Answers to homework

Section 4.3　　30，44，48，53, 56

Section 4-3

30.

a. $\Phi(c) = .9100 \Rightarrow c \approx 1.34$, since .9099 is the entry in the 1.3 row, .04 column.

b. Since the standard normal distribution is symmetric about $z = 0$, the 9^{th} percentile = $-$[the 91^{st} percentile] = -1.34.

c. $\Phi(c) = .7500 \Rightarrow c \approx .675$, since .7486 and .7517 are in the .67 and .68 entries, respectively.

d. Since the standard normal distribution is symmetric about $z = 0$, the 25^{th} percentile = $-$[the 75^{th} percentile] = $-.675$.

e. $\Phi(c) = .06 \Rightarrow c \approx -1.555$, since .0594 and .0606 appear as the -1.56 and -1.55 entries, respectively.

44.

a. $P(\mu - 1.5\sigma \le X \le \mu + 1.5\sigma) = P(-1.5 \le Z \le 1.5) = \Phi(1.50) - \Phi(-1.50) = .8664$.

b. $P(X < \mu - 2.5\sigma \text{ or } X > \mu + 2.5\sigma) = 1 - P(\mu - 2.5\sigma \le X \le \mu + 2.5\sigma)$
$= 1 - P(-2.5 \le Z \le 2.5) = 1 - .9876 = .0124$.

c. $P(\mu - 2\sigma \le X \le \mu - \sigma \text{ or } \mu + \sigma \le X \le \mu + 2\sigma) = P(\text{within 2 sd's}) - P(\text{within 1 sd}) =$
$P(\mu - 2\sigma \le X \le \mu + 2\sigma) - P(\mu - \sigma \le X \le \mu + \sigma) = .9544 - .6826 = .2718$.

48.

a. By symmetry, $P(-1.72 \le Z \le -.55) = P(.55 \le Z \le 1.72) = \Phi(1.72) - \Phi(.55)$.

b. $P(-1.72 \le Z \le .55) = \Phi(.55) - \Phi(-1.72) = \Phi(.55) - [1 - \Phi(1.72)]$.

No, thanks to the symmetry of the z curve about 0.

53. $p = .5 \Rightarrow \mu = 12.5$ & $\sigma^2 = 6.25$; $p = .6 \Rightarrow \mu = 15$ & $\sigma^2 = 6$; $p = .8 \Rightarrow \mu = 20$ and $\sigma^2 = 4$. These mean and standard deviation values are used for the normal calculations below.

a. For the binomial calculation, $P(15 \le X \le 20) = B(20; 25, p) - B(14; 25, p)$.

p	$P(15 \le X \le 20)$	$P(14.5 \le \text{Normal} \le 20.5)$
.5	$= .212$	$= P(.80 \le Z \le 3.20) = .2112$
.6	$= .577$	$= P(-.20 \le Z \le 2.24) = .5668$
.8	$= .573$	$= P(-2.75 \le Z \le .25) = .5957$

b. For the binomial calculation, $P(X \le 15) = B(15; 25, p)$.

p	$P(X \le 15)$	$P(\text{Normal} \le 15.5)$
.5	$= .885$	$= P(Z \le 1.20) = .8849$
.6	$= .575$	$= P(Z \le .20) = .5793$
.8	$= .017$	$= P(Z \le -2.25) = .0122$

c. For the binomial calculation, $P(X \ge 20) = 1 - B(19; 25, p)$.

p	$P(X \ge 20)$	$P(\text{Normal} \ge 19.5)$
.5	$= .002$	$= P(Z \ge 2.80) = .0026$
.6	$= .029$	$= P(Z \ge 1.84) = .0329$
.8	$= .617$	$= P(Z \ge -0.25) = .5987$

56. Let z_{1-p} denote the $(100p)$th percentile of a standard normal distribution. The claim is the $(100p)$th percentile of a $N(\mu, \sigma)$ distribution is $\mu + z_{1-p}\sigma$. To verify this,

$P(X \le \mu + z_{1-p}\sigma) = P\left(\frac{X-\mu}{\sigma} \le z_{1-p}\right) = P\left(Z \le z_{1-p}\right) = p$ by definition of z_{1-p}. That establishes $\mu + z_{1-p}\sigma$ as the $(100p)$th percentile.