

INFSEN02-1 Sample exam

The INFDEV@HR Team

1 Exam

1.1 Question 1

Given the following lambda program, and a series of relevant delta rules, show the beta reductions for this program.

```
if FALSE then 1 else 0
```

1.2 Relevant delta rules

If-then-else

```
(λp th el→((p th) el))
```

Boolean false

```
(λt f→f)
```

Integer one (0)

```
(λs z→z)
```

Integer one (1)

```
(λs z→(s z))
```

1.3 Answer 1 (note: you do not need to write all this detail yourself, it is only included for completeness)

```
if FALSE then 1 else 0
```

```
(((if-then-else) FALSE) 1) 0)
```

```
((((λp th el→((p th) el)) FALSE) 1) 0)
```

```
((((λp th el→((p th) el)) FALSE) 1) 0)
```

```
(((((λp th el→((p th) el)) (λt f→f)) 1) 0)
```

```
(((((λp th el→((p th) el)) (λt f→f)) 1) 0)
```

```
(((((λp th el→((p th) el)) (λt f→f)) (λs z→(s z))) 0)
```

```
(((((λp th el→((p th) el)) (λt f→f)) (λs z→(s z))) 0)
```

```
(((((λp th el→((p th) el)) (λt f→f)) (λs z→(s z)))  
  (λs z→z))
```

```
(((((λp th el→((p th) el)) (λt f→f)) (λs z→(s z))) (λs z→z))
```

```
((((λth el→((λt f→f) th) el)) (λs z→(s z))) (λs z→z))
```

```
((((λth el→(((λt f→f) th) el)) (λs z→(s z))) (λs z→z))
```

```
((λel→(((λt f→f) (λs z→(s z))) el)) (λs z→z))
```

```
((λel→(((λt f→f) (λs z→(s z))) el)) (λs z→z))
```

```
((((λt f→f) (λs z→(s z))) (λs z→z))
```

```
((((λt f→f) (λs z→(s z))) (λs z→z))
```

```
((λf→f) (λs z→z))
```

```
((λf→f) (λs z→z))
```

```
(λs z→z)
```

```
(λs z→z)
```

```
0
```

1.4 Question 2

Given the following lambda calculus program, and a series of relevant delta rules, give the full typing derivation for the program.

```
Λβ⇒(λ(p:Boolean) (th:β) (el:β)→(((p β) th) el))
```

1.5 Relevant delta rules

Boolean type:

$$(\forall \alpha \Rightarrow ((\alpha \rightarrow \alpha) \rightarrow \alpha \rightarrow \alpha))$$

1.6 Answer 2 (note: you do not need to write all this detail yourself, it is only included for completeness)

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow ((p \ \beta) \text{ th) el}))$$

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow (((p \ \beta) \text{ th) el}))$$

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow (((\text{Boolean} \ \beta) \text{ th) el}))$$

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow (((\text{Boolean} \ \beta) \text{ th) el}))$$

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow (((\text{Boolean} \ \beta) \ \beta) \text{ el}))$$

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow (((\text{Boolean} \ \beta) \ \beta) \text{ el}))$$

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow (((\text{Boolean} \ \beta) \ \beta) \ \beta))$$

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow (((\text{Boolean} \ \beta) \ \beta) \ \beta))$$

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow (((\forall \alpha \Rightarrow (\alpha \rightarrow \alpha \rightarrow \alpha)) \ \beta) \ \beta) \ \beta))$$

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow (((\forall \alpha \Rightarrow (\alpha \rightarrow \alpha \rightarrow \alpha)) \ \beta) \ \beta) \ \beta))$$

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow (((\beta \rightarrow \beta \rightarrow \beta) \ \beta) \ \beta))$$

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow (((\beta \rightarrow \beta \rightarrow \beta) \ \beta) \ \beta))$$

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow ((\beta \rightarrow \beta) \ \beta))$$

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow ((\beta \rightarrow \beta) \ \beta))$$

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow \beta)$$

$$\Lambda \beta \Rightarrow (\lambda (p : \text{Boolean}) \text{ (th : } \beta) \text{ (el : } \beta) \rightarrow \beta)$$

$\Lambda\beta \Rightarrow (\lambda(p:\text{Boolean}) \ (\text{th}:\beta) \rightarrow (\beta \rightarrow \beta))$

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$\Lambda\beta \Rightarrow (\text{Boolean} \rightarrow \beta \rightarrow \beta \rightarrow \beta)$

$\Lambda\beta \Rightarrow (\text{Boolean} \rightarrow \beta \rightarrow \beta \rightarrow \beta)$

$(\forall \beta \Rightarrow (\text{Boolean} \rightarrow \beta \rightarrow \beta \rightarrow \beta))$