Physics FSE (2021-22) Homework 4

Please send the completed file to my mailbox yy.lam@qq.com by May 11, with using the filename format:

`2021xxxxxx_yourname_fse_hw4`

Please answer the questions by filling on these sheets. It would be perfect if you use e-pen directly writing on the sheets. If you do not have the appropriate hardware, you may handle the questions as usual by using pieces of blank papers, then take photos and paste them onto these question sheets.

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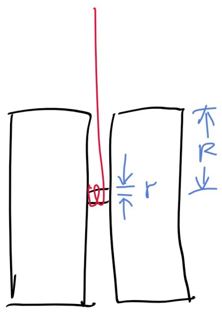
1. A yo-yo of total mass m consists of two solid cylinders of radius R, connected by a small spindle of negligible mass and radius r. The top of the string is held motionless while the string unrolls from the spindle freely under gravity. Given the angular momentum of a cylinder $L = \pi m R^2/T$, find the linear acceleration of the yo-yo. *(7 marks)*



Solution. *The energy conservation of the system gives*

$$KE_R + KE = PE$$

where KE_R, KE and PE are the rotational, the linear kinetic energies and the potential energy of the yo-yo. Let v be the falling velocity of the yo-yo, using the formula $v^2 = 2as$ we have

$$KE_R + \frac{1}{2}mv^2 = mg\frac{v^2}{2a} \tag{1}$$

where a is the falling acceleration of the yo-yo. For I is the moment of inertia of the yo-yo, $L = I\dot{\theta}$ where $\dot{\theta}$ is the angular velocity. Thus,

$$KE_R = \frac{1}{2}I\dot{\theta}^2 = \frac{1}{2}L\dot{\theta}.$$

The tangential velocity of the spindle is equal to the linear falling velocity of the yo-yo $r\dot{\theta} = 2\pi r/T = v$ where T being the period is not a constant in the system. Using the given angular momentum $L = \pi m R^2/T$,

$$KE_R = \frac{1}{2}L\dot{\theta} = \frac{1}{2}\frac{\pi m R^2}{T}\dot{\theta} = \frac{mR^2}{4r^2}v^2.$$

Therefore, the conservation equation (1) becomes

$$\frac{mR^2}{4r^2}v^2 + \frac{1}{2}mv^2 = mg\frac{v^2}{2a} \quad \Rightarrow \quad \frac{R^2}{2r^2} + 1 = \frac{g}{a} \quad \Rightarrow \quad a = \frac{g}{1 + \frac{R^2}{2r^2}}$$

2. Estimate the pressure at the centre of the earth in terms of the gravitational constant G, mass and radius of the earth M and R, respectively, assuming it is of constant density throughout.

Solution. *Gravitational force at the centre of the the earth is zero, however, the pressure is the maximum due to the non-directional property of pressure. We may use the familiar formula* $P = \rho\bar{g}R$ *to estimate the pressure at the centre where* R *is the height which is just the radius of the earth. Since the gravitational acceleration* g *is not a constant which varies along the radial direction. We have put* $\bar{g}$ *as the mean value of the gravitational accelerations over the radial interval. The gravitational acceleration function* $g(r)$ *can be found as*

$$g(r) = \frac{GMr}{R^3}$$

which is measured at r from the centre of the earth. For the average gravitational field $\bar{g}$*, integrate the field over the interval and divide by the interval:*

$$\bar{g} = \frac{1}{R}\int_0^R \frac{GMr}{R^3}\mathrm{d}r = \frac{1}{R}\frac{GMr^2}{2R^3}\bigg|_0^R = \frac{GM}{2R^2}$$

Thus the pressure at the centre of the earth is

$$P = \rho\bar{g}R = \left(\frac{M}{\frac{4}{3}R^3\pi}\right)\left(\frac{GM}{2R^2}\right)R = \frac{3GM^2}{8\pi R^3}.$$

3. A thin spherical shell with a mass of 2.5 kg and a diameter of 0.2 m is filled with helium (density 0.18 kg/m^3). It is then released from rest on the bottom of a pool of water that is 5 m deep. (a) Neglecting the fluid friction, show that the shell rises with constant acceleration and determine the value of that acceleration. (b) How long does it take for the **top** of the shell to reach the water's surface?

Solution. *(a) Since the upward net force equals to the product of its mass and acceleration, i.e.,*

$$\rho_w V g - mg - \rho_h V g = (m + \rho_h V)a \quad \Rightarrow \quad a = \frac{(\rho_w - \rho_h)Vg - mg}{m + \rho_h V}$$

where ρ_w *and* ρ_h *are the densities of water and helium, respectively.* m *and* V *are the mass and volume* $(4\pi r^3/3)$ *of the sphere. Since all terms on the right hand side are constants, the sphere rises with constant acceleration. Inserting the values into the equation, we get*

$$a = \frac{(1000 - 0.18)(4\pi(0.1)^3/3) \times 9.8 - 2.5 \times 9.8}{2.5 + 0.18 \times (4\pi(0.1)^3/3)} = 6.615\ ms^{-2}.$$

(b) We may use $s = ut + \frac{1}{2}at^2$ *for the time taken* t*. Putting* $s = 5 - 0.2 = 4.8$ *with* $u = 0$ *and the obtained* a *from part (a) we get*

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 4.8}{6.615}} = 1.22\ s.$$

4. Firemen use a hose of inside diameter 5.0 cm to deliver 800 litres of water per minute. A nozzle is attached to the hose, and they want to squirt the water up to a window 25 m above the nozzle. (a) With what speed must the water leave the nozzle? (b) What is the inside diameter of the nozzle? (c) What pressure in the hose is required?

Solution. *(a) Assume water leaving from the nozzle upward without air resistance. The Bernoulli's equation gives*

$$\frac{1}{2}\rho u^2 + 0 = \frac{1}{2}\rho(0)^2 + \rho g h$$

where we have ignored the difference of pressures between the altitudes $P_a \approx P_a + P_{a-h}$. *The equation equivalent to the suvat-equation gives*

$$v = \sqrt{2 \times 9.8 \times 25} = 22.14\ ms^{-1}.$$

(b) Given the flow $0.8/60\ m^3 s^{-1}$, *the radius inside of the nozzle is*

$$\frac{0.8}{60} = r^2 \pi \times 22.14 \quad \Rightarrow \quad r = 0.0138\ m.$$

Thus, the diameter is approximately 2.7 cm. (c) The pressure P ***inside*** *the hose is given by*

$$P + \frac{1}{2}\rho \left(\frac{flow}{area}\right)^2 = 1\ atm + \frac{1}{2}\rho \times 22.14^2$$

Using density of water $1000\ kgm^{-3}$, *1 atmospheric pressure being* $10^5\ Pa$ *for the calculation we get*

$$P = 10^5 + \frac{1}{2} \times 1000 \times 22.14^2 - \frac{1}{2} \times 1000 \times \left(\frac{0.8/60}{0.025^2 \pi}\right)^2 \approx 322,000\ Pa.$$

Therefore, the pressure is 3.22 atmospheric pressure inside the hose.

5. Oil of density 750 kgm^{-3} is poured on top of a tank of water, and it floats on the water without mixing. A block of plastic of density 830 kgm^{-3} is placed in the tank, and it is about floating at the interface of the two liquids (completely immerses in the liquids). What fraction of the block's volume is immersed in water?

   Solution. *Let* x *be the fraction of the volume of the wooden block in water. Thus the volume* xV *is in water, and volume* $(1-x)V$ *is in oil, where* V *is the volume of the block. We have,*

   $$\rho V g = \rho_w x V g + \rho_o (1-x) V g$$

   where ρ, ρ_w, ρ_o *are the densities of the block, water and oil. Therefore,*

   $$x = \frac{\rho - \rho_o}{\rho_w - \rho_o} = \frac{830 - 750}{1000 - 750} = 0.32.$$

6. When a person sits erect from lying to increase the vertical position of the brain by 33 cm, the heart must continue to pump blood to the brain at the same rate. (a) What is the gain in gravitational potential energy for 95 mL of blood raised 33 cm? (b) What is the drop in pressure, neglecting any losses due to friction? (Given the density of blood 1.05 g cm^{-3})

   Solution. *(a) The gain of potential energy is*

   $$1.05 \times 10^3 \times 95 \times 10^{-6} \times 33 \times 10^{-2} \times 9.8 = 0.32\ J$$

   (b) The pressure drop is the energy in (a) divided by the volume of the blood, i.e.,

   $$\frac{0.32}{95 \times 10^{-3}} = 3.4\ Pa.$$

7. To 120 g of water ($c = 1$ cal/g °C) at 12°C is added 170 g of iron ($c = 0.11$ cal/g °C) at 95°C and 65 g of marble ($c = 0.21$ cal/g °C) at 22°C. What is the final temperature of the mixture?

   Solution. *Let T be the final temperature of the system at thermal equilibrium. The total heat transfer is just zero*

$$m_i c_i(T-95) + m_w c_w(T-12) + m_m c_m(T-22) = 0$$

   Substituting the values gives

$$170 \times 0.11(T-95) + 120 \times 1(T-12) + 65 \times 0.21(T-22) = 0 \quad \Rightarrow \quad T = 23.08\,^\circ C$$

8. A physicist makes a cup of instant coffee and notices that, as the coffee cools, its level drops 2.80 mm in the glass cup. (a) Show that this decrease cannot be due to thermal contraction by calculating the decrease in level if the 320 cm^3 of coffee is in a 7.50-cm-diameter cup and decreases in temperature from 95.0 °C to 45 °C. (You may find the coefficients on page 480 of the College Physics e-book.) (b) Then why does the coffee (water) level drop?

   Solution. *(a) The coefficient of volume expansion of glass is much smaller than water, $27 \times 10^{-6} < 210 \times 10^{-6}\ K^{-1}$, we may only calculate the linear expansion of water in vertical direction. The change of height of water is*

$$\Delta L = \frac{320}{(7.5/2)^2\pi} \cdot \frac{210 \times 10^{-6}}{3}(45-95) = -0.0254\ cm = -0.254\ mm$$

   which is greatly smaller than the 2.80 *mm drop even though we did not count the expansion of the glass cup and the fact that the change of coffee (water) level will even be smaller if the calculation included. Notice that the coefficient of linear expansion of water has been used by dividing the coefficient for volume by 3. (b) The most obvious reason of the decrease of the coffee level is the water becoming water vapour evaporating to air.*

9. Given the coefficient of conductivity of human tissue 18 Cal cm/m^2-hr-°C, calculate the heat transfer in 2.8 hours by conduction in the unit of kilo-Joule. Assume that the thickness of tissue is 2.6 cm, the average area of the cross section is 1.65 m^2 and the temperature difference is 2.2 °C between inner body and skin.

   Solution. *The heat flow is*

$$H = \frac{K_c A \Delta T}{L} = \frac{18 \times 1.65 \times 2.2}{2.6} = 25.13\ Cal/hr.$$

   In 2.2 hours, the heat energy is $25.13 \times 2.8 = 70.36$ *Cal. In term of kilo-Joule it is*

$$70.36 \times 4.186 = 294.5\ kJ$$