Hybrid Energy Storage System Optimization Model

Nomenclature

Sets

$t \in \mathcal{T}$	Set of time periods, $\mathcal{T} = \{1, \dots, T\}$
$i \in \{A, B, C\}$	Index for Electrical Energy Storage Systems (ESS)

Decision Variables — Electrical

$pBt_{i,t}$	Net basepoint power of ESS i at time t
$pBtch_{i,t}$	Charging power of ESS i at time t
$pBtdis_{i,t}$	Discharging power of ESS i at time t
$zch_{i,t}$	Binary variable: 1 if ESS i is charging at t
$zdis_{i,t}$	Binary variable: 1 if ESS i is discharging at t
$et_{i,t}$	State of charge of ESS i at time t
$et0_i$	Initial state of charge of ESS i
E_t^{Imp}	Energy imported from the grid at time t
E_t^{Exp}	Energy exported to the grid at time t
P_t^{Imp}	Power imported from the grid at time t
P_t^{Exp}	Power exported to the grid at time t
P_t^{Grid}	Power exchanged with grid at time t
p_t^{RC}	Power generated from Rankine cycle at time t
P_t^{Gen}	Total power generation at time t
P_t^{Load}	Total electrical load at time t
P_t^{Cbu}	Electrical load of controllable building units at time t
P_t^{Cebu}	Electrical load of controllable equipment building units at time t
P_t^{EV}	EV charging/discharging power at time t
$P_t^{Pelletizer}$	Power consumption of pelletizer at time t
P_t^{PV}	Power from photovoltaics at time t
P_t^{Wind}	Power from wind at time t
$CostESS_i$	Operation and management cost for the ESS i

Parameters — Electrical

CI_t^{el}	CO_2 intensity of imported electricity at time t
$EnPrice_t$	Energy price at time t
P_i	Power capacity limit of ESS i
δ_i	Static energy loss rate for ESS i
η_i^{ch}	Charging efficiency of ESS i
η_i^{dis}	Discharging efficiency of ESS i
E_i	Energy capacity of ESS i
$aEmin_i$	Minimum state of energy ratio for ESS i
$aEmax_i$	Maximum state of energy ratio for ESS i
dt	Time step duration (typically 1 hour)
η_{Rankine}	Efficiency of the Rankine cycle for converting heat to electricity

Thermal Energy Storage (TES)

Decision Variables — Thermal

Qt_t	State of thermal energy at time t
$Qt0$	Initial state of thermal energy
qBt_t	Net thermal basepoint power at time t
$qBtch_t$	Charging power into TES at time t
$qBtdis_t$	Discharging power from TES at time t
$HeatPrice_t$	Heat price at time t
CI_t^{heat}	CO_2 intensity of imported thermal energy at time t
$zqch_t$	Binary variable: 1 if TES is charging at time t
$zqdis_t$	Binary variable: 1 if TES is discharging at time t
$q_t^{TES\to Elec}$	Thermal energy used in Rankine cycle at time t
q_t^{TPS}	Heat generated by tower solar plant at time t
q_t^{CSP}	Heat generated by concentrated solar power at time t
Q_t^{Imp}	Imported thermal energy at time t
Q_t^{Export}	Exported thermal energy at time t
q_t^{Imp}	Imported thermal power at time t
q_t^{Export}	Exported thermal power at time t
q_t^{Gen}	Total thermal generation at time t
q_t^{Grid}	Thermal power exchanged with grid at time t
q_t^{Demand}	Total thermal demand at time t
q_t^{Ctbu}	Electricity-driven thermal load of buildings at time t
$P_t^{Imp,\max}$	Maximum allowed power import from the electrical grid at time t
$P_t^{Exp,\max}$	Maximum allowed power export to the electrical grid at time t

Parameters — Thermal

δ^{th}	Static energy loss rate for TES
η_{th}^{ch}	Charging efficiency of TES
η_{th}^{dis}	Discharging efficiency of TES
Q	Energy capacity of TES
$aQmin$	Minimum state of energy ratio for TES
$aQmax$	Maximum state of energy ratio for TES
$q_{th}^{\max}$	Power capacity of TES
c^{th}	Cost coefficient for TES charging
$HeatValue_t$	Heat discharging value per unit at time t
$q_t^{Imp,\max}$	Maximum allowed heat import from the thermal grid at time t
$q_t^{Exp,\max}$	Maximum allowed heat export to the thermal grid at time t

Parameters — General

w	Weighting factor between cost and emissions
$CO2Price$	Cost of CO_2 emissions per unit

Objective Function

$$\begin{aligned} \min \quad & w \cdot \Bigg(\sum_{t\in\mathcal{T}} EnPrice_t \cdot E_t^{Grid} + \sum_{t\in\mathcal{T}} HeatPrice_t \cdot Q_t^{Grid} \\ & + \sum_i \sum_{t\in\mathcal{T}} CostESS_i \cdot (pBtch_{i,t} + pBtdis_{i,t}) \cdot dt \Bigg) \\ & + (1-w) \cdot \sum_{t\in\mathcal{T}} \text{CO2Price} \cdot \Bigg(\underbrace{CI_t^{el} \cdot E_t^{Imp}}_{\text{CO}_2 \text{ from grid electricity}} + \underbrace{CI_t^{heat} \cdot Q_t^{Imp}}_{\text{CO}_2 \text{ from imported heat}} \Bigg) \end{aligned}$$

System-Wide Balance Equations

Electricity Balance

$$P_t^{Gen} + P_t^{Grid} = \sum_i pB_{i,t} + P_t^{Load} \qquad \forall t \in \mathcal{T}$$

$$P_t^{Load} = P_t^{Cbu} + P_t^{Cebu} + P_t^{EV} + P_t^{Pelletizer} \qquad \forall t \in \mathcal{T}$$

$$P_t^{Gen} = P_t^{PV} + P_t^{Wind} + P_t^{RC} \qquad \forall t \in \mathcal{T}$$

Thermal Energy Balance

$$q_t^{Grid} + q_t^{Gen} = qBt_t + q_t^{Demand} \qquad \forall t \in \mathcal{T}$$

$$q_t^{Demand} = q_t^{TES \to Elec} + q_t^{Ctbu} \qquad \forall t \in \mathcal{T}$$

$$q_t^{Gen} = q_t^{TPS} + q_t^{CSP} \qquad \forall t \in \mathcal{T}$$

Thermal to Electrical Balance

$$p_t^{RC} = \eta_{\text{Rankine}} \cdot q_t^{TES \to Elec} \qquad \forall t \in \mathcal{T}$$

Constraints

Basepoint Power Decomposition

$$pBt_{i,t} = pBtch_{i,t} - pBtdis_{i,t} \qquad \forall i, t$$

Charging/Discharging Binary Constraints

$$\begin{aligned} pBtch_{i,t} &\leq zch_{i,t} \cdot P_i && \forall i, t \\ pBtdis_{i,t} &\leq zdis_{i,t} \cdot P_i && \forall i, t \\ zch_{i,t} + zdis_{i,t} &\leq 1 && \forall i, t \end{aligned}$$

State of Charge Dynamics

$$\begin{aligned} et_{i,1} &= (1 - \delta_i) \cdot et0_i + \eta_i^{ch} \cdot pBtch_{i,1} \cdot dt - \frac{1}{\eta_i^{dis}} \cdot pBtdis_{i,1} \cdot dt \\ et_{i,t} &= (1 - \delta_i) \cdot et_{i,t-1} + \eta_i^{ch} \cdot pBtch_{i,t} \cdot dt - \frac{1}{\eta_i^{dis}} \cdot pBtdis_{i,t} \cdot dt && \forall t > 1 \end{aligned}$$

State of Charge Constraints

$$\begin{aligned} et0_i &= et_{i,T} \\ aEmin_i \cdot E_i \leq et_{i,t} &\leq aEmax_i \cdot E_i && \forall i, t \end{aligned}$$

TES Constraints

Thermal Basepoint Power Definition

$$qBt_t = qBtch_t - qBtdis_t \qquad \forall t \in \mathcal{T}$$

Charging/Discharging Binary Constraints

$$
\begin{aligned}
qBtch_t &\le zqch_t \cdot q_{th}^{\max} && \forall t \in \mathcal{T} \\
qBtdis_t &\le zqdis_t \cdot q_{th}^{\max} && \forall t \in \mathcal{T} \\
zqch_t + zqdis_t &\le 1 && \forall t \in \mathcal{T}
\end{aligned}
$$

State of Thermal Energy Dynamics

$$
\begin{aligned}
Qt_1 &= (1-\delta^{th}) \cdot Qt0 + \eta_{th}^{ch} \cdot qBtch_1 \cdot dt - \frac{1}{\eta_{th}^{dis}} \cdot qBtdis_1 \cdot dt \\
Qt_t &= (1-\delta^{th}) \cdot Qt_{t-1} + \eta_{th}^{ch} \cdot qBtch_t \cdot dt - \frac{1}{\eta_{th}^{dis}} \cdot qBtdis_t \cdot dt && \forall t > 1
\end{aligned}
$$

State of Thermal Energy Constraints

$$
aQmin \cdot Q \le Qt_t \le aQmax \cdot Q \qquad \forall t \in \mathcal{T}
$$

Terminal Condition

$$
Qt_T = Qt_1
$$

Grid Import/Export Power Constraints — Electrical

$$
-P_t^{Grid,\max} \le P_t^{Grid} \le P_t^{Grid,\max} \qquad \forall t \in \mathcal{T}
$$

Grid Import/Export Power Constraints — Thermal

$$
-q_t^{Grid,\max} \le q_t^{Grid} \le q_t^{Grid,\max} \qquad \forall t \in \mathcal{T}
$$

Appendix: Glossary of Symbols

Symbol	Description	Unit
t	Time index	[h]
i	Index for ESS units	[A,B,C]
$pBt_{i,t}$	Net basepoint power of ESS i at time t	[kW]
$pBtch_{i,t}$	Charging power of ESS i at time t	[kW]
$pBtdis_{i,t}$	Discharging power of ESS i at time t	[kW]
$et_{i,t}$	State of charge of ESS i at time t	[kWh]
P_t^{Imp}	Power imported from the grid at time t	[kW]
Q_t^{Imp}	Heat imported from the thermal grid at time t	[kWh]
η_i^{ch}	Charging efficiency of ESS i	[–]
δ^{th}	Energy loss rate in TES	[1/h]
w	Weighting factor between cost and emissions	[–]
$CO2Price$	CO_2 emission cost per unit	[€/kg CO_2]
$HeatPrice_t$	Heat price at time t	[€/kWh]
CI_t^{el}	CO_2 intensity of electricity at time t	[kg CO_2/kWh]
CI_t^{heat}	CO_2 intensity of heat at time t	[kg CO_2/kWh]