The fifth time

Section 3.3 29， 33， 38， 41

Section 3.3

29.

a. $E(X) = \sum_{\text{all } x} xp(x) = 1(.05) + 2(.10) + 4(.35) + 8(.40) + 16(.10) = 6.45$ GB.

b. $V(X) = \sum_{\text{all } x} (x-\mu)^2 p(x) = (1-6.45)^2(.05) + (2-6.45)^2(.10) + \ldots + (16-6.45)^2(.10) = 15.6475.$

c. $\sigma = \sqrt{V(X)} = \sqrt{15.6475} = 3.956$ GB.

d. $E(X^2) = \sum_{\text{all } x} x^2 p(x) = 1^2(.05) + 2^2(.10) + 4^2(.35) + 8^2(.40) + 16^2(.10) = 57.25$. Using the shortcut formula, $V(X) = E(X^2) - \mu^2 = 57.25 - (6.45)^2 = 15.6475.$

33.

a. $E(X^2) = \sum_{x=0}^{1} x^2 \cdot p(x) = 0^2(1-p) + 1^2(p) = p.$

b. $V(X) = E(X^2) - [E(X)]^2 = p - [p]^2 = p(1-p).$

c. $E(X^{79}) = 0^{79}(1-p) + 1^{79}(p) = p$. In fact, $E(X^n) = p$ for any non-negative power n.

38. $(1/3.5) = \$.286$, while $E[h(X)] = E\left(\frac{1}{X}\right) = \sum_{x=1}^{6}\left(\frac{1}{x}\right)\cdot p(x) = \sum_{x=1}^{6}\left(\frac{1}{x}\right)\cdot\frac{1}{6} = \frac{1}{6}\sum_{x=1}^{6}\frac{1}{x} = \$.408.$

So you expect to win more if you gamble.

Note: In general, if $h(x)$ is concave up then $E[h(X)] > h(E(X))$, while the opposite is true if $h(x)$ is concave down.

41.

$$\begin{aligned}
\mathrm{V}(\mathrm{a}X + \mathrm{b}) &= \sum[aX + b - E(aX+b)]^2 \cdot P(X) \\
&= \sum[aX + b - aE(X) - b]^2 \cdot P(X) \\
&= \sum[aX - aE(X)]^2 \cdot P(X) \\
&= a^2 \cdot \sum[X - E(X)]^2 \cdot P(X) \\
&= a^2 \cdot \sigma_X^2
\end{aligned}$$