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A Study of Angular Motion of the *Chibis-M* Microsatellite with Three-Axis Flywheel Control

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Abstract—We study the controlled angular motion of the *Chibis-M* microsatellite. Executive elements are three pairs of flywheels, whose axes are mutually perpendicular. The task of the control system is realization of a required program motion and support of its asymptotic stability. In this paper, we synthesize a control algorithm and study the evolution of the angular momentum of flywheels on long time intervals. The attitude accuracy is estimated for the case when disturbances act upon the spacecraft.

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INTRODUCTION

The problem of attitude control is important for most spacecraft. In this paper, we discuss the controlled motion of the *Chibis-M* microsatellite. It is designed to study atmospheric lightning discharges. The scientific instrumentation of the satellite requires its three-axis orientation relative to the orbital reference frame. Damping of the satellite's initial angular velocity after its separation from the carrier is realized by electromagnetic coils interacting with the geomagnetic field. Then, the spacecraft is stabilized in the required position by means of flywheels. A magnetometer, a set of solar sensors with linear CCD arrays[1], and three single-axis angular velocity sensors are used as sensors of attitude determination.

The problem of spacecraft attitude control is solved using a three-axis flywheel attitude control system. Flywheel control is based on the PD controller[2], which ensures the existence of the required equilibrium position and its asymptotic stability. This type of control is widely used in the active flywheel control [1–6]. This paper presents the method for selecting the control parameters. We study the processes of evolution of the flywheel angular momentum on a long time interval. The influence of various disturbances on the spacecraft motion relative to the center of mass close to equilibrium position is analyzed.

[1] Specialized analog integrated chip consisting of a light-sensitive silicon photodiodes and using the CCD (charge-coupled device) technology.

[2] PD (proportional derivative) is the control according to a mismatch in position and velocity.

1. THE EQUATIONS OF CONTROLLED MOTION

The following reference frames are used for studying the system's dynamics:

$OY_1Y_2Y_3$ is the nonrotating frame: the axis OY_2 is perpendicular to the equatorial plane, OY_3 is directed to the vernal equinox, OY_1 complements this system to the right-handed orthogonal frame, the origin of coordinates O is located at the center of mass of the satellite;

$Ox_1x_2x_3$ is the body frame, whose axes are the principal central axes of inertia of the spacecraft;

$OX_1X_2X_3$ is the orbital frame: the axis OX_3 is directed along local vertical, OX_2 along normal to the orbit plane, and OX_1 complements the system to the right-handed orthogonal frame. This system rotates around the axis OX_2 with the angular velocity equal to ω_{orb}. In what follows we assume that the orbit is circular, and therefore, $\omega_{orb} = \text{const}$.

The relationship between frames $Ox_1x_2x_3$ and $OX_1X_2X_3$ is given in two ways: by a set of angles α, β, and γ (Fig. 1) (Euler angles), and by matrix $\mathbf{A}$ of direction cosines.

In the case of using the angles, a conversion from the system $OX_1X_2X_3$ to the system $Ox_1x_2x_3$ is made through three sequential rotations (Fig. 1). The first rotation is realized relative to the axis OX_2 by the angle α, the second around the axis Ox_3' (in which the axis OX_3 transfers after the first rotation) by the angle β, and the third around the axis Ox_1 by the angle γ.

The matrix of direction cosines

$$\mathbf{A} = \begin{pmatrix} \cos\alpha\cos\beta & \sin\beta & -\sin\alpha\cos\beta \\ -\cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma & \cos\beta\cos\gamma & \sin\alpha\sin\beta\cos\gamma + \cos\alpha\sin\gamma \\ \cos\alpha\sin\beta\sin\gamma + \sin\alpha\cos\gamma & -\cos\beta\sin\gamma & -\sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma \end{pmatrix},$$

whose elements are expressed in terms of introduced angles, describes the conversion from the orbital coordinate system $OX_1X_2X_3$ to the body-fixed frame $Ox_1x_2x_3$.

The spacecraft motion is given by the Euler's dynamical equations

$$\frac{d\mathbf{K}}{dt} + \omega_{\text{abs}} \times \mathbf{K} = \mathbf{M}_{\text{ext}} + \mathbf{M}_{\text{c}} \tag{1.1}$$

and by the kinematic relationships

$$\dot{\mathbf{A}} = \mathbf{WA} \tag{1.2a}$$

or

$$\begin{gathered} \dot{\alpha} = \frac{1}{\cos\beta}(\omega_2\cos\gamma - \omega_3\sin\gamma), \\ \dot{\beta} = \omega_2\sin\gamma + \omega_3\cos\gamma, \\ \dot{\gamma} = \omega_1 - \text{tg}\beta(\omega_2\cos\gamma - \omega_3\sin\gamma). \end{gathered} \tag{1.2b}$$

Here, $\mathbf{K} = \mathbf{J}\boldsymbol{\omega}_{\text{abs}}$ is the angular momentum of the spacecraft; $\boldsymbol{\omega}_{\text{abs}} = \boldsymbol{\omega}_{\text{rel}} + \mathbf{A}\boldsymbol{\omega}_0$ is the vector of its absolute angular velocity; $\mathbf{J}$ is the inertia tensor of the spacecraft with flywheels; $\mathbf{M}_{\text{c}}$ is the control torque; $\mathbf{M}_{\text{ext}}$ is the external forces torque; $\omega_{\text{rel}} = (\omega_1\ \omega_2\ \omega_3)^T$ is the angular velocity vector of the system $Ox_1x_2x_3$ relative to $OX_1X_2X_3$ written in projections onto the axes of the system $Ox_1x_2x_3$; $\boldsymbol{\omega}_0 = (0,\ \omega_{\text{orb}}\ 0)^T$ is the absolute angular velocity vector of the orbital frame in projections onto the axes of this system; the so-called angular velocity matrix $\mathbf{W}$ has the form

$$\mathbf{W} = \begin{pmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{pmatrix}.$$

Let us require that the control would realize the equilibrium position $\boldsymbol{\omega}_{\text{rel}} = 0$, $\mathbf{A} = \text{diag}(1;\ 1;\ 1)$ and ensure its asymptotic stability. As was already mentioned, the control is based on the PD controller. In order to specify a concrete type of controller we use the following Lyapunov function:

$$\begin{gathered} V = \frac{1}{2}(\boldsymbol{\omega}_{\text{rel}}, \mathbf{J}\boldsymbol{\omega}_{\text{rel}}) \\ + k_a[(1 - a_{11}) + (1 - a_{22}) + (1 - a_{33})]. \end{gathered} \tag{1.3}$$

Here, a_{ii} are diagonal elements of the matrix of direction cosines ($i = 1, 2, 3$); k_a is a positive coefficient of proportionality (its dimensionality $[k_a] = $ Nm). Function (1.3) satisfies the conditions, which are imposed on the Lyapunov function by conditions of the Barbashin–Krasovskii theorem [7], namely: $V = 0$ at $\omega_{\text{rel}} = 0$ and $\mathbf{A} = \text{diag}(1, 1, 1)$; $V > 0$ at all other values of ω_{rel} and $\mathbf{A}$. We write $\dot{V}$ in view of equations (1.1) and (1.2a)

$$\begin{gathered} \dot{V} = (\boldsymbol{\omega}_{\text{rel}}, [\mathbf{M}_{\text{c}} + \mathbf{M}_{\text{ext}} - \boldsymbol{\omega}_{\text{abs}} \\ \times \mathbf{K} - \mathbf{JWA}\boldsymbol{\omega}_0 - \mathbf{A}\dot{\boldsymbol{\omega}}_0 - k_a\mathbf{S}]). \end{gathered} \tag{1.4}$$

Here, we have introduced the designation $\mathbf{S} = (a_{23} - a_{32};\ a_{31} - a_{13};\ a_{12} - a_{21})^T$. We require that the control ensured the fulfillment of the equality $\dot{V} = -k_\omega\omega_{\text{rel}}^2$, then all conditions of the Barbashin–Krasovskii theorem on asymptotic stability ($k_\omega > 0$, $[k_\omega] = $ N m s) will be met. Controlling torque ensuring this equality takes on the form

$$\begin{gathered} \mathbf{M}_{\text{c}} = -k_\omega\boldsymbol{\omega}_{\text{rel}} - k_a\mathbf{S} + \mathbf{A}\dot{\boldsymbol{\omega}}_0 \\ + \mathbf{JWA}\boldsymbol{\omega}_0 + \boldsymbol{\omega}_{\text{abs}} \times \mathbf{K} - \mathbf{M}_{\text{ext}}. \end{gathered}$$

Let us take into account the fact that the orbit is circular, then

$$\mathbf{M}_{\text{c}} = -k_\omega\boldsymbol{\omega}_{\text{rel}} - k_a\mathbf{S} + \mathbf{JWA}\boldsymbol{\omega}_0 + \boldsymbol{\omega}_{\text{abs}} \times \mathbf{K} - \mathbf{M}_{\text{ext}}.$$

As was already mentioned, the controlling torque is created by three mutually orthogonal flywheels. We present the controlling torque in the form

$$\mathbf{M}_{\text{c}} = -\dot{\mathbf{H}} - \boldsymbol{\omega}_{\text{abs}} \times \mathbf{H}.$$

As a result, the control must satisfy the equation

$$\begin{gathered} \dot{\mathbf{H}} + \boldsymbol{\omega}_{\text{abs}} \times \mathbf{H} = k_\omega\boldsymbol{\omega}_{\text{rel}} + k_a\mathbf{S} \\ - \mathbf{JWA}\boldsymbol{\omega}_0 - \boldsymbol{\omega}_{\text{abs}} \times \mathbf{K} + \mathbf{M}_{\text{ext}}, \end{gathered}$$

and the closed system of equations describing the controlled motion of the spacecraft with three flywheels can be represented as

$$\begin{gathered} \mathbf{J}\dot{\boldsymbol{\omega}}_{\text{rel}} + k_a\mathbf{S} + k_\omega\boldsymbol{\omega}_{\text{rel}} = 0, \quad \dot{\mathbf{A}} = \mathbf{WA}, \\ \dot{\mathbf{H}} + \boldsymbol{\omega}_{\text{abs}} \times \mathbf{H} = k_\omega\boldsymbol{\omega}_{\text{rel}} \\ + k_a\mathbf{S} - \mathbf{JWA}\boldsymbol{\omega}_0 - \boldsymbol{\omega}_{\text{abs}} \times \mathbf{K} + \mathbf{M}_{\text{ext}}. \end{gathered} \tag{1.5}$$

Here, the first and second equations describe the controlled motion and the third equation describes the control realizing this motion.

Further analysis will use the equations in dimensionless parameters. Let us introduce the designations

$$H_0 = H_{\max}, \quad m = \dot{H}_{\max}, \quad t_0 = \frac{H_0}{m}. \tag{1.6}$$

Here, $H_{\max}$ and $\dot{H}_{\max}$ are characteristics of flywheels, maximum angular momentum and maximum reaction moment created by the flywheel, respectively. Then, we go over to dimensionless variables and parameters:

$$\boldsymbol{\omega}_{\mathrm{rel}} = \frac{1}{t_0}\boldsymbol{\Omega},\quad \mathbf{H} = H_0\mathbf{h},\quad \dot{\mathbf{H}} = m\dot{\mathbf{h}},\quad t = t_0\tau,$$
$$\mathbf{I} = J_1\,\mathrm{diag}(1\ \theta_1\ \theta_2),\quad \boldsymbol{\omega}_0 = \omega_{\mathrm{orb}}\boldsymbol{\Omega}_0. \tag{1.7}$$

In these variables, the first equation from (1.4) takes the form

$$\mathbf{I}\boldsymbol{\Omega}' + K_a\mathbf{S} + K_\omega\boldsymbol{\Omega} = 0, \tag{1.8}$$

where we introduced the designations $K_a = t_0^2 k_a/J_1$, $K_\omega = t_0 k_\omega/J_1$. In further consideration, equation (1.8) will be used in order to choose controlling parameters K_a and K_ω, and it will serve as a basis for estimating the attitude accuracy at different disturbances.

2. THE CHOICE OF CONTROLLING PARAMETERS

In this section the choice of the control parameters is considered. We look for the parameters, which ensure the maximum degree of stability of the characteristic equation of the linearized system [8–11]. For this purpose, we linearize equation (1.8) in the neighborhood of the equilibrium position

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},\ \boldsymbol{\Omega} = (0\ 0\ 0),$$

supplement it by linearized kinematic relations (1.2b) and then obtain

$$\begin{gathered} \Omega_1' + K_\omega\Omega_1 + 2K_a\gamma = 0, \\ \theta_1\Omega_2' + K_\omega\Omega_2 + 2K_a\alpha = 0, \\ \theta_2\Omega_3' + K_\omega\Omega_3 + 2K_a\beta = 0, \\ \alpha' = \Omega_2,\quad \beta' = \Omega_3,\quad \gamma' = \Omega_1. \end{gathered} \tag{2.1}$$

Now the characteristic equation of this system has the form

$$(\lambda^2 + K_\omega\lambda + 2K_a)(\theta_1\lambda^2 + K_\omega\lambda + 2K_a) \times (\theta_2\lambda^2 + K_\omega\lambda + 2K_a) = 0.$$

Its roots are

$$\lambda_{1,2} = -\frac{1}{2}K_\omega \pm \frac{1}{2}\sqrt{K_\omega^2 - 8K_a},$$
$$\lambda_{3,4} = -\frac{1}{2}\frac{K_\omega}{\theta_1} \pm \frac{1}{2\theta_1}\sqrt{K_\omega^2 - 8\theta_1 K_a},$$
$$\lambda_{5,6} = -\frac{1}{2}\frac{K_\omega}{\theta_2} \pm \frac{1}{2\theta_2}\sqrt{K_\omega^2 - 8\theta_2 K_a}.$$

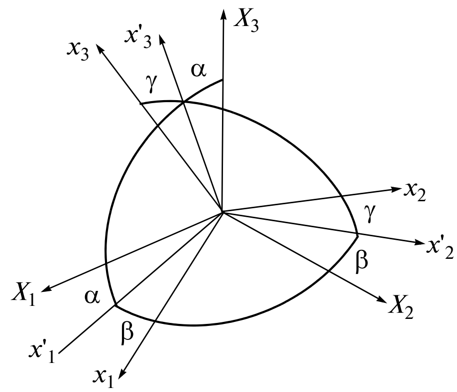

Fig. 1. Euler aircraft angles.

Let us find such relations of controlling parameters K_a and K_ω that the rightmost root of the characteristic equation would lie as much as possible to the left (to reach the maximum degree of stability). Let us divide the range of parameters into four parts (we assume that the condition $\theta_2 > \theta_1 > 1$ is fulfilled):

$$0 < K_\omega^2 \le 8K_a,\quad 8K_a < K_\omega^2 \le 8\theta_1 K_a,$$
$$8\theta_1 K_a < K_\omega^2 \le 8\theta_2 K_a,\quad 8\theta_2 K_a < K_\omega^2.$$

Let us consider sequentially these four ranges. Taking into account that $\theta_2 > \theta_1 > 1$ and at $K_\omega^2 \le 8K_a$, the degree of stability is determined by the expression $\xi_1 = K_\omega^2/2\theta_2$.

When $8\theta_2 K_a < K_\omega^2$, the degree of stability is determined by the expression

$$\xi_2 = -\frac{1}{2}K_\omega + \frac{1}{2}\sqrt{K_\omega^2 - 8K_a}.$$

When $8K_a < K_\omega^2 \le 8\theta_2 K_a$, the degree of stability is equal to either ξ_1 or ξ_2. The equality $\xi_1 = \xi_2$ is valid under the condition

$$K_a = \frac{2\theta_2 - 1}{8\theta_2^2}K_\omega^2. \tag{2.2}$$

Thus, we can construct (Fig. 2) broken lines that are isolines with equal degrees of stability. In Fig. 2, found parabola (2.2) is also constructed.

The straight line in Fig. 2 describes the limitation on the maximum control torque of flywheels. It is specified by the equation

$$H_0 t_0/J_1 = 2\delta_{\max}K_a + \Omega_{\max}K_\omega. \tag{2.3}$$

Here, $\delta_{\max}$ and $\Omega_{\max}$ are maximum deviations in orientation and angular velocity. Relation (2.3) gives the estimation of the maximum control torque required

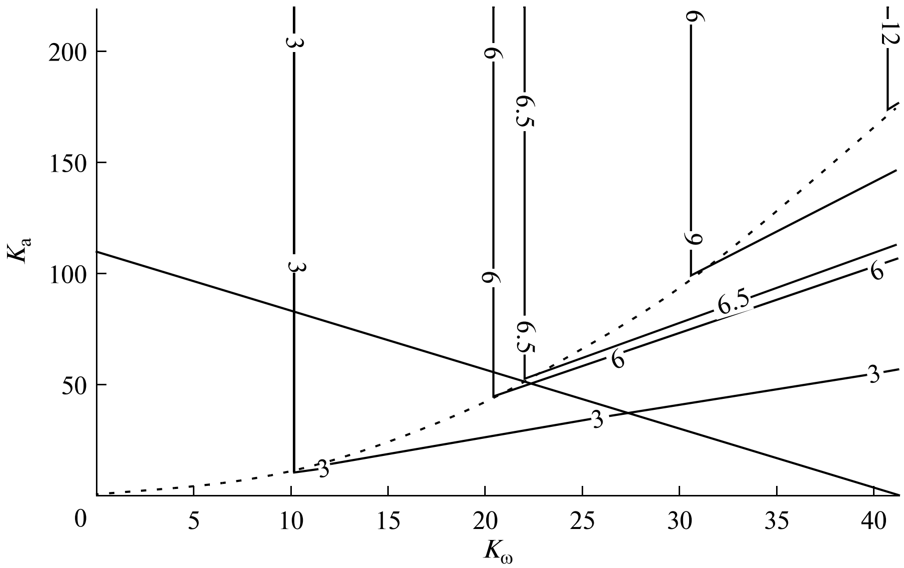

Fig. 2. Broken lines represent isolines with equal degree of stability, the straight line shows the constraint on control, and dashed line is a parabola on which the sought-for parameters of control are located.

for the spacecraft reorientation. It was obtained by linearization of the last equation of (1.5) taking into account the smallness of the external forces torque. All points on the plane (K_ω, K_a) should lie below this straight line. Thus, desired values lie at the intersection of parabola (2.2) and straight line (2.3). In this case, K_ω is found from the expression

$$K_\omega = -\frac{2\Omega_{\max}}{\delta_{\max}}\frac{\theta_2^2}{2\theta_2 - 1} + \frac{1}{\delta_{\max}}\sqrt{\left(2\Omega_{\max}\frac{\theta_2^2}{2\theta_2 - 1}\right)^2 + 4\frac{H_0 t_0}{J_1}\frac{\theta_2^2}{2\theta_2 - 1}},$$

and K_a at known K_ω can be found, for example, from (2.2). Thus, the expressions for the parameters providing for the maximum degree of stability are found.

3. THE STUDY OF EVOLUTION OF THE ANGULAR MOMENTUM OF FLYWHEELS IN THE NOMINAL MODE

In the nominal mode, the axes of the body-fixed and orbital frames coincide. Therefore, the gravitational moment is equal to zero and we will consider as the external disturbing torque the aerodynamic and magnetic torques. Let us investigate their influence. Since in this mode

$$\boldsymbol{\omega}_{\text{отн}} = 0, \quad \mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

the first and second equations of system (1.5) become identities. Let us write the third equation. It describes evolution of the angular momentum of flywheels when the spacecraft is hold in equilibrium position under the action of the magnetic moment caused by the residual magnetization and of the aerodynamic moment

$$\dot{\mathbf{H}} + \boldsymbol{\omega}_0 \times \mathbf{H} = \mathbf{M}_{\text{m}} + \mathbf{M}_{\text{a}} - \boldsymbol{\omega}_0 \times \mathbf{J}\boldsymbol{\omega}_0. \tag{3.1}$$

Let us take the following expressions for the aerodynamic and magnetic moments [12]

$$\mathbf{M}_{\text{a}} = -\frac{1}{2}\rho C_x S v \mathbf{r}_a \times \mathbf{v}, \quad \mathbf{M}_{\text{m}} = \mathbf{L} \times \mathbf{B}$$

respectively. Here, ρ is the atmospheric density; C_x is the satellite drag coefficient, S is the characteristic area of its cross-section; v is the velocity of orbital motion; $\mathbf{r}_a$ is the radius vector of the center of pressure in the body-fixed frame; $\mathbf{L}$ is the residual magnetic moment of the spacecraft; $\mathbf{B}$ is the induction vector of the external magnetic field (we use the direct dipole field as a model of the geomagnetic field). We rewrite (3.1) in the coordinate form

$$\begin{aligned} \dot{H}_1 + \omega_{\text{orb}} H_3 &= -2L_2 B_0 \sin\omega_{\text{orb}} t \sin i - L_3 B_0 \cos i, \\ \dot{H}_2 &= -\frac{1}{2}\rho C_x S v^2 r_{a3} + L_3 B_0 \cos\omega_{\text{orb}} t \sin i \\ &\quad + 2L_1 B_0 \sin\omega_{\text{orb}} t \sin i, \\ \dot{H}_3 - \omega_{\text{orb}} H_1 &= \frac{1}{2}\rho C_x S v^2 r_{a2} \\ &\quad + L_1 B_0 \cos i - L_2 B_0 \cos\omega_{\text{orb}} t \sin i. \end{aligned} \tag{3.2}$$

The solution to this system can be written as follows:

$$\begin{aligned}
H_1 &= H_{10}\cos\omega_0 t + \left(\frac{L_1 B_0 \cos i}{\omega_0} + \frac{1}{2}\frac{\rho C_x S v^2 r_{a2}}{\omega_0}\right) \\
&\times(\cos\omega_{\text{orb}}t - 1) - \left(H_{30} + \frac{L_3 B_0\cos i}{\omega_0}\right)\sin\omega_{\text{orb}}t \\
&- \frac{1}{2}L_2 B_0 t\sin\omega_{\text{orb}}t\sin i, \\
H_2 &= -\frac{1}{2}\rho C_x S v^2 r_{a3} t - \frac{1}{\omega_0}L_3 B_0\sin\omega_{\text{orb}}t\sin i \\
&+ \frac{2}{\omega_0}L_1 B_0\sin i(\cos\omega_{\text{orb}}t - 1) + H_{20}, \\
H_3 &= \left(H_{10} + \frac{\mathfrak{M}_1 B_0\cos i}{\omega_0} + \frac{1}{2}\frac{\rho C_x S v^2 r_{a2}}{\omega_0}\right)\sin\omega_{\text{orb}}t \\
&+ H_{30}\cos\omega_{\text{orb}}t + \frac{L_3 B_0\cos i}{\omega_0}(\cos\omega_{\text{orb}}t - 1) \\
&+ \frac{1}{2}L_2 B_0 t\cos\omega_{\text{orb}}t\sin i - \frac{3}{2}\frac{L_2 B_0\sin\omega_{\text{orb}}t\sin i}{\omega_0}.
\end{aligned} \tag{3.3}$$

It allows us to estimate the time, in which saturation of the flywheel system is reached (in fact, this is time, when one needs to unload the flywheels). Let us write down separately expressions for the saturation time t_a specified by the aerodynamic moment and the saturation time t_{m} specified by the residual magnetization

$$t_{\text{a}} = \frac{2H_0}{\rho C_x S v^2 r}, \quad t_{\text{m}} = \frac{2H_0}{LB\sin i}.$$

These formulas are used to estimate the saturation time, while expressions (3.3) describe the exact (for the adopted model of disturbances) evolution of the angular momentum of flywheels in time.

4. ACCOUNTING FOR DISTURBANCES ACTING UPON THE SPACECRAFT

The control law in (1.5) suggests that the control moment is formed taking into account the magnitude of the external forces torque. However, as a rule, this moment is either unknown or its determination requires installation of additional sensors and creates additional load on the onboard computer. In addition, other disturbances may also affect the spacecraft motion, for example, connected with natural inaccuracies introduced when manufacturing the spacecraft .

We rewrite expression (1.4) in the form

$$\begin{aligned}
\dot V &= -(\boldsymbol{\omega}_{\text{rel}}, [\dot{\mathbf{H}} + \boldsymbol{\omega}_{\text{abs}}\times\mathbf{H} + \boldsymbol{\omega}_{\text{abs}} \\
&\times \mathbf{K} + \mathbf{JWA}\boldsymbol{\omega}_0 - k_a\mathbf{S}]) + (\boldsymbol{\omega}_{\text{rel}}, \mathbf{M}_{\text{d}}),
\end{aligned} \tag{4.1}$$

where $\mathbf{M}_{\text{d}}$ is the disturbing torque, whose existence may be caused by both external actions and internal factors. The first summand of (4.1) includes the terms, which are calculated on the basis of known information, the second summand is determined by the disturbances. We will construct such control, which is calculated on the basis of the first summand of (4.1). Similarly, let us require the fulfillment of the equality

$$\dot{\mathbf{H}} + \boldsymbol{\omega}_{\text{abs}}\times\mathbf{H} + \boldsymbol{\omega}_{\text{abs}}\times\mathbf{K} + \mathbf{JWA}\boldsymbol{\omega}_0 - k_a\mathbf{S} = k_\omega\boldsymbol{\omega}_{\text{rel}},$$

then the first summand in (4.1) will be less than zero. This means that at $\mathbf{M}_{\text{d}} = 0$ the control ensures the asymptotic stability of the nominal motion. Hence, the control moment $\dot{\mathbf{H}} + \boldsymbol{\omega}_{\text{abs}}\times\mathbf{H}$ has the form

$$\dot{\mathbf{H}} + \boldsymbol{\omega}_{\text{abs}}\times\mathbf{H} = k_\omega\boldsymbol{\omega}_{\text{rel}} + k_a\mathbf{S} - \boldsymbol{\omega}_{\text{abs}}\times\mathbf{K} - \mathbf{JWA}\boldsymbol{\omega}_0. \tag{4.2}$$

In this case, the control is formed only on the basis of information on the motion of the orbital frame $OX_1X_2X_3$ and the data on the motion of the *body frame* $Ox_1x_2x_3$ relative to $OX_1X_2X_3$. The drawback of this control is the fact that, approaching to the equilibrium position, one cannot guarantee the validity of the inequality $\dot V < 0$, and hence, asymptotic stability of the required position. One can make an estimation of the minimum angular velocity, at which the condition $\dot V < 0$ is still valid with guarantee. Let us write (4.2) in the form

$$\dot V = -k_\omega\boldsymbol{\omega}_{\text{rel}}^2 + (\boldsymbol{\omega}_{\text{rel}}, \mathbf{M}_{\text{d}}).$$

The worst case is when $(\boldsymbol{\omega}_{\text{rel}}, \mathbf{M}_{\text{d}}) = \omega_{\text{rel}}M_{\text{d}}$, then $-k_\omega\boldsymbol{\omega}_{\text{rel}}^2 + \omega_{\text{rel}}M_{\text{d}} < 0$.

Hence, we obtain an estimate $\omega_{\text{rel}} > M_{\text{d}}/k_\omega$. Here, the case in point is absolute values, and M_{d} is the maximum disturbing torque.

Let us describe an approach, which is used in order to estimate the attitude accuracy at acting disturbances. We write system (1.5) in dimensionless variables (1.7) with a new control law taking into account the fact that $\mathbf{M}_{\text{d}} = \mu\mathbf{M}(\mathbf{A}, \boldsymbol{\Omega}_{\text{rel}}, \mathbf{h})J_1/t_0^2$ $(\mu \ll 1)$:

$$\begin{aligned}
&\mathbf{I}\frac{d}{d\tau}\boldsymbol{\Omega}_{\text{rel}} + K_a\mathbf{S} + K_\omega\boldsymbol{\Omega}_{\text{rel}} = \mu\mathbf{M}(\mathbf{A}, \boldsymbol{\Omega}_{\text{rel}}, \mathbf{h}), \\
&\frac{d}{d\tau}\mathbf{A} = \mathbf{WA}, \\
&\frac{d}{d\tau}\mathbf{h} + \boldsymbol{\Omega}_{\text{abs}}\times\mathbf{h} = K_\omega\boldsymbol{\Omega}_{\text{rel}} + K_a\mathbf{S} \\
&\quad - \omega_{\text{orb}}t_0\mathbf{IwA}\boldsymbol{\Omega}_0 - \boldsymbol{\Omega}_{\text{abs}}\times\mathbf{I}\boldsymbol{\Omega}_{\text{abs}}.
\end{aligned} \tag{4.3}$$

The motion of the system at $\mu = 0$ and $\tau \to \infty$ will tend to the motion described by the equalities

$$\begin{aligned}
&\mathbf{q} = 0, \quad \boldsymbol{\Omega}_{\text{rel}} = 0, \\
&h_1 = h_{10}\cos\Omega_{\text{orb}}\tau - h_{30}\sin\Omega_{\text{orb}}\tau, \quad h_2 = h_{20}, \\
&h_3 = h_{10}\sin\Omega_{\text{orb}}\tau + h_{30}\cos\Omega_{\text{orb}}\tau.
\end{aligned} \tag{4.4}$$

Here, $\mathbf{q} = (\gamma\ \alpha\ \beta)^T$; h_{10}, h_{20}, and h_{30} are constants. This follows from asymptotic stability of the position $\mathbf{q} = 0$, $\boldsymbol{\Omega}_{\text{rel}} = 0$. In this case, as was shown in [13], at $\mu \ll 1$ the motion of the disturbed system will tend to the motion in the vicinity of the equilibrium position. To construct this motion, we make use of the Poincare method. The solution is sought in the form

$$\begin{gathered} \mathbf{q} = \mathbf{q}^{(0)} + \mu\mathbf{q}^{(1)} + \ldots \\ \mathbf{\Omega}_{\text{rel}} = \mathbf{\Omega}_{\text{rel}}^{(0)} + \mu\mathbf{\Omega}_{\text{rel}}^{(1)} + \ldots \\ \mathbf{h} = \mathbf{h}^{(0)} + \mu\mathbf{h}^{(1)} + \ldots \end{gathered} \tag{4.5}$$

Since $\mathbf{q}^{(0)} = 0$ and $\mathbf{\Omega}_{\text{rel}}^{(0)} = 0$, the equations for $\mathbf{\Omega}_{\text{rel}}^{(1)}$ and $\mathbf{q}^{(1)}$ take the form

$$\mathbf{I}\frac{d}{d\tau}\mathbf{\Omega}_{\text{rel}}^{(1)} + K_\omega\mathbf{\Omega}_{\text{rel}}^{(1)} + 2K_a\mathbf{q}^{(1)} = \mathbf{M}(E, 0, \mathbf{h}^{(0)}),$$

$$\frac{d}{d\tau}\mathbf{q}^{(1)} = \mathbf{\Omega}_{\text{rel}}^{(1)}.$$

Or

$$\begin{gathered} \mathbf{I}\frac{d^2}{d\tau^2}\mathbf{q}^{(1)} + K_\omega\frac{d}{d\tau}\mathbf{q}^{(1)} + 2K_a\mathbf{q}^{(1)} = \mathbf{M}(E, 0, \mathbf{h}^{(0)}), \\ \mathbf{q}_0^{(1)} = 0, \quad \mathbf{\Omega}_{\text{rel},0}^{(1)} = 0. \end{gathered} \tag{4.6}$$

Here, we introduced the designation $\mathbf{E} = \text{diag}(1, 1, 1)$. Since the general solution to the homogeneous system is damping on the strength of asymptotic stability $\mathbf{q} = 0$, $\mathbf{\Omega}_{\text{rel}} = 0$, then in order to estimate the accuracy it is sufficient to find a particular solution to inhomogeneous system (4.6) Let us now turn to concrete examples.

4.1. The Influence of External Disturbances

Firstly, we consider the influence of external disturbances on the accuracy of the attitude control system. As disturbing torque, we consider, as in chapter 3, the aerodynamic and magnetic torques. Then, the disturbing torque

$$\mathbf{M}_{\text{d}} = \frac{t_0^2}{J_1}\left(-\frac{1}{2}\rho C_x S v^2 r_a \mathbf{e}_a \times \mathbf{A}\frac{\mathbf{v}}{v} + LB\mathbf{l} \times \mathbf{b}\right).$$

As a small parameter, we consider here the following ratios

$$\mu_1 = -\frac{1}{2}\rho C_x S v^2 r_a \frac{t_0^2}{J_1}, \quad \mu_2 = LB\frac{t_0^2}{J_1}.$$

Both here and below, the smallness of parameters means that the control moment prevails over disturbing ones ($\mu \ll 1$). Since the orbit is circular, $\mathbf{v}/v = (1\ 0\ 0)^T$, to estimate the attitude accuracy we use, as a model of the magnetic field, the model of the direct dipole field. In this case, the disturbing torque in (4.6) takes the form

$$\mathbf{M} = \begin{pmatrix} \left(-2l_2\sin\omega_0 t\sin i - l_3\cos i\right) \\ -1 + \left(l_3\cos\omega_0 t\sin i + 2l_1\sin\omega_0 t\sin i\right) \\ 1 + \left(l_1\cos i - l_2\cos\omega_0 t\sin i\right) \end{pmatrix}.$$

As was already mentioned, the estimation of the accuracy of orientation is determined by the partial solution to (4.6)

$$\mathbf{q}_{\text{d}} = \frac{a_i\cos\Omega_{\text{orb}}\tau + b_i\sin\Omega_{\text{orb}}\tau}{\left(-\Omega_{\text{orb}}^2 I_i + 2K_a\right)^2 + K_\omega^2\Omega_{\text{orbl}}^2} + \frac{c_i}{2K_a}, \quad i = 1, 2, 3.$$

Here,

$$\mathbf{a} = \mu_2\begin{pmatrix} 2l_2K_\omega\Omega_{\text{orb}}\sin i \\ \left(-\Omega_{\text{orb}}^2 I_2 + 2K_a\right)l_3\sin i - 2l_1K_\omega\Omega_{\text{orb}}\sin i \\ -\left(-\Omega_{\text{orb}}^2 I_3 + 2K_a\right)l_2\sin i \end{pmatrix},$$

$$\mathbf{b} = \mu_2\begin{pmatrix} 2\left(-\Omega_{\text{orb}}^2 + 2K_a\right)l_2\sin i \\ 2l_1\sin i\left(-\Omega_{\text{orb}}^2 I_2 + 2K_a\right) - l_3K_\omega\Omega_{\text{orb}}\sin i \\ -l_2K_\omega\Omega_{\text{orb}}\sin i \end{pmatrix},$$

$$\mathbf{c} = \begin{pmatrix} -\mu_2 l_3\cos i \\ -\mu_1 \\ \mu_1 + \mu_2 l_1\cos i \end{pmatrix}.$$

Thus, the obtained final formulas allow us to estimate the attitude accuracy at existing external disturbances.

4.2. The Influence of Off-diagonal Elements of the Inertia Tensor

Let us now consider the case, when the inertia tensor of the spacecraft has off-diagonal elements. In practice, this situation arises due to the fact that the principal axes of the spacecraft are determined inaccurately, which leads to an error in the realization of control. In this chapter, we consider the influence of such an error on the attitude accuracy. Let us present the inertia tensor

$$\mathbf{J} = \begin{pmatrix} J_1 & -J_{12} & -J_{13} \\ -J_{12} & J_2 & -J_{23} \\ -J_{13} & -J_{23} & J_3 \end{pmatrix}$$

in the form $\mathbf{J} = \mathbf{J}_{\text{p}} + \mathbf{J}_{\text{cent}}$,

where $\mathbf{J}_{\text{p}} = \begin{pmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{pmatrix}$, and $\mathbf{J}_{\text{cent}} = \begin{pmatrix} 0 & -J_{12} & -J_{13} \\ -J_{12} & 0 & -J_{23} \\ -J_{13} & -J_{23} & 0 \end{pmatrix}$.

We assume that the off-diagonal elements are much smaller than the principal moments of inertia, i.e., $\mathbf{J} = J_1(\mathbf{I}_{\text{p}} + \mu_1\mathbf{I}_{\text{cent}})$.

For the sake of simplicity of calculations, we assume that

$$\frac{J_{12}}{J_1} = \frac{J_{13}}{J_1} = \frac{J_{23}}{J_1} = \mu_3 \ll 1.$$

In this case, $\mathbf{K} = J_1(\mathbf{I}_{\text{p}} + \mu_1\mathbf{I}_{\text{cent}})\boldsymbol{\omega}_{\text{abs}} + \mathbf{H}$.

Equations of motion (4.3), in this case, can be written in the form

$$\mathbf{I}_\mathrm{p}\dot{\mathbf{\Omega}}_\mathrm{rel} + K_a\mathbf{S} + K_\omega\mathbf{\Omega}_\mathrm{rel} = \mu_3(t_0^2K_a\mathbf{I}_\mathrm{cent}\mathbf{I}_\mathrm{p}^{-1}S + \mathbf{I}_\mathrm{cent}\mathbf{I}_\mathrm{p}^{-1}\mathbf{\Omega}_\mathrm{rel}$$
$$- \mathbf{\Omega}_\mathrm{abs}\times\mathbf{I}_\mathrm{cent}\mathbf{\Omega}_\text{абс} - t_0\omega_\mathrm{orb}\mathbf{I}_\mathrm{cent}\mathbf{wA\Omega}_0),$$
$$\dot{\mathbf{A}} = \mathbf{WA},$$
$$\dot{\mathbf{h}} + \mathbf{\Omega}_\mathrm{abs}\times\mathbf{h} = K_\omega\mathbf{\Omega}_\mathrm{rel} + K_a\mathbf{S}$$
$$- \omega_\mathrm{orb}t_0\mathbf{I}_\mathrm{p}\mathbf{wA\Omega}_0 - \mathbf{\Omega}_\mathrm{abs}\times\mathbf{I}_\mathrm{p}\mathbf{\Omega}_\mathrm{abs}.$$

Here, when deriving the disturbing torque, we use a series expansion [14]

$$\left(\mathbf{E} + \mu_3\mathbf{I}_\mathrm{p}^{-1}\mathbf{I}_\mathrm{cent}\right)^{-1} = \mathbf{E} - \mu_3\mathbf{I}_\mathrm{p}^{-1}\mathbf{I}_\mathrm{cent} + \ldots,$$

and retain in the equation only terms of first order in μ_3. Let us designate $\mu_4 = \mu_3t_0^2\omega_\mathrm{orb}^2$.

Let us write the disturbing torque for (4.6)

$$\mathbf{M} = -\mathbf{\Omega}_0\times\mathbf{I}_\mathrm{cent}\mathbf{\Omega}_0.$$

Similarly to the previous case, we obtain

$$\mathbf{q}_\mathrm{d} = \begin{pmatrix}\dfrac{\mu_4}{2K_a} & 0 & -\dfrac{\mu_4}{2K_a}\end{pmatrix}^T.$$

Thus, we have obtained estimates of the attitude accuracy in the case of a noncoincidence of the construction and principal axes of inertia of the spacecraft.

4.3. The Influence of Deviation of the Flywheel Axis from a Specified Position

Let us study the influence of deviation of the flywheel axis relative to a specified position on the dynamics of angular motion of the spacecraft. We consider that the angular momentum of the flywheels can be written in the form

$$\mathbf{H}_\mathrm{d} = (\mathbf{E} + \mu_5\mathbf{N})\mathbf{H},$$

where $\mathbf{H}$ is the angular momentum, which is calculated based on equation (4.2), matrix $\mathbf{N}$ contains only off-diagonal elements, and parameter $\mu_5 \ll 1$ is responsible for characteristic values of the deviations. In this case, the disturbing torque in (4.3) takes on the form

$$\mathbf{M} = -\mu_5\mathbf{N}(-\mathbf{\Omega}_\mathrm{abs}\times\mathbf{h} + K_\omega\mathbf{\Omega}_\mathrm{rel} + K_a\mathbf{S}$$
$$- \omega_\mathrm{orb}t_0\mathbf{IwA\Omega}_0 - \mathbf{\Omega}_\mathrm{abs}\times\mathbf{I\Omega}_\mathrm{abs}) - \mu_5\mathbf{\Omega}_\mathrm{abs}\times\mathbf{Nh},$$

and in (4.6)

$$\mathbf{M} = \mathbf{N}\left(\mathbf{\Omega}_0\times\mathbf{h}^{(0)}\right) - \mathbf{\Omega}_0\times\mathbf{Nh}^{(0)}.$$

For simplicity of calculations, we assume that matrix $\mathbf{N}$ has the form

$$\mathbf{N} = \begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}.$$

Then, a partial solution to inhomogeneous equation (4.6) determining the estimation of accuracy has the form

$$q_i = a_i\left(\frac{(J_i\omega_\mathrm{orb}^2 - k_a)\sin\omega_\mathrm{orb}t - k_\omega\omega_\mathrm{orb}\cos\omega_\mathrm{orb}t}{(J_i\omega_\mathrm{orb}^2 - k_a)^2 + k_\omega^2\omega_\mathrm{orb}^2}\right)$$
$$+ b_i\left(\frac{k_\omega\omega_\mathrm{orb}\sin\omega_\mathrm{orb}t + (J_i\omega_\mathrm{orb}^2 - k_a)\cos\omega_\mathrm{orb}t}{(J_i\omega_\mathrm{orb}^2 - k_a)^2 + k_\omega^2\omega_\mathrm{orb}^2}\right)$$
$$+ \frac{c_i}{2k_a},\quad i = 1, 2, 3.$$

Here,

$$\mathbf{a} = \omega_0\begin{pmatrix}-H_{10}\\H_{10}\\H_{30}\end{pmatrix}, \mathbf{b} = \begin{pmatrix}H_{30}\\H_{10}\\H_{30}\end{pmatrix}, \mathbf{c} = \begin{pmatrix}-H_{20}\\0\\0\end{pmatrix}.$$

It is seen that except for the dependence of the attitude accuracy on a small parameter, there exists a dependence on the value of accumulated angular momentum of the flywheels H_{10}, H_{20}, and H_{30}.

4.4. The Influence of Errors in Determining Orientation and Angular Velocity

Let us consider the influence of systematic errors in determining the state vector on the accuracy of the spacecraft orientation. Let the vector of absolute angular velocity be determined to a systematic error $\Delta\boldsymbol{\omega}$:

$$\boldsymbol{\omega}_\mathrm{abs}^\mathrm{meas} = \boldsymbol{\omega}_\mathrm{abs} + \Delta\boldsymbol{\omega},$$

and the orientation to a systematic error $\Delta\alpha = \Delta\beta = \Delta\gamma$: $\mathbf{A}^\mathrm{meas} = \mathbf{A} + \Delta\mathbf{A}$, where $\Delta\mathbf{A} = \dfrac{\partial\mathbf{A}}{\partial\alpha}\Delta\alpha + \dfrac{\partial\mathbf{A}}{\partial\beta}\Delta\beta + \dfrac{\partial\mathbf{A}}{\partial\gamma}\Delta\gamma$.

In this case, control (4.2) takes the form

$$\dot{\mathbf{H}} = -(\boldsymbol{\omega}_\mathrm{abs} + \Delta\boldsymbol{\omega})\times\mathbf{H} + k_\omega(\boldsymbol{\omega}_\mathrm{rel} + \Delta\boldsymbol{\omega} - \Delta\mathbf{A}\boldsymbol{\omega}_0)$$
$$+ k_a(\mathbf{S} + \Delta\mathbf{S}) - (\boldsymbol{\omega}_\mathrm{abs} + \Delta\boldsymbol{\omega})\times\mathbf{J}(\boldsymbol{\omega}_\mathrm{abs} + \Delta\boldsymbol{\omega})$$
$$- \mathbf{J}(\mathbf{W} + \Delta\mathbf{W})(\mathbf{A} + \Delta\mathbf{A})\boldsymbol{\omega}_0,$$

then, the dimensionless disturbing torque looks like

$$\mathbf{M}_\mathrm{d} = \mu(\Delta\mathbf{\Omega}\times\mathbf{h} - K_\omega(\Delta\mathbf{\Omega} - \Delta\mathbf{A\Omega}_0) - K_a\Delta\mathbf{S}$$
$$- \Delta\mathbf{\Omega}\times\mathbf{J\Omega}_\mathrm{abs} - \mathbf{\Omega}_\mathrm{abs}\times\mathbf{J}\Delta\mathbf{\Omega} - \mathbf{Jw}\Delta\mathbf{A\Omega}_0 - \mathbf{J}\Delta\mathbf{wA\Omega}_0).$$

Here, $\mu = t_0\Delta\omega = \Delta\alpha = \Delta\beta = \Delta\gamma$ is the quantity characterizing an error (it is accepted $\Delta\omega_1 = \Delta\omega_2 = \Delta\omega_3$ in order to reduce calculations). The disturbing torque in equation (4.6) can be written as

$$\mathbf{M} = \Delta\boldsymbol{\Omega}\times\mathbf{h}^{(0)} - K_\omega(\Delta\boldsymbol{\Omega} - \Delta\mathbf{A}\boldsymbol{\Omega}_0) - K_a\Delta\mathbf{S} - \Delta\boldsymbol{\Omega} \times \mathbf{J}\boldsymbol{\Omega}_0 - \boldsymbol{\Omega}_0\times\mathbf{J}\Delta\boldsymbol{\Omega} - \mathbf{J}\Delta\mathbf{w}\boldsymbol{\Omega}_0.$$

This expression can be rewritten as follows:

$$\mathbf{M} = \Delta\boldsymbol{\Omega}\times\mathbf{h}^{(0)} + \Delta\mathbf{M}.$$

Here, $\Delta\mathbf{M}$ is a part of the disturbing torque that does not depend on time. The first term gives the forced oscillations and, as a result, an estimate of the accuracy is as follows:

$$q_i = a_i\left(\frac{(J_i\omega_{\text{orb}}^2 - k_a)\sin\omega_{\text{orb}}t - k_\omega\omega_{\text{orb}}\cos\omega_{\text{orb}}t}{(J_i\omega_{\text{orb}}^2 - k_a)^2 + k_\omega^2\omega_{\text{orb}}^2}\right) + b_i\left(\frac{k_\omega\omega_{\text{orb}}\sin\omega_{\text{orb}}t + (J_i\omega_{\text{orb}}^2 - k_a)\cos\omega_{\text{orb}}t}{(J_i\omega_{\text{orb}}^2 - k_a)^2 + k_\omega^2\omega_{\text{orb}}^2}\right) + \frac{c_i}{2k_a} + \frac{\Delta M_i}{2k_a},\ i = 1, 2, 3.$$

Here,

$$\mathbf{a} = \begin{pmatrix} H_{10}\Delta\omega_1 \\ -H_{10}\Delta\omega_1 - H_{30}\Delta\omega_3 \\ H_{30}\Delta\omega_2 \end{pmatrix},\ \mathbf{b} = \begin{pmatrix} H_{30}\Delta\omega_2 \\ H_{10}\Delta\omega_3 - H_{30}\Delta\omega_1 \\ -H_{10}\Delta\omega_2 \end{pmatrix},\ \mathbf{c} = \begin{pmatrix} -H_{20}\Delta\omega_3 \\ 0 \\ H_{20}\Delta\omega_1 \end{pmatrix}.$$

As in the previous chapter, there are forced oscillations, whose amplitude depends on values of the accumulated angular momentum.

5. ANALYSIS OF THE ANGULAR MOTION OF *CHIBIS-M* MICROSATELLITE

Let us illustrate the obtained results by calculations for the *Chibis-M* satellite [15]. The tensor of inertia of the spacecraft is $\mathbf{J} = \text{diag}(1.03\ 1.51\ 1.75)$ kg m^2. The inclination of the orbit is $i = 56.7°$, and its altitude $h =$ 550 km. Since the pre-damping is executed, at the instant of the flywheel activation the magnitude of initial relative angular velocity does not exceed 1 deg/s. Flywheel parameters are $H_0 = 0.072$ N m s and $m =$ 0.00023 N m.

Let us first calculate the control parameters. For small velocities and deviations $\delta_{\max} = 0.1$ rad, $\omega_{\max} =$ 0.001 s^{-1}, whuch corresponds to the motion in the vicinity of the equilibrium position

$$K_\omega = -\frac{2\Omega_{\max}}{\delta_{\max}}\frac{\theta_2^2}{2\theta_2 - 1} + \frac{1}{\delta_{\max}}\sqrt{\left(2\Omega_{\max}\frac{\theta_2^2}{2\theta_2 - 1}\right)^2 + 4\frac{mt_0^2}{J_1}\frac{\theta_2^2}{2\theta_2 - 1}} \approx 22.1,$$

$$K_a \approx 50.8,\quad k_\omega \approx 0.07\ \text{Nms},\quad k_a = 0.00053\ \text{Nm}.$$

These parameters will be used to make estimations. In order to estimate the attitude accuracy we accept the following values: $C_x = 2.1$, $S = 0.1$ m^2. We consider that the center of pressure is shifted from the center of mass by 10% of the characteristic size, which is determined by the characteristic area $r_a = 0.05$ m. As the density of the atmosphere, we choose the average density for an orbit of the altitude of 550 km, $\rho = 2.86 \cdot 10^{-13}$ kg/m^3. The residual dipole moment $L = 0.1$ A m^2. The magnetic field strength $B = 5 \cdot 10^{-5}$ T. Then, estimations of the attitude accuracy under the action of external disturbances are as follows:

$$\alpha_0 = \frac{1}{2k_a}\left(\frac{1}{2}\rho C_x S v^2 r_a + LB\right) \approx 0.25°,$$

$$\beta_0 = \frac{1}{2k_a}\left(\frac{1}{2}\rho C_x S v^2 r_a + LB\right) \approx 0.25°,$$

$$\gamma_0 = \frac{1}{2k_a}LB \approx 0.25°.$$

To calculate the estimations of the accuracy of orientation in the case of off-diagonal inertia tensor let us accept $\mu_1 = 0.01$ ($\omega_{\text{orb}}t_0 \approx 0.34$, $\mu \approx 0.001$). This corresponds to the accuracy of determination of the principal central axes of inertia of the *Chibis-M* satellite. In this case, our calculations give

$$\alpha_0 = 0,\quad \beta_0 = \mu\frac{J_1\omega_{\text{orb}}^2}{2k_a} \approx 5\cdot 10^{-4}\ \text{deg},$$

$$\gamma_0 = -\mu\frac{J_1\omega_{\text{orb}}^2}{2k_a} \approx -5\cdot 10^{-4}\ \text{deg}.$$

When calculating an estimation of the attitude accuracy in the case of deviation of the flywheel axes from the rated positions, we take $H_{10} = H_{20} = H_{30} =$ $H_{\max} = 0.072$ N m s.

In this case, the attitude accuracy is a quantity of order of 0.03°. For calculations we used a value $\mu_1 =$ 0.01 (this corresponds to deviation of the axis by 0.6°, $\mu \approx 0.003$).

In the latter case, for an error of determination of the angular velocity $\Delta\omega = 0.01$ deg/s and orientation $\Delta\alpha = \Delta\beta = \Delta\gamma = 1°$, the maximum deviation from the equilibrium position being about 2.5°.

Thus, among the above disturbances the largest influence on deviation from the equilibrium position

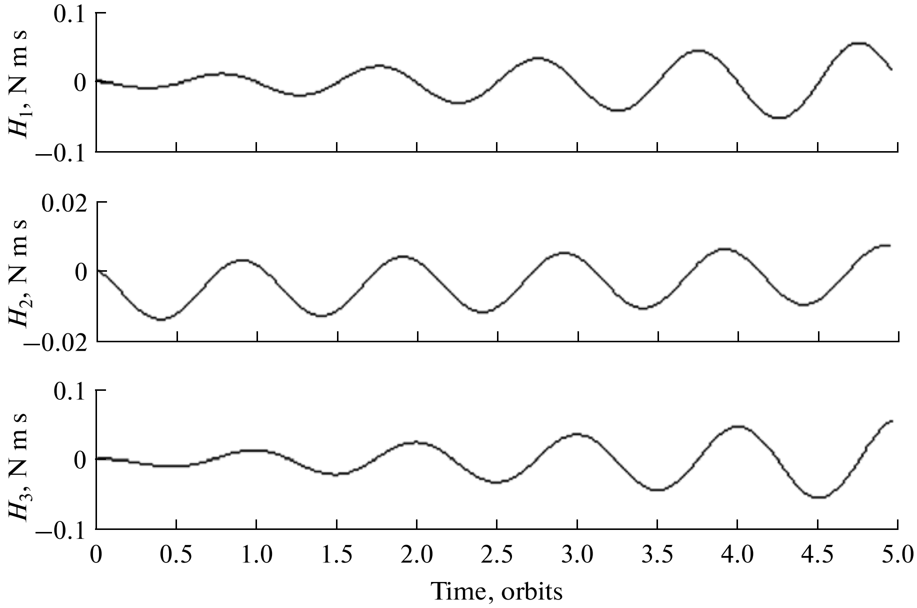

Fig. 3. Evolution of the angular momenta of flywheels in the established regime under the action of an aerodynamic moment.

have errors in determining orientation and angular velocity. In addition, external disturbances appreciably influence the attitude accuracy.

Let us now find the time, in which the flywheel saturation occurs owing to external disturbances. In view of the fact that flywheels have the maximum angular momentum $H_0 = 0.072$ N m s, the saturation time is $t_m \approx 8$ h, and $t_a \approx 4.5$ days. Figure 3 shows the plots illustrating the evolution of the angular momentum of flywheels (2.7).

It is clear that the time to reach maximum angular momentum of the flywheels is small. This is connected with too large magnitude of the residual magnetic moment. One can find the way out of this situation, for example, compensating it by a moment created by electromagnetic coils.

CONCLUSIONS

In this paper, we have investigated the controlled motion of the *Chibis-M* satellite with flywheels. We have found parameters of the control algorithm of the flywheel system. The final relationships are obtained allowing us to estimate the attitude accuracy under acting disturbances. Estimations are made for the given parameters of the *Chibis-M* microsatellite. These estimations have shown that at given parameters of the microsatellite the major influence on a deviation from the required position will have errors in determining the orientation and angular velocity.

The processes of evolution of the angular momenta of flywheels over long time intervals are studied. As a result of the calculations, it was found that for specified values of the residual magnetic moment of the spacecraft, the saturation time of the flywheel can be only a few hours.

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