

60080079 Introduction to Statistical Methods
Semester 2 2023-2024
Homework Assignment 5

21 CST H3Art

1. Which in the following is a correct statement?

- 1) A random sample of size 40 is taken from a population that is assumed to have a standard deviation of 9. The standard deviation of the sample mean is $9/40$.
- 2) A researcher tests the following null hypothesis: $H_0: \bar{X} = 6.9$.
- 3) A researcher tests the hypothesis $H_0: \mu = 50$ and concludes that H_0 is accepted, which means the population mean is equal to 50.
- 4) A researcher tests the hypothesis $H_0: \mu = 50$, and concludes that H_0 is rejected, which means there is enough evidence showing μ is not 50.

Write your answer as a single-digit number.

Answer: 4

2. In each of the following situations, a significance test for a population mean μ is called for. State the null hypothesis H_0 and the alternative hypothesis H_A in each case.

2.1 Experiments on learning in animals sometimes measure how long it takes a mouse to find its way through a maze. The mean time is 18 seconds for one particular maze. A researcher thinks that a loud noise will cause the mice to complete the maze faster. She measures how long each of 10 mice takes with a noise as stimulus. The null and alternative hypotheses are ____ and ____.

- 1) $\mu = 18$
- 2) $\mu > 18$
- 3) $\bar{X} > 18$
- 4) $\bar{X} \neq 18$
- 5) $\mu < 18$
- 6) $\mu \neq 18$

Part I: Write your answer as a two-digit number.

Answer: 15

2.2 The examinations in a large history class are scaled after grading so that the mean score is 50. A self-confident teaching assistant thinks that his students have a higher mean score than the class as a whole. His students this semester can be considered a sample from the population of all students he might teach, so he compares their mean score with 50. The null and alternative hypotheses are ____ and ____.

- 1) $\mu = 50$
- 2) $\mu > 50$
- 3) $\bar{X} > 50$

- 4) $\bar{X} \neq 50$
- 5) $\mu < 50$
- 6) $\mu \neq 50$

Part II: Write your answer as a two-digit number.

Answer: 12

2.3 The Census Bureau reports that households spend as average of 31% of their total spending on housing. A homebuilder's association in Cleveland wonders if the national finding applies in their area. They interview a sample of 40 households in the Cleveland metropolitan area to learn what percent of their spending goes toward housing. The null and alternative hypotheses are ____ and ____.

- 1) $\mu = 31\%$
- 2) $\mu > 31\%$
- 3) $\bar{X} > 31\%$
- 4) $\bar{X} \neq 31\%$
- 5) $\mu < 31\%$
- 6) $\mu \neq 31\%$

Part III: Write your answer as a two-digit number.

Answer: 16

3. The significance level of a hypothesis testing is selected as 0.01. For 3.1 – 3.3, use the following options:

- 1) -2.58
- 2) 2.58
- 3) 2.33
- 4) -2.33

3.1 What are the critical values if the alternative is $H_A: \mu \neq \mu_0$?

Part I: Write your answer as a two-digit number.

Answer: 12

3.2 What is the critical value if the alternative is $H_A: \mu > \mu_0$?

3.3 What is the critical value if the alternative is $H_A: \mu < \mu_0$?

Part II: Write your answer as a two-digit number.

Answer: 34

4. Statistics can help decide the authorship of literary works. Sonnets by a certain Elizabethan poet are known to contain an average of $\mu = 6.9$ new words (words not used in the poet's other works). The standard deviation of the number of new words is $\sigma = 2.7$.

Now a manuscript with five new sonnets has come to light and scholars are debating whether it is the poet's work. The new sonnets contain an average of $\bar{X} = 11.2$ words now used in the poet's known works. We expect poems by another author to contain more new words, so to see if we have evidence that the new sonnets are not by our poet we test: $H_0: \mu = 6.9$ vs. $H_A: \mu > 6.9$.

4.1 Under H_0 , which means $\mu = 6.9$ is assumed, the distribution of \bar{X} is ____.

- 1) $N(0, 1)$
- 2) $N(6.9, 2.7)$
- 3) $N(6.9, 1.21)$
- 4) $N(11.2, 1.21)$

4.2 The Z test statistic is ____.

- 1) 2.85
- 2) 3.55
- 3) 1.59
- 4) -1.59

Part I: Write your answer as a two-digit number.

Answer: 32

4.3 If $\alpha = 0.05$ is selected, the critical value is ____.

- 1) 1.96
- 2) -1.96
- 3) 1.65
- 4) -1.65

4.4 Based on 4.2 and 4.3, your conclusion is that ____.

- 1) H_0 is rejected
- 2) H_0 is retained
- 3) H_A is rejected
- 4) there is not enough information to make a decision

Part II: Write your answer as a two-digit number.

Answer: 31

5. A test of the null hypothesis $H_0: \mu = \mu_0$ gives test statistic $Z = 1.5$. For Problems 5.1 – 5.3, choose from the following options:

- 1) 0.9332
- 2) 0.0668
- 3) 0.1336

4) 0.8664

5.1 If the alternative is $H_A: \mu > \mu_0$, the p-value is ____.

5.2 If the alternative is $H_A: \mu < \mu_0$, the p-value is ____.

5.3 If the alternative is $H_A: \mu \neq \mu_0$, the p-value is ____.

Write your answer as a three-digit number.

Answer: 213

6. The p-value for a two-tailed test of the null hypothesis $H_0: \mu = 10$ is 0.06. For Problems 6.1 and 6.2, answer either 1) Yes or 2) No.

6.1 Does the 95% confidence interval include the value 10?

6.2 Does the 90% confidence interval include the value 10?

Write your answer as a two-digit number.

Answer: 12

7. You are told that a significance test is significant (rejected) at the 5% level. If the significance level is changed to 1%, the test ____.

- 1) is significant
- 2) is not significant
- 3) may or may not be significant

Write your answer as a single-digit number.

Answer: 3

8. You measure the weights of 24 male runners. These runners are not a random sample from a population, but you are willing to assume that their weights represent weights of similar runners. Here are their weights in kilograms:

| | | | | | | | |
|------|------|------|------|------|------|------|------|
| 67.8 | 61.9 | 63.0 | 53.1 | 62.3 | 59.7 | 55.4 | 58.9 |
| 60.9 | 69.2 | 63.7 | 68.3 | 64.7 | 65.6 | 56.0 | 57.8 |
| 66.0 | 62.9 | 53.6 | 65.0 | 55.8 | 60.4 | 69.3 | 61.7 |

Assume that the population standard deviation is 4.5 kg, and the sample mean is 61.8 kg. The goal is to find the 95% confidence interval based on the sample mean.

8.1 The margin of error = ____.

- 1) 1.96
- 2) 8.82

- 3) 1.8
- 4) 0.92

8.2 The 95% confidence interval is ____.

- 1) (-1.8, 1.8)
- 2) (-1.96, 1.96)
- 3) (60, 63.6)
- 4) we are not able to find the interval because the hypothesized population mean (μ_0) is unknown.

Write your answer as a two-digit number.

Answer: 33

9. Use the data and confidence interval obtained in Problem 8 to answer following questions.

9.1 Now, you would like to test, $H_0: \mu = 61.3$ kg vs. $H_A: \mu \neq 61.3$ kg, at $\alpha = 0.05$, based on the confidence interval you obtained above. The null hypothesis is ____ because ____.

- 1) rejected
- 2) retained
- 3) 61.3 is less than the sample mean, 61.8
- 4) 61.3 is within the confidence interval
- 5) 61.3 is out of the confidence interval

Part I: Write your answer as a two-digit number.

Answer: 24

9.2 Also based on the confidence interval you obtained in above, the test, $H_0: \mu = 64$ kg vs. $H_A: \mu \neq 64$ kg, at $\alpha = 0.05$, is ____ because ____.

- 1) Rejected
- 2) retained
- 3) 64 is greater than the sample mean, 61.8
- 4) 64 is within the confidence interval
- 5) 64 is out of the confidence interval

Part II: Write your answer as a two-digit number.

Answer: 15

10. Free trial samples are offered to shoppers in a local supermarket. The probability that a shopper will buy a packet of crackers after tasting the free sample is **0.200**.

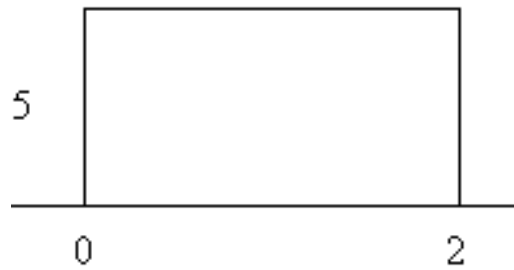
If \hat{p} is the proportion of the next **100** shoppers that buy a packet of the crackers after tasting a free sample, then its approximately distributed as:

1. $N\left(.2, \sqrt{\frac{.2 \times .8}{100}}\right)$
2. $N\left(.2, \frac{\sqrt{.2 \times .8}}{100}\right)$
3. $N(.2, .2 \times .8 \times 100)$

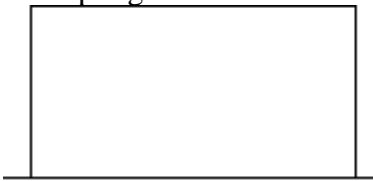
Write your answer as a single-digit number.

Answer: 1

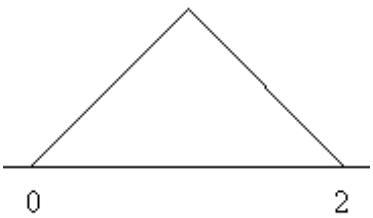
11. Suppose that the random variable X has a distribution that looks like the following:



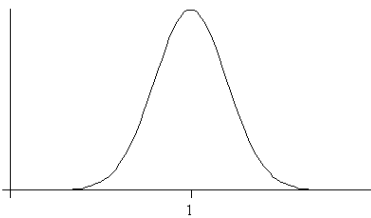
The sampling distribution \bar{X} of with $n=100$ from this distribution will look most like ____.



1) 0



2)



3)

Write your answer as a single-digit number.

Answer: 3