Answers to Exercises

Section 4.1 2, 5, 8

Section 4.2 12, 17, 22, 23

Section 4-1

2. F(x) = $\frac{1}{10}$ for $-5 \le x \le 5$, and = 0 otherwise

a. $P(X < 0) = \int_{-5}^{0} \frac{1}{10} dx = .5$

b. $P(-2.5 < X < 2.5) = \int_{-2.5}^{2.5} \frac{1}{10} dx = .5$

c. $P(-2 \le X \le 3) = \int_{-2}^{3} \frac{1}{10} dx = .5$

d. $P(k < X < k+4) = \int_{k}^{k+4} \frac{1}{10} dx = \frac{x}{10}\Big]_{k}^{k+4} = \frac{1}{10}[(k+4) - k] = .4$

5.

a. $1 = \int_{-\infty}^{\infty} f(x)dx = \int_{0}^{2} kx^2 dx = k\left(\frac{x^3}{3}\right)\Big]_{0}^{2} = k\left(\frac{8}{3}\right) \Rightarrow k = \frac{3}{8}$

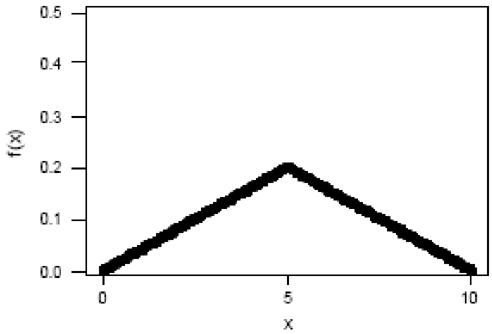
b. $P(0 \le X \le 1) = \int_{0}^{1} \frac{3}{8} x^2 dx = \frac{1}{8} x^3 \Big]_{0}^{1} = \frac{1}{8} = .125$

c. $P(1 \le X \le 1.5) = \int_{1}^{1.5} \frac{3}{8} x^2 dx = \frac{1}{8} x^3 \Big]_{1}^{1.5} = \frac{1}{8}\left(\frac{3}{2}\right)^3 - \frac{1}{8}(1)^3 = \frac{19}{64} \approx .2969$

d. $P(X \ge 1.5) = 1 - \int_{0}^{1.5} \frac{3}{8} x^2 dx = \frac{1}{8} x^3 \Big]_{0}^{1.5} = 1 - \left[\frac{1}{8}\left(\frac{3}{2}\right)^3 - 0\right] = 1 - \frac{27}{64} = \frac{37}{64} \approx .5781$

8.

a.



b. $\int_{-\infty}^{\infty} f(y)dy = \int_0^5 \frac{1}{25} y dy + \int_5^{10} (\frac{2}{5} - \frac{1}{25} y)dy = \frac{y^2}{50}\Big]_0^5 + \left(\frac{2}{5}y - \frac{1}{50}y^2\right)\Big]_5^{10}$

$= \frac{1}{2} + \left[(4-2) - (2 - \frac{1}{2})\right] = \frac{1}{2} + \frac{1}{2} = 1$

c. $P(Y \le 3) = \int_0^3 \frac{1}{25} y dy = \frac{y^2}{50}\Big]_0^5 = \frac{9}{50} \approx .18$

d. $P(Y \le 8) = = \int_0^5 \frac{1}{25} y dy + \int_5^8 (\frac{2}{5} - \frac{1}{25} y)dy = \frac{23}{25} \approx .92$

e. $P(3 \le Y \le 8) = P(Y \le 8) - P(Y < 3) = \frac{46}{50} - \frac{9}{50} = \frac{37}{50} = .74$

f. $P(Y < 2 \text{ or } Y > 6) = = \int_0^3 \frac{1}{25} y dy + \int_6^{10} (\frac{2}{5} - \frac{1}{25} y)dy = \frac{2}{5} = .4$

Section 4-2

12.

a. P(X < 0) = F(0) = .5

b. P(-1 ≤ X ≤ 1) = F(1) – F(-1) = $\frac{11}{16}$ = .6875

c. P(X > .5) = 1 – P(X ≤ .5) = 1 – F(.5) = 1 - .6836 = .3164

d. $F(x) = F'(x) = \frac{d}{dx}\left(\frac{1}{2} + \frac{3}{32}\left(4x - \frac{x^3}{3}\right)\right) = 0 + \frac{3}{32}\left(4 - \frac{3x^2}{3}\right) = .09375\left(4 - x^2\right)$

e. $F(\tilde{\mu}) = .5$ by definition. F(0) = .5 from **a** above, which is as desired.

17

a. $F(X) = \frac{x - A}{B - A} = p \quad \Rightarrow \quad$ x = (100p)th percentile = A + (B - A)p

b. $E(X) = \int_A^B x \cdot \frac{1}{B - A} dx = \frac{1}{B - A} \cdot \frac{x^2}{2}\Big]_A^B = \frac{1}{2} \cdot \frac{1}{B - A} \cdot \left(B^2 - A^2\right) = \frac{A + B}{2}$

$$E(X^2) = \frac{1}{3} \cdot \frac{1}{B - A} \cdot \left(B^3 - A^3\right) = \frac{A^2 + AB + B^2}{3}$$

$$V(X) = \left(\frac{A^2 + AB + B^2}{3}\right) - \left(\frac{(A + B)}{2}\right)^2 = \frac{(B - A)^2}{12}, \quad \sigma_x = \frac{(B - A)}{\sqrt{12}}$$

c. $E(X^n) = \int_A^B x^n \cdot \frac{1}{B - A} dx = \frac{B^{n+1} - A^{n+1}}{(n + 1)(B - A)}$

22.

a. For $1 \le x \le 2$, $F(x) = \int_1^x 2\left(1 - \frac{1}{y^2}\right)dy = 2\left(y + \frac{1}{y}\right)\Big]_1^x = 2\left(x + \frac{1}{x}\right) - 4$, so the cdf is

$$F(x) = \begin{cases} 0 & x < 1 \\ 2\left(x + \frac{1}{x}\right) - 4 & 1 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

b. Set $F(x) = p$ and solve for x: $2\left(x + \frac{1}{x}\right) - 4 = p \Rightarrow 2x^2 - (p+4)x + 2 = 0 \Rightarrow$

$\eta(p) = x = \frac{(p+4) + \sqrt{(p+4)^2 - 4(2)(2)}}{2(2)} = \frac{p + 4 + \sqrt{p^2 + 8p}}{4}$. (The other root of the quadratic gives solutions outside the interval [1, 2].) To find the median $\tilde{\mu}$, set $p = .5$: $\tilde{\mu} = \eta(.5) = \ldots = 1.640$.

c. $E(X) = \int_1^2 x \cdot 2\left(1 - \frac{1}{x^2}\right)dx = 2\int_1^2\left(x - \frac{1}{x}\right)dx = 2\left(\frac{x^2}{2} - \ln(x)\right)\Big]_1^2 = 1.614$. Similarly,

$E(X^2) = 2\int_1^2\left(x^2 - 1\right)dx = 2\left(\frac{x^3}{3} - x\right)\Big]_1^2 = \frac{8}{3} \Rightarrow V(X) = .0626.$

d. The amount left is given by $h(x) = \max(1.5 - x, 0)$, so

$E(h(X)) = \int_1^2 \max(1.5 - x, 0) f(x)dx = 2\int_1^{1.5}(1.5 - x)\left(1 - \frac{1}{x^2}\right)dx = .061$.

23. With X = temperature in °C, temperature in °F = $\frac{9}{5}X + 32$, so

$E\left[\frac{9}{5}X + 32\right] = \frac{9}{5}(120) + 32 = 248, \quad Var\left[\frac{9}{5}X + 32\right] = \left(\frac{9}{5}\right)^2 \cdot (2)^2 = 12.96$,

so $\sigma = 3.6$