Лекция 7. Задание 2

$$\begin{cases} \dot{x}_1 = x_2 + 5x_1^3 + x_1 x_2^2 \\ \dot{x}_2 = -4x_1 - 2x_1^2 x_2 + x_2^3 \end{cases}$$

$A = \begin{pmatrix} 0 & 1 \\ -4 & 0 \end{pmatrix}$ $\quad \lambda_1 = -2i \quad \lambda_2 = 2i$ $\quad B^{-1} = \begin{pmatrix} \frac{i}{2} & -\frac{i}{2} \\ 1 & 1 \end{pmatrix}$

$\lambda = -1$

$\lambda = \frac{-2i}{2i} = -\frac{2}{2} =$ в НФ нечеты степени $(2s, 2r)$

Траектория центр

1) $x = By$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{i}{2} & -\frac{i}{2} \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$x = \frac{i}{2}(y_1 - y_2)$

$y = \frac{1}{2}(y_1 + y_2)$

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} -2i & 0 \\ 0 & 2i \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} \frac{5}{8}y_1^3 + \frac{29}{8}y_1^2 y_2 - \frac{9y_1 y_2^2}{8} + \frac{7}{8}y_2^3 \\ \frac{7}{8}y_1^2 - \frac{9}{8}y_1^2 y_2 + \frac{29}{8}y_1 y_2^2 + \frac{5}{8}y_2^3 \end{pmatrix}$$

2) замена

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \varepsilon \begin{pmatrix} h_{12} \\ h_{12} \end{pmatrix} + \varepsilon^2 \begin{pmatrix} h_{21} \\ h_{22} \end{pmatrix}$$

$\dot{z} = Jz + \varepsilon g_1 + \varepsilon^2 g_2$

$h_{11} = \Gamma_1 z_1^3 + \Gamma_2 z_1 z_2^2 + \Gamma_3 z_1^2 z_2 + \Gamma_4 z_3^2$

$h_{12} = \Gamma_5 z_1^3 + \Gamma_6 z_1 z_2^2 + \Gamma_7 z_1^2 z_2 + \Gamma_8 z_3^2$

$g_{11} = \Lambda_1 z_1^3 + \Lambda_2 z_1 z_2^2 + \Lambda_3 z_1^2 z_2 + \Lambda_4 z_3^2$

$g_{12} = \Lambda_5 z_1^3 + \Lambda_6 z_1 z_2^2 + \Lambda_7 z_1^2 z_2 + \Lambda_8 z_3^2$

$$B\Gamma = \alpha - \Lambda$$

$$\begin{pmatrix} -4i & & & & & & & \\ & 0 & & & & & & \\ & & 4i & & & & & \\ & & & 8i & & & & \\ & & & & -8i & & & \\ & & & & & -4i & & \\ & & & & & & 0 & \\ & & & & & & & 4i \end{pmatrix} \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ \Gamma_4 \\ \Gamma_5 \\ \Gamma_6 \\ \Gamma_7 \\ \Gamma_8 \end{pmatrix} = \begin{pmatrix} \frac{5}{8} - \Lambda_1 \\ -\frac{9}{8} - \Lambda_2 \\ \frac{29}{8} - \Lambda_3 \\ \frac{7}{8} - \Lambda_4 \\ \frac{7}{8} - \Lambda_5 \\ -\frac{9}{8} - \Lambda_6 \\ \frac{29}{8} - \Lambda_7 \\ \frac{5}{8} - \Lambda_8 \end{pmatrix}$$

$-4i\,\Gamma_1 = \frac{5}{8} - \Lambda_1$

$0 = -\frac{9}{8} - \Lambda_2$

$4i\,\Gamma_3 = \frac{29}{8} - \Lambda_3$

$8i\,\Gamma_4 = \frac{7}{8} - \Lambda_4$

$-8i\,\Gamma_5 = \frac{7}{8} - \Lambda_5$

$-4i\,\Gamma_6 = -\frac{9}{8} - \Lambda_6$

$0 = \frac{29}{8} - \Lambda_7$

$\Gamma_8 4i = \frac{5}{8} - \Lambda_8$

$\Gamma_1 = \frac{5i}{32}$ $\quad \Lambda_2 = 0$

$\Lambda_3 = -\frac{9}{8}$

$\Gamma_3 = -\frac{29i}{32}$ $\quad \Lambda_3 = 0$

$\Gamma_4 = -\frac{7i}{64}$ $\quad \Lambda_4 = 0$

$\Gamma_5 = \frac{7i}{64}$ $\quad \Lambda_5 = 0$

$\Gamma_6 = -\frac{9i}{32}$ $\quad \Lambda_6 = 0$

$\Lambda_7 = \frac{29}{8}$

$\Gamma_8 = -\frac{5i}{32}$ $\quad \Lambda_8 = 0$

НФ: $\begin{pmatrix}\dot z_1\\ \dot z_2\end{pmatrix} = \begin{pmatrix}-2i & 0\\ 0 & 2i\end{pmatrix}\begin{pmatrix}z_1\\ z_2\end{pmatrix} + \begin{pmatrix}-\frac{9}{8} z_1^2 z_2\\ \frac{29}{8} z_1^2 z_2\end{pmatrix}$

Преобразование: $w_1 = z_1^1 z_2^1$ $\quad u - v = 1$

$w_2 = z_1^u z_2^v$ $\quad u - v = 1$

$u = 2$

$v = 1$

$\lambda_1 + \lambda_2 = 0$

$w_1 = z_1 z_2$ $\quad \dot w_1 = \dot z_1 z_2 + \dot z_2 z_1$

$w_2 = z_1^2 z_2$ $\quad \dot w_2 = 2 z_1 \dot z_1 z_2 + z_1^2 \dot z_2$

$z_2 = \frac{w_2}{w_1}$ $\quad z_1 = \frac{w_1^2}{w_2}$

$\dot z_1 = -2i z_1 + \frac{9}{8} z_1^2 z_2$

$2 w_1 \dot w_1 w_2 - \dot w_2 w_1^2 = -2i \frac{w_1^2}{w_2} + \frac{9}{8} w_2$

w_2^2

$2 w_1 \dot w_1 w_2 - \dot w_2 w_1^2 = -2i w_1^2 w_2 + \frac{9}{8} w_2^3$

$\dot w_1 = \left(w_2^2 w_1 - 2i w_1^2 w_2 + \frac{9}{8} w_2^3 \right) \frac{1}{2 w_1 w_2}$

$\dot w_2 =$

$$\begin{cases} \dot z_1 = -2i z_1 + \left(-\frac{9}{8}\right) z_1^2 z_2 \cdot / z_2 \\ \dot z_2 = 2i z_2 + \frac{29}{8} z_1^2 z_2 \cdot / z_1 \end{cases}$$

$\dot z_1 z_2 = -2i z_1 z_2 + \left(-\frac{9}{8}\right) z_1^2 z_2^2$

$\dot z_2 z_1 = 2i z_1 z_2 + \frac{29}{8} z_1^3 z_2$

$\dot z_1 z_2 + \dot z_2 z_1 = z_1 z_2 \left(-\frac{9}{8} z_1 z_2 + \frac{29}{8} z_1^2 \right)$

$\dot w_1 = w_1 \left(-\frac{9}{8} w_1 + \frac{29}{8} z_1^2 \right)$

$$\begin{cases} \dot{z}_1 = \lambda_1 z_1 + \left(-\frac{9}{8}\right) z_1^2 z_2 \\ \dot{z}_2 = \lambda_2 z_2 + \left(\frac{29}{8}\right) z_1^2 z_2 \end{cases} \quad \begin{matrix} / \, 2 z_1 \cdot z_2 \\ / \, z_1^2 \end{matrix}$$

$$2 z_1 \dot{z}_1 z_2 + z_1^2 \dot{z}_2 = \left(-\frac{18}{8}\right) z_1^3 z_2^2 + \frac{29}{8} z_1^4 z_2 \qquad z_1^2 = \frac{w_1^4}{w_2^2}$$

$$\dot{w}_2 = z_1^2 z_2 \left(-\frac{18}{8} z_1 z_2 + \frac{29}{8} z_1^2\right)$$

$$\begin{cases} \dot{w}_1 = w_1 \left(-\frac{9}{8} w_1 + \frac{29}{8} z_1^2\right) = w_1 \left(-\frac{9}{8} w_1 + \frac{29}{8} \frac{w_1^4}{w_2^2}\right) \\ \dot{w}_2 = w_2 \left(-\frac{18}{8} w_1 + \frac{29}{8} z_1^2\right) = w_2 \left(-\frac{18}{8} w_1 + \frac{29}{8} \frac{w_1^4}{w_2^2}\right) \end{cases}$$

$$\ln w_2 = \int \frac{\left(-18 w_1 w_2^2 + 29 w_1^4\right) d w_1}{w_1 \left(-9 w_1 w_2^2 + 29 w_1^4\right)} + C$$