

Recursion and F# translations

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Practicum begins now

## Recursion and F# translations

The INFDEV@HR Team

Hogeschool Rotterdam Rotterdam, Netherlands



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# Introduction



## Introduction

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## Lecture topics

- Recursion (let-rec)
- F# translations of lambda programs so far



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# Recursive functions and let-rec



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#### ldea

- The lambda calculus has no while loops
- This means that we need to emulate them with recursion



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#### Idea

- This is a bit of an issue
- A function is just a lambda term, which does not have a name
- If the function does not have a name, how do we call it from its own body?



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#### Idea

- We can define a recursive function as a function with an extra parameter
- Calling the extra parameter will result in calling the function itself



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Practicum begins now For example, the factorial function becomes

(
$$\lambda f \ n \rightarrow if \ (n = 0)$$
 then 1 else ((f (n - 1))  $\times$  n))



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- We now need an external operator that handles recursive functions properly
- This must ensure that a recursive function gets itself as a parameter, in an endless chain
- This combinator is known as fixpoint operator

$$(\lambda f \rightarrow ((\lambda x \rightarrow (f (x x))) (\lambda x \rightarrow (f (x x)))))$$



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```
((fix (\lambdaf n\rightarrowif (n = 0) then 1 else (f (n - 1) ))) 2)
```



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((fix (
$$\lambda$$
f n $\rightarrow$ if (n = 0) then 1 else (f (n - 1) ))) 2)

((
$$\underline{\text{fix}}$$
 ( $\lambda f$  n $\rightarrow$ if (n = 0) then 1 else (f (n - 1)))) 2)



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((
$$\underline{\text{fix}}$$
 ( $\lambda \text{f n} \rightarrow \text{if (n = 0) then 1 else (f (n - 1)))}$ ) 2)



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$$((\underline{\text{fix}}\ (\lambda f\ n \rightarrow if\ (n=0)\ then\ 1\ else\ (f\ (n-1))))$$
 2)

((
$$\lambda n \rightarrow \text{if (n = 0) then 1 else (} \dots \text{ (n - 1)))}$$
 2)



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) 2)



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) 2)

$$\underline{\text{(($\lambda$n$$$\rightarrow$if (n = 0) then 1 else (... (n - 1))) 2)}}$$



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$$\underline{\text{(($\lambda$n$$$\rightarrow$if (n = 0) then 1 else (... (n - 1))) 2)}}$$



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) 2)

if 
$$(2 = 0)$$
 then 1 else  $(... (2 - 1))$ 



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if 
$$(2 = 0)$$
 then 1 else  $(... (2 - 1))$ 



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if 
$$(2 = 0)$$
 then 1 else  $(... (2 - 1))$ 

if 
$$(2 = 0)$$
 then 1 else (...  $(2 - 1)$ )



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 then 1 else (...  $(2 - 1)$ )



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if 
$$(2 = 0)$$
 then 1 else (...  $(2 - 1)$ )

```
if FALSE then 1 else (\ldots (2-1))
```



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```
if FALSE then 1 else (... (2 - 1))
```



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```
if FALSE then 1 else (... (2-1))
```

```
if FALSE then 1 else (... (2-1))
```



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Practicum begins now if FALSE then 1 else (... (2-1))



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if FALSE then 1 else 
$$(...$$
  $(2-1))$ 

$$(... (2-1))$$



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$$(...(2-1))$$



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$$(... (2 - 1))$$

((
$$\underline{\text{fix}}$$
  $\underline{(\lambda \text{f n} \rightarrow \text{if (n = 0) then 1 else (f (n - 1)))}}$ ) (2 - 1))



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$$((\underline{\text{fix}} \ \underline{(\lambda f \ n \rightarrow \text{if} \ (n=0) \ \text{then} \ 1 \ \text{else} \ (f \ (n-1)))}) \ (2$$



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$$((\underline{\text{fix}} \ (\lambda f \ n \rightarrow \text{if } (n = 0) \ \text{then 1 else } (f \ (n - 1))))$$
 (2

((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) (2 - 1))



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((
$$\lambda$$
n $\to$ if (n = 0) then 1 else (... (n - 1))) (2 - 1))

((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) (2 - 1))



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```
((\lambdan\rightarrowif (n = 0) then 1 else (... (n - 1))) (2-1)
```



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$$((\lambda n \rightarrow \text{if (n = 0) then 1 else (... (n - 1))})$$



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```
((\lambdan	oif (n = 0) then 1 else (... (n - 1))) 1)
```



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((
$$\lambda$$
n $ightarrow$ if (n = 0) then 1 else (... (n - 1))) 1)

$$\underline{\text{(($\lambda$n$$$\rightarrow$if (n = 0) then 1 else (... (n - 1))) 1)}}$$



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$$\underline{\text{(($\lambda$n$$$\rightarrow$if (n = 0) then 1 else (... (n - 1))) 1)}}$$



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$$((\lambda n \rightarrow \text{if } (n = 0) \text{ then } 1 \text{ else } (\dots (n - 1))) \text{ } 1)$$

if 
$$(1 = 0)$$
 then 1 else  $(... (1 - 1))$ 



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if 
$$(1 = 0)$$
 then 1 else  $(... (1 - 1))$ 



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if 
$$(1 = 0)$$
 then 1 else  $(... (1 - 1))$ 

if 
$$(1 = 0)$$
 then 1 else  $(... (1 - 1))$ 



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if 
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 then 1 else (...  $(1 - 1)$ )



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```
if (1 = 0) then 1 else (... (1 - 1))
```

```
if FALSE then 1 else (...(1-1))
```



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```
if FALSE then 1 else (... (1 - 1))
```



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```
if FALSE then 1 else (... (1-1))
```

```
if FALSE then 1 else (... (1-1))
```



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Practicum begins now if FALSE then 1 else (... (1-1))



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if FALSE then 1 else 
$$(... (1-1))$$



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$$(...(1-1))$$



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$$(...(1-1))$$

((
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$$((\underline{\text{fix}} \ \underline{(\lambda f \ n \rightarrow \text{if } (n=0) \ \text{then 1 else } (f \ (n-1)))}) \ (1$$



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$$((\underline{\text{fix}} \ (\lambda \text{f n} \rightarrow \text{if (n = 0) then 1 else (f (n - 1))})) \ (1 - 1))$$

((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) (1 - 1))



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((
$$\lambda$$
n $\to$ if (n = 0) then 1 else (... (n - 1))) (1 - 1))

((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1))) (1 - 1))



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```
((\lambda \mathtt{n} {\rightarrow} \mathtt{if} \ (\mathtt{n} = \mathtt{0}) \ \mathtt{then} \ \mathtt{1} \ \mathtt{else} \ (\dots \ (\mathtt{n} - \mathtt{1}))) \\ \underline{(\mathtt{1} - \mathtt{1})})
```



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else (... (n - 1)))  $\underline{(1-1)}$ )

```
((\lambda n \rightarrow \text{if (n = 0) then 1 else (... (n - 1)))}
```



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((
$$\lambda$$
n $ightarrow$ if (n = 0) then 1 else (... (n - 1))) 0)



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((
$$\lambda$$
n $ightarrow$ if (n = 0) then 1 else (... (n - 1))) 0)

$$\underline{\text{(($\lambda$n$$$\rightarrow$if (n = 0) then 1 else (... (n - 1))) 0)}}$$



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$$((\lambda n \rightarrow \text{if } (n = 0) \text{ then } 1 \text{ else } (\dots (n - 1))) \text{ 0})$$



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$$\boxed{ ((\lambda \mathtt{n} \! \to \! \mathtt{if} \ (\mathtt{n} = \mathtt{0}) \ \mathtt{then} \ \mathtt{1} \ \mathtt{else} \ (\dots \ (\mathtt{n} - \mathtt{1}))) \ \mathtt{0}) }$$

if 
$$(0 = 0)$$
 then 1 else  $(... (0 - 1))$ 



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```
if (0 = 0) then 1 else (... (0 - 1))
```



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if 
$$(0 = 0)$$
 then 1 else  $(... (0 - 1))$ 

if 
$$(0 = 0)$$
 then 1 else  $(... (0 - 1))$ 



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if 
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 then 1 else (...  $(0 - 1)$ )



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```
if (0 = 0) then 1 else (... (0 - 1))
```

```
if TRUE then 1 else (... (0-1))
```



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```
if TRUE then 1 else (\dots (0-1))
```



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```
if TRUE then 1 else (... (0-1))
```

```
if TRUE then 1 else (\dots (0-1))
```



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Practicum begins now  $\underline{\text{if TRUE then 1 else }(\dots\ (0\ -\ 1))}$ 



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Practicum begins now if TRUE then 1 else (... (0-1))

1



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Practicum begins now We can now try our hand at a factorial computation

```
((fix (\lambdaf n\rightarrowif (n = 0) then 1 else ((f (n - 1)) \times n))) 2)
```



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```
((fix (\lambdaf n\rightarrowif (n = 0) then 1 else ((f (n - 1)) 	imes n))) 2)
```



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((fix (
$$\lambda$$
f n $\rightarrow$ if (n = 0) then 1 else ((f (n - 1))  $imes$  n))) 2)



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else ((... (n - 1))  $\times$  n)) 2)



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```
((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) 	imesn)) 2)
```



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((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else ((... (n - 1))  $imes$ n)) 2)

$$(\underline{\lambda n} \rightarrow \underline{if (n = 0) then 1 else ((... (n - 1)) \times n)}$$



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$$(\underline{\lambda n} \rightarrow \underline{if} \ (n=0) \ then \ 1 \ else \ ((\dots \ (n-1)) \ \times \ n)$$
  $\underline{2})$ 



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if 
$$(2 = 0)$$
 then 1 else  $((... (2 - 1)) \times 2)$ 



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if (2 = 0) then 1 else ((... 
$$(2 - 1)) \times 2$$
)



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if (2 = 0) then 1 else ((... 
$$(2-1)) \times 2$$
)

if 
$$\underline{(2=0)}$$
 then 1 else ((...  $(2-1)$ )  $\times$  2)



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if 
$$\underline{(2=0)}$$
 then 1 else ((...  $(2-1)$ )  $\times$  2)



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if 
$$(2=0)$$
 then 1 else ((...  $(2-1)) \times 2$ )

```
if FALSE then 1 else ((... (2-1)) \times 2)
```



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```
if FALSE then 1 else ((... (2-1)) \times 2)
```



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```
if FALSE then 1 else ((... (2-1)) \times 2)
```

```
if FALSE then 1 else ((... (2-1)) \times 2)
```



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```
if FALSE then 1 else ((... (2-1)) \times 2)
```



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if FALSE then 1 else ((... 
$$(2-1)) \times 2$$
)

$$((... (2-1)) \times 2)$$



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$$((... (2 - 1)) \times 2)$$



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$$((...(2-1)) \times 2)$$

$$(((\underbrace{\text{fix}} \ \underline{\lambda f} \rightarrow \ \underline{n} \rightarrow \\ \underline{\text{if } (n = 0) \text{ then 1 else } ((f (n - 1)) \times n)}) (2 - 1)) \times 2)$$



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```
(((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) (2 - 1)) \times 2)
```



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(((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else ((... (n - 1))  $\times$  n)) (2 - 1))  $\times$  2)

```
(((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) (2-1)) \times 2)
```



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```
(((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) (2-1) \times 2)
```



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(((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else ((... (n - 1))  $\times$  n))  $\underline{(2-1)}$ )  $\times$  2)

```
(((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) 	imes n)) \frac{1}{1}) 	imes 2)
```



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```
(((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) 1) \times 2)
```



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(((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else ((... (n - 1))  $\times$  n)) 1)  $\times$  2)



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$$\begin{array}{c} \text{(($\underline{\lambda}{\tt n}$\rightarrow$ $\to$ $if (n=0)$ then 1 else ((... (n-1)) $\times n$)} \\ \underline{1}) \times 2) \end{array}$$

(if 
$$(1 = 0)$$
 then 1 else  $((... (1 - 1)) \times 1)$   $\times 2)$ 



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```
(if (1 = 0) then 1 else ((... (1 - 1)) \times 1) \times 2)
```



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(if (1 = 0) then 1 else ((... (1 - 1)) 
$$\times$$
 1)  $\times$  2)

(if 
$$(1=0)$$
 then 1 else ((...  $(1-1)$ )  $\times$  1)  $\times$  2)



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(if 
$$(1=0)$$
 then 1 else ((...  $(1-1)$ )  $\times$  1)  $\times$  2)



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(if 
$$(1=0)$$
 then 1 else ((...  $(1-1)$ )  $\times$  1)  $\times$  2)

```
(if FALSE then 1 else ((... (1 - 1)) \times 1) \times 2)
```



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```
(if FALSE then 1 else ((... (1 - 1)) \times 1) \times 2)
```



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```
(if FALSE then 1 else ((... (1-1)) \times 1) \times 2)
```

(if FALSE then 1 else ((... 
$$(1-1)) \times 1) \times 2$$
)



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```
(if FALSE then 1 else ((... (1-1)) \times 1) \times 2)
```



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(if FALSE then 1 else ((... 
$$(1-1)) \times 1) \times 2$$
)

$$(((... (1-1)) \times 1) \times 2)$$



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$$(((... (1 - 1)) \times 1) \times 2)$$



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$$(((... (1 - 1)) \times 1) \times 2)$$



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```
\begin{array}{c} ((((\underbrace{\text{fix}}\ \underline{\lambda f} \rightarrow \ \underline{n} \rightarrow \\ \underline{\text{if (n = 0) then 1 else ((f (n - 1)) } \times n)}) \ \ (1 \ - \\ \underline{1)) \ \times \ 1) \ \times \ 2) \end{array}
```



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```
(((((\lambda n \rightarrow \text{if } (n = 0) \text{ then } 1 \text{ else } ((\underbrace{\dots} (n - 1)) \times n)) (1 - 1)) \times 1) \times 2)
```



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```
((((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) (1 - 1)) \times 1) \times 2)
```

```
(((((\lambda n \rightarrow if (n = 0) then 1 else ((... (n - 1)) \times n)) (1 - 1)) \times 1) \times 2)
```



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```
(((((\lambda n \rightarrow \text{if } (n=0) \text{ then 1 else } ((\dots (n-1)) \times n)) \ \underline{(1-1)} \times 1) \times 2)
```



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((((
$$\lambda$$
n $\rightarrow$ if (n = 0) then 1 else ((... (n - 1))  $\times$  n))  $\underline{(1-1)}$   $\times$  1)  $\times$  2)

```
(((((\lambda n \rightarrow \text{if } (n = 0) \text{ then } 1 \text{ else } ((\dots (n - 1)) \times n)) 0) \times 1) \times 2)
```



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```
((((\lambdan\rightarrowif (n = 0) then 1 else ((... (n - 1)) \times n)) 0) \times 1) \times 2)
```



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$$\begin{array}{c} (((\underbrace{\lambda \mathtt{n} \rightarrow} \rightarrow \underbrace{\mathtt{if} \ (\mathtt{n} = \mathtt{0}) \ \mathtt{then} \ \mathtt{1} \ \mathtt{else} \ ((\dots \ (\mathtt{n} - \mathtt{1})) \ \times \ \mathtt{n})} \ \ \underline{\mathtt{0}}) \ \ \times } \\ \underline{\mathtt{1}) \ \ \times \ \mathtt{2}) \end{array}$$



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```
 \begin{array}{c} \text{((($\underline{\lambda}$n$ $\to $})$} \\ \underline{\text{if (n = 0) then 1 else ((... (n - 1)) $\times $n)}} \ \underline{0}) \ \times \\ 1) \ \times \ 2) \end{array}
```



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((if (
$$0 = 0$$
) then 1 else ((... ( $0 - 1$ )) ×  $0$ ) × 1) × 2)



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((if (0 = 0) then 1 else ((... (0 - 1)) 
$$\times$$
 0)  $\times$  1)  $\times$  2)



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((if 
$$(0 = 0)$$
 then 1 else ((...  $(0 - 1)) \times 0$ )  $\times$  1)  $\times$  2)



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((if 
$$(0=0)$$
 then 1 else ((...  $(0-1)$ )  $\times$  0)  $\times$  1)  $\times$  2)



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((if 
$$(0=0)$$
 then 1 else ((...  $(0-1)$ )  $\times$  0)  $\times$  1)  $\times$  2)

```
((if TRUE then 1 else ((... (0-1)) \times 0) \times 1) \times 2)
```



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```
((if TRUE then 1 else ((... (0 - 1)) \times 0) \times 1) \times 2)
```



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```
((if TRUE then 1 else ((... (0-1)) \times 0) \times 1) \times 2)
```

```
((\underline{\text{if TRUE then 1 else }((\dots (0-1)) \times 0)} \times 1) \times 2)
```



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((if TRUE then 1 else ((... 
$$(0-1)) \times 0$$
)  $\times$  1)  $\times$  2)



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((if TRUE then 1 else ((... 
$$(0-1)) \times 0$$
)  $\times$  1)  $\times$  2)

$$((1 \times 1) \times 2)$$



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Practicum begins now  $((1 \times 1) \times 2)$ 



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((1 
$$\times$$
 1)  $\times$  2)

$$(\underline{(1 \times 1)} \times 2)$$



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$$(\underline{(1 \times 1)} \times 2)$$



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$$(\underline{(1 \times 1)} \times 2)$$

$$(1 \times 2)$$



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Practicum begins now  $(1 \times 2)$ 



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$$(1 \times 2)$$

$$(1 \times 2)$$



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Practicum begins now  $(1 \times 2)$ 

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Practicum begins now (1 × 2)

2



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# Translating to F#



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#### Overview

- Each and every one of the constructs we have seen so far has a direct translation in F#<sup>a</sup>
- All constructs have exactly the same behaviour as in the lambda calculus, but with a slightly less mathematical syntax for ASCII keyboards
- A few operators are menemonically friendlier or just plain more readable, but the essence remains exactly the same
- F# is indentation-sensitive, like Python: pay a lot of attention to how terms are indented!

<sup>a</sup>Haskell is a bit different, so we leave it for later

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Practicum begins now Integers, booleans, floats, strings have the usual meaning, both in the lambda calculus, F#, and the languages you are used to:

$$((2 + 3) - 4)$$



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- Conditionals behave just like in the lambda calculus
- This means that they return the evaluation of either of the two branches
- This differs from imperative languages, where we just jump into either of the two branches

```
if (true && false) then
else
```



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Practicum begins now Functions look very similar, with fun instead of  $\lambda$ 

$$(fun x f \rightarrow (f x))$$



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Conclusion

Practicum begins now Functions look very similar, with fun instead of  $\lambda$ 

$$(fun x f \rightarrow (f x))$$

Just like function application

$$(((fun x f \rightarrow (f x)) 3) (fun x \rightarrow (3 + x)))$$



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Practicum begins now We can give names to functions, and code becomes much prettier as a result

```
let apply =
  fun x f -> (f x)
((apply 3) (fun x -> (3 + x)))
```



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Practicum begins now We can also give names to recursive functions by using let rec instead of let, and code becomes much more readable than with fix

```
let rec fact =
  fun n ->
  if (n = 0) then
    1
  else
    ((fact (n - 1)) * n)
```



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Practicum begins now We can define tuples by just putting a comma between the values, with or without nesting for more than two values is done for us

(1, true)



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#### **Pairs**

- Nested tuples are the same as in the lambda calculus, and of course they also work in F#
- They are, in essence, pairs of pairs, such as: (1,(2,(3,4) or (((1,2),3),4), which are not the same
- Non-nested tuples are a slight extension for practical ease of work in F#
- They are, in essence, tuples with as many elements as we want, such as: (1,2,3,4)
- All the examples in this slide are valid in F#, but are not identical objects



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- Functions such as  $\pi_1$  and  $pi_2$ , which both extract one item of a pair, also exist in F#
- They are called, respectively, fst and snd

```
(fst (1, true))
```

```
(snd (1, true))
```



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- Of course F#, just like the lambda calculus, imposes no limitation on composition
- Why not build a tuple of functions, then? Why not indeed!



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Practicum begins now  Of course F#, just like the lambda calculus, imposes no limitation on composition

• Why not build a tuple of functions, then? Why not indeed!

```
((fun x \rightarrow (1 + x)), (fun x \rightarrow (2 * x)))
```



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Practicum begins now  Of course F#, just like the lambda calculus, imposes no limitation on composition

• Why not build a tuple of functions, then? Why not indeed!

```
((fun x \rightarrow (1 + x)), (fun x \rightarrow (2 * x)))
```

Could we also build a tuple of tuples of numbers and functions returning tuples of functions?



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```
    Of course F#, just like the lambda calculus, imposes no
limitation on composition
```

• Why not build a tuple of functions, then? Why not indeed!

```
((fun x \rightarrow (1 + x)), (fun x \rightarrow (2 * x)))
```

Could we also build a tuple of tuples of numbers and functions returning tuples of functions?

Yes, but we are not going to do it :)



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- F# also offers built-in discriminated unions
- Functions such as inl and inr, which both embed one item into the union, also exist in F#
- They are called, respectively, Choice10f2 and Choice20f2

(Choice10f2 1)

(Choice20f2 true)



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Practicum begins now We can, of course, perform matches on discriminated unions

```
match (Choice10f2 1) with
| Choice10f2 x ->
    (Choice10f2 x)
| Choice20f2 y ->
    (Choice20f2 y)
```



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Practicum begins now We can, of course, perform matches on discriminated unions

```
match (Choice10f2 1) with
| Choice10f2 x ->
    (Choice10f2 x)
| Choice20f2 y ->
    (Choice20f2 y)
```

```
let i =
  match (Choice10f2 1) with
  | Choice10f2 x ->
        x
  | Choice20f2 y ->
        0

(i * 2)
```



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Practicum begins now Discriminated unions, and their corresponding matches, can be nested as deep as we need

```
match (Choice10f2 (Choice20f2 true)) with
| Choice10f2 x ->
   match x with
| Choice10f2 x ->
        (Choice10f2 x)
| Choice20f2 y ->
        (Choice20f2 y)
| Choice20f2 y ->
   (Choice20f2 y)
```



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#### Structural equality

- Composable data types make it possible for F# to build a series of useful functions automatically
- These functions allow automatic comparison of data structures with the same shape (for example, tuples with tuples or lists with lists)
- Comparison can work with values made up of primitive expressions, tuples, unions, lists, records, and any composition of them
- With the only exclusion of functions: functions, or composite values containing functions, may not be compared



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#### Interoperability

- One of the practical strengths of F# is its ability to fully interoperate with the .Net framework
- Any feature or library available, even if written in other languages like C#, can be used from inside F#
- Moreover, any F# library can be used from other languages like C#
- This means that applications could be the internal logic in F#, and the UI and database store in C#



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#### Interoperability

- For example, we can call functions such as System.Console.WriteLine, System.Console.ReadLine, System.Int32.Parse, etc.
- This gives a whole other dimension of usefulness to F#
- Ranging from games to web applications, the language knows no intrinsic limitation
- This also comes with full IDE, autocompletion, and debugger support in Visual Studio and a few other IDE's
- Scala is to some extent comparable



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#### Interoperability

A short F# demo might be in order



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#### Forbidden features

- F# is a hybrid language
- It also contains OO features, mutability (variables), interfaces, classes, inheritance, etc.
- You may only use these features in the assignments if there is no alternative to access a library you need
- Mutability may only be used in the main program, which then calls regular functions that only use the lambda calculus derived core
- Failure to comply will result in an automatic insufficient grade



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#### About types

- A major difference between F# and the lambda calculus is that F# is, in fact, a statically typed language
- Even though the language does not require type declarations like Java or C#, the compiler will still ensure that the types make sense
- This means that we will get compiler errors because we mix types
- Code like fst 1 will not compile, for example, because 1 is not a pair and therefore fst cannot be applied to it
- We will cover the type system of F# in the next lecture



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#### Ways to exercise

- We have added a ton of F# homework, with solutions
  - It is all on GitHub
- It is not mandatory, but it is a good idea to do it until you feel more sure
- You may discuss it during one of the two practicums



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### **Conclusion**



#### Conclusion

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#### Recap

- ullet The lambda calculus can be translated, term to term, into F#
- F# is therefore just a practical lambda calculus with a series of handy extensions and slightly more readable



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# Practicum begins now



#### This is it!

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Practicum begins now The best of luck, and thanks for the attention!