Page No.
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# Tutorial - 6

1) let R denote the Relation on the set of order set on +ve int such that $\langle x,y\rangle R \langle u,v\rangle$ iff $xv = yu$ so that R is an Equivalance Relation.

→

Reflexive property:

$$\langle x,y\rangle R \langle x,y\rangle$$
$$\Leftrightarrow \quad xy = yx$$

thus R is Reflexive

→ symmeteric :

$$\langle x,y\rangle R \langle u,v\rangle$$
$$\Leftrightarrow \quad xv = yu \quad \text{--- (1)}$$
$$\Leftrightarrow \quad \langle u,v\rangle R \langle x,y\rangle$$
$$\Leftrightarrow \quad uy = vx \quad \text{--- (2)}$$

(1) = (2)

thus R is symmetric

→ transitive :

$$\langle x,y\rangle R \langle u,v\rangle$$
$$\Leftrightarrow \quad xV = yu \quad \text{--- (1)}$$

$$\langle u,v\rangle R \langle p,q\rangle$$
$$\Leftrightarrow \quad uq = vp$$
$$u = \frac{vp}{q} \quad \text{--- (2)}$$

(2) → (1)

$$xV = y\left(\frac{vp}{q}\right)$$
$$xq = yp$$
$$\Leftrightarrow \langle x,y\rangle R \langle p,q\rangle$$

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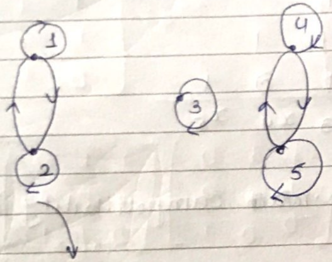
(2) given a set $S = \{1,2,3,4,5\}$, find the equivalence relation on $x$ which generate a partition $\{\{1,2\}, \{3\}, \{4,5\}\}$ also draw the graph of Relation

→



$R = \{\underbrace{\langle 1,1\rangle, \langle 2,2\rangle}_{\text{for reflexive}}, \underbrace{\langle 1,2\rangle, \langle 2,1\rangle}_{\text{for symmetric}}\}$

& indirectly transitivy property is satisfied

$R = \{\langle 3,3\rangle\}$

for only 1 single point this $\langle 3,3\rangle$ satisfy all property

$R = \{\langle 4,4\rangle, \langle 5,5\rangle, \langle 5,4\rangle, \langle 4,5\rangle\}$

same as above point 1 & 2

so $R = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle 1,2\rangle, \langle 2,1\rangle, \langle 3,3\rangle,$
$\langle 4,4\rangle, \langle 5,5\rangle, \langle 5,4\rangle, \langle 4,5\rangle\}$

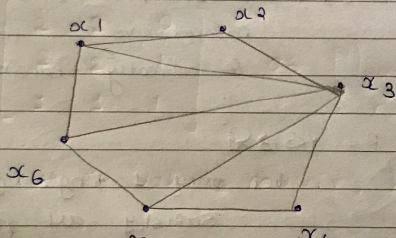
this R is Equivalance Relation of partition $\{\{1,2\}, \{3\}, \{4,5\}\}$

3) let the compatibility Relation on the set $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ be the given by matrix.

| | | | | | |
|---|---|---|---|---|---|
| $x_2$ | 1 | | | | |
| $x_3$ | 1 | 1 | | | |
| $x_4$ | 0 | 0 | 1 | | |
| $x_5$ | 0 | 0 | 1 | 1 | |
| $x_6$ | 1 | 0 | 1 | 0 | 1 |
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |

find the maximum campatibility block of the Relation

matrix → graph




partition in such a way that we get $\cup$ of all partition
union

$\{x_1, x_2, x_3\}$
$\{x_1, x_3, x_6\}$
$\{x_6, x_3, x_5\}$
$\{x_5, x_3, x_4\}$

Maximum compatibility block 1 – $\{x_1, x_2, x_3\}$
2 – $\{x_1, x_3, x_6\}$
3 – $\{x_3, x_5, x_6\}$
4 – $\{x_3, x_4, x_5\}$

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4) Given the Relation Matrix $M_R$ of a Relation R on the set {a, b, c}, find the Relation Relation Matrices $\tilde{R}$, $R^2 = R \circ R$, $R^3 = R \circ R \circ R$ & $R \circ \tilde{R}$

composition ★ Multiplicate

Where $M_R = \begin{array}{c} a \\ b \\ c \end{array}\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$\qquad\qquad\quad a \quad b \quad c$

→

$M_{\tilde{R}}$ → transpose of $M_R$ { row ⇄ col. }

$$M_{\tilde{R}} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\rightarrow M_{R^2} = M_{R \circ R} = M_R \cdot M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+1 & 0+0+1 & 1+0+1 \\ 1+1+0 & 0+1+0 & 1+0+0 \\ 1+1+1 & 0+1+1 & 1+0+1 \end{bmatrix}$$

Ans: $\begin{bmatrix} 2 & 1 & 2 \\ 2 & 1 & 1 \\ 3 & 2 & 2 \end{bmatrix}$

$$\rightarrow M_{R^3} = M_{R \circ R \circ R} = M_R \cdot M_R \cdot M_R = M_{R^2} \cdot M_R$$

$$= \begin{bmatrix} 2 & 1 & 2 \\ 2 & 1 & 1 \\ 3 & 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2+1+2 & \cancel{0+1+2} \; 0+1+2 & 2+0+2 \\ 1+1+1 & \cancel{0+1+1} & 1+0+1 \\ 3+2+2 & \cancel{0+2+2} & 3+0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 & 4 \\ 4 & 2 & 3 \\ 7 & 4 & 5 \end{bmatrix}$$

$$\rightarrow M_{R \circ \tilde{R}} = M_R \cdot M_{\tilde{R}} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix} \qquad = \begin{bmatrix} 1+0+1 & 1+0+0 & 1+0+1 \\ & & \\ & & \end{bmatrix}$

Q-5 two equivalence relation R and S are given by their relation matrix $M_R$ and $M_S$. Show that ROS is not an equivalence relation

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad M_S = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$M_{ROS} = M_R \cdot M_S = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\tilde{M}_{ROS} = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

as $M_{ROS} \neq \tilde{M}_{ROS}$
So not symmetric

$\therefore$ it's not Equivalence matrix, hence proved

Q-6 Draw the Hasse diagrams of the following sets under the partial ordering relation "divides" and indicate those which are totally ordered.

$\{2, 6, 24\}$ $\{3, 5, 15\}$ $\{1, 2, 3, 6, 12\}$
$\{2, 4, 8, 16\}$ $\{3, 9, 27, 54\}$

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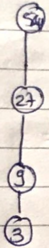
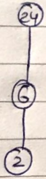
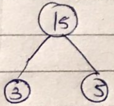
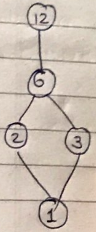
16
8
4
2


54
27
9
3


24
6
2


15
3
5


12
6
2
3
1