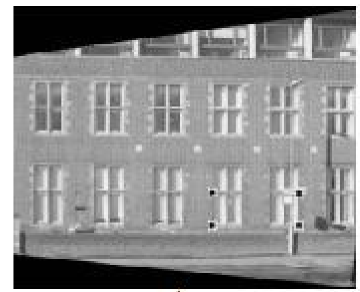
Upcoming Assignment

- Removing the projective distortion from a perspective image of a plane

a



b

- Use different measures
- Use Matlab

Parameter estimation

Multiple View Geometry

Parameter estimation

- 2D homography(单应)
  Given a set of points in P^2 (x_i,x_i'), compute the projective transformation H such that ($x_i'=Hx_i$)
- 3D to 2D camera projection
  Given a set of (X_i,x_i), compute P ($x_i=PX_i$)
- Fundamental matrix
  Given a set of (x_i,x_i'), compute F ($x_i'^TFx_i=0$)
- Trifocal tensor
  Given a set of (x_i,x_i',x_i''), compute T

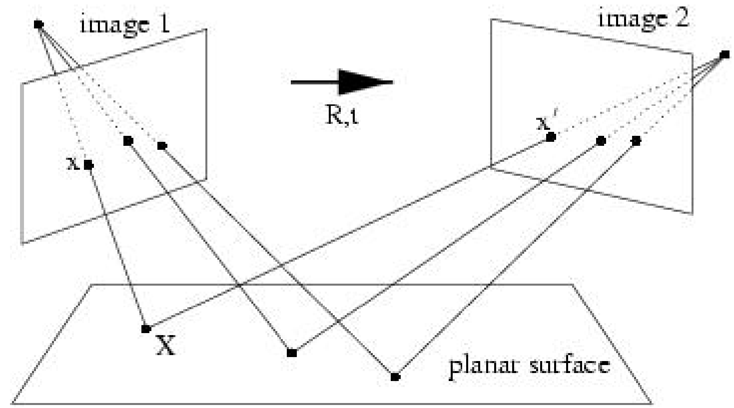
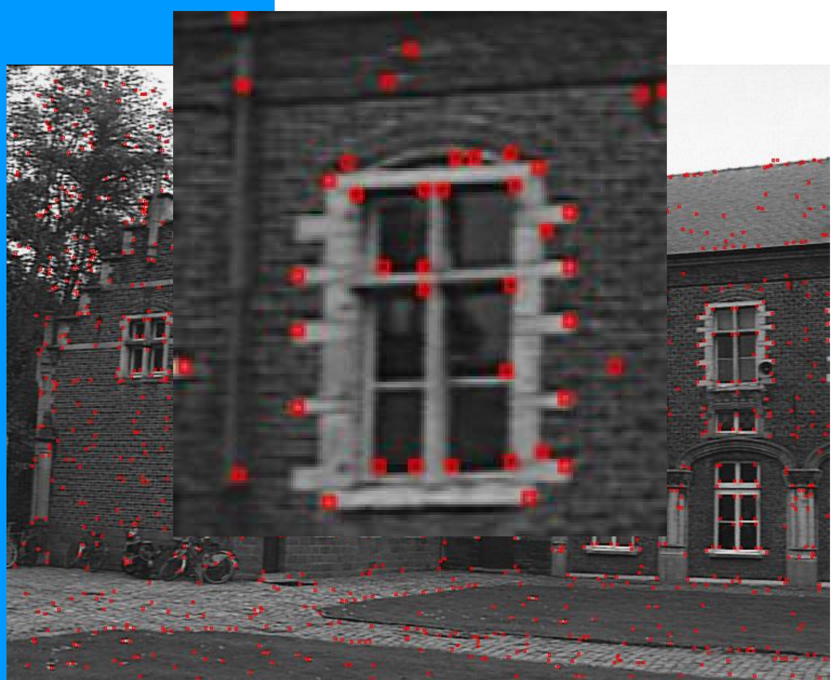
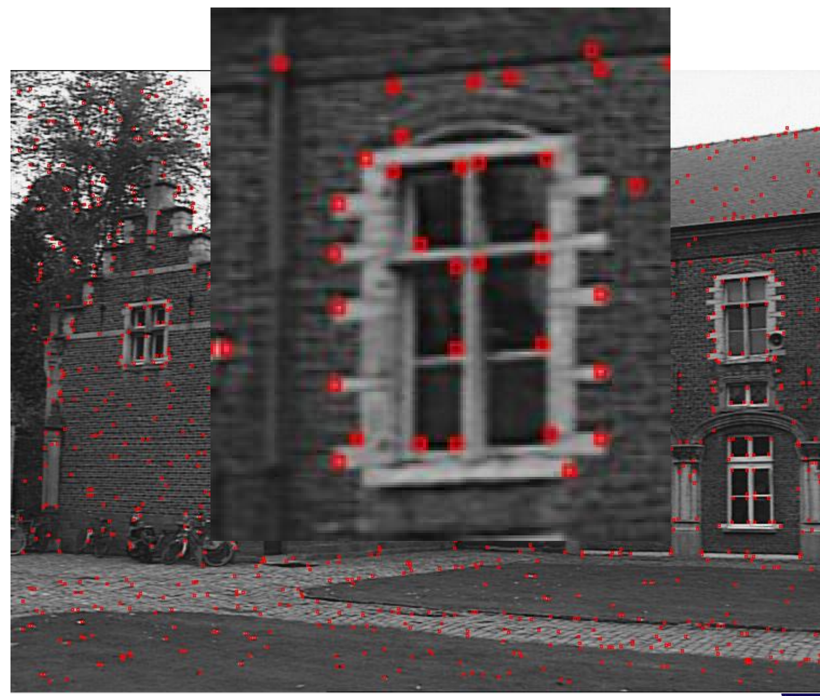
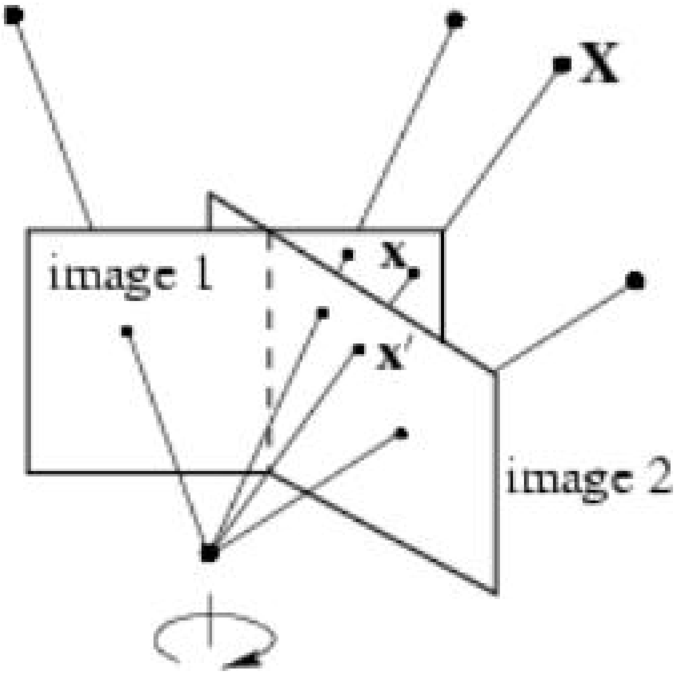


image 1
R,t
image 2
x'
x
X
planar surface






X
image 1
x
x'
image 2

Number of measurements required

- At least as many independent equations as degrees of freedom required
- Example:

$$\mathbf{x}' \cong \mathrm{H}\mathbf{x}$$

$$\lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2 independent equations / point
8 degrees of freedom

$4 \mathrm{x} 2 \geq 8$

$$\mathbf{x}'_i \times \mathrm{H}\mathbf{x}_i = 0 \qquad \mathrm{x}'_i = (x'_i, y'_i, w'_i)^\top \qquad \mathrm{Hx}_i = \begin{pmatrix} \mathrm{h}^{1\top}\mathrm{x}_i \\ \mathrm{h}^{2\top}\mathrm{x}_i \\ \mathrm{h}^{3\top}\mathrm{x}_i \end{pmatrix}$$

$$\mathrm{x}'_i \times \mathrm{Hx}_i = \begin{pmatrix} y'_i\mathrm{h}^{3\top}\mathrm{x}_i - w'_i\mathrm{h}^{2\top}\mathrm{x}_i \\ w'_i\mathrm{h}^{1\top}\mathrm{x}_i - x'_i\mathrm{h}^{3\top}\mathrm{x}_i \\ x'_i\mathrm{h}^{2\top}\mathrm{x}_i - y'_i\mathrm{h}^{1\top}\mathrm{x}_i \end{pmatrix}$$

$$\begin{bmatrix} 0^\top & -w'_i\mathrm{x}_i^\top & y'_i\mathrm{x}_i^\top \\ w'_i\mathrm{x}_i^\top & 0^\top & -x'_i\mathrm{x}_i^\top \\ -y'_i\mathrm{x}_i^\top & x'_i\mathrm{x}_i^\top & 0^\top \end{bmatrix} \begin{pmatrix} \mathrm{h}^1 \\ \mathrm{h}^2 \\ \mathrm{h}^3 \end{pmatrix} = 0$$

$$\mathrm{A}_i\mathrm{h} = 0$$

- Equations are linear in $\mathbf{h}$

$$\mathrm{A}_i \mathbf{h} = 0$$

- Only 2 out of 3 are linearly independent (indeed, 2 eq/pt)

$$\begin{bmatrix} \mathbf{0}^\top & -w_i'\mathbf{x}_i^\top & y_i'\mathbf{x}_i^\top \\ w_i'\mathbf{x}_i^\top & \mathbf{0}^\top & -x_i'\mathbf{x}_i^\top \\ -y_i'\mathbf{x}_i^\top & x_i'\mathbf{x}_i^\top & \mathbf{0}^\top \end{bmatrix} \begin{pmatrix} \mathbf{h}^1 \\ \mathbf{h}^2 \\ \mathbf{h}^3 \end{pmatrix} = 0$$

$$\begin{bmatrix} \mathbf{0}^\top & -w_i'\mathbf{x}_i^\top & y_i'\mathbf{x}_i^\top \\ w_i'\mathbf{x}_i^\top & \mathbf{0}^\top & -x_i'\mathbf{x}_i^\top \end{bmatrix} \begin{pmatrix} \mathbf{h}^1 \\ \mathbf{h}^2 \\ \mathbf{h}^3 \end{pmatrix} = 0$$

$$x_i'\mathrm{A}_i^1 + y_i'\mathrm{A}_i^2 + w_i'\mathrm{A}_i^3 = 0$$

(only drop third row if $w_i' \neq 0$)

- Holds for any homogeneous representation, e.g. $(x_i', y_i', 1)$

Approximate solutions

- Minimal solution

  4 points yield an exact solution for H

- More points
  - No exact solution, because measurements are inexact ("noise")
  - Search for "best" according to some cost function
  - Algebraic or geometric/statistical cost

Direct Linear Transformation (DLT)

- Solving for H

$$\begin{bmatrix} \mathrm{A}_1 \\ \mathrm{A}_2 \\ \mathrm{A}_3 \\ \mathrm{A}_4 \end{bmatrix} \mathrm{A}\mathrm{h} = 0$$

size A is 8x9 or 12x9, but rank 8

Trivial solution is $\mathrm{h}=0_9^{\mathrm{T}}$ is not interesting

1-D null-space yields solution of interest

pick for example the one with $\|\mathrm{h}\| = 1$

Direct Linear Transformation (DLT)

- Over-determined solution

$$\begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{bmatrix} \mathbf{h} = \mathbf{0}$$

No exact solution because of inexact measurement i.e. “noise”

Find approximate solution

- Additional constraint needed to avoid 0, e.g. $\|h\| = 1$
- $Ah = 0$ not possible, so minimize $\|Ah\|$

DLT algorithm

Objective

Given $n \geq 4$ 2D to 2D point correspondences $\{x_i \leftrightarrow x_i'\}$, determine the 2D homography matrix H such that $x_i' = Hx_i$

Algorithm

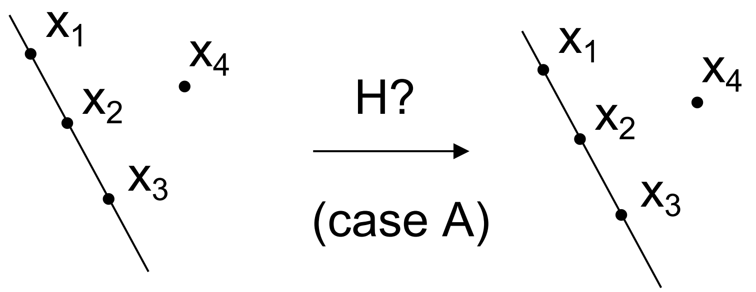
(i) For each correspondence $x_i \leftrightarrow x_i'$ compute A_i. Usually only two first rows needed.

(ii) Assemble n 2x9 matrices A_i into a single $2n$x9 matrix A

(iii) Obtain SVD of A. Solution for h is last column of V

(iv) Determine H from h

Degenerate configurations



Constraints: $\mathrm{x}_i' \times \mathrm{H}\mathrm{x}_i = 0$ $\quad i$=1,2,3,4

Cost functions

- Algebraic distance $\|\mathrm{Ah}\|$
- Geometric distance
- Reprojection error

Algebraic distance

DLT minimizes $\|\mathrm{Ah}\|$

$e = \mathrm{Ah}$ residual vector

e_i partial vector for each $(\mathrm{x}_i \leftrightarrow \mathrm{x}_i')$

algebraic error vector

$$d_{\mathrm{alg}}\left(\mathrm{x}_i', \mathrm{H}\mathrm{x}_i\right)^2 = \left\|e_i\right\|^2 = \left\| \begin{bmatrix} 0^\mathsf{T} & -w_i'\mathrm{x}_i^\mathsf{T} & -y_i'\mathrm{x}_i^\mathsf{T} \\ -w_i'\mathrm{x}_i^\mathsf{T} & 0^\mathsf{T} & -x_i'\mathrm{x}_i^\mathsf{T} \end{bmatrix} \mathrm{h} \right\|^2$$

algebraic distance

$$d_{\mathrm{alg}}\left(\mathrm{x}_1, \mathrm{x}_2\right)^2 = a_1^2 + a_2^2 \text{ where } \mathrm{a} = \left(a_1, a_2, a_3\right)^\mathrm{T} = \mathrm{x}_1 \times \mathrm{x}_2$$

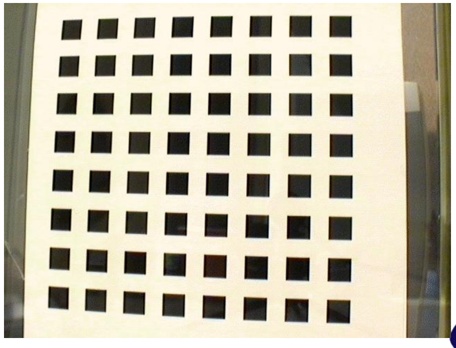
$$\sum_i d_{\mathrm{alg}}\left(\mathrm{x}_i', \mathrm{H}\mathrm{x}_i\right)^2 = \sum_i \left\|e_i\right\|^2 = \left\|\mathrm{Ah}\right\|^2 = \left\|e\right\|^2$$

Not geometrically/statistically meaningfull, but given good normalization it works fine and is very fast (use for initialization)

Different Geometric Errors

x measured coordinates
$\hat{\mathrm{x}}$ estimated coordinates
$\bar{\mathrm{x}}$ true coordinates
$d(.,.)$ Euclidean distance (in image)

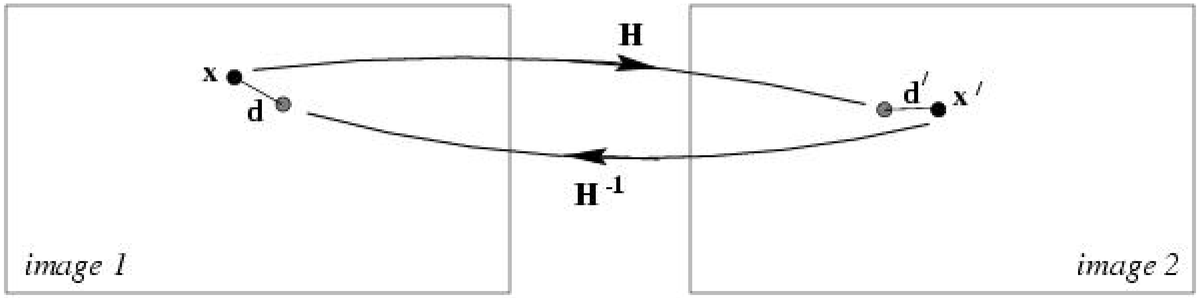
e.g. calibration pattern



1. Error in one image

$$\hat{\mathrm{H}} = \underset{\mathrm{H}}{\operatorname{argmin}} \sum_i d\left(\mathrm{x}'_i, \mathrm{H}\bar{\mathrm{x}}_i\right)^2$$

Reprojection error

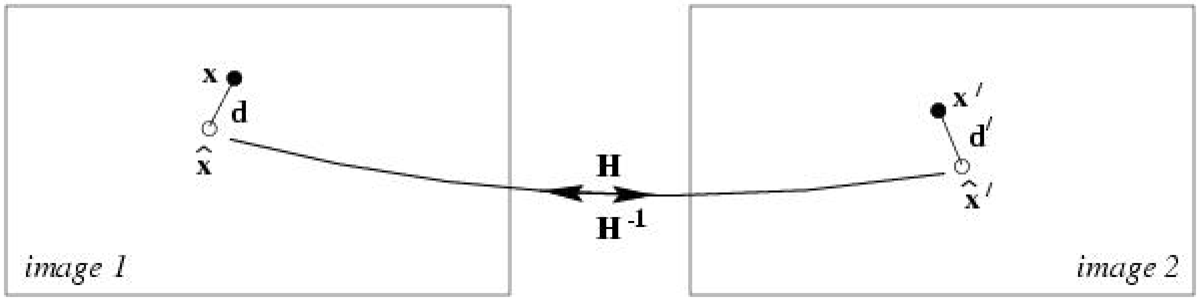


$$d\left(\mathrm{x},\mathrm{H}^{-1}\mathrm{x}'\right)^2+d\left(\mathrm{x}',\mathrm{H}\mathrm{x}\right)^2$$

2. Symmetric transfer error

$$\hat{\mathrm{H}}=\underset{\mathrm{H}}{\operatorname{argmin}}\sum_i d\left(\mathrm{x}_i,\mathrm{H}^{-1}\mathrm{x}'_i\right)^2+d\left(\mathrm{x}'_i,\mathrm{H}\mathrm{x}_i\right)^2$$

Reprojection error



$$d\left(\mathrm{x}, \hat{\mathrm{x}}\right)^2 + d\left(\mathrm{x}', \hat{\mathrm{x}}'\right)^2$$

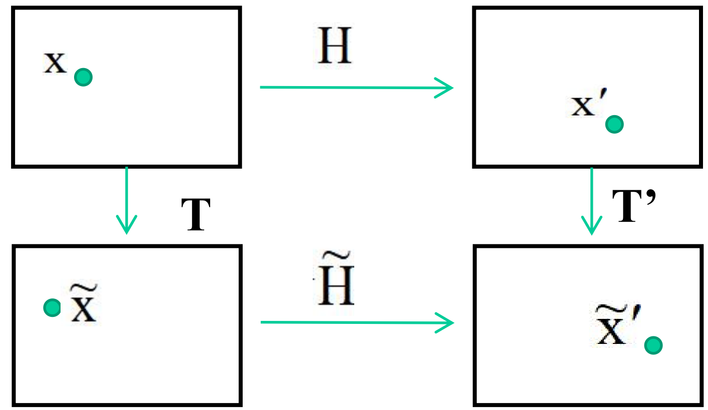
3. Reprojection error

$$\left(\hat{\mathrm{H}}, \hat{\mathrm{x}}_i, \hat{\mathrm{x}}_i'\right) = \underset{\mathrm{H}, \hat{\mathrm{x}}_i, \hat{\mathrm{x}}_i'}{\operatorname{argmin}} \sum_i d\left(\mathrm{x}_i, \hat{\mathrm{x}}_i\right)^2 + d\left(\mathrm{x}_i', \hat{\mathrm{x}}_i'\right)^2$$

$$\text{subject to } \hat{\mathrm{x}}_i' = \hat{\mathrm{H}}\hat{\mathrm{x}}_i$$

Normalized DLT Algorithm

Invariance to transforms ?



$$x' = Hx$$

$$\tilde{x} = Tx$$

$$\tilde{x}' = T'x'$$

$$\tilde{x}' = \tilde{H}\tilde{x}$$

$$T'x' = \tilde{H}Tx$$

$$x' = T'^{-1}\tilde{H}Tx$$

$$H \overset{?}{=} T'^{-1}\tilde{H}T$$

Will result change for the DLT algorithm?

Normalizing transformations

- Since DLT is not invariant,
  what is a good choice of coordinates?

  e.g.
  - The points are translated so that their centroid is at the origin
  - The points are then isotropicly scaled so that the average distance from the origin is equal to $\sqrt{2}$
  - This transformation is applied to each of the two images independently.

Normalized DLT algorithm

<u>Objective</u>

Given n≥4 2D to 2D point correspondences $\{x_i \leftrightarrow x_i'\}$, determine the 2D homography matrix H such that $x_i' = Hx_i$

<u>Algorithm</u>

(i) Normalize points $\tilde{x}_i = T_{norm} x_i, \tilde{x}_i' = T'_{norm} x_i'$

(ii) Apply DLT algorithm to $\tilde{x}_i \leftrightarrow \tilde{x}_i'$,

(iii) Denormalize solution $H = T'^{-1}_{norm} \tilde{H} T_{norm}$