Delta rules

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Introduction

Introduction

Lecture topics

- Make it pretty: delta rules
- Booleans, boolean logic operators, if-then-else
- Naturals, arithmetic operators, comparison operators
- Let-binding and let rec
- Tuples
- Discriminated unions
- Lists

Introduction

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(2 + 1)
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Introduction


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(2 + 1)
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((+ 2) 1)
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Introduction

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((+ 2) 1)
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((+ 2) 1)
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    z)))) 1)
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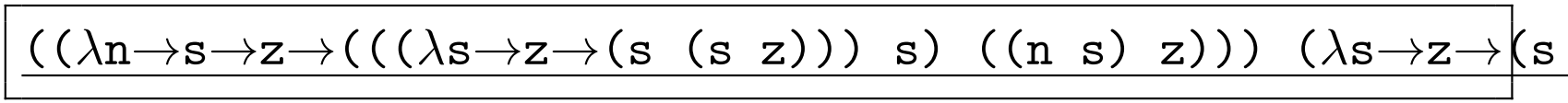
Introduction



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Introduction

$$(\texttt{TRUE} \wedge \texttt{TRUE})$$

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$$(\texttt{TRUE} \wedge \texttt{TRUE})$$

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$$((\lambda a \rightarrow b \rightarrow ((a\ b)\ a))\ (\lambda t \rightarrow f \rightarrow t))\ (\lambda t \rightarrow f \rightarrow t))$$

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$$((\lambda a \rightarrow b \rightarrow ((a\ b)\ a))\ (\lambda t \rightarrow f \rightarrow t))\ (\lambda t \rightarrow f \rightarrow t))$$

$$((\lambda b \rightarrow (((\lambda t \rightarrow f \rightarrow t)\ b)\ (\lambda t \rightarrow f \rightarrow t)))\ (\lambda t \rightarrow f \rightarrow t))$$

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((λb→(((λt→f→t) b) (λt→f→t))) (λt→f→t))
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((λb→(((λt→f→t) b) (λt→f→t))) (λt→f→t))
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$$\underline{((\lambda b \rightarrow (((\lambda t \rightarrow f \rightarrow t)\ b)\ (\lambda t \rightarrow f \rightarrow t)))\ (\lambda t \rightarrow f \rightarrow t))}$$

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$$\underline{((\lambda b \rightarrow (((\lambda t \rightarrow f \rightarrow t)\ b)\ (\lambda t \rightarrow f \rightarrow t)))\ (\lambda t \rightarrow f \rightarrow t))}$$

$$(((\lambda t \rightarrow f \rightarrow t)\ (\lambda t \rightarrow f \rightarrow t))\ (\lambda t \rightarrow f \rightarrow t))$$

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((λf→t→f→t) (λt→f→t))
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$$(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})$$

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(λt→f→t)
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$(\lambda t \rightarrow f \rightarrow t)$

$\underline{(\lambda t \rightarrow f \rightarrow t)}$

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$(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})$

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$(\lambda t \rightarrow f \rightarrow t)$

TRUE

This is it!

The best of luck, and thanks for the attention!