The sixth time

Section 3.4 46，47，48，54

Section 3.5 68, 69，72，75

Section 3.6 79, 84，86，87

Section 3-4

46.

a. $b(3;8,.35) = \binom{8}{3}(.35)^3(.65)^5 = .279.$

b. $b(5;8,.6) = \binom{8}{5}(.6)^5(.4)^3 = .279.$

c. $P(3 \le X \le 5) = b(3;7,.6) + b(4;7,.6) + \mathrm{b}(5;7,.6) = .745.$

d. $P(1 \le X) = 1 - P(X = 0) = 1 - \binom{9}{0}(.1)^0(.9)^9 = 1 - (.9)^9 = .613.$

47.

a. $B(4;15,.3) = .515.$

b. $b(4;15,.3) = B(4;15,.3) - B(3;15,.3) = .219.$

c. $b(6;15,.7) = B(6;15,.7) - B(5;15,.7) = .012.$

d. $P(2 \le X \le 4) = B(4;15,.3) - B(1;15,.3) = .480.$

e. $P(2 \le X) = 1 - P(X \le 1) = 1 - B(1;15,.3) = .965.$

f. $P(X \le 1) = B(1;15,.7) = .000.$

g. $P(2 < X < 6) = P(2 < X \le 5) = B(5;15,.3) - B(2;15,.3) = .595.$

48. $X \sim \text{Bin}(25, .05)$

a. $P(X \le 2) = B(2;25,.05) = .873.$

b. $P(X \ge 5) = 1 - P(X \le 4) = 1 - B(4;25,.05) = .1 - .993 = .007.$

c. $P(1 \le X \le 4) = P(X \le 4) - P(X \le 0) = .993 - .277 = .716.$

d. $P(X = 0) = P(X \le 0) = .277.$

e. $E(X) = np = (25)(.05) = 1.25,\ SD(X) = \sqrt{np(1-p)} = \sqrt{25(.05)(.95)} = 1.09.$

54. Let X equal the number of customers who choose an oversize racket, so $X \sim \text{Bin}(10, .60)$.

a. $P(X \ge 6) = 1 - P(X \le 5) = 1 - B(5;20,.60) = 1 - .367 = .633.$

b. $\mu = np = 10(.6) = 6$ and $\sigma = \sqrt{10(.6)(.4)} = 1.55$, so $\mu \pm \sigma = (4.45, 7.55)$.
$P(4.45 < X < 7.55) = P(5 \le X \le 7) = P(X \le 7) - P(X \le 4) = .833 - .166 = .667.$

c. This occurs iff between 3 and 7 customers want the oversize racket (otherwise, one type will run out early). $P(3 \le X \le 7) = P(X \le 7) - P(X \le 2) = .833 - .012 = .821.$

Section 3.5

68.

a. There are 20 items total, 12 of which are "successes" (two slots). Among these 20 items, 6 have been randomly selected to be put under the shelf. So, the random variable X is hypergeometric, with $N = 20$, $M = 12$, and $n = 6$.

b. $$P(X=2) = \frac{\binom{12}{2}\binom{20-12}{6-2}}{\binom{20}{6}} = \frac{\binom{12}{2}\binom{8}{4}}{\binom{20}{6}} = \frac{(66)(70)}{(38760)} = .1192.$$

$$P(X \le 2) = P(X=0) + P(X=1) + P(X=2) = \frac{\binom{12}{0}\binom{8}{6}}{\binom{20}{6}} + \frac{\binom{12}{1}\binom{8}{5}}{\binom{20}{6}} + .1192 =$$

$.0007 + .0174 + .1192 = .1373.$
$P(X \ge 2) = 1 - P(X \le 1) = 1 - [P(X=0) + P(X=1)] = 1 - [.0007 + .0174] = .9819.$

c. $$E(X) = n \cdot \frac{M}{N} = 6 \cdot \frac{12}{20} = 6 \cdot (.6) = 3.6;\ V(X) = \left(\frac{20-6}{20-1}\right) \cdot 6(.6)(1-.6) = 1.061;\ \sigma = 1.030.$$

69. According to the problem description, X is hypergeometric with $n = 6$, $N = 12$, and $M = 7$.

a. $P(X = 5) = \dfrac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}} = \dfrac{105}{924} = .114$.

b. $P(X \le 4) = 1 - P(X > 4) = 1 - [P(X = 5) + P(X = 6)] = 1 - \left[\dfrac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}} + \dfrac{\binom{7}{6}\binom{5}{0}}{\binom{12}{6}}\right] =$

$1 - [.114 + .007] = 1 - .121 = .879$.

c. $E(X) = n \cdot \dfrac{M}{N} = 6 \cdot \dfrac{7}{12} = 3.5$; $V(X) = \left(\dfrac{12-6}{12-1}\right)6\left(\dfrac{7}{12}\right)\left(1 - \dfrac{7}{12}\right) = 0.795$; $\sigma = 0.892$. So, $P(X > \mu + \sigma) = P(X > 3.5 + 0.892) = P(X > 4.392) = P(X = 5 \text{ or } 6) = .121$ (from part **b**).

d. We can approximate the hypergeometric distribution with the binomial if the population size and the number of successes are large. Here, $n = 15$ and $M/N = 40/400 = .1$, so $h(x;15, 40, 400) \approx b(x;15, .10)$. Using this approximation, $P(X \le 5) \approx B(5; 15, .10) = .998$ from the binomial tables. (This agrees with the exact answer to 3 decimal places.)

72.

a. There are $N = 11$ candidates, $M = 4$ in the "top four" (obviously), and $n = 6$ selected for the first day's interviews. So, the probability x of the "top four" are interviewed on the first day equals $h(x; 6, 4, 11) =$ $\dfrac{\binom{4}{x}\binom{7}{6-x}}{\binom{11}{6}}$.

b. With $X =$ the number of "top four" interview candidates on the first day, $E(X) = n \cdot \dfrac{M}{N} = 6 \cdot \dfrac{4}{11} = 2.18$.

75.

a. With $S =$ a female child and $F =$ a male child, let $X =$ the number of F's before the 2nd S. Then $P(X = x) = nb(x; 2, .5) = \binom{x+2-1}{2-1}(.5)^2(1-.5)^x = (x+1)(.5)^{x+2}$.

b. $P(\text{exactly 4 children}) = P(\text{exactly 2 males} = P(X = 2) = nb(2; 2, .5) = (2 + 1)(.5)^4 = .188$.

c. $P(\text{at most 4 children}) = P(X \le 2) = \sum_{x=0}^{2} nb(x;2,.5) = .25 + .25 + .188 = .688$.

d. $E(X) = \dfrac{r(1-p)}{p} = \dfrac{2(1-.5)}{.5} = 2$, so the expected number of children is equal to $E(X + 2) = E(X) + 2 = 4$.

Section 3.6

79. All these solutions are found using the cumulative Poisson table, $F(x; \mu) = F(x; 5)$.

a. $P(X \le 8) = F(8; 5) = .932$.

b. $P(X = 8) = F(8; 5) - F(7; 5) = .065$.

c. $P(X \ge 9) = 1 - P(X \le 8) = .068$.

d. $P(5 \le X \le 8) = F(8; 5) - F(4; 5) = .492$.

e. $P(5 < X < 8) = F(7; 5) - F(5; 5) = .867 - .616 = .251$.

84.

a. The experiment is binomial with $n = 10{,}000$ and $p = .001$, so $\mu = np = 10$ and $\sigma = \sqrt{npq} = \sqrt{10000(.001)(.999)} = 3.16$.

b. X has approximately a Poisson distribution with $\mu = 10$, so $P(X > 10) \approx 1 - F(10; 10) = 1 - .583 = .417$.

c. Using the same Poisson approximation, $P(X = 0) \approx \dfrac{e^{-10}10^0}{0!} = e^{-10} = .0000454$.

86.

a. $P(X = 4) = \dfrac{e^{-5}5^4}{4!} = .175$.

b. $P(X \ge 4) = 1 - P(X \le 3) = 1 - F(3; 5) = 1 - .265 = .735$.

c. Arrivals occur at the rate of 5 per hour, so for a 45-minute period the mean is $\mu = (5)(.75) = 3.75$, which is the expected number of arrivals in a 45-minute period.

87

a. For a two hour period the parameter of the distribution is $\lambda t = (4)(2) = 8$, so $P(X = 10) = F(10;8) - F(9;8) = .099$.

b. For a 30 minute period, $\lambda t = (4)(.5) = 2$, so $P(X = 0) = F(0,2) = .135$

c. $E(X) = \lambda t = 2$