

SIM & Treynor-Black

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Single Index Model (Sharpe)

- ▶ Rendement

$$R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t)$$

- ▶ Espérance de rendement

$$E(R_i(t)) = \alpha_i + \beta_i E(R_M(t))$$

- ▶ Variance du rendement

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma(e_i)^2$$

- ▶ Covariance des rendements

$$\text{cov}(R_i, R_j) = \beta_i \beta_j \sigma_M^2$$

Portefeuille selon le SIM

Soit un portefeuille de n actifs avec $w_i = \frac{1}{n}$.

$$R_P(t) = \alpha_P + \beta_P R_M(t) + e_P(t)$$

$$\sigma_P^2 = \beta_P^2 \sigma_M^2 + \sigma(e_P)^2$$

Division du travail en Gestion de Portefeuille

- Espérance de rendement

$$E(R_i(t)) = \alpha_i + \beta_i E(R_M(t))$$

- Variance du rendement

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma(e_i)^2$$

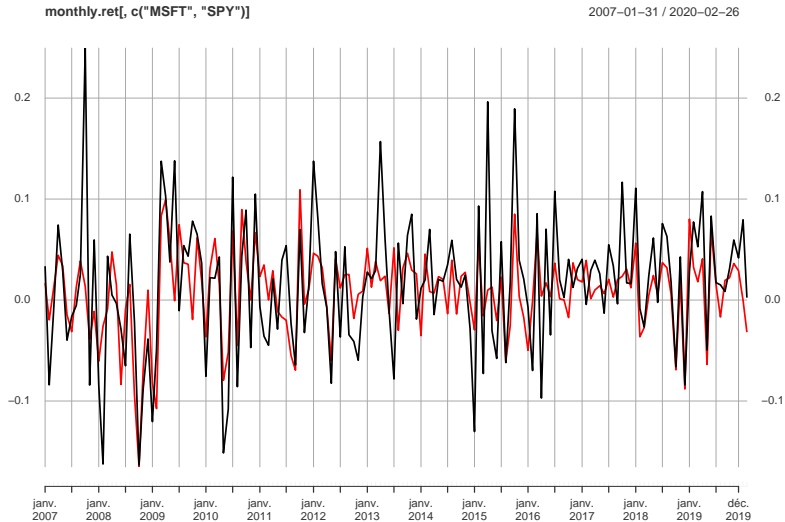
Allocation Treynor-Black

Voir note pour le détail des calculs.

Données

| | AAPL | AMZN | MSFT | F | SPY | QQQ | XOM | MMM | HD | PG | KO |
|-----------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Observations | 158.0000 | 158.0000 | 158.0000 | 158.0000 | 158.0000 | 158.0000 | 158.0000 | 158.0000 | 158.0000 | 158.0000 | 158.0000 |
| NAs | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Minimum | -0.3296 | -0.2540 | -0.1634 | -0.5788 | -0.1652 | -0.1558 | -0.1423 | -0.1498 | -0.1652 | -0.1161 | -0.1668 |
| Quartile 1 | -0.0213 | -0.0303 | -0.0280 | -0.0456 | -0.0144 | -0.0159 | -0.0265 | -0.0230 | -0.0227 | -0.0211 | -0.0168 |
| Median | 0.0291 | 0.0256 | 0.0203 | -0.0022 | 0.0128 | 0.0192 | 0.0022 | 0.0144 | 0.0178 | 0.0081 | 0.0104 |
| Arithmetic Mean | 0.0254 | 0.0298 | 0.0152 | 0.0115 | 0.0076 | 0.0123 | 0.0017 | 0.0079 | 0.0151 | 0.0074 | 0.0100 |
| Geometric Mean | 0.0214 | 0.0252 | 0.0130 | 0.0020 | 0.0067 | 0.0110 | 0.0004 | 0.0063 | 0.0133 | 0.0065 | 0.0090 |
| Quartile 3 | 0.0792 | 0.0830 | 0.0545 | 0.0470 | 0.0324 | 0.0455 | 0.0411 | 0.0455 | 0.0608 | 0.0361 | 0.0405 |
| Maximum | 0.2377 | 0.5413 | 0.2495 | 1.2738 | 0.1091 | 0.1317 | 0.1128 | 0.1734 | 0.1605 | 0.1161 | 0.1419 |
| SE Mean | 0.0071 | 0.0079 | 0.0053 | 0.0120 | 0.0033 | 0.0040 | 0.0040 | 0.0045 | 0.0049 | 0.0034 | 0.0035 |
| LCL Mean (0.95) | 0.0114 | 0.0143 | 0.0047 | -0.0121 | 0.0010 | 0.0044 | -0.0062 | -0.0010 | 0.0055 | 0.0006 | 0.0031 |
| UCL Mean (0.95) | 0.0394 | 0.0454 | 0.0257 | 0.0352 | 0.0142 | 0.0201 | 0.0095 | 0.0168 | 0.0248 | 0.0141 | 0.0170 |
| Variance | 0.0079 | 0.0098 | 0.0045 | 0.0226 | 0.0018 | 0.0025 | 0.0025 | 0.0032 | 0.0037 | 0.0019 | 0.0020 |
| Stdev | 0.0889 | 0.0990 | 0.0670 | 0.1504 | 0.0419 | 0.0502 | 0.0502 | 0.0566 | 0.0612 | 0.0430 | 0.0442 |
| Skewness | -0.6572 | 0.7403 | 0.0788 | 3.6175 | -0.7421 | -0.6247 | -0.3969 | -0.3662 | -0.3033 | -0.2344 | -0.2589 |
| Kurtosis | 1.9491 | 4.0251 | 0.8958 | 31.4460 | 1.5521 | 0.9798 | 0.0867 | 0.5184 | 0.4291 | 0.0619 | 1.1258 |

MSFT & SPY



Calcul de β

```
beta_roll <- removeNA(rollapply(data=monthly.ret$MSFT, Rb=r  
                                FUN=CAPM.beta, width=36, by
```



Calcul de α

```
alpha_roll <- removeNA(rollapply(data=monthly.ret$MSFT, Rb=
                                FUN=CAPM.alpha, width=36,
```



Bibliographie

Grinhold, R.C. and Kahn, R. Active Portfolio Management, McGraw-Hill, 2000