Course 9

Gamma Distribution伽马分布

For $\alpha > 0$, the gamma function $\Gamma(\alpha)$ is defined by:

$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}\mathrm{d}x$

- For any $\alpha > 1, \Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1)$
- For any positive integer, $n, \Gamma(n) = (n-1)!$
- $\Gamma(\frac{1}{2}) = \sqrt{\pi}$
- **pdf** of X:

$$f(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^\alpha\Gamma(\alpha)}x^{\alpha-1}e^{-\frac{x}{\beta}} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- Let $\beta = 1$, it becomes a **standard Gamma distribution**:

$$f(x;\alpha) = \begin{cases} \frac{x^{\alpha-1}e^{-x}}{\Gamma(\alpha)} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- **cdf** of standard Gamma distribution X: Refer to **Appendix Table A.4**:

$$F(x;\alpha) = \begin{cases} \int_0^x \frac{y^{\alpha-1}e^{-y}}{\Gamma(\alpha)}\mathrm{d}y & x \geq 0 \\ 0 & x < 0 \end{cases}$$

- β代表伽马分布曲线在x轴上的拉升和压缩，α代表曲线的走向
    - β越大，x轴上的范围越长
    - $\alpha \leq 1$时，减函数
    - $\alpha > 1$时，先增后减
- $E(X) = \mu = \alpha\beta$
- $V(X) = \sigma^2 = \alpha\beta^2$
- For any Gamma distribution with parameter α and β, its **cdf** is $P(X \leq x) = F(x;\alpha,\beta) = F(\frac{x}{\beta};\alpha)$(like standard Gamma distribution)

Exponential Distribution指数分布

Let $\alpha = 1$, $\beta = \frac{1}{\lambda}(\lambda > 0)$, Gamma distribution becomes an exponential distribution

- **pdf** of X:

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- $E(X) = \frac{1}{\lambda}$
- $V(X) = \frac{1}{\lambda^2}$
- **cdf** of X:

$$F(x;\lambda) = \begin{cases} 0 & x < 0 \\ 1 - e^{-\lambda x} & x \geq 0 \end{cases}$$

Chi-Squared Distribution卡方分布

Let $\alpha = \frac{v}{2}(v > 0)$, $\beta = 2$, Gamma distribution becomes an Chi-Squared distribution

Weibull Distribution韦布尔分布

A random variable X is said to have a Weibull distribution with parameters α and β ($\alpha > 0$, $\beta > 0$) if the pdf of X is:

$$f(x;\alpha,\beta)=\begin{cases}\frac{\alpha}{\beta^\alpha}x^{\alpha-1}e^{-(x/\beta)^\alpha} & x\geq 0\\ 0 & x<0\end{cases}$$

When $\alpha=1$, the pdf of Weibull distribution reduces to the exponential distribution (with $\beta=\dfrac{1}{\lambda}$), so the exponential distribution is a special case of both the gamma and Weibull distributions

- Mean: $\mu=\beta\Gamma(1+\dfrac{1}{\alpha})$
- Variance: $\sigma^2=\beta^2\left\{\Gamma\left(1+\frac{2}{\alpha}\right)-\left[\Gamma\left(1+\frac{1}{\alpha}\right)\right]^2\right\}$
- cdf:

$$F(x;\alpha,\beta)=\begin{cases}0 & x<0\\ 1-e^{-(x/\beta)^\alpha} & x\geq 0\end{cases}$$

Lognormal Distribution对数分布

Beta Distribution贝塔分布

Probability Plots概率绘图
Select the dot (x,y) by $x=[100(i-0.5)/n]\mathrm{th}$ percentile of the distribution, $y=i\mathrm{th}$ smallest sample observation

- 只有满足正态分布才会绘制出一条近似直线

Homework

Section 4.4 59, 67, 70
Section 4.6 87, 88