

## Quiz 2

1. Suppose the pdf of the magnitude  $X$  of a dynamic load on a bridge is given by

$$f(x) = \begin{cases} \frac{1}{8} + \frac{3}{8}x, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Find  $F(x)$ ,  $P(1 \leq X \leq 1.5)$  and  $P(X > 1)$ .

2 The following pdf of  $X$  is essentially the one suggested in “The Statistical Properties of Freeway Traffic”.

$$f(x) = \begin{cases} 0.15e^{-0.15(x-0.5)} & x \geq 0.5 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

(A) The formula (1) satisfy the pdf conditions?

(B) The probability that headway time is at most 5 sec is ?

3. Let  $X$  denote the number of creatures of a particular type captured in a trap during a given time period. Suppose that  $X$  has a Poisson distribution with  $\lambda=4.5$ , so on average traps will contain 4.5 creatures.

(A) What is the probability that a trap contains exactly five creatures?

(B) What is the probability that a trap has at most five creatures?

4. The breakdown voltage of a randomly chosen diode of a particular type is known to be normally distributed. What is the probability that a diode's breakdown voltage is within 1 standard deviation of its mean value?

5. A bank operates both a drive-up facility and a walk-up window. On a randomly selected day, let  $X$  = the proportion of time that the drive-up facility is in use,  $Y$  = the proportion of time that the walk-up window is in use. Let the joint pdf of  $(X,Y)$  be

$$f(x, y) = \begin{cases} \frac{6}{5}(x + y^2) & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(A) Determine the probability  $P(0 \leq X \leq \frac{1}{4}, 0 \leq Y \leq \frac{1}{4})$

(B) Find  $f_x(x)$  and  $f_y(y)$

(C) X and Y are independent or not independent? why?

**6. IQ in a particular population is known to be approximately normally distributed with  $\mu = 100$  and  $\sigma = 15$ . What is the probability that a randomly selected individual has an IQ of at least 125?**

**7. Suppose the response time X at a certain on-line computer terminal (the elapsed time between the end of a user's inquiry and the beginning of the system's response to inquiry) has an exponential distribution with expected response time equal to 5 sec.**

(A) The probability that the response time is at most 10 sec is

(B) The probability that response time is between 5 and 10 sec is

**8. If  $X_1, \dots, X_n$  represent the lifetime of  $n$  components, the components operate independently of one another, and each lifetime is exponentially distributed with parameter  $\lambda$ .**

(A) Joint pdf is?

(B) If there  $n$  components constitute a system that will fail as soon as a single component fails, then the probability that the system lasts past  $t$  time is?

**9. The time that it takes a randomly selected rat of a certain subspecies to find its way through a maze is a normally distributed rv with  $\mu = 1.5$  min and  $\sigma = .35$  min. Suppose five rats are selected. Let  $X_1, X_2, \dots, X_5$  denote their times in the maze. Assuming the  $X_i$ 's to be a random sample from this normal distribution.**

(A) What is the probability that the total time  $T_o = X_1 + X_2 + \dots + X_5$  for the five is between 6 and 8 min?

(B) Determine the probability that the sample average time  $\bar{X}$  is at most 2.0 min.

**10. When a batch of a certain chemical product is prepared, the amount of a particular impurity in the batch is a random variable with mean value 4.0g and standard deviation 1.5g. If 50 batches are independently prepared, what is the (approximate) probability that the sample average amount of impurity  $\bar{X}$  is between 3.5 and 3.8g?**

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