Introduction to Statistics Note

2024 Spring Semester

21 CST H3Art

Chapter 5: Sampling Distributions

5.1 The Sampling Distribution of a Sample Mean

Mean of a sampling distribution of a sample mean:
There is no tendency for a sample mean to fall systematically above or below μ, even if the distribution of the raw data is skewed. Thus, the sample mean is an **unbiased estimate** of the population mean μ.

Standard deviation of a sampling distribution of a sample mean:
The standard deviation of the sampling distribution measures how much the sample statistic varies from sample to sample. It is smaller than the standard deviation of the population by a factor of $\sqrt{n}$.

The Central Limit Theorem:
Draw an SRS of size n from any population with mean μ and standard deviation σ. The **central limit theorem (CLT)** says that when n is sufficiently large, the sampling distribution of the **sample mean is approximately Normal**, specifically,

$$\bar{X}\ N(\mu, \frac{\sigma}{\sqrt{n}})$$

5.2 Sampling Distributions for Proportions

Choose an SRS of size n from a population with p as the true proportion of success $\rightarrow$ it follows that the population standard deviation is $\sqrt{p(1-p)}$

- The mean of the sampling distribution of $\hat{p}$ is $\mu_{\hat{p}} = p$
- The standard deviation of the sampling distribution of $\hat{p}$ is $\sigma_{\hat{p}} = \sqrt{p(1-p)/n}$

As n **increases**, the sampling distribution **becomes approximately Normal**.

For sufficiently large n:

$$\hat{p}\ N(p, \sqrt{p(1-p)/n})$$