

Haskell

The  
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Team

Translating  
Lambda  
calculus to  
Haskell

Lazy  
evaluation

The "do"  
notation and  
IO monad

Conclusion

# Haskell

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# Translating Lambda calculus to Haskell

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## Overview

- Haskell can be mapped to the lambda calculus as we did for F#
- It is slightly different than F# with respect to let-bindings
- It uses a different evaluation strategy than the straightforward beta reduction seen so far

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Conclusion

Integers, booleans, floats, strings have the usual meaning, both in the lambda calculus, Haskell, and the languages you are used to:

```
((2 + 3) - 4)
```

```
(True && False)
```

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- Conditionals behave just like in the lambda calculus
- This means that they return the evaluation of either of the two branches
- This differs from imperative languages, where we just jump into either of the two branches

```
if (True && False) then
  0
else
  1
```

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## Haskell

```
if (True && False) then  
  0  
else  
  1
```

## Lambda calculus

```
if (TRUE  $\wedge$  FALSE) then 0 else 1
```

```
((((( $\lambda p$  th  $e1 \rightarrow ((p$  th)  $e1$ )) (TRUE  $\wedge$  FALSE)) 0)  
  1)
```

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Conclusion

Functions look very similar, with  $\backslash$  instead of  $\lambda$

```
(\x f -> (f x))
```

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Conclusion

Functions look very similar, with `\` instead of  $\lambda$

```
(\x f -> (f x))
```

Just like function application

```
(((\x f -> (f x)) 3) (\x -> (3 + x)))
```



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## Haskell

```
(((\x f -> (f x)) 3) (\x -> (3 + x)))
```

## Lambda calculus

```
(((\lambda x f \rightarrow (f x)) 3) (\lambda x \rightarrow (3 + x)))
```

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We can give names to functions, and code becomes much prettier as a result

```
let apply =  
  \ x f -> (f x) in  
((apply 3) (\x -> (3 + x)))
```

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## Haskell

```
let apply =  
  \ x f -> (f x) in  
((apply 3) (\x -> (3 + x)))
```

## Lambda calculus

```
let apply = ( $\lambda x f \rightarrow (f x)$ ) in ((apply 3) ( $\lambda x \rightarrow (3 + x)$ ))
```

```
(( $\lambda apply \rightarrow ((apply 3) (\lambda x \rightarrow (3 + x)))$ ) ( $\lambda x f \rightarrow (f x)$ ))
```

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Conclusion

- In Haskell, unlike F#, there is no distinction between `let` and `let rec`. Everything is `let rec`.
- `let-in` is used to define bindings locally into a function body
- Global `let` bindings are simply defined by defining the function name and can be used recursively

```
fact =  
  \ n ->  
    if (n == 0) then  
      1  
    else  
      ((fact (n - 1)) * n)  
  
(fact 2)
```

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## Haskell

```
fact =  
  \ n ->  
    if (n == 0) then  
      1  
    else  
      ((fact (n - 1)) * n)  
  
(fact 2)
```

## Lambda calculus

```
let fact = (fix (λf n→if (n = 0) then 1 else  
  ((f (n - 1)) × n))) in (fact 2)
```

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Conclusion

We can define tuples by just putting a comma between the values, with or without nesting for more than two values is done for us. Unlike F#, brackets are mandatory when defining tuples in Haskell

```
(1, True)
```

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**Haskell**

```
(1, True)
```

**Lambda calculus**

```
(1, TRUE)
```

```
((((λx y → (λf → ((f x) y))) 1) TRUE)
```

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Conclusion

- Functions such as  $\pi_1$  and  $\pi_2$ , which both extract one item of a pair, also exist in Haskell
- They are called, respectively, `fst` and `snd`

```
(fst (1, True))
```

```
(snd (1, True))
```



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Conclusion

## Haskell

```
(fst (1, True))
```

```
(snd (1, True))
```

## Lambda calculus

```
 $(\pi_1 (1, \text{TRUE}))$ 
```

```
 $((\lambda p \rightarrow (p (\lambda x y \rightarrow x))) (1, \text{TRUE}))$ 
```

```
 $(\pi_2 (1, \text{TRUE}))$ 
```

```
 $((\lambda p \rightarrow (p (\lambda x y \rightarrow y))) (1, \text{TRUE}))$ 
```

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Conclusion

- Haskell also offers built-in discriminated unions
- Functions such as `inl` and `inr`, which both embed one item into the union, also exist in Haskell
- They are called, respectively, `Left` and `Right`

```
(Left 1)
```

```
(Right True)
```

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Conclusion

## Haskell

```
(Left 1)
```

```
(Right True)
```

## Lambda calculus

```
(inl 1)
```

```
((λx→ (λf g→(f x))) 1)
```

```
(inr TRUE)
```

```
((λy→ (λf g→(g y))) TRUE)
```

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Conclusion

We can, of course, perform matches on discriminated unions

```
case (Left 1) of
  Left x ->
    (Left x)
  Right y ->
    (Right y)
```

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Conclusion

We can, of course, perform matches on discriminated unions

```
case (Left 1) of
  Left x ->
    (Left x)
  Right y ->
    (Right y)
```

```
let i =
  case (Left 1) of
    Left x ->
      x
    Right y ->
      0
in
(i * 2)
```

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Conclusion

Discriminated unions, and their corresponding matches, can be nested as deep as we need

```
case (Left (Right True)) of
  Left x ->
    case x of
      Left x ->
        (Left x)
      Right y ->
        (Right y)

  Right y ->
    (Right y)
```

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Conclusion

- Type annotations in haskell are quite different from F#
- The declaration of the function types is separated from the body

```
fact :: Integral -> Integral
fact = (\n ->
    if (n == 0) then
        1
    else
        ((fact (n - 1)) * n)
)
```

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Conclusion

- It is possible to use pattern matching to define functions instead of using a match or an if-then-else
- It is done just repeating the function definition with the specific arguments

```
length=(\[] -> 0)
length=(\(x:xs) -> (1 + (length xs)))
```



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Conclusion

Since the type declaration is separated from the function body, type variables for generics can be written as normal variables

Note that in Haskell the type of a list is written as `[a]` where `a` is a concrete type or a type variable

```
length :: [a] -> Integral
length = (\[] -> 0)
length = (\(x:xs) -> (1 + (length xs)))
```

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# Lazy evaluation

# Lazy evaluation

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Conclusion

- Haskell uses a mechanism of evaluation for expressions called *lazy evaluation*
- When binding an expression to a variable the expression is not evaluated immediately
- The binding contains a “recipe” to evaluate the expression
- The evaluation is delayed until the binding is actually used in the program
- Unevaluated values are called *thunks*

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Conclusion

Consider the following code:

```
let (x,y) =  
    ((length [1,2]), (reverse [1,2])) in  
    ...
```

- The variables `x` and `y` initially contain `thunks`, until at some point in the `in` body they are used
- If the values are never used, they will never be evaluated

Consider the following code:

```
1 let
2   z = (length [1,2], reverse [1,2])
3   (n,s) = z
4 in ...
```

- At line 1 line `z` is simply a thunk
- At line 2 the compiler must know if `z` is actually a pair, because the pattern must match the `let` binding
- The compiler does not need to evaluate the content of the pair
- Thus `(n,s)` becomes a pair of thunks, i.e. `z = (thunk,thunk)`

# Lazy evaluation

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Consider the following code:

```
1 let
2   z = (length [1,2], reverse [1,2])
3   (n,s) = z
4   (1::ss) = s
5 in ...
```

- At line 4 the compiler must know if `s` is a list with the number 1 as head to match the pattern
- The compiler needs to know if `s` is a list, thus it evaluates the result of `reverse` as a list with a thunk as a head and another thunk as a tail, so we have `thunk:thunk`
- The compiler needs to know if the head of `s` matches the number 1, thus we have `1:thunk`

# Lazy evaluation

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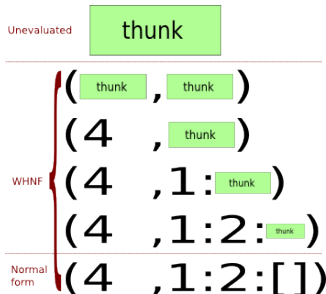
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Conclusion

- The figure below shows the possible evaluation of  $(2, [1,2])$
- WHNF = *Weak head normal form*, i.e. when the evaluation contains both values and thunks
- NF = *Normal form*, i.e. when the evaluation contains only values and no thunks



# Lazy evaluation

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Conclusion

- In the Haskell standard library we have a value called `undefined` which is used to capture errors in the program
- When the program evaluates `undefined`, execution halts and an error is returned
- Now consider the following code:

```
let
  failMiserably = \x -> undefined
  (x,y) = (4,failMiserably "Please crash")
in
  x
```

Does it crash?



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```
1 let
2   failMiserably = \x -> undefined
3   (x,y) = (4,failMiserably "Please crash")
4 in
5   x
```

The answer is no!

- At line 3  $(x,y) = (\text{thunk}, \text{thunk})$
- At line 5 the expression only uses  $x$ , thus only 4 is evaluated.
- $y$  is still a thunk, so the program will never know that it contains undefined
- You might have an evaluation that actually failed but you will never know because of the lazy evaluation!

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# The "do" notation and IO monad

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Conclusion

- Haskell, unlike F#, is a pure functional language
- This means we cannot make calls to imperative functions just like in F#. For example we cannot call something like `printf` because that is an imperative function
- How can we print a value to the standard output?

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Conclusion

- In Haskell the main function is always<sup>a</sup> defined as a do block
- For example:

---

<sup>a</sup>For a large enough value of *a*lways

```
main = do
  putStr("Velociraptor\n")
  print (velociraptor 30.0 10)
```

The code allows you to print things on the shell

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Conclusion

**You are cheating!!! That is imperative code!!! So all this course is about nothing because you cannot have pure functional programming!**

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**You are cheating!!! That is imperative code!!! So all this course is about nothing because you cannot have pure functional programming!**

- No. The do notation is syntax to hide a functional structure called *Monad*
- We do not have time to explain monads in detail in this course, but they are structures that only use lambdas and a composition of lambdas to produce a result.
- It is possible to express imperative behaviours only with monads.
- If you are interested take a look at the State monad.
- In particular the IO Monad allows you to handle side effects, thus print on the shell

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## Closing up

- Haskell can be mapped to the lambda calculus as we did for F#: it looks mostly the same
- It does not feature non-recursive let-bindings
- It uses a lazy evaluation strategy that delays expanding values for as long as possible



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The best of luck, and thanks for the  
attention!