Stat 254 - Chapter 6 Practice Problems - Solutions

1. X = number of students that buy snacks at school. $X \sim Binom(70, 0.4)$ but $np = 70(0.4) = 28 > 5$ and $n(1-p) = 70(0.6) = 42 > 5$ so we may use the normal approximation:

   $X \sim N(np, \sqrt{np(1-p)}) = N(70(0.4), \sqrt{70(0.4)(0.6)}) = N(28, 4.0988)$

   $P(X \leq 20)$ with the continuity correction is

   $P(X < 20.5) = P\left(Z < \dfrac{20.5 - 28}{\sqrt{(70(0.4)(0.6))}}\right) = P(Z < -1.83)$

   Answer between 2.5% and 16% is (c) 0.0336

2. X = area of the ozone hole on one day and follows unknown distribution shape with $\mu = 17.9$ and $\sigma = 5.6$.

   For $n = 33 > 30$ we can say $\bar{X}$ = the average ozone hole area for 33 days and $\bar{X} \sim N(17.9, \frac{5.6}{\sqrt{33}})$

   (a) $P(\bar{X} < 15.5) = P\left(Z < \dfrac{15.5 - 17.9}{5.6/\sqrt{33}}\right) = P(Z < -2.46)$

   Answer between 0.15% and 2.5% is (a) 0.0069

   (b) We want a such that $P(\bar{X} > a) = 0.10$. This must occur between 1 and 2 standard deviations above the mean.

   $\mu_{\bar{X}} + \sigma_{\bar{X}} = 17.9 + \frac{5.6}{\sqrt{33}} = 18.8$

   $\mu_{\bar{X}} + 2\sigma_{\bar{X}} = 17.9 + 2\frac{5.6}{\sqrt{33}} = 19.9$

   The answer between 18.8 and 19.9 million square km is (c) 19.1

3. $\hat{p}$ = the sample proportion of Canadians that visit a food bank, out of 600. Since the expected counts $np = 600(0.024) = 14.4 > 5$ and $n(1-p) = 600(0.976) = 585.6 > 5$ we can assume $\hat{p} \sim N(p, \sqrt{\frac{p(1-p)}{n}}) = N(0.024, \sqrt{\frac{0.024(0.976)}{600}})$

   Then, $P(\hat{p} > \dfrac{17}{600}) = P\left(Z > \dfrac{\frac{17}{600} - 0.024}{\sqrt{\frac{0.024(0.976)}{600}}}\right) = P(Z > 0.69)$

   The answer between 16% and 50% is (d) 0.2440

   This result is fairly likely to occur so it is reasonable to believe that 2.4% of all Victoria residents use the food bank, just like the national proportion.

4. $\hat{p}$ = the sample proportion that do not use landlines. Since the expected counts $np = 36(0.25) = 9 > 5$ and $n(1-p) = 36(0.75) = 27 > 5$ we can assume $\hat{p} \sim N(p, \sqrt{\frac{p(1-p)}{n}}) = N(0.25, \sqrt{\frac{0.25(0.75)}{36}})$

   (a) $P(0.2 < \hat{p} < 0.3) = P\left(\dfrac{0.2 - 0.25}{\sqrt{\frac{0.25(0.75)}{36}}} < Z < \dfrac{0.3 - 0.25}{\sqrt{\frac{0.25(0.75)}{36}}}\right) = P(-0.69 < Z < 0.69)$

   The answer that is smaller than 68% (the proportion between -1 and +1) is (c) 0.5116

(b) We want a such that $P(\hat{p} < a) = 0.2$. The lowest 20% corresponds to a z score between -1 and 0. For our distribution that will be between $0.25 - \sqrt{\frac{0.25(0.75)}{36}} = 0.1778$ and 0.25. The answer is (b) 0.1893.

(c) NOTE: There was a typo in this question, but you would still select the same answer. We want a such that $P(\hat{p} < -a) + P(\hat{p} > a) = 0.01$ so $P(\hat{p} > a) = 0.005$. To cut off the outermost 0.5%, we should have a z score of between 2 and 3 standard deviations away from the mean.

Lower End: $\mu - 3\sigma = p - 3(\sqrt{\frac{p(1-p)}{n}}) = 0.25 - 3\sqrt{\frac{0.25(0.75)}{36}} = 0.0334$

$\mu - 2\sigma = p - 2(\sqrt{\frac{p(1-p)}{n}}) = 0.25 - 2\sqrt{\frac{0.25(0.75)}{36}} = 0.106$

Upper End: $\mu + 2\sigma = p + 2(\sqrt{\frac{p(1-p)}{n}}) = 0.25 + 2\sqrt{\frac{0.25(0.75)}{36}} = 0.394$

$\mu + 3\sigma = p + 3(\sqrt{\frac{p(1-p)}{n}}) = 0.25 + 3\sqrt{\frac{0.25(0.75)}{36}} = 0.467$

The values in the lowest 0.5% will be cut off in $3.3\% < \hat{p} < 10.6\%$ and the highest 0.5% must be cut off between $39.4\% < \hat{p} < 46.7\%$

The answer is (b) but the correct values are actually "more than 43.6% or less than 6.4%"

5. $\hat{p}$ = sample proportion of female subscribers to female-lead comics. The expected number of successes and failures is $np = 500(0.46) = 230 > 5$ and $n(1-p) = 500(0.54) = 270 > 5$. This is large enough to assume that $\hat{p} \sim N\left(p, \sqrt{\frac{p(1-p)}{n}}\right) = N\left(0.46, \sqrt{\frac{0.46(0.54)}{500}}\right)$

$$P\left(\hat{p} > \frac{265}{500}\right) = P\left(Z > \frac{\frac{265}{500} - 0.46}{\sqrt{\frac{0.46(0.54)}{500}}}\right) = P(Z > 3.14)$$

The probability that is smaller than 0.15% is (a) 0.0008

If the proportion of female subscribers was 46% for female-lead comics, we would expect a result like ours only 0.08% of the time. This sample was highly unlikely to occur with the assumption of $p = 46\%$ which suggests that the proportion of female subscribers may be different for female-lead comics

6. (a) X = the systolic blood pressure of one person. $X \sim N(120, 5.6)$

$$P(X > 130) = P\left(Z > \frac{130 - 120}{5.6}\right) = P(Z > 1.79)$$

The answer between 2.5% and 16% is (b) 0.0371

(b) We want a such that $P(X > a) = 0.05$. This requires a z between +1 and +2 so our X value must be between

$\mu + \sigma = 120 + 5.6 = 125.6$ and $\mu + 2\sigma = 120 + 2(5.6) = 131.2$

The cut off is (a) 129.2

(c) $\bar{X}$ = average blood pressure of 20 healthy adults. Since X is normally distributed, $\bar{X} \sim N(120, 5.6/\sqrt{20})$

$$P(\bar{X} > 124) = P\left(Z > \frac{124 - 120}{5.6/\sqrt{20}}\right) = P(Z > 3.19)$$

The answer smaller than 0.15% is (a) 0.0007

(d) $P(117.5 < \bar{X} < 122.5) = P\left(\frac{117.5 - 120}{5.6/\sqrt{20}} < Z < \frac{122.5 - 120}{5.6/\sqrt{20}}\right) = P(-2.45 < Z < 2.45)$

The probability should be between 95% and 99.7%. The answer is (c) 0.9855

7. $\bar{X}$ = average amount spent by 49 customers. SInce $n = 49 > 30$ $\bar{X} \sim N(30, \frac{8}{\sqrt{49}})$

   $P(\bar{X} < 28) = P\left(Z < \frac{28 - 30}{8/\sqrt{49}}\right) = P(Z < -1.75)$

   The probability between 2.5% and 16% is (a) 0.0401

   This result suggests that if the average amount spent by shoppers really is \$30 as the manager claimed then we would expect results like ours about 4% of the time. This result is fairly unlikely, so we would have reason to doubt the claim made and suspect that the mean may actually be lower than \$30.