

Delta rules

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The INFDEV@HR Team

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Lecture topics

- Make it pretty: delta rules
- Booleans, boolean logic operators, if-then-else
- Naturals, arithmetic operators, comparison operators

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Introduction

- We can decide that some specific lambda terms have special meanings
- For example, we could decide that a given lambda term means TRUE, another FALSE, etc.
- The important thing is that we choose terms that behave as we wish

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As we wish?

- Suppose we define some lambda terms for TRUE, FALSE, and AND
- We expect these terms to reduce^a following our expectations of boolean logic
- We can use truth tables to encode our expectations

^aThat is, computed according to \rightarrow_β

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We want to formulate TRUE, FALSE, and AND so that

- $\text{TRUE} \wedge \text{TRUE} \rightarrow_{\beta} \text{TRUE}$
- $\text{TRUE} \wedge \text{FALSE} \rightarrow_{\beta} \text{FALSE}$
- $\text{FALSE} \wedge \text{TRUE} \rightarrow_{\beta} \text{FALSE}$
- $\text{FALSE} \wedge \text{FALSE} \rightarrow_{\beta} \text{FALSE}$

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Choice terms

- Terms with special meaning essentially make a choice when given parameters
- The choice is expressed by either returning, or applying, the parameters

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- We wish to use special symbols to these terms with special meaning
- We define a series of delta rules, which are transformation from pretty symbols into lambda terms (and vice-versa)

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This means that we will be able to write lambda programs such as $5+3$, that will then be translated into the appropriate lambda terms

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Idea

- Boolean operators such as TRUE and FALSE must be defined so as to identify themselves
- The choice is expressed by returning their identity from a choice of two options

TRUE is defined as a selector of the representative for true, that is the first argument^a

^aby arbitrary convention

$$(\lambda t \ f \rightarrow t)$$

FALSE is defined as a selector of the representative for false, that is the second argument^a

^aby arbitrary convention, as long as different from the previous

$$(\lambda t \ f \rightarrow f)$$

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```
((TRUE bit1) bit0)
```

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```
((TRUE bit1) bit0)
```

```
((TRUE bit1) bit0)
```


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```
((TRUE bit1) bit0)
```

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```
((TRUE bit1) bit0)
```

```
(( $\lambda t f \rightarrow t$ ) bit1) bit0)
```

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```
((λt f→t) bit1) bit0)
```

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```
((λt f→t) bit1) bit0)
```

```
((λt f→t) bit1) bit0)
```

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```
(((λt f→t) bit1) bit0)
```

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```
(((λt f→t) bit1) bit0)
```

```
((λf→ bit1) bit0)
```

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```
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```

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```
((λf→bit1) bit0)
```

```
((λf→bit1) bit0)
```


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```
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```
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```

```
bit1
```

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AND

- The conjunction^a of two terms is a function that takes as input two booleans and returns a boolean
- Since we just defined booleans to be two-parameter functions, we know that the two input booleans can be applied to each other
- Given two booleans p and q , their conjunction is q if p was true, or false otherwise

$$(\lambda p \ q \rightarrow ((p \ q) \ p))$$

^aAND, or \wedge

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AND

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$$(TRUE \wedge TRUE)$$

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$$(\text{TRUE} \wedge \text{TRUE})$$
$$((\wedge \text{TRUE}) \text{ TRUE})$$

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$$((\underline{\wedge} \text{ TRUE}) \text{ TRUE})$$

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$$((\underline{\wedge} \text{ TRUE}) \text{ TRUE})$$
$$(((\lambda p \ q \rightarrow ((p \ q) \ p)) \text{ TRUE}) \text{ TRUE})$$

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$$(((\lambda p \ q \rightarrow ((p \ q) \ p)) \ \text{TRUE}) \ \text{TRUE})$$

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$$(((\lambda p \ q \rightarrow ((p \ q) \ p)) \ \text{TRUE}) \ \text{TRUE})$$
$$(((\lambda p \ q \rightarrow ((p \ q) \ p)) \ \underline{\text{TRUE}}) \ \text{TRUE})$$

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$$(((\lambda p \ q \rightarrow ((p \ q) \ p))) \underline{\text{TRUE}}) \text{ TRUE})$$
$$(((\lambda p \ q \rightarrow ((p \ q) \ p))) (\lambda t \ f \rightarrow t)) \text{ TRUE})$$

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$$(((\lambda p \ q \rightarrow ((p \ q) \ p))) (\lambda t \ f \rightarrow t)) \text{ TRUE})$$

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$$(((\lambda p \ q \rightarrow ((p \ q) \ p))) (\lambda t \ f \rightarrow t)) \underline{\text{TRUE}})$$

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$$(((\lambda p \ q \rightarrow ((p \ q) \ p))) (\lambda t \ f \rightarrow t)) (\lambda t \ f \rightarrow t))$$

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$$((\underline{\lambda q \rightarrow (((\lambda t \ f \rightarrow t) \ q) (\lambda t \ f \rightarrow t))}) \underline{(\lambda t \ f \rightarrow t)})$$

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It works, but it is probably only because of black magic.

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It works, but it is probably only because of black magic.

Or is it? Let's see if we can get lucky again...

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OR

- The disjunction^a of two terms is a function that takes as input two booleans and returns a boolean
- Like with conjunction, remember that the two input booleans can be applied to one another
- Given two booleans p and q , their disjunction is true if p was true, or q otherwise

$$(\lambda p \ q \rightarrow ((p \ p) \ q))$$

^aOR, or \vee

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OR

Let us begin to with $\text{TRUE} \vee \text{TRUE} \rightarrow_{\beta} \text{TRUE}$

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$$(TRUE \vee TRUE)$$

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$$(\text{TRUE} \vee \text{TRUE})$$
$$((\underline{\vee} \text{ TRUE}) \text{ TRUE})$$

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$$((\underline{\vee} \text{ TRUE}) \text{ TRUE})$$
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$$((\lambda q \rightarrow (((\lambda t \ f \rightarrow t) \ (\lambda t \ f \rightarrow t)) \ q)) \ \underline{(\lambda t \ f \rightarrow t)})$$

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$$((\lambda q \rightarrow (((\lambda t \ f \rightarrow t) \ (\lambda t \ f \rightarrow t)) \ q)) \ (\lambda t \ f \rightarrow t))$$
$$(((\lambda t \ f \rightarrow t) \ (\lambda t \ f \rightarrow t)) \ (\lambda t \ f \rightarrow t))$$

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if-then-else

- The conditional operator `if-then-else` chooses one of two parameters based on the value of the input condition
- Given a boolean `c` and two values `th` and `el`, the result is `th` if `c` was true, or `el` otherwise
- Since `c` is a boolean, it already performs this choice!

```
(λp th el → ((p th) el))
```

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if-then-else

Let us try with if $\text{TRUE} \vee \text{FALSE}$ then A else $B \rightarrow_{\beta} A$

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```
if TRUE then A else B
```

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```
if TRUE then A else B
```

```
((if-then-else TRUE) A) B)
```

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```
((if-then-else TRUE) A) B)
```

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```
(((if-then-else TRUE) A) B)
```

```
((((λp th el→((p th) el)) TRUE) A) B)
```

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$$((((\lambda p \text{ th } e1 \rightarrow ((p \text{ th}) e1)) \underline{\text{TRUE}}) A) B)$$

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```
((((λp th el → ((p th) el)) TRUE) A) B)
```

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$$(((\lambda \text{th } e1 \rightarrow ((\lambda t f \rightarrow t) \text{th}) e1)) A) B)$$

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Idea

- Natural numbers such as 3 and 0 must be defined so as to identify themselves
- Their identity is determined by how many times they perform an action
- The only action we have available is applying a function to a term

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Idea

- We will use unary numbers
- A number is defined by how many times it applies a function to a given term
- Zero applications are also possible, in this case we default to the given term

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0, 1, etc.

A number is defined as an applicator of a term identifying as successor to another term identifying as zero^a

^afirst and second arguments by arbitrary convention

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0 will thus look like

$$(\lambda s \ z \rightarrow z)$$

1 will look like

$$(\lambda s \ z \rightarrow (s \ z))$$

7 will look like

$$(\lambda s \ z \rightarrow (s \ (s \ (s \ (s \ (s \ (s \ (s \ z))))))))))$$

etc.

Addition

- Adding numbers is a function that takes as input two numbers (say m and n), and returns a number
- The first number applies its first parameter m times to its second parameter
- The second number applies its first parameter n times to its second parameter
- We can use the second number as the second parameter to the first, therefore obtaining something that applies $m+n$ times

$$(\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ s) ((n \ s) z))))$$

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Addition

Let us try it out to $2 + 1 \rightarrow_{\beta} 3$

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$$(2 + 1)$$

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$$(2 + 1)$$
$$((\pm 2) 1)$$

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$$((+ \ 2) \ 1)$$

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$$((+ \ 2) \ 1)$$
$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ s) ((n \ s) z)))) \ 2) \ 1)$$

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$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ s) ((n \ s) z)))) \ 2) \ 1)$$

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$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ s) ((n \ s) z)))) \ 2) \ 1)$$
$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ s) ((n \ s) z)))) \ \underline{2}) \ 1)$$

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$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ s) ((n \ s) z)))) \underline{2}) \ 1)$$

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$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ s) ((n \ s) z)))) \underline{2}) \ 1)$$
$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ s) ((n \ s) z)))) \\ (\lambda s \ z \rightarrow (s \ (s \ z)))) \ 1)$$

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$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ s) ((n \ s) z)))) (\lambda s \ z \rightarrow (s \ (s \ z)))) 1)$$

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$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ s) ((n \ s) z)))) (\lambda s \ z \rightarrow (s \ (s \ z)))) \underline{1})$$

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$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ s) ((n \ s) z)))) (\lambda s \ z \rightarrow (s \ (s \ z)))) (\lambda s \ z \rightarrow (s \ z)))$$

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$$((\underline{(\lambda m \rightarrow \ n \rightarrow (\lambda s \ z \rightarrow ((m \ s) ((n \ s) z)))}) \ \underline{(\lambda s \ z \rightarrow (s \ (s \ z)))}) (\lambda s \ z \rightarrow (s \ z)))$$

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$$\frac{((\lambda m \rightarrow \underline{n \rightarrow (\lambda s \ z \rightarrow ((m \ s) ((n \ s) \ z)))}) \ (\lambda s \ z \rightarrow (s \ (s \ z))))}{(\lambda s \ z \rightarrow (s \ z))}$$

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$$\frac{((\lambda m \rightarrow \underline{n} \rightarrow (\lambda s \ z \rightarrow ((m \ s) ((n \ s) \ z)))) \ (\lambda s \ z \rightarrow (s \ (s \ z))))}{(\lambda s \ z \rightarrow (s \ z))}$$
$$\frac{((\lambda n \ s \ z \rightarrow ((\lambda s \ z \rightarrow (s \ (s \ z))) \ s) ((n \ s) \ z))) \ (\lambda s \ z \rightarrow (s \ z))}{(\lambda s \ z \rightarrow (s \ z))}$$

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$$((\lambda n \ s \ z \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ s) \ ((n \ s) \ z))) \ (\lambda s \ z \rightarrow (s \ z)))$$

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$$((\lambda n \ s \ z \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ s) \ ((n \ s) \ z))) \ (\lambda s \ z \rightarrow (s \ z)))$$

$$(\underline{\lambda n \rightarrow} \ \underline{s z \rightarrow} \ \underline{(((\lambda s \ z \rightarrow (s \ (s \ z))) \ s) \ ((n \ s) \ z))})$$

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$$\frac{(\lambda n \rightarrow \underline{s z \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ s) \ ((n \ s) \ z)))}{(\lambda s \ z \rightarrow (s \ z))}$$

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$$\frac{(\lambda n \rightarrow \underline{s z \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ s) \ ((n \ s) \ z)))}{(\lambda s \ z \rightarrow (s \ z))}$$
$$(\lambda s \ z \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ s) \ ((\lambda s \ z \rightarrow (s \ z)) \ s) \ z)))$$

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$$(\lambda s \ z \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ s) \ (((\lambda s \ z \rightarrow (s \ z)) \ s) \ z)))$$

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$$(\lambda s \ z \rightarrow ((\lambda z \rightarrow (\textcolor{yellow}{s} \ (\textcolor{yellow}{s} \ z))) \ (((\lambda s \ z \rightarrow (s \ z)) \ s) \ z)))$$

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$$(\lambda s \ z \rightarrow ((\lambda z \rightarrow (s \ (s \ z))) \ (\underline{((\lambda s \ z \rightarrow (s \ z)) \ s)} \ z)))$$

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$$(\lambda s \ z \rightarrow ((\lambda z \rightarrow (s \ (s \ z))) \ ((\lambda z \rightarrow (\mathbf{s} \ z)) \ z)))$$

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$$(\lambda s \ z \rightarrow ((\lambda z \rightarrow (s \ (s \ z))) \ ((\lambda z \rightarrow (s \ z)) \ z)))$$

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$$(\lambda s \ z \rightarrow ((\lambda z \rightarrow (s \ (s \ z))) \ \underline{((\lambda z \rightarrow (s \ z)) \ z)}))$$

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$$(\lambda s \ z \rightarrow ((\lambda z \rightarrow (s \ (s \ z))) \ (s \ \mathbf{z})))$$

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$$(\lambda s \ z \rightarrow \underline{((\lambda z \rightarrow (s \ (s \ z))) \ (s \ z))})$$

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Multiplication

- Multiplying numbers is a function that takes as input two numbers (say m and n), and returns a number
- The first number applies its first parameter m times to its second parameter
- The second number applies its first parameter n times to its second parameter
- We can use the second number as the first parameter to the first, therefore obtaining something that applies $n + m$ times, starting from z

$$(\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ (n \ s)) \ z)))$$

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Multiplication

Let us try it out to $2 \times 2 \rightarrow_{\beta} 4$

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$$(2 \times 2)$$

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$$(2 \times 2)$$
$$((\underline{\times} \ 2) \ 2)$$

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$$((\underline{\times} \ 2) \ 2)$$

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$$((\underline{x} \ 2) \ 2)$$
$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ (n \ s)) \ z))) \ 2) \ 2)$$

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$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ (n \ s)) \ z))) \ 2) \ 2)$$

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$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ (n \ s)) \ z))) \ \underline{2}) \ 2)$$

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$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ (n \ s)) \ z))) \ \underline{2}) \ 2)$$

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$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ (n \ s)) \ z))) \ \underline{2}) \ 2)$$
$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ (n \ s)) \ z))) \\ (\lambda s \ z \rightarrow (s \ (s \ z))) \) \ 2)$$

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$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ (n \ s)) \ z))) \ (\lambda s \ z \rightarrow (s \ (s \ z)))) \ \underline{2})$$

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$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ (n \ s)) \ z))) \ (\lambda s \ z \rightarrow (s \ (s \ z)))) \ (\lambda s \ z \rightarrow (s \ (s \ z))))$$

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$$(((\lambda m \ n \rightarrow (\lambda s \ z \rightarrow ((m \ (n \ s)) \ z)))) \ (\lambda s \ z \rightarrow (s \ (s \ z)))) \ (\lambda s \ z \rightarrow (s \ (s \ z))))$$

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$$((\underline{(\lambda m \rightarrow \underline{n} \rightarrow (\lambda s \ z \rightarrow ((m \ (n \ s)) \ z)))} \ \underline{(\lambda s \ z \rightarrow (s \ (s \ z)))}) \ (\lambda s \ z \rightarrow (s \ (s \ z))))$$

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$$((\lambda n \ s \ z \rightarrow ((\lambda s \ z \rightarrow (s \ (s \ z))) \ (n \ s)) \ z)) \ (\lambda s \ z \rightarrow (s \ (s \ z))))$$

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$$(\lambda n \rightarrow \underline{s z \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ (n \ s)) \ z)} \\ \underline{(\lambda s \ z \rightarrow (s \ (s \ z)))})$$

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$$\frac{(\lambda n \rightarrow \underline{s z \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ (n \ s))) \ z})}{(\lambda s \ z \rightarrow (s \ (s \ z)))}$$

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$$\frac{(\lambda n \rightarrow \underline{s z \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ (n \ s))) \ z})}{(\lambda s \ z \rightarrow (s \ (s \ z)))}$$
$$(\lambda s \ z \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ ((\lambda s \ z \rightarrow (s \ (s \ z))) \ s) \ z)))$$

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$$(\lambda s \ z \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ ((\lambda s \ z \rightarrow (s \ (s \ z))) \ s \\)) \ z))$$

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$$(\lambda s \ z \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ \underline{((\lambda s \ z \rightarrow (s \ (s \ z))) \ s)}) \\ z))$$

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$$(\lambda s \ z \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ (\lambda z \rightarrow (s \ (s \ z)))) \ z))$$

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$$(\lambda s \ z \rightarrow \underline{(((\lambda s \ z \rightarrow (s \ (s \ z))) \ (\lambda z \rightarrow (s \ (s \ z)))) \ z)})$$

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$$(\lambda s \ z \rightarrow ((\lambda z \rightarrow (\text{(\lambda z \rightarrow (s (s z)))} \ (\text{(\lambda z \rightarrow (s (s z)))} \ z)) \ z))$$

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$$(\lambda s \ z \rightarrow ((\lambda z \rightarrow ((\lambda z \rightarrow (s \ (s \ z))) \ ((\lambda z \rightarrow (s \ (s \ z))) \ z))) \ \underline{z})))$$

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$$(\lambda s \ z \rightarrow ((\lambda z \rightarrow (s \ (s \ z))) \ (s \ (s \ z))))$$

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$$\underline{(\lambda s \ z \rightarrow (s \ (s \ (s \ (s \ z)))))}$$

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Zero checking

- We might wish to verify whether or not a number is zero
- We can simply pass the number parameters that fail the check (s) and pass it (z)

- $(\lambda m \ n \rightarrow ((m \ (\lambda x \rightarrow \text{FALSE})) \ \text{TRUE}))$

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Zero checking

Let us try it out to $0 = 2 \rightarrow_{\beta} \text{FALSE}$

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$$(2 = 0)$$

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$(2 = 0)$

$(\underline{0} \neq 2)$

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(0? 2)

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$(0? \ 2)$

$((\lambda m \ n \rightarrow ((m \ (\lambda x \rightarrow \text{FALSE})) \ \text{TRUE})) \ 2)$

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$$((\lambda m \ n \rightarrow ((m \ (\lambda x \rightarrow \text{FALSE})) \ \text{TRUE}))) \ 2)$$

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$$((\lambda m \ n \rightarrow ((m \ (\lambda x \rightarrow \text{FALSE})) \ \text{TRUE}))) \ \underline{2})$$

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```
((λm n→((m (λx→FALSE)) TRUE)) 2)
```

```
((λm n→((m (λx→FALSE)) TRUE))  
  (λs z→(s (s z))))
```

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$$((\lambda m \ n \rightarrow ((m \ (\lambda x \rightarrow \text{FALSE})) \ \text{TRUE}))) \ (\lambda s \ z \rightarrow (s \ (s \ z) \)))$$

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$$((\lambda m \ n \rightarrow ((m \ (\lambda x \rightarrow \text{FALSE})) \ \text{TRUE})) \ (\lambda s \ z \rightarrow (s \ (s \ z))))$$
$$(\lambda n \rightarrow ((\lambda s \ z \rightarrow (s \ (s \ z))) \ (\lambda x \rightarrow \text{FALSE})) \ \text{TRUE}))$$

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$$(\lambda n \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ (\lambda x \rightarrow \text{FALSE})) \ \text{TRUE})))$$

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$$(\lambda n \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ (\lambda x \rightarrow \text{FALSE})) \ \text{TRUE}))$$
$$(\lambda n \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ (\lambda x \rightarrow \underline{\text{FALSE}})) \ \text{TRUE}))$$

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$$(\lambda n \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ (\lambda x \rightarrow \underline{FALSE})) \ TRUE))$$

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$$(\lambda n \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ (\lambda x \rightarrow \underline{\text{FALSE}})) \ \text{TRUE})))$$
$$(\lambda n \rightarrow (((\lambda s \ z \rightarrow (s \ (s \ z))) \ (\lambda x \rightarrow (\lambda t \ f \rightarrow f))) \ \text{TRUE})))$$

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$$(\lambda n \rightarrow (\underline{((\lambda s \ z \rightarrow (s \ (s \ z))) \ (\lambda x \rightarrow (\lambda t \ f \rightarrow f)))}) \ \text{TRUE}))$$

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$$(\lambda n \rightarrow ((\lambda z \rightarrow ((\lambda x \rightarrow (\lambda t \ f \rightarrow f)) \ ((\lambda x \rightarrow (\lambda t \ f \rightarrow f)) \ z))) \ \text{TRUE}))$$

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$$(\lambda n \rightarrow ((\lambda z \rightarrow ((\lambda x \rightarrow (\lambda t \ f \rightarrow f)) ((\lambda x \rightarrow (\lambda t \ f \rightarrow f)) z))$$
$$)) \text{ TRUE}))$$

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$$)) \text{ TRUE}))$$
$$(\lambda n \rightarrow ((\lambda z \rightarrow ((\lambda x \rightarrow (\lambda t \ f \rightarrow f)) ((\lambda x \rightarrow (\lambda t \ f \rightarrow f)) z))$$
$$)) \underline{\text{TRUE}}))$$

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$$(\lambda n \rightarrow ((\lambda z \rightarrow ((\lambda x \rightarrow (\lambda t \ f \rightarrow f)) ((\lambda x \rightarrow (\lambda t \ f \rightarrow f)) z) \\)) \ \underline{\text{TRUE}}))$$

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$$)) \ \underline{\text{TRUE}}))$$
$$(\lambda n \rightarrow ((\lambda z \rightarrow ((\lambda x \rightarrow (\lambda t \ f \rightarrow f)) ((\lambda x \rightarrow (\lambda t \ f \rightarrow f)) z))$$
$$)) \ (\lambda t \ f \rightarrow t))$$

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$$)) (\lambda t \rightarrow f \rightarrow t)))$$

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$$)) (\lambda t \ f \rightarrow t)))$$
$$(\lambda n \rightarrow (\underline{\lambda z \rightarrow} \rightarrow ((\underline{\lambda x \rightarrow (\lambda t \ f \rightarrow f)}) ((\underline{\lambda x \rightarrow (\lambda t \ f \rightarrow f)}) z))$$
$$(\underline{\lambda t \ f \rightarrow t})))$$

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$$(\lambda n \rightarrow (\lambda z \rightarrow \rightarrow ((\lambda x \rightarrow (\lambda t \ f \rightarrow f)) \ (\lambda x \rightarrow (\lambda t \ f \rightarrow f)) \ z)) \ (\lambda t \ f \rightarrow t)))$$

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$$(\lambda n \rightarrow ((\lambda x \ t \ f \rightarrow f) \ ((\lambda x \ t \ f \rightarrow f) \ (\lambda t \ f \rightarrow t))))$$

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$$(\lambda n \rightarrow ((\lambda x \ t \ f \rightarrow f) \ \underline{((\lambda x \ t \ f \rightarrow f) \ (\lambda t \ f \rightarrow t))}))$$

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$$(\lambda n \rightarrow ((\lambda x \ t \ f \rightarrow f) \ (\lambda t \ f \rightarrow f)))$$

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Other arithmetic operators

- Division, subtraction, and all manners of comparison operators can be defined similarly
- The level of detail of the specification can be compared to that of a very high level CPU
- This means that we are, to an extent, programming in a sort of assembly
- This is the reason why the traces have been so verbose so far

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Other arithmetic operators

- We could also define numbers in base two instead of base one
- This would save processing time, but would result in a slighter more complex specification
- We will just ignore these engineering details: we only focus on **what** can be done, not the best way to do it

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Recap

- Lambda terms can be used to encode arbitrary basic data types
- The terms are always lambda expression which, when they get parameters passed in, identify themselves somehow
- Identification can be done by applying something (possibly even a given number of times), or returning one of the parameters

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Recap

- There are many encodings of data types, but they all behave in the same way by producing the same outputs for the same inputs
- From now on we will start ignoring the reduction steps for simple terms such as $3+3$
- We will instead focus on more complex data structures, such as tuples, discriminated unions, and even lists

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```
((FALSE bit1) bit0)
```


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```
((FALSE bit1) bit0)
```

```
((FALSE bit1) bit0)
```

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```
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```

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```
((FALSE bit1) bit0)
```

```
((( $\lambda t$  f  $\rightarrow$  f) bit1) bit0)
```

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```
((λt f→f) bit1) bit0)
```

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```
((λt f→f) bit1) bit0)
```

```
((λt f→f) bit1) bit0)
```

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```
(((λt f→f) bit1) bit0)
```

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```
(((λt f→f) bit1) bit0)
```

```
((λf→f) bit0)
```

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$$((\lambda f \rightarrow f) \text{ bit0})$$

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$((\lambda f \rightarrow f) \text{ bit0})$

$((\lambda f \rightarrow f) \text{ bit0})$

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bit0

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(TRUE \wedge FALSE)

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$(\text{TRUE} \wedge \text{FALSE})$

$((\wedge \text{TRUE}) \text{ FALSE})$

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$((\underline{\quad} \text{ TRUE}) \text{ FALSE})$

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```
((^ TRUE) FALSE)
```

```
((( $\lambda p\ q \rightarrow ((p\ q)\ p)$ ) TRUE) FALSE)
```


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$$(((\lambda p \ q \rightarrow ((p \ q) \ p))) \underline{\text{TRUE}}) \text{ FALSE})$$

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$((\lambda p \ q \rightarrow ((p \ q) \ p)) \ (\lambda t \ f \rightarrow t)) \ \text{FALSE})$

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$$(((\lambda p \ q \rightarrow ((p \ q) \ p))) (\lambda t \ f \rightarrow t)) \underline{\text{FALSE}})$$

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```
(( (λp q → ((p q) p)) (λt f → t)) FALSE)
```

```
(( (λp q → ((p q) p)) (λt f → t)) (λt f → f) )
```


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$$(((\lambda p \ q \rightarrow ((p \ q) \ p))) (\lambda t \ f \rightarrow t)) (\lambda t \ f \rightarrow f))$$

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$$\underline{((\lambda p \ q \rightarrow ((p \ q) \ p)) \ (\lambda t \ f \rightarrow t)) \ (\lambda t \ f \rightarrow f))}$$

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$$((\underline{\lambda q \rightarrow (((\lambda t \ f \rightarrow t) \ q) (\lambda t \ f \rightarrow t))}) \underline{(\lambda t \ f \rightarrow f)})$$

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$$((\lambda f \ t \ f \rightarrow f) \ (\lambda t \ f \rightarrow t))$$

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$$((\wedge \text{FALSE}) \text{ TRUE})$$

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```

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```
(( (λp q → ((p q) p)) (λt f → f)) FALSE)
```

```
(( (λp q → ((p q) p)) (λt f → f)) (λt f → f) )
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$$((\underline{\vee} \text{ TRUE}) \text{ FALSE})$$

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((∇ TRUE) FALSE)
```

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(( (λp q→((p p) q)) TRUE) FALSE)
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((λp q→((p p) q)) TRUE) FALSE)
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((λp q→((p p) q)) TRUE) FALSE)
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```
((((λp q → ((p p) q)) (λt f → f)) FALSE))
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(( (λp q → ((p p) q)) (λt f → f)) FALSE)
```

```
(( (λp q → ((p p) q)) (λt f → f)) (λt f → f) )
```

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Remaining numeral derivations

Let us try out $0 = 0 \rightarrow_{\beta} \text{TRUE}$

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$$(0 = 0)$$

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$$(0 = 0)$$
$$(\underline{0?} \ 0)$$

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(0? 0)

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$(\underline{0?} \ 0)$

$((\lambda m \ n \rightarrow ((m \ (\lambda x \rightarrow \text{FALSE})) \ \text{TRUE})) \ 0)$

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$$((\lambda m \ n \rightarrow ((m \ (\lambda x \rightarrow \text{FALSE})) \ \text{TRUE})) \ 0)$$

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The best of luck, and thanks for the
attention!