HOGESCHOOL ROTTERDAM

Data structures

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Introduction

Introduction

Lecture topics

- Let
- Tuples
- Discriminated unions (polymorphism)

Let-in

Let-in

Idea

- Sometimes we wish to give a name to a value or a computation, to reuse later
- This construct is called `let-in`
- We could then say something like `let age = 9 in age + age`
- We can nest `let-in` constructs, and then say something like `let age = 9 in (let x = 2 in age * x)`

Let-in

Idea

- Sometimes we wish to give a name to a value or a computation, to reuse later
- This construct is called `let-in`
- We could then say something like `let age = 9 in age + age`
- We can nest `let-in` constructs, and then say something like `let age = 9 in (let x = 2 in age * x)`
- This makes code significantly more readable, as it looks like a series of declarations top-to-bottom

Let-in

Idea

- Lets are simply translated to function applications
- `let x = t in u` simply becomes $(\lambda x \rightarrow u)\ t$

Let-in


```
let age = 9 in (age + age)
```

Let-in


```
let age = 9 in (age + age)
```

```
let age = 9 in (age + age)
```

Let-in

```
let age = 9 in (age + age)
```

Let-in


```
let age = 9 in (age + age)
```

$$((\lambda age \rightarrow (age + age))\ 9)$$

Let-in


```
((λage→(age + age)) 9)
```

Let-in


```
((λage→(age + age)) 9)
```

```
((λage→(age + age)) 9)
```

Let-in

$((\lambda \text{age} \rightarrow (\text{age} + \text{age}))\ 9)$

Let-in

$((\lambda \texttt{age} \rightarrow (\texttt{age} + \texttt{age}))\ 9)$

$(9 + 9)$

Let-in

```
(9 + 9)
```

Let-in

```
(9 + 9)
```

```
(9 + 9)
```

Let-in

```
(9 + 9)
```

Let-in

```
(9 + 9)
```

```
18
```

Data types

Data types

Overview

- We now move on to ways to define data types
- The definitions will be both **minimal** and **composable**
- Classes, polymorphism, etc. can all be rendered under our definitions, so we miss nothing substantial

Data types

Overview

Notice: from now on we will start ignoring the reduction steps for simple terms such as 3+3, `x = 0`, etc. for brevity

Minimality

- The lambda calculus has so far proven very powerful, despite its size
- We do not need hundreds of different operators, we can simply build them[a]
- The only extension needed is purely syntactic in nature to make it more mnemonic, but this is only skin-deep and requires no change to the underlying mechanisms of the lambda calculus

[a] and more

Data types

Minimality

- In defining data types we wish to maintain this minimality
- We do not want dozens of separate, competing data types all slightly overlapping

Data types

Fundamental scenarios

- **Tuples**: storing multiple things together at the same time, like the fields and methods in a class
- **Unions**: storing either one of various things at a time, like an interface that is exactly one of its concrete implementors

The importance of composition

- We just need to cover the case of two items, higher numbers come through composition
- For example, given the ability to store a pair, we can build a pair of pairs to create arbitrary tuples
- Similarly, given the ability to store either of two values, we can build either of many values with nesting

Tuples

Tuples

- A pair of values is defined simply as something that stores these two values
- We can extract them by giving the pair a function that will receive the values

$$(\lambda \mathtt{x}\ \ \mathtt{y} \rightarrow\ (\lambda \mathtt{f} \rightarrow ((\mathtt{f}\ \ \mathtt{x})\ \ \mathtt{y})))$$

Tuples

```
(1, 2)
```

Tuples


```
(1, 2)
```

```
(((,) 1) 2)
```

Tuples

```
(( (,)  1)  2)
```

Tuples


```
(( (,)  1)  2)
```

$((\ (\lambda x\ y \rightarrow\ (\lambda f \rightarrow ((f\ x)\ y)))\ \ 1)\ \ 2)$

Tuples

```
(((λx y→ (λf→((f x) y))) 1) 2)
```

Tuples

```
(((λx y→ (λf→((f x) y))) 1) 2)
```

```
( ((λx y→ (λf→((f x) y))) 1) 2)
```

```
( ((λx y→ (λf→((f x) y))) 1) 2)
```

Tuples


```
( ((λx y→ (λf→((f x) y))) 1) 2)
```

```
((λy f→((f 1) y)) 2)
```

Tuples

```
((λy f→((f 1) y)) 2)
```

Tuples

$((\lambda y\ f \rightarrow ((f\ 1)\ y))\ 2)$

$((\lambda y\ f \rightarrow ((f\ 1)\ y))\ 2)$

Tuples

```
((λy f→((f 1) y)) 2)
```

Tuples


```
((λy f→((f 1) y)) 2)
```

```
(λf→((f 1) 2))
```

Tuples

- We can define two utility functions that, given a pair, extract the first or second value
- They are usually called π_1 and π_2, or `fst` and `snd`

```
(λp→(p (λx y→x)))
```

```
(λp→(p (λx y→y)))
```

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Tuples

$(\pi_1 \ (1, \ 2))$

Tuples

$(\pi_1 \ (1, \ 2))$

$(\boxed{\pi_1} \ (1, \ 2))$

$$(\boxed{\pi_1}\ (1,\ 2))$$

Tuples

$(\pi_1\ (1,\ 2))$

$((\lambda p \rightarrow (p\ (\lambda x\ y \rightarrow x)))\ (1,\ 2))$

Tuples

```
((λp→(p (λx y→x))) (1, 2))
```

Tuples

```
((λp→(p (λx y→x))) (1, 2))
```

```
((λp→(p (λx y→x))) (((,) 1) 2))
```

Tuples

```
((λp→(p (λx y→x))) (( (,) 1) 2))
```

Tuples

```
((λp→(p (λx y→x))) (( (,)  1) 2))
```

```
((λp→(p (λx y→x))) (( (λx y→ (λf→((f x) y)))  1)
    2))
```

```
((λp→(p (λx y→x))) (((λx y→ (λf→((f x) y)))
    1) 2))
```

Tuples

$$((\lambda p \rightarrow (p\ (\lambda x\ y \rightarrow x)))\ (((\lambda x\ y \rightarrow\ (\lambda f \rightarrow ((f\ x)\ y)))\ 1)\ 2))$$

$$((\lambda p \rightarrow (p\ (\lambda x\ y \rightarrow x)))\ (\boxed{((\lambda x\ y \rightarrow\ (\lambda f \rightarrow ((f\ x)\ y)))\ 1)}\ 2))$$

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Tuples

```
((λp→(p (λx y→x))) (
    ((λx y→ (λf→((f x) y))) 1) 2))
```

Tuples

```
((λp→(p (λx y→x))) (
    ((λx y→ (λf→((f x) y))) 1) 2))
```

```
((λp→(p (λx y→x))) ((λy f→((f 1) y)) 2))
```

Tuples

$$\boxed{\texttt{((}\lambda\texttt{p}\rightarrow\texttt{(p (}\lambda\texttt{x y}\rightarrow\texttt{x))) ((}\lambda\texttt{y f}\rightarrow\texttt{((f 1) y)) 2))}}$$

Tuples


```
((λp→(p (λx y→x))) ((λy f→((f 1) y)) 2))
```

```
((λp→(p (λx y→x))) ((λy f→((f 1) y)) 2))
```

Tuples


```
((λp→(p (λx y→x))) ((λy f→((f 1) y)) 2))
```

Tuples


```
((λp→(p (λx y→x))) ((λy f→((f 1) y)) 2))
```

```
((λp→(p (λx y→x))) (λf→((f 1) 2)))
```

Tuples

$$\boxed{((\lambda p \rightarrow (p\ \ (\lambda x\ \ y \rightarrow x)))\ \ (\lambda f \rightarrow ((f\ \ 1)\ \ 2)))}$$

Tuples


```
((λp→(p (λx y→x))) (λf→((f 1) 2)))
```

```
((λp→(p (λx y→x))) (λf→((f 1) 2)))
```

Tuples

$((\lambda p \rightarrow (p\ (\lambda x\ y \rightarrow x)))\ (\lambda f \rightarrow ((f\ 1)\ 2)))$

Tuples


```
((λp→(p (λx y→x))) (λf→((f 1) 2)))
```

```
( (λf→((f 1) 2)) (λx y→x))
```

Tuples


```
((λf→((f 1) 2)) (λx y→x))
```

Tuples

```
((λf→((f 1) 2)) (λx y→x))
```

```
((λf→((f 1) 2)) (λx y→x))
```

Tuples

$$((\lambda \mathtt{f} \rightarrow ((\mathtt{f}\ 1)\ 2))\ (\lambda \mathtt{x}\ \mathtt{y} \rightarrow \mathtt{x}))$$

Tuples

$((\lambda f \to ((f\ 1)\ 2))\ (\lambda x\ y \to x))$

$((\ (\lambda x\ y \to x)\ \ 1)\ \ 2)$

Tuples

$$(((\lambda x\ \ y \rightarrow x)\ \ 1)\ \ 2)$$

Tuples


```
(((λx y→x) 1) 2)
```

```
( ((λx y→x) 1) 2)
```

Tuples


```
( ((λx y→x) 1) 2)
```

Tuples

($((\lambda$x y$\rightarrow$x) 1) 2)

$((\lambda$y$\rightarrow$ 1) 2)

Tuples

```
((λy→1) 2)
```

Tuples

$((\lambda y \rightarrow 1)\ \ 2)$

$((\lambda y \rightarrow 1)\ \ 2)$

Tuples


```
((λy→1) 2)
```

Tuples


```
((λy→1) 2)
```

```
1
```

Pair of values

We should expect that π_1 and π_2 are inverse operations to constructing a pair, as they destroy it

Tuples

$\texttt{let p = (1, 2) in ((}\pi_1\texttt{ p), (}\pi_2\texttt{ p))}$

Tuples


```
let p = (1, 2) in ((π1 p), (π2 p))
```

```
((π1 (1, 2)), (π2 (1, 2)))
```

Tuples

$$((\pi_1 \ \texttt{(1, 2)}), (\pi_2 \ \texttt{(1, 2)}))$$

Tuples

$$((\pi_1 \; (1,\; 2)),\; (\pi_2 \; (1,\; 2)))$$

$$((\pi_1 \; (1,\; 2)),\; (\pi_2 \; (1,\; 2)))$$

Tuples

$((\pi_1 \ (1, \ 2)) , \ (\pi_2 \ (1 , \ 2)))$

Tuples

$$((\pi_1\ (1,\ 2)),\ (\pi_2\ (1,\ 2)))$$

$$(1,\ (\pi_2\ (1,\ 2)))$$

Tuples


```
( 1 , (π₂ (1 , 2)))
```

Tuples

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$$(\boxed{1} , (\pi_2 (1, 2)))$$

$$(1, (\pi_2 (1, 2)))$$

Tuples

$$(1\ ,\ (\pi_2\ (1,\ 2))\ )$$

Tuples

$(1\,,\ (\pi_2\ (1,\ 2))\ )$

$(1\,,\ 2\,)$

Tuples


```
(1, 2)
```

Tuples


```
(1, 2)
```

```
(1, 2)
```

Discriminated unions

Discriminated unions

- A choice of values is defined simply as something that stores either of two possible values
- We call such a choice a **discriminated union**
- We build a discriminated union with either of two functions to build the first or the second value
- They are usually called `inl` and `inr`[a]

[a] *in* stands for injection, and *l* and *r* stand for left and right

$$(\lambda x \rightarrow (\lambda f\ g \rightarrow (f\ x)))$$

$$(\lambda y \rightarrow (\lambda f\ g \rightarrow (g\ y)))$$

Discriminated unions

```
(inl 1)
```

Discriminated unions


```
(inl 1)
```

```
( inl  1)
```

```
( inl  1)
```

Discriminated unions


```
( inl  1)
```

```
( (λx→ (λf g→(f x)))  1)
```

Discriminated unions

$$((\lambda x \rightarrow (\lambda f\ g \rightarrow (f\ x)))\ 1)$$

Discriminated unions

$((\lambda x \rightarrow (\lambda f\ g \rightarrow (f\ x)))\ 1)$

$((\lambda x \rightarrow (\lambda f\ g \rightarrow (f\ x)))\ 1)$

Discriminated unions

$((\lambda x \rightarrow (\lambda f\ g \rightarrow (f\ x)))\ 1)$

Discriminated unions


```
((λx→ (λf g→(f x))) 1)
```

```
(λf g→(f 1))
```

Discriminated unions


```
(λf  g→(f  1))
```

Discriminated unions

$$(\lambda \texttt{f g} \rightarrow (\texttt{f 1}))$$

$$(\lambda \texttt{f g} \rightarrow (\texttt{f 1}))$$

Discriminated unions

$(\lambda \mathtt{f\ g} \rightarrow (\mathtt{f\ 1}))$

Discriminated unions

```
(λf g→(f 1))
```

```
(inl 1)
```

```
(inr TRUE)
```

Discriminated unions


```
(inr TRUE)
```

```
(inr TRUE)
```

```
( inr  TRUE)
```

Discriminated unions

(inr TRUE)

((λy→ (λf g→(g y))) TRUE)

Discriminated unions

$$((\lambda y \rightarrow (\lambda f\ g \rightarrow (g\ y)))\ \texttt{TRUE})$$

Discriminated unions

$((\lambda y \rightarrow\ (\lambda f\ \ g \rightarrow (g\ \ y)))\ \ \mathtt{TRUE})$

$((\lambda y \rightarrow\ (\lambda f\ \ g \rightarrow (g\ \ y)))\ \ \mathtt{TRUE})$

Discriminated unions

$((\lambda y \rightarrow (\lambda f\ g \rightarrow (g\ y)))\ \mathtt{TRUE})$

```
((λy→ (λf g→(g y))) TRUE)
```

```
(λf g→(g TRUE))
```

Discriminated unions

$$(\lambda f\ \ g \rightarrow (g\ \ \mathtt{TRUE}))$$

Discriminated unions

$(\lambda \texttt{f g} \rightarrow (\texttt{g TRUE}))$

$(\lambda \texttt{f g} \rightarrow (\texttt{g TRUE}))$

Discriminated unions

$(\lambda \texttt{f g} \rightarrow (\texttt{g TRUE}))$

Discriminated unions


```
(λf g→(g TRUE))
```

```
(inr TRUE)
```

Discriminated unions

- Extracting the input of a discriminated union is a process known as `match`[a]
- Given a union and two functions (one per case), if the union was the first case we apply the first function, otherwise we apply the second function

[a] which is a sort of `switch`, just on steroids

```
(λu→ (λf g→((u f) g)))
```

Discriminated unions


```
(((match (inl 1)) (λx→(x + 1))) (λy→(y ∧
    FALSE)))
```

```
(((match (inl 1)) (λx→(x + 1))) (λy→(y ∧
    FALSE)))
```

```
(((match (inl 1)) (λx→(x + 1))) (λy→(y ∧
    FALSE)))
```

Discriminated unions


```
(((match (inl 1)) (λx→(x + 1))) (λy→(y ∧
    FALSE)))
```

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Discriminated unions

$$(((\texttt{match}\ (\texttt{inl}\ 1))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \texttt{FALSE})))$$

$$(((\ (\lambda u \rightarrow\ (\lambda f\ g \rightarrow ((u\ f)\ g)))\ \ (\texttt{inl}\ 1))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \texttt{FALSE})))$$

Discriminated unions


```
((((λu→ (λf g→((u f) g))) (inl 1)) (λx→(x +
    1))) (λy→(y ∧ FALSE)))
```

Discriminated unions

$$((((\lambda u \rightarrow (\lambda f\ g \rightarrow ((u\ f)\ g)))\ (\mathtt{inl}\ 1))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))$$

$$((((\lambda u \rightarrow (\lambda f\ g \rightarrow ((u\ f)\ g)))\ (\boxed{\mathtt{inl}}\ 1))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))$$

Discriminated unions


```
((((λu→ (λf g→((u f) g))) (inl 1)) (λx→(x +
     1))) (λy→(y ∧ FALSE)))
```

Discriminated unions

```
((((λu→ (λf g→((u f) g))) ( inl 1)) (λx→(x +
    1))) (λy→(y ∧ FALSE)))
```

```
((((λu→ (λf g→((u f) g))) (
    (λx→ (λf g→(f x))) 1)) (λx→(x + 1))) (λy→(
    y ∧ FALSE)))
```

Discriminated unions

$$\boxed{\texttt{((((}\lambda\texttt{u}\rightarrow\ \texttt{(}\lambda\texttt{f g}\rightarrow\texttt{((u f) g))) ((}\lambda\texttt{x}\rightarrow\ \texttt{(}\lambda\texttt{f g}\rightarrow\texttt{(f x))) 1)) (}\lambda\texttt{x}\rightarrow\texttt{(x + 1))) (}\lambda\texttt{y}\rightarrow\texttt{(y}\wedge\texttt{ FALSE)))}}$$

Discriminated unions

$$((((\lambda u \rightarrow (\lambda f\ g \rightarrow ((u\ f)\ g)))\ ((\lambda x \rightarrow (\lambda f\ g \rightarrow (f\ x)))\ 1))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \mathrm{FALSE})))$$

$$((((\lambda u \rightarrow (\lambda f\ g \rightarrow ((u\ f)\ g)))\ ((\lambda x \rightarrow (\lambda f\ g \rightarrow (f\ x)))\ 1))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \mathrm{FALSE})))$$

Discriminated unions


```
((((λu→ (λf g→((u f) g)))
     ((λx→ (λf g→(f x))) 1)) (λx→(x + 1))) (λy→
     (y ∧ FALSE)))
```

Discriminated unions


```
((((λu→ (λf g→((u f) g)))
    ((λx→ (λf g→(f x))) 1) ) (λx→(x + 1))) (λy→
    (y ∧ FALSE)))
```

```
((((λu→ (λf g→((u f) g))) (λf g→(f 1 ))) (λx
    →(x + 1))) (λy→(y ∧ FALSE)))
```

Discriminated unions

$$\boxed{\texttt{((((}\lambda\texttt{u}\rightarrow\texttt{ (}\lambda\texttt{f g}\rightarrow\texttt{((u f) g))) (}\lambda\texttt{f g}\rightarrow\texttt{(f 1))) (}\lambda\texttt{x}\rightarrow\texttt{(x + 1))) (}\lambda\texttt{y}\rightarrow\texttt{(y }\wedge\texttt{ FALSE)))}}$$

Discriminated unions


```
((((λu→ (λf g→((u f) g))) (λf g→(f 1))) (λx→
    (x + 1))) (λy→(y ∧ FALSE)))
```

```
((((λu→ (λf g→((u f) g))) (λf g→(f 1))) (λx→(x +
    1))) (λy→(y ∧ FALSE)))
```

Discriminated unions

$$((\ (((\lambda u \rightarrow\ (\lambda f\ g \rightarrow ((u\ f)\ g)))\ (\lambda f\ g \rightarrow (f\ 1)))\ (\lambda x \rightarrow (x\ +\ 1)))\ (\lambda y \rightarrow (y\ \wedge\ \mathtt{FALSE})))$$

Discriminated unions


```
(( ((λu→ (λf g→((u f) g))) (λf g→(f 1))) (λx→(x +
    1))) (λy→(y ∧ FALSE)))
```

```
((((λf g→(( (λf g→(f 1)) f) g)) (λx→(x + 1))) (
   λy→(y ∧ FALSE)))
```

Discriminated unions

```
(((λf g→(((λf g→(f 1)) f) g)) (λx→(x + 1)))
    (λy→(y ∧ FALSE)))
```

Discriminated unions


```
(((λf g→(((λf g→(f 1)) f) g)) (λx→(x + 1)))
    (λy→(y ∧ FALSE)))
```

```
( ((λf g→(((λf g→(f 1)) f) g)) (λx→(x + 1))) (λy→
    (y ∧ FALSE)))
```

Discriminated unions

$$(((\lambda f\ g \rightarrow (((\lambda f\ g \rightarrow (f\ 1))\ f)\ g))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))$$

Discriminated unions

$$(((\lambda f\ g \rightarrow (((\lambda f\ g \rightarrow (f\ 1))\ f)\ g))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))$$

$$((\lambda g \rightarrow (((\lambda f\ g \rightarrow (f\ 1))\ (\lambda x \rightarrow (x + 1)))\ g))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))$$

Discriminated unions

```
((λg→(((λf g→(f 1)) (λx→(x + 1))) g)) (λy→(
    y ∧ FALSE)))
```

Discriminated unions


```
((λg→(((λf g→(f 1)) (λx→(x + 1))) g)) (λy→(
    y ∧ FALSE)))
```

```
((λg→(((λf g→(f 1)) (λx→(x + 1))) g)) (λy→(y ∧ FALSE)
```

Discriminated unions

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$$((\lambda\mathtt{g}\rightarrow(((\lambda\mathtt{f}\ \ \mathtt{g}\rightarrow(\mathtt{f}\ \ 1))\ \ (\lambda\mathtt{x}\rightarrow(\mathtt{x}\ +\ 1)))\ \ \mathtt{g}))\ \ (\lambda\mathtt{y}\rightarrow(\mathtt{y}\ \wedge\ \mathtt{FALSE})$$

Discriminated unions


```
((λg→(((λf g→(f 1)) (λx→(x + 1))) g)) (λy→(y ∧ FALSE)
```

```
(((λf g→(f 1)) (λx→(x + 1)))
    (λy→(y ∧ FALSE)) )
```

Discriminated unions

$$(((\lambda \mathtt{f}\ \mathtt{g} \rightarrow (\mathtt{f}\ 1))\ (\lambda \mathtt{x} \rightarrow (\mathtt{x} + 1)))\ (\lambda \mathtt{y} \rightarrow (\mathtt{y} \wedge \mathtt{FALSE})))$$

Discriminated unions

```
(((λf g→(f 1)) (λx→(x + 1))) (λy→(y ∧ FALSE)
    ))
```

```
( ((λf g→(f 1)) (λx→(x + 1))) (λy→(y ∧ FALSE)))
```

Discriminated unions

$$(((\lambda f\ g \rightarrow (f\ 1))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))$$

Discriminated unions

$$\texttt{(} \colorbox{orange}{$\texttt{((}\lambda\texttt{f g}\rightarrow\texttt{(f 1)) (}\lambda\texttt{x}\rightarrow\texttt{(x} + \texttt{1)))}$} \texttt{ (}\lambda\texttt{y}\rightarrow\texttt{(y} \wedge \texttt{FALSE)))}$$

$$\texttt{((}\lambda\texttt{g}\rightarrow\texttt{(} \colorbox{yellow}{$\texttt{(}\lambda\texttt{x}\rightarrow\texttt{(x} + \texttt{1))}$} \texttt{ 1)) (}\lambda\texttt{y}\rightarrow\texttt{(y} \wedge \texttt{FALSE)))}$$

Discriminated unions

$$\boxed{((\lambda g \rightarrow ((\lambda x \rightarrow (x + 1))\ 1))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))}$$

Discriminated unions

$$((\lambda g \rightarrow ((\lambda x \rightarrow (x + 1))\ 1))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))$$

$$((\lambda g \rightarrow ((\lambda x \rightarrow (x + 1))\ 1))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))$$

Discriminated unions

$$((\lambda g \rightarrow ((\lambda x \rightarrow (x + 1))\ 1))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))$$

Discriminated unions

$((\lambda g \rightarrow ((\lambda x \rightarrow (x + 1))\ 1))\ (\lambda y \rightarrow (y \wedge \text{FALSE})))$

$((\lambda x \rightarrow (x + 1))\ 1)$

Discriminated unions


```
((λx→(x + 1)) 1)
```

Discriminated unions

$$((\lambda x \rightarrow (x + 1))\ 1)$$

$$((\lambda x \rightarrow (x + 1))\ 1)$$

Discriminated unions

$((\lambda x \rightarrow (x + 1))\ 1)$

Discriminated unions


```
((λx→(x + 1)) 1)
```

```
( 1 + 1)
```

Discriminated unions

```
(1 + 1)
```

Discriminated unions


```
(1 + 1)
```

```
(1 + 1)
```

Discriminated unions


```
(1 + 1)
```

Discriminated unions


```
(1 + 1)
```

```
2
```

Discriminated unions

Choice between a pair of values

We should expect that `inl` and `inr` are inverse operations to `match`

Discriminated unions

```
(((match (inl 1)) inl) inr)
```

Discriminated unions


```
(((match (inl 1)) inl) inr)
```

```
(( (match (inl 1)) inl) inr)
```

Discriminated unions


```
(( (match (inl 1)) inl) inr)
```

Discriminated unions


```
(( (match (inl 1)) inl) inr)
```

```
(inl 1 )
```

Discriminated unions

```
(((match (inr TRUE)) inl) inr)
```

Discriminated unions


```
(((match (inr TRUE)) inl) inr)
```

```
(( (match (inr TRUE)) inl) inr)
```

Discriminated unions


```
(( (match (inr TRUE)) inl) inr)
```

Discriminated unions

```
(( (match (inr TRUE)) inl) inr)
```

```
(inr TRUE )
```

Conclusion

Recap

- Lambda terms can be used to encode arbitrary basic data types
- The terms are always lambda expression which, when they get parameters passed in, identify themselves somehow
- Identification can be done by applying something (possibly even a given number of times), or returning one of the parameters

Conclusion

Recap

- The data types we have seen cover an impressive range of applications
- Tuples cover grouping data together (like the fields of a class)
- Unions cover choosing different things (like the polymorphism of an interface that might be implemented by various concrete classes)
- Combining these two covers all possible programming needs, even for more complex data structures

This is it!

The best of luck, and thanks for the attention!