

# INFSEN02-1 Sample exam

The INFDEV@HR Team

## 1 Exam

### 1.1 Question 1

Given the following lambda program, and a series of relevant delta rules, show the beta reductions for this program.

(TRUE  $\vee$  FALSE)

### 1.2 Relevant delta rules

Boolean or:

$(\lambda p \ q \rightarrow ((p \ p) \ q))$

True

$(\lambda t \ f \rightarrow t)$

False

$(\lambda t \ f \rightarrow f)$

### 1.3 Answer 1 (note: you do not need to write all this detail yourself, it is only included for completeness)

(TRUE  $\vee$  FALSE)

(( $\underline{\vee}$  TRUE) FALSE)

(( $(\lambda p \ q \rightarrow ((p \ p) \ q))$  TRUE) FALSE)

(( $((\lambda p \ q \rightarrow ((p \ p) \ q)) \underline{\text{TRUE}})$  FALSE)

(( $((\lambda p \ q \rightarrow ((p \ p) \ q)) (\lambda t \ f \rightarrow t))$  FALSE)

$((\lambda p \ q \rightarrow ((p \ p) \ q)) \ (\lambda t \ f \rightarrow t)) \ \underline{\text{FALSE}}$

$((\lambda p \ q \rightarrow ((p \ p) \ q)) \ (\lambda t \ f \rightarrow t)) \ (\lambda t \ f \rightarrow f)$

$((\lambda p \ q \rightarrow ((p \ p) \ q)) \ (\lambda t \ f \rightarrow t)) \ (\lambda t \ f \rightarrow f)$

$((\lambda q \rightarrow ((\lambda t \ f \rightarrow t) \ (\lambda t \ f \rightarrow t)) \ q)) \ (\lambda t \ f \rightarrow f)$

$((\lambda q \rightarrow ((\lambda t \ f \rightarrow t) \ (\lambda t \ f \rightarrow t)) \ q)) \ (\lambda t \ f \rightarrow f)$

$((\lambda t \ f \rightarrow t) \ (\lambda t \ f \rightarrow t)) \ (\lambda t \ f \rightarrow f)$

$((\lambda t \ f \rightarrow t) \ (\lambda t \ f \rightarrow t)) \ (\lambda t \ f \rightarrow f)$

$((\lambda f \ t \ f \rightarrow t) \ (\lambda t \ f \rightarrow f))$

$((\lambda f \ t \ f \rightarrow t) \ (\lambda t \ f \rightarrow f))$

$(\lambda t \ f \rightarrow t)$

$(\lambda t \ f \rightarrow t)$

$\text{TRUE}$

## 1.4 Question 2

Given the following lambda calculus program, and a series of relevant delta rules, give the full typing derivation for the program.

$(\lambda(p:\text{Boolean}) \ (q:\text{Boolean}) \rightarrow (((p \ \text{Boolean}) \ p) \ q))$

## 1.5 Relevant delta rules

Boolean type:

$(\forall \alpha \Rightarrow (\alpha \rightarrow \alpha \rightarrow \alpha))$

1.6 Answer 2 (note: you do not need to write all this detail yourself, it is only included for completeness)

$$(\lambda(p:\text{Boolean}) \ (q:\text{Boolean}) \rightarrow ((p \ \text{Boolean}) \ p) \ q))$$
$$(\lambda(p:\text{Boolean})\ (q:\text{Boolean}) \rightarrow ((p \ \text{Boolean})\ p)\ q))$$

```
(λ(p:Boolean) (q:Boolean)→((Boolean Boolean) Boolean) q))
```

$$(\lambda(p:\text{Boolean})(q:\text{Boolean}) \rightarrow (((\text{Boolean Boolean}) \text{ Boolean}) q))$$

```
(λ(p:Boolean) (q:Boolean)→(((Boolean Boolean) Boolean)
Boolean))
```

```
(λ(p:Boolean) (q:Boolean)→(((Boolean Boolean) Boolean)
  Boolean))
```

```
(λ(p:Boolean) (q:Boolean)→((( (∀α ⇒(α→α→α)) Boolean)
  Boolean) Boolean))
```

$$(\lambda(p:\text{Boolean})\ (q:\text{Boolean})\rightarrow(((\forall\alpha\ \Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))\ \underline{\text{Boolean}})\ \text{Boolean})$$

```
(λ(p:Boolean) (q:Boolean)→(((∀α ⇒(α→α→α))
(∀α ⇒(α→α→α))) Boolean) Boolean))
```

$$(\lambda(p:\text{Boolean})\ (q:\text{Boolean})\rightarrow((\underbrace{((\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))\ (\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha)))}_{\text{Boolean}})\ \text{Boolean}))$$

```
(λ(p:Boolean) (q:Boolean)→((
  ((∀α ⇒(α→α→α))→(∀α ⇒(α→α→α))→(∀α ⇒(α→α→α)))
) Boolean)) Boolean
```

$$(\lambda(p:\text{Boolean})\ (q:\text{Boolean})\rightarrow(((\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))\rightarrow(\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))\rightarrow(\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))))\ \text{Boolean})\ \text{Boolean}))$$
$$(\lambda(p:\text{Boolean})\ (q:\text{Boolean})\rightarrow(((\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))\rightarrow(\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))\rightarrow(\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha)))\ (\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha)))\ \text{Boolean}))$$
$$(\lambda(p:\text{Boolean})\ (q:\text{Boolean})\rightarrow(((\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))\rightarrow(\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))\rightarrow(\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha)))\ (\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha)))\ \text{Boolean}))$$

```
(λ(p:Boolean) (q:Boolean)→(  
  ((∀α ⇒(α→α→α))→(∀α ⇒(α→α→α))) Boolean))
```

```
(λ(p:Boolean) (q:Boolean)→(((∀α ⇒(α→α→α))→(∀α ⇒(α→α→α))  
  ) Boolean))
```

```
(λ(p:Boolean) (q:Boolean)→(((∀α ⇒(α→α→α))→(∀α ⇒(α→α→α))  
  ) (∀α ⇒(α→α→α))) )
```

```
(λ(p:Boolean) (q:Boolean)→(((∀α ⇒(α→α→α))→(∀α ⇒(α→α→α)))  
  (∀α ⇒(α→α→α))))
```

```
(λ(p:Boolean) (q:Boolean)→ (∀α ⇒(α→α→α)) )
```

```
(λ(p:Boolean) (q:Boolean)→(∀α ⇒(α→α→α)))
```

```
(λ(p:Boolean) (q:Boolean)→ Boolean )
```

```
(λ(p:Boolean) (q:Boolean)→Boolean)
```

```
(λ(p:Boolean)→ (Boolean→Boolean) )
```

```
(λ(p:Boolean)→(Boolean→Boolean))
```

```
(Boolean→Boolean→Boolean)
```