Solutions to Chapter 7 Problems

1.
$$\begin{aligned} E(W_1) &= E\left(\frac{X_1+X_2+X_3}{7}\right) \\ &= \frac{1}{7}E(X_1+X_2+X_3) \\ &= \frac{1}{7}[E(X_1)+E(X_2)+E(X_3)] \\ &= \frac{1}{7}(\mu+2\mu+4\mu) = \frac{7\mu}{7} = \mu \end{aligned}$$

$$\begin{aligned} E(W_2) &= E(X_1 - 2X_2 + X_3) \\ &= E(X_1) - 2E(X_2) + E(X_3) \\ &= \mu - 2(2\mu) + 4\mu = \mu \end{aligned}$$

Since $E(W_1) = \mu$ and $E(W_2) = \mu$ both are unbiased estimators for μ

2. (a) For $\nu = 36 - 1$ and 95% confidence, the critical value is $t^* = 2.03$

```
> qt(0.025,35)
[1] -2.030108
```

(b) $\mu =$ the mean selling price of all homes in Victoria. Since $n = 36 > 30$, we may use the t interval to estimate μ

$\bar{x} \pm t^* \frac{s}{\sqrt{n}} = 605850 \pm (2.03)\frac{133183}{\sqrt{36}} = 605850 \pm 45062.65 = (560787.35, 650912.65)$

We are 95% confident that the mean selling price of all homes in Victoria is between \$560 787.35 and \$650 912.65.

(c) The claimed value of \$675 000 is outside our interval and we have reason to doubt the realtor.

3. (a) False: higher confidence results in a larger critical value and thus a larger margin of error

(b) True

(c) False: Since the margin of error depends on $\sqrt{n}$ if we want to cut the margin of error in half we will need to quadruple the sample size.

4. (i) $t^*_{20} = 2.086$ (ii) $t^*_{76} = 2.642$

```
> qt(0.025,20)
[1] -2.085963
> qt(0.005,76)
[1] -2.642078
```

5. $\mu =$ the mean weight of all football players on opposing teams. Since $n = 50 > 30$ we can use the t interval to estimate μ.

For 99% confidence, $\nu = 49$ the critical value is $t^* = 2.67995$

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}} = 104 \pm (2.67995)\frac{7}{\sqrt{50}} = 104 \pm 2.65 = (101.3, 106.7)$$

We are 99% confident the mean weight of all players on opposing teams is between 101.3 and 106.7 kg. Since 102kg is within the interval, there is no reason to doubt the coach's claim.

6. For $p =$ the proportion of hospitalizations for all crashes involving cars equipped with airbags since $n\hat{p} = 50 > 5$ and $n(1-\hat{p}) = 905 - 50 = 855 > 5$ we can use the z interval to estimate p.

   The critical value for 95% confidence is $z^* = 1.96$

   $$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \frac{50}{905} \pm 1.96\sqrt{\frac{\frac{50}{905}\left(\frac{855}{905}\right)}{905}} == 0.0552 \pm 0.01488 = (0.0403, 0.07008)$$

   We are 95% confident that for cars equipped with airbags between 4% and 7% of crashes result in hospitalization. Since this interval is strictly lower than 7.3%, we can conclude that the hospitalization rate is lower for cars with airbags than for those without.

7. (a) $p =$ the proportion of all cardholders that register for the offer with the new campaign. The interval is `(0.02198100,0.02550612)`.

   We are 99% confident that between 2.2% and 2.6% of all customers will register for the offer when using the new campaign.

   (b) The rate is strictly higher than 2% so it is worth switching to the new campaign.

8. (a) $n = df + 1 = 14$, $\bar{x} = 157.8571$, $s^2 = 2002.474$, $s = \sqrt{s^2} = 44.749$

   (b) The interval is $(132.0181, 183.6962)$. We are 95% confident that the mean calorie content for all brands of vanilla yogurt is between 132 and 183.7 calories per serving.

   (c) For the mean test, we require $n \geq 30$ or normally distributed data and for the variance test we require normally distributed data. In this case, $n = 14$ is small, but the normal probability plot is roughly linear which indicates that the calorie content of yogurt is approximately normally distributed.

   (d) The estimate for the mean is between 132 and 183.6 calories. This is strictly higher than the 100 calorie estimate given by My Fitness Pal. Thus, we have reason to disbelieve their estimated value.

9. To estimate the parameter $p =$ the proportion of all children in England that have vitamin D deficiency, we have observed $\hat{p} = 0.2$. Since $n\hat{p} = 2700(0.2) = 540 > 5$ and $n(1-\hat{p}) = 2700(0.8) = 2160 > 5$ we may use the z interval for estimating p.

   The critical value for 90% confidence is $z^* = 1.644854$

   $$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.2 \pm 1.644854\sqrt{\frac{0.2(0.8)}{2700}} == 0.2 \pm 0.013 = (0.187, 0.213)$$

We are 90% confident that between 18.7% and 21.3% of all children in England have a vitamin D deficiency.

10. Since we have no prior knowledge to use for this study, we will use $\hat{p} = 0.5$. Then for $B = 0.03$ and 95% confidence $z^* = 1.96$ we get the minimum sample size required as

    $$n = \hat{p}(1-\hat{p})\left(\frac{z^*}{B}\right)^2 = 0.5(0.5)\left(\frac{1.96}{0.03}\right)^2 = 1067.1$$

    We require a sample of at least 1068 students.