# Financial Time Series

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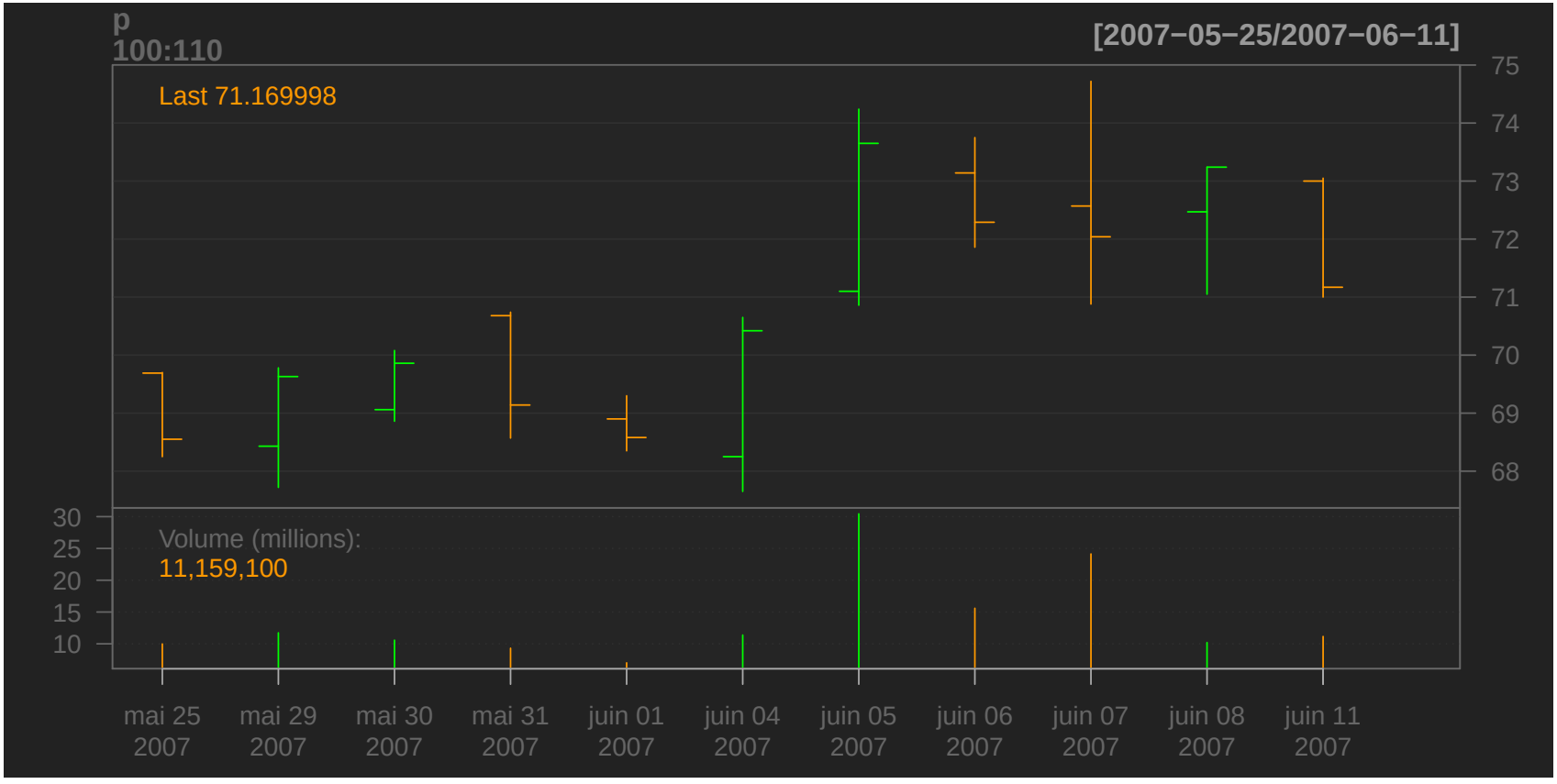
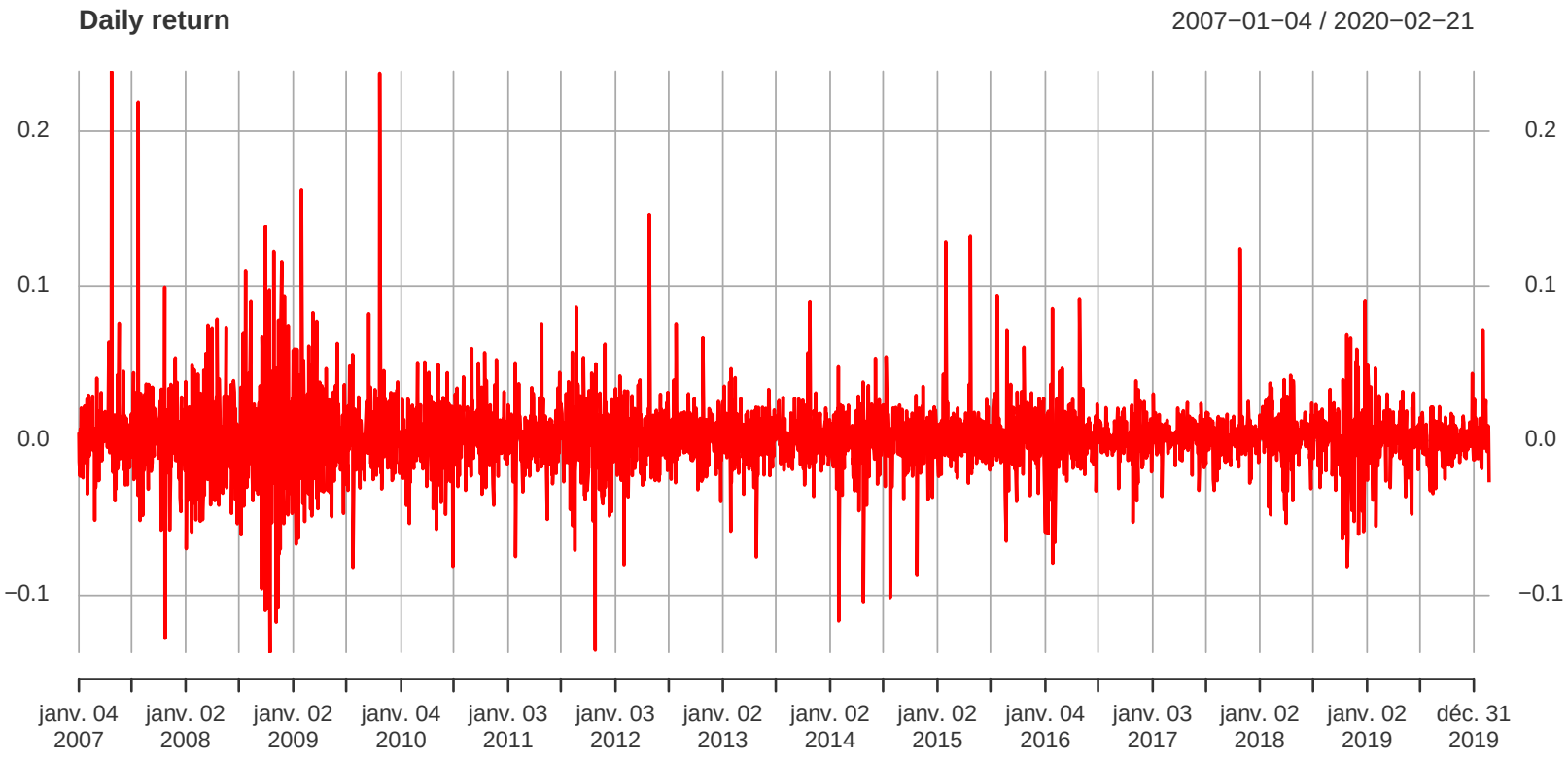
# Financial Time Series (daily OHLC)

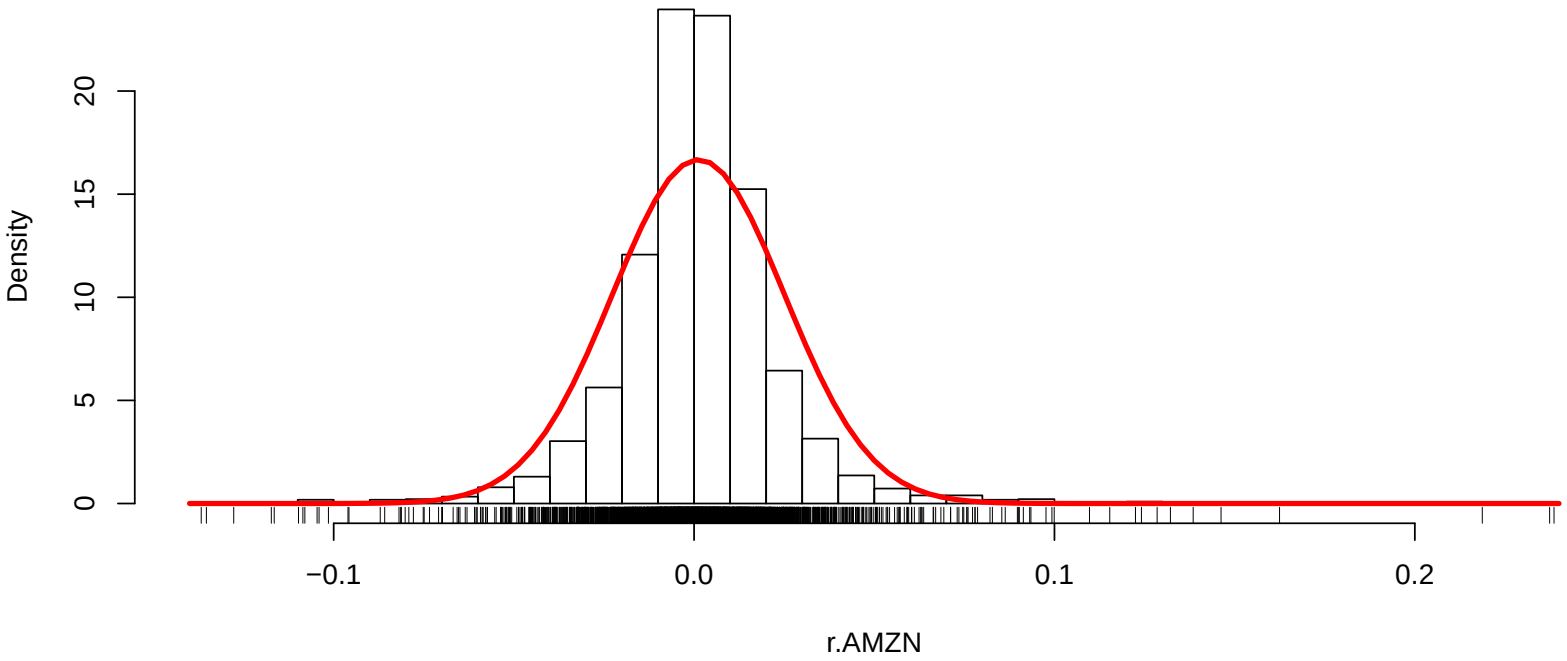


# Daily Return - AMZN

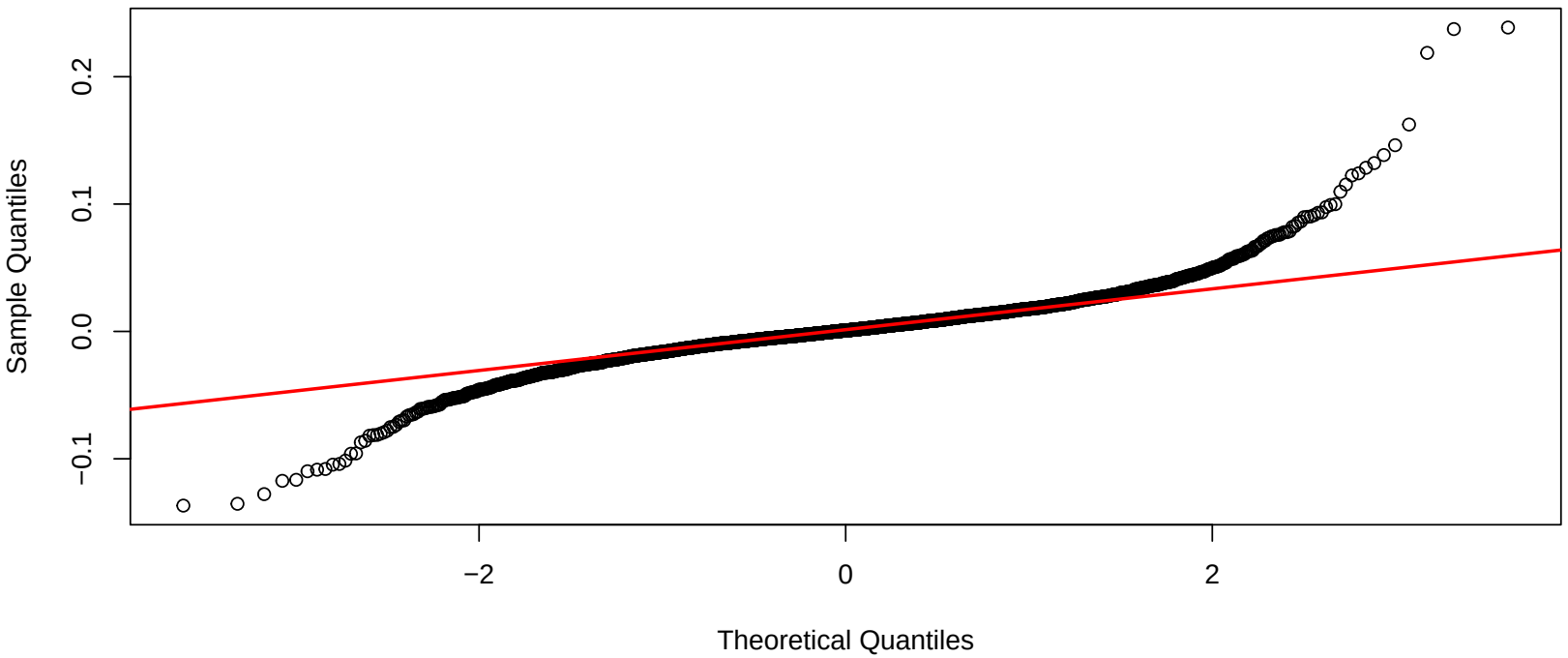
$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right)$$



# Histogram of daily return - AMZN



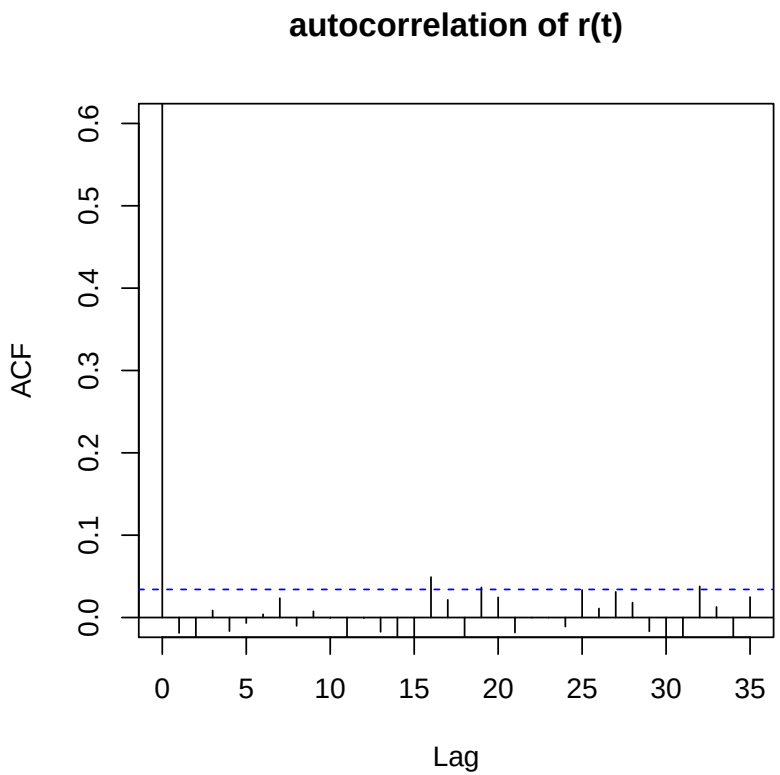
# Analysis of return distribution - AMZN

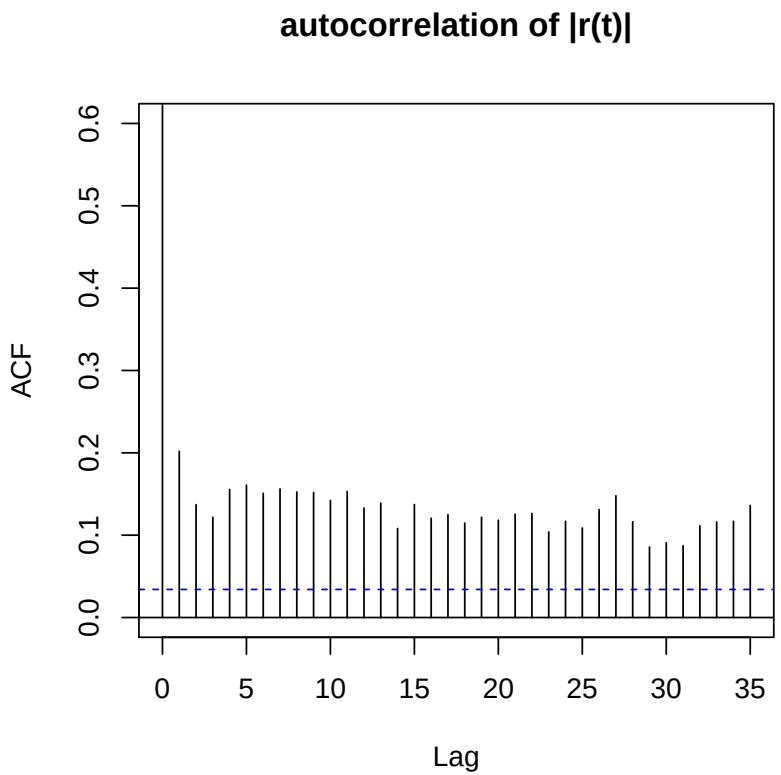


# Moments of daily returns

| | mean | std dev | skewness | kurtosis |
|---|---|---|---|---|
| AMZN | 0.0012075 | 0.0239215 | 0.9526121 | 12.849686 |
| GOOG | 0.0005604 | 0.0178051 | 0.5008926 | 11.495964 |
| AAPL | 0.0010301 | 0.0196726 | -0.4678985 | 7.176276 |
| QQQ | 0.0005412 | 0.0130159 | -0.1803480 | 6.985970 |
| DIA | 0.0003516 | 0.0114022 | 0.2150189 | 15.397181 |
| SPY | 0.0003397 | 0.0121101 | -0.1557741 | 14.109873 |
| PG | 0.0003238 | 0.0109203 | -0.1052228 | 7.846785 |
| KO | 0.0004433 | 0.0112445 | 0.2436121 | 12.923579 |

# Autocorrelation of Returns (AMZN)



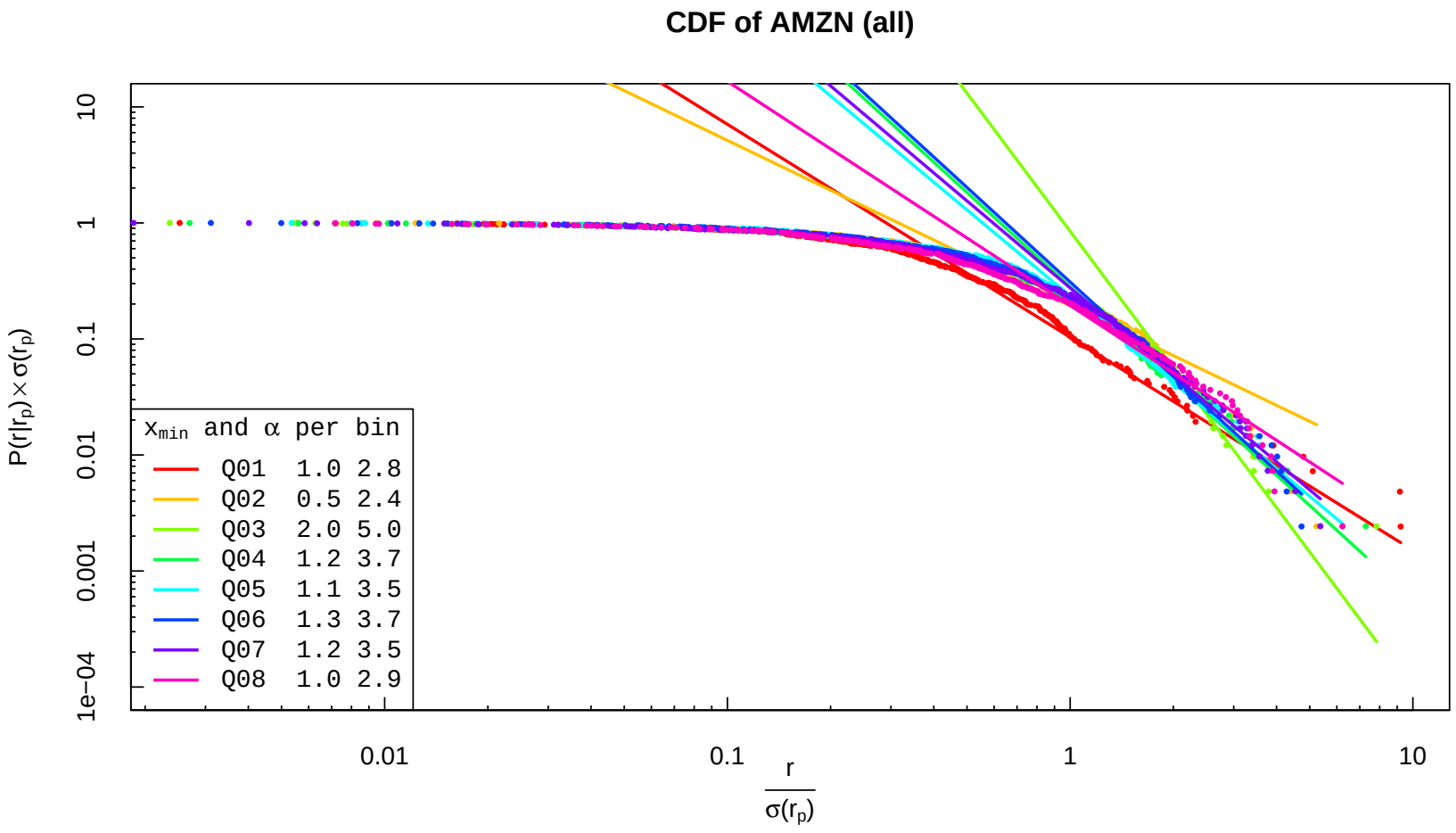
# Rescaling daily return by $\sigma(r_{t-1})$ (Chen, Jayaprakash, and Yuan 2008)

$$z_t = \frac{r_t}{\sigma(r_{t-1})}$$

$$\left.\begin{aligned} p(z_t) &= \frac{\alpha - 1}{z_{min}} \left(\frac{z_t}{z_{min}}\right)^{-\alpha} \\ Pr(z_t > x) &= \left(\frac{z_t}{z_{min}}\right)^{-\alpha+1} \end{aligned}\right\} z_t > z_{min}$$

The density of $z_t$ can be approximated by a power law. See paper for details of calculation.

# Rescaling of daily return by $\sigma(r_{t-1})$



# Unconditional distribution of return

The Johnson family of distributions is formed by various transformations of the normal density. Let $X$ be the observed data, an define $Z$ by:

$$Z = \gamma + \delta \ln\left(g\left(\frac{X-\xi}{\lambda}\right)\right)$$

where:

$$g(u) = \begin{cases} u & SL \\ u + \sqrt{1+u^2} & SU \\ \frac{u}{1-u} & SB \\ e^u & SN \end{cases}$$

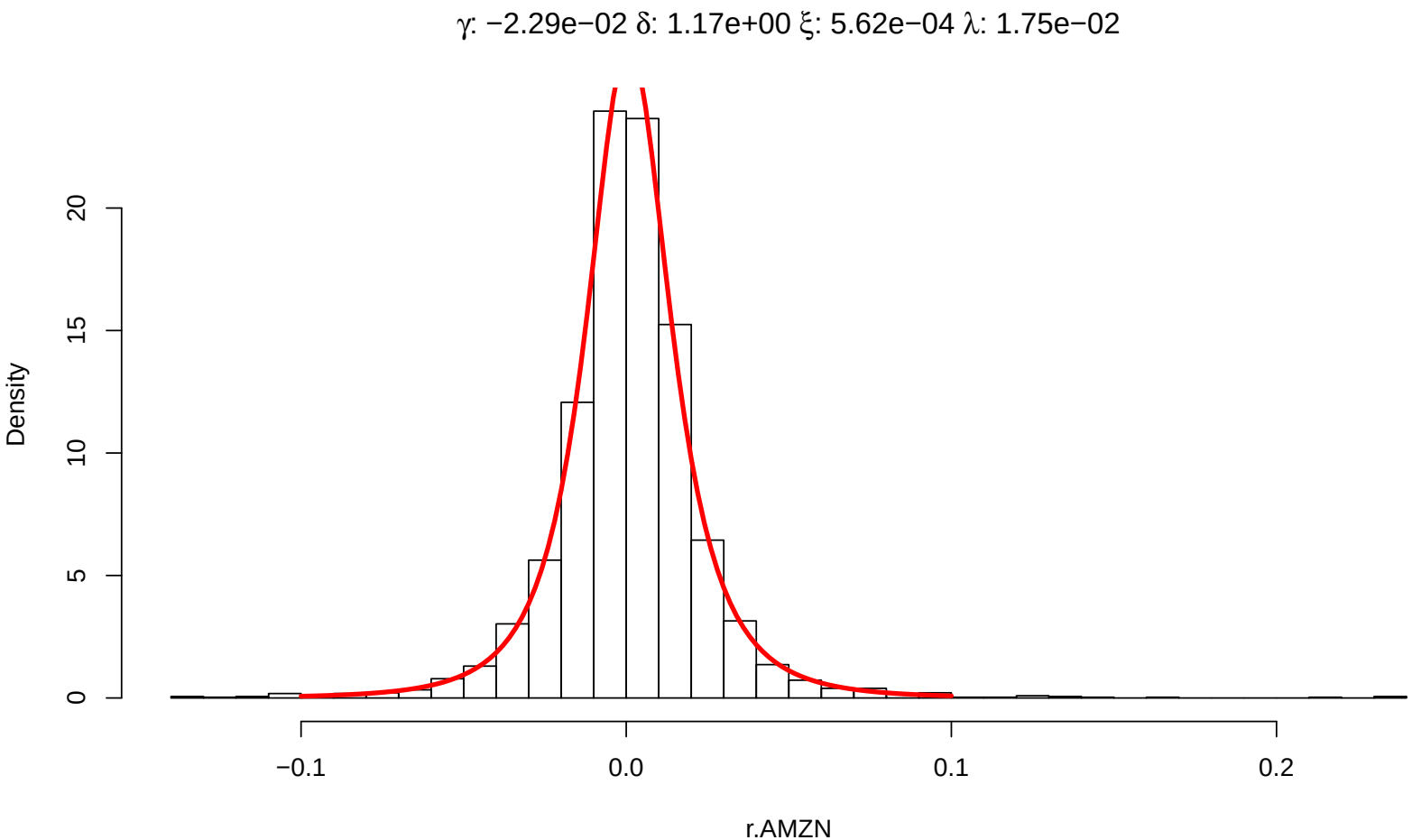
$X$ follows a Johnson distribution if $Z$ is normal.

# Fitted Johnson SU distribution - AMZN (1)

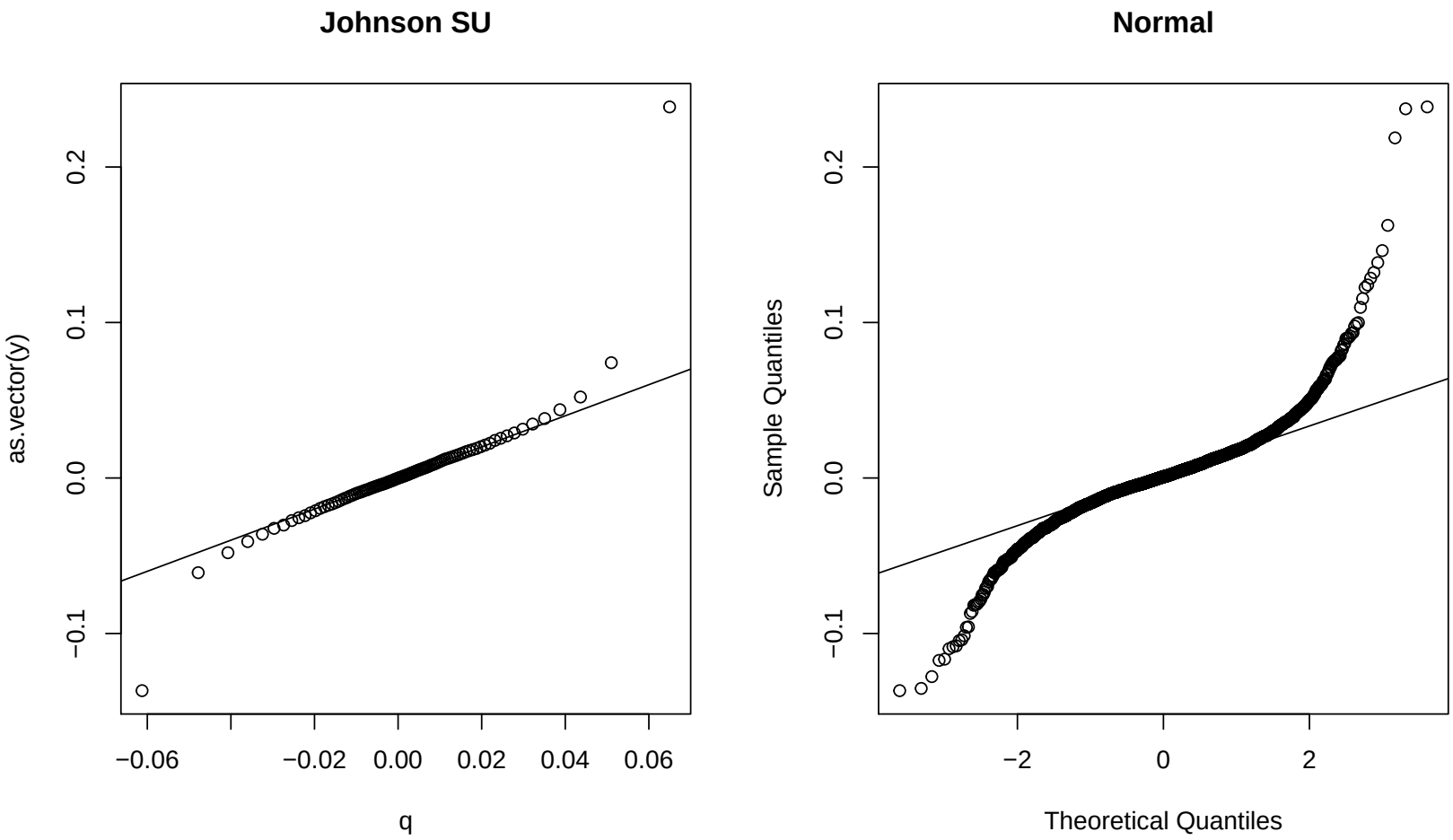
| gamma | delta | xi | lambda | type |
|---|---|---|---|---|
| -0.0228945 | 1.16685 | 0.0005621 | 0.0174527 | SU |

| | sample | johnson |
|---|---|---|
| mean | 0.0012075 | 0.0010565 |
| sigma | 0.0239179 | 0.0225752 |
| skew | 0.9530444 | -0.1098671 |
| kurt | 12.8592791 | 12.3022551 |

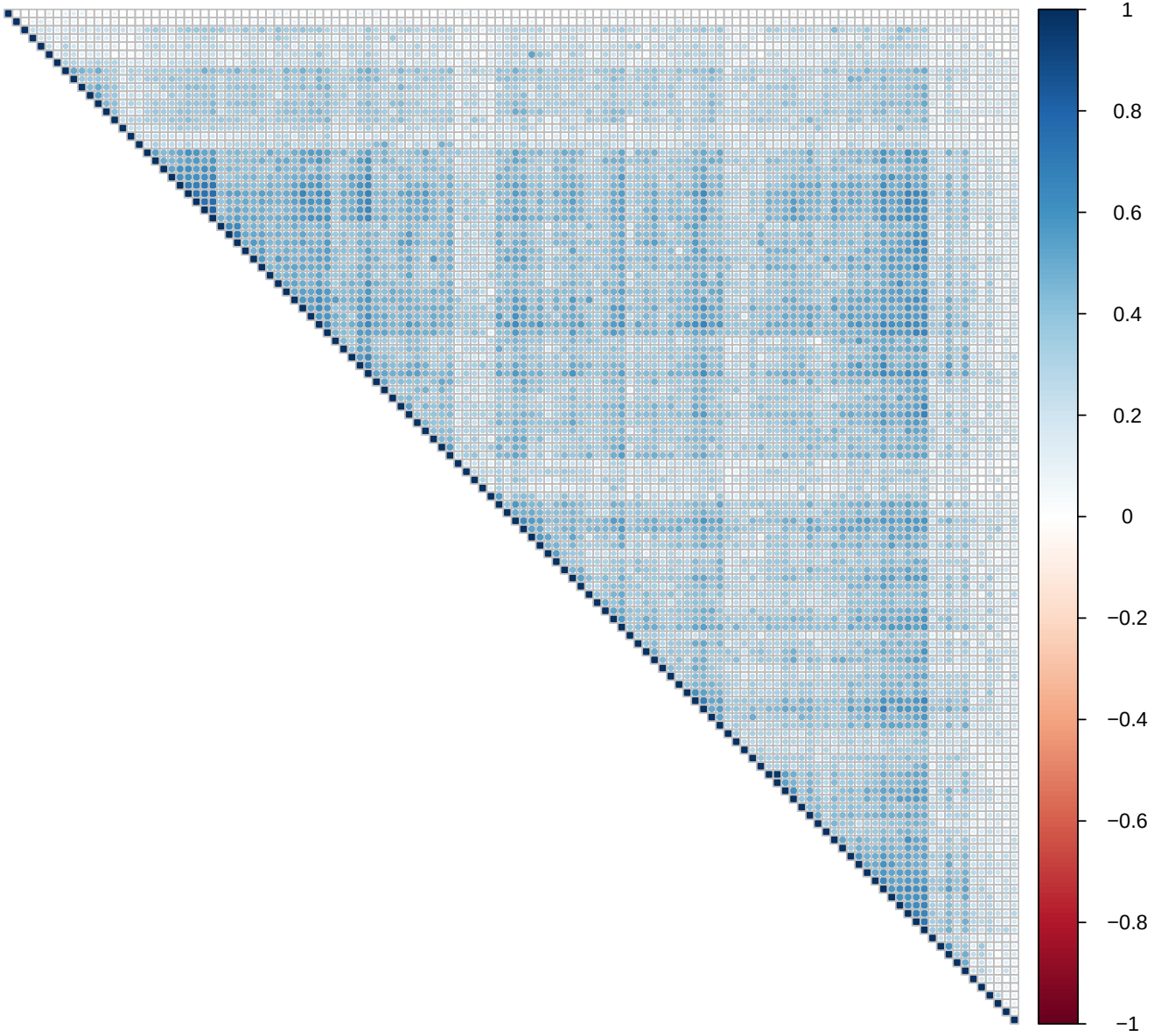
# Fitted Johnson SU distribution - AMZN (2)



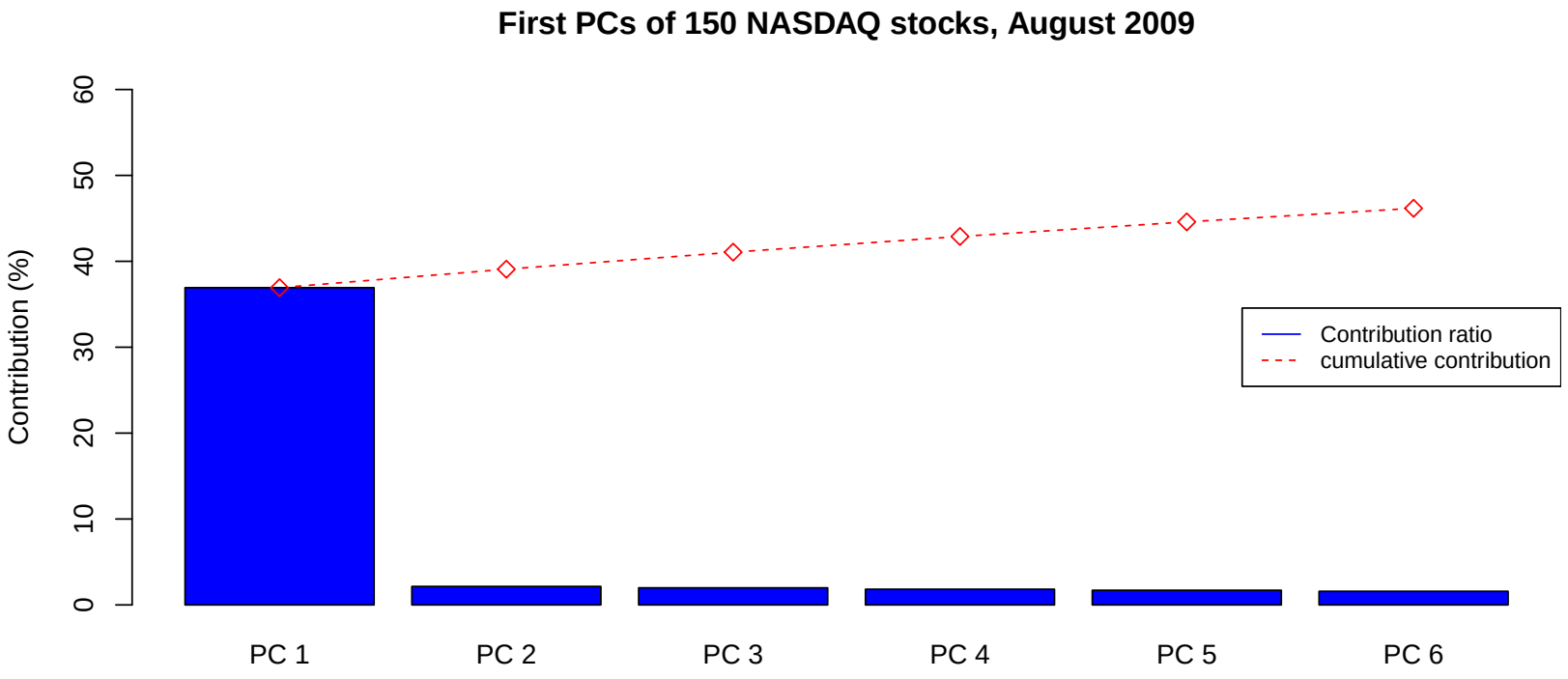
# Fitted Johnson SU distribution - AMZN (3)



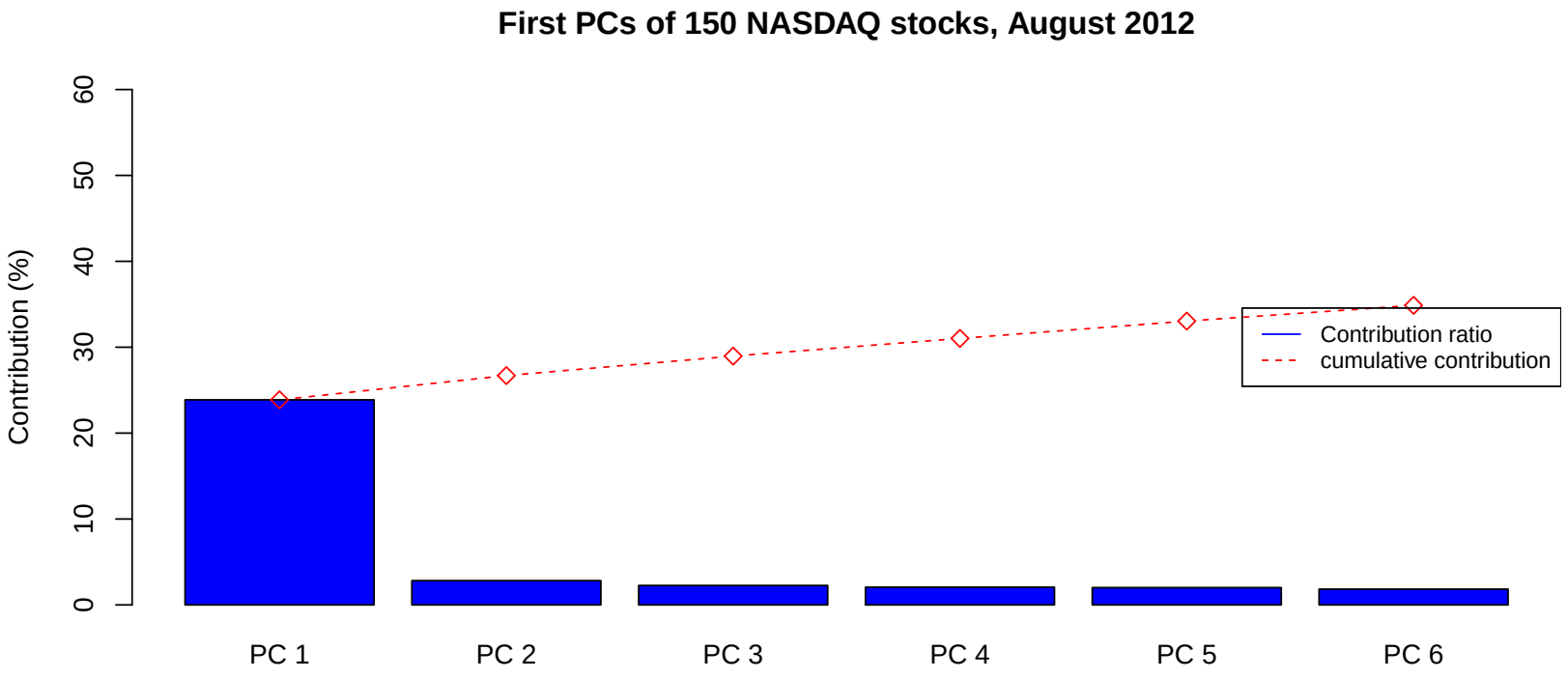
# Correlation between assets (NASDAQ)



# Correlation between assets

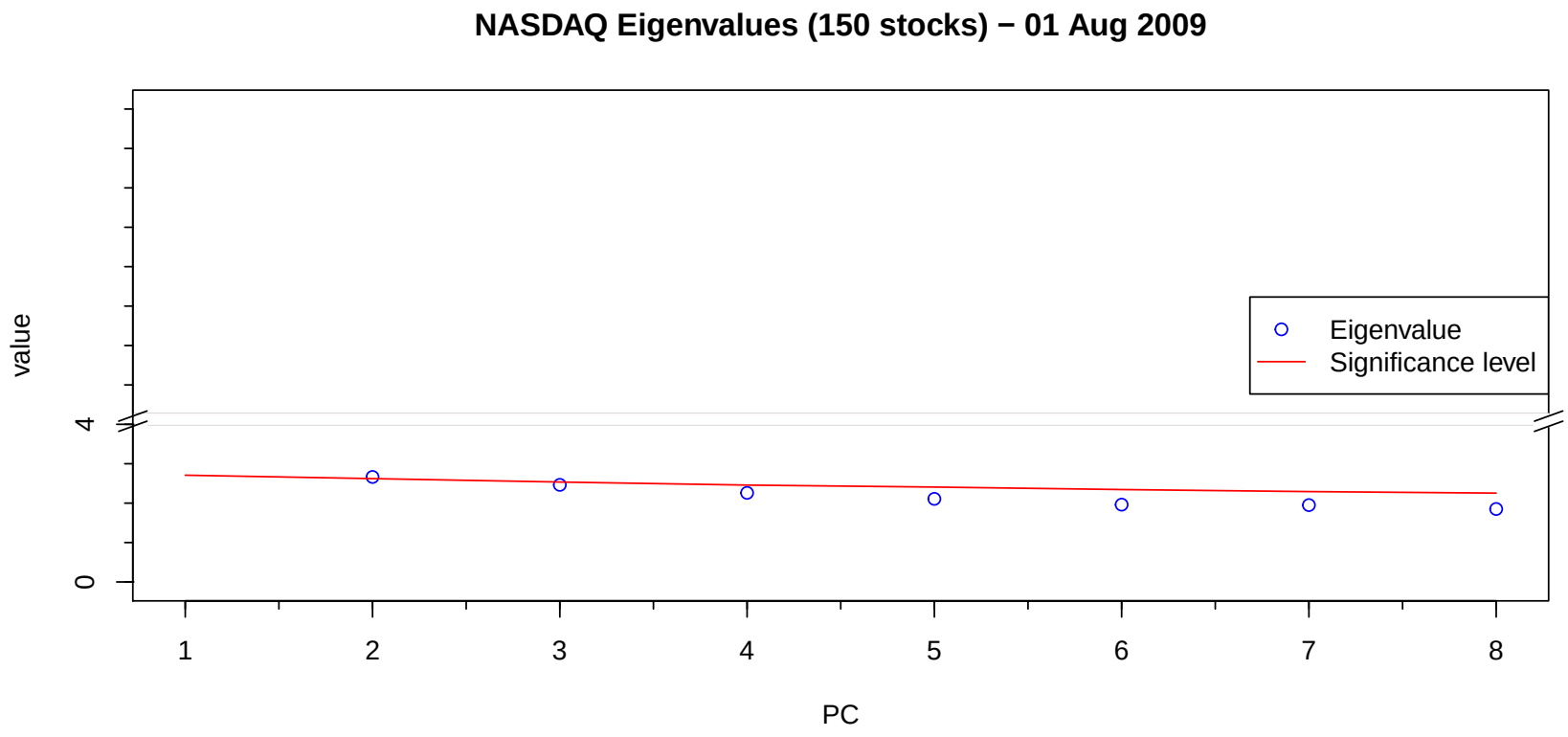


# Correlation between assets



# How many dimensions in a market?

Significance level (95%) for eigenvalues (252 observations, 127 variables):



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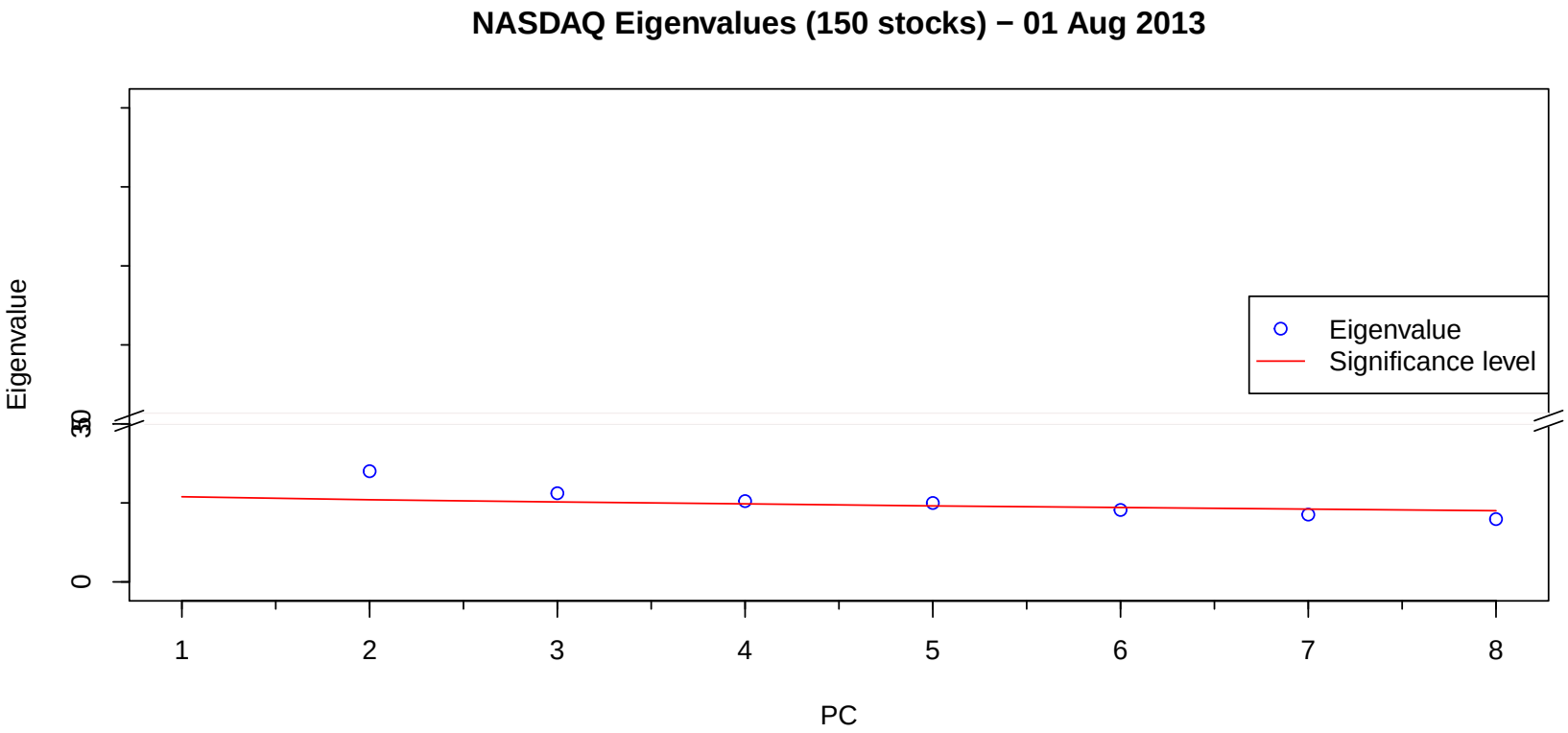

# Summary

To summarize, empirical observations show that the distribution of returns exhibit features that strongly repart from the classical hypothesis of independence and normality. We find:

1. no evidence of linear autocorrelation of return, however,
2. there is an observable autocorrelation of $|r_t|$ and $r_t^2$, suggesting autocorrelation in the volatility of return,
3. we also observe large excess kurtosis, which is incompatible with normal density,
4. The rank of a broad stock market such as the NASDAQ is probably much lower than the number of stocks.

# Bibliography

Chen, Kan, C Jayaprakash, and Baosheng Yuan. 2008. "Conditional Probability as a Measure of Volatility Clustering in Financial Time Series." *Physica A*, 1–5. http://arxiv.org/abs/0503157v2.