Course 4

independence独立: Two event A and B are independence **if $P(A|B) = P(A)$** and are dependent otherwise

- $P(B|A) = P(B)$
- A' and B, A and B', A' and B' are also independent
- disjoint($P(A \cap B) = P(\emptyset) = 0$) $\rightarrow$ dependant 不相容是有关系的(互斥)，而不是相对独立
- A and B are independent **if and only if** $P(A \cap B) = P(A)P(B)$

> **Proof Example**:
> $$P(A) = \frac{1}{2}, P(B) = \frac{1}{2}$$
> $$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{3} \neq P(A)$$
> So A and B are dependant
>
> **Proof disjoint/mutually exclusive -> dependent**:
>
> Since A and B are disjoint/mutually exclusive, then $A \cap B = \emptyset, P(A \cap B) = 0, P(A|B) = \frac{P(A \cap B)}{P(B)} = 0 \neq P(A)$

discrete random variable离散随机变量

- 一般记作$X()$，是一个离散变量函数，括号中的内容可以是某个事件

probability distribution for discrete random variable离散随机变量的概率分布

x	x_1	x_2	$\dots$	x_n
$P(x)$	$P(x_1)$	$P(x_2)$	$\dots$	$P(x_n)$

Find the probability distribution of x_i

probability mass function(pmf)概率质量函数

- 记作$P(x)$
- $p_i = P(x_i) \geq 0$
- $\sum p_i = 1$

Parameter of a Probability Distribution概率分布的参数: Suppose $p(x)$ depends on a quantity that can be assigned any one of a number of possible values, with each different value determining a different probability distribution. Such a quantity is called a parameter of the distribution

概率分布的参数是用于描述和确定概率分布特性的数值或者符号

> **Generate by GPT-4**:
>
> 二项分布(Binomial Distribution)：二项分布是描述在n次独立伯努利试验中成功次数的概率分布。它有两个参数：n(试验次数)和p(每次试验成功的概率)
>
> 正态分布(Normal Distribution)：正态分布，也称为高斯分布，是一种连续概率分布，用于描述许多自然现象。它有两个参数：μ(均值，表示分布的中心)和σ^2(方差，表示分布的离散程度)
>
> 指数分布(Exponential Distribution)：指数分布用于描述在恒定平均速率下，两个独立随机事件之间的时间间隔。它有一个参数：λ(平均速率或强度，即单位时间内事件发生的次数)

Bernoulli random variable伯努利随机变量: outcomes 0 or 1

x	0	1
$p(x)$	$1 - \alpha$	α

cumulative distribution function(cdf)累计分布函数

- 记作$F(x) = P(X \leq x) = \sum_{y \leq x} p(y)$

$F(1) = P(y \leq 1) = p(1)$
$F(2) = P(y \leq 2) = p(1) + p(2)$
$F(3) = P(y \leq 3) = p(1) + p(2) + p(3)$
$\vdots$
$F(n) = P(y \leq n) = p(1) + p(2) + p(3) + \cdots + p(n) = 1$

Homework

Section 2.5 71, 72, 80, 84
Section 3.1 4, 5, 8, 10
Section 3.2 12, 23, 25