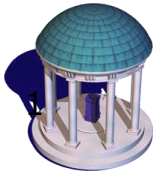
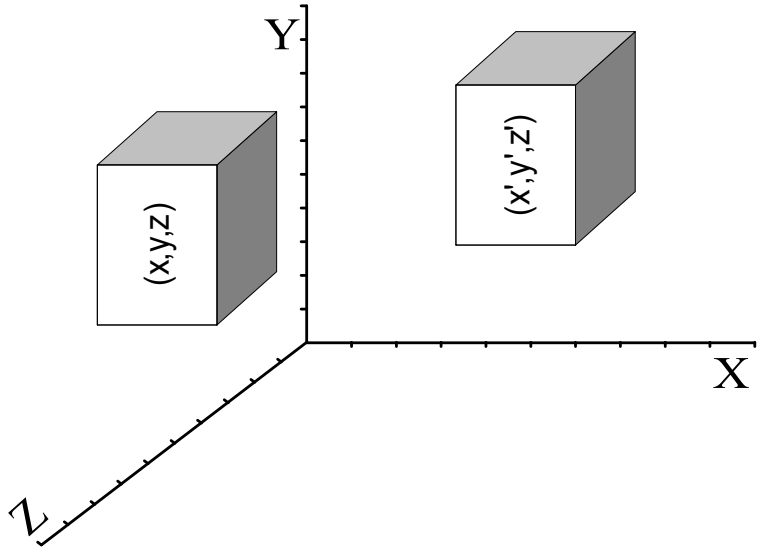
Basic 3D transformations

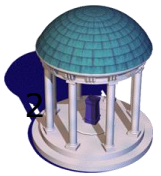
- *3D basic transformations are relative to the axis and the original point*
- Transformation matrix is a 4 by 4 non-singular matrix



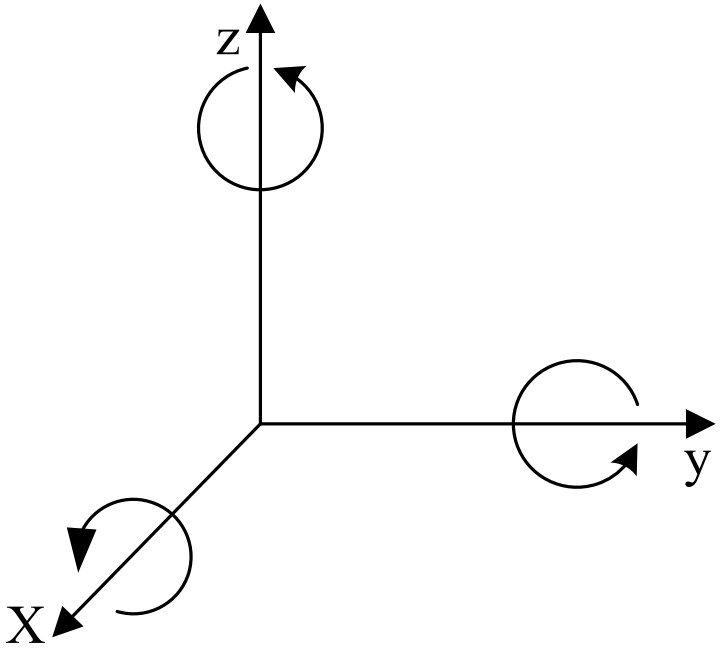
Basic transformation: Translation

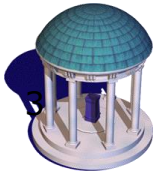
$$T_t = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





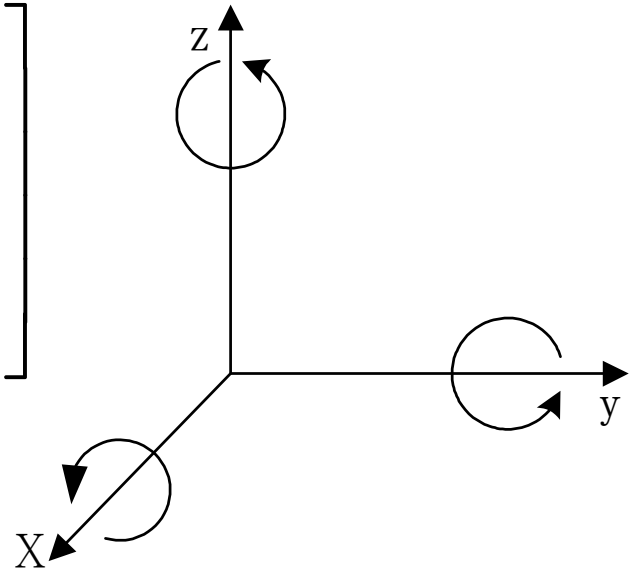
Basic transformation: Rotation

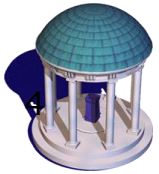




- Rotate around the z axis

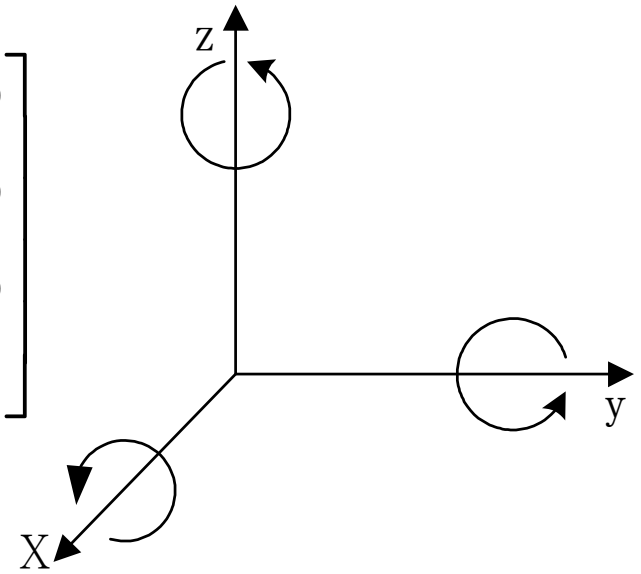
$$T_{RZ} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

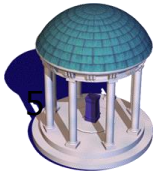




- Rotate around the x axis

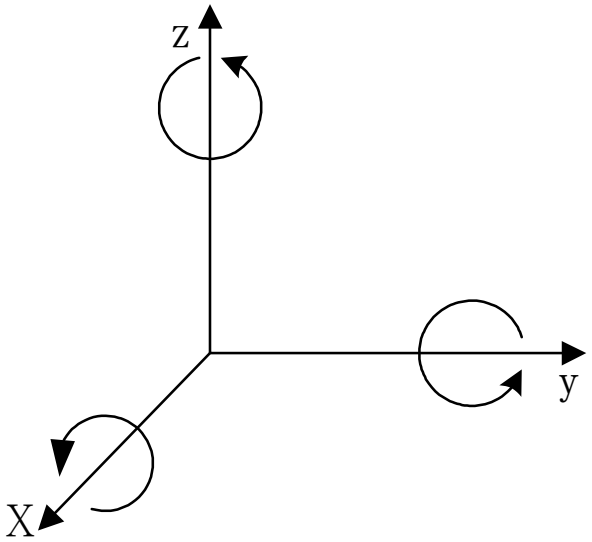
$$T_{RX} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

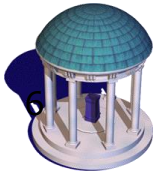




- Rotate around the y axis

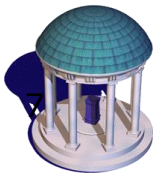
$$T_{RY} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





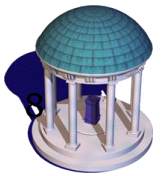
Basic transformation: Scaling

$$T_s = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & e & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



- Isotropic scaling

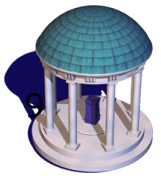
$$T_S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & s \end{bmatrix}$$



Basic transformation: symmetric

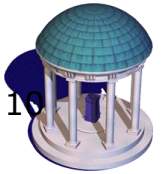
- symmetric about XOY plane

$$T_{Fxy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



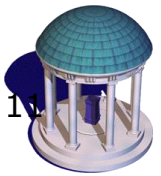
- About YOZ plane

$$T_{Fyz} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



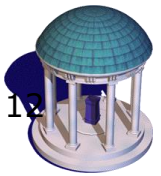
- About ZOX plane

$$T_{Fzx} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



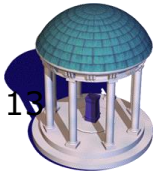
- About x axis

$$T_{Fx} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



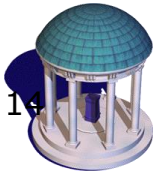
- About y axis:

$$T_{Fy} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



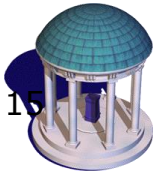
- About z axis

$$T_{Fz} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



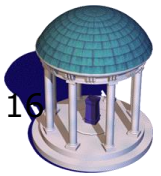
- About the original point

$$T_{Fxy} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

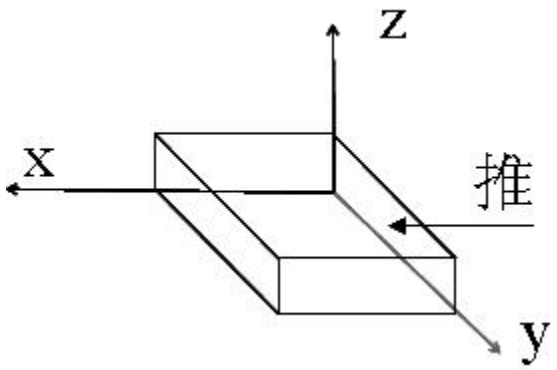


Basic transformation: miscut (错切变换)

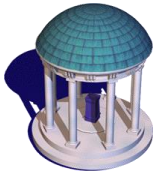
$$T_{SH} = \begin{bmatrix} 1 & b & c & 0 \\ d & 1 & f & 0 \\ g & h & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



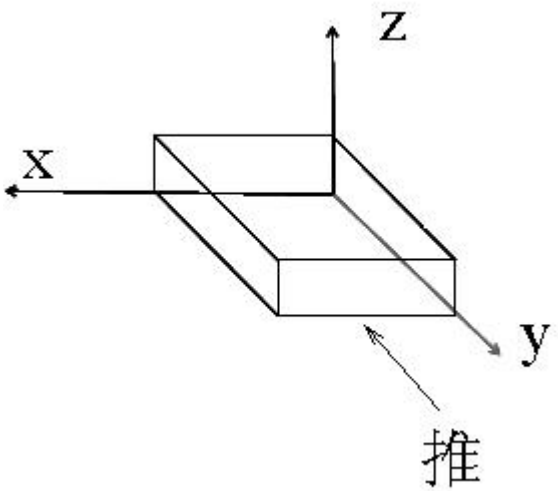
(1)Along x direction



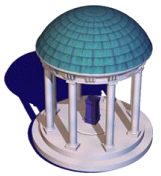
$$T_{SHx} = \begin{bmatrix} 1 & d & g & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



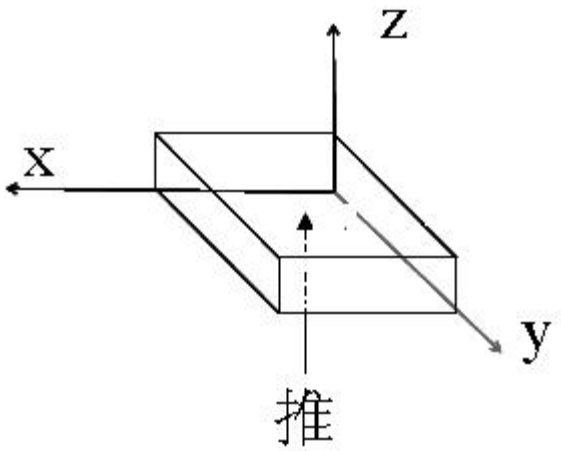
(2)Along y direction



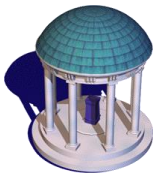
$$T_{SHy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ b & 1 & h & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



(3)Along z direction

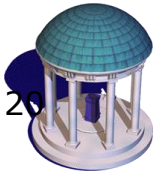


$$T_{SHz} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ c & f & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



3D transformation matrix

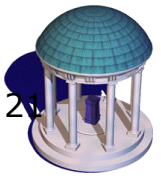
$$\left[\begin{array}{ccc|c} a & b & c & p \\ d & e & f & q \\ h & i & j & r \\ \hline l & m & n & s \end{array}\right]$$



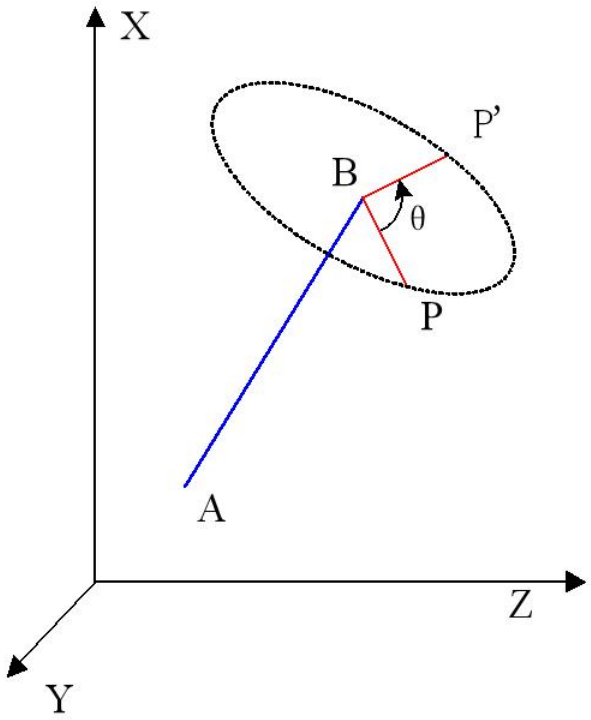
3D compound transformation

- 3D compound transformation refers to the transformation of graphics more than once, and the transformation result is the product of each transformation matrix.

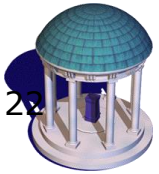
$$P' = T \cdot P = (T_n \cdots T_3 \cdot T_2 \cdot T_1) \cdot P \qquad (n > 1)$$



3D rotation around any axis



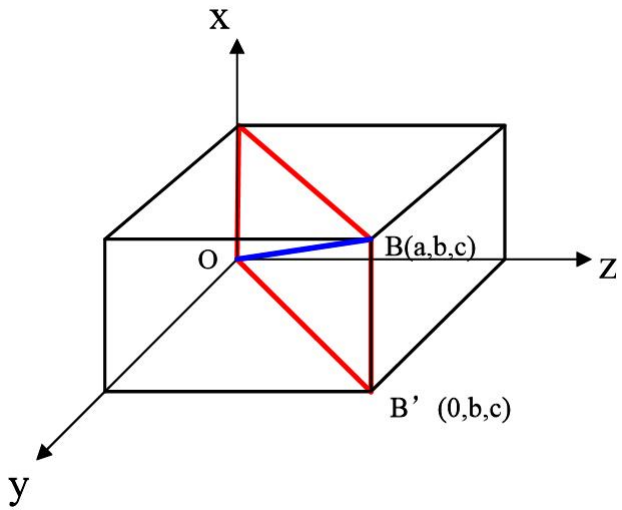
Rotate around the line AB

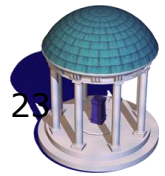


3D rotation around any axis point

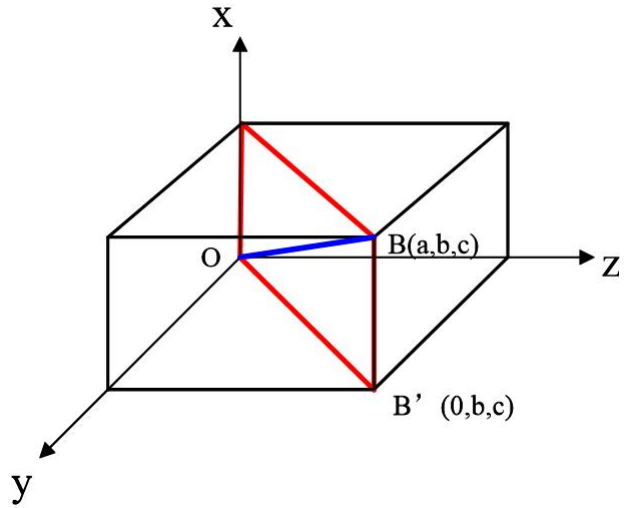
(1) Translate A to the original point

$$T_A = \begin{bmatrix} 1 & 0 & 0 & -x_A \\ 0 & 1 & 0 & -y_A \\ 0 & 0 & 1 & -z_A \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



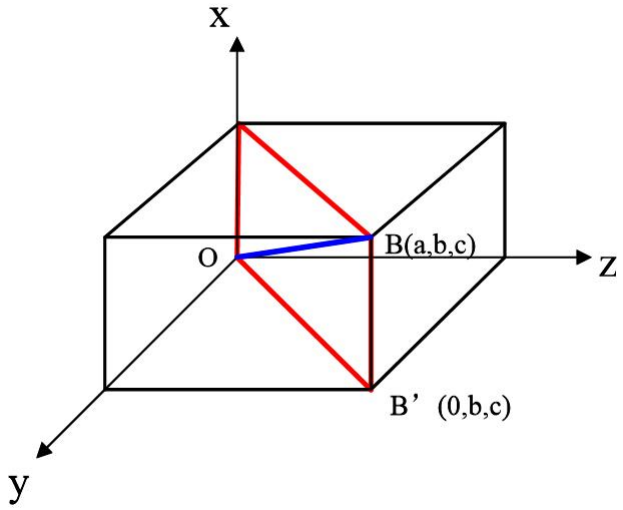


(2) Rotate around x axis such that OB on the plane y=0

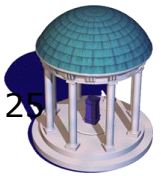


$$T_{Rx} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3) Rotate around y axis such that OB align with the z axis



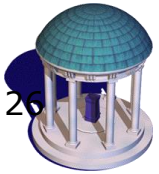
$$T_{Ry} = \begin{bmatrix} \cos(-\beta) & 0 & \sin(-\beta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-\beta) & 0 & \cos(-\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



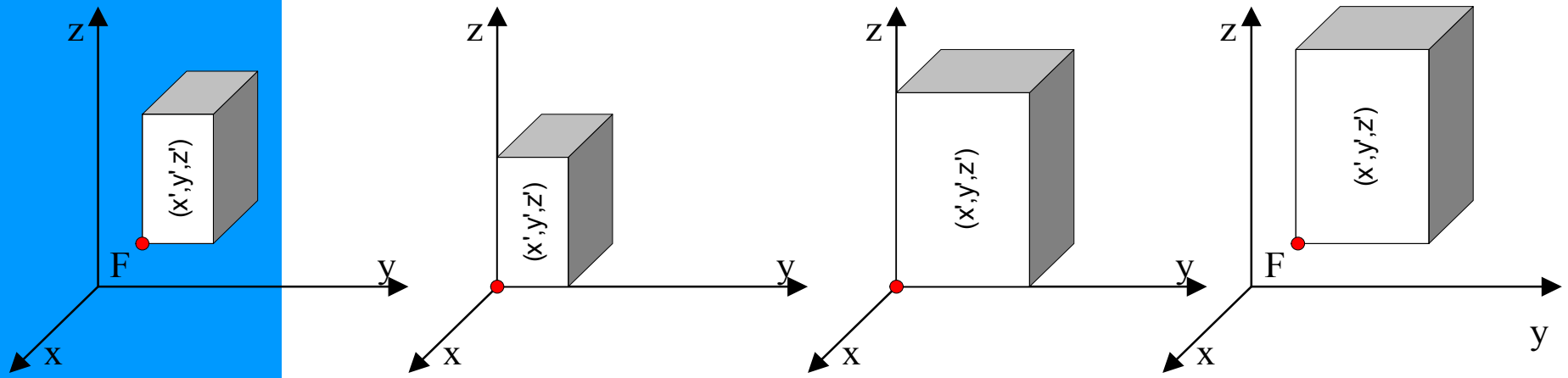
(4) Rotate around z axis

(5) Rotate and translate back to AB

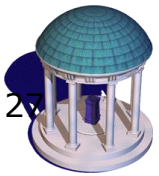
$$T = T^{-1}_{A} \cdot T^{-1}_{Rx} \cdot T^{-1}_{Ry} \cdot T_{R} \cdot T_{Ry} \cdot T_{Rx} \cdot T_{A}$$



3D transformation relative to any reference point



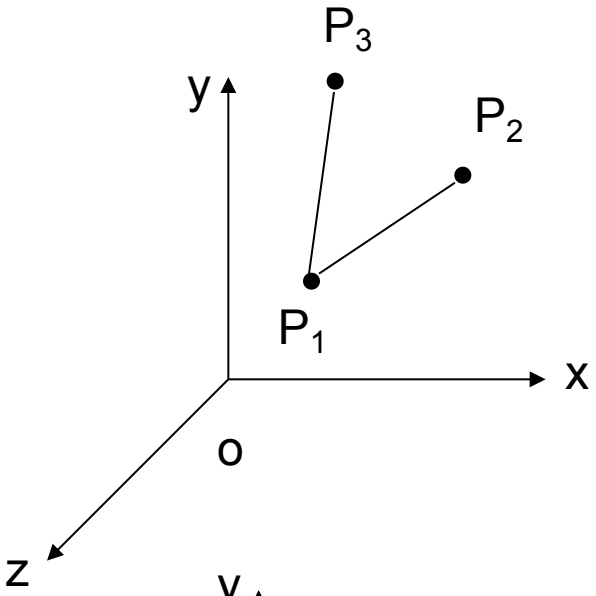
(a)原图　(b)移至坐标原点　(c)基本比例变换　(d)移回F点原来位置

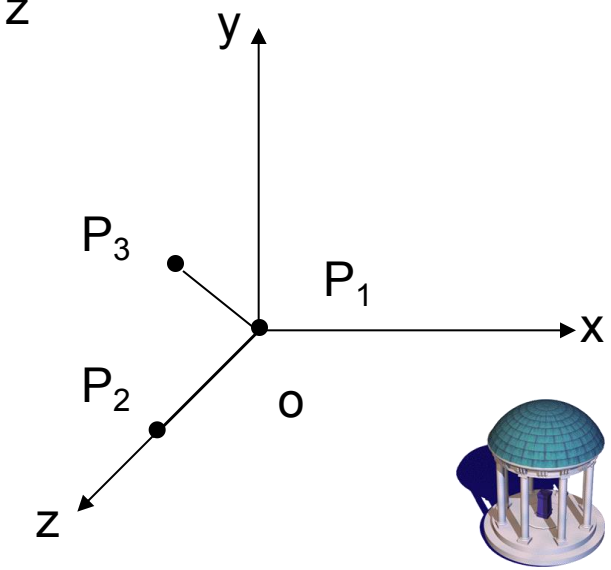


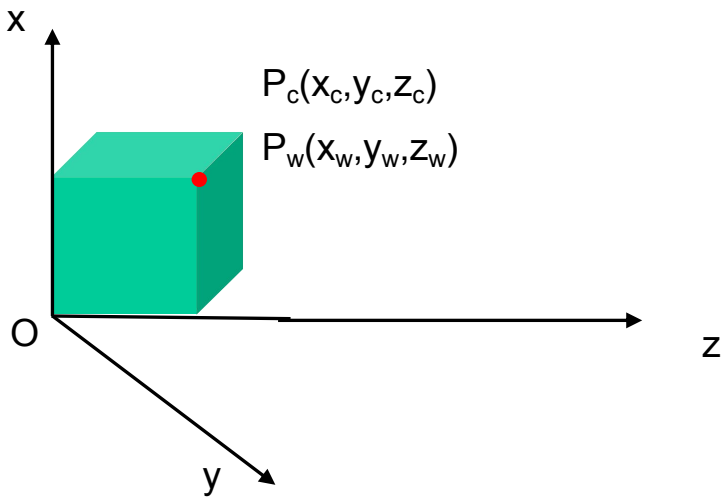
Example:

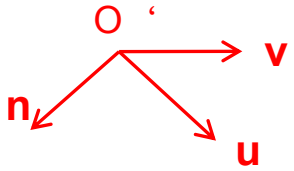
Transform the directed line segments P1P2 and P1P3 from the initial position in the figure above to the final position in the figure below, and the length of the line segments does not change.

(The line segment P1P2 coincides with the x axis, and the line segment P1P3 lies on the yoz plane).






x
$P_c(x_c,y_c,z_c)$
$P_w(x_w,y_w,z_w)$
O
z
y


O ‘
v
n
u