60080079 Introduction to Statistical Methods
Semester 2 2023-2024
Solutions 6

1. PART I: Write your answer as a three-digit number: 151
PART II: Write your answer as a two-digit number: 41

1.2. Recall the margin of error is the critical value multiplied by the standard error.
For the data, $\bar{x} = 531$ and s=82.792, $df = 9$ and value from Table D is 2.262.
A 95% confidence interval for the mean rent is

$$\bar{x} \pm t^* s/\sqrt{n} = 531 \pm 2.262 \times 82.792/\sqrt{10} = 531 \pm 59.222 = (471.78, 590.22)$$

2. PART I: Write your answer as a two-digit number: 21
PART II: Write your answer as a three-digit number: 213

2.3. From Table D, the p-value of the *t* statistic is between the 0.15 and 0.10 columns.

3. PART I: Write your answer as a two-digit number: 42
PART II: Write your answer as a four-digit number: 2462

4. Write your answer as a four-digit number: 3213

5. PART I: Write your answer as a two-digit number: 21
PART II: Write your answer as a two-digit number: 22

5.1.

One-Sample Test

Test Value = 0					
t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
				Lower	Upper
20.282	9	.000	531.00000	471.7745	590.2255

5.2.

One-Sample Test

Test Value = 500					
t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
				Lower	Upper
1.184	9	.267*	31.00000	-28.2255	90.2255

The final p-value is half of this because the alternative is one-tailed.

6. PART I: Write your answer as a four-digit number: 3241
PART II: Write your answer as a three-digit number: 315
PART III: Write your answer as a two-digit number: 43

6.2.

$$\bar{d} = 1.450, \; s_d = 3.203, \; s_{\bar{d}} = SE(\bar{d}) = s_d / \sqrt{n} = 3.203/\sqrt{20} = 0.716$$

$$t = \frac{\bar{d} - \mu_d}{s_{\bar{d}}} = \frac{1.450 - 0}{.716} = 2.02$$

6.4.

$$\bar{d} \pm t^* s_d / \sqrt{n} = 1.450 \pm 1.729 \times 3.203 / \sqrt{20} = 1.450 \pm 1.238 = (.212, 2.688)$$

7. PART I: Write your answer as a three-digit number: 213
PART II: Write your answer as a two-digit number: 52
PART III: Write your answer as a three-digit number: 415

7.2.

	1 Bedroom	2 Bedrooms
$\bar{x}$	531	609
s^2	6854.44	7976.67
n	10	10

$$S_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(10-1)6854.44 + (10-1)7976.67}{10+10-2}} = \sqrt{\frac{7415.56}{18}} = 86.1126$$

7.3. The 95% confidence interval for the difference of the means $\mu_2 - \mu_1$ (which represents the additional cost) is given by $(\bar{x}_2 - \bar{x}_1) \pm t^* s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$. Here, the t-statistic follows $t(n_1 + n_2 - 2) = t(18)$ distribution. Hence, $t^* = 2.101$.

7.4.

$$(609 - 531) \pm 2.101(86.1126)\sqrt{\frac{1}{10} + \frac{1}{10}} = 78 \pm 80.9120 = (-2.91, 158.91)$$

We are 95% confident that the true difference lies between -$2.91 and $158.91. Note that the interval includes 0 so we cannot discount the possibility that the true difference is zero (i.e., on the average, two-bedroom apartments are not more expensive than one-bedroom apartments.)

8. Write your answer as a three-digit number: 132

8.2. Note that it is not too uncommon that even if the alternative hypothesis is stated as $H_a : \mu_1 > \mu_2$, the statistic is computed with $\bar{x}_2 - \bar{x}_1$ in the formula (perhaps by habit) that the resulting t-statistic would be negative (as to be expected from the alternative hypothesis). We should interpret the p-value as in (a), and still reject the null hypothesis.

9. PART I: Write your answer as a three-digit number: 232
PART II: Write your answer as a three-digit number: 235
PART III: Write your answer as a four-digit number: 1413

9.2.

$$S_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(23-1)1.7^2+(19-1)1.8^2}{23+19-2}} = \sqrt{\frac{121.9}{40}} = 1.7457$$

$$t = \frac{(\bar{x}_1-\bar{x}_2)-(\mu_2-\mu_1)}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{(13.3-12.4)-0}{1.7457\sqrt{\frac{1}{23}+\frac{1}{19}}} = \frac{.9}{.5412} = 1.66$$

9.4.

$$(\bar{x}_2 - \bar{x}_1)\ ?\ t^* s_p\sqrt{\frac{1}{n_1}\quad\frac{1}{n_2}} = .9\ ?\ 2.021(.5412)\quad(-\ .19,\ 1.99)$$

10. Write your answer as a three-digit number: 652

Paired Samples Test

Paired Differences					t	df	Sig. (2-tailed)
			90% Confidence Interval of the Difference				
Mean	Std. Deviation	Std. Error Mean	Lower	Upper			
1.4500	3.2032	.7163	.2115	2.6885	2.024	19	.057*

11. Write your answer as a two-digit number: 17

Independent Samples Test

		t-test for Equality of Means				
		t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Rent	Equal variances assumed	-2.025	18	.058	-78.00000	38.51118
	Equal variances not assumed	-2.025	17.898	.058	-78.00000	38.51118