Course 6

Some important distribution of discrete random variable:

1. **geometric distribution几何分布**

   > 有放回抽样

   1. Definition: The probability that the first $k-1$ times are all failures, and the probability that the k times are successes
   2. pmf of geometry distribution = $(1-p)^{x-1}p$
   3. cdf of geometry distribution = $1-(1-p)^x$
   4. $E(X)=\dfrac{1}{p}$

   > Proof:
   >
   > $$E(X)=\sum_D x\cdot p(x)=\sum_{x=1}^{\infty}xp(1-p)^{x-1}=p\sum_{x=1}^{\infty}\left[-\frac{\mathrm{d}}{\mathrm{d}p}(1-p)^x\right]$$
   >
   > $$=-p\frac{\mathrm{d}}{\mathrm{d}p}\sum_{x=1}^{\infty}(1-p)^x=-p\frac{\mathrm{d}\left[\frac{1-p}{1-(1-p)}\right]}{\mathrm{d}p}=-p\left[\frac{1-p}{p}\right]'=\frac{1}{p}$$

   5. $V(X)=\dfrac{1-p}{p^2}$

2. **hypergeometric distribution超几何分布**

   > **不放回抽样!**

   1. Sample consists N individuals, objects or elements
   2. Each individual can be characterized as a S(success) or F(failure), thera are M success in the population
   3. **A sample of n individuals** is selected **without replacement** in such a way that each subset of size n is equally likely to be chosen
   4. X = the number of S's in the sample
   5. $P(X=x)=h(x;n,M,N)=\dfrac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$
   6. $E(X)=np$, where $p=\dfrac{M}{N}$
   7. $V(X)=\left(\dfrac{N-n}{N-1}\right)np(1-p)$, where $p=\dfrac{M}{N}$

      跟二项分布的$V(X)$就相差$\left(\dfrac{N-n}{N-1}\right)$，这个项被称为有限总体校正因子(finite population correction factor)

3. **negative binomial distribution负二项分布**

   > 有放回抽样

   1. The experiment consists of a sequence of **independent** trials
   2. Only 2 results: S(success) or F(faliure)
   3. $P(S)=p$(p is a constant)
   4. The experiment continues (trials are performed) until a total of r **successes** have been observed
   5. X = the number of F's in the sample
   6. pmf of negative binomial distribution = $nb(x;r,p)=\dbinom{x+r-1}{r-1}p^r(1-p)^x$
   7. cdf of negative binomial distribution = $nB(x;r,p)$
   8. $E(X)=\dfrac{r(1-p)}{p}$
   9. $V(X)=\dfrac{r(1-p)}{p^2}$

4. **Poisson probability distribution泊松分布**

   > 有放回抽样

1. $\lambda = \mu = E(X)$
2. $p(x, \lambda) = \dfrac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, 3, \ldots$
3. $e \approx 2.71828$

> Since
>
> $$e^{\lambda} = 1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots = \sum_{i=0}^{\infty} \frac{\lambda^i}{i!}$$
>
> $$\sum_{i=0}^{\infty} \frac{e^{-\lambda}\lambda^i}{i!} = e^{-\lambda}e^{\lambda} = 1$$

4. When $n \geq 100$, $p \leq 0.01$ and $np \leq 20$, a binomial distribution can be appximately seen as a Poisson distribution, λ is $E(X)$
5. $E(X) = V(X) = \lambda$

Homework

Section 3.4 46, 47, 48, 54

Section 3.5 68, 69, 72, 75

Section 3.6 79, 84, 86, 87