Software Engineering 1

Lambda Calculus exercises
Beta reduction

1 Exercise 1

Given the following lambda program, complete the empty beta reduction steps for this program.

```
((λx→(x y)) (λz→z))
```

1.1 Answer 1

```
((λx→(x y)) (λz→z))
```

```
((λx→(x y)) (λz→z))
```

```
( (λz→z) y)
```

```
((λz→z) y)
```

```
y
```

2 Exercise 2

Given the following lambda program, complete the empty beta reduction steps for this program.

```
((λx→((λy→(x y)) x)) (λz→w))
```

2.1 Answer 2

```
((λx→((λy→(x y)) x)) (λz→w))
```

```
((λx→((λy→(x y)) x)) (λz→w))
```

```
((λy→( (λz→w) y)) (λz→w) )
```

```
((λy→((λz→w) y)) (λz→w))
```

```
((λz→w) (λz→w) )
```

```
((λz→w) (λz→w))
```

```
w
```

3 Exercise 3

Given the following lambda program, complete the empty beta reduction steps for this program.

```
((((λf g x→((f x) (g x))) (λm n→(n m))) (λn→z)) p)
```

3.1 Answer 3

```
((((λf g x→((f x) (g x))) (λm n→(n m))) (λn→z)) p)
```

```
(( ((λf g x→((f x) (g x))) (λm n→(n m))) (λn→z)) p)
```

```
(((λg x→(( (λm n→(n m)) x) (g x))) (λn→z)) p)
```

```
( ((λg x→(((λm n→(n m)) x) (g x))) (λn→z)) p)
```

```
((λx→(((λm n→(n m)) x) ( (λn→z) x))) p)
```

```
((λx→(((λm n→(n m)) x) ((λn→z) x))) p)
```

```
(((λm n→(n m)) p) ((λn→z) p))
```

```
( ((λm n→(n m)) p) ((λn→z) p))
```

```
((λn→(n p)) ((λn→z) p))
```

```
((λn→(n p)) ((λn→z) p) )
```

```
((λn→(n p)) z)
```

```
((λn→(n p)) z)
```

```
( z p)
```

4 Exercise 4

Given the following lambda program, complete the empty beta reduction steps for this program.

```
((λf→((λg→((f f) g)) (λh→(k h)))) (λx y→y))
```

4.1 Answer 4

```
((λf→((λg→((f f) g)) (λh→(k h)))) (λx y→y))
```

```
((λf→((λg→((f f) g)) (λh→(k h)))) (λx y→y))
```

```
((λg→(( (λx y→y) (λx y→y) ) g)) (λh→(k h)))
```

```
((λg→(((λx y→y) (λx y→y)) g)) (λh→(k h)))
```

```
(((λx y→y) (λx y→y)) (λh→(k h)) )
```

```
( ((λx y→y) (λx y→y)) (λh→(k h)))
```

```
((λy→y) (λh→(k h)))
```

```
((λy→y) (λh→(k h)))
```

```
(λh→(k h))
```

5 Exercise 5

Given the following lambda program, complete the empty beta reduction steps for this program.

```
(((λz→z) (λy→(y y))) ((λx→x) a))
```

5.1 Answer 5

```
(((λz→z) (λy→(y y))) ((λx→x) a))
```

```
( ((λz→z) (λy→(y y))) ((λx→x) a))
```

```
( (λy→(y y)) ((λx→x) a))
```

```
((λy→(y y)) ((λx→x) a) )
```

```
((λy→(y y)) a )
```

```
((λy→(y y)) a)
```

```
( a a )
```

6 Exercise 6

Given the following lambda program, complete the empty beta reduction steps for this program.

```
(((λz→z) ((λy→y) y)) ((λz→z) y))
```

6.1 Answer 6

```
(((λz→z) ((λy→y) y)) ((λz→z) y))
```

```
(((λz→z) ((λy→y) y) ) ((λz→z) y))
```

```
(((λz→z) y ) ((λz→z) y))
```

```
( ((λz→z) y) ((λz→z) y))
```

```
( y ((λz→z) y))
```

```
(y ((λz→z) y) )
```

```
(y y )
```

7 Exercise 7

Given the following lambda program, complete the empty beta reduction steps for this program.

```
(((λx y→(x (y y))) (λa→a)) b)
```

7.1 Answer 7

```
(((λx y→(x (y y))) (λa→a)) b)
```

```
(((λx y→(x (y y))) (λa→a)) b)
```

```
((λy→((λa→a) (y y))) b)
```

```
((λy→((λa→a) (y y))) b)
```

```
((λa→a) (b b))
```

```
((λa→a) (b b))
```

```
(b b)
```

8 Exercise 8

Given the following lambda program, complete the empty beta reduction steps for this program.

```
(((λx y→(x (y y))) (λy→y)) y)
```

8.1 Answer 8

```
(((λx y→(x (y y))) (λy→y)) y)
```

```
(((λx y→(x (y y))) (λy→y)) y)
```

```
((λy→((λy→y) (y y))) y)
```

```
((λy→((λy→y) (y y))) y)
```

```
((λy→y) (y y))
```

```
((λy→y) (y y))
```

```
(y y)
```

9 Exercise 9

Given the following lambda program, complete the empty beta reduction steps for this program.

```
((((λx→x) x) ((λy→y) x)) z)
```

9.1 Answer 9

```
((((λx→x) x) ((λy→y) x)) z)
```

```
(( ((λx→x) x) ((λy→y) x)) z)
```

```
(( x ((λy→y) x)) z)
```

```
((x ((λy→y) x)) z)
```

```
((x x) z)
```

10 Exercise 10

Given the following lambda program, complete the empty beta reduction steps for this program.

```
(((λx y→(x y)) y) z)
```

10.1 Answer 10

```
(((λx y→(x y)) y) z)
```

```
( ((λx y→(x y)) y) z)
```

```
((λy→( y y)) z)
```

```
((λy→(y y)) z)
```

```
( z z)
```

11 Exercise 11

Given the following lambda program, complete the empty beta reduction steps for this program.

```
(((λx→(x x)) (λy→y)) (λy→y))
```

11.1 Answer 11

```
(((λx→(x x)) (λy→y)) (λy→y))
```

```
( ((λx→(x x)) (λy→y)) (λy→y))
```

```
(( (λy→y) (λy→y)) (λy→y))
```

```
( ((λy→y) (λy→y)) (λy→y))
```

```
( (λy→y) (λy→y))
```

```
((λy→y) (λy→y))
```

```
(λy→y)
```

12 Exercise 12

Given the following lambda program, complete the empty beta reduction steps for this program.

```
(((λx y→(x y)) (λy→y)) w)
```

12.1 Answer 12

```
(((λx y→(x y)) (λy→y)) w)
```

```
( ((λx y→(x y)) (λy→y)) w)
```

```
((λy→( (λy→y) y)) w)
```

```
((λy→((λy→y) y)) w)
```

```
((λy→y) w)
```

```
((λy→y) w)
```

```
w
```