# Course 13

**Confidence Interval(CI)置信区间**: The probability of the interval **cover the true value** is confidence level(CL)

Problem: Given CL, find CI

- **Case1**: Population distribution is normal, $\sigma$ is known, estimate $\mu$
  - Since $X \sim N(\mu, \sigma^2)$, then $\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$, $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$
  - $P(-Z_{\frac{\alpha}{2}} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < Z_{\frac{\alpha}{2}}) = 1-\alpha = P(\bar{X} - \frac{\sigma}{\sqrt{n}} Z_{\frac{\alpha}{2}} < \mu < \bar{X} + \frac{\sigma}{\sqrt{n}} Z_{\frac{\alpha}{2}})$
- **Case2**: Any population distribution(normal or abnormal), $\sigma$ is unknown, $n$ is sufficiently large($n \geq 40$)
  - Use CLT(Central Limit Theorem)
  - $S \to \sigma (s \approx \sigma)$
  - Sample variance: $S^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$
  - $Z = \frac{\bar{X}-\mu}{S/\sqrt{n}}$
  - $\mu = \bar{X} \pm \frac{S}{\sqrt{n}} - Z_{\frac{\alpha}{2}}$
- **Case3**: Population is normal, both $\mu$ and $\sigma$ are unknown, and $n$ is small
  - Use **T distribution**: $T = \frac{\bar{X}-\mu}{S/\sqrt{n}}$
    - The only one parameter in T is **the number of df(degrees of freedom)**: $v = n-1$, when $n \geq 40, T \sim N(0,1)$