Answers to Exercises

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Section 5-3

38.

a. Since each X is 0 or 1 or 2, the possible values of T_o are 0, 1, 2, 3, 4.
$P(T_o = 0) = P(X_1 = 0 \text{ and } X_2 = 0) = (.2)(.2) = .04$ since X_1 and X_2 are independent.
$P(T_o = 1) = P(X_1 = 1 \text{ and } X_2 = 0, \text{ or } X_1 = 0 \text{ and } X_2 = 1) = (.5)(.2) + (.2)(.5) = .20$.
Similarly, $P(T_o = 2) = .37$, $P(T_o = 3) = .30$, and $P(T_o = 4) = .09$. These values are displayed in the pmf table below.

t_o	0	1	2	3	4
$p(t_o)$	.04	.20	.37	.30	.09

b. $E(T_o) = 0(.04) + 1(.20) + 2(.37) + 3(.30) + 4(.09) = 2.2$. This is exactly twice the population mean: $E(T_o) = 2\mu$.

c. First, $E(T_o^2) = 0^2(.04) + 1^2(.20) + 2^2(.37) + 3^2(.30) + 4^2(.09) = 5.82$. Then $V(T_o) = 5.82 - (2.2)^2 = .98$. This is exactly twice the population variance: $V(T_o) = 2\sigma^2$.

d. Assuming the pattern persists (and it does), when $T_o = X_1 + X_2 + X_3 + X_4$ we have $E(T_o) = 4\mu = 4(1.1) = 4.4$ and $V(T_o) = 4\sigma^2 = 4(.49) = 1.96$.

e. The event $\{T_o = 8\}$ occurs iff we encounter 2 lights on all four trips; i.e., $X_i = 2$ for each X_i. So, assuming the X_i are independent,
$P(T_o = 8) = P(X_1 = 2 \cap X_2 = 2 \cap X_3 = 2 \cap X_4 = 2) = P(X_1 = 2) \cdots P(X_4 = 2) = (.3)^4 = 0081$.
Similarly, $T_o = 7$ iff exactly three of the X_i are 2 and the remaining X_i is 1. The probability of that event is $P(T_o = 7) = (.3)(.3)(.3)(.5) + (.3)(.3)(.5)(.3) + \ldots = 4(.3)^3(.5) = .054$. Therefore, $P(T_o \geq 7) = P(T_o = 7) + P(T_o = 8) = .054 + .0081 = .0621$.

41. The tables below delineate all 16 possible (x_1, x_2) pairs, their probabilities, the value of $\bar{x}$ for that pair, and the value of r for that pair. Probabilities are calculated using the independence of X_1 and X_2.

(x_1, x_2)	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4
probability	.16	.12	.08	.04	.12	.09	.06	.03
$\bar{x}$	1	1.5	2	2.5	1.5	2	2.5	3
r	0	1	2	3	1	0	1	2

(x_1, x_2)	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
probability	.08	.06	.04	.02	.04	.03	.02	.01
$\bar{x}$	2	2.5	3	3.5	2.5	3	3.5	4
r	2	1	0	1	3	2	1	2

a. Collecting the $\bar{x}$ values from the table above yields the pmf table below.

$\bar{x}$	1	1.5	2	2.5	3	3.5	4
$p(\bar{x})$	.16	.24	.25	.20	.10	.04	.01

b. $P(\bar{X} \le 2.5) = .16 + .24 + .25 + .20 = .85$.

c. Collecting the r values from the table above yields the pmf table below.

r	0	1	2	3
$p(r)$	.30	.40	.22	.08

d. With $n = 4$, there are numerous ways to get a sample average of at most 1.5, since $\bar{X} \le 1.5$ iff the sum of the X_i is at most 6. Listing out all options, $P(\bar{X} \le 1.5) = P(1,1,1,1) + P(2,1,1,1) + \ldots + P(1,1,1,2) + P(1,1,2,2) + \ldots + P(2,2,1,1) + P(3,1,1,1) + \ldots + P(1,1,1,3)$
$= (.4)^4 + 4(.4)^3(.3) + 6(.4)^2(.3)^2 + 4(.4)^2(.2)^2 = .2400.$

Section 5-4

46.

a. The sampling distribution of $\bar{X}$ is centered at $E(\bar{X}) = \mu = 12$ cm, and the standard deviation of the $\bar{X}$ distribution is $\sigma_{\bar{X}} = \frac{\sigma_{\bar{X}}}{\sqrt{n}} = \frac{.04}{\sqrt{16}} = .01$ cm.

b. With $n = 64$, the sampling distribution of $\bar{X}$ is still centered at $E(\bar{X}) = \mu = 12$ cm, but the standard deviation of the $\bar{X}$ distribution is $\sigma_{\bar{X}} = \frac{\sigma_{\bar{X}}}{\sqrt{n}} = \frac{.04}{\sqrt{64}} = .005$ cm.

c. $\bar{X}$ is more likely to be within .01 cm of the mean (12 cm) with the second, larger, sample. This is due to the decreased variability of $\bar{X}$ that comes with a larger sample size.

51. Individual times are given by $X \sim N(10, 2)$. For day 1, $n = 5$, and so

$$P(\bar{X} \le 11) = P\left(Z \le \frac{11-10}{2/\sqrt{5}}\right) = P(Z \le 1.12) = .8686\ .$$

For day 2, $n = 6$, and so

$$P(\bar{X} \le 11) = P(\bar{X} \le 11) = P\left(Z \le \frac{11-10}{2/\sqrt{6}}\right) = P(Z \le 1.22) = .8888\ .$$

Finally, assuming the results of the two days are independent (which seems reasonable), the probability the sample average is at most 11 min on both days is $(.8686)(.8888) = .7720$.

55.

a. 11 P.M. – 6:50 P.M. = 250 minutes. With $T_o = X_1 + \ldots + X_{40}$ = total grading time, $\mu_{T_o} = n\mu = (40)(6) = 240$ and $\sigma_{T_o} = \sigma \cdot \sqrt{n} = 37.95$, so $P(T_o \le 250) \approx$
$P\left(Z \le \dfrac{250-240}{37.95}\right) = P(Z \le .26) = .6026.$

b. The sports report begins 260 minutes after he begins grading papers.
$P(T_0 > 260) = P\left(Z > \dfrac{260-240}{37.95}\right) = P(Z > .53) = .2981.$

Section 5-5

58.

a. $E(27X_1 + 125X_2 + 512X_3) = 27E(X_1) + 125E(X_2) + 512E(X_3)$
$= 27(200) + 125(250) + 512(100) = 87{,}850.$
$V(27X_1 + 125X_2 + 512X_3) = 27^2\ V(X_1) + 125^2\ V(X_2) + 512^2\ V(X_3)$
$= 27^2\ (10)^2 + 125^2\ (12)^2 + 512^2\ (8)^2 = 19{,}100{,}116.$

b. The expected value is still correct, but the variance is not because the covariances now also contribute to the variance.

70.

a. $E(Y_i) = \dfrac{1}{2}$, so $E(W) = \sum_{i=1}^{n} i \cdot E(Y_i) = \dfrac{1}{2}\sum_{i=1}^{n} i = \dfrac{n(n+1)}{4}$.

b. $V(Y_i) = \dfrac{1}{4}$, so $V(W) = \sum_{i=1}^{n} i^2 \cdot V(Y_i) = \dfrac{1}{4}\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{24}$.

73.

a. Both are approximately normal by the Central Limit Theorem.

b. The difference of two rvs is just an example of a linear combination, and a linear combination of normal rvs has a normal distribution, so $\bar{X} - \bar{Y}$ has approximately a normal distribution with $\mu_{\bar{X}-\bar{Y}} = 5$ and $\sigma_{\bar{X}-\bar{Y}} = \sqrt{\dfrac{8^2}{40} + \dfrac{6^2}{35}} = 1.621$.

c. $P\left(-1 \le \bar{X} - \bar{Y} \le 1\right) \approx P\left(\dfrac{-1-5}{1.6213} \le Z \le \dfrac{1-5}{1.6213}\right) = P(-3.70 \le Z \le -2.47) \approx .0068.$

d. $P\left(\bar{X} - \bar{Y} \ge 10\right) \approx P\left(Z \ge \dfrac{10-5}{1.6213}\right) = P(Z \ge 3.08) = .0010.$ This probability is quite small, so such an occurrence is unlikely if $\mu_1 - \mu_2 = 5$, and we would thus doubt this claim.