Physics FSE (2021-22) Homework 5

Send the completed file to my mailbox yy.lam@qq.com by June 15th, with using the filename format:

`2021xxxxxx_yourname_fse_hw5`

Please answer the questions by filling on these sheets.

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1. One of the string of a violin is 30 cm long and has a mass per unit length of 0.0028 kgm^{-1}. If the string tension of the string is 78.2 N. What is the frequency of the seventh harmonics of the string?

   Solution. *Since the two ends of the string are fixed, the length of the string is 7/2 of wavelength of the seventh harmonics. Thus,*

$$\begin{aligned}\nu &= \frac{v}{2L/7} \\ &= \frac{7}{2L}\sqrt{\frac{T}{\mu}} \\ &= \frac{7}{2 \times 0.3}\sqrt{\frac{78.2}{0.0028}} \\ &= 1950 \ Hz\end{aligned}$$

2. What is the intensity at a distance of 4.2 m from a point source emitting sound uniformly in all directions at a power rating of 120 W?

   Solution. *The intensity is the power per unit area. For a point source emitting sound wave, applying the spherical area we have*

$$I = \frac{P}{4\pi r^2} = \frac{120}{4\pi \times 4.2^2} = 0.541 \ Wm^{-2}.$$

3. A machine operates at about 62 dB measured from 1.8 m away. What is the intensity measured at 3.3 m away from the source?

   Solution. *Since*

$$62 = 10\log_{10}\frac{I}{10^{-12}},$$

   thus, $I = 10^{6.2} \times 10^{-12} \ Wm^{-2}$. *At 3.3 m away from the source the intensity will be*

$$10^{6.2} \times 10^{-12} \times \frac{1.8^2}{3.3^2} = 4.715 \times 10^{-7} \ Wm^{-2}.$$

4. Find the length of an one end open and one end closed organ pipe if the difference of frequencies between two successive resonances and the speed of sound are 220 Hz and 343 ms^{-1}, respectively.

Solution. *Let* λ_1, λ_2 *be the wavelengths of two of the successive resonances. We have the conditions for the successive harmonics:*

$$\frac{L}{n} = \frac{\lambda_1}{4} \quad \textit{and} \quad \frac{L}{n+2} = \frac{\lambda_2}{4}$$

There only exist the odd harmonics, where $n = 1, 3, 5 \ldots$. *Eliminating* n *we get*

$$\frac{L}{4L/\lambda_1 + 2} = \frac{\lambda_2}{4} \quad \Rightarrow \quad L = \frac{\lambda_1 \lambda_2}{2(\lambda_1 - \lambda_2)}$$

Using $\lambda = v/\nu$, *we obtain*

$$L = \frac{v}{2(\nu_2 - \nu_1)} \quad \Rightarrow \quad L = \frac{v}{2\Delta\nu}$$

Inserting the numerical values, we obtain

$$L = \frac{343}{2 \times 220} = 0.78 \ m.$$

5. At a distance of 3.2 m from a source the sound level is 84 dB. How far away has the level dropped to 65 dB?

   Solution. *According to the inverse square law,* $\frac{d^2}{3.2^2} = \frac{I_{3.2}}{I_d}$ *where* d *is the distance to be determined,* I_d, $I_{3.2}$ *are the corresponding intensities measured at* d *and at 3.2 metre away from the source, respectively. As* I_0 *stands for the threshold of hearing, we get,*

$$\begin{aligned} 65 - 84 &= 10\left(\log\frac{I_d}{I_0} - \log\frac{I_{3.2}}{I_0}\right) \\ &= 10\log\frac{I_d}{I_{3.2}} \\ -1.9 &= \log\frac{3.2^2}{d^2} \\ 10^{-1.9} &= \frac{3.2^2}{d^2} \\ d &= 28.5 \ m \end{aligned}$$

6. An 8-hour exposure to a sound intensity level of 89 dB may cause hearing damage. What energy in joules falls on a 0.77 cm diameter eardrum so exposed?

   Solution. *The intensity at 89 dB is*

$$89 = 10\log\frac{I}{10^{-12}} \quad \Rightarrow \quad 10^{8.9} = \frac{I}{10^{-12}} \quad \Rightarrow \quad I = 10^{-3.1} \ Wm^{-2}$$

The total energy contribution on the eardrum is

$$10^{-3.1} \times 8 \times 60 \times 60 \times (0.0077/2)^2\pi = 1.065 \times 10^{-3} \ J$$

7. Calculate the first overtone in an ear canal, which resonates like a 2.38 cm long tube closed at one end, by taking air temperature to be $35.0°C$. Is the ear particularly sensitive to such a frequency? (Given the speed of sound in air at $0\,°\mathrm{C}$ is 331 ms^{-1})

Solution. *The first overtone of an one end open one end closed tube also means the third harmonics, the wavelength is*

$$\frac{3}{4}\lambda = 0.0238 \quad \Rightarrow \quad \lambda = 0.0317\ m.$$

The velocity of sound in air at $35\,^{\circ}C$ *is*

$$v = 331 \times \sqrt{1 + \frac{35}{273}} = 351.6\ ms^{-1}.$$

The frequency of the first overtone is

$$\nu = \frac{351.6}{0.0317} \approx 11\ kHz.$$

This frequency is well higher than the resonant frequency of our ear canal ($1000 \sim 3000$ *Hz) and we are not sensitive to that frequency. In most case we simply ignore it involuntarily unless the intensity is very high.*

8. Ultrasound that has a frequency of 2.40 MHz is sent toward blood in an artery that is moving away the source at 20.0 cm/s. Use the speed of sound in human tissue as 1540 m/s. (a) What frequency does the blood receive? (b) What frequency returns to the source?

   Solution. *(a) As the blood moving away from the source, it is the case of moving observer with stationary sound source. Thus the the blood receive the frequency with*

   $$\nu\frac{v - v_o}{v} = 2.4 \times \frac{1540 - 0.2}{1540} = 2.399688\ MHz.$$

   (b) The blood here becomes the moving (away) sound source due to the reflection of sound, the original sound source acts as an observer receiving

   $$\nu\frac{v}{v + v_s} = 2.399688 \times \frac{1540}{1540 + 0.2} = 2.399376\ MHz.$$

   Note that the 'new source' frequency $\nu = 2.399688$ *is used from the result of part (a).*