Physics FSE (2021-22) Homework 2

Please send the completed file to my mailbox yy.lam@qq.com by March 30th, with using the filename format:

2021xxxxxx_yourname_fse_hw2

Please answer the questions by filling on these sheets. It would be perfect if you use e-pen directly writing on the sheets. If you do not have the appropriate hardware, you may handle the questions as usual by using pieces of blank papers, then take photos and paste them onto these question sheets.

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1. Given the distance of the centre to centre distance of Earth and the Moon 3.84×10^5 km, the time interval for a month 27.3 days, (a) find the acceleration due to Earth's gravity at the distance of the moon. (b) Given the radius of Earth 6370 km, calculate the period of an artificial satellite orbiting at an average altitude of 1500 km above Earth's surface.

   Solution. *Suppose that we do not know the numerical values of the gravitational constant G and the mass of Earth M, we only use the given information for the computation. (a) The gravitational acceleration is*

$$a = r\omega^2 = 3.84 \times 10^8 \times \left(\frac{2\pi}{27.3 \times 24 \times 60 \times 60}\right)^2 = 2.72 \times 10^{-3}\ ms^{-2}$$

   (b) Since the gravitational acceleration (centripetal acceleration) obeys the inverse square law

$$a \propto \frac{1}{r^2} \quad \Rightarrow \quad r\omega^2 \propto \frac{1}{r^2}$$

   with $T = 2\pi/\omega$ gives

$$\frac{T^2}{r^3} = constant$$

   Let r_m, T_m be the given distance of the moon and the period, r_s the average altitude of the satellite, thus the period of the satellite is

$$T = T_m \sqrt[3]{\frac{R_E + r_s}{r_m}} = 27.3 \times 24 \times 60 \times 60 \times \sqrt[3]{\frac{(6370 + 1500) \times 10^3}{3.84 \times 10^8}} = 6920\ s$$

   or 115 minutes 20 seconds.

2. A person is located on Earth's surface at a latitude α. Calculate the centripetal acceleration of the person resulting from the rotation of Earth around its polar axis. Express your answer in terms of α, the radius R_E of Earth, and time T for one rotation of Earth. Compare your answer with g for $\alpha = 60°$.

   Solution. *The distance (radius) between the person and the rotational axis of Earth is $R_E \cos\alpha$. The angular speed of Earth is just $2\pi/T$. Therefore, the centripetal acceleration of the person towards the rotational axis is*

$$r\omega^2 = R_E \cos\alpha \left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2 R_E \cos\alpha}{T^2}$$

Putting $g = GM/R_E^2$ into the expression the acceleration is

$$\frac{4\pi^2 \cos\alpha}{T}\sqrt{\frac{GM}{g}} = \frac{4\pi^2 \cos\alpha}{T}\sqrt{\frac{GM}{g^2}}g$$

Inserting $T = 24 \times 60^2$ s, G, M and $\alpha = 60°$ into the expression, we get the centripetal acceleration being 0.17% of g.

3. The driver presses down on the brake of a truck being moving at 80 kmh^{-1} while entering a circular curve of radius 120 m. If the speed of the truck is decreasing at a rate of 7.5 kmh^{-1} each second, what is the magnitude of the acceleration of the car at the instant its speed is 55 kmh^{-1}?

   Solution. *The tangential acceleration is -7.5 km/h^{-2}. That is*

   $$a_t = -\frac{7.5 \times 10^3}{60^2} = -2.08\ ms^{-2}.$$

   The radial acceleration at the moment when the speed 55 km/h is

   $$a_r = \frac{v^2}{r} = \left(\frac{55 \times 10^3}{60^2}\right)^2 \frac{1}{120} = 1.95\ ms^{-2}.$$

   The resultant is

   $$a = \sqrt{a_r^2 + a_t^2} = \sqrt{1.95^2 + (-2.08)^2} = 2.85\ ms^{-2}.$$

4. If you wish to reduce the stress (which is related to centripetal force) on high-speed tires, would you use large or small diameter tires? Explain. (*Hint: Analyse the stress formula*)

   Solution. *The shear deformation is defined by*

   $$\Delta x = \frac{F}{SA}L_0$$

   where S is the shear modulus, F is the applied force (centripetal force here), the L_0 is the radius of the tire, and Δx is the longitudinal displacement in F direction. Thus, we have

   $$Stress = \frac{F}{A} \propto \frac{1}{L_0}.$$

   A larger diameter tire is able to reduce the stress.

5. Discuss the relative effectiveness of dieting and exercise in losing weight, noting that most athletic activities consume food energy at a rate of 400 to 500 W, while a single cup of yogurt can contain 1250 kJ. Is it likely that exercise alone will be sufficient to lose weight?

   Solution. *Since the rate of consuming food energy is equivalent to the rating for power, the time taken for an athlete doing exercise to consume the mentioned merely a single cup of yogurt may around*

   $$\frac{1250 \times 10^3}{500} \sim \frac{1250 \times 10^3}{400}, \qquad i.e. \quad 2500 \sim 3125\ s$$

   or in other words, $42 \sim 52$ min. A person in protracted dieting could take a longer time to consume the same amount of food energy due to lower metabolism. The best way for losing weight seems to be both doing exercise and taking appropriate dieting.

6. A hockey puck of mass 0.18 kg is shot across a rough floor with the roughness different at different places, which can be described by a position-dependent coefficient of kinetic friction. For a puck moving along the x-axis the coefficient of kinetic friction is the following function of x, where x is in m: $\mu(x) = 0.1 + 0.05x$. Find the work done by the kinetic friction force on the hockey p when it has moved (a) from $x = 0$ to $x = 1$ m, and (b) from $x = 1$ m to $x = 3$ m.

Solution. *The frictional force also becomes a function of x as*

$$f = -9.8 \times 0.18(0.1 + 0.05x) = -0.088(2 + x).$$

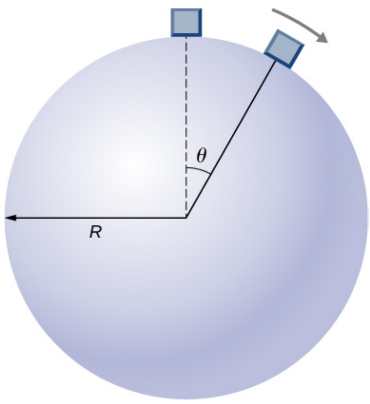
(a) The work done by the kinetic friction is

$$-\int_0^1 0.088(2+x)\mathrm{d}x = -0.088\left[2x + \frac{x^2}{2}\right]_0^1 = -0.22 \; J.$$

(b) The work done for a different interval,

$$-\int_1^3 0.088(2+x)\mathrm{d}x = -0.088\left[2x + \frac{x^2}{2}\right]_1^3 = -0.70 \; J.$$

7. A body of mass m and negligible size starts from rest and slides down the surface of a frictionless solid sphere of radius R as shown. What is the angle θ while the body leaves the sphere?



Solution. *Before the mass sliding out from the sphere, it keeps a circular motion in*

$$\frac{mv^2}{R} = mg\cos\theta.$$

*As the angle θ increases as well as the increasing of v due to gravity, above equation at some point no long holds. The tangential velocity v becomes the velocity for **linear** kinetic energy which equals to the potential difference between the top and the breaking position, i.e.*

$$\frac{1}{2}mv^2 = mgR(1 - \cos\theta)$$

Inserting the first equation while at the threshold condition into the second equation we get

$$\frac{1}{2}(Rmg\cos\theta) = mgR(1 - \cos\theta)$$

$$\frac{1}{2}\cos\theta = 1 - \cos\theta$$

$$\theta = \cos^{-1}\frac{2}{3}$$