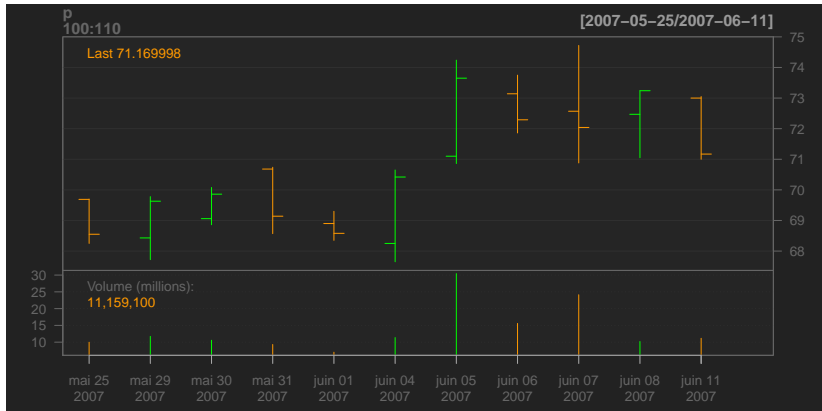# Financial Time Series

P. Hénaff
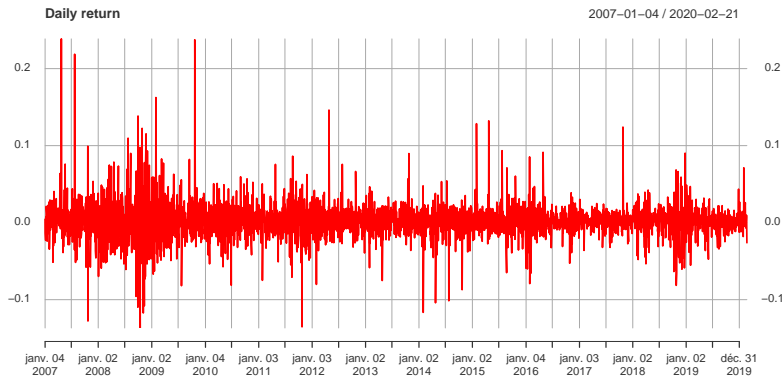
Version: 26 déc. 2021

Financial Time Series (daily OHLC)
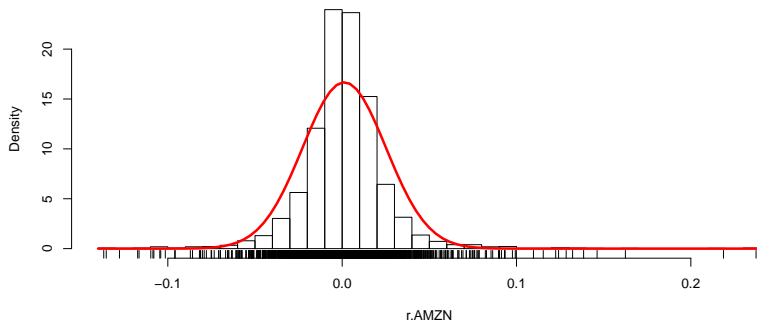
Daily Return - AMZN

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right)$$
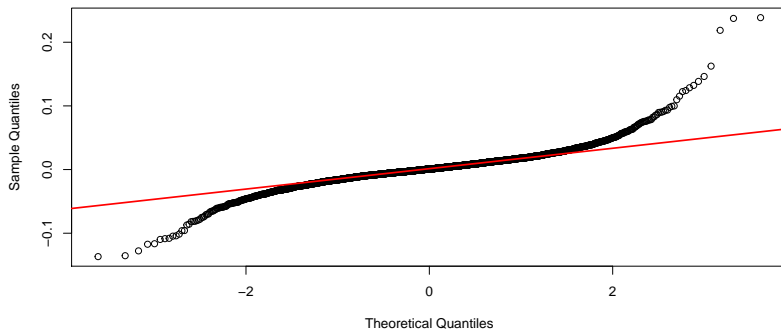

Daily return   2007–01–04 / 2020–02–21

Histogram of daily return - AMZN

Analysis of return distribution - AMZN

Moments of daily returns

	mean	std dev	skewness	kurtosis
AMZN	0.0012075	0.0239215	0.9526121	12.849686
GOOG	0.0005604	0.0178051	0.5008926	11.495964
AAPL	0.0010301	0.0196726	-0.4678985	7.176276
QQQ	0.0005412	0.0130159	-0.1803480	6.985970
DIA	0.0003516	0.0114022	0.2150189	15.397181
SPY	0.0003397	0.0121101	-0.1557741	14.109873
PG	0.0003238	0.0109203	-0.1052228	7.846785
KO	0.0004433	0.0112445	0.2436121	12.923579

Autocorrelation of Returns (AMZN)
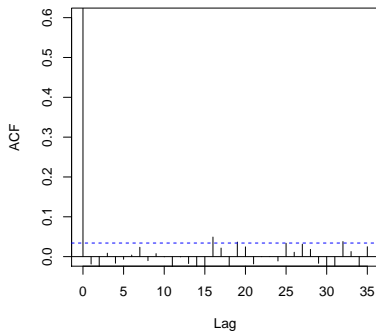
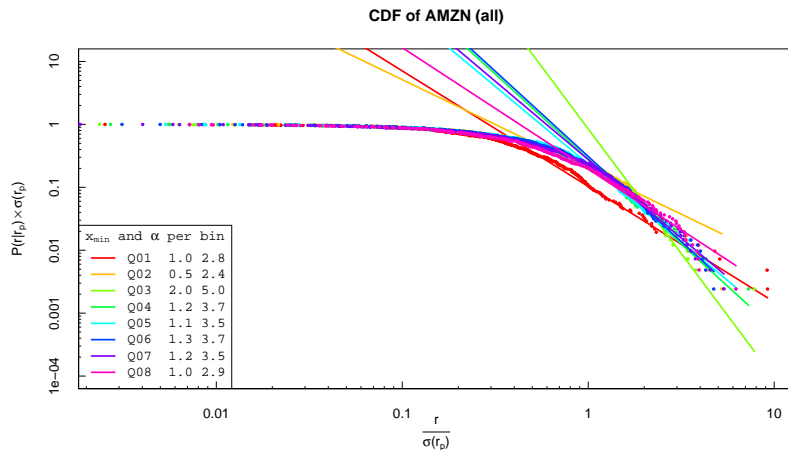autocorrelation of r(t)   autocorrelation of |r(t)|
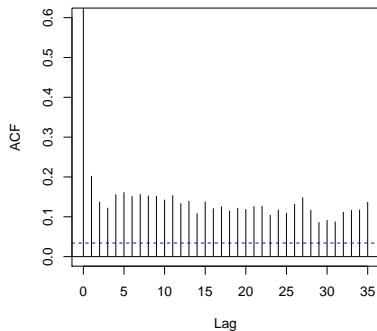
Rescaling daily return by $\sigma(r_{t-1})$ (Chen, Jayaprakash, and Yuan 2008)

$$z_t = \frac{r_t}{\sigma(r_{t-1})}$$

$$\left. \begin{array}{l} p(z_t) = \dfrac{\alpha - 1}{z_{min}} \left(\dfrac{z_t}{z_{min}} \right)^{-\alpha} \\[3mm] Pr(z_t > x) = \left(\dfrac{z_t}{z_{min}} \right)^{-\alpha + 1} \end{array} \right\} z_t > z_{min}$$

The density of z_t can be approximated by a power law. See paper for details of calculation.

Rescaling of daily return by $\sigma(r_{t-1})$

CDF of AMZN (all)

x_{min} and α per bin		
Q01	1.0	2.8
Q02	0.5	2.4
Q03	2.0	5.0
Q04	1.2	3.7
Q05	1.1	3.5
Q06	1.3	3.7
Q07	1.2	3.5
Q08	1.0	2.9

Unconditional distribution of return

The Johnson family of distributions is formed by various transformations of the normal density. Let X be the observed data, an define Z by:

$$Z = \gamma + \delta \ln \left(g \left(\frac{X - \xi}{\lambda} \right) \right)$$

where:

$$g(u) = \begin{cases} u & SL \\ u + \sqrt{1 + u^2} & SU \\ \frac{u}{1-u} & SB \\ e^u & SN \end{cases}$$
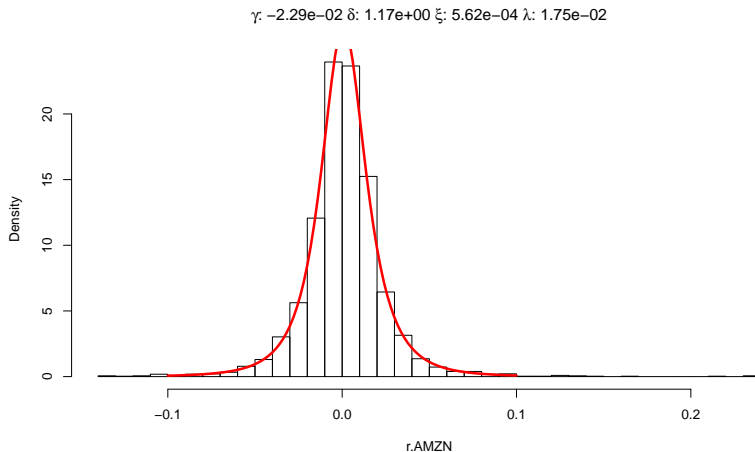
X follows a Johnson distribution if Z is normal.

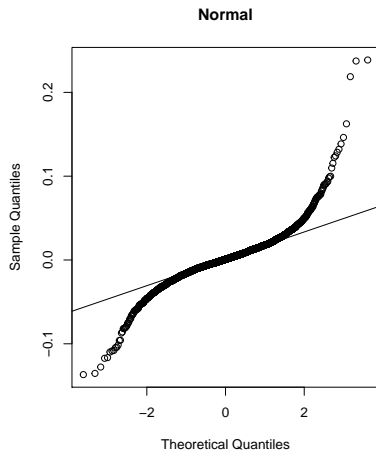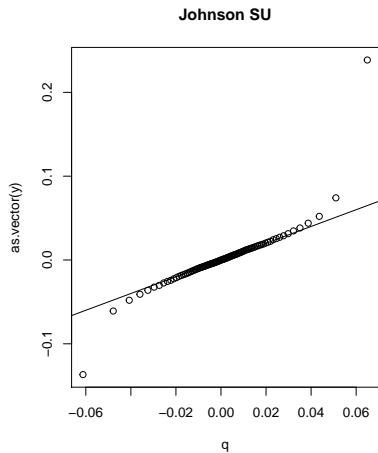## Fitted Johnson SU distribution - AMZN (1)

gamma	delta	xi	lambda	type
-0.0228945	1.16685	0.0005621	0.0174527	SU

	sample	johnson
mean	0.0012075	0.0010565
sigma	0.0239179	0.0225752
skew	0.9530444	-0.1098671
kurt	12.8592791	12.3022551

Fitted Johnson SU distribution - AMZN (2)

γ: −2.29e−02 δ: 1.17e+00 ξ: 5.62e−04 λ: 1.75e−02

Fitted Johnson SU distribution - AMZN (3)

Johnson SU

Normal

Correlation between assets (NASDAQ)

Correlation between assets

First PCs of 150 NASDAQ stocks, August 2009
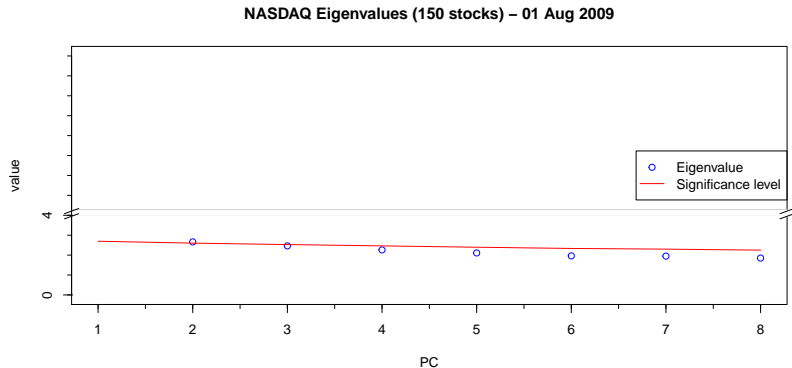
Correlation between assets

First PCs of 150 NASDAQ stocks, August 2012
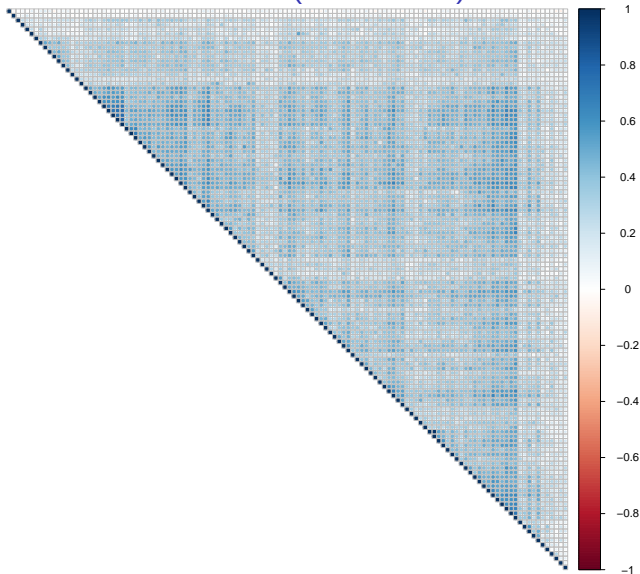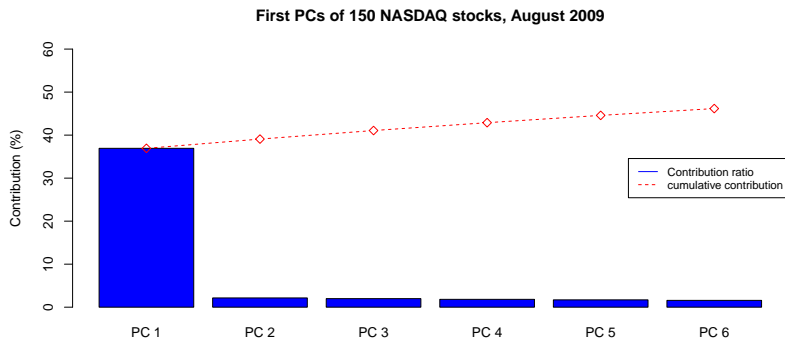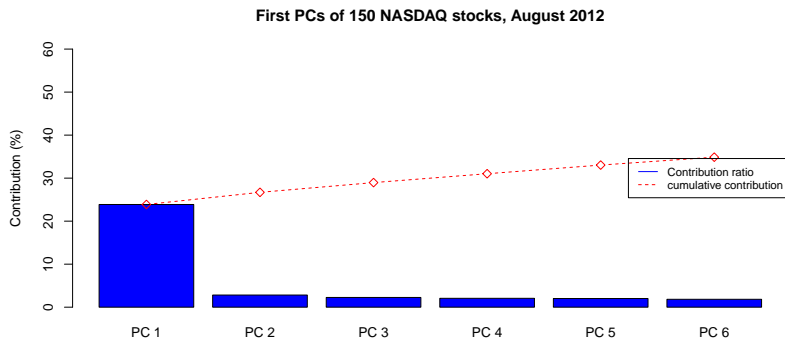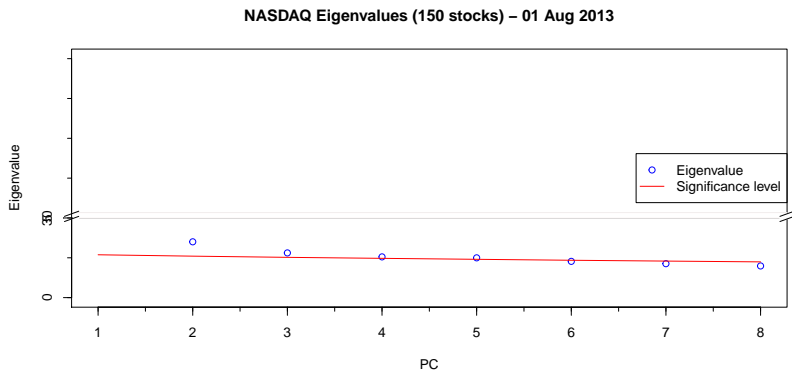
How many dimensions in a market?

Significance level (95%) for eigenvalues (252 observations, 127 variables):

NASDAQ Eigenvalues (150 stocks) – 01 Aug 2009

How many dimensions in a market?

NASDAQ Eigenvalues (150 stocks) – 01 Aug 2013

Summary

To summarize, empirical observations show that the distribution of returns exhibit features that strongly repart from the classical hypothesis of independence and normality. We find:

1. no evidence of linear autocorrelation of return, however,
2. there is an observable autocorrelation of $|r_t|$ and r_t^2, suggesting autocorrelation in the volatility of return,
3. we also observe large excess kurtosis, which is incompatible with normal density,
4. The rank of a broad stock market such as the NASDAQ is probably much lower than the number of stocks.

Bibliography

Chen, Kan, C Jayaprakash, and Baosheng Yuan. 2008.
  "Conditional Probability as a Measure of Volatility Clustering in
  Financial Time Series." *Physica A*, 1–5.
  http://arxiv.org/abs/0503157v2.