in charge between point E(Left) and the other points on the sphere. Points A, C and D all have a roughly neutral charge(0.44 V, 0.28 V, 0.02 V), while points B and E have a negative charge(-1 V, -6.28 V), and point F has a slightly positive charge(0.76 V).

2.3.3 Part 3

Table 3: : Results From Two Conductive Spheres 1 cm apart, after momentarily grounding the sampling sphere

	CHARGE (V)							
POINTS	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	**Average**	**MR**	**ME**
A (Top)	-1.1	-1.5	-0.1	-0.3	-0.7	-0.74	0.76	0.339882333
B (Bottom)	-2.5	-0.8	-1.3	-1.1	-1.6	-1.46	1.04	0.465102139
C (Front)	-1.3	-1.4	-1.2	-0.9	-0.3	-1.02	0.72	0.321993789
D (Back)	-0.8	-1.4	-0.7	-0.6	-2.6	-1.22	1.38	0.617154762
E (Left)	-8	-5.5	-7.4	-8.5	-6.7	-7.22	1.72	0.769207384
F (Right)	-0.4	0.5	0.5	0.6	0.7	0.38	0.78	0.348826604

The results from two conductive spheres 1 cm apart after momentarily grounding the sampling sphere, as seen in Table **??**, show a significant charge disparity between point E(Left) and the other points on the sphere. Point E is significantly more negative than other points, with an average of -7.22 V. Point F is the only positive point with an average of 0.78 V. Points A, B, C, and D are all roughly similarly changed (-.074 V, -1.46 V, -1.02 V, -1.22 V).

2.3.4 Part 4

The results from two conductive spheres 50 cm apart that were momentarily grounded while next to the charged sphere, as seen in Table **??**, show a uniform charge distribution with

Table 4: : Results From Two Conductive Spheres 50 cm apart, after having momentarily grounded the sampling sphere in the previous step

	CHARGE (V)							
POINTS	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	**Average**	**Residual**	**ME**
A (Top)	-1.2	-1.8	-1.5	-0.8	-1.8	-1.42	0.62	0.277272429
B (Bottom)	-1.9	-1.7	-2.5	-2	-2	-2.02	0.48	0.214662526
C (Front)	-1.3	-1.5	-1.8	-1.2	-1.7	-1.5	0.3	0.134164079
D (Back)	-0.5	-1.5	-2	-1.8	-1.3	-1.42	0.92	0.411436508
E (Left)	-1.5	-1.6	-1.8	-1.5	-1.5	-1.58	0.22	0.098386991
F (Right)	-0.9	-1.6	-1.7	-1.3	-2	-1.5	0.6	0.268328157
Overall Average	-1.573333333							
Overall MR	1.073333333							
Overall ME	0.195963							

a net negative charge. All points have a roughly equal charge with an overall average of -1.57V.

2.4 Experiment 2 - Conical Shape

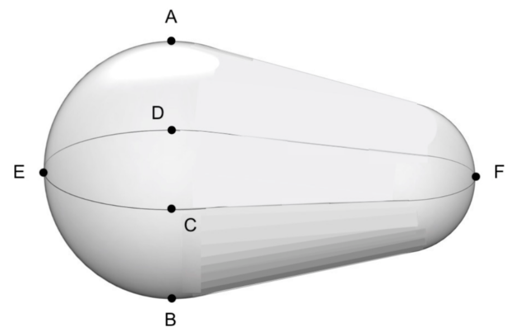

Fig. 3. Experiment 2: Conical Shape Diagram

Table 5: : Results From The Conductive Conical Shape

	CHARGE (V)							
POINTS	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	**Average**	**MR**	**ME**
A (Top)	5.2	4.9	5.9	5.7	6	5.54	0.64	0.286216701
B (Bottom)	4.8	5.4	4.9	4.7	4	4.76	0.76	0.339882333
C (Front)	6.9	5.6	4.6	5.3	4.9	5.46	1.44	0.643987578
D (Back)	6.4	4.8	5.2	5.8	5.3	5.5	0.9	0.402492236
E (Left -Larger end)	4.8	6	5.9	4.8	5.9	5.48	0.68	0.304105245
F (Right - Narrow end)	8	7.8	7.6	6.6	7.2	7.44	0.84	0.37565942

The results from the conductive conical shape, as seen in Table 5, show a uniform charge on the larger end and a larger charge density on the narrower end. Points A, C, D and E have roughly similar charge densities (5.54 V, 5.46 V, 5.5 V, 5.48 V). Point B has a slightly lower charge density of 4.76 V. Point F(Right - Narrow end) has a higher charge density with an average of 7.44 V.

2.5 Experiment 3 - Hollow Sphere

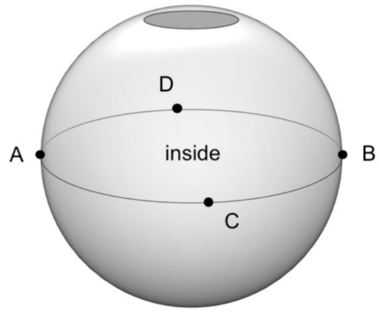

Fig. 4. Experiment 3: Hollow Sphere Diagram

Table 6: : Results From The Conductive Hollow Sphere

	CHARGE (V)							
POINTS	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	**Average**	**MR**	**ME**
A (Right - Outside)	4.7	4.2	4.7	4.8	4.9	4.66	0.46	0.205718254
B (Left - Outside)	5.4	5.3	6.8	7.8	6.5	6.36	1.44	0.643987578
C (Front - Outside)	5.9	5.5	5.6	5.3	5.1	5.48	0.42	0.18782971
D (Back - Outside)	6.4	6.7	6.7	6	5.2	6.2	1	0.447213595
Inside	-0.1	-0.1	-0.2	-0.1	-0.4	-0.18	0.22	0.098386991

The results from the conductive hollow sphere, as seen in Table 6, show a nearly uniform charge density outside the sphere. The inside of the sphere had an average charge density very close to zero (-0.18 V). Points B, C and D have similar charge densities (6.36 V, 5.48 V, 6.2 V), whereas point A has a slightly lower charge of 4.66 V.

3 Discussion

3.1 Experiment 1 : Charging by induction

Part 1

When the distance between the charged and sampling sphere was 50 cm, it was expected that the charged sphere would have little to no effect on the sampling sphere. This expectation was accurate, as the charge on the sampling sphere was uniform, as seen in Table 1. The results were, however, slightly noisy and variable; with the maximum overall residual of the average of the entire sphere being 0.68 V. In contrast, the maximum error was small, being 0.12 V. Point E, closest to the charged sphere, had a charge of 0.08 V, while point F, furthest from the charged sphere, had a charge of 0.48 V. This difference can be attributed to the sources of error discussed in the results section.

Part 2

The distance between the spheres was reduced to 1 cm for this part of the experiment. The charged sphere was now close enough to attract the charge of the sampling sphere. Therefore, it was expected that the electrons on the surface of the sampling sphere would be attracted to the positive charge of the other sphere, resulting in the side closest to the charged sphere (point E) to gain a negative charge distribution. Due to the charged sphere, more electrons would be situated on the side closest to the charged sphere than on the rest of the sampling sphere, causing an electric dipole. This electric dipole means that the point furthest away from the charged sphere (point F) is the most positively charged, and the point closest to the charged sphere (point E) is the most negatively charged.

The results, which can be seen in Table 2, support the expected theoretical values. The left side of the sampling sphere (point E) had an average charge of -6.28 V, and the right side (point F) had an average positive charge of +0.76 V. This non-uniform charge density is in line with the expected result. Points A, B, C and D had average voltages of +0.44, -1, +0.28 and +0.02 volts respectively. However, the negative charge on point B is understandable, because it was measured at the bottom left (closer to the charged sphere), as the stand prevented proper measurement. Due to the noisy results, the greatest maximum error and maximum absolute residual out of all the points were reasonable values of 0.44V and 0.98V, respectively. Therefore, the results match the theoretical expectations where a larger and denser negative charge is closest to the charged sphere, and a more distributed positive charge is furthest from the sphere.

Part 3

For this part of the experiment, the distance between the two spheres was kept to 1cm, but the sampling sphere was momentarily grounded whilst still close to the charged sphere. The electric dipole of the sampling sphere meant the electrons and ground had a difference in electric potential, so current flowed from the ground into the sampling sphere, imparting a net charge on the sampling sphere. It was due to this phenomenon, that the sampling sphere was taken a distance from the charged sphere in-between trials to be correctly grounded and have a net charge of zero. Since the charged sphere was grounded on the side farthest from the charged sphere (Point F), the net charge on the sphere was expected to be negative. However, the effects described in the previous step still applied (i.e. there would still be an electric dipole on the charged sphere due to the proximity of the charged sphere).

The results, which can be seen in Table 3, support the expected theoretical results. The point closest to the charged sphere (Point E) had an average voltage of -7.22V, and the point farthest away (Point F) had an average voltage of +0.38V. Compared to the results and scenario in Part 2, points A, B, C and D were more negatively charged with an average of -1.42V, -2.02V, -1.5V and -1.42V, respectively. The net charge of the sampling sphere is negative despite point F being positive. Point F is still positive due to the electric dipole of the sphere, induced by the charged sphere. Due to variable results, the maximum residual and maximum error was high, being at 1.72 V and 0.77 V, respectively.

Part 4

In this part of the experiment, the sampling sphere received a net charge from the charged sphere by being in close proximity to it. Like part 3, it was momentarily grounded while close to the charged sphere. The sampling sphere was then moved away from the charged sphere to allow accurate measuring of the net charge imparted upon the sphere in the previous step. Due to the significant distance between the spheres and the negative net charge on the sphere, the sampling sphere was expected to have a uniform net negative charge.

The results support the theoretical expectation as they show a uniform net negative charge on the sphere, as seen in Table 4. The maximum average voltage was from points A and D (-1.42 V); in comparison, the minimum average voltage was from point B (-2.02 V). Overall, the sampling sphere was roughly uniformly charged, where the maximum absolute residual and maximum error for this scenario was rather high, being 1.07V and 0.20V, respectively.

3.2 Experiment 2 : Conductive Conical Shape

Experiment 2 involved charging a conductive conical shape to measure the charge density that arises due to the object's shape. Five trials were performed for this experiment to allow for accurate data collection. In each trial, six points on the conical shape were measured using a proof plane and a Faraday ice pail. The equipment remained the same as in experiment 1, and the sources of error discussed in the results section still apply.

The results in Table 5 show that the narrower end of the conical shape had the highest charge density with an average charge of 7.44 V. Whereas; the larger end had an average charge of 5.48 V. Points A, B, C and D had average voltages of 5.54, 4.76, 5.46, 5.5 volts. The highest maximum error for this experiment was 0.644V, which is reasonable considering the large measured values..

Consider a charged conductor made out of two spheres of radii R_1 and R_2, connected with a conducting wire. Assume that $R_1 < R_2$, and that the spheres are far apart so that effects of electrostatic interactions between the spheres can be neglected. Then, the surface charge density, the quantity that describes how crowded the charges are, is higher at the smaller sphere.

To see why, remember that, since this system is a conductor, its surface is an equipotential. In particular, the electric potential on the surface of two spheres is the same, $V_1 = V_2$, which implies that

$$\frac{q_1}{R_1} = \frac{q_2}{R_2} \Rightarrow \frac{q_1}{q_2} = \frac{R_1}{R_2} < 1,$$

i.e., that most of the charge is in the bigger sphere. However, the ratio of the surface charge densities behaves the opposite way:

$$\frac{\sigma_1}{\sigma_2} = \frac{\frac{q_1}{4\pi R_1^2}}{\frac{q_2}{4\pi R_2^2}} = \frac{q_1}{q_2} \cdot \frac{R_2^2}{R_1^2} = \frac{R_2}{R_1} > 1.$$

This means that the surface charge density of the smaller sphere is larger, i.e., that the charge is more crowded (you find more charge per unit area) on the smaller sphere.

A generalization of this argument shows that charges are more crowded at pointy parts of a conductor, as opposed to the more gently-curved parts.

3.3 Experiment 3 : Conductive Hollow Sphere

The charge distributed on the outside of the sphere was observed to be a uniform positive charge of around 5.7 V. The inside of the sphere contained a uniform neutral charge of zero volts. However, these results were slightly noisy and variable, where the greatest maximum absolute residual was 1.44V, and the greatest maximum error was 0.64V, likely a byproduct of the experimental error discussed in the results section. In short, these results are consistent with the expected theoretical results, where the outside surface of the hollow sphere contained the object's net charge, and the sphere's inside had no charge.

The reason for the lack of charge on the inside surface can be explained using Gauss' law. Consider a Gaussian sphere with a small radius placed within the hollow sphere.

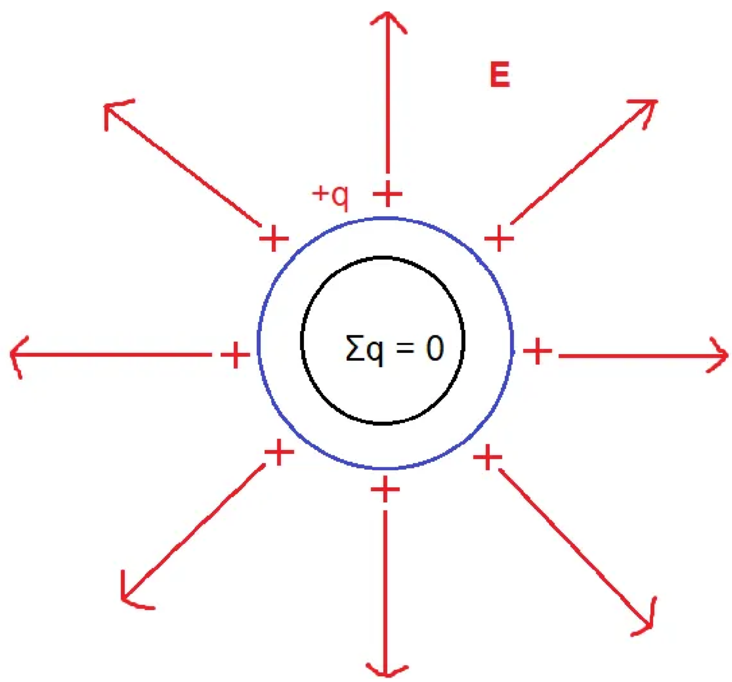

Fig. 5. Gaussian surface placed within hollow sphere (Tushar, 2020)

According to Gauss' law, any electric field experienced on the surface of this sphere would be proportional to the amount of charge enclosed within the Gaussian surface. In the case of this hypothetical sphere, there is no enclosed charge.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl.}}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{A} = 0$$

for $\oint \vec{E} \cdot d\vec{A}$ to equal 0, one of the two must be true: 1) the electric field is 0, and thus no charge exists on the inside surface of the hollow sphere, or 2) the dot product of the two vectors is 0, meaning they are perpendicular. Since both the hollow and Gaussian spheres are spherical, all $\vec{E}$ and $d\vec{A}$ vectors are parallel. This can only mean that the electric field is 0, and there would, in theory, be no charge on the inside surface of the hollow sphere.

input