Reviewing Exercises

Chapter One

1. The domain of the function $f(x)=\sqrt{x^2-1}+\ln(4-x^2)$ is ________________.

2. The domain of $y=\dfrac{x+2}{4-\sqrt{x^2-9}}$ is ____________________.

3. The domain of the function $y=\dfrac{\log(2-x)}{\sqrt{|x|-1}}$ is ___________________.

4. The domain of the function $y=\dfrac{\log(3-x)}{\sqrt{|x|-2}}$ is __________________.

5. The domain of the function $y=\ln(1-x)+\arccos(|x|-1)$ is __________.

6. The domain of the function $y=\log(2-x)+\sqrt{x^2-1}$ is _______________.

Chapter Two

7. $\lim\limits_{x\to-1}(-x^3+2x-5)=$_____________

8. $\lim\limits_{x\to\infty} x\sin\dfrac{1}{x}=$______________

9. $\lim\limits_{x\to+\infty}(x\sin\dfrac{1}{x}+\dfrac{1}{x}\sin x)=$______________

10. $\lim\limits_{x\to+\infty}(\sqrt{x+2}-\sqrt{x})=$____________

11. $\lim\limits_{x\to+\infty}(\sqrt{x+2}-\sqrt{x+1})=$______________

12. $\lim\limits_{n\to\infty}\left(\sqrt{n+4}-\sqrt{n-4}\right)=$ ________

13. $\lim\limits_{n\to\infty}(\sqrt{n+2}-\sqrt{n+1})=$____________

14. $\lim\limits_{x\to\infty}\left(\dfrac{1+x}{x}\right)^{3x}=$_______

15. $\lim\limits_{x\to\infty}\left(\dfrac{1+x}{x}\right)^{2x}=$________

16. $\lim\limits_{x\to 0}(1+2x)^{\frac{1}{x}} =$ _______

17. $\lim\limits_{x\to 0}\sqrt[x]{1-2x} =$ _________ .

18. $\lim\limits_{n\to\infty}\left(1-\dfrac{2}{n}\right)^{n} =$ __________

19. $\lim\limits_{n\to\infty} 3^{n}\sin\dfrac{\pi}{3^{n}} =$ _________

20. $\lim\limits_{n\to\infty} 2^{n}\sin\dfrac{\pi}{2^{n}} =$ ______

21. If $\lim\limits_{x\to 2}\dfrac{x^{2}+ax+b}{x^{2}-x-2} = 2$, then ( )

A. $a=-8,\ b=2$; B. $a=2,\ b=-8$;

C. $a=2$, b is arbitrary; D. a, b are arbitrary.

22. $\lim\limits_{n\to\infty}\left(1-\dfrac{1}{n}\right)^{n} =$ ( )

A. e; B. e^{-1}; C. 1; D. ∞.

23. For the following limits, ( ) exists

A. $\lim\limits_{x\to 0}\dfrac{1}{e^{x}-1}$; B. $\lim\limits_{x\to\infty}\dfrac{x^{2}}{1-x^{2}}$; C. $\lim\limits_{x\to\infty}\sin x$; D. $\lim\limits_{x\to 0} e^{\frac{1}{x}}$.

24. If for any x, $h(x)\le f(x)\le g(x)$, $\lim\limits_{x\to\infty}[g(x)-h(x)]=0$, then $\lim\limits_{x\to\infty} f(x)$ ( )

A. exists and the limit is 0; B. exists but the limit is not 0; C. doesn't exist; D. may exist.

25. Find the following limits:

(1) $\lim\limits_{x\to 2}(-x^{2}+5x-2)$

(2) $\lim\limits_{t\to 1}\dfrac{t^{2}+t-2}{t^{2}-1}$

(3) $\lim\limits_{x\to -2}\dfrac{-2x-4}{x^{3}+2x^{2}}$

(4) $\lim\limits_{x\to 9}\dfrac{\sqrt{x}-3}{x-9}$

(5) $\lim\limits_{x \to 1} \dfrac{x-1}{\sqrt{x+3}-2}$

(6) $\lim\limits_{x \to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}$

26. Find the following limits:

(1) $\lim\limits_{y \to 0} \dfrac{\sin 3y}{4y}$

(2) $\lim\limits_{x \to 0} \dfrac{\sin 5x}{\sin 4x}$

(3) $\lim\limits_{x \to 0} \dfrac{x + x\cos x}{\sin x \cos x}$

27. The discontinuous points of the function $f(x) = \dfrac{x+1}{x^2-4x+3}$ are ______

28. The function $f(x) = \dfrac{x}{x^2+4x-5}$ is discontinuous at _____________.

29. The discontinuous points of the function $y = \dfrac{x-1}{(x-1)(x-2)}$ are _________

30. If $f(x) = \begin{cases} x\sin\dfrac{1}{x}, & x > 0 \\ a + x^2, & x \le 0 \end{cases}$ is continuous on $(-\infty, +\infty)$, then $a =$__________

31. If $f(x) = \begin{cases} 1 + x\sin\dfrac{1}{x}, & x > 0 \\ a + x^2, & x \le 0 \end{cases}$ is continuous on $(-\infty, +\infty)$, then $a =$_______

32. If $f(x) = \begin{cases} 4x+1, & x \ge 1 \\ x^2 + k, & x < 1 \end{cases}$ is continuous at $x = 1$, then $k =$_____ .

33. If $f(x) = \begin{cases} x^2 + a, & x \ne 0 \\ 4, & x = 0 \end{cases}$ is continuous at $x = 0$, then *a*= (D).

A. 1 B. 2 C. 3 D. 4

34. If $f(x) = \begin{cases} x+3, & x < 0 \\ x^2 - 2x + k, & x \ge 0 \end{cases}$ is continuous on R, then k =(   )

A. 3 B. 2 C. 1 D. 0

35. If $f(x)=\begin{cases} x\sin\dfrac{1}{x}, & x>0 \\ \ln(a+x^2), & x\le 0 \end{cases}$ is continuous on $(-\infty,+\infty)$, then $a=$ (   )

A. $\dfrac{1}{e}$; B. e; C. 2; D. 1

36. If $f(x)=\dfrac{e^{\frac{1}{x}}+e}{e^{\frac{1}{x}}-e}$, then (   )

A. $x=0$ is the jump discontinuity; B. $x=0$ is the removable discontinuity;
C. $x=1$ is the jump discontinuity; D. $x=1$ is the removable discontinuity.

37. If $f(x)=\begin{cases} x^2\sin\dfrac{1}{x}, & x\ne 0 \\ 0, & x=0 \end{cases}$, then at $x=0$, $f(x)$ is (   )

A. discontinuous; B. continuous but non-differentiable;

C. continuous and differentiable; D. discontinuous but differentiable.

38. For what value of *a* is

$$f(x)=\begin{cases} x^2-1, & x<3 \\ 2ax, & x\ge 3 \end{cases}$$

continuous at every *x*?

39. For what value of *b* is

$$g(x)=\begin{cases} x, & x<-2 \\ bx^2, & x\ge -2 \end{cases}$$

continuous at every *x*?

40. For the function $f(x)=\dfrac{x^2+2x-3}{x^2-3x+2}$, the horizontal asymptote of its graph is ________,

the vertical asymptote is ________.

41. The horizontal asymptote of the curve $y=\dfrac{x^2}{x^2-1}$ is _______

42. Given $f(x)=\dfrac{x^2-9}{x^2+3x}$, the vertical asymptote is ______, the horizontal asymptote is___

43. The horizontal asymptote of the curve $y=\dfrac{x^2+2x-3}{2x^2-x+1}$ is __________

44. If $f(x)=\dfrac{\sqrt{x^2+1}}{3x-5}$, find all the horizontal asymptotes and vertical asymptotes of its

graph.

45. Find the following limits:

(1) $\lim\limits_{x\to\infty}\dfrac{2x^3+7}{x^3-x^2+x+7}$

(2) $\lim\limits_{x\to\infty}\dfrac{x+1}{x^2+3}$

(3) $\lim\limits_{x\to+\infty}(\sqrt{x+9}-\sqrt{x+4})$

46. Find the limits of the following functions.

(1) $\lim\limits_{x\to-1}\dfrac{x^2+2x+1}{x^2-2x-3}$

(2) $\lim\limits_{x\to0}\dfrac{x^2}{1-\sqrt{1+x^2}}$

(3) $\lim\limits_{x\to0}\dfrac{\tan x-\sin x}{x^3}$

(4) Find $\lim\limits_{n\to\infty}\left(\dfrac{1}{n^2+\pi}+\dfrac{2}{n^2+2\pi}+\cdots+\dfrac{n}{n^2+n\pi}\right)$

(5) $\lim\limits_{n\to\infty}\left(\dfrac{1}{\sqrt{n^2+\pi}}+\dfrac{1}{\sqrt{n^2+2\pi}}+\cdots+\dfrac{1}{\sqrt{n^2+n\pi}}\right)$

(6) Find $\lim\limits_{n\to\infty}\left(\dfrac{1}{n^2+\dfrac{1}{n}}+\dfrac{2}{n^2+\dfrac{2}{n}}+\dfrac{3}{n^2+\dfrac{3}{n}}+\cdots+\dfrac{n}{n^2+1}\right)$

47. If $f(x)\in C([0,\ 2])$, and $f(0)=f(2)=1$, show that there exists a $\xi\in[0,\ 2]$, such that $f(\xi)=\xi$.

Chapter Three

48. If $f(x) = -x^2 + \frac{2}{\sqrt{x}} - \pi$, then $f'(x) =$ ______

49. If $f'(x_0) = 5$, then $\lim\limits_{\varepsilon \to 0} \frac{f(x_0 - 2\varepsilon) - f(x_0)}{5\varepsilon} =$ ______.

50. If $f'(x_0) = 5$, then $\lim\limits_{\varepsilon \to 0} \frac{f(x_0 - 3\varepsilon) - f(x_0)}{5\varepsilon} =$ ______.

51. If $f'(x_0) = 1$, then $\lim\limits_{\varepsilon \to 0} \frac{f(x_0 - 5\varepsilon) - f(x_0)}{3\varepsilon} =$ ______.

52. If $f'(x_0) = 1$, then $\lim\limits_{\varepsilon \to 0} \frac{f(x_0 - 3\varepsilon) - f(x_0)}{5\varepsilon} =$ ______.

53. If $f'(x_0) = 2$, then $\lim\limits_{\varepsilon \to 0} \frac{f(x_0 - 2\varepsilon) - f(x_0)}{5\varepsilon} =$ ______.

54. If $f(x) = x(x+1)(x+2)\cdots(x+n)$, $(n \geq 2)$, then $f'(0) =$ ________

55. If $f(x) = x(x+1)(x+2)\cdots(x+100)$, then $f'(0) =$ ________

56. If $y = 2\ln\sqrt{x}$, then $y' =$ ______

57. The derivative of $y = x^{\frac{1}{x}}$ $(x > 0)$ is ______

58. The derivative of $y = x^x$ $(x > 0)$ is ______

59. If $f(x)$ has second derivative at any x, $y = f(\ln x)$, then $y'' =$ ______

60. The slope of the tangent line to the curve $y = 2^x + x$ at the point (0,1) is________

61. The equation for the line that is tangent to the curve $y = x^3 - x$ at the point $(-1, 0)$ is ________.

62. At the point where the curve of $y = \ln x$ intersects the line $x = e$, the tangent line of curve $y = \ln x$ is ( )

A. $x - ey = 0$; B. $x - ey - 2 = 0$; C. $ex - y = 0$; D. $ex - y - 2 = 0$

63. If $f(x)$ is differentiable at $x = a$, then $f'(a) =$ ( )

A. $\lim\limits_{h \to 0} \frac{f(a) - f(a+h)}{h}$; B. $\lim\limits_{h \to 0} \frac{f(a-h) - f(a)}{h}$;

C. $\lim\limits_{h\to 0}\dfrac{f(a+2h)-f(a)}{h}$;　　　D. $\lim\limits_{h\to 0}\dfrac{f(a+2h)-f(a+h)}{h}$.

64. Suppose that $f(x)$ has second derivative, $y=f(\ln x)$, then $y''=($ D $)$

A. $f''(\ln x)$;　　　B. $\dfrac{1}{x^2}f''(\ln x)$;

C. $\dfrac{1}{x^2}[f''(\ln x)+f'(\ln x)]$;　　　D. $\dfrac{1}{x^2}[f''(\ln x)-f'(\ln x)]$

65. If $\lim\limits_{x\to 0}\dfrac{f(x)}{1-\cos x}=-1$, and $f(x)$ is continuous at $x=0$, then at $x=0$ (　)

A. $f(x)$ is not differentiable;　　　B. $f(x)$ is differentiable, but $f'(0)\neq 0$;

C. $f(x)$ attains its relative minimum;　D. $f(x)$ attains its relative maximum.

66. If $f(x)$ is continuous at $x=0$ and $\lim\limits_{x\to 0}\dfrac{f(x)}{x^2}=-1$, then at $x=0$, $f(x)$ (　)

A. is non-differentiable;　　　B. is differentiable and $f'(0)\neq 0$;

C.takes local maximum;　　　D. takes local minimum.

67. If $f(x)=\begin{cases} x^2+a, & x\neq 0 \\ 4, & x=0 \end{cases}$ is continuous at $x=0$, then $a=(D)$.

A. 1　　　B. 2　　　C. 3　　　D. 4

68. If $f(x)=xe^x$, then $f'(0)=($　$)$

A. 1　　　B . 2　　　C. 3　　　D. 4

69. Find $\dfrac{d}{dx}\ln(x^2+9)$.

70. Find the derivative of $f(x)=\begin{cases} x^2\sin\dfrac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$

71. Find the derivative of the following functions:

(1) $f(x)=\sin x+\sqrt{x}+\arctan x-e^{-x}+\ln x$

(2) $f(x)=\dfrac{1}{x}+\cos x-\arcsin x+\sin 1+2^x$

(3) $y=(x^2+1)(x+5+\frac{1}{x})$

(4) $y=\frac{2x+5}{3x-2}$

(5) $y=e^{-x}\cos 2x$

(6) $y=\sqrt{3x^2-4x+6}$

(7) $y=\frac{\ln t}{t}$

(8) $y=\sqrt{x(x+1)}$

(9) $y=x^x$

(10) If $f(x)=-x^2+\frac{2}{\sqrt{x}}-\pi$, find $f'(x)$.

(11) If $y=(2x+1)(\sqrt[3]{x}-x^3)$, find y'.

(12) If $f(t)=\frac{t^2+2t}{\sqrt{t}-1}$, find $\frac{df}{dt}$.

(13) If $\rho=\theta\sin\theta+\frac{1}{2}\cos\theta$, find $\left.\frac{d\rho}{d\theta}\right|_{\theta=\frac{\pi}{4}}$.

(14) $f(x)=\frac{1}{3^{x^2+2x}}+\log_2(1-2x+x^3)$

(15) $g(\mathrm{x})=3x\sqrt{2x^2+3}$

(16) $g(x)=x^{\cos x}+\arctan e^x$

(17) $h(\mathrm{x})=\left(\frac{1}{x^2}-5\right)^{-2}$

72. If $2y=1+xe^{xy}$, then $\left.\frac{dy}{dx}\right|_{x=0}=\underline{\frac{1}{2}}$

73. Given $y-xy^2+x^2+1=0$. Find y'.

74. If r is a function of θ which is defined by the equation $\cos r+\cot\theta=e^{r\theta}$, find $\frac{dr}{d\theta}$.

75. If the equation $y-xe^y=1$ define the function y of x implicitly, find $y'|_{x=0}$.

Chapter Four

76. For $f(x)=\dfrac{x-1}{x^2-x+2}$, the y intercept of its graph is ( )

A. (1, 0) B. $(-0.5,\ 1)$ C. $(0,\ -0.5)$ D. (0, 1)

77. Let $g(x)=e^x-x-3$, the intervals where the graph of g is increasing is ( ).

A. $(-\infty,0]$ B. $(-\infty,-3]$ C. $[0,+\infty)$ D. $(-\infty,+\infty)$

78. If (1, 3) is the inflection point of the curve $f(x)=ax^3+bx^2$, then ( )

A. $a=-\dfrac{3}{2},\ b=\dfrac{9}{2}$; B. $a=\dfrac{3}{2},\ b=-\dfrac{9}{2}$; C. $a=-\dfrac{3}{2},\ b=-\dfrac{9}{2}$; D. $a=\dfrac{3}{2},\ b=\dfrac{9}{2}$

79. Finding limits:

(1) $\lim\limits_{x\to 0}\dfrac{(1+x)^{\frac{1}{x}}-e}{x}$

(2) $\lim\limits_{x\to 0}\dfrac{e^x-e^{-x}-2x}{x-\sin x}$

(3) $\lim\limits_{x\to 1}\left(\dfrac{x}{x-1}-\dfrac{1}{\ln x}\right)$

(4) Find $\lim\limits_{x\to 0}(\dfrac{1}{e^x-1}-\dfrac{1}{x})$

(5) $\lim\limits_{x\to 0}\left(\dfrac{a_1^x+a_2^x}{2}\right)^{\frac{1}{x}}$ $(a_1>0, a_2>0)$

80. (1) Determine the intervals where $y=x^{\frac{1}{x}}$, $(x>0)$ is increasing and where is decreasing.

(2) Find the largest term of the sequence $\left\{\sqrt[n]{n}\right\}$

(3) Find $\lim\limits_{x\to+\infty} x^{\frac{1}{x}}$

81. If the function $f(x)=\begin{cases}\dfrac{\ln(1+kx)}{2x}, & x\neq 0\\ -1, & x=0\end{cases}$ is differentiable at $x=0$, find k and

$f'(0)$

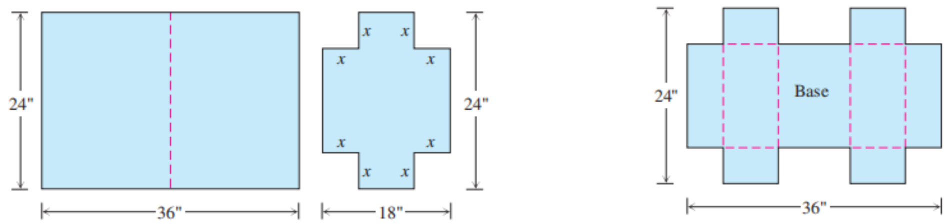
82. For $f(x) = -2x^3 + 6x^2 - 3$

(1) Find all critical points.
(2) Find the intervals on which the function is increasing and decreasing.
(3) Identify the function's local extreme values, if any, saying where they occur.
(4) Find where the graph of f is concave up and where it is concave down.
(5) Find all inflection points.

83. For $f(x) = \dfrac{x^2 - 4}{x^2 - 2}$

(1) Find all critical points.
(2) Find the open intervals on which the function is increasing and decreasing.
(3) Identify the function's local extreme values, if any, saying where they occur.
(4) Find where the graph of f is concave up and where it is concave down.
(5) Find all inflection points.
(6) Find all asymptotes.

84. **Designing a suitcase** A 24-in.-by-36-in. sheet of cardboard is folded in half to form a 24-in.-by-18-in. rectangle as shown in the accompanying figure. Then four congruent squares of side length x are cut from the corners of the folded rectangle. The sheet is unfolded, and the six tabs are folded up to form a box with sides and a lid. Find the value of x such that the box holds as much as possible.



85. A rectangular storage container with an open top is to have a volume of 10 cubic meters. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.

86. If $f(x)$ is continuous on $[0,1]$, and differentiable on $(0,1)$,

$$f(0) = 0,\ f(1) = \frac{1}{2},\ f\left(\frac{1}{2}\right) = 1,$$

then show that

(1) There exists at least a $\xi_1 \in (\frac{1}{2},\ 1)$ such that $f(\xi_1) = \xi_1$.

(2) There exists at least a $\xi \in (0,1)$ such that $f'(\xi) = 1$.

87. If $f(x)$ is continuous on [0, 1], and differentiable on (0, 1), $f(0)=0$, show that there exists at least a $\xi \in (0,1)$ such that

$$f'(\xi)=\frac{3f(\xi)}{1-\xi}.$$

Chapter Five

88. $\int (2x-\sqrt{x})dx =$ ________

89. $\int (\frac{1}{x}-3\sqrt{x}+\frac{1}{\sqrt{1-x^2}})dx =$ __________

90. $\int (\sqrt{x}-\sin x+\frac{1}{1+x^2})dx =$ __________

91. If $\int f(x)dx = 2xe^x + C,$ then $f(x)=$ ( )

A. $2xe^x$; **B.** $x\,e^x$;

C. $x+e^x$ **D.** $2e^x(1+x)$

92. If $\boldsymbol{f(x)}$ is continuous, and $\boldsymbol{\int f(x)dx = F(x)+C}$, then ( )

A. $\boldsymbol{\int f(2x)dx = F(2x)+C}$; B. $\boldsymbol{\int f(x^2)x\,dx = F(x^2)+C}$;

C. $\boldsymbol{\int f(e^x)e^x\,dx = F(e^x)+C}$; D. $\boldsymbol{\int f(\cos x)\sin x\,dx = F(\cos x)+C}$

93. If $f'(e^x)=x$, then $f(e^x)=$ ( )

A. $\frac{1}{2}x^2$ B. $\frac{1}{2}x^2+C$ C. xe^x+e^x+C D. xe^x-e^x+C

94. If $\frac{\sin x}{x}$ is one anti-derivative of $f(x)$, then $\int xf'(x)dx=$ ( )

A. $\frac{\sin x}{x}+C$; B. $\frac{1+\sin x}{x^2}+C$; C. $\cos x-\frac{2\sin x}{x}+C$; D. $\cos x+\frac{2\sin x}{x}+C$

95. Find the following integrals.

(1) $\int (\sqrt{x}-\sin x+2^x-\frac{1}{1+x^2}+1)dx$

(2) $\int (\frac{1}{x} - \cos x + x - \frac{1}{\sqrt{1-x^2}} + 2)dx$

(3) $\int (\frac{2}{x} + \frac{1}{\sqrt{x}} - \sin x + 2 - \frac{1}{1+x^2})\,\mathrm{d}\,x$

(4) $\int (2\cos 2x - 3\sin 3x)dx$

96. Finding the following indefinite integrals:

(1) $\int \frac{12+5t-3t^2}{t^3}dt$

(2) $\int \frac{2x+2}{x^2+2x}dx$

(3) $\int 2t^2(t^3+4)^{-2}\,dt$

(4) $\int x\sqrt{x-1}dx$

(5) $\int \frac{x^2}{\sqrt{4x^3-1}}dx$

97. Finding the following indefinite integrals:

(1) $\int \frac{e^x}{e^x+1}dx$

(2) $\int \frac{1}{1+e^x}\,dx$

(3) Find $\int x \arctan x dx$

(4) $\int \frac{x+2}{x^2-2x+5}dx$

(5) $\int \frac{x+\sqrt{\arctan x}}{1+x^2}\,dx$

(6) Find $\int x \arctan x dx$

(7) $\int \frac{\arctan e^x}{e^x} \mathrm{d}x$

(8) $\frac{\sin x}{x}$ is one anti-derivative of $f(x)$, find $\int x f'(x) dx$