Course 5

expect value期望值: Given **pmf** of X, then

- $E(X) = \mu_X = \sum_{x \in D} x \cdot p(x)$
- $E[h(X)] = \sum h(x)p(x)$
- $E[aX + b] = aE(X) + b$

> **Proof** $E[aX + b] = aE(X) + b$
>
> $$\begin{aligned} & E[aX+b] \\ = & \sum_D (ax+b)p(x) \\ = & a\sum_D xp(x) + b\sum_D p(x) \\ = & aE(X) + b \end{aligned}$$

the variance of X X的方差

- $\sigma^2 = V(X) = \sum (x - \mu_X)^2 \cdot p(x)$
- $\sigma = \sqrt{\sigma^2}$
- $V(aX + b) = a^2 V(X)$
- $\sigma_{aX+b} = |a|\sigma_X$

> **Proof** $\sigma^2 = V(X) = \sum (x - \mu_X)^2 \cdot p(x) = E(X^2) - [E(X)]^2$
>
> $$\begin{aligned} & \sum (x-\mu)^2 \cdot p(x) \\ = & \sum (x^2 - 2x\mu + \mu^2) \cdot p(x) \\ = & \sum x^2 \cdot p(x) - 2\mu \sum x \cdot p(x) + \mu^2 \sum p(x) \end{aligned}$$
>
> **Because:** $\sum x \cdot p(x) = \mu$, $\sum p(x) = 1$
>
> $$\begin{aligned} = & E(X^2) - 2\mu\mu + \mu^2 \\ = & E(X^2) - \mu^2 \\ = & E(X^2) - [E(X)]^2 \end{aligned}$$

Some important distribution of discrete random variable:

1. **binomial distribution二项分布**
   > 有放回抽样

   1. Definition: Given a binomial experiment consisting of n trails, the binomial random variable X associated with this experiment is defined as
      X = the number of S's among the n trials
   2. . n trials(n is fixed in advance)
   3. Only 2 results: S(success) or F(faliure)
   4. Each trial is independent——有放回的标志
   5. $P(S) = p$(p is a constant)
   6. If the sample size(number of trials n) is at most 5% of the total result $\rightarrow$ can be seen as binomial distribution
      不放回试验的次数为样本量的5%及以下，可以近似视为二项分布处理
   7. **pmf** of binomial distribution = $b(x; n, p)$
      注意字母b的大小写，此处为小写
   8. **cdf** of binomial distribution = $B(x; n, p)$
   9. $b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$, when $x = 0, 1, 2, \ldots, n$
   10. $B(x; n, p) = \sum_{i=0}^{x} b(i; n, p)$（查表）

11. $E(X) = np$
12. $V(X) = np(1-p) = npq$, where $q = 1 - p$
13. For $n = 1$, the binomial distribution became the **Bernoulli distribution**. The mean value of Bernoulli variables is $\mu = p$

Homework

Section 3.3 29, 33, 38, 41