Physics FSE (2021-22) Homework 1

Please send the completed file to my mailbox yy.lam@qq.com by March 16th, with using the filename format:

`2021xxxxxx_yourname_fse_hw1`

Please answer the questions by filling on these sheets. It would be perfect if you use e-pen directly writing on the sheets. If you do not have the appropriate hardware, you may handle the questions as usual by using pieces of blank papers, then take photos and paste them onto these question sheets.

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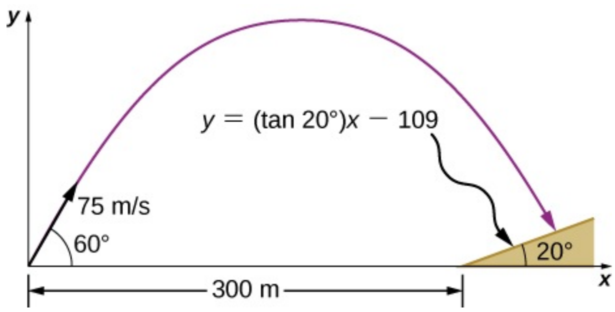
1. A projectile is shot at a hill, the base of which is 300 m away. The projectile is shot at 60° above the horizontal with an initial speed of 75 m/s. The hill can be approximated by a plane sloped air 20° to the horizontal. Relative to the coordinate system shown in the following figure, the equation of the this straight line is $y = (tan 20°)x - 109$. Where on the hill does the projectile land?



Solution. *We have to find the intersection point of the projectile and the straight line. The horizontal and vertical components of the projectile is*

$$x = 75t\cos 60 \quad \textit{and} \quad y = 75t\sin 60 + \frac{1}{2}(-9.8)t^2$$

Eliminating t gives

$$y = x\tan 60 - \frac{4.9x^2}{(75\cos 60)^2}.$$

Putting the straight line y into the left hand side of the equation gives

$$x\tan 60 - 109 = x\tan 60 - \frac{4.9x^2}{(75\cos 60)^2} \quad \Rightarrow \quad x^2 - 391x - 31143 = 0$$

Solving the quadratic equation we get

$$x = 459\ m, \quad y = 58\ m$$

in which we only took the positive value of the solutions. The horizontal distance from the foothill is $459 - 300 = 159\ m$.

2. Find the unit vector of the cross product of the vectors $2\hat{\mathbf{i}}+3\hat{\mathbf{j}}$ and $\hat{\mathbf{i}}+2\hat{\mathbf{j}}-4\hat{\mathbf{k}}$.

Solution. *The cross product of them is*

$$(2\hat{\mathbf{i}}+3\hat{\mathbf{j}})\times(\hat{\mathbf{i}}+2\hat{\mathbf{j}}-4\hat{\mathbf{k}}) = -12\hat{\mathbf{i}}+8\hat{\mathbf{j}}+\hat{\mathbf{k}}$$

The magnitude is $\sqrt{(-12)^2+8^2+1^2}=\sqrt{209}$. *Thus the unit vector is*

$$\frac{1}{\sqrt{209}}(-12\hat{\mathbf{i}}+8\hat{\mathbf{j}}+\hat{\mathbf{k}})$$

3. A phone is accidently dropped from a hot-air balloon that is 320 m above the ground and rising at 6.50 m/s upward. For the phone, find (a) the maximum height reached, (b) its position and velocity 3.50 s after being released, and (c) the time before it hits the ground.

Solution. *(a) The vertical displacement of the phone from the position of releasing to the maximum height is*

$$0 = 6.5^2 + 2(-9.8)s \quad\Rightarrow\quad s = 2.16\ m.$$

The maximum height reached from the ground is $320+2.16=322.16$ *m. (b) After 3.5 seconds,* $6.5\times 3.5+\frac{1}{2}(-9.8)3.5^2=-37.28$ *m, the height is* $320-37.3=282.7$ *m. The velocity is* $v=u+at=6.5+(-9.8)3.5=-27.8\ ms^{-1}$, *downward of course. (c) Recall that we set the vertical postion of the air balloon at 0 while we drop the phone. The equation* $-320=6.5t+(-9.8)t^2/2$ *gives the quadratic equation for the roots;*

$$4.9t^2-6.5t-320=0 \quad\Rightarrow\quad t=\frac{6.5\pm\sqrt{6.5^2-4(4.9)(-320)}}{2(4.9)}=8.77\ s$$

where only positive time interval has been taken.

4. Given a vector $\mathbf{v}$ in 2-dimensional plane written in terms of the orthonormal basis in polar coordinates
$$\mathbf{v} = 3t^2\mathbf{e}_r + t\mathbf{e}_\theta,$$
Find the derivative of $\mathbf{v}$ with respect to t.

   Solution.
$$\begin{aligned}\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} &= 6t\mathbf{e}_r + 3t^2\dot{\mathbf{e}}_r + \mathbf{e}_\theta + t\dot{\mathbf{e}}_\theta \\ &= 6t\mathbf{e}_r + 3t^2\dot{\theta}\mathbf{e}_\theta + \mathbf{e}_\theta - t\dot{\theta}\mathbf{e}_r \\ &= (6-\dot{\theta})t\mathbf{e}_r + (3t^2\dot{\theta}+1)\mathbf{e}_\theta\end{aligned}$$

5. Assuming earth is a rigid sphere with a constant density. Find the gravitational acceleration at a point p inside earth. Express your results in terms of the radius of earth R, the distance r of the point p from the centre, and M the mass of earth.

   Solution. *We simply treat the gravitational acceleration as being created by the centre of mass of the earth as usual. The acceleration g_p depends on the position p for taking values however,*
$$g_p = \frac{GM_p}{r^2}$$
where M_p, r are the corresponding mass and the radius of the measuring sphere. Thus,
$$g_p = \frac{GM_p}{r^2} = \frac{G4\pi r^3\rho}{3r^2} = \frac{G4\pi R^3 r\rho}{3R^3} = \frac{GMr}{R^3}$$
where M and R are the mass and radius of earth.

6. The velocity of a particle moving horizontally along a straight line varies in time according to the expression $v = (30 - 2t^2)\ ms^{-1}$, where t is in seconds. (a) Find the average acceleration in the time interval $t = 0$ to $t = 2$ s along the direction $25°$ from the moving straight line. (b) Determine the acceleration at $t = 2$ s.

Solution. *(a) The average acceleration is the change of the velocity within a given time interval. Along the x-axis it is*

$$\textit{Average acceleration} = \frac{30 - 2 \cdot 2^2 - (30 - 0)}{2 - 0} = -4\ ms^{-2}.$$

The required acceleration is

$$-4 \cos 25 = -3.63\ ms^{-2}.$$

(b) The instantaneous acceleration at $t = 2$ *is*

$$\left[\frac{\mathrm{d}}{\mathrm{d}t}(30 - 2t^2)\right]_{t=2} = -4t\Big|_{t=2} = -8\ ms^{-2}.$$

7. A physical quantity called the 'Planck length' l_P with dimension as length is the multiplications with some powers of Newton's gravitational constant G, Planck's constant $\hbar$ (with the unit $\mathrm{kg\,m^2 s^{-1}}$) and the speed of light c, respectively, namely

$$[l_P] = [G]^a [\hbar]^b [c]^c$$

where a, b and c are some numbers. Use dimensional analysis to derive the expression for l_P.

Solution.

$$\begin{aligned} [l_P] = L &= [G]^a [\hbar]^b [c]^c \\ &= (M^{-1} L^3 T^{-2})^a (M L^2 T^{-1})^b (L T^{-1})^c \end{aligned}$$

where $a, b,$ *and* c *are some numbers that we have to find. Comparing the dimensions of the both sides gives*

$$L^1 = L^{3a+2b+c}, \quad M^0 = M^{-a+b}, \quad T^0 = T^{-2a-b-c}$$

Solving for the powers we get

$$a = b = \frac{1}{2}, \quad c = -\frac{3}{2}$$

Thus, the Planck length is

$$l_P \approx \sqrt{\frac{G\hbar}{c^3}}.$$

8. A car is traveling east at 90 km/h. At an intersection 36 km ahead, a truck is traveling north at 55 km/h. (a) How long after this moment will the vehicles be closest to each other? (b) How far apart will they be at that point?

Solution. *The position vector of the truck relative to the car is* $\mathbf{r}_{CT} = (-36 + 90t)\hat{\mathbf{i}} + 55\hat{\mathbf{j}}$, *Pythagorean theorem gives the distance between the vehicles*

$$|\mathbf{r}_{TC}|^2 = (-36 + 90t)^2 + (55t)^2.$$

For the minimum distance, differentiating the distance gives

$$2r\frac{\mathrm{d}r}{\mathrm{d}t} = 2(-36 + 90t)(90) + 2(55)^2 t \quad \Rightarrow \quad \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{r}[(-36 + 90t)(90) + 55^2 t]$$

Setting it to zero we get minimum time taken

$$90(-36 + 90t) + 55^2 t = 0 \quad \Rightarrow \quad t = 0.291\ hr = 17.47\ min$$

Inserting into the Pythagorean theorem, we obtain the minimum distance $r = 18.77$ *km.*