Answers to Exercises

Chapter 2

Section 2.1	2, 4, 9
Section 2.2	12 , 18, 27
Section 2.3	30, 38,40

Chapter 2

2.

a. $A = \{RRR, LLL, SSS\}$.

b. $B = \{RLS, RSL, LRS, LSR, SRL, SLR\}$.

c. $C = \{RRL, RRS, RLR, RSR, LRR, SRR\}$.

d. $D = \{RRL, RRS, RLR, RSR, LRR, SRR, LLR, LLS, LRL, LSL, RLL, SLL, SSR, SSL, SRS, SLS, RSS, LSS\}$

e. Event D' contains outcomes where either all cars go the same direction or they all go different directions:
$D' = \{RRR, LLL, SSS, RLS, RSL, LRS, LSR, SRL, SLR\}$.
Because event D totally encloses event C (see the lists above), the compound event $C \cup D$ is just event D:
$C \cup D = D = \{RRL, RRS, RLR, RSR, LRR, SRR, LLR, LLS, LRL, LSL, RLL, SLL, SSR, SSL, SRS, SLS, RSS, LSS\}$.
Using similar reasoning, we see that the compound event $C \cap D$ is just event C:
$C \cap D = C = \{RRL, RRS, RLR, RSR, LRR, SRR\}$.

4.

a. The $2^4 = 16$ possible outcomes have been numbered here for later reference.

	Home Mortgage Number			
Outcome	1	2	3	4
1	F	F	F	F
2	F	F	F	V
3	F	F	V	F
4	F	F	V	V
5	F	V	F	F
6	F	V	F	V
7	F	V	V	F
8	F	V	V	V
9	V	F	F	F
10	V	F	F	V
11	V	F	V	F
12	V	F	V	V
13	V	V	F	F
14	V	V	F	V
15	V	V	V	F
16	V	V	V	V

b. Outcome numbers 2, 3, 5, 9 above.

c. Outcome numbers 1, 16 above.

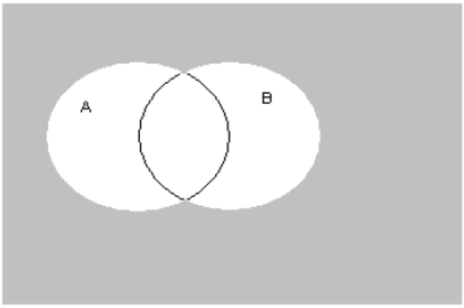
d. Outcome numbers 1, 2, 3, 5, 9 above.

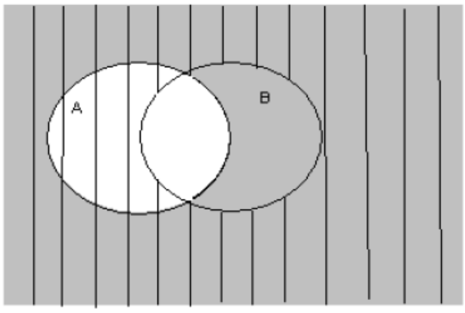
e. In words, the union of (c) and (d) is the event that either all of the mortgages are variable, or that at most one of them is variable-rate: outcomes 1, 2, 3, 5, 9, 16. The intersection of (c) and (d) is the event that all of the mortgages are fixed-rate: outcome 1.

f. The union of (b) and (c) is the event that either exactly three are fixed, or that all four are the same: outcomes 1, 2, 3, 5, 9, 16. The intersection of (b) and (c) is the event that exactly three are fixed and all four are the same type. This cannot happen (the events have no outcomes in common), so the intersection of (b) and (c) is $\varnothing$.

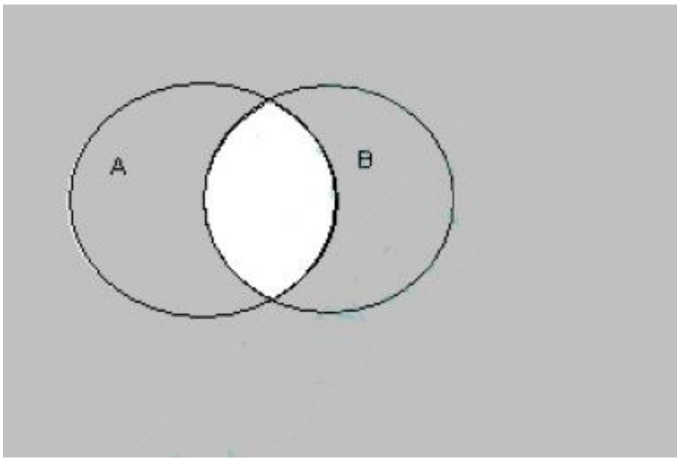
9.

a. In the diagram on the left, the shaded area is $(A \cup B)'$. On the right, the shaded area is A', the striped area is B', and the intersection $A' \cap B'$ occurs where there is BOTH shading and stripes. These two diagrams display the same area.





b. In the diagram below, the shaded area represents $(A \cap B)'$. Using the diagram on the right above, the union of A' and B' is represented by the areas that have either shading or stripes or both. Both of the diagrams display the same area.



12.

a. $P(A \cup B) = .50 + .40 - .25 = .65$.

b. $P(\text{neither } A \text{ nor } B) = P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - .65 = .35$.

c. The event of interest is $A \cap B'$; from a Venn diagram, we see $P(A \cap B') = P(A) - P(A \cap B) = .50 - .25 = .25$.

18. This situation requires the complement concept. The only way for the desired event NOT to happen is if a 75 W bulb is selected first. Let event A be that a 75 W bulb is selected first, and P(A)=4/15. Then the desired event is event A'.

So P(A')=1-P(A)=1-4/15=11/15

27. There are 10 equally likely outcomes: {A, B} {A, Co} {A, Cr} {A,F} {B, Co} {B, Cr} {B, F} {Co, Cr} {Co, F} and {Cr, F}.

a. $P(\{A, B\}) = \frac{1}{10} = .1$.

b. P(at least one C) = P({A, Co} or {A, Cr} or {B, Co} or {B, Cr} or {Co, Cr} or {Co, F} or {Cr, F}) = $\frac{7}{10} = .7$.

c. Replacing each person with his/her years of experience, P(at least 15 years) = P({3, 14} or {6, 10} or {6, 14} or {7, 10} or {7, 14} or {10, 14}) = $\frac{6}{10} = .6$.

30.

a. Because order is important, we'll use $P_{3,8} = (8)(7)(6) = 336$.

b. Order doesn't matter here, so we use $\binom{30}{6} = 593{,}775$.

c. The number of ways to choose 2 zinfandels from the 8 available is $\binom{8}{2}$. Similarly, the number of ways to choose the merlots and cabernets are $\binom{10}{2}$ and $\binom{12}{2}$, respectively. Hence, the total number of options (using the Fundamental Counting Principle) equals $\binom{8}{2}\binom{10}{2}\binom{12}{2} = (28)(45)(66) = 83{,}160$.

d. The numerator comes from part **c** and the denominator from part **b**: $\frac{83{,}160}{593{,}775} = .140$.

e. We use the same denominator as in part **d**. The number of ways to choose all zinfandel is $\binom{8}{6}$, with similar answers for all merlot and all cabernet. Since these are disjoint events, P(all same) = P(all zin) + P(all merlot) + P(all cab) = $\dfrac{\binom{8}{6}+\binom{10}{6}+\binom{12}{6}}{\binom{30}{6}} = \dfrac{1162}{593{,}775} = .002$.

38.

a. There are 6 75W bulbs and 9 other bulbs. So, P(select exactly 2 75W bulbs) = P(select exactly 2 75W bulbs and 1 other bulb) $= \frac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}} = \frac{(15)(9)}{455} = .2967$.

b. P(all three are the same rating) = P(all 3 are 40W or all 3 are 60W or all 3 are 75W) =

$$\frac{\binom{4}{3}+\binom{5}{3}+\binom{6}{3}}{\binom{15}{3}} = \frac{4+10+20}{455} = .0747.$$

c. P(one of each type is selected) $= \frac{\binom{4}{1}\binom{5}{1}\binom{6}{1}}{\binom{15}{3}} = \frac{120}{455} = .2637$.

d. It is necessary to examine at least six bulbs if and only if the first five light bulbs were all of the 40W or 60W variety. Since there are 9 such bulbs, the chance of this event is

$$\frac{\binom{9}{5}}{\binom{15}{5}} = \frac{126}{3003} = .042.$$

40.

a. If the A's were distinguishable from one another, and similarly for the B's, C's and D's, then there would be 12! possible chain molecules. Six of these are:

$A_1A_2A_3B_2C_3C_1D_3C_2D_1D_2B_3B_1$ $\quad A_1A_3A_2B_2C_3C_1D_3C_2D_1D_2B_3B_1$

$A_2A_1A_3B_2C_3C_1D_3C_2D_1D_2B_3B_1$ $\quad A_2A_3A_1B_2C_3C_1D_3C_2D_1D_2B_3B_1$

$A_3A_1A_2B_2C_3C_1D_3C_2D_1D_2B_3B_1$ $\quad A_3A_2A_1B_2C_3C_1D_3C_2D_1D_2B_3B_1$

These 6 (=3!) differ only with respect to ordering of the 3 A's. In general, groups of 6 chain molecules can be created such that within each group only the ordering of the A's is different. When the A subscripts are suppressed, each group of 6 "collapses" into a single molecule (B's, C's and D's are still distinguishable).

At this point there are (12!/3!) different molecules. Now suppressing subscripts on the B's, C's, and D's in turn gives $\frac{12!}{(3!)^4} = 369{,}600$ chain molecules.

b. Think of the group of 3 A's as a single entity, and similarly for the B's, C's, and D's. Then there are 4! = 24 ways to order these triplets, and thus 24 molecules in which the A's are contiguous, the B's, C's, and D's also. The desired probability is $\frac{24}{369{,}600} = .00006494$.