

Exercice Modèle Black-Litterman

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```
library(xts)
library(hornpa)
library(lubridate)
library(xtable)
library(PerformanceAnalytics)
library(TTR)
library(lubridate)
library(roll)
library(Hmisc)
library(nFactors)
library(kableExtra)
#library(broom)
library(quadprog)
```

L'objet de cet exercice est de combiner l'approche de Black-Litterman et le modèle moyenne-variance classique pour imposer des contraintes à la solution.

Rappel

Distribution ex=ante des rendements:

$$r \sim \mathcal{N}(\mu, \Sigma)$$

Rendements espérés d'équilibre

$$\Pi = \delta \Sigma w_{eq}$$

Distribution de l'espérance de rendement:

$$\mu = \Pi + \epsilon^{(e)}$$

avec

$$\epsilon^{(e)} \sim \mathcal{N}(0, \tau \Sigma)$$

where τ is a scalar.

Expression des vues:

$$P\mu = Q + \epsilon^{(v)}$$

avec

$$\epsilon^{(v)} \sim \mathcal{N}(0, \Omega)$$

Solution ex-post:

After algebraic manipulations:

Espérance de rendement

$$\mu^* = [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1} [(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q]$$

Covariance des rendements

$$M^{-1} = [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}$$

Distribution ex-post des rendements:

$$r \sim \mathcal{N}(\mu^*, \Sigma^*)$$

avec $\Sigma^* = \Sigma + M^{-1}$.

Données

Données de He & Litterman:

```
data =  
'1,0.4880,0.4780,0.5150,0.4390,0.5120,0.4910  
0.4880,1,0.6640,0.6550,0.3100,0.6080,0.7790  
0.4780,0.6640,1,0.8610,0.3550,0.7830,0.6680  
0.5150,0.6550,0.8610,1,0.3540,0.7770,0.6530  
0.4390,0.3100,0.3550,0.3540,1,0.4050,0.3060  
0.5120,0.6080,0.7830,0.7770,0.4050,1,0.6520  
0.4910,0.7790,0.6680,0.6530,0.3060,0.6520,1'  
  
Corrmat = matrix( as.double(spl( gsub('\n', ',', data), ',')),  
                  nrow = length(spl(data, '\n')), byrow=TRUE)  
  
stdevs = c(16.0, 20.3, 24.8, 27.1, 21.0, 20.0, 18.7)/100  
w.eq = c(1.6, 2.2, 5.2, 5.5, 11.6, 12.4, 61.5)/100  
# Prior covariance of returns  
Sigma = Corrmat * (stdevs %*% t(stdevs))
```

Rendements d'équilibre

```
# risk aversion parameter  
delta = 2.5  
Pi = delta * Sigma %*% w.eq
```

Assets	Std Dev	Weq	PI
Australia	16	1.6	3.9
Canada	20.3	2.2	6.9
France	24.8	5.2	8.4
Germany	27.1	5.5	9
Japan	21	11.6	4.3
UK	20	12.4	6.8
USA	18.7	61.5	7.6

Calculs

Résoudre le programme d'optimisation en imposant la vue #1 (le marché allemand sur-performe) tels que $w_i \geq 0$.