

Software Engineering 1

Lambda Calculus exercises Beta reduction

1 Exercise 1

Given the following lambda program, complete the empty beta reduction steps for this program.

$((\lambda x.(x\ y))\ (\lambda z.z))$

1.1 Answer 1

$((\lambda x.(x\ y))\ (\lambda z.z))$

$((\lambda x.(x\ y))\ (\lambda z.z))$

$((\lambda z.z)\ y)$

$((\lambda z.z)\ y)$

y

2 Exercise 2

Given the following lambda program, complete the empty beta reduction steps for this program.

$((\lambda x.((\lambda y.(x\ y))\ x))\ (\lambda z.w))$

2.1 Answer 2

$((\lambda x.((\lambda y.(x\ y))\ x))\ (\lambda z.w))$

$((\lambda x.((\lambda y.(x\ y))\ x))\ (\lambda z.w))$

$((\lambda y.((\lambda z.w)\ y))\ (\lambda z.w))$

$((\lambda y.((\lambda z.w)\ y))\ (\lambda z.w))$

$((\lambda z.w)\ (\lambda z.w))$

$((\lambda z.w)\ (\lambda z.w))$

w

3 Exercise 3

Given the following lambda program, complete the empty beta reduction steps for this program.

$((((\lambda f \ g \ x.((f \ x) \ (g \ x))) \ (\lambda m \ n.(n \ m))) \ (\lambda n.z)) \ p)$

3.1 Answer 3

$((((\lambda f \ g \ x.((f \ x) \ (g \ x))) \ (\lambda m \ n.(n \ m))) \ (\lambda n.z)) \ p)$

$(((\lambda f \ g \ x.((f \ x) \ (g \ x))) \ (\lambda m \ n.(n \ m))) \ (\lambda n.z)) \ p)$

$(((\lambda g \ x.((\lambda m \ n.(n \ m)) \ x) \ (g \ x))) \ (\lambda n.z)) \ p)$

$(((\lambda g \ x.((\lambda m \ n.(n \ m)) \ x) \ (g \ x))) \ (\lambda n.z)) \ p)$

$(((\lambda x.(((\lambda m \ n.(n \ m)) \ x) \ ((\lambda n.z) \ x))) \ p)$

$(((\lambda x.(((\lambda m \ n.(n \ m)) \ x) \ ((\lambda n.z) \ x))) \ p)$

$((((\lambda m \ n.(n \ m)) \ p) \ ((\lambda n.z) \ p))$

$(((\lambda m \ n.(n \ m)) \ p) \ ((\lambda n.z) \ p))$

$(((\lambda n.(n \ p)) \ ((\lambda n.z) \ p))$

$(((\lambda n.(n \ p)) \ ((\lambda n.z) \ p))$

$(((\lambda n.(n \ p)) \ z)$

$(((\lambda n.(n \ p)) \ z)$

$((z \ p)$

4 Exercise 4

Given the following lambda program, complete the empty beta reduction steps for this program.

$((\lambda f.((\lambda g.((f \ f) \ g)) \ (\lambda h.(k \ h)))) \ (\lambda x \ y.y))$

4.1 Answer 4

$((\lambda f.((\lambda g.((f \ f) \ g)) \ (\lambda h.(k \ h)))) \ (\lambda x \ y.y))$

$((\lambda f.((\lambda g.((f \ f) \ g)) \ (\lambda h.(k \ h)))) \ (\lambda x \ y.y))$

$(((\lambda g.((\lambda x \ y.y) \ (\lambda x \ y.y)) \ g)) \ (\lambda h.(k \ h)))$

$(((\lambda g.((\lambda x \ y.y) \ (\lambda x \ y.y)) \ g)) \ (\lambda h.(k \ h)))$

$(((\lambda x \ y.y) \ (\lambda x \ y.y)) \ (\lambda h.(k \ h)))$

```
((λx y.y) (λx y.y)) (λh.(k h))
```

```
((λy.y) (λh.(k h)))
```

```
((λy.y) (λh.(k h)))
```

```
(λh.(k h))
```

5 Exercise 5

Given the following lambda program, complete the empty beta reduction steps for this program.

```
((λz.z) (λy.(y y))) ((λx.x) a)
```

5.1 Answer 5

```
((λz.z) (λy.(y y))) ((λx.x) a)
```

```
((λz.z) (λy.(y y))) ((λx.x) a)
```

```
((λy.(y y)) ((λx.x) a))
```

```
((λy.(y y)) ((λx.x) a))
```

```
((λy.(y y)) a)
```

```
((λy.(y y)) a)
```

```
(a a)
```

6 Exercise 6

Given the following lambda program, complete the empty beta reduction steps for this program.

```
((λz.z) ((λy.y) y)) ((λz.z) y)
```

6.1 Answer 6

```
((λz.z) ((λy.y) y)) ((λz.z) y)
```

```
((λz.z) ((λy.y) y)) ((λz.z) y)
```

```
((λz.z) y) ((λz.z) y)
```

```
((λz.z) y) ((λz.z) y)
```

```
(y ((λz.z) y))
```

```
(y ((λz.z) y))
```

```
(y y)
```

7 Exercise 7

Given the following lambda program, complete the empty beta reduction steps for this program.

```
((λx y.(x (y y))) (λa.a) b)
```

7.1 Answer 7

```
((λx y.(x (y y))) (λa.a) b)
```

```
((λx y.(x (y y))) (λa.a) b)
```

```
((λy.(λa.a) (y y)) b)
```

```
((λy.(λa.a) (y y)) b)
```

```
((λa.a) (b b))
```

```
((λa.a) (b b))
```

```
(b b)
```

8 Exercise 8

Given the following lambda program, complete the empty beta reduction steps for this program.

```
((λx y.(x (y y))) (λy.y) y)
```

8.1 Answer 8

```
((λx y.(x (y y))) (λy.y) y)
```

```
((λx y.(x (y y))) (λy.y) y)
```

```
((λy.(λy.y) (y y)) y)
```

```
((λy.(λy.y) (y y)) y)
```

```
((λy.y) (y y))
```

```
((λy.y) (y y))
```

```
(y y)
```

9 Exercise 9

Given the following lambda program, complete the empty beta reduction steps for this program.

```
((λx.x) ((λy.y) x) z)
```

9.1 Answer 9

$((((\lambda x.x) x) ((\lambda y.y) x)) z)$

$((((\lambda x.x) x) ((\lambda y.y) x)) z)$

$((x ((\lambda y.y) x)) z)$

$((x ((\lambda y.y) x)) z)$

$((x x) z)$

10 Exercise 10

Given the following lambda program, complete the empty beta reduction steps for this program.

$((((\lambda x y.(x y)) y) z)$

10.1 Answer 10

$((((\lambda x y.(x y)) y) z)$

$((((\lambda x y.(x y)) y) z)$

$((\lambda y.(y y)) z)$

$((\lambda y.(y y)) z)$

$((z z)$

11 Exercise 11

Given the following lambda program, complete the empty beta reduction steps for this program.

$((((\lambda x.(x x)) (\lambda y.y)) (\lambda y.y))$

11.1 Answer 11

$((((\lambda x.(x x)) (\lambda y.y)) (\lambda y.y))$

$((((\lambda x.(x x)) (\lambda y.y)) (\lambda y.y))$

$(((\lambda y.y) (\lambda y.y)) (\lambda y.y))$

$((((\lambda y.y) (\lambda y.y)) (\lambda y.y))$

$(((\lambda y.y) (\lambda y.y))$

$((\lambda y.y) (\lambda y.y))$

$(\lambda y.y)$

12 Exercise 12

Given the following lambda program, complete the empty beta reduction steps for this program.

$((\lambda x \ y. (x \ y)) (\lambda y. y)) \ w$

12.1 Answer 12

$((\lambda x \ y. (x \ y)) (\lambda y. y)) \ w$

$((\lambda x \ y. (x \ y)) (\lambda y. y)) \ w$

$((\lambda y. (\lambda y. y) \ y)) \ w$

$((\lambda y. ((\lambda y. y) \ y)) \ w)$

$((\lambda y. y) \ w)$

$((\lambda y. y) \ w)$

w