

INFSEN02-1 Sample exam

The INFDEV@HR Team

1 Exam

1.1 Question 1

Given the following lambda program, and a series of relevant delta rules, show the beta reductions for this program.

(TRUE \wedge FALSE)

1.2 Relevant delta rules

Boolean and:

($\lambda p \ q \rightarrow ((p \ q) \ p)$)

True

($\lambda t \ f \rightarrow t$)

False

($\lambda t \ f \rightarrow f$)

1.3 Answer 1 (note: you do not need to write all this detail yourself, it is only included for completeness)

(TRUE \wedge FALSE)

(($\underline{\wedge}$ TRUE) FALSE)

((($\lambda p \ q \rightarrow ((p \ q) \ p)$) TRUE) FALSE)

((($\lambda p \ q \rightarrow ((p \ q) \ p)$) TRUE) FALSE)

((($\lambda p \ q \rightarrow ((p \ q) \ p)$) ($\lambda t \ f \rightarrow t$)) FALSE)

$((\lambda p \ q \rightarrow ((p \ q) \ p)) \ (\lambda t \ f \rightarrow t)) \ \underline{\text{FALSE}}$

$((\lambda p \ q \rightarrow ((p \ q) \ p)) \ (\lambda t \ f \rightarrow t)) \ (\lambda t \ f \rightarrow f)$

$((\lambda p \ q \rightarrow ((p \ q) \ p)) \ (\lambda t \ f \rightarrow t)) \ (\lambda t \ f \rightarrow f)$

$((\lambda q \rightarrow ((\lambda t \ f \rightarrow t) \ q) \ (\lambda t \ f \rightarrow t)) \ (\lambda t \ f \rightarrow f))$

$((\lambda q \rightarrow (((\lambda t \ f \rightarrow t) \ q) \ (\lambda t \ f \rightarrow t))) \ (\lambda t \ f \rightarrow f))$

$((\lambda t \ f \rightarrow t) \ (\lambda t \ f \rightarrow f)) \ (\lambda t \ f \rightarrow t)$

$((\lambda t \ f \rightarrow t) \ (\lambda t \ f \rightarrow f)) \ (\lambda t \ f \rightarrow t)$

$((\lambda f \ t \ f \rightarrow f) \ (\lambda t \ f \rightarrow t))$

$((\lambda f \ t \ f \rightarrow f) \ (\lambda t \ f \rightarrow t))$

$(\lambda t \ f \rightarrow f)$

$(\lambda t \ f \rightarrow f)$

FALSE

1.4 Question 2

Given the following lambda calculus program, and a series of relevant delta rules, give the full typing derivation for the program.

$(\lambda(p:\text{Boolean}) \ (q:\text{Boolean}) \rightarrow (((p \ \text{Boolean}) \ q) \ p))$

1.5 Relevant delta rules

Boolean type:

$(\forall \alpha \Rightarrow (\alpha \rightarrow \alpha \rightarrow \alpha))$

1.6 Answer 2 (note: you do not need to write all this detail yourself, it is only included for completeness)

$$(\lambda(p:\text{Boolean}) \ (q:\text{Boolean}) \rightarrow ((p \ \text{Boolean}) \ q) \ p))$$
$$(\lambda(p:\text{Boolean})\ (q:\text{Boolean}) \rightarrow ((p \ \text{Boolean})\ q)\ p))$$

```
(λ(p:Boolean) (q:Boolean)→((Boolean Boolean) q) Boolean))
```

$$(\lambda(p:\text{Boolean})(q:\text{Boolean}) \rightarrow ((\text{Boolean Boolean})\ q)\ \text{Boolean}))$$

```
(λ(p:Boolean) (q:Boolean)→(((Boolean Boolean) Boolean) Boolean))
```

```
(λ(p:Boolean) (q:Boolean)→(((Boolean Boolean) Boolean)
  Boolean))
```

```
(λ(p:Boolean) (q:Boolean)→((( (∀α ⇒(α→α→α)) Boolean)
Boolean) Boolean))
```

$$(\lambda(p:\text{Boolean})\ (q:\text{Boolean})\rightarrow(((\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))\ \underline{\text{Boolean}})\ \text{Boolean})$$

```
(λ(p:Boolean) (q:Boolean)→(((∀α ⇒(α→α→α))
(∀α ⇒(α→α→α))) Boolean) Boolean))
```

$$(\lambda(p:\text{Boolean})\ (q:\text{Boolean}) \rightarrow ((\underbrace{((\forall \alpha \Rightarrow (\alpha \rightarrow \alpha \rightarrow \alpha))\ (\forall \alpha \Rightarrow (\alpha \rightarrow \alpha \rightarrow \alpha)))}_{\text{Boolean}})\ \text{Boolean})))$$

```
(λ(p:Boolean) (q:Boolean)→((
  ((∀α ⇒(α→α→α))→(∀α ⇒(α→α→α))→(∀α ⇒(α→α→α)))
) Boolean)) Boolean
```

$$(\lambda(p:\text{Boolean})\ (q:\text{Boolean})\rightarrow(((\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))\rightarrow(\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))\rightarrow(\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))))\ \text{Boolean}\ \text{Boolean}))$$
$$(\lambda(p:\text{Boolean})\ (\lambda(q:\text{Boolean})\rightarrow(((\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))\rightarrow(\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))\rightarrow(\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))))\ (\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha)))\ \text{Boolean}))$$
$$(\lambda(p:\text{Boolean})\ (q:\text{Boolean})\rightarrow(((\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))\rightarrow(\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha))\rightarrow(\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha)))\ (\forall\alpha\Rightarrow(\alpha\rightarrow\alpha\rightarrow\alpha)))\ \text{Boolean}))$$

```
(λ(p:Boolean) (q:Boolean)→(  
  ((∀α ⇒(α→α→α))→(∀α ⇒(α→α→α))) Boolean))
```

```
(λ(p:Boolean) (q:Boolean)→(((∀α ⇒(α→α→α))→(∀α ⇒(α→α→α))  
  ) Boolean))
```

```
(λ(p:Boolean) (q:Boolean)→(((∀α ⇒(α→α→α))→(∀α ⇒(α→α→α))  
  ) (∀α ⇒(α→α→α))) )
```

```
(λ(p:Boolean) (q:Boolean)→(((∀α ⇒(α→α→α))→(∀α ⇒(α→α→α)))  
  (∀α ⇒(α→α→α))))
```

```
(λ(p:Boolean) (q:Boolean)→ (∀α ⇒(α→α→α)) )
```

```
(λ(p:Boolean) (q:Boolean)→(∀α ⇒(α→α→α)))
```

```
(λ(p:Boolean) (q:Boolean)→ Boolean )
```

```
(λ(p:Boolean) (q:Boolean)→Boolean)
```

```
(λ(p:Boolean)→ (Boolean→Boolean) )
```

```
(λ(p:Boolean)→(Boolean→Boolean))
```

```
(Boolean→Boolean→Boolean)
```