**Solutions to Chapter 8 Problems**

1. (a) One tailed: we want to see if visibility improves (not just changes).
   Hypotheses: $H_o$ : visibility is the same $H_a$ : visibility improves

   (b) Type I Error: say "visibility improves" when it is actually the same. In this case, we would switch to using the new signs even though they are not any better than the old signs. There would be unnecessary cost.

   (c) Type II Error: say "visibility is the same" when it is actually improved by the new signs. In this case, we would continue using the old signs even though we could have improved visibility by using the new signs. Likely, we could have prevented accidents but have failed to do so.

   (d) Power is the probability of the test showing that the new signs have improved visibility when they truly do.

2. (a) False. It is possible to make a type I error.

   (b) False. You can also use the z test if $n \geq 30$. However, you must also know $\sigma$

   (c) False. The *smaller* the p-value, the stronger the evidence against the null hypothesis.

   (d) False. The p-value is the conditional probability of observing a result at least as extreme as your data, given that the null hypothesis is actually true.

   (e) False. This means that if we took many additional samples and, from each, computed a 95% confidence interval for $\mu$, approximately 95% of these intervals would contain $\mu$.

   (f) True.

3. (a) A 99% confidence interval will have a larger critical value and therefore a larger margin of error than a a 95% confidence interval.

   (b) If sample size increases, the margin of error decreases, and therefore the confidence interval gets narrower.

   (c) Significant at 5% means $p-value < 0.05$. Then it will also be true that $p-value < 0.10$, thus the results are significant at 10%. However, it is impossible to tell if the $p - value < 0.01$; the test may or may not be significant at 1%.

   (d) The probability of type I error increases.

   (e) The probability of type II error decreases.

   (f) You started by assuming the the null hypothesis is true. You can only fail to prove that it is false.

   (g) **p-value** is the conditional probability of observing a result at least as extreme as your data, given that the null hypothesis is actually true.
   **Power** is the probability of rejecting the null hypothesis when it is actually false.

**Significance level** is the probability of rejecting the null hypothesis when it is actually true.

4. **Parameter**: $p =$ proportion of crashes involving midsize cars equipped with airbags that result in hospitalization.

   **Hypotheses**: $H_o : p = 0.073$ $H_a : p < 0.073$

   **Conditions**: $n\hat{p} = 905(50/905) = 50 \geq 5$ and $n(1-\hat{p}) = 905(1 - 50/905) = 855 \geq 5$

   **Statistic**:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{\frac{50}{905} - 0.073}{\sqrt{\frac{0.073(1-0.073)}{905}}} = -2.05$$

   **p-value**: $p - value = P(Z < -2.05) = 0.02$

   **Conclusion**: $p - value < \alpha$

   There is strong evidence to reject the null hypothesis. It is reasonable to assume that the proportion of crashes that result in hospitalization is lower than 7.3% for cars equipped with airbags.

5. **Parameter**: $p =$ proportion of all clients that will register for the offer

   **Hypotheses**: $H_o : p = 0.02$ $H_a : p > 0.02$

   **Conditions**: $n\hat{p} = 1184 \geq 5$ and $n(1-\hat{p}) = 48816 \geq 5$

   **Statistic**: "X-squared = 34.359" means $z \approxeq \sqrt{34.359} = 5.86$

   **p-value**: $p - value = 2.291 \times 10^{-9}$

   **Conclusion**: $p - value < \alpha$

   There is highly significant evidence that more than 2% of all clients will register for the offer. This gives us good reason to switch to the new campaign.

6. **Parameter**: $p =$ proportion of all adults that drink milk

   **Hypotheses**: $H_o : p = 0.9$ $H_a : p \neq 0.9$

   **Conditions**: $n\hat{p} = 657 \geq 5$ and $n(1-\hat{p}) = 93 \geq 5$

   **Statistic**:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{\frac{657}{750} - 0.9}{\sqrt{\frac{0.9(1-0.9)}{750}}} = -2.19$$

   **p-value**: $p - value = 2 \times P(Z < -2.19) = 0.028$

   **Conclusion**: $p - value < \alpha$

   There is strong evidence to reject the null hypothesis. There is reason to believe that the proportion of adults that drink milk was misreported by the newsletter and is not actually 90%

7. **Parameter**: $p$ = proportion of wells found by the dowser that are less than 100m deep.

   **Hypotheses**: $H_o : p = 0.3$ $H_a : p > 0.3$

   **Conditions**: $n\hat{p} = 27 \geq 5$ and $n(1-\hat{p}) = 53 \geq 5$

   **Statistic**:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{\frac{27}{80} - 0.3}{\sqrt{\frac{0.3(1-0.3)}{80}}} = 0.73$$

   **p-value**: $p - value = P(Z > 0.73) = 0.2327$

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> 1-pnorm(0.73,0,1)
[1] 0.2326951
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   **Conclusion**: $p - value > \alpha$

   There is insufficient evidence to reject the null hypothesis. It is reasonable to assume that the dowsing method does not find wells less than 100m deep more than 30% of the time, and thus is no better than other methods.

8. If the mean really was 25, we would expect data like ours to occur about 4% of the time. Such unlikely results gives us reason to put our faith in the data moreso than the claim. In other words, we have reason to believe the mean is actually lower than 25.

9. C

10. **Parameter**: $mu$ = true mean weight of all opposing team members

    **Hypotheses**: $H_o : \mu = 102$ $H_a : \mu \neq 102$

    **Conditions**: $n = 50 > 30$

    **Statistic**:

$$t_o = \frac{\bar{x} - \mu_o}{s\sqrt{n}} = \frac{104 - 102}{7/\sqrt{50}} = 2.02$$
$$\nu = 49$$

    **p-value**: $p - value = 0.0489$ from `2*pt(2.02,49,lower.tail=FALSE)`

    **Conclusion**: $p - value < \alpha$

    There is sufficient evidence to reject the null hypothesis. We have reason to believe the mean weight of opposing players is not 102kg as the coach at claimed.

11. **Parameter**: $\mu =$ mean calories in all vanilla yogurt brands

    **Hypotheses**: $H_o : \mu = 100$ $H_a : \mu > 100$

    **Conditions**: $n = 14 < 30$ but normal probability plot provided in chapter 7 indicates the data are normally distributed

    **Statistic**: $t = 4.8374$ with $\nu = 13$

    **p-value**: 0.0001622

    **Conclusion**: $p - value < 0.01$

    There is highly significant evidence to suggest the mean calorie content of vanilla yogurt is actually higher than 100 calories.