

Recursion and F# translations

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# Recursion and F# translations

The INFDEV@HR Team

Hogeschool Rotterdam Rotterdam, Netherlands



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# Introduction



# Introduction

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# Lecture topics

- Recursion (let-rec)
- F# translations of lambda programs so far



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# Recursive functions and let-rec



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#### ldea

- The lambda calculus has no while loops
- This means that we need to emulate them with recursion



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#### Idea

- This is a bit of an issue
- A function is just a lambda term, which does not have a name
- If the function does not have a name, how do we call it from its own body?



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#### Idea

- We can define a recursive function as a function with an extra parameter
- Calling the extra parameter will result in calling the function itself



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Practicum begins now For example, the factorial function becomes

(
$$\lambda f \ n \rightarrow if \ (n = 0)$$
 then 1 else ((f (n - 1))  $\times$  n))



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- We now need an external operator that handles recursive functions properly
- This must ensure that a recursive function gets itself as a parameter, in an endless chain
- This combinator is known as fixpoint operator

$$(\lambda f \rightarrow ((\lambda x \rightarrow (f (x x))) (\lambda x \rightarrow (f (x x)))))$$



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```
((fix (\lambdaf n	oif (n = 0) then 1 else (f (n - 1) ))) 2)
```



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```
((fix (\lambdaf n\rightarrowif (n = 0) then 1 else (f (n - 1)))) 2)
```

```
( (fix (\lambdaf n\rightarrowif (n = 0) then 1 else (f (n - 1)))) 2)
```



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```
( (fix (\lambdaf n\rightarrowif (n = 0) then 1 else (f (n - 1)))) 2)
```



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```
(fix (\lambdaf n\rightarrowif (n = 0) then 1 else (f (n - 1)))) 2)
```

```
((\lambda n \rightarrow if (n = 0) then 1 else (... (n - 1)))
2)
```



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) 2)



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) 2)

((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) 2)



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((
$$\lambda n \rightarrow \text{if } (n = 0) \text{ then } 1 \text{ else } (\dots (n - 1)))$$
 2)



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) 2)

if 
$$(2 = 0)$$
 then 1 else  $(... (2 - 1))$ 



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if 
$$(2 = 0)$$
 then 1 else  $(... (2 - 1))$ 



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if 
$$(2 = 0)$$
 then 1 else  $(... (2 - 1))$ 

if 
$$(2 = 0)$$
 then 1 else  $(... (2 - 1))$ 



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Practicum begins now if (2 = 0) then 1 else (... (2 - 1))



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if 
$$(2 = 0)$$
 then 1 else (...  $(2 - 1)$ )

```
if FALSE then 1 else (... (2-1))
```



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```
if FALSE then 1 else (... (2-1))
```



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```
if FALSE then 1 else (... (2-1))
```

```
if FALSE then 1 else (\ldots (2-1))
```



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Practicum begins now if FALSE then 1 else (... (2-1))



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if FALSE then 1 else (... 
$$(2-1)$$
)

$$(... (2-1))$$



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$$(...(2-1))$$



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$$(... (2 - 1))$$

( (fix (
$$\lambda$$
f n $\rightarrow$ if (n = 0) then 1 else (f (n - 1)))) (2 - 1))



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```
( (fix (\lambdaf n\rightarrowif (n = 0) then 1 else (f (n - 1)))) (2 - 1))
```



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(fix (
$$\lambda$$
f n $\rightarrow$ if (n = 0) then 1 else (f (n - 1)))) (2 - 1))

```
((\lambdan\rightarrowif (n = 0) then 1 else (... (n - 1))) (2 - 1))
```



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) (2 - 1))



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) (2 - 1))

$$((\lambda ext{n} o ext{if (n = 0) then 1 else (... (n - 1)))}$$



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```
((\lambda \mathtt{n} \! \! \rightarrow \! \mathtt{if} \ (\mathtt{n} = \mathtt{0}) \ \mathtt{then} \ \mathtt{1} \ \mathtt{else} \ (\ldots \ (\mathtt{n} - \mathtt{1})))
```



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$$((\lambda \mathtt{n} \! \! \rightarrow \! \mathtt{if} \ (\mathtt{n} = \mathtt{0}) \ \mathtt{then} \ \mathtt{1} \ \mathtt{else} \ (\ldots \ (\mathtt{n} - \mathtt{1})))$$

((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1)))



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```
((\lambdan
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```



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) 1)

((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) 1)



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Practicum begins now (( $\lambda$ n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) 1)



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) 1)

if 
$$(1 = 0)$$
 then 1 else  $(... (1 - 1))$ 



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if 
$$(1 = 0)$$
 then 1 else  $(... (1 - 1))$ 



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if 
$$(1 = 0)$$
 then 1 else  $(... (1 - 1))$ 

if 
$$(1 = 0)$$
 then 1 else (...  $(1 - 1)$ )



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```
if (1 = 0) then 1 else (... (1 - 1))
```



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```
if (1 = 0) then 1 else (... (1 - 1))
```

```
if FALSE then 1 else (... (1 - 1))
```



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```
if FALSE then 1 else (... (1-1))
```



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```
if FALSE then 1 else (... (1-1))
```

```
if FALSE then 1 else (\dots (1-1))
```



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$$(... (1-1))$$



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$$(...(1-1))$$



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$$(...(1-1))$$

( (fix (
$$\lambda$$
f n $\rightarrow$ if (n = 0) then 1 else (f (n - 1)))) (1 - 1))



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```
( (fix (\lambdaf n\rightarrowif (n = 0) then 1 else (f (n - 1)))) (1 - 1))
```



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(fix (
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f n $\rightarrow$ if (n = 0) then 1 else (f (n - 1)))) (1 - 1))

```
((\lambdan\rightarrowif (n = 0) then 1 else (... (n - 1))) (1 - 1))
```



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((
$$\lambda$$
n $\to$ if (n = 0) then 1 else (... (n - 1))) (1 - 1))



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) (1 - 1))

$$((\lambda n \rightarrow \text{if (n = 0) then 1 else (... (n - 1))})$$



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```
((\lambda n \rightarrow \text{if } (n=0) \text{ then 1 else } (\dots (n-1)))
```



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$$((\lambda \mathtt{n} \! \! \rightarrow \! \mathtt{if} \ (\mathtt{n} = \mathtt{0}) \ \mathtt{then} \ \mathtt{1} \ \mathtt{else} \ (\ldots \ (\mathtt{n} - \mathtt{1})))$$

((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1)))



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) 0)



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) 0)

((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) 0)



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Practicum begins now (( $\lambda n \rightarrow \text{if } (n = 0) \text{ then 1 else } (\dots (n - 1))) 0$ )



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) 0)

if 
$$(0 = 0)$$
 then 1 else  $(... (0 - 1))$ 



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if 
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if 
$$(0 = 0)$$
 then 1 else  $(... (0 - 1))$ 

if 
$$(0 = 0)$$
 then 1 else (...  $(0 - 1)$ )



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if 
$$(0 = 0)$$
 then 1 else (...  $(0 - 1)$ )

```
if TRUE then 1 else (... (0-1))
```



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```
if TRUE then 1 else (... (0-1))
```



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```
if TRUE then 1 else (... (0-1))
```

```
if TRUE then 1 else (\dots (0-1))
```



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Practicum begins now if TRUE then 1 else (... (0-1))



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Practicum begins now if TRUE then 1 else (... (0-1))

1



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Practicum begins now We can now try our hand at a factorial computation

```
((fix (\lambdaf n\rightarrowif (n = 0) then 1 else ((f (n - 1)) \times n))) 2)
```



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```
((fix (\lambdaf n\rightarrowif (n = 0) then 1 else ((f (n - 1)) 	imes n))) 2)
```



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```
((fix (\lambdaf n\rightarrowif (n = 0) then 1 else ((f (n - 1)) \times n))) 2)
```

```
(fix (\lambdaf n\toif (n = 0) then 1 else ((f (n - 1)) 	imes n) 2)
```



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```
(fix (\lambdaf n	oif (n = 0) then 1 else ((f (n - 1)) 	imes n) 2)
```

```
((\lambdan\rightarrowif (n = 0) then 1 else ((\dots (n - 1)) \times n)) 2)
```



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```
((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) 2)
```



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else ((... (n - 1))  $imes$  n)) 2)

((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else ((... (n - 1))  $imes$  n)) 2)



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((
$$\lambda n \rightarrow \text{if } (n = 0) \text{ then 1 else } ((\dots (n - 1)) \times n))$$
 2)



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((
$$\lambda n{
ightarrow} if$$
 (n = 0) then 1 else ((... (n - 1))  $imes$  n)) 2)

if 
$$(2 = 0)$$
 then 1 else  $((... (2 - 1)) \times 2)$ 



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if (2 = 0) then 1 else ((... 
$$(2 - 1)) \times 2$$
)



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if 
$$(2 = 0)$$
 then 1 else  $((... (2 - 1)) \times 2)$ 

if 
$$(2 = 0)$$
 then 1 else  $((... (2 - 1)) \times 2)$ 



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if 
$$(2 = 0)$$
 then 1 else  $((... (2 - 1)) \times 2)$ 

```
if FALSE then 1 else ((... (2-1)) \times 2)
```



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```
if FALSE then 1 else ((... (2-1)) \times 2)
```



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```
if FALSE then 1 else ((... (2 - 1)) \times 2)
```

```
if FALSE then 1 else ((... (2-1)) \times 2)
```



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if FALSE then 1 else ((... 
$$(2-1)) \times 2$$
)

$$((... (2-1)) \times 2)$$



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$$((... (2 - 1)) \times 2)$$



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```
((... (2 - 1)) \times 2)
```

```
((
\frac{(\text{fix } (\lambda \text{f n} \rightarrow \text{if } (\text{n} = 0) \text{ then 1 else } ((\text{f } (\text{n} - 1)) \times \text{n})}{(2 - 1)) \times 2)}
```



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```
((    (fix (\lambdaf n\toif (n = 0) then 1 else ((f (n - 1)) \times n) (2 - 1)) \times 2)
```



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```
(((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) (2 - 1)) \times 2)
```



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```
(((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) (2 - 1)) \times 2)
```



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(((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else ((... (n - 1))  $\times$  n)) (2 - 1))  $\times$  2)



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```
(((\lambda n \rightarrow \text{if } (n = 0) \text{ then 1 else } ((\dots (n - 1)) \times n)) \times 2)
```



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```
(((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) (2 - 1) \times 2)
```

```
(((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) 	imes n)) \frac{1}{1}) 	imes 2)
```



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```
(((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) 1) \times 2)
```



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```
(((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) 1) \times 2)
```



```
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```

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```
((\lambdan\toif (n = 0) then 1 else ((... (n - 1)) \times n)) 1 \times 2)
```



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```
(if (\frac{1}{2} = 0) then 1 else ((... (\frac{1}{2} - 1)) \times \frac{1}{2}) \times 2)
```



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```
(if (1 = 0) then 1 else ((... (1 - 1)) \times 1) \times 2)
```



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(if 
$$(1 = 0)$$
 then 1 else  $((... (1 - 1)) \times 1) \times 2)$ 

(if 
$$(1 = 0)$$
 then 1 else ((...  $(1 - 1)) \times 1) \times 2$ 



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```
(if (1 = 0) then 1 else ((... (1 - 1)) \times 1) \times 2
```



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```
(if (1=0) then 1 else ((... (1-1)) \times 1) \times 2)
```

```
(if FALSE then 1 else ((... (1-1)) \times 1) \times 2)
```



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```
(if FALSE then 1 else ((... (1 - 1)) \times 1) \times 2)
```



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```
(if FALSE then 1 else ((... (1 - 1)) 	imes 1) 	imes
   2)
```

```
(if FALSE then 1 else ((... (1-1)) \times 1) \times 2)
```



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```
(if FALSE then 1 else ((... (1-1)) \times 1) \times 2)
```



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(if FALSE then 1 else ((... 
$$(1-1)) \times 1) \times 2$$
)

$$(((... (1-1)) \times 1) \times 2)$$



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$$(((... (1 - 1)) \times 1) \times 2)$$



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```
(((... (1 - 1)) \times 1) \times 2)
```

```
(((

(fix (\lambdaf n\rightarrowif (n = 0) then 1 else ((f (n - 1)) \times n)

(1 - 1)) \times 1) \times 2)
```



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```
((((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) (1 - 1)) \times 1) \times 2)
```



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```
((((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) (1 - 1)) \times 1) \times 2)
```



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((((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else ((... (n - 1))  $\times$  n)) (1 - 1))  $\times$  1)  $\times$  2)

```
((((\lambda n \rightarrow \text{if } (n = 0) \text{ then } 1 \text{ else } ((\dots (n - 1)) \\ \times n)) (1 - 1)) \times 1) \times 2)
```



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```
(((((\lambda n \rightarrow if (n = 0) then 1 else ((... (n - 1)) \times n)) (1 - 1) \times 1) \times 2)
```



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$$(((((\lambda n \rightarrow \text{if } (n = 0) \text{ then } 1 \text{ else } ((\dots (n-1)) \times n))$$

```
((((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) \frac{0}{0}) \times 1) \times 2)
```



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```
((((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) 0) \times 1) \times 2)
```



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```
(((((\lambda n \rightarrow \text{if } (n=0) \text{ then 1 else } ((\dots (n-1)) \times n)) 0) \times 1) \times 2)
```



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```
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```

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```
((\lambda n \rightarrow if (n = 0) then 1 else ((... (n - 1)) \times n)) 0 \times 1) \times 2)
```

```
((if (0 = 0) then 1 else ((... (0 - 1)) × 0) × 1) × 2)
```



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```
((if (0 = 0) then 1 else ((... (0 - 1)) \times 0) \times 1) \times 2)
```



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((if (0 = 0) then 1 else ((... (0 - 1)) 
$$\times$$
 0)  $\times$  1)  $\times$  2)

((if 
$$(0 = 0)$$
 then 1 else ((...  $(0 - 1)) \times 0$ )  $\times 1) \times 2$ )



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```
((if (0 = 0) then 1 else ((... (0 - 1)) \times 0) \times 1) \times 2
```



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((if 
$$(0 = 0)$$
 then 1 else ((...  $(0 - 1)) \times 0$ )  $\times$  1)  $\times$  2)

```
((if TRUE then 1 else ((... (0-1)) \times 0) \times 1) \times 2)
```



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```
((if TRUE then 1 else ((... (0-1)) \times 0) \times 1) \times 2)
```



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```
((if TRUE then 1 else ((... (0 - 1)) \times 0) \times 1) \times 2)
```

```
((if TRUE then 1 else ((... (0-1)) \times 0) \times 1) \times 2)
```



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```
(( if TRUE then 1 else ((... (0-1)) \times 0) \times 1) \times 2)
```



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(( if TRUE then 1 else ((... 
$$(0-1)) \times 0$$
)  $\times$  1)  $\times$  2)

$$((1 \times 1) \times 2)$$



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$$((1 \times 1) \times 2)$$



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((1 
$$\times$$
 1)  $\times$  2)

$$(\begin{array}{c|c} (1 \times 1) \times 2)$$



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Practicum begins now  $((1 \times 1) \times 2)$ 



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$$(1 \times 1) \times 2$$

 $(1 \times 2)$ 



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Practicum begins now  $(1 \times 2)$ 



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Practicum begins now  $(1 \times 2)$ 

 $(1 \times 2)$ 



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Practicum begins now (1 × 2)



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(1 × 2)

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# Translating to F#



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#### Overview

- Each and every one of the constructs we have seen so far has a direct translation in F#<sup>a</sup>
- All constructs have exactly the same behaviour as in the lambda calculus, but with a slightly less mathematical syntax for ASCII keyboards
- A few operators are menemonically friendlier or just plain more readable, but the essence remains exactly the same
- F# is indentation-sensitive, like Python: pay a lot of attention to how terms are indented!

<sup>a</sup>Haskell is a bit different, so we leave it for later

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Practicum begins now Integers, booleans, floats, strings have the usual meaning, both in the lambda calculus, F#, and the languages you are used to:

$$((2 + 3) - 4)$$



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- Conditionals behave just like in the lambda calculus
- This means that they return the evaluation of either of the two branches
- This differs from imperative languages, where we just jump into either of the two branches

```
if (true && false) then
else
```



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Conclusion

Practicum begins now Functions look very similar, with fun instead of  $\lambda$ 

$$(fun x f \rightarrow (f x))$$



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Conclusion

Practicum begins now Functions look very similar, with fun instead of  $\lambda$ 

$$(fun x f \rightarrow (f x))$$

Just like function application

$$(((fun x f \rightarrow (f x)) 3) (fun x \rightarrow (3 + x)))$$



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Practicum begins now We can give names to functions, and code becomes much prettier as a result

```
let apply =
  fun x f -> (f x)
((apply 3) (fun x -> (3 + x)))
```



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Practicum begins now We can also give names to recursive functions by using let rec instead of let, and code becomes much more readable than with fix

```
let rec fact =
  fun n ->
  if (n = 0) then
    1
  else
    ((fact (n - 1)) * n)
```



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Practicum begins now We can define tuples by just putting a comma between the values, with or without nesting for more than two values is done for us

(1, true)



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#### **Pairs**

- Nested tuples are the same as in the lambda calculus, and of course they also work in F#
- They are, in essence, pairs of pairs, such as: (1,(2,(3,4) or (((1,2),3),4), which are not the same
- Non-nested tuples are a slight extension for practical ease of work in F#
- They are, in essence, tuples with as many elements as we want, such as: (1,2,3,4)
- All the examples in this slide are valid in F#, but are not identical objects



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- Functions such as  $\pi_1$  and  $pi_2$ , which both extract one item of a pair, also exist in F#
- They are called, respectively, fst and snd

```
(fst (1, true))
```

```
(snd (1, true))
```



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- Of course F#, just like the lambda calculus, imposes no limitation on composition
- Why not build a tuple of functions, then? Why not indeed!



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Conclusion

Practicum begins now  Of course F#, just like the lambda calculus, imposes no limitation on composition

• Why not build a tuple of functions, then? Why not indeed!

```
((fun x \rightarrow (1 + x)), (fun x \rightarrow (2 * x)))
```



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Conclusion

Practicum begins now  Of course F#, just like the lambda calculus, imposes no limitation on composition

• Why not build a tuple of functions, then? Why not indeed!

```
((fun x \rightarrow (1 + x)), (fun x \rightarrow (2 * x)))
```

Could we also build a tuple of tuples of numbers and functions returning tuples of functions?



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```
    Of course F#, just like the lambda calculus, imposes no
limitation on composition
```

• Why not build a tuple of functions, then? Why not indeed!

```
((fun x \rightarrow (1 + x)), (fun x \rightarrow (2 * x)))
```

Could we also build a tuple of tuples of numbers and functions returning tuples of functions?

Yes, but we are not going to do it :)



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- F# also offers built-in discriminated unions
- Functions such as inl and inr, which both embed one item into the union, also exist in F#
- They are called, respectively, Choice10f2 and Choice20f2

(Choice10f2 1)

(Choice20f2 true)



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Practicum begins now We can, of course, perform matches on discriminated unions

```
match (Choice10f2 1) with
| Choice10f2 x ->
    (Choice10f2 x)
| Choice20f2 y ->
    (Choice20f2 y)
```



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Practicum begins now We can, of course, perform matches on discriminated unions

```
match (Choice10f2 1) with
| Choice10f2 x ->
    (Choice10f2 x)
| Choice20f2 y ->
    (Choice20f2 y)
```

```
let i =
  match (Choice10f2 1) with
  | Choice10f2 x ->
        x
  | Choice20f2 y ->
        0

(i * 2)
```



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Practicum begins now Discriminated unions, and their corresponding matches, can be nested as deep as we need

```
match (Choice10f2 (Choice20f2 true)) with
| Choice10f2 x ->
   match x with
| Choice10f2 x ->
        (Choice10f2 x)
| Choice20f2 y ->
        (Choice20f2 y)
| Choice20f2 y ->
   (Choice20f2 y)
```



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#### Structural equality

- Composable data types make it possible for F# to build a series of useful functions automatically
- These functions allow automatic comparison of data structures with the same shape (for example, tuples with tuples or lists with lists)
- Comparison can work with values made up of primitive expressions, tuples, unions, lists, records, and any composition of them
- With the only exclusion of functions: functions, or composite values containing functions, may not be compared



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#### Interoperability

- One of the practical strengths of F# is its ability to fully interoperate with the .Net framework
- Any feature or library available, even if written in other languages like C#, can be used from inside F#
- Moreover, any F# library can be used from other languages like C#
- This means that applications could be the internal logic in F#, and the UI and database store in C#



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#### Interoperability

- For example, we can call functions such as System.Console.WriteLine, System.Console.ReadLine, System.Int32.Parse, etc.
- This gives a whole other dimension of usefulness to F#
- Ranging from games to web applications, the language knows no intrinsic limitation
- This also comes with full IDE, autocompletion, and debugger support in Visual Studio and a few other IDE's
- Scala is to some extent comparable



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#### Interoperability

A short F# demo might be in order



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#### Forbidden features

- F# is a hybrid language
- It also contains OO features, mutability (variables), interfaces, classes, inheritance, etc.
- You may only use these features in the assignments if there is no alternative to access a library you need
- Mutability may only be used in the main program, which then calls regular functions that only use the lambda calculus derived core
- Failure to comply will result in an automatic insufficient grade



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#### About types

- A major difference between F# and the lambda calculus is that F# is, in fact, a statically typed language
- Even though the language does not require type declarations like Java or C#, the compiler will still ensure that the types make sense
- This means that we will get compiler errors because we mix types
- Code like fst 1 will not compile, for example, because 1 is not a pair and therefore fst cannot be applied to it
- We will cover the type system of F# in the next lecture



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#### Homework

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#### Ways to exercise

- We have added a ton of F# homework, with solutions
  - It is all on GitHub
- It is not mandatory, but it is a good idea to do it until you feel more sure
- You may discuss it during one of the two practicums



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### **Conclusion**



#### Conclusion

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#### Recap

- ullet The lambda calculus can be translated, term to term, into F#
- F# is therefore just a practical lambda calculus with a series of handy extensions and slightly more readable



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# Practicum begins now



#### This is it!

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Practicum begins now The best of luck, and thanks for the attention!