Answers to homework

Chapter 2

Section 2.5 71, 72, 80,84

Chapter 3

Section 3.1 4 , 5, 8, 10

Section 3.2 12, 23, 25

Section 2-5

71

a. Since the events are independent, then A' and B' are independent, too. (See the paragraph below Equation 2.7.) Thus, $P(B'|A') = P(B') = 1 - .7 = .3$.

b. Using the addition rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .4 + .7 - (.4)(.7) = .82$. Since A and B are independent, we are permitted to write $P(A \cap B) = P(A)P(B) = (.4)(.7)$.

c. $P(AB' \mid A \cup B) = \dfrac{P(AB' \cap (A \cup B))}{P(A \cup B)} = \dfrac{P(AB')}{P(A \cup B)} = \dfrac{P(A)P(B')}{P(A \cup B)} = \dfrac{(.4)(1-.7)}{.82} = \dfrac{.12}{.82} = .146$.

72. $P(A_1 \cap A_2) = .11$ while $P(A_1)P(A_2) = .055$, so A_1 and A_2 are not independent.
$P(A_1 \cap A_3) = .05$ while $P(A_1)P(A_3) = .0616$, so A_1 and A_3 are not independent.
$P(A_2 \cap A_3) = .07$ and $P(A_2)P(A_3) = .07$, so A_2 and A_3 are independent.

80.

Let A_i denote the event that component #i works (i = 1, 2, 3, 4). Based on the design of the system, the event "the system works" is $(A_1 \cup A_2) \cup (A_3 \cap A_4)$. We'll eventually need $P(A_1 \cup A_2)$, so work that out first: $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = (.9) + (.9) - (.9)(.9) = .99$. The third term uses independence of events. Also, $P(A_3 \cap A_4) = (.9)(.9) = .81$, again using independence.

Now use the addition rule and independence for the system:

$$\begin{aligned} P((A_1 \cup A_2) \cup (A_3 \cap A_4)) &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P((A_1 \cup A_2) \cap (A_3 \cap A_4)) \\ &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P(A_1 \cup A_2) \times P(A_3 \cap A_4) \\ &= (.99) + (.81) - (.99)(.81) = .9981 \end{aligned}$$

(You could also use deMorgan's law in a couple of places.)

84

Let A_i denote the event that vehicle #i passes inspection (i = 1, 2, 3).

a. $P(A_1 \cap A_2 \cap A_3) = P(A_1) \times P(A_2) \times P(A_3) = (.7)(.7)(.7) = (.7)^3 = .343$.

b. This is the complement of part **a**, so the answer is 1 – .343 = .657.

c. $P([A_1 \cap A_2' \cap A_3'] \cup [A_1' \cap A_2 \cap A_3'] \cup [A_1' \cap A_2' \cap A_3]) = (.7)(.3)(.3) + (.3)(.7)(.3) + (.3)(.3)(.7) = 3(.3)^2(.7)$ = .189. Notice that we're using the fact that if events are independent then their complements are also independent.

d. P(at most one passes) = P(zero pass) + P(exactly one passes) = P(zero pass) + .189. For the first probability, P(zero pass) = $P(A_1' \cap A_2' \cap A_3') = (.3)(.3)(.3) = .027$. So, the answer is .027 + .189 = .216.

e. We'll need the fact that P(at least one passes) = 1 – P(zero pass) = 1 – .027 = .973. Then,

$$P(A_1 \cap A_2 \cap A_3 \mid A_1 \cup A_2 \cup A_3) = \frac{P([A_1 \cap A_2 \cap A_3] \cap [A_1 \cup A_2 \cup A_3])}{P(A_1 \cup A_2 \cup A_3)} = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cup A_2 \cup A_3)} = \frac{.343}{.973} = .3525.$$

Section 3-1

4. In my perusal of a zip code directory, I found no 00000, nor did I find any zip codes with four zeros, a fact which was not obvious. Thus possible X values are 2, 3, 4, 5 (and not 0 or 1). As examples, X = 5 for the outcome 15213, X = 4 for the outcome 44074, and X = 3 for 90022.

5. No. In the experiment in which a coin is tossed repeatedly until a H results, let Y = 1 if the experiment terminates with at most 5 tosses and Y = 0 otherwise. The sample space is infinite, yet Y has only two possible values. See the back of the book for another example.

8. The least possible value of Y is 3; all possible values of Y are 3, 4, 5, 6, ….
Y = 3: *SSS*; Y = 4: *FSSS*; Y = 5: *FFSSS*, *SFSSS*;
Y = 6: *SSFSSS*, *SFFSSS*, *FSFSSS*, *FFFSSS*;
Y = 7: *SSFFSSS*, *SFSFSSS*, *SFFFSSS*, *FSSFSSS*, *FSFFSSS*, *FFSFSSS*, *FFFFSSS*

10.

a. Possible values of T are: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

b. Possible values of X are: –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6.

c. Possible values of U are: 0, 1, 2, 3, 4, 5, 6.

d. Possible values of Z are: 0, 1, 2.

Section 3-2

12.

a. Since there are 50 seats, the flight will accommodate all ticketed passengers who show up as long as there are no more than 50. $P(Y \le 50) = .05 + .10 + .12 + .14 + .25 + .17 = .83$.

b. This is the complement of part **a**: $P(Y > 50) = 1 - P(Y \le 50) = 1 - .83 = .17$.

c. If you're the first standby passenger, you need no more than 49 people to show up (so that there's space left for you). $P(Y \le 49) = .05 + .10 + .12 + .14 + .25 = .66$. On the other hand, if you're third on the standby list, you need no more than 47 people to show up (so that, even with the two standby passengers ahead of you, there's still room). $P(Y \le 47) = .05 + .10 + .12 = .27$.

23

a. $p(2) = P(X = 2) = F(3) - F(2) = .39 - .19 = .20$.

b. $P(X > 3) = 1 - P(X \le 3) = 1 - F(3) = 1 - .67 = .33$.

c. $P(2 \le X \le 5) = F(5) - F(2{-}1) = F(5) - F(1) = .92 - .19 = .78$.

d. $P(2 < X < 5) = P(2 < X \le 4) = F(4) - F(2) = .92 - .39 = .53$.

25

$p(0) = P(Y = 0) = P(B \text{ first}) = p$;
$p(1) = P(Y = 1) = P(G \text{ first, then } B) = (1 - p)p$;
$p(2) = P(Y = 2) = P(GGB) = (1 - p)^2 p$;
Continuing, $p(y) = P(y\ G\text{s and then a } B) = (1 - p)^y p$ for $y = 0,1,2,3,\ldots$.