Course 7

He said this chapter will be tough🥵🥵🥵

Continuous Random Variables连续随机变量

Definition: Value is a entire interval of numbers

eg: PH(酸碱度) $\longrightarrow$ [0, 14]

Probability Distribution概率分布

- $f(x)$ represents the probability in exact x, namely, the **pdf**
- $f(x) > 0$ for all x
- $P(a \leq X \leq b) = \int_a^b f(x)\mathrm{d}x$
- $\int_{-\infty}^{\infty} f(x)\mathrm{d}x = 1$
- Difference between **discrete** & **continuous** RV
  - For discrete RV: $P(X = c) = p(c)$
  - For continuous RV: $P(X = c) = \int_c^c f(x)\mathrm{d}x = 0$, therefore $P(a \leq X \leq b) = P(a \leq X < b) = P(a < X \leq b) = P(a < X < b)$
- **cdf**: $F(X) = \int_{-\infty}^{x} f(y)\mathrm{d}y$
- **Uniform distribution**:

$$P(X, A, B) = \begin{cases} \frac{1}{B-A} & A \leq X \leq B \\ 0 & \text{otherwise} \end{cases}$$

$$F(X, A, B) = \begin{cases} 0 & X < A \\ \frac{x-A}{B-A} & A \leq X < B \\ 1 & X \geq B \end{cases}$$

  Caution: When the range is given, you need to write down all the conditions like above.
- Given $F(X)$ to find $P(X)$:
  $P(X > a) = 1 - F(a)$
  $P(a \leq X \leq b) = F(b) - F(a)$

Percentiles of a Continuous Distribution连续随机变量分布的百分位表示

Let p be a number between 0 and 1. The $(100p)$th percentile of the distribution of a continuous rv X, denoted by $\eta(p)$, $p = F(\eta(p)) = \int_{-\infty}^{\eta(p)} f(y)\mathrm{d}y$

> **Eg**:
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> 根据“有6亿人每个月的收入也就1000元”这句话的内容，我们可知，在有大约14亿人口的国家，要统计个人的收入数据，有以下的结论
>
> 月收入1000这个数字，有这样的关系$F(1000) = \dfrac{6}{14} \approx 43\%$，这代表，1000是当$p \approx 0.43$时的结果，百分位数是$\eta(0.43) = 1000$

Median中位数

The median of a continuous distribution, denoted by $\tilde{\mu}$, is the 50^{th} percentile. $F(\tilde{\mu}) = 0.5$. That is, half the area under the density curve is to the left of $\tilde{\mu}$ and to the right of $\tilde{\mu}$

- For symmetric distribution, $\mu = \tilde{\mu}$, but in general, $\mu \neq \tilde{\mu}$

expected value期望

- $\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\mathrm{d}x$

- $E(h(X)) = \int_{-\infty}^{\infty} h(x)f(x)\mathrm{d}x$
- $E(aX + b) = aE(X) + b$

variance & standard difference方差与标准差

- $\sigma^2 = V(X) = \int_{-\infty}^{\infty} (x - \mathrm{\mu})^2 f(x)\mathrm{d}x = E[(X - \mathrm{\mu})^2]$
- $V(aX + b) = a^2 V(X)$
- $V(X) = \int_{-\infty}^{\infty} [x - E(X)]^2 f(x)\mathrm{d}x$
- $V(X) = E(X^2) - [E(X)]^2$

Homework

Section 4.1 2, 5, 8
Section 4.2 12, 17, 22, 23