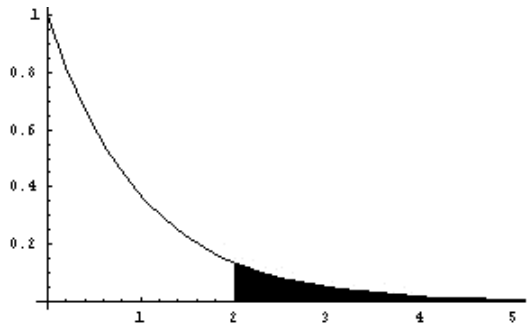
Practice Problems for Chapter 5

Course Pack P.162 # 1-4; P.168 #5.17, 5.19; P.177 #1-4

OpenIntro P.142 # 4.1,4.2,4.3 (For each problem, find an approximate interval for your solution or calculate using R)

Course Pack P.184 #5.24,5.25,5.26,5.27 (find intervals for each answer, then verify by checking the solutions)

1. The graph below represents the probability density function for some continuous random variable X. The shaded region corresponds to



   (a) $P(X < 5)$

   (b) $P(X \geq 2)$

   (c) $P(X > 2)$

   (d) Both B and C are correct

2. Rugs are made from various wools and woven in several different countries. The pile height (in millimetres) of a rug varies slightly and can be modelled by a uniform random variable X on the interval [6, 10].

   (a) Define the random variable and state its distribution.

   (b) Sketch a graph of the probability density curve for the random variable X.

   (c) What is the probability that a randomly selected rug

      (i) will have a pile height less than 7.5 mm?

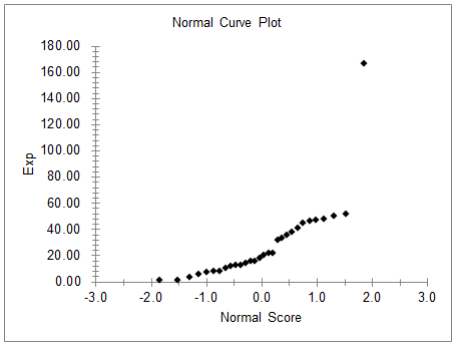
      (ii) will have a pile height between 6.4 and 8.1 mm?

      (iii) will have a pile height more than 8.5 mm?

   (d) Find the height h such that the rug's pile height is more than 80% of all rugs.

3. Suppose X has pdf $f(x) = \begin{cases} 2e^{-2x} & 0 \leq x < \infty \\ 0 & \text{otherwise} \end{cases}$. Find the mean, variance, and median.

4. The graph below gives the normal probability plot for a variable. Comment on the plot.



5. Using the standard normal distribution find each of the following.

   (a) $P(Z > 1.22)$ (a) 0.2401 (b) 0.0211 (c) 0.1112 (d) 0.4003

   (b) $P(Z \leq 1.45)$ (a) 0.8311 (b) 0.9984 (c) 0.0833 (d) 0.9265

   (c) $P(Z = -0.68)$ (a) 0.2482 (b) 0.0000 (c) 0.7517 (d) 0.4134

   (d) $P(-2.22 < Z < -0.03)$ (a) 0.4748 (b) 0.9984 (c) 0.1033 (d) 0.6834

   (e) the 21^{st} percentile (a) -2.03 (b) 2.03 (c) 0.81 (d) -0.81

   (f) the z-score that cuts off the highest 18%. (a) 0.92 (b) -0.92 (c) 1.92 (d) -1.92

6. Movie trailers are designed to entice audiences by showing scenes from coming attractions. The length of a movie trailer is normally distributed with a mean of 2.5 min and a standard deviation of 0.5 min.

   (a) Define the random variable and state its distribution.

   (b) What is the probability that a randomly selected trailer lasts more than 3.2 minutes? (a) 0.0134 (b) 0.1712 (c) 0.0808 (d) 0.5331

   (c) What is the probability that a randomly selected trailer lasts less than 1.1 minutes? (a) 0.0003 (b) 0.0026 (c) 0.0542 (d) 0.1336

   (d) What is the probability that a randomly selected trailer lasts between 2.6 and 3.7 minutes? (a) 0.9534 (b) 0.4125 (c) 0.1002 (d) 0.5713

   (e) Find the time t such that 90% of all trailers are less than t minutes. (a) 1.74 (b) 2.03 (c) 3.14 (d) 3.62

7. The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes. What is the probability that a person will be served in less than 3 minutes on at least 4 of the next 6 days?

8. Suppose that a study of a certain computer system reveals that the response time, in seconds, has an exponential distribution with a mean of 3 seconds.

   (a) What is the probability that the response time exceeds 5 seconds?

   (b) What is the probability that the response time exceeds 10 seconds?

9. Suppose that some random variable has the probability distribution

$$f(x) = \begin{cases} x & \text{for } 0 \leq x \leq 1 \\ 2 - x & \text{for } 1 < x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Verify this is a valid probability density function by finding the area under the entire curve.

   (b) Find $P(X < 1.3)$

   (c) Find $P(0.3 < X < 1.7)$

   (d) Find the mean and variance of X

10. The average life of a certain type of small motor is normally distributed with a mean of 10 years and standard deviation 2 years.

   (a) What is the probability of a randomly selected motor lasting more than 13 years?
   (a) 0.0668 (b) 0.1734 (c) 0.0202 (d) 0.5433

   (b) The manufacturer would like to set the warranty time so that only 3% of motors will have to be replaced. For how many years should the warranty last?
   (a) 5 (b) 3.1 (c) 6.2 (d) 9

11. The daily amount of coffee, in litres, dispensed by a machine located in an airport lobby is uniformly distributed between 7L and 10L. On a randomly selected day

   (a) What is the mean and standard deviation for the amount of coffee dispensed?

   (b) what is the probability they will dispense less than 8.8L?

   (c) what is the 95th percentile for coffee dispensed?

   (d) what is the probability that they will dispense between 7.4L and 9.5L?

12. Suppose that some random variable has the probability distribution

$$f(x) = \begin{cases} k\sqrt{x} & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the value of k

   (b) Find $P(0.3 < X < 0.6)$

   (c) Find the mean and variance of X

   (d) Define the cdf of X