Data structures

The INFDEV@HR Team

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Introduction

Introduction

Lecture topics

- Tuples
- Discriminated unions (polymorphism)
- Lists

Data types

Data types

Overview

- We now move on to ways to define data types
- The definitions will be both **minimal** and **composable**
- Classes, polymorphism, etc. can all be rendered under our definitions, so we miss nothing substantial

Data types

Overview

Notice: from now on we will start ignoring the reduction steps for simple terms such as 3+3, `x = 0`, etc. for brevity

Data types

Minimality

- The lambda calculus has so far proven very powerful, despite its size
- We do not need hundreds of different operators, we can simply build them[a]
- The only extension needed is purely syntactic in nature to make it more mnemonic, but this is only skin-deep and requires no change to the underlying mechanisms of the lambda calculus

[a] and more

Minimality

- In defining data types we wish to maintain this minimality
- We do not want dozens of separate, competing data types all slightly overlapping

Data types

Fundamental scenarios

- **Tuples**: storing multiple things together at the same time, like the fields and methods in a class
- **Unions**: storing either one of various things at a time, like an interface that is exactly one of its concrete implementors

Data types

The importance of composition

- We just need to cover the case of two items, higher numbers come through composition
- For example, given the ability to store a pair, we can build a pair of pairs to create arbitrary tuples
- Similarly, given the ability to store either of two values, we can build either of many values with nesting

Tuples

Tuples

- A pair of values is defined simply as something that stores these two values
- We can extract them by giving the pair a function that will receive the values

```
(λx→y→f→((f x) y))
```

Tuples


```
(1, 2)
```

Tuples


```
(1, 2)
```

```
(((,) 1) 2)
```

Tuples


```
(((,) 1) 2)
```

Tuples

$$(((,)\ 1)\ 2)$$

$$(((\lambda x \rightarrow y \rightarrow f \rightarrow ((f\ x)\ y))\ 1)\ 2)$$

Tuples

$$(((\lambda x \rightarrow y \rightarrow f \rightarrow ((f\ x)\ y))\ 1)\ 2)$$

Tuples

$$(((\lambda x \to y \to f \to ((f\ x)\ y))\ 1)\ 2)$$

$$(\underline{((\lambda x \to y \to f \to ((f\ x)\ y))\ 1)}\ 2)$$

Tuples

$$(\underline{((\lambda x \rightarrow y \rightarrow f \rightarrow ((f\ x)\ y))\ 1)}\ 2)$$

Tuples

$$\texttt{(}\underline{\texttt{((}\lambda\texttt{x}\rightarrow\texttt{y}\rightarrow\texttt{f}\rightarrow\texttt{((f x) y)) 1)}}\texttt{ 2)}$$

$$\texttt{((}\lambda\texttt{y}\rightarrow\texttt{f}\rightarrow\texttt{((f 1) y)) 2)}$$

Tuples

$$((\lambda \mathtt{y} \rightarrow \mathtt{f} \rightarrow ((\mathtt{f}\ \ 1)\ \ \mathtt{y}))\ \ 2)$$

Tuples

$$((\lambda y \rightarrow f \rightarrow ((f\ 1)\ y))\ 2)$$

$$\underline{((\lambda y \rightarrow f \rightarrow ((f\ 1)\ y))\ 2)}$$

Tuples

$$\underline{((\lambda y \rightarrow f \rightarrow ((f\ 1)\ y))\ 2)}$$

Tuples

$\underline{((\lambda \mathtt{y} \rightarrow \mathtt{f} \rightarrow ((\mathtt{f}\ \ 1)\ \ \mathtt{y}))\ \ 2)}$

$(\lambda \mathtt{f} \rightarrow ((\mathtt{f}\ \ 1)\ \ 2))$

Tuples

- We can define two utility functions that, given a pair, extract the first or second value
- They are usually called π_1 and π_2, or `fst` and `snd`

```
(λp→(p (λx→y→x)))
```

```
(λp→(p (λx→y→y)))
```

Tuples

$(\pi_1\ (1,\ 2))$

Tuples

$(\pi_1\ (1,\ 2))$

$(\underline{\pi_1}\ (1,\ 2))$

Tuples

$$(\underline{\pi_1}\ \ \texttt{(1 , 2))}$$

Tuples

$(\underline{\pi_1}\ (1,\ 2))$

$((\lambda p \to (p\ (\lambda x \to y \to x)))\ (1,\ 2))$

Tuples

$$((\lambda p \rightarrow (p\ (\lambda x \rightarrow y \rightarrow x)))\ (1,\ 2))$$

Tuples

$$((\lambda p \rightarrow (p\ (\lambda x \rightarrow y \rightarrow x)))\ (1,\ 2))$$

$$((\lambda p \rightarrow (p\ (\lambda x \rightarrow y \rightarrow x)))\ (((\underline{,})\ 1)\ 2))$$

Tuples

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$$\boxed{\texttt{((}\lambda\texttt{p}\rightarrow\texttt{(p (}\lambda\texttt{x}\rightarrow\texttt{y}\rightarrow\texttt{x))) ((}\underline{\texttt{(,)}}\texttt{ 1) 2))}}$$

Tuples


```
((λp→(p (λx→y→x))) (((,) 1) 2))
```

```
((λp→(p (λx→y→x))) ((((λx→y→f→((f x) y)) 1)
     2))
```

Tuples


```
((λp→(p (λx→y→x))) (((λx→y→f→((f x) y)) 1)
    2))
```

Tuples

$$\texttt{((}\lambda\texttt{p}\rightarrow\texttt{(p (}\lambda\texttt{x}\rightarrow\texttt{y}\rightarrow\texttt{x))) (((}\lambda\texttt{x}\rightarrow\texttt{y}\rightarrow\texttt{f}\rightarrow\texttt{((f x) y)) 1) 2))}$$

$$\texttt{((}\lambda\texttt{p}\rightarrow\texttt{(p (}\lambda\texttt{x}\rightarrow\texttt{y}\rightarrow\texttt{x))) (}\underline{\texttt{((}\lambda\texttt{x}\rightarrow\texttt{y}\rightarrow\texttt{f}\rightarrow\texttt{((f x) y)) 1)}}\texttt{ 2))}$$

Tuples

$$((\lambda p \rightarrow (p\ \ (\lambda x \rightarrow y \rightarrow x)))\ \ (\underline{((\lambda x \rightarrow y \rightarrow f \rightarrow ((f\ \ x)\ \ y))\ \ 1)}\ \ 2)\ )$$

Tuples

$$\texttt{((}\lambda\texttt{p}\rightarrow\texttt{(p (}\lambda\texttt{x}\rightarrow\texttt{y}\rightarrow\texttt{x))) (}\underline{\texttt{((}\lambda\texttt{x}\rightarrow\texttt{y}\rightarrow\texttt{f}\rightarrow\texttt{((f x) y)) 1)}}\texttt{ 2))}$$

$$\texttt{((}\lambda\texttt{p}\rightarrow\texttt{(p (}\lambda\texttt{x}\rightarrow\texttt{y}\rightarrow\texttt{x))) ((}\lambda\texttt{y}\rightarrow\texttt{f}\rightarrow\texttt{((f 1) y)) 2))}$$

Tuples

$$\boxed{\texttt{((}\lambda\texttt{p}\rightarrow\texttt{(p (}\lambda\texttt{x}\rightarrow\texttt{y}\rightarrow\texttt{x))) ((}\lambda\texttt{y}\rightarrow\texttt{f}\rightarrow\texttt{((f 1) y)) 2))}}$$

Tuples


```
((λp→(p (λx→y→x))) ((λy→f→((f 1) y)) 2))
```


```
((λp→(p (λx→y→x))) ((λy→f→((f 1) y)) 2))
```

Tuples

$$\boxed{\texttt{((}\lambda\texttt{p}\rightarrow\texttt{(p (}\lambda\texttt{x}\rightarrow\texttt{y}\rightarrow\texttt{x))) ((}\lambda\texttt{y}\rightarrow\texttt{f}\rightarrow\texttt{((f 1) y)) 2))}}$$

Tuples

$$\texttt{((}\lambda\texttt{p}\rightarrow\texttt{(p (}\lambda\texttt{x}\rightarrow\texttt{y}\rightarrow\texttt{x))) ((}\lambda\texttt{y}\rightarrow\texttt{f}\rightarrow\texttt{((f 1) y)) 2))}$$

$$\underline{\texttt{((}\lambda\texttt{p}\rightarrow\texttt{(p (}\lambda\texttt{x}\rightarrow\texttt{y}\rightarrow\texttt{x))) ((}\lambda\texttt{y}\rightarrow\texttt{f}\rightarrow\texttt{((f 1) y)) 2))}}$$

Tuples

$$\underline{((\lambda p \rightarrow (p\ (\lambda x \rightarrow y \rightarrow x)))\ ((\lambda y \rightarrow f \rightarrow ((f\ 1)\ y))\ 2))}$$

Tuples

$$\boxed{\underline{((\lambda p \rightarrow (p\ (\lambda x \rightarrow y \rightarrow x)))\ ((\lambda y \rightarrow f \rightarrow ((f\ 1)\ y))\ 2))}}$$

$$\boxed{(((\lambda y \rightarrow f \rightarrow ((f\ 1)\ y))\ 2)\ (\lambda x \rightarrow y \rightarrow x))}$$

Tuples

$$(((\lambda y \rightarrow f \rightarrow ((f\ 1)\ y))\ 2)\ (\lambda x \rightarrow y \rightarrow x))$$

Tuples

$$\boxed{(((\lambda y \rightarrow f \rightarrow ((f\ 1)\ y))\ 2)\ (\lambda x \rightarrow y \rightarrow x))}$$

$$\boxed{(\underline{((\lambda y \rightarrow f \rightarrow ((f\ 1)\ y))\ 2)}\ (\lambda x \rightarrow y \rightarrow x))}$$

Tuples

$$\texttt{(}\underline{\texttt{((}\lambda\texttt{y}\rightarrow\texttt{f}\rightarrow\texttt{((f 1) y)) 2)}}\texttt{ (}\lambda\texttt{x}\rightarrow\texttt{y}\rightarrow\texttt{x))}$$

Tuples

```
(((λy→f→((f 1) y)) 2) (λx→y→x))
```

```
((λf→((f 1) 2)) (λx→y→x))
```

Tuples


```
((λf→((f 1) 2)) (λx→y→x))
```

Tuples


```
((λf→((f 1) 2)) (λx→y→x))
```

$\underline{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{((f 1) 2)) (}\lambda\texttt{x}\rightarrow\texttt{y}\rightarrow\texttt{x))}}$

Tuples

$$\underline{((\lambda f \rightarrow ((f\ 1)\ 2))\ (\lambda x \rightarrow y \rightarrow x))}$$

Tuples

$\underline{((\lambda f \rightarrow ((f\ 1)\ 2))\ (\lambda x \rightarrow y \rightarrow x))}$

$(((\lambda x \rightarrow y \rightarrow x)\ 1)\ 2)$

Tuples

$$(((\lambda x \rightarrow y \rightarrow x)\ 1)\ 2)$$

Tuples

$$(((\lambda x \rightarrow y \rightarrow x)\ 1)\ 2)$$

$$(\underline{((\lambda x \rightarrow y \rightarrow x)\ 1)}\ 2)$$

Tuples

$$(\underline{((\lambda x \rightarrow y \rightarrow x)\ 1)}\ 2)$$

Tuples

$$(\underline{((\lambda x \rightarrow y \rightarrow x)\ 1)}\ 2)$$

$$((\lambda y \rightarrow 1)\ 2)$$

Tuples

$$((\lambda \mathtt{y} \rightarrow \mathtt{1})\ \ \mathtt{2})$$

Tuples

$$((\lambda y \rightarrow 1)\ \ 2)$$

$$\underline{((\lambda y \rightarrow 1)\ \ 2)}$$

Tuples

$\underline{((\lambda y \rightarrow 1)\ \ 2)}$

Tuples

$\underline{((\lambda y \rightarrow 1)\ \ 2)}$

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Tuples

Pair of values

We should expect that π_1 and π_2 are inverse operations to constructing a pair, as they destroy it

Tuples

let p = (1, 2) in ((π_1 p), (π_2 p))

Tuples


```
let p = (1, 2) in ((π₁ p), (π₂ p))
```



```
let p = (1, 2) in ((π₁ p), (π₂ p))
```

Tuples

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let p = (1, 2) in ((π_1 p), (π_2 p))

Tuples

$\texttt{let p = (1, 2) in ((}\pi_1\texttt{ p), (}\pi_2\texttt{ p))}$

$\texttt{((}\pi_1\texttt{ (1, 2)), (}\pi_2\texttt{ (1, 2)))}$

Tuples

$$((\pi_1 \ (1,\ 2)),\ (\pi_2 \ (1,\ 2)))$$

Tuples

$((\pi_1 \;\; (1\,, \;\; 2))\,, \;\; (\pi_2 \;\; (1\,, \;\; 2)))$

$(\underline{(\pi_1 \;\; (1, \;\; 2))}\,, \;\; (\pi_2 \;\; (1\,, \;\; 2)))$

Tuples

$$(\underline{(\pi_1\ (1,\ 2))},\ (\pi_2\ (1,\ 2)))$$

Tuples

$$\boxed{(\underline{(\pi_1\ (1,\ 2))},\ (\pi_2\ (1,\ 2)))}$$

$$\boxed{(1,\ (\pi_2\ (1,\ 2)))}$$

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Tuples

$$(1\ ,\ \ (\pi_2\ \ (1\ ,\ \ 2)))$$

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Tuples

$$\texttt{(1 , (}\pi_2\texttt{ (1 , 2)))}$$

$$\texttt{(1 , }\underline{\texttt{(}\pi_2\texttt{ (1, 2))}}\texttt{)}$$

Tuples

$$(1\ ,\ \underline{(\pi_2\ (1,\ 2))})$$

Tuples

$(1\ ,\ \underline{(\pi_2\ (1,\ 2))})$

$(1\ ,\ 2)$

Tuples


```
(1, 2)
```

Tuples


```
(1, 2)
```

```
(1, 2)
```

Discriminated unions

Discriminated unions

- A choice of values is defined simply as something that stores either of two possible values
- We build a discriminated union with either of two functions to build the first or the second value
- They are usually called `inl` and `inr`[a]

[a] *in* stands for injection, and *l* and *r* stand for left and right

$$(\lambda x \rightarrow f \rightarrow g \rightarrow (f\ \ x))$$

$$(\lambda y \rightarrow f \rightarrow g \rightarrow (g\ \ y))$$

Discriminated unions

```
(inl 1)
```

Discriminated unions

```
(inl 1)
```

```
(inl 1)
```

Discriminated unions


```
(inl 1)
```

Discriminated unions

```
(inl 1)
```

```
((λx→f→g→(f x)) 1)
```

Discriminated unions

$$((\lambda x \rightarrow f \rightarrow g \rightarrow (f\ \ x))\ \ 1)$$

```
((λx→f→g→(f  x))  1)
```

```
((λx→f→g→(f x)) 1)
```

Discriminated unions

$$((\lambda x \rightarrow f \rightarrow g \rightarrow (f\ \ x))\ \ 1)$$

Discriminated unions

$\underline{((\lambda x \rightarrow f \rightarrow g \rightarrow (f\ x))\ 1)}$

$(\lambda f \rightarrow g \rightarrow (f\ 1))$

Discriminated unions

$$\boxed{(\lambda f \rightarrow g \rightarrow (f\ \ 1))}$$

Discriminated unions

$$(\lambda \mathtt{f} \rightarrow \mathtt{g} \rightarrow (\mathtt{f}\ \ 1))$$

$$\underline{(\lambda \mathtt{f} \rightarrow \mathtt{g} \rightarrow (\mathtt{f}\ \ 1))}$$

Discriminated unions

$(\lambda f \rightarrow g \rightarrow (f\ 1))$

Discriminated unions

$\underline{(\lambda \mathtt{f} \rightarrow \mathtt{g} \rightarrow (\mathtt{f}\ \ 1))}$

```
(inl 1)
```

Discriminated unions


```
(inr TRUE)
```

Discriminated unions


```
(inr TRUE)
```

```
(inr TRUE)
```

Discriminated unions

```
(inr TRUE)
```

Discriminated unions


```
(inr TRUE)
```

```
((λy→f→g→(g y)) TRUE)
```

Discriminated unions

$$((\lambda y \rightarrow f \rightarrow g \rightarrow (g\ \ y))\ \ \mathtt{TRUE})$$

Discriminated unions

$((\lambda y \rightarrow f \rightarrow g \rightarrow (g\ \ y))\ \ \mathtt{TRUE})$

$\underline{((\lambda y \rightarrow f \rightarrow g \rightarrow (g\ \ y))\ \ \mathtt{TRUE})}$

Discriminated unions

$\underline{((\lambda \texttt{y} \rightarrow \texttt{f} \rightarrow \texttt{g} \rightarrow (\texttt{g y}))\ \texttt{TRUE})}$

Discriminated unions

$\underline{((\lambda y \rightarrow f \rightarrow g \rightarrow (g\ y))\ \texttt{TRUE})}$

$(\lambda f \rightarrow g \rightarrow (g\ \texttt{TRUE}))$

Discriminated unions


```
(λf→g→(g TRUE))
```

Discriminated unions


```
(λf→g→(g  TRUE))
```

$\underline{(\lambda f \rightarrow g \rightarrow (g\ \ TRUE))}$

Discriminated unions


```
(λf→g→(g TRUE))
```

Discriminated unions

$\underline{(\lambda \mathtt{f} \rightarrow \mathtt{g} \rightarrow (\mathtt{g}\ \mathtt{TRUE}))}$

```
(inr TRUE)
```

Discriminated unions

- Extracting the input of a discriminated union is a process known as `match`[a]
- Given a union and two functions (one per case), if the union was the first case we apply the first function, otherwise we apply the second function

[a] which is a sort of `switch`, just on steroids

$$(\lambda \mathtt{u} \rightarrow \mathtt{f} \rightarrow \mathtt{g} \rightarrow ((\mathtt{u}\ \mathtt{f})\ \mathtt{g}))$$

Discriminated unions


```
(((match (inl 1)) (λx→(x + 1))) (λy→(y ∧
    FALSE)))
```

Discriminated unions


```
(((match (inl 1)) (λx→(x + 1))) (λy→(y ∧
    FALSE)))
```

```
(((match (inl 1)) (λx→(x + 1))) (λy→(y ∧
    FALSE)))
```

Discriminated unions

$$(((\underline{\texttt{match}}\ (\texttt{inl}\ 1))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \texttt{FALSE})))$$

Discriminated unions


```
(((match (inl 1)) (λx→(x + 1))) (λy→(y ∧
    FALSE)))
```

```
(((((λu→f→g→((u f) g)) (inl 1)) (λx→(x + 1))
    ) (λy→(y ∧ FALSE)))
```

Discriminated unions

$$\boxed{((((\lambda u \rightarrow f \rightarrow g \rightarrow ((u\ f)\ g))\ (\mathtt{inl}\ 1))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))}$$

Discriminated unions

$$((((\lambda u \rightarrow f \rightarrow g \rightarrow ((u\ f)\ g))\ (\mathtt{inl}\ 1))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))$$

$$((((\lambda u \rightarrow f \rightarrow g \rightarrow ((u\ f)\ g))\ (\underline{\mathtt{inl}}\ 1))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))$$

Discriminated unions

$$\texttt{((((}\lambda\texttt{u}\rightarrow\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{((u f) g)) (}\underline{\texttt{inl}}\texttt{ 1)) (}\lambda\texttt{x}\rightarrow\texttt{(x }+\texttt{ 1)))}$$
$$\texttt{(}\lambda\texttt{y}\rightarrow\texttt{(y }\wedge\texttt{ FALSE)))}$$

Discriminated unions

$$((((\lambda u \rightarrow f \rightarrow g \rightarrow ((u\ f)\ g))\ (\underline{\texttt{inl}}\ 1))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \texttt{FALSE})))$$

$$((((\lambda u \rightarrow f \rightarrow g \rightarrow ((u\ f)\ g))\ ((\lambda x \rightarrow f \rightarrow g \rightarrow (f\ x))\ 1))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \texttt{FALSE})))$$

Discriminated unions

$$((((\lambda u \rightarrow f \rightarrow g \rightarrow ((u\ f)\ g))\ ((\lambda x \rightarrow f \rightarrow g \rightarrow (f\ x))\ 1))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))$$

Discriminated unions

$$((((\lambda u \rightarrow f \rightarrow g \rightarrow ((u\ \ f)\ \ g))\ \ ((\lambda x \rightarrow f \rightarrow g \rightarrow (f\ \ x))\ \ 1))\ \ (\lambda x \rightarrow (x\ + \ 1)))\ \ (\lambda y \rightarrow (y\ \wedge\ \mathtt{FALSE})))$$

$$((((\lambda u \rightarrow f \rightarrow g \rightarrow ((u\ \ f)\ \ g))\ \ \underline{((\lambda x \rightarrow f \rightarrow g \rightarrow (f\ \ x))\ \ 1)})\ \ (\lambda x \rightarrow (x\ + \ 1)))\ \ (\lambda y \rightarrow (y\ \wedge\ \mathtt{FALSE})))$$

Discriminated unions

$$\boxed{((((\lambda u \rightarrow f \rightarrow g \rightarrow ((u\ f)\ g))\ \underline{((\lambda x \rightarrow f \rightarrow g \rightarrow (f\ x))\ 1)})\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))}$$

Discriminated unions

$$\texttt{((((}\lambda\texttt{u}\rightarrow\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{((u f) g))}\ \underline{\texttt{((}\lambda\texttt{x}\rightarrow\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{(f x)) 1)}}\texttt{)}\ \texttt{(}\lambda\texttt{x}\rightarrow\texttt{(x + 1)))}\ \texttt{(}\lambda\texttt{y}\rightarrow\texttt{(y}\wedge\texttt{FALSE)))}$$

$$\texttt{((((}\lambda\texttt{u}\rightarrow\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{((u f) g))}\ \texttt{(}\lambda\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{(f 1)))}\ \texttt{(}\lambda\texttt{x}\rightarrow\texttt{(x + 1)))}\ \texttt{(}\lambda\texttt{y}\rightarrow\texttt{(y}\wedge\texttt{FALSE)))}$$

Discriminated unions

$$\texttt{((((}\lambda\texttt{u}\rightarrow\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{((u f) g)) (}\lambda\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{(f 1))) (}\lambda\texttt{x}\rightarrow\texttt{(x} + \texttt{1))) (}\lambda\texttt{y}\rightarrow\texttt{(y} \wedge \texttt{FALSE)))}$$

Discriminated unions


```
((((λu→f→g→((u f) g)) (λf→g→(f 1))) (λx→(x
    + 1))) (λy→(y ∧ FALSE)))
```



```
((((λu→f→g→((u f) g)) (λf→g→(f 1))) (λx→(x
    + 1))) (λy→(y ∧ FALSE)))
```

Discriminated unions

$$\texttt{((((}\lambda\texttt{u}\rightarrow\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{((u f) g)) (}\lambda\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{(f 1))) (}\lambda\texttt{x}\rightarrow\texttt{(x} + \texttt{1))) (}\lambda\texttt{y}\rightarrow\texttt{(y} \wedge \texttt{FALSE)))}$$

Discriminated unions

$$\texttt{((((}\lambda \texttt{u}\rightarrow\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{((u f) g)) (}\lambda\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{(f 1))) (}\lambda\texttt{x}\rightarrow\texttt{(x + 1))) (}\lambda\texttt{y}\rightarrow\texttt{(y} \wedge \texttt{FALSE)))}$$

$$\texttt{((}\underline{\texttt{((}\lambda \texttt{u}\rightarrow\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{((u f) g)) (}\lambda\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{(f 1)))}}\texttt{ (}\lambda\texttt{x}\rightarrow\texttt{(x + 1))) (}\lambda\texttt{y}\rightarrow\texttt{(y} \wedge \texttt{FALSE)))}$$

Discriminated unions

$$\boxed{\texttt{((}\underline{\texttt{((}\lambda\texttt{u}\rightarrow\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{((u f) g)) (}\lambda\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{(f 1)))}}\texttt{ (}\lambda\texttt{x}\rightarrow\texttt{(x + 1))) (}\lambda\texttt{y}\rightarrow\texttt{(y }\wedge\texttt{ FALSE)))}}$$

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$$\texttt{((}\underline{\texttt{((}\lambda\texttt{u}\rightarrow\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{((u f) g)) (}\lambda\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{(f 1)))}}\texttt{ (}\lambda\texttt{x}\rightarrow\texttt{(x + 1))) (}\lambda\texttt{y}\rightarrow\texttt{(y }\wedge\texttt{ FALSE)))}$$

$$\texttt{(((}\lambda\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{(((}\lambda\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{(f 1)) f) g)) (}\lambda\texttt{x}\rightarrow\texttt{(x + 1))) (}\lambda\texttt{y}\rightarrow\texttt{(y }\wedge\texttt{ FALSE)))}$$

Discriminated unions

$$\boxed{\begin{array}{l}(((\lambda \mathtt{f} \rightarrow \mathtt{g} \rightarrow (((\lambda \mathtt{f} \rightarrow \mathtt{g} \rightarrow (\mathtt{f}\ \ \mathtt{1}))\ \ \mathtt{f})\ \ \mathtt{g}))\ \ (\lambda \mathtt{x} \rightarrow (\mathtt{x}\ \ +\ \ \mathtt{1})))\\ \qquad (\lambda \mathtt{y} \rightarrow (\mathtt{y}\ \ \wedge\ \ \mathtt{FALSE})))\end{array}}$$

Discriminated unions

$$\boxed{(((\lambda f \rightarrow g \rightarrow (((\lambda f \rightarrow g \rightarrow (f\ \ 1))\ \ f)\ \ g))\ \ (\lambda x \rightarrow (x\ +\ 1)))\ \ (\lambda y \rightarrow (y\ \wedge\ \mathtt{FALSE})))}$$

$$\boxed{(\underline{((\lambda f \rightarrow g \rightarrow (((\lambda f \rightarrow g \rightarrow (f\ \ 1))\ \ f)\ \ g))\ \ (\lambda x \rightarrow (x\ +\ 1)))}\ \ (\lambda y \rightarrow (y\ \wedge\ \mathtt{FALSE})))}$$

$$\boxed{\begin{array}{l}(\underline{((\lambda f \rightarrow g \rightarrow (((\lambda f \rightarrow g \rightarrow (f\ 1))\ f)\ g))\ (\lambda x \rightarrow (x + 1)))}\ (\lambda y \rightarrow \\ \quad (y \wedge \mathtt{FALSE})))\end{array}}$$

Discriminated unions

$$(\underline{((\lambda f \rightarrow g \rightarrow (((\lambda f \rightarrow g \rightarrow (f\ 1))\ f)\ g))\ (\lambda x \rightarrow (x + 1)))}\ (\lambda y \rightarrow (y \wedge \texttt{FALSE})))$$

$$((\lambda g \rightarrow (((\lambda f \rightarrow g \rightarrow (f\ 1))\ (\lambda x \rightarrow (x + 1)))\ g))\ (\lambda y \rightarrow (y \wedge \texttt{FALSE})))$$

Discriminated unions

$$((\lambda g \rightarrow (((\lambda f \rightarrow g \rightarrow (f\ 1))\ (\lambda x \rightarrow (x + 1)))\ g))\ (\lambda y \rightarrow (y \wedge \texttt{FALSE})))$$

Discriminated unions

$$((\lambda g \rightarrow (((\lambda f \rightarrow g \rightarrow (f\ 1))\ (\lambda x \rightarrow (x + 1)))\ g))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))$$

$$\underline{((\lambda g \rightarrow (((\lambda f \rightarrow g \rightarrow (f\ 1))\ (\lambda x \rightarrow (x + 1)))\ g))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE}}$$

Discriminated unions

$$((\lambda g \rightarrow (((\lambda f \rightarrow g \rightarrow (f\ 1))\ (\lambda x \rightarrow (x + 1)))\ g))\ (\lambda y \rightarrow (y \wedge \texttt{FALSE}$$

Discriminated unions

```
((λg→(((λf→g→(f 1)) (λx→(x + 1))) g)) (λy→(y ∧ FALSE
```

```
(((λf→g→(f 1)) (λx→(x + 1))) (λy→(y ∧ FALSE
    )))
```

Discriminated unions

$$(((\lambda f \rightarrow g \rightarrow (f\ 1))\ (\lambda x \rightarrow (x + 1)))\ (\lambda y \rightarrow (y \wedge \texttt{FALSE})))$$

Discriminated unions

$$\texttt{(((}\lambda\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{(f 1)) (}\lambda\texttt{x}\rightarrow\texttt{(x + 1))) (}\lambda\texttt{y}\rightarrow\texttt{(y} \wedge \texttt{FALSE)))}$$

$$\texttt{(}\underline{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{(f 1)) (}\lambda\texttt{x}\rightarrow\texttt{(x + 1)))}}\texttt{ (}\lambda\texttt{y}\rightarrow\texttt{(y} \wedge \texttt{FALSE)))}$$

Discriminated unions

$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{(f 1)) (}\lambda\texttt{x}\rightarrow\texttt{(x + 1)))}}\texttt{ (}\lambda\texttt{y}\rightarrow\texttt{(y }\wedge\texttt{ FALSE)))}}$$

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$$\texttt{(}\underline{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{g}\rightarrow\texttt{(f 1)) (}\lambda\texttt{x}\rightarrow\texttt{(x}+\texttt{1)))}}\texttt{ (}\lambda\texttt{y}\rightarrow\texttt{(y}\wedge\texttt{FALSE)))}$$

$$\texttt{((}\lambda\texttt{g}\rightarrow\texttt{((}\lambda\texttt{x}\rightarrow\texttt{(x}+\texttt{1)) 1)) (}\lambda\texttt{y}\rightarrow\texttt{(y}\wedge\texttt{FALSE)))}$$

Discriminated unions

$$\boxed{\texttt{((}\lambda\texttt{g}\rightarrow\texttt{((}\lambda\texttt{x}\rightarrow\texttt{(x + 1)) 1)) (}\lambda\texttt{y}\rightarrow\texttt{(y}\ \wedge\ \texttt{FALSE)))}}$$

Discriminated unions

$$((\lambda g \rightarrow ((\lambda x \rightarrow (x + 1))\ 1))\ (\lambda y \rightarrow (y \wedge \texttt{FALSE})))$$

$$\underline{((\lambda g \rightarrow ((\lambda x \rightarrow (x + 1))\ 1))\ (\lambda y \rightarrow (y \wedge \texttt{FALSE})))}$$

Discriminated unions

$$((\lambda g \rightarrow ((\lambda x \rightarrow (x + 1))\ 1))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))$$

Discriminated unions

Data structures

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$$\underline{((\lambda g \rightarrow ((\lambda x \rightarrow (x + 1))\ 1))\ (\lambda y \rightarrow (y \wedge \mathtt{FALSE})))}$$

$$((\lambda x \rightarrow (x + 1))\ 1)$$

Discriminated unions

$$((\lambda x \rightarrow (x + 1))\ 1)$$

Discriminated unions

$$((\lambda x \rightarrow (x + 1))\ 1)$$

$$\underline{((\lambda x \rightarrow (x + 1))\ 1)}$$

Discriminated unions

$\boxed{\underline{((\lambda x \rightarrow (x + 1))\ 1)}}$

Discriminated unions

$\underline{((\lambda x \rightarrow (x + 1))\ 1)}$

$(1 + 1)$

Discriminated unions


```
(1 + 1)
```

Discriminated unions


```
(1 + 1)
```

```
(1 + 1)
```

Discriminated unions


```
(1 + 1)
```

Discriminated unions

```
(1 + 1)
```

```
2
```

Discriminated unions

Choice between a pair of values

We should expect that `inl` and `inr` are inverse operations to `match`

Discriminated unions


```
(((match (inl 1)) inl) inr)
```

Discriminated unions


```
(((match (inl 1)) inl) inr)
```

```
(((match (inl 1)) inl) inr)
```

Discriminated unions


```
(((match (inl 1)) inl) inr)
```

Discriminated unions


```
(((match (inl 1)) inl) inr)
```

```
(inl 1)
```

Discriminated unions


```
(((match (inr TRUE)) inl) inr)
```

Discriminated unions

```
(((match (inr TRUE)) inl) inr)
```

```
(((match (inr TRUE)) inl) inr)
```

Discriminated unions

```
(((match (inr TRUE)) inl) inr)
```

```
(((match (inr TRUE)) inl) inr)
```

```
(inr TRUE)
```

Conclusion

Conclusion

Recap

- Lambda terms can be used to encode arbitrary basic data types
- The terms are always lambda expression which, when they get parameters passed in, identify themselves somehow
- Identification can be done by applying something (possibly even a given number of times), or returning one of the parameters

Conclusion

Recap

- The data types we have seen cover an impressive range of applications
- Tuples cover grouping data together (like the fields of a class)
- Unions cover choosing different things (like the polymorphism of an interface that might be implemented by various concrete classes)
- Combining these two covers all possible programming needs, even for more complex data structures

This is it!

The best of luck, and thanks for the attention!