Practical: Robot modeling and Control

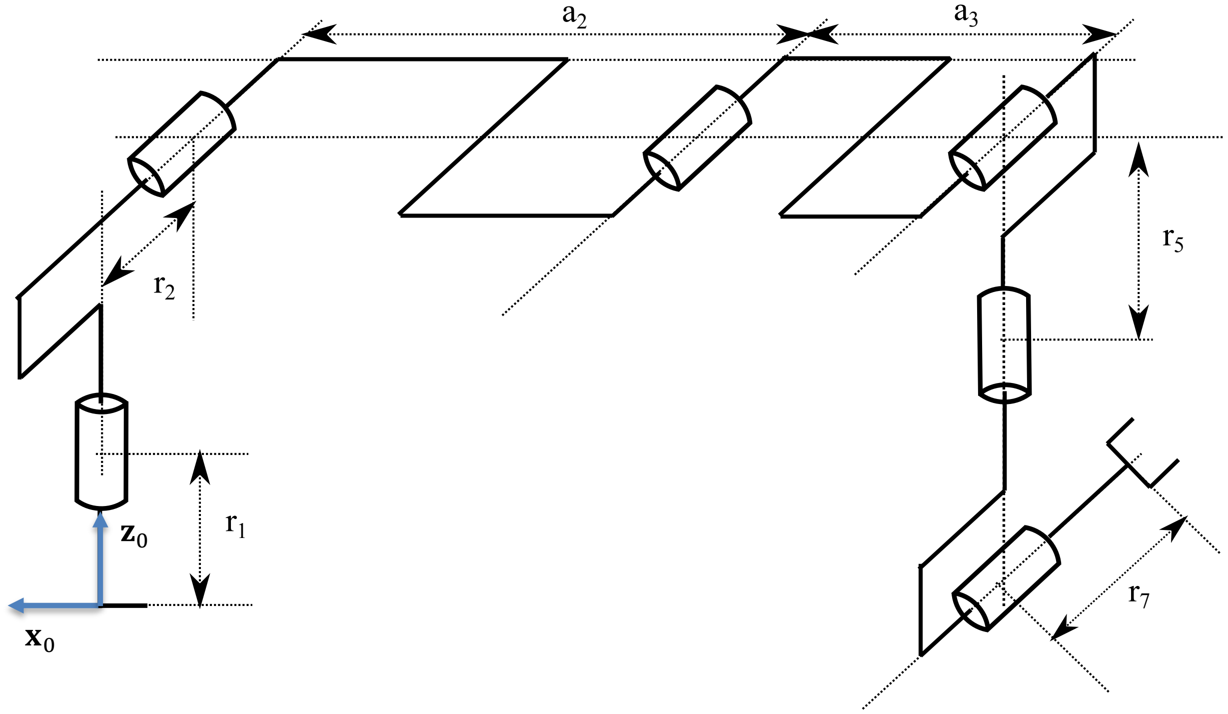

Figure 1 : Architecture of the robot

The study focuses on the modeling and control of the UR10 robot whose architecture is given in Figure 1. The robot has six revolute joints. In order to model tis robot, we adopt the modified Denavit-Hartenberg convention seen in the course. We note by ${}^{i}\mathbf{T}_j$ the homogenous transformation matrix between the frame $\mathcal{R}_i$ and the frame $\mathcal{R}_j$.

The kinematic parameters of the robot are:

$$\begin{cases} \mathrm{a}_2 = 0.612 \\ \mathrm{a}_3 = 0.5723 \\ \mathrm{r}_1 = 0.1273 \\ \mathrm{r}_2 = 0.163941 \\ \mathrm{r}_5 = 0.1157 \\ \mathrm{r}_7 = 0.0922 \end{cases}$$

Q1: Place the frames according to the modified Denavit-Hartenberg convention and determine DH parameters. Write a function that defines the transformation matrix
def MatrixTransformation(uj):

return T

This function receives as an argument the vector $u_j = [\alpha_{j-1} \quad \mathrm{a}_{j-1} \quad \theta_j \quad r_j]^T$ with j the joint number and returns the transformation matrix ${}^{j-1}\mathbf{T}_j$.

Q2: Compute the position and the orientation of the end effector in the reference frame $\mathcal{R}_0$. If the computations are done correctly, you should obtain for $\theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = \theta_6 = 0$:

$$
{}^{0}\mathbf{T}_6 = \begin{pmatrix} 1 & 0 & 0 & 1.1843 \\ 0 & 0 & -1 & 0.1639 \\ 0 & 1 & 0 & 0.0116 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

The orientation of the end-effector should be expressed using axis and angle representation.
Important: the questions 1, 2 and 3 must be treated in the same function that will be named: *def kinematics(joint_pos):*

return T06, J

This function receives the joints positions of the robot and returns the transformation matrix ${}^{0}\mathbf{T}_6$ and the Jacobian matrix **J.**

Q3: Determine the differential kinematic model of the robot. If your computations are correct, you should obtain for $\theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = \theta_6 = 0$:

$$
\mathbf{J} = \begin{pmatrix} -0.0717 & 0.1157 & 0.1157 & 0.1157 & -0.0922 & 0 \\ 1.1843 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.1843 & 0.5723 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}
$$

Q4: Generate the trajectory of the robot using the 5th order polynomial equation. The desired orientation should be considered using the convention axis and angle.

$$
\mathbf{x}^d(\mathrm{t}) = \mathbf{x}^{\mathbf{i}} + \mathrm{r}(\mathrm{t})\left(\mathbf{x}^{\mathrm{f}} - \mathbf{x}^{\mathrm{i}}\right)
$$
$$
\mathbf{R}^d(\mathrm{t}) = \mathbf{R}^{\mathrm{i}}\ \mathbf{rot}(\mathbf{u},\ \mathrm{r}(\mathrm{t})\vartheta)
$$

With:

$$
\mathbf{rot}(\mathbf{u},\ \mathrm{r}(\mathrm{t})\theta) = \begin{pmatrix} u_x{}^2(1-\cos(\mathrm{r}\vartheta)) + \cos(\mathrm{r}\vartheta) & u_x u_y(1-\cos(\mathrm{r}\vartheta)) - u_z\sin(\mathrm{r}\vartheta) & u_x u_z(1-\cos(\mathrm{r}\vartheta)) + u_y\sin(\mathrm{r}\vartheta) \\ u_x u_y(1-\cos(\mathrm{r}\vartheta)) + u_z\sin(\mathrm{r}\vartheta) & u_y{}^2(1-\cos(\vartheta)) + \cos(\mathrm{r}\vartheta) & u_y u_z(1-\cos(\mathrm{r}\vartheta)) - u_x\sin(\mathrm{r}\vartheta) \\ u_x u_z(1-\cos(\mathrm{r}\vartheta)) - u_y\sin(\mathrm{r}\vartheta) & u_y u_z(1-\cos(\mathrm{r}\vartheta)) + u_x\sin(\mathrm{r}\vartheta) & u_z{}^2(1-\cos(\mathrm{r}\vartheta)) + \cos(\mathrm{r}\vartheta) \end{pmatrix}
$$

Q5: Compute the position error $\mathbf{e}_p = \mathbf{x}^d - \mathbf{x}$ as well as the orientation error using the following relationship:

$$
\mathbf{e}_O = \frac{1}{2}(\mathbf{n}^e \wedge \mathbf{n}^d + \mathbf{s}^e \wedge \mathbf{s}^d + \mathbf{a}^e \wedge \mathbf{a}^d)
$$

Where $(\mathbf{n}^e \quad \mathbf{s}^e \quad \mathbf{a}^e)$ are the columns of the current orientation matrix $\mathbf{R}^e$. This latter is retrieved from the transformation matrix ${}^{0}\mathbf{T}_6$.
$(\mathbf{n}^d \quad \mathbf{s}^d \quad \mathbf{a}^d)$ represent the column of the desired orientation matrix $\mathbf{R}^{\boldsymbol{d}}$, that is computed using the trajectory generation.

Q6: Generate the control law of the robot using the following relationship:

$$\dot{\mathbf{q}}^{d} = \mathbf{J}^{-1}(\mathbf{q}) \begin{bmatrix} \dot{\mathbf{x}}^{d} + \mathbf{K}_{p}\boldsymbol{e}_{p} \\ \mathbf{L}^{-1}(\mathbf{L}^{T}\boldsymbol{\omega}_{d} + \mathbf{K}_{O}\boldsymbol{e}_{O}) \end{bmatrix}$$

With: $\boldsymbol{\omega}_{d} = \mathbf{R}^{\mathbf{i}}\, \dot{\mathbf{r}}\, (\boldsymbol{t})\, \boldsymbol{\vartheta}\, \mathbf{u}$