

Practice Problems for Chapter 7

OpenIntro P.187 #5.7,5.8,5.10; P.259 #7.1, 7.5, 7.11b

Course Pack P.237 #3,5,11,13,19; P.230 #5, 9, 11, 15

1. Let X_1, X_2, X_3 be independent random variables with means $\mu, 2\mu, 4\mu$ and standard deviations $\sigma, 2\sigma, \sigma$, respectively.

Define two estimators $W_1 = \frac{X_1 + X_2 + X_3}{7}$ and $W_2 = X_1 - 2X_2 + X_3$. Show that both W_1 and W_2 are unbiased estimators

2. A random sample of 36 homes sold in Victoria had an average of \$605,850 and standard deviation of \$133,183

- (a) Choose the critical value required for a 95% confidence interval

<code>> qt(0.95,35)</code>	<code>> qt(0.025,35)</code>	<code>> qt(0.95,36)</code>	<code>> qnorm(0.975,0,1)</code>
<code>[1] 1.689572</code>	<code>[1] -2.030108</code>	<code>[1] 1.688298</code>	<code>[1] 1.959964</code>

- (b) Construct and interpret the appropriate 95% confidence interval to estimate the mean selling price of all homes in Victoria

- (c) A real estate agent claims that the average price of a single-family home in Victoria is \$675,000. Does your interval from part (a) give you reason to doubt the realtor's claim?

3. For each of the following, determine whether the statement is true or false. If it is false, explain why.

- (a) Higher confidence means a smaller margin of error.

- (b) Larger samples provide smaller margins of error.

- (c) For a given confidence level, halving the margin of error requires a sample twice as large.

4. Use R to find the t critical value for (i) df=20, 95% confidence, (ii) df=76, 99% confidence

5. A football coach claims that the average weight of all opposing team members is 102 kg. To test the claim, a sample of 50 players is taken from all opposing teams. The mean is found to be 104 kg and standard deviation is 7 kg. Construct and interpret the appropriate 99% confidence interval and use it to test check the coach's claim. (You may choose the critical value from the list below)

<code>> qnorm(0.995,0,1)</code>	<code>> qnorm(0.99,0,1)</code>	<code>> qt(0.995,49)</code>	<code>> qt(0.99,49)</code>
<code>[1] 2.575829</code>	<code>[1] 2.326348</code>	<code>[1] 2.679952</code>	<code>[1] 2.404892</code>

6. The hospitalization rate for people who are involved in crashes in midsize cars not equipped with airbags is 7.3%. In a study of airbag effectiveness, it was found that in 905 crashes of midsize cars equipped with airbags, 50 of the crashes resulted in hospitalization of the drivers. Construct a 95% confidence interval to estimate the proportion of hospitalizations resulting in cars equipped with airbags. Is it lower than those not equipped with airbags?

```
> qt(0.95,100)      > qt(0.975,100)      > qnorm(0.975,0,1) > pnorm(0.975,0,1)
[1] 1.660234         [1] 1.983972         [1] 1.959964         [1] 0.8352199
```

7. A major credit card company is planning a new offer for their current cardholders. The offer will give double airline miles on purchases for the next six months if the cardholder goes online and registers for the offer. To test effectiveness of the campaign, the company sent out offers to a random sample of 50000 cardholders. Of those, 1184 registered.

- (a) Using the R output below, identify and interpret the confidence interval

```
> prop.test(1184,50000,p=NULL,alternative="two.sided",conf.level=0.99)
1-sample proportions test with continuity correction
```

```
data: 1184 out of 50000, null probability 0.5
X-squared = 45374, df = 1, p-value < 2.2e-16
alternative hypothesis: true p is not equal to 0.5
99 percent confidence interval:
0.02198100 0.02550612
sample estimates:
p
0.02368
```

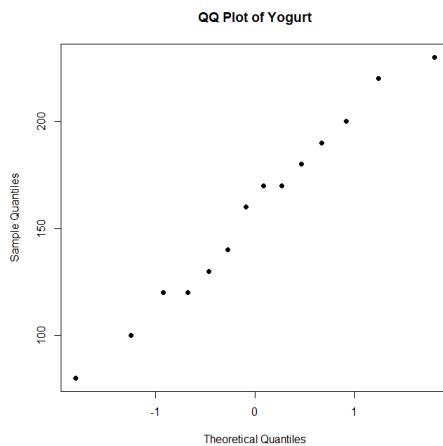
- (b) If the acceptance rate is only 2% or less, the campaign won't be worth the expense. Given the interval, would you choose to run the campaign?

8. Consumer reports tested several brands of vanilla yogurt and recorded the calories per serving. Use the R output below to answer the following questions
- (a) Identify the sample size, mean, variance, and standard deviation from the output.
 - (b) Identify and interpret the confidence interval for the mean.
 - (c) Check the conditions and comment on the validity of both confidence intervals.
 - (d) *My Fitness Pal* estimates 100 calories per serving for vanilla yogurt. Does this estimate look reasonable? Justify your answer using the confidence interval.

```
> t.test(yogurt, alternative="two.sided", conf.level=0.95)
```

One Sample t-test

```
data: yogurt
t = 13.198, df = 13, p-value = 6.62e-09
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
132.0181 183.6962
sample estimates:
mean of x
157.8571
```



9. Vitamin D, whether ingested as a dietary supplement or produced naturally when the skin is exposed to sunlight, is essential for strong, healthy bones. The bone disease rickets was largely eliminated in England during the 1950s, but now there is concern that a generation of children more likely to watch television or play computer games than spend time outdoors is at increased risk. A recent study of 2700 children randomly selected from all parts of England found 20% of them deficient in vitamin D. Find and interpret a 90% confidence interval, selecting your critical value(s) from the list below.

```
> qnorm(0.9,0,1)      > qnorm(0.95,0,1)    > qt(0.95,2699)      > qt(0.9,2699)
[1] 1.281552           [1] 1.644854           [1] 1.645418           [1] 1.281865
```

10. We wish to conduct a survey of Camosun students to estimate the true proportion that are in favour of having designated smoking areas on campus. How many people must be asked in order to estimate the true proportion to within 0.03 with 95% confidence if we have no idea what the true proportion is? You may select the critical value(s) from the list below.

```
> qt(0.95,100)        > qt(0.975,100)       > qnorm(0.975,0,1)   > pnorm(0.975,0,1)
[1] 1.660234           [1] 1.983972           [1] 1.959964           [1] 0.8352199
```