

# Introduction to functional programming and lambda calculus

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## Lecture topics

- Course introduction
- Exam and practicum
- Semantics of traditional programming languages
- Basic lambda calculus

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## Course topics

- We will discuss a completely new paradigm for expressing programs
- This paradigm, functional programming, is based on different premises on computation
- It gives guarantees of correctness in complex places, like parallelism or separation of concerns
- It requires a radical conceptual shift in the way you think about programming

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## Course topics

- We will begin with a short discussion on traditional programming language extbfsemantics
- We will then show the extbflambda calculus, which is the foundation for functional languages
- 
- We will then bridge the gap between theory and practice
- We will translate the lambda calculus into two mainstream functional languages: F# and Haskell
- This will cover a huge chunk of possible applications in countless other languages and libraries, from C# LINQ to Java streams, to Scala, Scheme, Closure, etc.

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## Exam structure

- There is a theoretical exam, where you show understanding of the basic principles
- There is a practical exam, where you show understanding of their concrete applications



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## Theoretical exam

- One question on a lambda calculus program execution
- One question on the type system of a lambda calculus program,  $F\#$  program, or Haskell program
- Both questions must be answered correctly to get a `extbfvoldoende`

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Build, in groups of max four, any of the following applications in either Haskell or F#:

- A 2D simulation of a supermarket with customers, cash registers, and various aisles
- A 2D simulation of a supply chain with trucks, containers, and ships
- An interpreter for a Python-like language (with a parser for an extra challenge)
- An interpreter for the lambda calculus (with a parser for an extra challenge)

We will get together at the end of the course, and the teacher(s) will ask you to **individually** perform some activities on the code to prove understanding and familiarity.

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# Semantics of traditional programming languages

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- Traditional, imperative programming languages are based on sharing memory through instructions
- This means that subsequent instructions are not independent from each other
- Any function call makes use of the available memory

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For example, consider the semantic rules that describe the working of “;”

First we run  $s_1$  with the initial memory, then we run  $s_2$  with the modified memory.

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For example, consider the semantic rules that describe the working of “;”

First we run  $s_1$  with the initial memory, then we run  $s_2$  with the modified memory.

$$\frac{\langle s_1, S, H \rangle \rightarrow \langle S_1, H_1 \rangle \wedge \langle s_2, S_1, H_1 \rangle \rightarrow \langle S_2, H_2 \rangle}{\langle (s_1; s_2), S, H \rangle \rightarrow \langle S_2, H_2 \rangle}$$

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What does “*first we run  $s_1$  with the initial memory, then we run  $s_2$  with the modified memory*” imply?.

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What does “*first we run  $s_1$  with the initial memory, then we run  $s_2$  with the modified memory*” imply?.

- The same instructions, executed at different moments, will produce **different results**.
- Change the order of some method calls, and some weird dependence might cause bugs or break things.



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## Goals

- Our goal is to ensure that behaviour of code is consistent.
- Change the order of some method calls, and the results remain the same.
- This makes it easier to test, parallelize, and in general ensure correctness.

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*How do we achieve this?*

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*How do we achieve this?*

We give (shared) memory up: every piece of code is a function which output only depends on input.

This very important property is called **referential transparency**.

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# Basic lambda calculus

## Introduction

- The (basic) lambda calculus is an alternative mechanism to Turing Machines and the Von Neumann architecture.
- It is very different, but has equivalent expressive power.
- It is the foundation of all functional programming languages.

## Substitution principle

- The (basic) lambda calculus is truly tiny when compared with its power.
- It is based on the substitution principle: calling a function with some parameters returns the function body with the variables replaced.
- There is no memory and no program counter: all we need to know is stored inside the body of the program itself.

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A lambda calculus program (just *program* from now on) is made up of three syntactic elements:

- Variables:  $x, y, \dots$
- Abstractions (function declarations with one parameter):  $\lambda x \rightarrow t$  where  $x$  is a variable and  $t$  is the function body (a program).
- Applications (function calls with one argument):  $t u$  where  $t$  is the function being called (a program) and  $u$  is its argument (another program).

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A simple example would be the identity function, which just returns whatever it gets as input

$$(\lambda x \rightarrow x)$$



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We can call this function with a variable as argument, by writing:

$$((\lambda x \rightarrow x) \ v)$$

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A lambda calculus program is computed by replacing lambda abstractions applied to arguments with the body of the lambda abstraction with the argument instead of the lambda parameter:

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A lambda calculus program is computed by replacing lambda abstractions applied to arguments with the body of the lambda abstraction with the argument instead of the lambda parameter:

$$\overline{(\lambda x \rightarrow t) u \rightarrow_{\beta} t[x \mapsto u]}$$

$t[x \mapsto u]$  means that we change variable  $x$  with  $u$  within  $t$

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$$((\lambda x \rightarrow x) \ v)$$

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$v$

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Multiple applications where the left-side is not a lambda abstraction are solved in a left-to-right fashion:



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Multiple applications where the left-side is not a lambda abstraction are solved in a left-to-right fashion:

$$\frac{t \rightarrow_{\beta} t' \wedge u \rightarrow_{\beta} u' \wedge t' u' \rightarrow_{\beta} v}{t u \rightarrow_{\beta} v}$$

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Variables cannot be further reduced, that is they stay the same:

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Variables cannot be further reduced, that is they stay the same:

$$\overline{x \rightarrow_{\beta} x}$$

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We can encode functions with multiple parameters by nesting lambda abstractions:

$$(\lambda x \rightarrow y \rightarrow (x \ y))$$

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The parameters are then given one at a time:

$$(((\lambda x \rightarrow y \rightarrow (x \ y)) \ A) \ B)$$

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$$(((\lambda x \rightarrow y \rightarrow (x \ y)) \ A) \ B)$$

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$$((\lambda y \rightarrow (A \ y)) \ B)$$
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## Example executions of (apparently) nonsensical programs

- We will now exercise with the execution of various lambda programs.
- Try to guess what the result of these programs is, and then we shall see what would have happened.

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*What is the result of this program execution?*

$$(((\lambda x \rightarrow y \rightarrow (x \ y)) (\lambda z \rightarrow (z \ z))) A)$$



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$$(((\lambda x \rightarrow y \rightarrow (x \ y)) \ (\lambda z \rightarrow (z \ z))) \ A)$$
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$$((\lambda y \rightarrow ((\lambda z \rightarrow (z \ z)) y)) A)$$

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$$\underline{((\lambda y \rightarrow ((\lambda z \rightarrow (z \ z)) \ y)) \ A)}$$

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*What is the result of this program execution? Watch out for the scope of the two “x” variables!*

$$(((\lambda x \rightarrow x \rightarrow (x \ x)) \ A) \ B)$$

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$(B \ B)$

The first “x” gets replaced with “A”, but the second “x” shadows it!

$$(((\lambda x \rightarrow x \rightarrow (x \ x)) \ A) \ B)$$

A better formulation, less ambiguous, would turn:

$$(((\lambda x \rightarrow x \rightarrow (x \ x)) \ A) \ B)$$

...into:

$$(((\lambda y \rightarrow x \rightarrow (x \ x)) \ A) \ B)$$

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*What is the result of this program execution? Is there even a result?*

$$((\lambda x \rightarrow (x \ x)) (\lambda x \rightarrow (x \ x)))$$



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```
((λx→(x x)) (λx→(x x)))
```

It never ends! Like a `while true: ...!`

Ok, I know what you are all thinking: what is this for sick joke?  
This is no real programming language!

- We have some sort of functions and function calls
- We do not have booleans and if's
- We do not have integers and arithmetic operators
- We do not have a lot of things!

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## Surprise!

With nothing but lambda programs we will show how to build all of these features and more.

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Stay tuned.

This will be a marvelous voyage.

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## The best of luck, and thanks for the attention!