



汇世国际书院
Wits International Academy

CIE Physics A2

Revision Handout

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Content 目录

1. Circular Motion 圆周运动	6
1.1 Radian 弧度	6
1.2 Angular speed 角速度	6
1.3 Centripetal force 向心力	6
1.4 Different models of circular motion 圆周运动模型	7
2. Fields 场	8
2.1 Gravitational Field 重力场	8
2.2 Electric Field 电场	11
2.3 Energy conservation 能量守恒	13
3. Thermodynamics 热力学	14
3.1 Thermodynamic scale 热力学温标	14
3.2 Thermometers 温度计	14
3.3 Sensitivity, range and linearity 灵敏度量程与线性	15
3.4 Internal Energy 内能	15
3.5 State Change of Matter 物态变化	15
3.6 Specific heat capacity 比热容	16
3.7 Specific latent heat 比潜热	16
3.8 Calculation of Thermodynamics 热力学计算	16
3.9 Thermodynamic 1 st Law 热力学第一定律	17



4. Ideal Gas 理想气体	18
4.1 Amount of substance 物质的量	18
4.2 Ideal Gas 理想气体	18
4.2.1 Kinetic model of ideal gas 理想气体动能模型	18
4.2.2 Root mean square 气体分子平均速度 RMS	21
5. Oscillation 振动	22
5.1 Oscillation Definitions 振动定义	22
5.2 Simple harmonic motion 简谐振动	22
5.3 Resonance 共振	24
5.4 Damping 阻尼	25
6. Capacitance 电容	26
6.1 Definition 定义	26
6.2 Why capacitor stores energy, not charge	26
6.3 Two type of capacitor 两种电容器	26
6.4 Graph and Energy Stored 图像和电容储存的能量	27
6.5 Capacitor in Series or Parallel 电容串并联	27
7. Magnetic Field 磁场	29
7.1 Definition 磁场定义	29
7.2 Magnetic Field Line 磁感线	29
7.3 The magnetic effect of a current 电流的磁效应	30
7.4 Force on a current-carrying conductor 通电导线在磁场中受力	31



7.5	D.C. Motor 直流电机.....	32
7.6	Force on a moving charge 运动的电荷受力.....	32
7.7	Circular motion of a moving charge 运动的电荷做圆周运动	34
7.8	Velocity (speed) selector 速度选择器	34
7.9	Hall Effect 霍尔效应	34
7.10	Hall Probe 霍尔探头.....	35
8.	Electromagnetic Induction 电磁感应	36
8.1	Magnetic Flux 磁通量	36
8.2	Magnetic Flux Linkage 磁通链	36
8.3	Lenz's Law 楞次定律	36
8.4	Faraday's Law 法拉第定律.....	37
8.5	A.C. Generator 交流发电机	37
9.	Alternating Current 交流电	40
9.1	Expression 表达式.....	40
9.2	Root-mean-square RMS 值	40
9.3	Rectification 整流（交流变直流）	41
9.4	Pros and Cons 交流电的优劣势.....	42
10.	Electronics 电子	43
10.1	Electronic system 电子系统.....	43
10.2	Sensing Device 传感器	43
10.3	Processor 处理器	44



10.4	Output Device 输出设备	49
11.	Nuclear Physics 核物理	50
11.1	Mass-Energy Conservation 质能守恒	50
11.2	Atomic Mass Unit 原子质量单位	50
11.3	Binding Energy 结合能	51
11.4	Nuclear fission and fusion 核裂变和核聚变	52
11.5	Properties of Decay 衰变的性质	52
11.6	Calculation of Decay 衰变的计算	52
11.7	Graph of Decay 衰变图像	53
12.	Quantum Physics 量子物理	54
12.1	Wave and Particle Model of Light 光的波动和粒子模型	54
12.2	Photon 光子	54
12.3	Photoelectric Effect 光电效应	55
12.4	Line Spectra 线谱图	56
12.5	Band Theory 能带理论	57
12.6	Particle nature of electron: Millikan Experiment 油滴实验	58
12.7	Wave nature of electron: electron diffraction 电子衍射	59
13.	Communication 通信	61
13.1	Signal 信号	61
13.2	Modulation 调制	61
13.3	Bandwidth 带宽	62



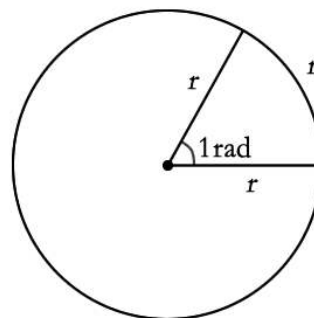
13.4	进制转换	63
13.5	ADC & DAC	64
13.6	Loss and Attenuation 损耗与衰减	65
13.7	Wire Communication 有线通信	65
13.8	Wireless Communication 无线通信	66
13.9	Satellite Communication 卫星通信	67
13.10	dB Calculation 分贝计算	68
14.	Medical Imaging 医学成像	69
14.1	X-ray	69
14.2	CT (Computerized axial Tomography)	73
14.3	NMRI (Nuclear Magnetic Resonance Imaging) 核磁共振	74
14.4	Ultrasound 超声波	75



1. Circular Motion 圆周运动

1.1 Radian 弧度

- 1) Definition 定义: angle subtended at the centre of a circle by an arc equal in length to the radius
- 2) Formula 公式: $\theta = \frac{l}{r}$, where l is length of arc, r is radius



1.2 Angular speed 角速度

- 1) Definition 定义: the rate of change of angular displacement swept out by radius
- 2) Formula 定义式: $\omega = \frac{\Delta\theta}{\Delta t}$, where θ is in radian
- 3) Other formulae 其他相关公式:
 - a) $\omega = \frac{2\pi}{T}$, where T is period
 - b) $\omega = 2\pi f$, where f is frequency
 - c) $v = \omega r$, where v is linear speed, r is radius

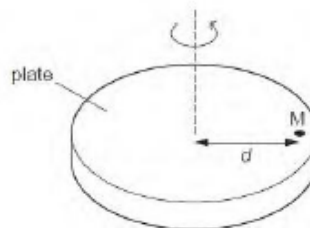
1.3 Centripetal force 向心力

- 1) 理解: Centripetal force 不是一个真实存在的具体的力, 是一个 resultant force, 需要被物体真实受的力比如 gravity, normal reaction 等来 provide, 方向是指向圆心, 作用是 change the direction of velocity,
- 2) Formula 公式: $F = m\omega^2 r = \frac{mv^2}{r}$

1.4 Different models of circular motion 圆周运动模型

- 1) 在水平方向上进行圆周运动，由某一个力提供向心力，如右图摩擦力

$$f = m\omega^2 r$$



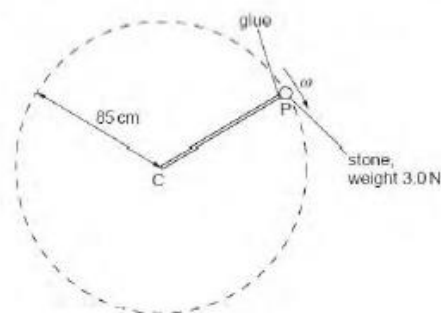
- 2) 在竖直平面上进行圆周运动，由重力和拉力共同提供向心力，如右图所示

在最高点拉力最小：

$$T + mg = m\omega^2 r$$

在最低点拉力最大：

$$T - mg = m\omega^2 r$$



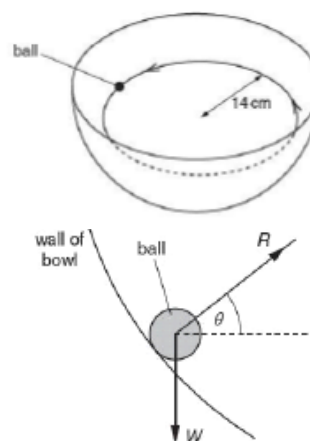
- 3) 在斜面上进行圆周运动，由支持力和重力的合力提供向心力，如下图所示

竖直方向平衡：

$$R \cdot \sin\theta = mg$$

水平方向提供向心力：

$$R \cdot \cos\theta = m\omega^2 r$$

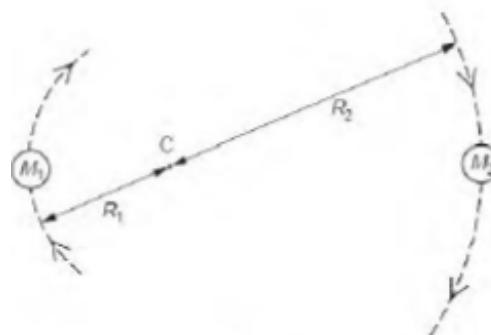


- 4) 万有引力提供向心力，绕天体旋转

$$m\left(\frac{2\pi}{T}\right)^2 r = \frac{GMm}{r^2}$$

- 5) Binary star 双星系统

$$M_1\omega^2 R_1 = M_2\omega^2 R_2 = \frac{GM_1 M_2}{(R_1 + R_2)^2}$$





2. Fields 场

2.1 Gravitational Field 重力场

1) Newton's law of gravitation 万有引力定律

force proportional to product of masses and inversely proportional to square of separation of the two **point** masses

2) Gravitational force 万有引力

公式: $F = \frac{GMm}{r^2}$, G is gravitation constant, r is separation of two point masses

3) Gravitational field strength 重力场强

a) 定义: the gravitational field strength at a point is the gravitational force exerted per unit mass on a small object placed at that point

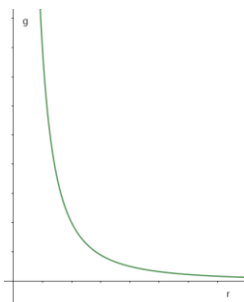
b) 公式: $g = \frac{F}{m} = \frac{GM}{r^2}$

c) 方向: 场强是 vector 所以具有方向, 场强的方向就是物体受力的方向

d) Field line 场线: direction of force on a small test mass

e) 从场线方面解释为什么地球重力场是 constant: Lines are radial, earth has large radius, so lines almost parallel, which means constant strength.

f) 重力场强 g 与距离 r 的图像: 平方反比图像



4) Gravitational potential energy 重力势能



a) 定义: ability to do work as a result of the position in a gravitational field

b) 公式: $E_p = -\frac{GMm}{r}$, 能量是 scalar, 负号只代表大小

c) Why gravitational potential energy is negative 为什么重力势能是负的:

- Gravitational potential energy at infinity is zero
- Force always attractive
- So work got out as object moves from infinity
- Therefore the gravitational potential energy is negative

d) 为什么不能用 mgh: 因为 gravitational field strength is not a constant

5) Gravitational potential 重力势:

a) 定义: work done bringing unit mass from infinity to the point in the gravitational field

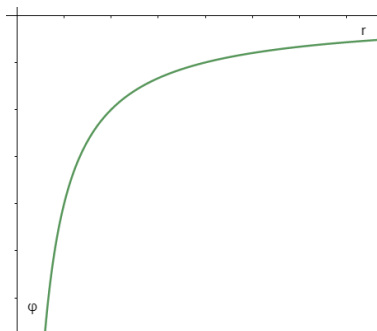
b) 公式: $\phi = \frac{E_p}{m} = -\frac{GM}{r}$, potential 是 scalar, 负号只代表大小

c) 单位: $\text{J}\cdot\text{kg}^{-1}$ 焦每千克

d) Why gravitational potential is negative 为什么重力势是负的:

- Gravitational potential at infinity is zero
- Force always attractive
- So work got out as object moves from infinity
- Therefore, the gravitational potential is negative

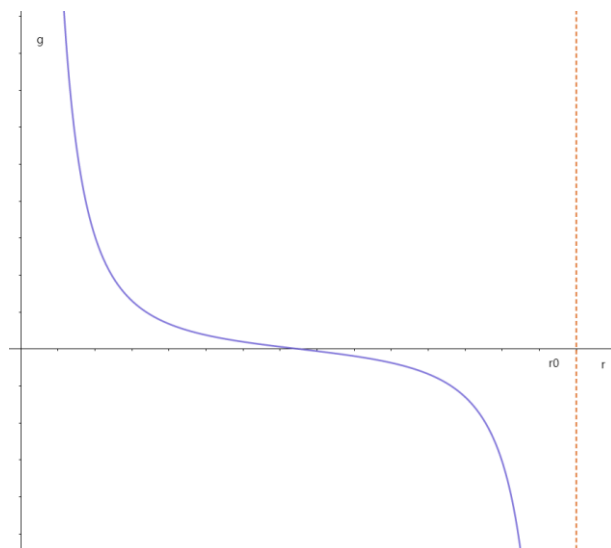
e) 重力势 ϕ 与距离 r 的图像: 反比图像



6) Superposition of gravitational field 重力场的叠加

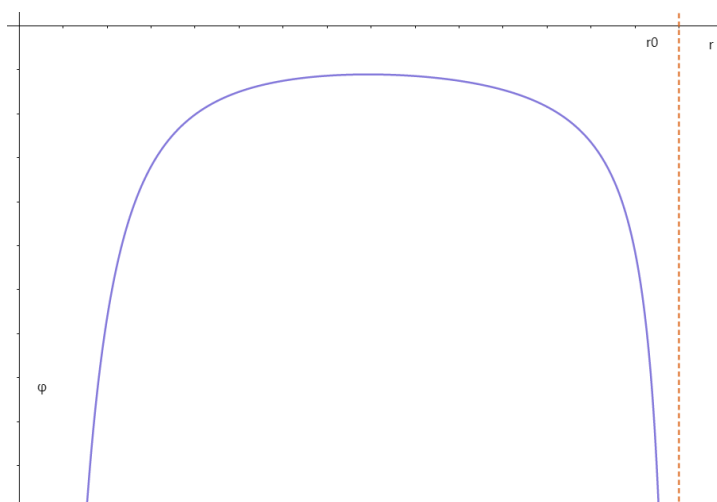


- a) 场强的叠加: 遵循 vector 的加法, 要考虑方向, 如图, 在 $r=0$ 和 $r=r_0$ 处分别有两个 point mass,



中间某点场强为零, 因为受到左边和右边的引力相同

- b) potential 的叠加: 遵循 scalar 的加法, 数值直接相加



Potential 始终是负的, 中间某个点最高



2.2 Electric Field 电场

1) Coulomb's Law 库仑定律

force proportional to product of charges and inversely proportional to square of separation of the two **point** charges

2) Electric force of point charge 点电荷的电场力

公式:
$$F = \frac{Qq}{4\pi\epsilon_0 r^2} = \frac{kQq}{r^2}$$

where $k = \frac{1}{4\pi\epsilon_0}$, ϵ_0 is permittivity in free space, r is separation of two point masses

3) Electric field strength 电场强

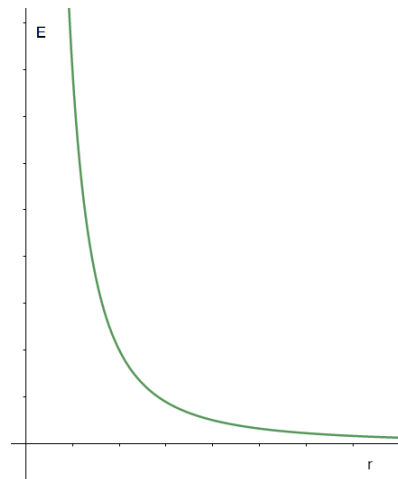
g) 定义: the electric field strength at a point is the electric force exerted per unit charge on a charge placed at that point

h) 公式: $E = \frac{F}{q} = \frac{kQ}{r^2}$, where $k = \frac{1}{4\pi\epsilon_0}$

i) 方向: 场强是 vector 所以具有方向, 场强的方向就是正电荷受力的方向

j) Field line 场线: direction of force on a small positive test charge

k) 电场强 E 与距离 r 的图像: 平方反比图像



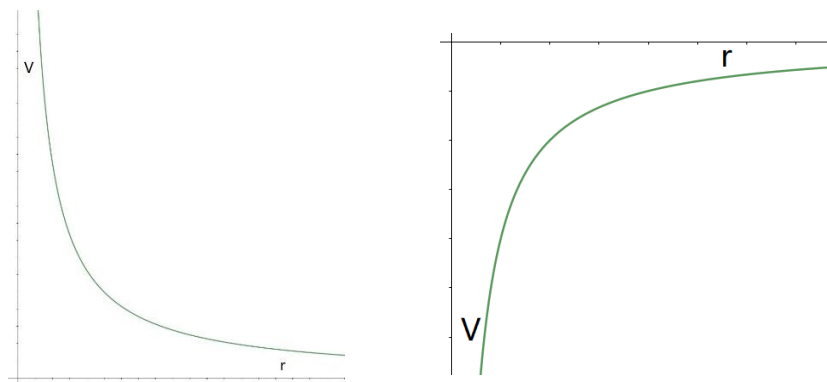


4) Electric potential energy 电势能

- a) 定义: ability to do work as a result of the position in a electric field
- b) 公式: $E_p = \frac{kQq}{r}$, 能量是 scalar, 带电同号为正, 异号为负, 符号代表大小

5) Electric potential 电势:

- a) 定义: work done bringing unit positive charge from infinity to the point in the electric field
- b) 公式: $V = \frac{E_p}{q} = \frac{kQ}{r}$, potential 是 scalar, 正电周围为正, 负电为负, 符号代表大小
- c) 单位: Volt 伏特, 写作 V
- d) Volt 定义: Joule per unit charge
- e) Why gravitational potential is negative 为什么正电产生的电势是正的:
- Electric potential at infinity is zero
 - Force always repulsive
 - So work is done to the positive test charge when it is moved from infinity
 - Therefore, the electric potential is positive
- f) 电势 V 与距离 r 的图像: 反比图像

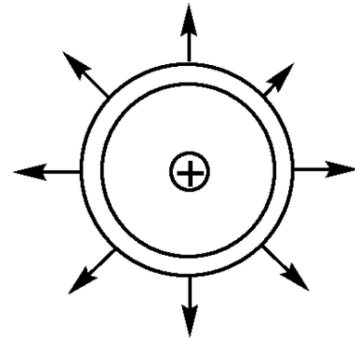


正电周围的电势为正, 负电周围的电势为负



6) Metal sphere 金属球壳:

金属球壳内部没有电场, 因为金属球壳内部的 charge 会受到来自四面八方的电场力互相抵消, 电荷在球壳内部移动的时候不会有电场力做功, 所以 potential 和球壳表面保持一致。



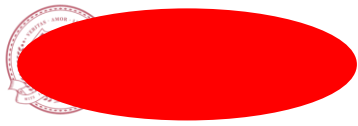
2.3 Energy conservation 能量守恒

- 1) 重力场: 先选取两个需要进行计算的点, 找出对应的两个 speed 和对应的 potential, 如果是 g 图像就数格子求面积看 potential difference, 如果是 ϕ 图像就直接读 potential difference $\phi_1 - \phi_2$

$$\text{列方程 } \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = m(\phi_1 - \phi_2)$$

- 2) 电场: 先选取两个需要进行计算的点, 找出对应的两个 speed 和对应的 potential, 如果是 E 图像就数格子求面积看 potential difference, 如果是 V 图像就直接读 potential difference $V_1 - V_2$

$$\text{列方程 } \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = q(V_1 - V_2)$$



3. Thermodynamics 热力学

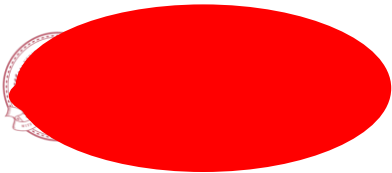
3.1 Thermodynamic scale 热力学温标

- 1) Thermodynamic (Kelvin) scale: theoretical scale that is independent of properties of any particular substance.
- 2) 公式: $K = ^\circ C + 273.15$
- 3) Absolute zero: temperature at which a system has minimum internal energy (not zero), impossible to remove any more energy, numerically equal to 0 K
- 4) Thermal Equilibrium: No net energy transfer between the bodies when the bodies are at the same temperature

3.2 Thermometers 温度计

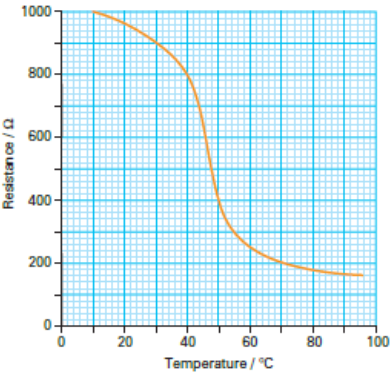
Principle 原理: A physical property that varies with temperature can be used for the measurement of temperature

	Thermistor	Thermocouple
Advantages	<ol style="list-style-type: none">1) Very robust2) Fast response3) Accurate4) Sensitive at low temps	<ol style="list-style-type: none">1) Faster response2) Wider range3) Small thermal capacity4) Physically small5) Readings taken at point6) Power supply not need
Disadvantages	<ol style="list-style-type: none">1) Narrower range2) Slower response time3) Larger thermal capacity4) Larger in size5) Hard to measure varying temperature.	High resistance voltmeter required



3.3 Sensitivity, range and linearity 灵敏度量程与线性

- 1) **Sensitivity** is the change in length per change in temperature. 图像一般体现为斜率
- 2) **Range** is the difference between maximum and minimum temperatures. 图像左右的跨度
- 3) **Linearity** is when a given change in temperature causes the same change in length. 图像是否为直线



3.4 Internal Energy 内能

- 1) 定义: Internal energy: sum of potential energy and kinetic energy of molecules in random motion
- 2) 宏观与微观体现:
 - a) Kinetic energy of molecules 宏观表现为 Temperature
 - b) Potential energy of molecules 宏观表现为 State of matter

3.5 State Change of Matter 物态变化

	Melting & Boiling 熔化和沸腾	Condensing & Freezing 液化和凝固
Kinetic energy of molecules 分子动能	unchanged (constant temperature)	unchanged (constant temperature)
Potential energy of molecules 分子势能	increase (separation between molecules increase)	decrease (separation between molecules decrease)
Internal Energy 内能	increase (absorb energy)	decrease (release energy)



3.6 Specific heat capacity 比热容

- 1) Specific heat capacity: the energy required per unit mass of the substance to raise the temperature by 1 K
- 2) 公式: $E = mc\Delta T$, where m is mass, c is specific heat capacity, ΔT is thermodynamic temperature

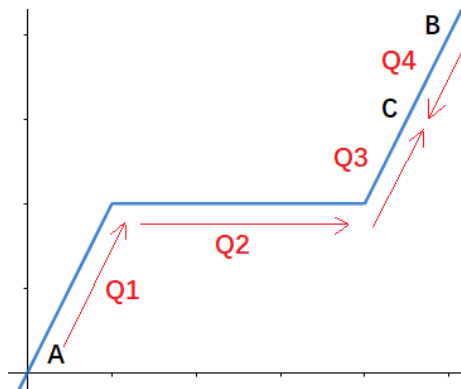
3.7 Specific latent heat 比潜热

- 1) Specific latent heat: quantity of (thermal) energy/heat to change state/phase of unit mass at constant temperature
- 2) 公式: $E = mL$, where m is mass, L is specific latent heat

3.8 Calculation of Thermodynamics 热力学计算

常见题型, A 和 B 两个不同温度的物体混合, 最终到达了温度 C。

- 1) 画物态变化的温度-时间图像, 找到 ABC 三点所对应的位置如下。



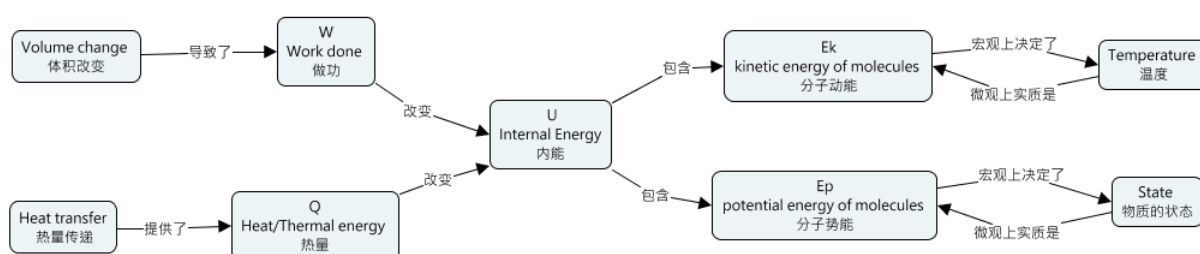
- 2) Total Energy Loss of B = Total Energy Gain of A
- 3) $Q1 + Q2 + Q3 = Q4$
- 4) $Q1, Q3, Q4$ 这类没发生物态变化的用 $E = mc\Delta T$
- 5) $Q2$ 这类发生物态变化的用 $E = mL$

3.9 Thermodynamic 1st Law 热力学第一定律

1) 定义: Increase in internal energy = energy supplied by heating + energy supplied by doing work

2) 公式: $\Delta U = Q + W$

3) 热力学第一定律的应用:



1) 内能变化的两个原因: 体积改变 + 热量传递

a) 体积改变带来做功 $W = p\Delta V$, 体积变大气体对外做功 W 为负, 体积变小气体被外界做功 W 为正

b) 热量传递包括两个方向, 向外散热 Q 为负, 被加热 Q 为正

2) 内能变化的两个表现: 温度 + 物质的状态

a) 温度变高意味着分子动能变大, 温度变低意味着分子动能变小

b) 固体到液体, 液体到气体意味着分子势能变大;

液体到固体, 气体到液体意味着分子势能变小。

4. Ideal Gas 理想气体

4.1 Amount of substance 物质的量

- 1) Mole 摩尔

定义: the amount of substance equal to the amount of molecules in 12g of carbon-12

符号: mol

- 2) Avogadro constant 阿伏伽德罗常数

定义: the number of atoms in 12g of carbon-12

符号: N_A

4.2 Ideal Gas 理想气体

- 1) 定义: gas that obeys the equation $pV/T = \text{constant}$, where p is pressure, V is volume and T is temperature

- 2) 公式 1: $pV = nRT$, R : universal molar gas constant, n 为气体的摩尔数

- 3) 公式 2: $pV = NkT$, k : Boltzmann constant, N 为气体分子的个数

- 4) 两个公式之间的联系:

a) n mol 气体的分子个数为 $n \cdot N_A$, 所以 $N = n \cdot N_A$

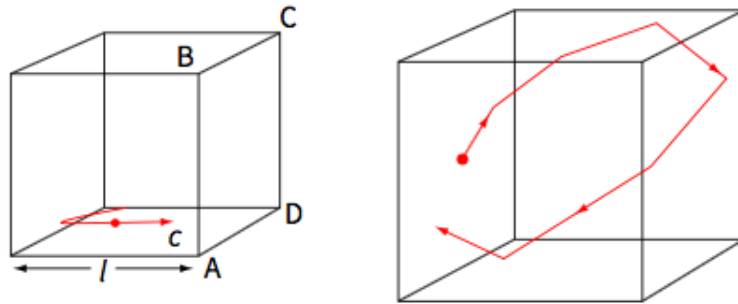
b) 因为 $nR = Nk$, 所以 $R = N_A \cdot k$

4.2.1 Kinetic model of ideal gas 理想气体动能模型

- 1) 前提: Average separation large compared with size of molecules \rightarrow No intermolecular forces \rightarrow No potential energy \rightarrow Internal Energy = Kinetic Energy \rightarrow Proportional to thermodynamic temperature (Kelvin)

2) 公式推导过程:

设单个分子的质量为 m , 直线运动速度为 c , 容器边长为 L , 分子数量为 N



a) 分子撞墙的 momentum 变化

$$\Delta p = 2mc$$

Assumption: Gas atoms/molecules behave as hard, identical spheres that are in continuous motion and undergo elastic collisions

解释: 弹性碰撞前后速度大小相等方向相反

b) 算出每个分子两次撞墙之间平均到时间 t 上的力

$$F = \frac{\Delta p}{\Delta t} = \frac{2mc}{\frac{2L}{c}} = \frac{mc^2}{L}$$

Assumption: Time of collisions negligible compared to time between collisions

解释: 因为只考虑分子往返的时间不考虑碰撞时间所以 $t = \frac{2L}{c}$

c) N 个分子撞墙产生这个力所产生的压强 p 的大小

$$p = N \frac{F}{A} = \frac{Nmc^2}{L^3} = \frac{Nmc^2}{V}$$

Assumption: Large number of molecule



解释：因为 N 数字很大，多个分子连续不断地撞墙所以产生了持续的压强

d) 三个维度的平均速度为 $\langle c^2 \rangle$ 带入

$$p = \frac{Nmc^2}{V} = \frac{Nm \langle c^2 \rangle}{3V}$$

e) 从步骤 d 可以获得关于密度的推论

$$p = \frac{1}{3} \rho \langle c^2 \rangle$$

解释：因为 m 是单个分子质量， Nm 是总质量， V 是总体积，所以 $\rho = Nm/V$

f) 回到步骤 d，变形得到

$$pV = \frac{Nm \langle c^2 \rangle}{3} = NkT$$

消掉 N ，得到

$$\frac{m \langle c^2 \rangle}{3} = kT$$

变形，得到

$$\frac{1}{2} m \langle c^2 \rangle = \frac{3}{2} kT$$

单个分子的平均动能为 $\frac{3}{2} kT$ ，总动能为 $\frac{3}{2} NkT$ ，也就是总内能

4.2.2 Root mean square 气体分子平均速度 RMS

c	1	2	3	4	5	6	7
c^2	1	4	9	16	25	36	49

- 1) 问题：假设有 7 个分子质量为 m ，速度分别为 1 到 7，用分子平均速度估算平均动能

方法 1：先直接对 v 平均，算出 mean speed，再用 $\frac{1}{2}mc^2$ ，结果乘 7

方法 2：先平方然后平均，算出 mean square speed $\langle c^2 \rangle$ ，然后再开根号算出 root mean square speed $\sqrt{\langle c^2 \rangle}$ ，再带入 $\frac{1}{2}m \langle c^2 \rangle$ ，结果乘 7

- 2) 结果：逐个计算出的实际总动能为 $70m$ ，用方法 1 平均估算出来的为 $56m$ ，用方法 2 平均估算出来的为 $70m$
- 3) 结论：直接平均算出的分子动能不准，用 rms speed 算出的分子动能是准的，原因是动能的公式里面带平方，所以不能先平均再平方，需要用 root mean square 来平均

5. Oscillation 振动

5.1 Oscillation Definitions 振动定义

- 1) Oscillation: back and forward motion
- 2) Angular frequency: angular frequency = 2π / Period

5.2 Simple harmonic motion 简谐振动

- 1) Definition 定义: simple harmonic motion (s.h.m.): acceleration proportional to distance from fixed point, and the direction towards fixed point
- 2) Formula 公式:

- a) 运动学时间相关的公式

x-t 位移时间: $x = A \cdot \sin \omega t$

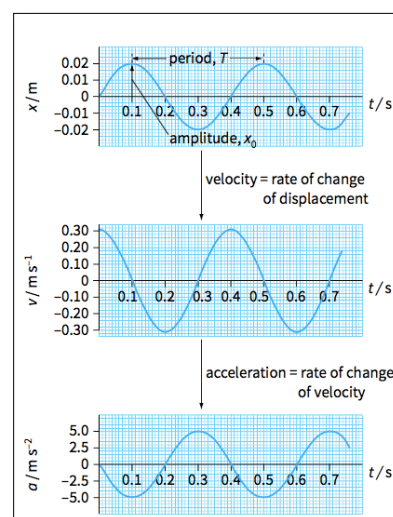
where A is amplitude

v-t 速度时间: $v = A\omega \cdot \cos \omega t$

where $A\omega$ is maximum speed

a-t 加速度时间: $a = -A\omega^2 \cdot \sin \omega t$

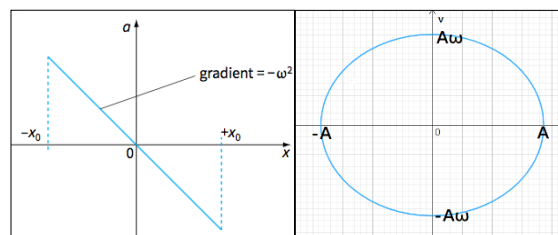
where $-A\omega^2$ is maximum acceleration



- b) 运动学时间无关的公式

a-x 加速度-位移: $a = -\omega^2 x$

x-v 位移-速度: $v = \pm \omega \sqrt{(A-x)^2}$



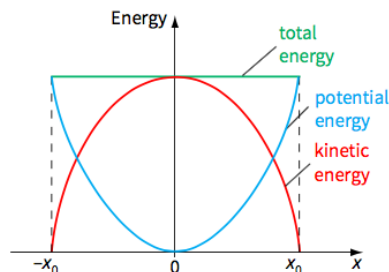
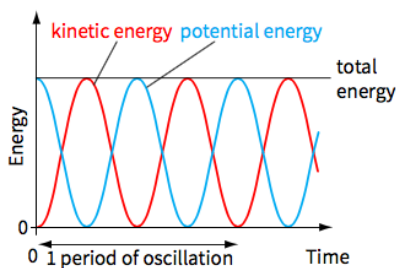


c) 能量公式

$$E_k = \frac{1}{2} m \omega^2 (A^2 - a^2)$$

因为能量守恒，系统能量 = 最大势能 = 最大动能

$$E_{\text{Total}} = E_{p_{\text{max}}} = E_{k_{\text{max}}} = \frac{1}{2} m v_{\text{max}}^2 = \frac{1}{2} m (A\omega)^2$$



3) How to prove s.h.m. 证明 SHM 的方法

- 找出 acceleration 和 displacement 之间的关系，写成 $a = -kx$ ，where k is constant
- Because k is constant, acceleration is proportional to displacement
- Negative sign shows directions of acceleration and displacement are opposite

4) Calculation of s.h.m. 简谐振动的计算

核心是先算出 ω ，然后带公式

- 从图像中找到周期 T ，用 $\omega = \frac{2\pi}{T}$ 算出 ω
- 将 a 和 x 的公式整理成 $a = -\omega^2 x$ ，开根号算出 ω

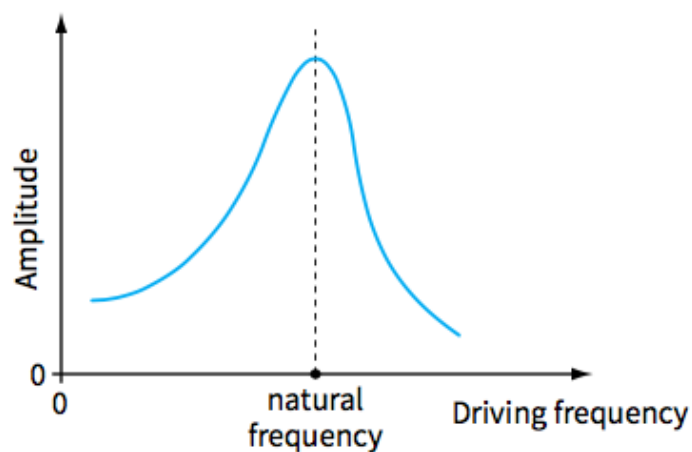


5.3 Resonance 共振

1) Basic Definition 基本定义

- a) Free oscillation 自由振动: body vibrates at natural frequency when there is no resultant external resistive force acting on it
- b) Natural frequency 固有频率: frequency at which body will vibrate when there is no resultant external resistive force acting on it
- c) Forced oscillation 受迫振动: object is made to oscillate
- d) Forced/driving frequency 受迫频率: frequency at which object is made to oscillate
- e) **Resonance 共振**: maximum amplitude of vibration of oscillating body when forced frequency equals natural frequency of vibration

2) A-f Graph 振幅频率图像



3) Resonance 的危害 & 用途

usage: musical instruments, heating by microwave through resonance with water molecules, MRI, tuning radio

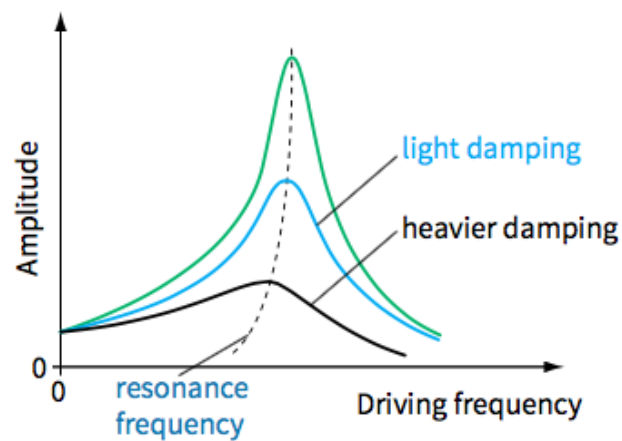
harm: bridge cracking, breaking glasses

5.4 Damping 阻尼

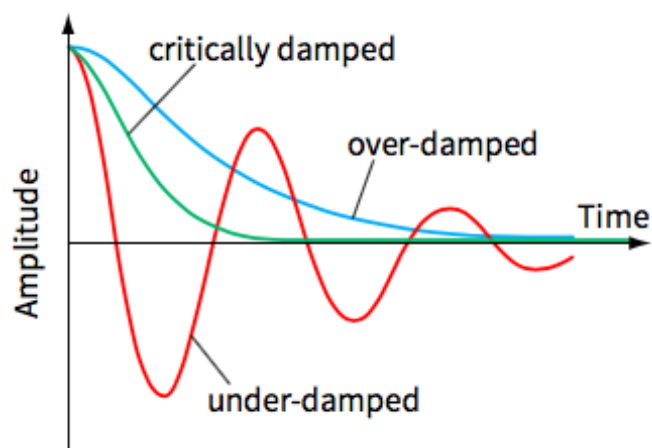
1) Definition 定义

- a) damping: reduction in energy and amplitude due to resistive forces
- b) light damping: amplitude decrease very gradually and oscillations continue for many oscillations
- c) critical damping: the minimum amount of damping required to return an oscillator to its equilibrium position without oscillating
- d) heavy damping: take a long time to return to its equilibrium position

2) A-f Graph 振幅频率图像



3) A-t Graph 振幅时间图像





6. Capacitance 电容

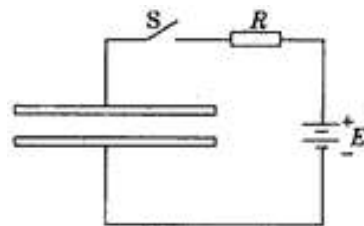
6.1 Definition 定义

- 1) 物理量定义: Capacitance = Q/V , where Q is the charge on one of the plates, V is the p.d. between the two plates
- 2) 公式: $C = \frac{Q}{V}$
- 3) 单位定义: Farad 法拉, Coulomb per Volt, 写作 F
- 4) 注意区分 capacitor 是电容器, 是一个电子元器件, 而 capacitance 是一个物理量。

6.2 Why capacitor stores energy, not charge

- 1) Energy is stored because work done is needed to separate the positive and negative charges. energy is released when the separated charges come together.
- 2) Charges on two plates are in same magnitude but opposite sign, therefore no net charge stored.

6.3 Two type of capacitor 两种 电容器

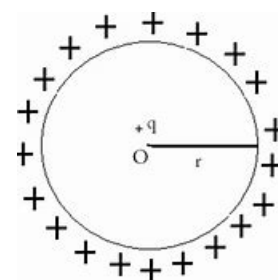


- 3) Parallel plate capacitor 平行板电容器
- 4) Metal sphere capacitor 球壳电容器

球壳电容推导过程:

$$\text{Potential at surface: } V = \frac{kQ}{r} = \frac{Q}{4\pi\epsilon_0 r}$$

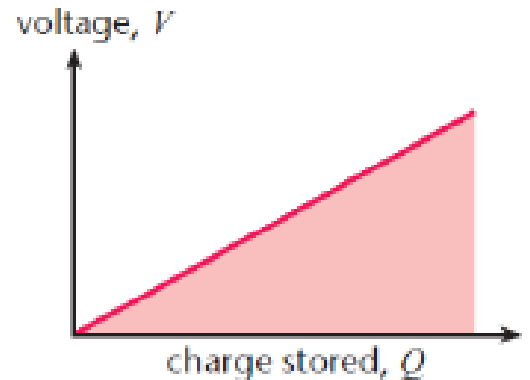
$$\text{Capacitance: } C = \frac{Q}{V} = 4\pi\epsilon_0 r$$



6.4 Graph and Energy Stored 图像和电容储存的能量

1) Graph 图像形状

根据电容公式 $Q = CV$ ，因为 C 是 constant
所以 VQ 成正比，所以 $V-Q$ 图像是一条过原点的
直线



2) Formula of energy 能量公式

$V-Q$ 图的面积代表能量 $E = \frac{1}{2}QV = \frac{1}{2}CV^2$

6.5 Capacitor in Series or Parallel 电容串并联

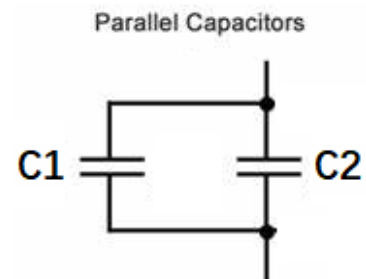
电容串并联以及分压规律与电阻完全相反

1) In parallel 电容并联

总电量: $Q_{total} = Q_1 + Q_2$

并联电压相同设为 V , $VC_{total} = VC_1 + VC_2$

消掉 V ，得到, $C_{total} = C_1 + C_2$



2) In series 电容串联

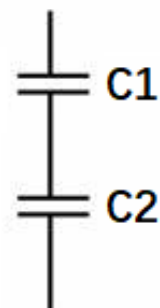
总电压: $V_{total} = V_1 + V_2$

C_1C_2 的电量相同设为 Q , $\frac{Q}{C_{total}} = \frac{Q}{C_1} + \frac{Q}{C_2}$

消掉 Q ，得到 $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$

$$C_{total} = \frac{C_1 C_2}{C_1 + C_2}$$

Series Capacitors



3) 解释串联电量相同: one plate with $+Q$ will induce $-Q$ on the opposite plate; by charge conservation, the capacitance nearby get $+Q$ of charge on one plate and induce $-Q$ again on its opposite plate... (C_1 上下板电量相同, C_1 下板和 C_2 上板相同...)



4) 串联电容分压规律

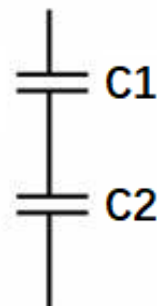
串联电路的 capacitor 根据 capacitance 的比例分压

电容越大，分压越小。

因为电量相同 $\frac{V1}{V2} = \frac{Q/C1}{Q/C2} = \frac{C2}{C1}$

所以如图情况下, $V1 = \frac{C2}{C1+C2}$; $V2 = \frac{C1}{C1+C2}$

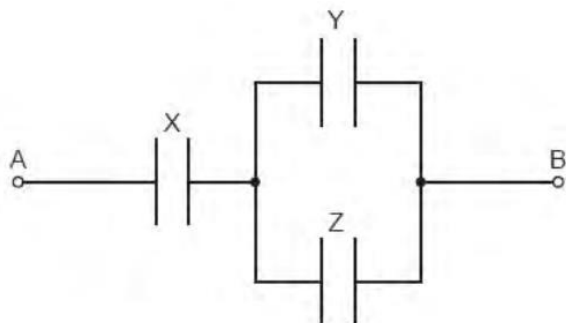
Series Capacitors



5) 电容的安全电压

每个电容有可以正常工作的最高电压限制，如果超出了，电容器会损坏。提高安全电压的方式是用多个电容串联，这样可以通过分压保证每个电容电压都在安全范围。

6) 串并联电容的安全电压计算



- 先计算出每个电容的分压比例，找到分压最多的电容。
- 将分压最多的电容分压设为最大安全电压。
- 利用比例算出其他电容电压，最后算出总电压。

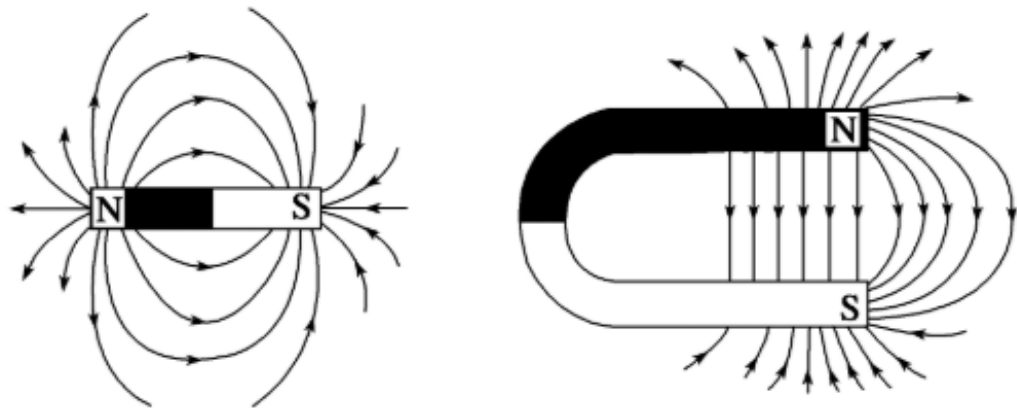
7. Magnetic Field 磁场

7.1 Definition 磁场定义

Region where there is a force on moving charge, magnetic poles or current carrying conductor.

7.2 Magnetic Field Line 磁感线

- 1) Direction 方向: The direction of a magnetic field line shows the direction of the force on a north pole at that point. (所以从图像上看是从 N 到 S)
- 2) Strength 强度: 磁感线越密集, 磁场越强
- 3) Graph 图像:

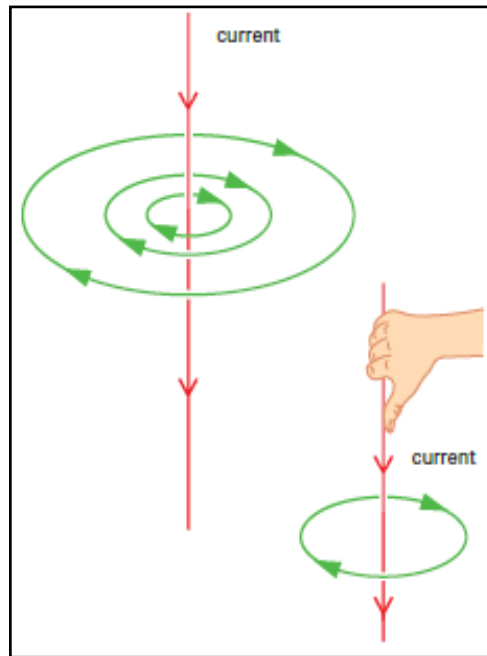


7.3 The magnetic effect of a current 电流的磁效应

1) Right-hand grip rule 右手螺旋法则

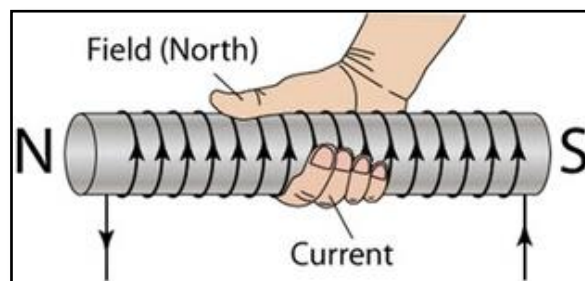
a) Straight conductor 通电直导线：直电流产生环形磁场

注意：靠中间的地方磁感线需要画的比较密集



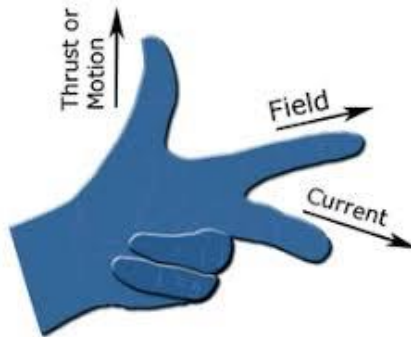
b) Solenoid coil 螺线管/线圈：环形电流产生直磁场

注意：产生的磁场和一个左 N 右 S 的条形磁体相同



7.4 Force on a current-carrying conductor 通电导线在磁场中受力

1) Fleming's Left-Hand Rule 左手定则



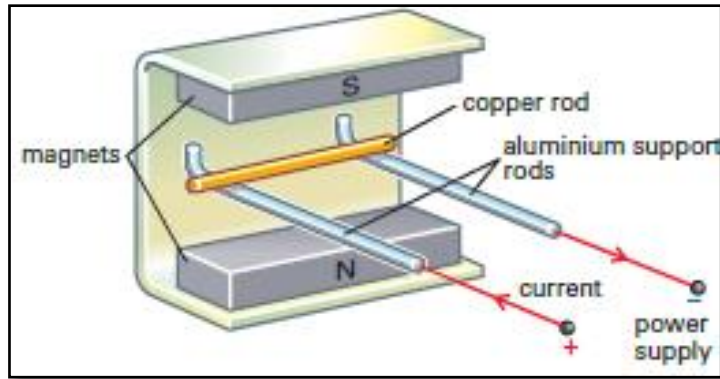
2) Formula 公式

$F = BIL\sin\theta$, where B is magnetic flux density, I is current, L is length of current-carrying conductor in the magnetic field, θ is angle between B and I

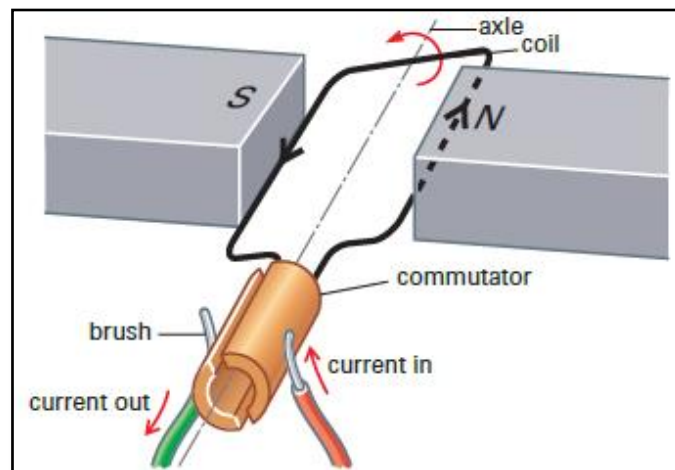
3) Magnetic flux density 磁通密度 (磁感应强度)

- a) 定义: Magnetic flux density: numerically equal to the force per unit length on straight conductor carrying unit current normal to the field
- b) 单位: Tesla 特斯拉, 写作 T
- c) 单位定义: Tesla: straight conductor carrying 1A current, wire normal to magnetic field of 1T, the force per unit length is 1Nm^{-1}

4) Example 例子: 铜线会向外运动

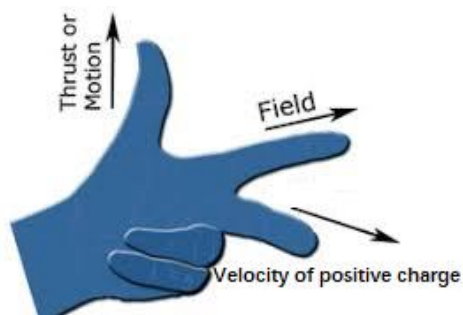


7.5 D.C. Motor 直流电机




7.6 Force on a moving charge 运动的电荷受力

1) Fleming's Left-Hand Rule 左手定则



注意此处电流方向为正电移动方向，也是负电移动的反方向

2) Formula 公式

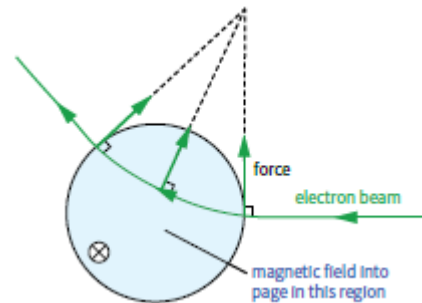


$F = Bqv\sin\theta$, where B is magnetic flux density, q is charge of the particle, v is the drift velocity of the moving charge, θ is angle between B and v

7.7 Circular motion of a moving charge 运动的电荷做圆周运动

- 1) 原理: Why circular motion?

The magnetic force is always perpendicular to the velocity of the particle, it does not change the magnitude of the particle, only change the direction, which providing the centripetal force needed



- 2) 公式: $Bqv = m \frac{v^2}{r}$

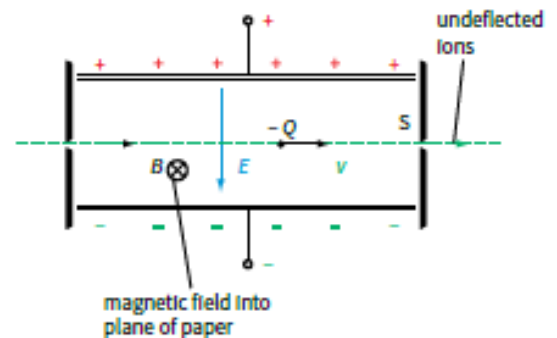
- 3) 荷质比 charge to mass ratio: $\frac{q}{m} = \frac{v}{Br}$

7.8 Velocity (speed) selector

速度选择器

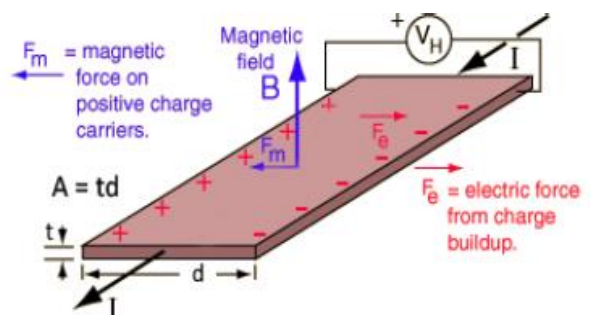
- 1) 公式: $Bqv = Eq$, $v = \frac{E}{B}$

- 2) 原理: 电场力和磁场力平衡, 当 E 和 B 确定好之后, 只有满足特定速度的粒子可以通过, 因为如果速度快就偏向磁场力那边, 如果速度慢就偏向电场力那边。



7.9 Hall Effect 霍尔效应

- 1) 原理: 电流通过的时候, 薄片里面的 charge carrier (可能是电子也可能是正电) 会因为磁场的存在而偏转到一侧, 从而产生一个电场, 当电场力和磁场力相等的时候达到平衡状态, 此时可以在电场的



两端检测到一个电压称为 V_H , hall voltage

2) 公式推导:

a) 磁场力等于电场力, $Bqv = Eq$

b) 电场公式: $E = \frac{V_H}{d}$

c) 联立: $Bqv = \frac{V_H}{d} q$

d) 消掉 q : $Bv = \frac{V_H}{d}$

e) 电流公式: $I = nAvq$

f) 将图像面积代换掉: $I = n(td)vq$

g) 把步骤 f 的 v 带到步骤 d 里面: $B \frac{I}{ntdq} = \frac{V_H}{d}$

h) 整理得到: $V_H = \frac{BI}{ntq}$

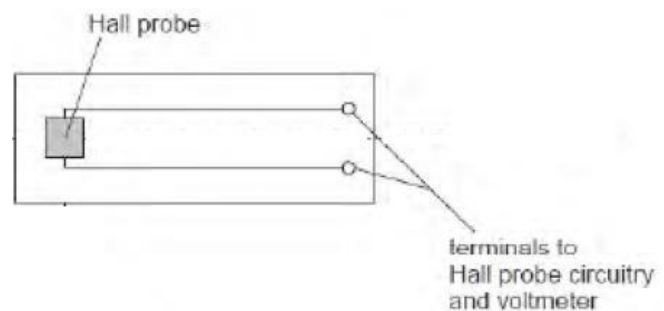
7.10 Hall Probe 霍尔探头

1) 公式: $V_H = \frac{BI}{ntq}$

2) 用途: 用于测量一个地方的磁场 flux density B

3) 使用方法: 将 hall probe 和要测量的磁场垂直放置, 见上一页霍尔效应的图。

4) 原理: 利用电压表的读数读出霍尔电压 V_H , 可以根据公式算出 B , 所以在某个点测出的 V_H 的大小一定代表着某个点的 flux density B 的大小





8. Electromagnetic Induction 电磁感应

8.1 Magnetic Flux 磁通量

- 5) 定义: the product of magnetic flux density normal to a circuit and the cross-sectional area of the circuit
- 6) 公式: $\Phi = BA$ (因为 B 是 vector, 所以 Φ 也是 vector)
- 7) 单位: Weber 韦伯, 写作 Wb
- 8) 单位定义: Tesla x metre²

8.2 Magnetic Flux Linkage 磁通链

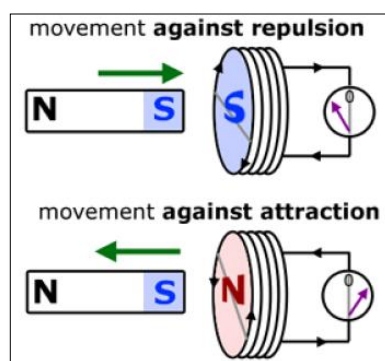
- 1) 定义: the product of magnetic flux and the number of turns
- 2) 公式: $n\Phi = nBA$
- 3) 单位: Weber 韦伯, 写作 Wb

8.3 Lenz's Law 楞次定律

- 1) 定义: An induced current or emf is in a direction so as to produce effects which oppose the change producing it.
- 2) 用途: 用于判断 induced emf 的方向

3) 常见案例：线圈（导体）和磁铁

- a) When magnet is approaching/leaving
- b) Flux linkage of the coil is changed
- c) Emf is induced in the coil
- d) Due to Lenz's Law, induced current produces a magnetic field against the magnet's motion

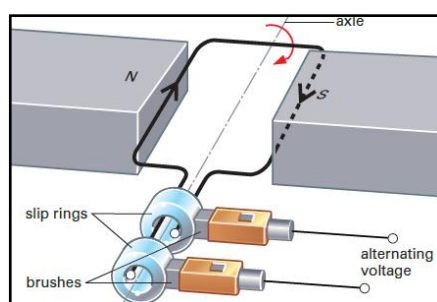


8.4 Faraday's Law 法拉第定律

- 1) 定义：Induced e.m.f. proportional to rate of change of flux linkage
- 2) 用途：用于判断 induced emf 的大小
- 3) 公式： $V = n \frac{\Delta\Phi}{\Delta t}$, where n is number of turns of the coil
- 4) 图像：induced emf V is the gradient of magnetic flux Φ (看斜率)
- 5) 注意事项： $\Delta\Phi$ 指的是 magnetic flux 的 change, 所以在计算的时候必须要用两个 flux 相减来计算, 而且 flux 为 vector, 在相减的时候注意考虑方向, 类似 momentum, 比如说 Φ 如果改变方向, $\Delta\Phi = 2\Phi$

8.5 A.C. Generator 交流发电机

- 1) 原理：利用 coil 在磁场中进行旋转, 通过改变 B 穿过的 Area 来改变 flux, 从而产生 induced emf



2) 图像及对应关系:

a) Flux/flux linkage 和 time 图像

因为是在不停旋转所以图像是 sin 或者 cos

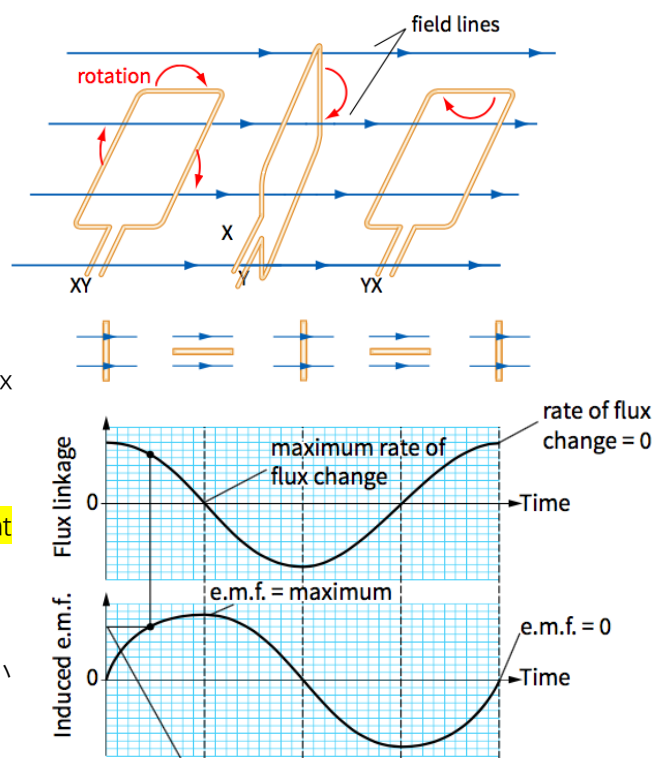
b) Induced emf 图像

为 flux 和 time 图像的 gradient, 如果 flux 是 sin, emf 就是 cos, 反之亦然。

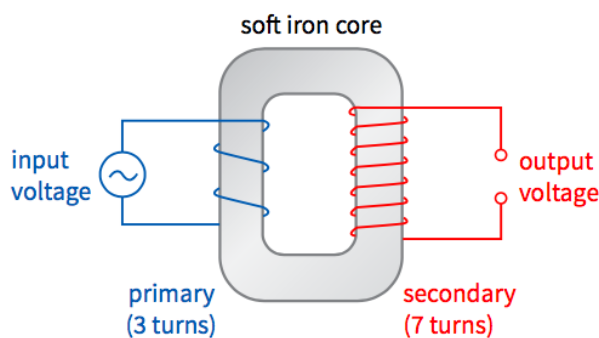
c) 结论: flux 和 induced emf 图像是 gradient

关系, phase difference 是 $\pi/2=90$ 度,

当 flux 最大的时候, emf 最小, flux 最小的时候, emf 最大。



1. Transformer 变压器



1) 各部分的作用

a) Primary coil: 输入电压 Secondary coil: 输出电压

b) Soft iron core 的作用: prevent the loss of magnetic field lines (concentrate magnetic flux)

c) 为什么 iron core 需要 laminated: reduce eddy current to prevent energy loss to heat



2) 原理

- a) A.C voltage in the primary coil produces changing magnetic field
- b) Magnetic flux linkage in secondary coil also changes
- c) Changing magnetic flux linkage in secondary coil induces output voltage

3) 公式

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1} \quad \text{因为 primary coil 和 secondary coil 的 power 一样}$$

$$\text{, 又因为 } P = VI, \text{ 所以 } \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

4) Application 应用

高压输电的目的是, 放大 V 减小 I, $Q=I^2Rt$, 小电流意味着发热消耗少

9. Alternating Current 交流电

9.1 Expression 表达式

$$V = V_p \sin \omega t \quad I = I_p \sin \omega t$$

Where I_p V_p are peak value, ω is angular frequency = $\frac{2\pi}{T}$

9.2 Root-mean-square RMS 值

1) Definition 定义

Root-mean-square value of alternating **current**: the value of constant current producing the same heating effect in a resistor

Root-mean-square value of alternating **voltage**: the value of constant current producing the same heating effect in a resistor

2) Calculation 计算

a) square wave 方波

分别用 $Q = I^2 R t = \frac{V^2}{R} t$ 计算出每一段产生的热量加起来，再反着把整体当成一段算回来 V 或者 I

b) sinusoidal wave 正弦波

$$V_{Peak} = \sqrt{2} V_{RMS}$$

3) Usage 用途

RMS value 是一种对交流电进行平均的方式，区别于 mean value.

当用来计算功率或者能量的时候，因为公式带平方，所以要用 RMS 来进行平均。

当用来计算电流或者电量的时候，因为公式不带平方，所以要用 mean 来进行平均。

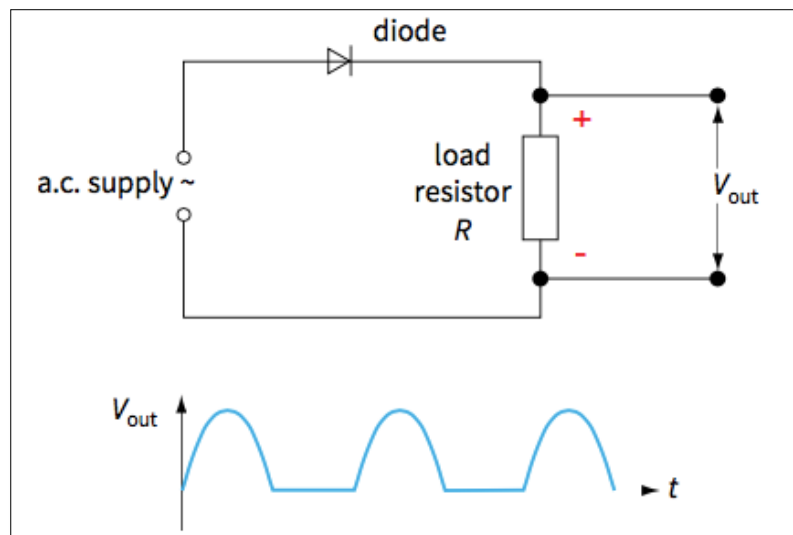
9.3 Rectification 整流 (交流变直流)

1) Definition 定义:

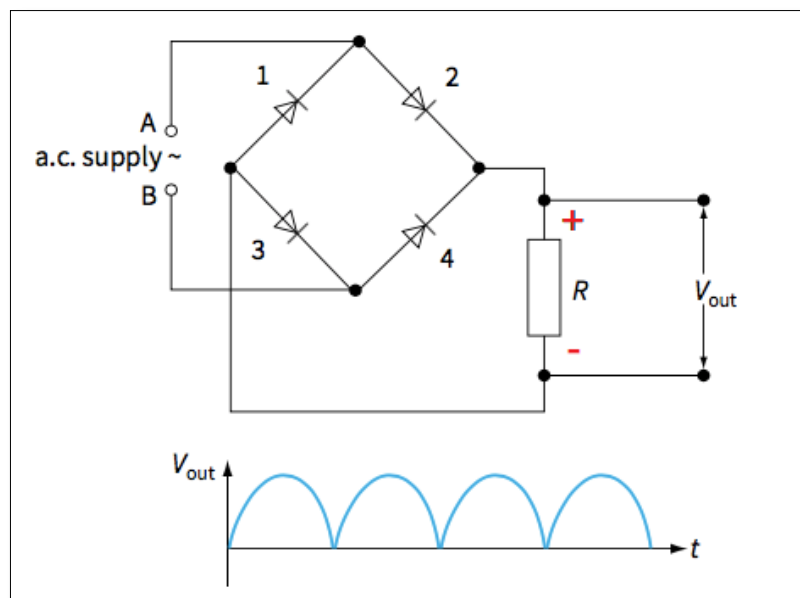
Rectification: the process during which a.c. is converted to d.c.

2) Method 方法

a) Diode 直接用二极管过滤掉反向电压，缺点是浪费了一半的能量。

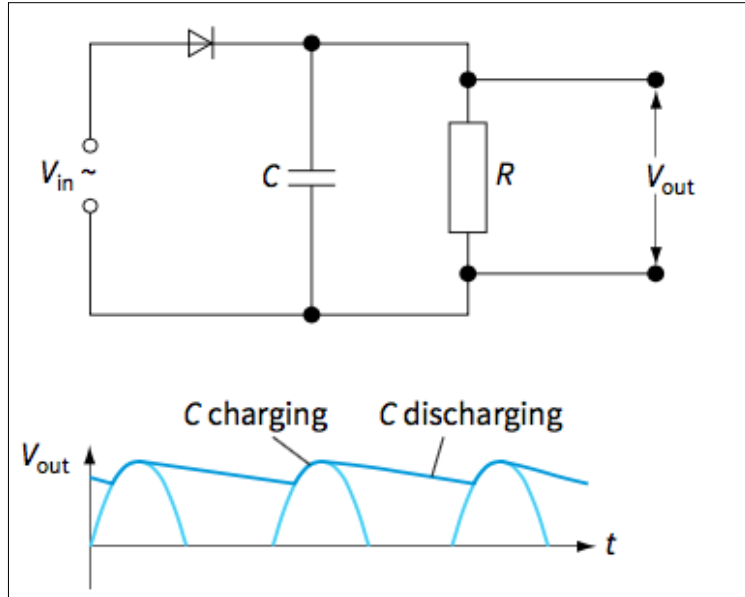


b) Bridge rectifier 桥电路



3) Smoothing 稳压

a) **Principle 原理:** 在用电器两端并联一个电容，电压升高时电容充电，电压降低时电容可以用做备用电源给用电器放电。



b) Better smoothing 增强稳压效果

- i) larger capacity (电容可以存更多的电用于释放)
- ii) larger resistance (减缓放电速度)

9.4 Pros and Cons 交流电的优劣势

1) Pros

Easy to change the voltage using transformer, can be changed into high voltage, reduce current in the cable, reduce power loss to heat, to saves money and fuel, and reduce the requirement for more power stations

2) Cons

- a) Cables need greater insulation
- b) Rectification needed before used in some devices

10. Electronics 电子

10.1 Electronic system 电子系统

一个完整的电子系统包括三个部分：



- 1) sensing device 传感器：负责从环境中获取 input 信号
- 2) processor 处理器：负责将 input 信号处理为 output 信号
- 3) output device 输出设备：负责用 output 信号控制设备执行

10.2 Sensing Device 传感器

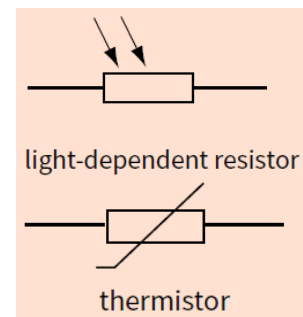
- 1) 定义：Sensing device is an electronic component with a property that changes when a physical quantity alters

- 2) 分类：
 - a) **LDR (Light-Dependent Resistor) 光敏电阻**

特点：电阻随着光强增加而降低

- b) **Thermistor 热敏电阻**

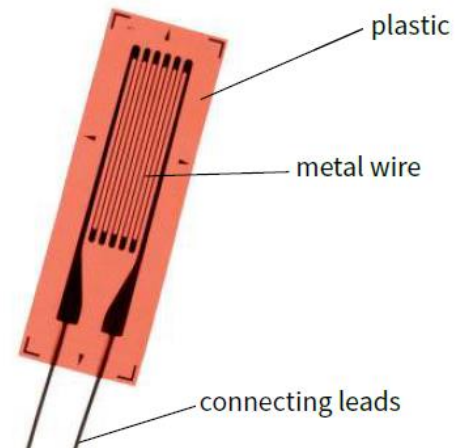
特点：电阻随着温度上升而降低



c) **Metal-wire strain gauge 金属丝应变计**

结构: Fine metal wire encapsulated by plastic envelope

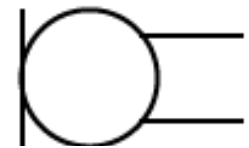
特点: 上下伸长电阻变大, 上下挤压电阻变小



d) **Piezo-electric transducer 压电式转换器**

结构: crystal consisting of positive and negative ions in a regular arrangement

特点: 当感受到压强的时候, 因为正负电分布不均会产生电压

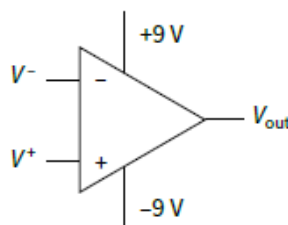


Symbol of microphone

用途: 做麦克风, 把声波振动转成电压信号

10.3 Processor 处理器

1) Op-Amp (Operational Amplifier) 运算放大器:



a) Op-Amp 是一种最简单的 processor, 用于放大 V^+ 和 V^- 两个 input 的电势差, 是一个有五个接口的电子元器件, 共有三个信号接口和两个电源接口。

b) 信号接口: 两个 input 接口 V^- (inverting input) 和 V^+ (non-inverting input) 以及一个 output 接口 V_{out} ,

c) 电源接口: 决定 V_{out} 能够输出的最大值和最小值, 确定 V_{out} 的范围。



2) Ideal Op-Amp 理想 Op-Amp 的特点:

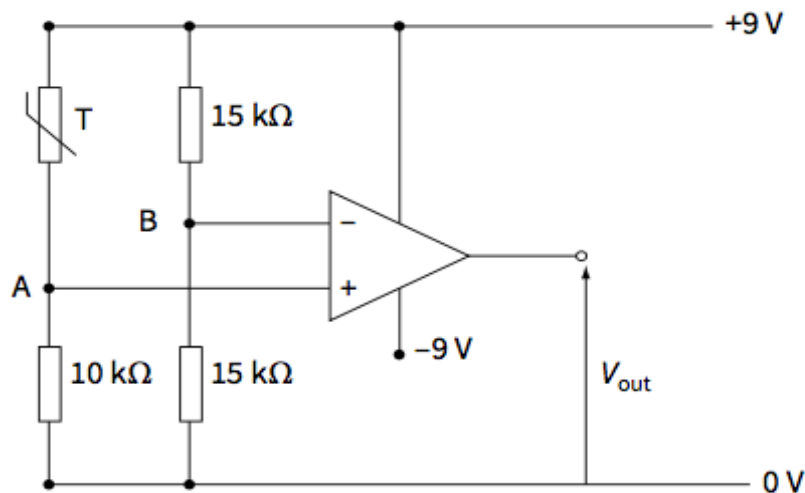
- a) Infinite open-loop gain
- b) zero output resistance (no lost volts out)
- c) infinite input resistance (no lost volts in, no current coming in)
- d) infinite bandwidth (amplify all frequencies by the same amount)
- e) infinite slew rate (once V_{in} changes, V_{out} will change instantaneously, no time delay)

3) Comparator 比较器 (Open-loop 开路用法):

Op-amp 的作用是放大 V_+ 和 V_- 的 difference, 放大倍数为 gain.

所以有:
$$V_{out} = gain (V_+ - V_-)$$

在理想情况下, Op-Amp 的 $gain \approx \infty$, 所以只要 $V_+ > V_-$, 放大完之后 V_{out} 为正无穷, 输出为电源能够提供的最大值。当 $V_+ < V_-$, 放大完之后 V_{out} 为负无穷, 输出为电源能够提供的最小值。这种情况叫做 saturate 饱和。当 $V_+ = V_-$ 的时候, $V_{out} = 0$.



如图所示, 热敏电阻 T 的阻值会随着温度改变而改变, 会导致 A 点分压不同 V_+ 发生改变, B 点始终是一半 $4.5V$ 所以 V_- 不变。所以 V_+ 和 V_- 之间的 difference 会



随着温度变化。当 $V_+ > V_-$ 时, V_{out} 输出最大值 9V, $V_+ = V_-$ 时, V_{out} 输出 0, 当 $V_+ < V_-$ 时, V_{out} 输出最小值 -9V

4) Negative feedback 负反馈电路 (Close-loop 闭路用法)

- 1) 定义: Part of the output is added into the inverting input
- 2) 优势:
 - a) Reduces gain
 - b) Increases bandwidth
 - c) Less distortion
 - d) Greater stability
- 3) 特点: 在负反馈电路中, 只有一个接口被用作 V_{in} 接受来自传感器的电压, V_+ 和 V_- 不是人为输入, 由 Op-Amp 自己调节为 $V_+ = V_-$

4) $V_+ = V_-$ 的解释:

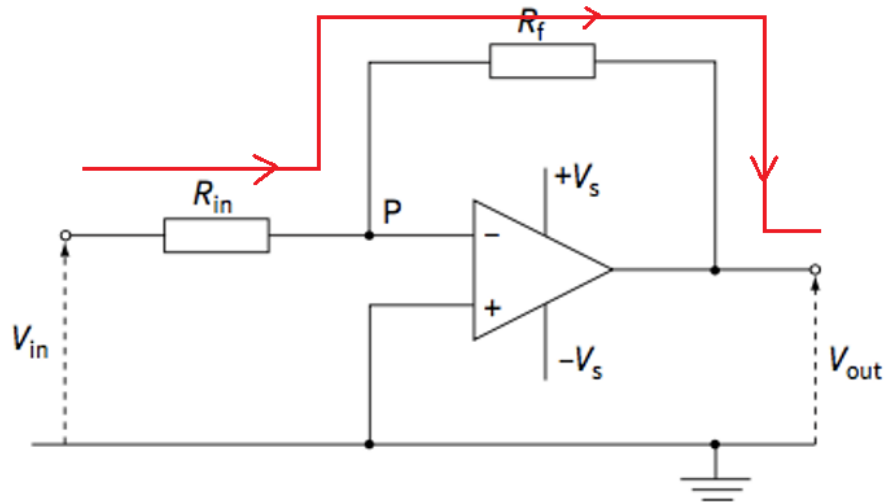
Gain of ideal amplifier is infinite, for the amplifier not to saturate, the difference between V_+ and V_- must be almost 0. So $V_+ = V_-$

5) Op-Amp 电路的通用计算方法:

- a) 利用 $V_+ = V_-$, 标出 V_+ 和 V_- 的电压, 以及所有与 V_+ 和 V_- 直接相连的电压
- b) 将所有连到 Op-Amp 接口上的电线划掉 (因为 infinite input resistance 所以没有任何电流流入 Op-amp)
- c) 剩下一条电路, 标好电流方向, 按电流方向列串联电流相等的式子

$$\frac{V1 - V2}{R1} = \frac{V3 - V4}{R2}$$

6) Inverting op-amp 电路 (第一种负反馈电路)



a) 在 inverting op-amp 电路中，P 点被称之为 virtual earth:

Gain of ideal amplifier is infinite, for the amplifier not to saturate, the difference between V_+ and V_- must be almost 0. So $V_- = V_+ = 0$.

b) 计算过程：第一步： $V_- = V_+ = 0$,

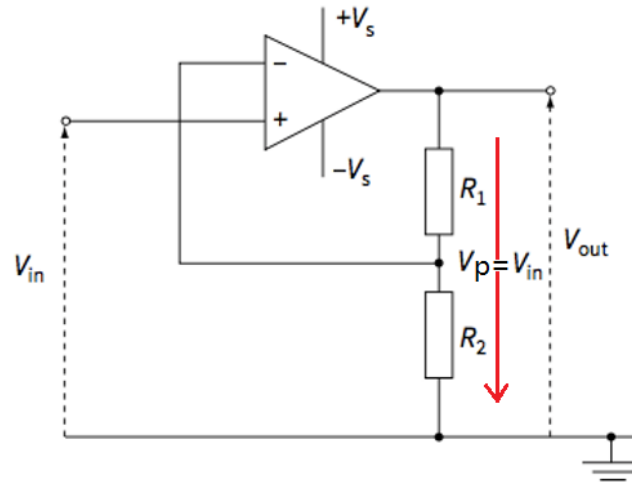
第二步：划掉多余的导线，剩下一条如图所示的电路

第三步：
$$\frac{V_{in}-0}{R_{in}} = \frac{0-V_{out}}{R_f}$$

第四步：得到新的放大倍数
$$\text{gain} = \frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$$

c) 由结果可见，gain 为负数，所以 V_{in} 和 V_{out} 符号相反，因此这种接法得名 inverting Op-Amp，如果用于放大交流电， V_{in} 和 V_{out} 为 antiphase

6) Non-inverting op-amp 电路（第二种负反馈电路）



a) 计算过程：第一步： $V_p = V_- = V_+ = V_{in}$

第二步：划掉多余的导线，剩下一条如图所示的电路

第三步：
$$\frac{V_{out} - V_{in}}{R_1} = \frac{V_{in} - 0}{R_2}$$

第四步：得到新的放大倍数 $gain = \frac{V_{out}}{V_{in}} = 1 + \frac{R_1}{R_2}$

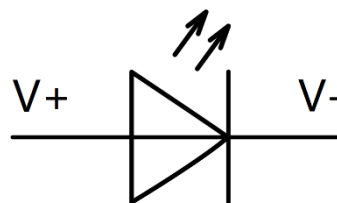
b) 由结果可见，gain 为正数，所以 V_{in} 和 V_{out} 符号相同，因此这种接法得名 non-inverting Op-Amp, 如果用于放大交流电, V_{in} 和 V_{out} 为 in phase

10.4 Output Device 输出设备

5) LED (Light-Emitting Diode) 发光二极管

特点：单向导通，只有 $V_+ > V_-$ 的时候才会亮，

用途：通常一端接地另一端接 V_{out} 通过是否亮来判断 V_{out} 大于零或者小于零

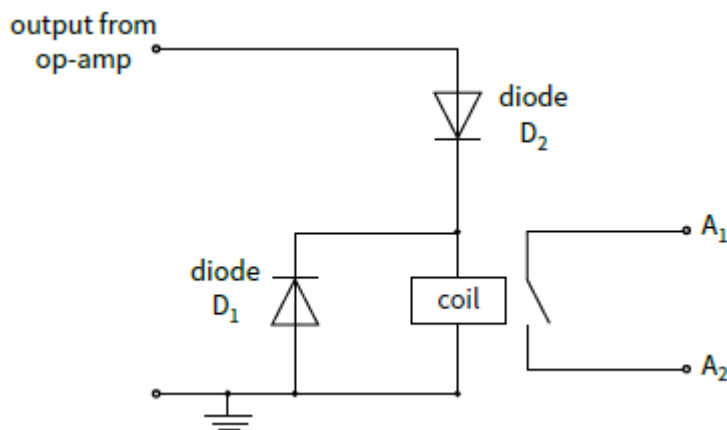


6) Calibrated Voltmeter 修改过刻度的电压表

用途：通常一端接地另一端接 V_{out} ，来测 V_{out} 的电压值，用指针的偏转作为对 input signal 的反映，如果 input 连热敏电阻通常为温度计，连光敏电阻通常为测光强的指示器，连 metal wire strain gauge 通常为检测伸长量的仪器

7) Relay 电磁继电器

用途：work as a remote switch, use low current low voltage circuit to control high current high voltage circuit



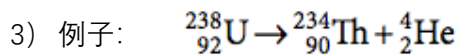
左边连从 Op-Amp 出来的 V_{out} ，右边 A1A2 的电路为需要被开关的电路。



11. Nuclear Physics 核物理

11.1 Mass-Energy Conservation 质能守恒

- 1) 公式: $E = mc^2$, c 为光速
- 2) 理解: 核反应如果反应后比反应前质量小, 说明质量转化成了能量, 所以是放能反应; 反之如果反应后质量变大, 说明给反应提供的能量被吸收转换为了物体的质量。



U-238 的 nucleus 质量为 $3.95283 \times 10^{-25}\text{kg}$

Th-234 和 He-4 的 nucleus 质量和为 $3.95276 \times 10^{-25}\text{kg}$

所以反应后比反应前少了 $7.0 \times 10^{-30}\text{kg}$

这些质量转化为能量就是 $7.0 \times 10^{-30} \times (3.0 \times 10^8)^2 = 6.3 \times 10^{-13}\text{J}$

(光速 $c = 3.0 \times 10^8$)

- 4) **Mass defect 质量亏损**: mass defect of a nucleus is equal to the difference between the total mass of the individual, separate nucleons and the mass of the nucleus

11.2 Atomic Mass Unit 原子质量单位

- 1) 定义: 1u is defined as 1/12 of the mass of a neutral atom of carbon-12, equal to $1.66 \times 10^{-27}\text{kg}$
- 2) 理解: u 是一个质量单位, 左上角的 nucleon number 是多少就代表这个原子核重多少 u, 比如 U-238 质量就是 238u, α -particle 的质量是 4u
- 3) Mass excess = mass – nucleon number



11.3 Binding Energy 结合能

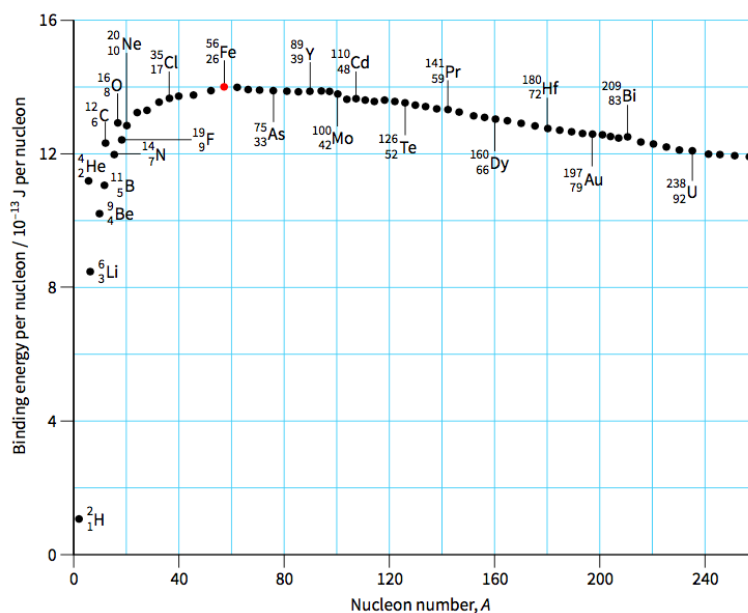
- 1) 定义: minimum energy needed to pull a nucleus apart into its separate nucleons
- 2) 理解: 因为把 nucleus 的 nucleon 全部拆开是需要我们提供能量的, 所以全部拆开之后总质量一定会变大, 有了这个质量变化, 我们可以用 $E = mc^2$ 求出能量。
- 3) Binding energy per nucleon: 因为拆开的越多需要的 binding energy 就一定越大, 但是为了比较, 我们需要计算平均每个 nucleon 需要多少能量用于拆开。

计算过程: a) 分散的 proton 和 neutron 的质量 - 原子核质量 = Δm

b) 用 $E = \Delta mc^2$ 算出总 binding energy

c) $E / \text{nucleon number}$ 就算出了 Binding energy per nucleon

- 4) Binding energy 原子核的稳定性: Binding energy 很大代表着需要更多能量去拆开原子核, 所以相应的原子就更稳定
- 5) Graph of Binding energy per nucleon 比结合能的图像



如图所示, Fe-56 拥有最大的 binding energy per nucleon, 意味着铁原子核最稳定。



11.4 Nuclear fission and fusion 核裂变和核聚变

- 1) **Fission 裂变**: Fission is the process in which a massive nucleus splits to form two smaller fragments
- 2) **Fusion 聚变**: Fusion is the process by which two very light nuclei join together to form a heavier nucleus
- 3) **发生裂变和聚变的条件**: 需要放出能量, 也就意味着生成物的 binding energy 需要比反应物的 binding energy 要大, 从图像上看, 聚变只能发生在铁左边, 裂变只能发生在铁右边, 两个反应的从反应物到生成物的方向都是朝着 Fe-56 的前进。

11.5 Properties of Decay 衰变的性质

1) Decay 定义

spontaneous emission of alpha beta particles and gamma ray by unstable nucleus

2) Random 随机性

- a) 定义: impossible to predict and each nucleus has the same probability of decaying per unit time
- b) 反映到图像上: graph will have fluctuations in count rate

3) Spontaneous 自发性

- a) 定义: not affected by external factors such as the presence of other nuclei, temperature and pressure
- b) 反映到图像上: graphs are the same under difference condition

11.6 Calculation of Decay 衰变的计算

1) Decay constant 衰变常数 λ

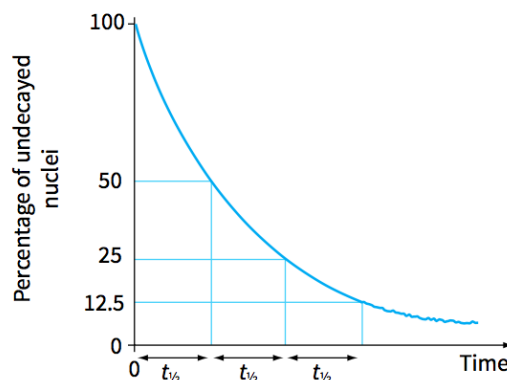
- a) 定义: The probability that an individual nucleus will decay per unit time interval
- b) 理解: 1 秒内该原子核可能会发生衰变的概率



- c) Decay constant 只与原子核的类型有关，和其他所有东西无关，因为 decay 是 spontaneous 的
- 2) Activity 放射性 A
- a) 定义: the rate at which nuclei decay
- b) 计算: $A = \lambda N$, λ 为 decay constant 衰变常数, N 为 number of undecayed nuclei 也就是还剩下多少原子核还没有衰变
- c) 单位: Bq, 代表每秒衰变多少次
- d) 理解: 从公式上看, 因为 λ 是 constant, 所以 activity 和剩下原子核的数量成正比。

11.7 Graph of Decay 衰变图像

- 1) number of undecayed nuclei 剩的原子核: $N = N_0 e^{-\lambda t}$ N_0 为最初原子核数量
- 2) activity 放射性: $A = A_0 e^{-\lambda t}$, A_0 为最初的放射性
- 3) **Half-life 半衰期 $t_{1/2}$:** in a time equal to one half-life, the number of undecayed nuclei is halved
- 4) 半衰期与 decay constant 的关系: 公式表里有
- $$\lambda = \frac{0.693}{t_{1/2}}$$



推导过程 $0.5A_0 = A_0 e^{-\lambda t_{1/2}} \rightarrow \lambda = \frac{\ln 2}{t_{1/2}}$

- 5) count rate 读数: 因为存在 background radiation, 所以 count rate 一定比实际的 activity 要大一些, 读图的时候要记得考虑到 background radiation 对 count rate 的影响。比如 count rate 最后降到 10 说明有 10Bq 的 background radiation, 当 count rate 为 100Bq 的时候, 其实 activity 只有 90Bq, 那如果要在图上读出半衰期, 需要找 activity 为 45Bq 的时间, 也就是 count rate 为 55Bq 的时间来计算。



12. Quantum Physics 量子物理

12.1 Wave and Particle Model of Light 光的波动和粒子模型

- 1) **Wave Model 波动模型**: Energy is continuous, intensity proportional to square of amplitude.
- 2) **Particle Model 粒子模型**: Energy is discrete, photon has packet of energy dependent on frequency.

12.2 Photon 光子

- 1) 定义: Packet of energy of electromagnetic radiation, equal to the product of Planck constant and frequency.
- 2) 公式: $E = hf$ where h is Planck constant, f is frequency
- 3) 理解: 公式算出的能量是单个光子的能量, 也就是每一份的能量由频率决定。
- 4) Intensity 和光子能量的关系:
 - a) Intensity 是每秒打到某块面积的总能量, photon 能量是单个光子的能量。
 - b) 所以在这块面积上的 intensity = 每一个 photon 能量 \times 光子数量

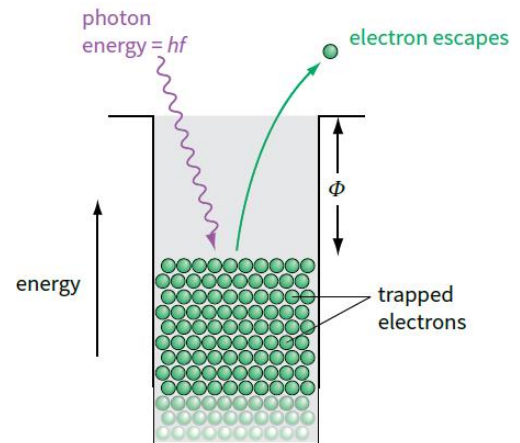
12.3 Photoelectric Effect 光电效应

- 1) 定义: When an electromagnetic radiation of sufficiently high frequency is incident on a metal surface, electrons are emitted.

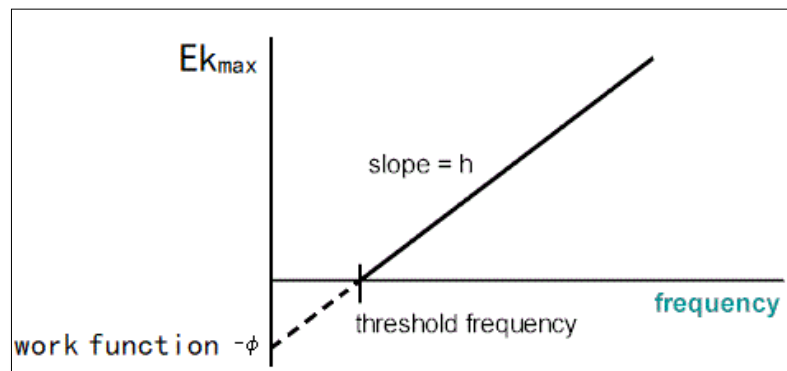
- 2) 公式: $E_{k_{max}} = hf - \phi$

where ϕ is work function of the metal,

$E_{k_{max}}$ is maximum kinetic energy of the electron



- 3) Work function 定义: minimum energy of photon needed to cause emission of photon from surface
- 4) Threshold frequency 临界频率: the lowest frequency of electromagnetic radiation giving rise to emission of electrons from surface

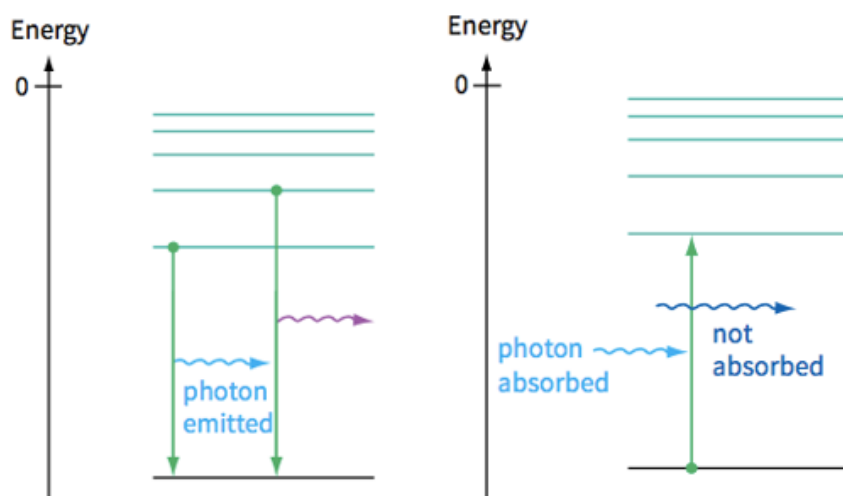
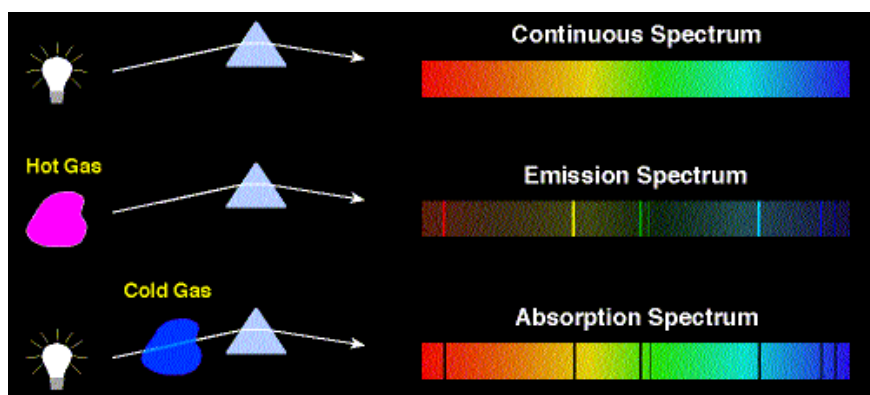


5) 实验现象及其解释:

Observation 现象	Wave Model 能否解释	Particle Model 能否解释
Instantaneous emission of electron (No time delay)	No, only very intense light should be needed to have immediate effect	Yes, a single photon is enough to release one electron instantaneously
Rate of emission proportional to intensity	Yes, greater intensity means more energy, so more electrons are released	Yes, greater intensity means more photons per second, so more electrons released per second

Maximum kinetic energy independent of intensity	No, greater intensity should mean electrons have more energy	Yes, greater intensity does not mean more energetic photons, so electrons cannot have more energy
A minimum threshold frequency of light is needed	No, low-frequency light should work if incident for a long time.	Yes, a photon in a low-frequency light beam has energy that is too small to release an electron
Maximum kinetic energy dependent on frequency	No, it should be increasing intensity, not frequency, that increases energy of electrons	Yes, higher frequency means more energetic photons; so electrons gain more energy and can move faster

12.4 Line Spectra 线谱图



- 1) Why **discrete electron energy levels** may be inferred from the line spectrum?

Each colored line corresponds to one frequency and thus photon of specific energy.

Since photon energy comes from energy change of electron, specific energy changes mean discrete electron energy levels.

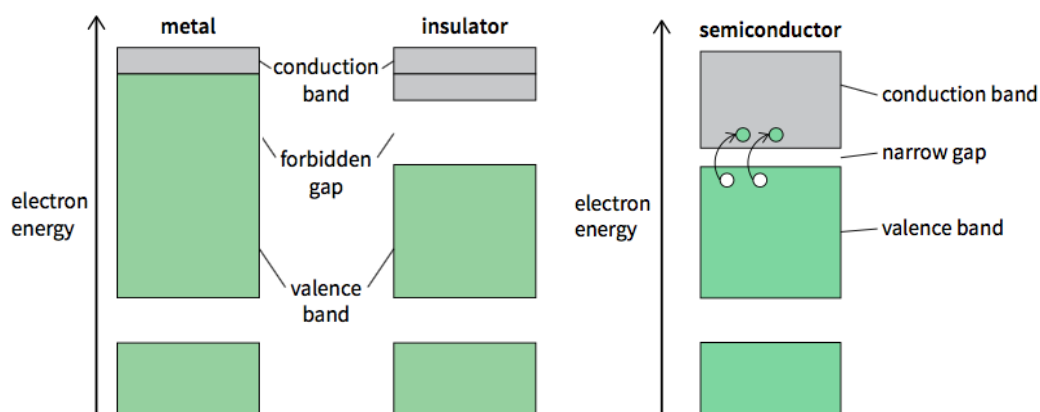
- 2) Why **emission spectrum is line** spectrum, not continuous spectrum?

Discrete electron energy levels cause specific energy change of electron, which emits photon of specific energy, each corresponding to one frequency of light, therefore there is line spectrum not continuous spectrum.

- 3) Why dark lines in **absorption spectrum**?

Photon is absorbed by electron if it has energy equal to difference of two energy levels, and the electron de-excites and emits photon in every direction. Therefore, those specific frequencies in the spectrum will be dark lines.

12.5 Band Theory 能带理论



- 1) Why energy band, not energy level in solid and liquid?

In liquid and solid, atoms are closer to each other and interacting with each other, making energy levels more complicated, resulting in many closely spaced energy levels, forming energy bands.

- 2) Conduction band & Valence band & Forbidden gap

Conduction band: 电子具有高能量，可以自由移动（可以导电）

Valence band: 电子具有低能量，无法自由移动（无法导电）

Forbidden gap: 在 conduction 和 valence band 中间, 没有电子处于这个能量范围

3) 用能带理论解释金属和半导体的导电

a) Metal 金属:

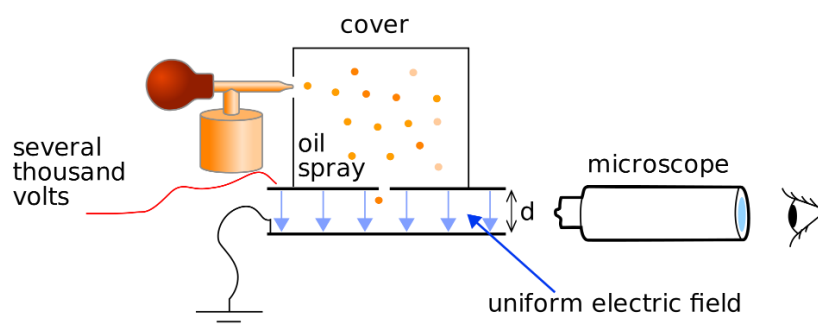
There is no forbidden gap in metal, because conduction and valence band overlap, therefore no change in charge carrier density when temperature rises. But high temperature means more lattice vibration, which resist the moving of electron, therefore resistance increase.

b) Thermistor/LDR 光敏/热敏电阻:

When temperature of thermistor rises/light incident on the LDR, electrons in valence band absorb energy to enter conduction band, leaving a hole in the valence band. Because both electron in the conduction band and holes in valence band are charge carriers, increasing charge carrier means low resistance.

12.6 Particle nature of electron: Millikan Experiment

油滴实验



1) Procedure 实验流程:

- Oil charged by friction
- Horizontal and parallel metal plates with adjustable p.d.

c) Adjust p.d. until oil drop is stationary

2) Calculation 计算:

油滴受到的向下的重力等于向上的电场力 $mg = Eq$, 计算出带电量 q

3) Result 结果:

Charges are all integral multiples of $1.6 \times 10^{-19} \text{C}$ 得出结果全是 $1.6 \times 10^{-19} \text{C}$ 的整数倍

4) Conclusion 结论:

Charge is quantized: charge exists in discrete and equal quantities of elementary charge $1.6 \times 10^{-19} \text{C}$

5) Wave-particle duality 波粒二象性:

这个实验说明了电子具有粒子性，是离散的粒子。

12.7 Wave nature of electron: electron diffraction 电子衍射

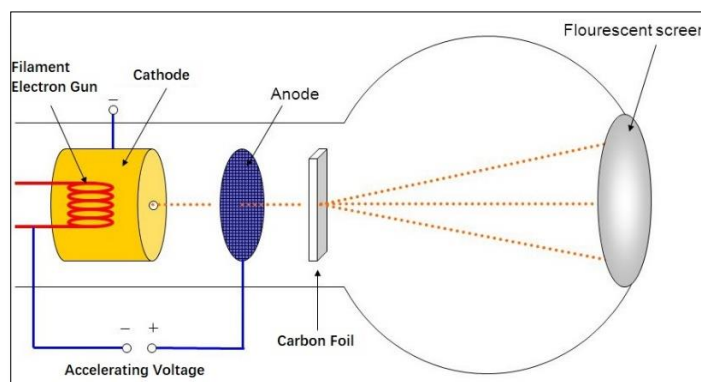
1) De Broglie wavelength 德布罗意波

a) 定义: wavelength associated with a particle that is moving

b) 公式: $\lambda = \frac{h}{p}$, where h is Planck constant, p is momentum, $p = mv$

c) 使用范围: 任何运动的物体，包括光子在内。

2) Electron diffraction 电子衍射实验





a) **Procedure 流程:** i) Accelerate electron beam in a vacuum

先在真空中加速电子 $\frac{1}{2}mv^2 = Ve$, 让电子的速度变快波长

变短, 利用 $\lambda = \frac{h}{mv}$ 可以求出波长

ii) Electrons incident on a thin carbon film/other material

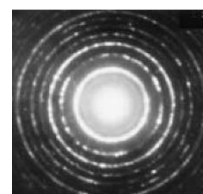
衍射发生的条件是 gap size \approx wavelength; 中间打到的 carbon film 或者其他材料原子之间的间隔接近 10^{-10}m 作为衍射发生的 gap, 当第一步中电子加速到让波长约等于 gap size 的时候再打到 carbon film 上发生最明显衍射。

iii) Diffraction pattern observed on the fluorescent screen

衍射图样根据穿过的物质不同分为三类如下

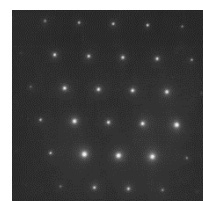
b) **Result 结果:** i) carbon film 或其他非晶体物质:

concentric rings 同心圆环



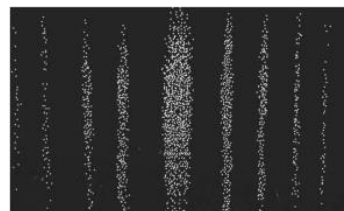
ii) crystal 晶体物质:

point matrix 点阵



iii) double slit 双缝:

fringes 条纹



c) **Conclusion 结论:**

This experiment shows the wave nature of electron. 实验证明了电子作为一个粒子同时也具有波动性。



13. Communication 通信

13.1 Signal 信号

1) 信号指的是被传递的信息，有两种传播方式：

- a) Analogue signals: signals that are continuously variable
- b) Digital signals: signals that consists of 1s and 0s

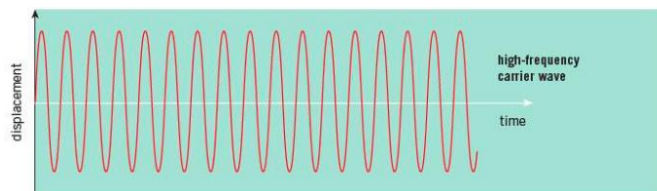
2) 用 digital signal 传递信息的优势：

noise can be eliminated/waveform can be regenerated, extra bits of data can be added to check for errors, cheaper/more reliable, greater rate of transfer of data

13.2 Modulation 调制

1) **Carrier Wave 载波：**

用来装载信息的电磁波，一般是 spectrum 里面的 radio wave



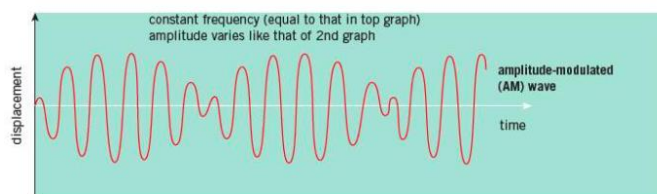
2) **Information Wave 信息：**

我们想要传递出去的信息内容，比如声音信号，频率一般较低。



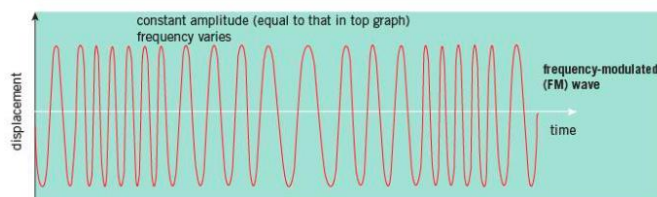
3) **AM (Amplitude Modulation 调幅)：**

amplitude of carrier wave varies in synchrony with displacement of information signal



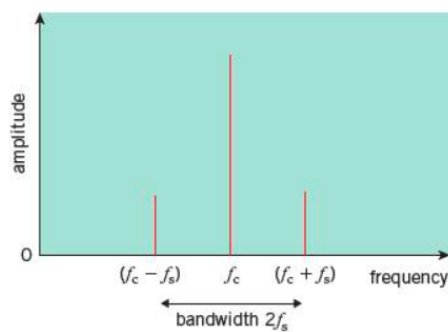
4) **FM (Frequency Modulation 调频)：**

frequency of carrier wave varies in synchrony with displacement of information signal

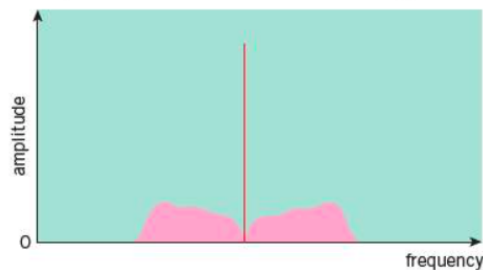


13.3 Bandwidth 带宽

- 1) 定义: the range of frequencies occupied by the modulated waveform
- 2) Greater bandwidth 的优缺点:
 - a) Advantage: Better quality, greater rate of transfer of data, less distortion
 - b) Disadvantage: fewer stations in any frequency range
- 3) AM信号的frequency spectrum以及bandwidth计算:



- 4) FM信号的frequency spectrum:



- 5) FM信号的相关计算:

- 1) Frequency deviation: 随着 information wave displacement 每增加 1V, FM 信号的 frequency 增加多少 Hz
- 2) Example: 已知 frequency deviation 为 3kHzV^{-1} , information wave 的 Amplitude 为 2V, 所以 FM 信号最大频率当 information wave displacement 最大为 2V 的时候, 频率增加 $2 \times 3 = 6\text{kHz}$, 所以 $f_{\max} = f_c + 6\text{k}$, 同理 $f_{\min} = f_c - 6\text{k}$



3) AM & FM 的比较

Signal 信号	Advantage 优点
FM	1. Less interference and noise 2. Greater bandwidth produces better quality
AM	1. Longer range 2. Smaller bandwidth required means more channels in given frequency 3. Less complex electronics needed (cheaper devices)

13.4 进制转换

- 1) 二进制转十进制方法：从右往左每一位下面依次写 1,2,4,8,16……，然后分别把竖列相乘加起来，如下例：

Q: 将 1011 转化成十进制

A: $1 \times 8 + 0 \times 4 + 1 \times 2 + 1 \times 1 = 11$

1	0	1	1
8	4	2	1

- 2) 十进制转二进制方法：从右往左依次写 1,2,4,8,16……，然后从原数上减掉能够找到的最大数，如果找到了就填 1，然后减掉，没有就填 0，再继续往右找，如下例

Q: 将 13 转化成二进制

A: 先找到 1 个 8，在 8 上填 1，剩 5

再找到 1 个 4，在 4 上填 1，剩 1

剩下 1 小于 2，在 2 上填 0，剩 1

1	1	0	1
8	4	2	1

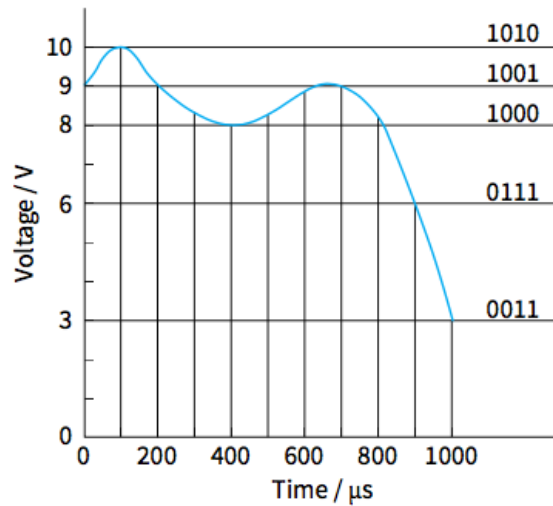
找到一个 1，在 1 上填 1，结束，答案是 1101



13.5 ADC & DAC

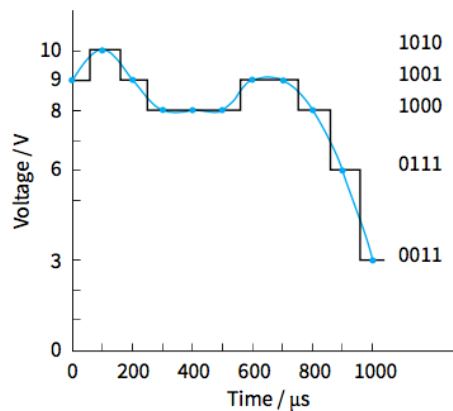
- 1) **ADC 全称 Analog to Digital Converter**, 将 analogue signal 转化为 digital signal

Function 功能: a) analogue signal is sampled at regular time intervals
b) sampled signal is converted into a binary number



- 2) **DAC 全称 Digital to Analogue Converter**, 将 digital signal 转化为 analogue signal

Function 功能: convert binary digital signal into analogue waveform



- 4) **Improve the quality and reproduction accuracy 提高质量和准确率:**

Sample more frequently to reduce "step depth",

Greater number of bits to reduce "step height"

13.6 Loss and Attenuation 损耗与衰减

- 1) **Crosstalk:** picking up of signal in one cable from a second cable
- 2) **Noise:** random unwanted signal that distorts transmitted signal
- 3) **Attenuation:** loss in power of the signal

13.7 Wire Communication 有线通信

1) wire-pairs 双绞线

Advantage: cheap



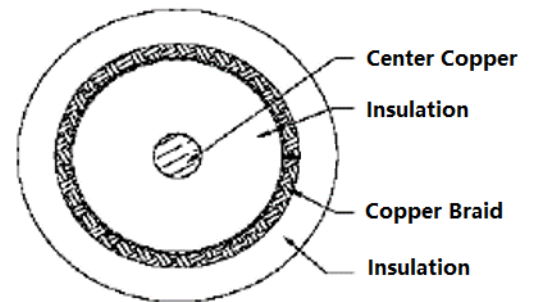
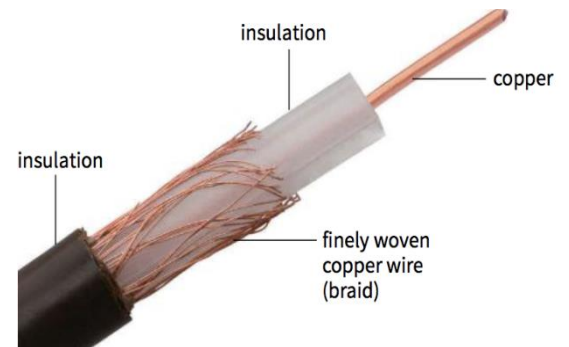
2) Coaxial cable 同轴电缆

Function of copper braid:

- a) act as return conductor for signal
- b) as shield from interference

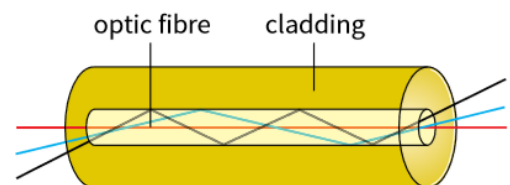
Advantage:

- a) greater bandwidth
- b) immune to e.m. interference
- c) radiates less power
- d) less crosstalk
- e) lower noise levels



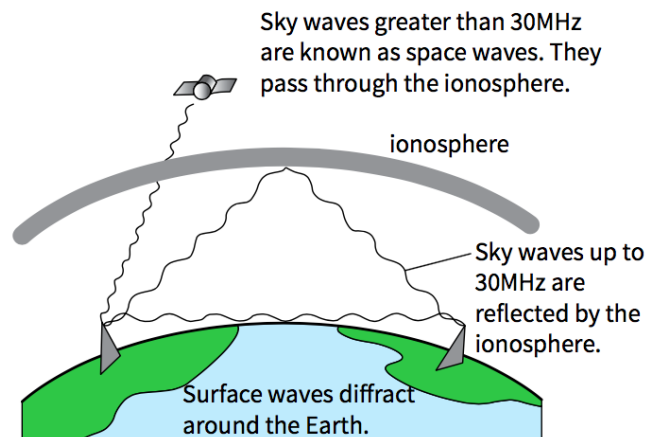
3) Optic fibre 光纤

- Advantage:**
- a) greater bandwidth
 - b) less signal attenuation
 - c) no electrical interference and crosstalk
 - d) less weight
 - e) cheaper



13.8 Wireless Communication 无线通信

1) 三种无线通信的方式:

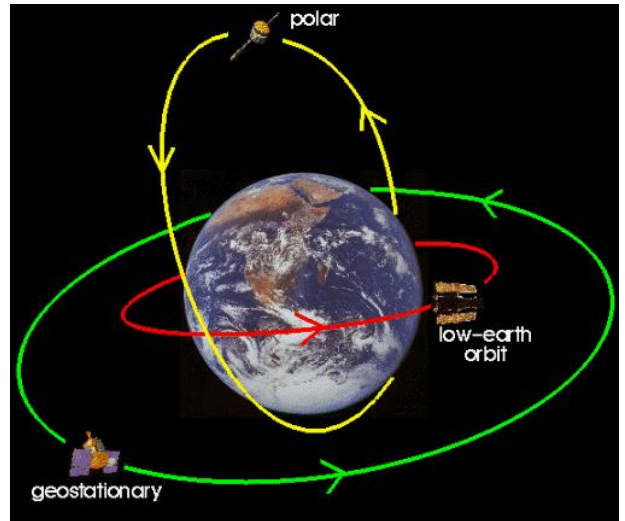


Type	f	λ	范围	传播方式
Surface wave	<3MHz	>100m	1000km	Diffraction
Sky wave	3-30MHz	10-100m	worldwide	Reflection at ionosphere
Space wave	>30MHz	<10m	Line-of-sight	Up to satellite and back down

13.9 Satellite Communication 卫星通信

1) Geostationary satellite 同步轨道卫星:

- a) **特点:** Geostationary satellites stay in equatorial orbit and rotate at a period of 1 day from west to east
- b) **优势:** Always at the same position above earth so the receiving disks do not need to rotate to track the satellite
- c) **用途:** suitable for communication



2) Polar-orbiting satellite 极地轨道卫星

- a) **特点:** Polar satellites are satellites in low orbits and pass over poles
- b) **优势:**
 - a) Smaller delay because of lower orbit (but not always available)
 - b) Can see more details
 - c) Can pass over whole earth in 24 hours
- c) **用途:** suitable for weather observing

3) Uplink and downlink

向上发给卫星的叫做 uplink, 卫星发回来的叫做 downlink, 这两个频率不同

- a) the signal is amplified because the received signal is already weak due to attenuation
- b) Uplink and downlink use different frequencies to avoid interference of uplink with downlink



13.10 dB Calculation 分贝计算

1) $\text{dB} = 10 \lg \frac{P_1}{P_2}$, 是一种表示比例关系的方式

2) 使用 dB 计算的好处:

a) Numbers involved are smaller

b) Calculation involve addition & subtraction rather than multiplication & division

3) signal to noise ratio 信噪比:

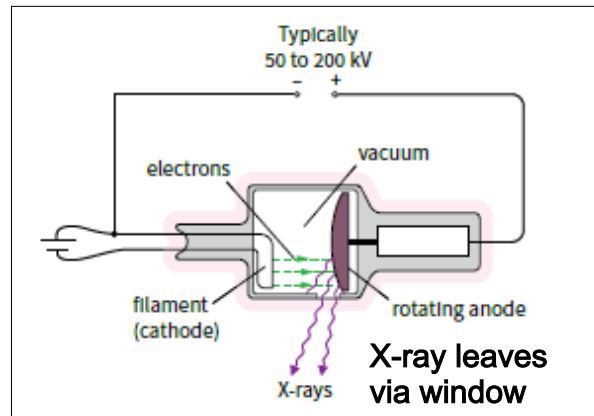
$$\text{S/N ratio} = 10 \lg \frac{P_{\text{signal}}}{P_{\text{noise}}} , \text{ 越大代表信号越强}$$

4) 计算方法:

如果是信号经过放大器加强就是+dB, 经过损耗减弱就-dB, 用直接加减来代替倍数的乘除运算。

14. Medical Imaging 医学成像

14.1 X-ray

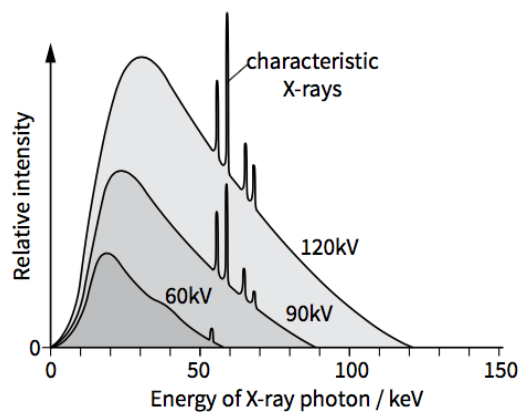


Rotating anode for a larger target area

Heated anode, thermoionic emission

- 1) X-ray tube 原理:
 - a) filament emits electrons
 - b) electrons are accelerated by high voltage
 - c) electrons hit anode and decelerate
 - d) photons are emitted due to reduction in kinetic energies

- 2) X-ray spectrum 光谱



- a) **Why a distribution of different frequencies of X-ray:** Radiation is produced when the electrons decelerate. Electrons hitting anode have a distribution of decelerations, therefore different energy of photons.
- b) **Why a largest energy/ highest frequency/ minimum wavelength:** When all the energy of an electron is consumed in one collision and produce one photon
- c) **Why do characteristic X-rays exist:** characteristic X-rays are caused by electron moving up and dropping down between energy levels, photon energy



equal to the difference in energy levels

3) Hardness 硬度

- a) 定义: hardness measures penetration of X-ray. high frequency = hard; low frequency=soft
- b) Hard 的优势: Hardness means larger penetration, so hard X-ray remains less in the human body and is thus safer; softer X-ray is more easily absorbed and is hazardous.
- c) Soft 的优势: When the contrast is low (e.g. distinguishing flesh and soft tissue), using softer ray and exposing for longer time can produce better image
- d) 控制 hardness 的方法: **i)** change the accelerating voltage of the X-ray tube, larger V, higher f, harder X-ray. **ii)** use an aluminum filter which absorbs lower frequency soft X-ray.

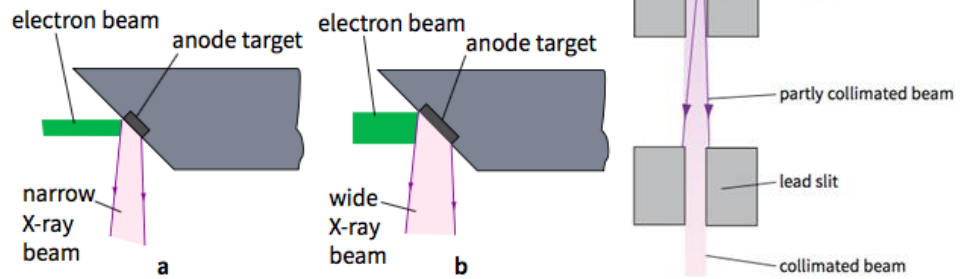
4) Intensity 强度

- a) 定义: the power per unit cross sectional area
- b) 控制 intensity 的方法: control the electric current in the electron source, more electrons bombarding each unit time, higher rate of emission of photons.

5) Sharpness 锐度

a) 定义: how well the edges of the structure are defined

b) 控制 sharpness 的方法: i) use anode with small area to produce narrow beam, ii) use lead grid to reduce scattering of light



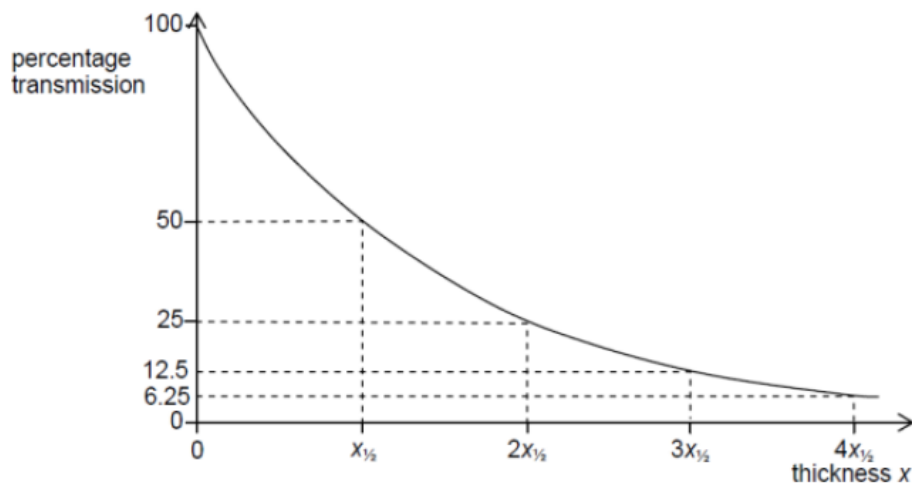
Cut-off wavelength: wavelength corresponding to **highest energy photons (highest energy of electron accelerated in the vacuum tube)**

6) Contrast 对比度

- a) 定义: difference in degree of blackening between structures
- a) 控制 contrast 的方法: i) Use contrast matter like barium meal ii) control hardness of X-ray: hard for bone, soft & longer exposure for soft tissue



7) Attenuation 衰减



- a) 公式: $I_0 = I_0 e^{-\mu x}$ where μ is absorption coefficient of medium, x is thickness of the medium
- b) Half-value thickness: thickness of the medium required to reduce the transmitted intensity to one half of its initial value.
- c) Half-value thickness 和 absorption coefficient 的关系 $x_{1/2} * \mu = \ln 2$

14.2 CT (Computerized axial Tomography)

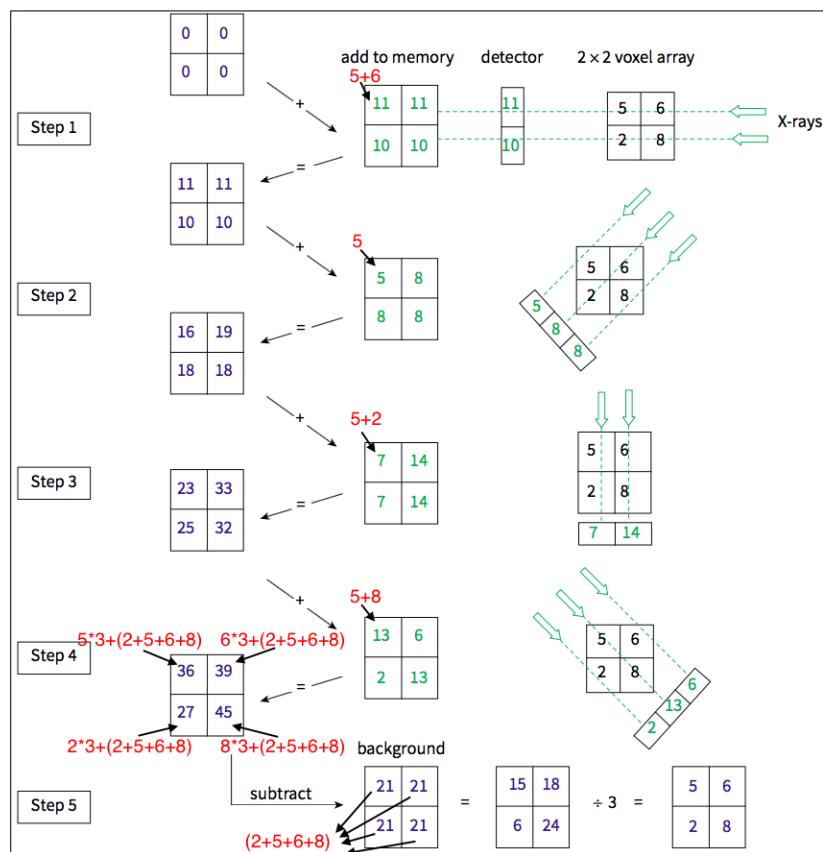
1) Advantage 优势

CT produces images that show three-dimensional relationships between different tissues. Also, it can distinguish tissues with quite similar attenuation coefficients.

2) Principle 原理：用多个角度的 X 光生成 3D 图像

CT scanning involves X-ray images taken from different angles of one slice, these images can be combined to give a 2D image of the slice. Then the images of successive slices are combined using a computer. The image formed is 3D image that can be rotated / viewed from different angles.

3) Calculation 计算

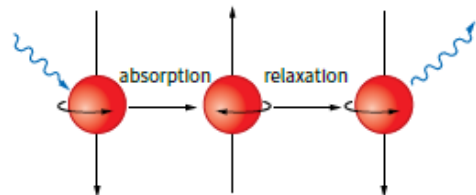


计算方法: $(\text{memory} - \text{background}) \div 3$

14.3 NMRI (Nuclear Magnetic Resonance Imaging) 核磁共振

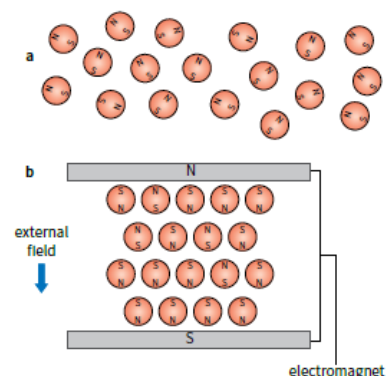
1) Principle of nuclear magnetic resonance imaging 造影流程&原理

- Strong uniform magnetic field aligns nuclei and make it rotate about field direction.
- Radio frequency pulse at Larmor frequency causes resonance, and the nuclei absorb energy
- On relaxation, nuclei emit r.f. pulse
- The emitted pulse is detected and processed
- Non-uniform field superposed on uniform field allows position of resonating nuclei to be determined and allows for location of detection to be changed



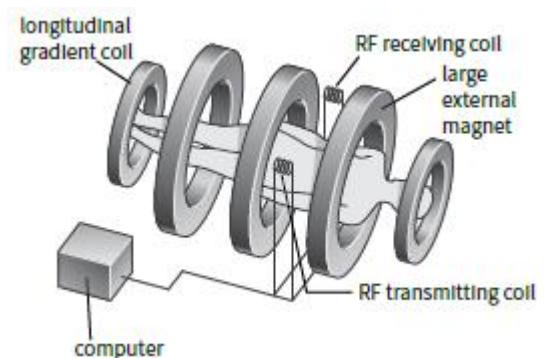
2) Function of large uniform field 强匀强磁场的作用

Large uniform field aligns nuclei and causes precession of nuclei about the direction of magnetic field. Larmor frequency depends on magnetic field strength, so large field means this frequency is in radio frequency range.

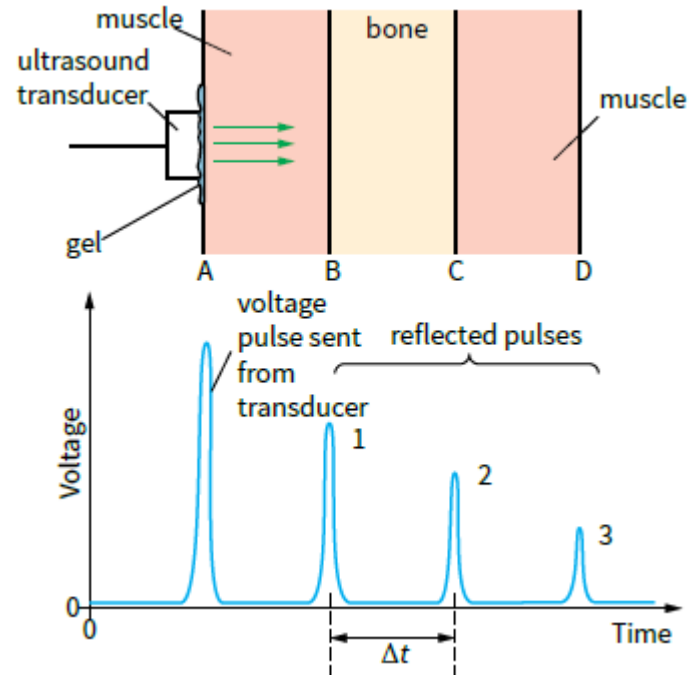


3) Function of non-uniform field 非匀强磁场的作用

Non-uniform field makes Larmor frequency different in different regions of subject. This enables location of precessing nuclei to be determined. And by changing the frequency of e.m. radiation, we can change the location of the slice to be detected.



14.4 Ultrasound 超声波



1) Generation 产生原理

p.d. across piezo-electric crystal causes crystal to change shape, alternating p.d. in ultrasound frequency range causes crystal to vibrate, producing ultrasound.

2) Detect 接收原理

to maximize the effect, the crystal is cut to produce resonance, when crystal made to vibrate by ultrasound wave, alternating p.d. is produced across the crystal

3) Process 整体检测流程

pulse of ultrasound produced by piezo-electric crystal

pulses reflected from boundaries

reflected pulse detected by the ultrasound transmitter

time delay gives information about depth

intensity of reflected pulse gives information about the boundary



gel used to minimize reflection at skin

degree of reflection depends on impedance of two media

4) Increase resolution 提高检测的分辨率

higher frequency, smaller wavelength, better resolution, smaller structures can be detected

5) Why pulse is needed in ultrasound 为什么需要 pulse 脉冲

Because the piezo-electric transducer works not only as emitter but also detector, so reflected ultrasound needs to be detected between emission of pulses.

6) Acoustic impedance 声阻抗

a) 定义: acoustic impedance is product of density of the medium and sound wave speed in the medium

b) 公式: $Z = \rho c$, where ρ is density, c is sound wave speed

7) Reflection coefficient 反射系数

a) 定义: reflected intensity / incident intensity 反弹回来的占的比例

b) 公式: $\alpha = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2} = \frac{I_{\text{reflected}}}{I_{\text{incident}}}$

8) Attenuation 衰减

a) 公式: $I = I_0 e^{-\mu x}$ where μ is absorption coefficient of medium, x is thickness of the medium

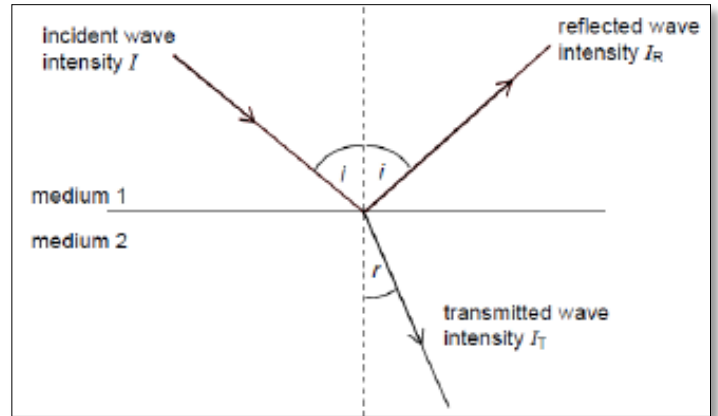
Application of gel: talk about difference in
A. impedance between air and gel hence
reflection coefficient.



9) Calculation 计算

在反射界面上: $I = I_R + I_T$

总入射强度=反射强度+折射强度



PET:

半甲板

Annihilation!