Chapter 6
放大器的频率特性
Frequency Response of Amplifiers

中科大微电子学院

黄鲁、程林

教材:模拟CMOS集成电路设计

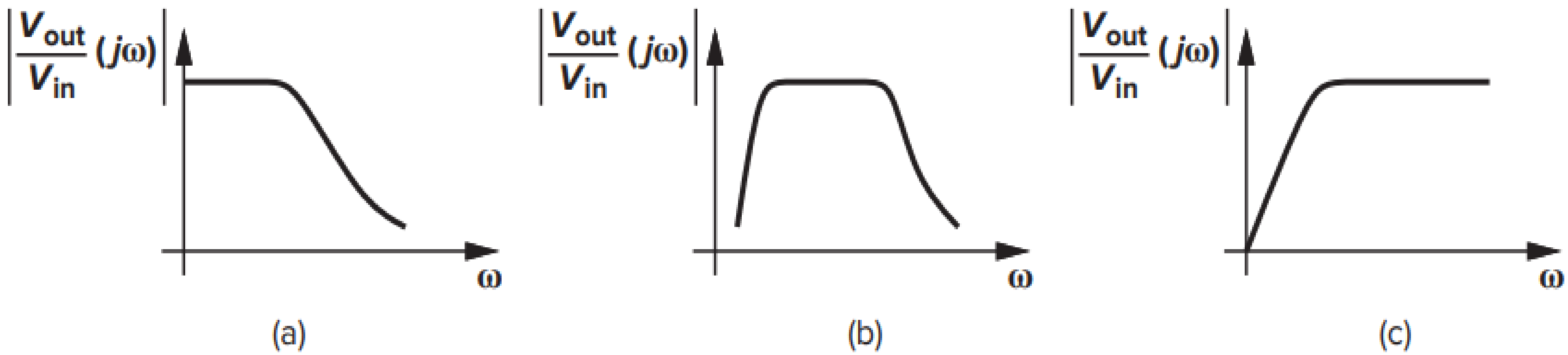
Behzad Razavi

第6章内容

6.1 概述

MOS管寄生电容主要有4个：C_{GS}、C_{GD}、C_{DB}、C_{SB}

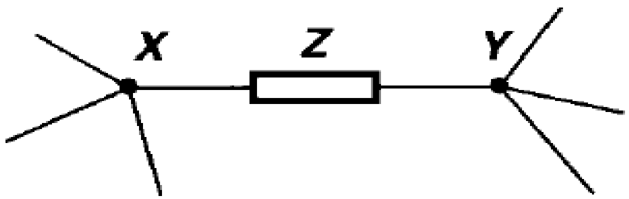
传递函数=与(复)频率相关的增益。



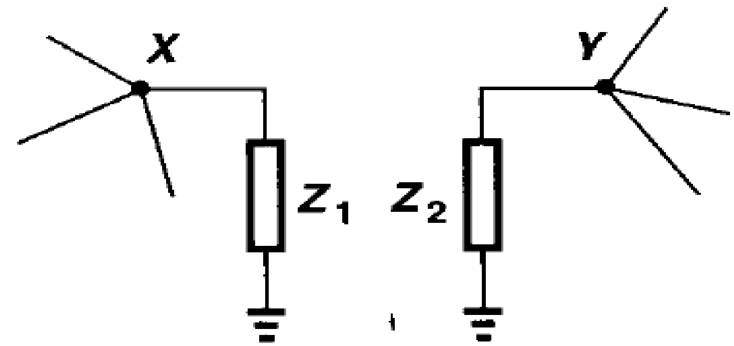
$s = \sigma + j\omega$ 拉氏变换，可解出包含初始过程的瞬态响应

$s = j\omega$ 傅里叶变换，代表稳态响应。频谱指稳态响应

6.1.1 密勒效应 与密勒近似计算



(a)



(b)

输入、输出阻抗？

Miller's theorem

设X和Y之间增益=VY/VX，由另一条支路提供（增益）

$$\frac{V_X - V_Y}{Z} = \frac{V_X}{Z_1}, \qquad \frac{V_Y - V_X}{Z} = \frac{V_Y}{Z_2}$$

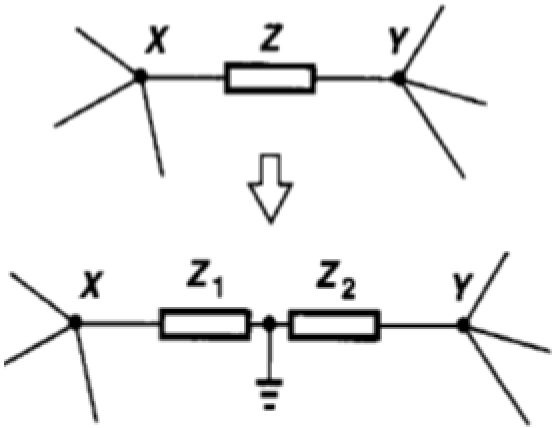
得 $Z_1 = \dfrac{Z}{1 - \dfrac{V_Y}{V_X}}$， $Z_2 = \dfrac{Z}{1 - \dfrac{V_X}{V_Y}}$，注意 $\dfrac{V_Y}{V_X}$ 为增益，正常为负

输入阻抗变小，一般不好

Miller effect (cont.)

于是 $\dfrac{Z_1}{Z} = \dfrac{V_X}{V_X - V_Y}$， $\dfrac{Z_2}{Z} = \dfrac{V_Y}{V_Y - V_X}$

$$\therefore \frac{Z_1}{Z} + \frac{Z_2}{Z} = \frac{V_X}{V_X - V_Y} + \frac{V_Y}{V_Y - V_X} = 1\text{，即} Z_1 + Z_2 = Z$$



推论：

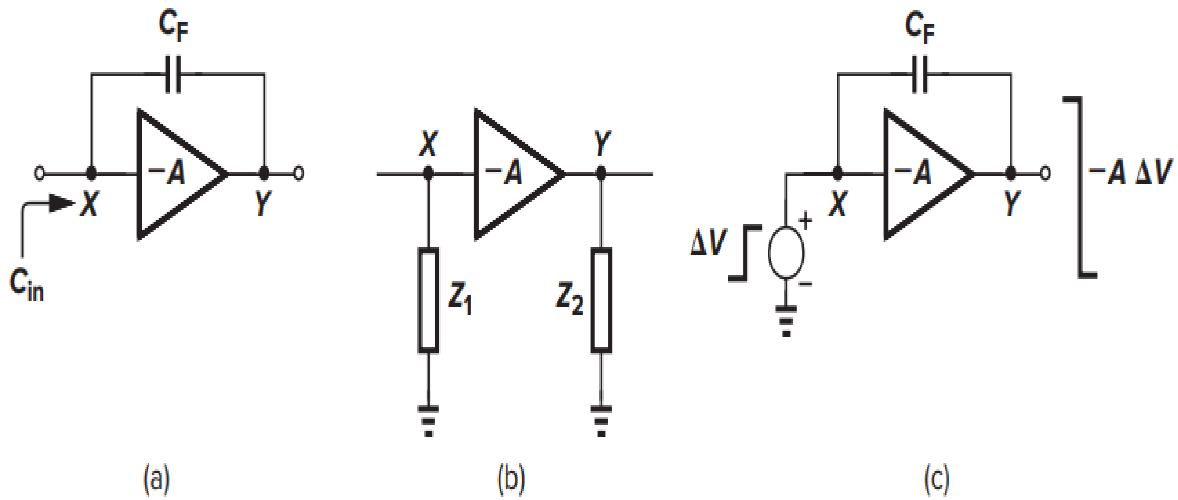
Z1+Z2之间某点可以接地，

则意味 **X与Y反向**

例6.1 计算图6.3(a)输入电容

$$Z=\frac{1}{C_F s}$$

$$\frac{V_Y}{V_X}=-A\text{，}A\text{为正}$$



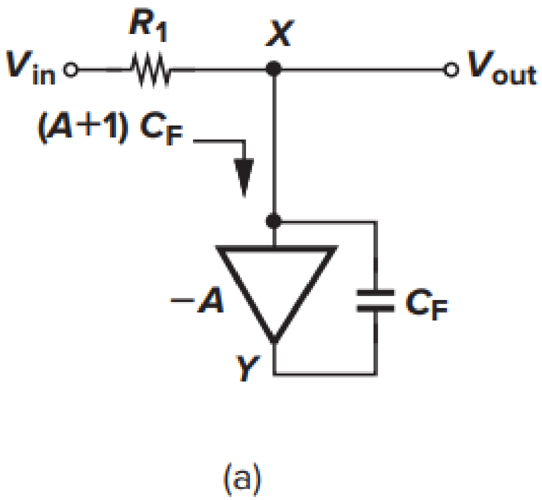
$$Z_1=\frac{Z}{1-\dfrac{V_Y}{V_X}}=\frac{\dfrac{1}{C_F s}}{1+A}=\frac{1}{(1+A)C_F s}$$

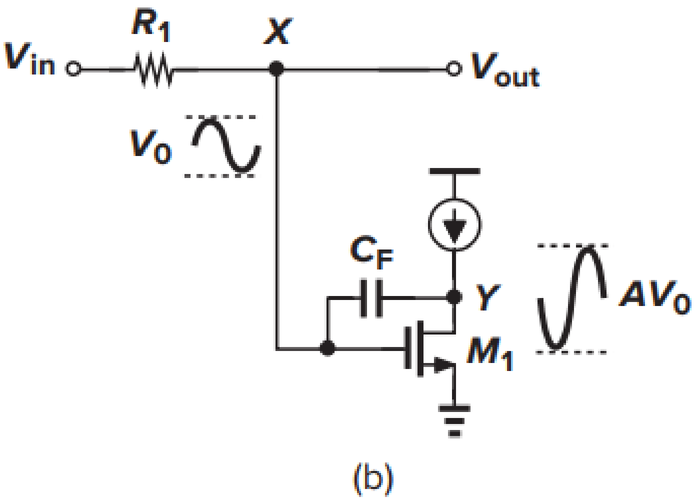
输入电容为CF(1+A)。
密勒效应的物理意义：
放大器的输入阻抗变小！

$$Z_2=\frac{Z}{1-\dfrac{V_X}{V_Y}}=\frac{\dfrac{1}{C_F s}}{1+\dfrac{1}{A}}=\frac{A}{(A+1)C_F s}\approx\frac{1}{C_F s}\text{，}\quad\text{大增益时}$$

直观估计方法：A>>1时，输入点X小信号电压很小

例6.2 利用密勒效应增大低频滤波器电容





设X点电压幅度为V0

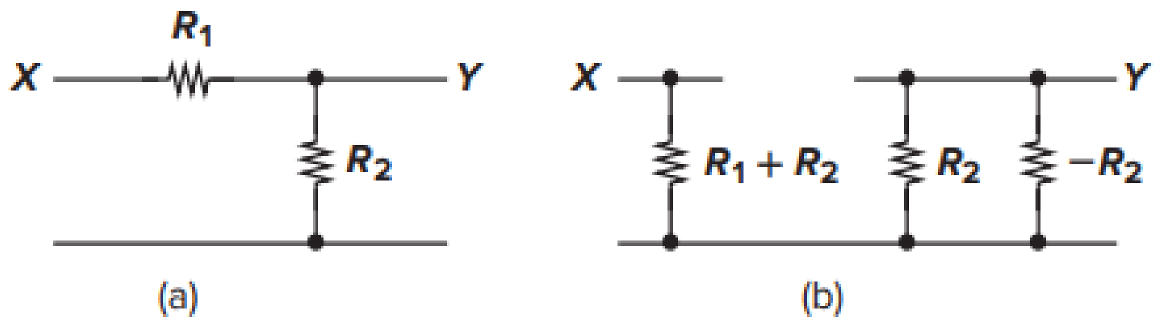
前提：（1）-A放大器输出幅度须满足A*V0，

（2）-A放大器直流输入电平与X点（Vin）相符。

Miller 效应计算输出阻抗的错误情况

如果X和Y点同相(即无另一条反向通路)，Miller 效应不能保证正确计算输出阻抗。

验证例：输入输出同向。拆掉R1



$$Z_1 = \frac{Z}{1 - \frac{V_Y}{V_X}} = \frac{R_1}{1 - \frac{R_2}{R_1 + R_2}} = R_1 + R_2$$

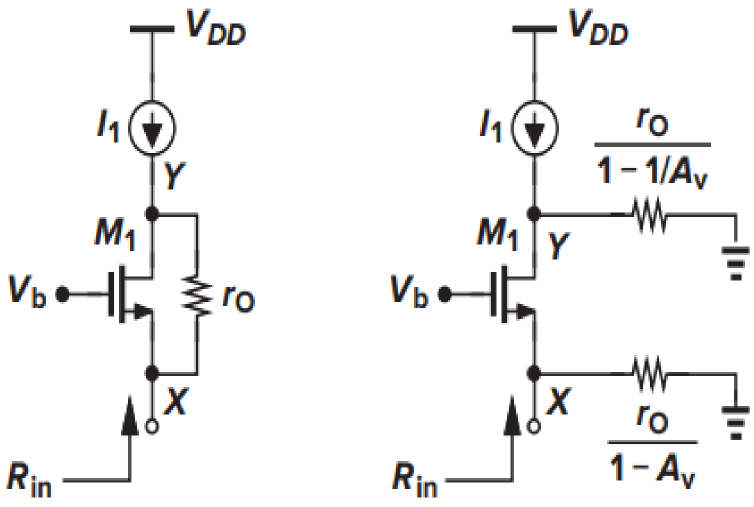
无论输入输出是同相或反相，密勒定律计算输入阻抗正确！

$$Z_2 = \frac{Z}{1 - \frac{V_X}{V_Y}} = \frac{R_1}{1 - \frac{R_1 + R_2}{R_2}} = -R_2$$

输入输出同相时，密勒定律计算输出阻抗可能错误！

应为R1||R2

例6.3 计算输入电阻（输入输出同相）



(a) (b)

$$A_V = \frac{V_{out}}{V_{in}} = \frac{r_o(g_m + g_{mb}) + 1}{r_o + (g_m + g_{mb})r_o R_S + R_S + R_D} R_D$$

式(3.111)

这里：负载$R_D = \infty$，信号源电阻$R_S = 0$

$\therefore\ A_V = r_o(g_m + g_{mb}) + 1$

从源极端向MOS 里看：$R_{in} = \dfrac{V_X}{I_X} = \dfrac{R_D + r_o}{1 + (g_m + g_{mb})r_o}$ （3.116）

用密勒效应计算：

$$R_{in} = \frac{r_O}{1 - [1 + (g_m + g_{mb})r_O]} \Bigg\| \frac{1}{g_m + g_{mb}}$$

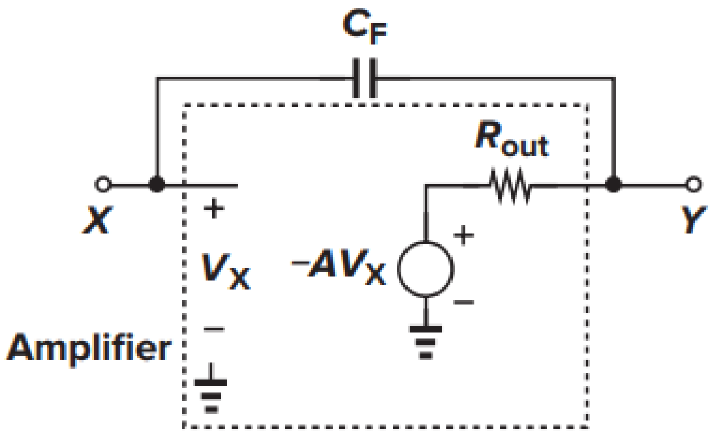
$$= \frac{-1}{g_m + g_{mb}} \Bigg\| \frac{1}{g_m + g_{mb}}$$

$$= \infty$$

与（3.116）计算一致。

实际的输入、输出阻抗与频率有关

密勒定律用于近似计算



高频时 $V_y \neq -AV_x$

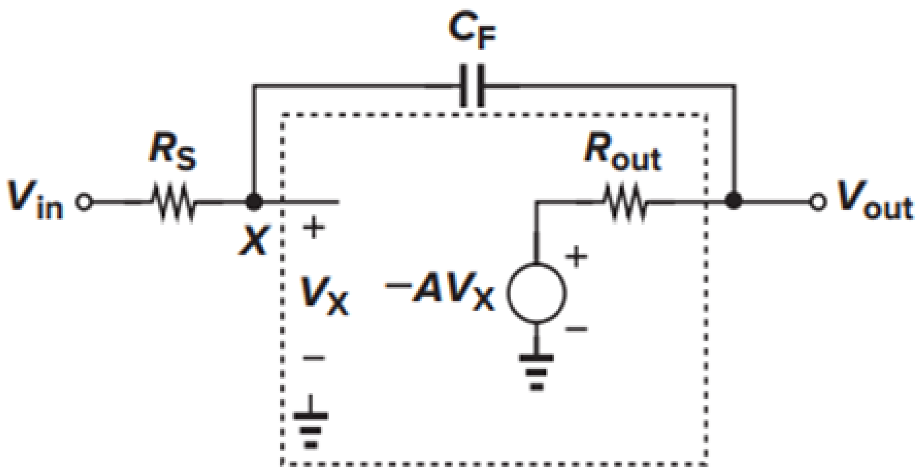
A为低频增益.

X点（交变）电压很小。

输入等效电容　Cx=CF(1+A)仅在低频时有效，
高频时增益幅值变小。

$$V_Y \approx -AV_X \frac{\frac{1}{C_F s}}{R_{out} + \frac{1}{C_F s}} = -AV_X \frac{1}{1 + R_{out} C_F s_X}, \quad \frac{V_Y}{V_X} = -\frac{A}{1 + R_{out} C_F s_X}$$

例6.4 求传递函数



$$\frac{V_{in}-V_X}{R_S}=\left(V_X-V_{out}\right)C_F s$$

即 $$\frac{V_{in}}{R_S}+C_F sV_{out}=\left(\frac{1}{R_S}+C_F s\right)V_X$$

$$\Rightarrow V_X=\frac{V_{in}+C_F R_S sV_{out}}{1+C_F R_S s}$$

$$\frac{V_{in}-V_X}{R_S}R_{out}-AV_X=V_{out} \qquad \Rightarrow V_X=\frac{V_{in}R_{out}-V_{out}R_S}{R_{out}+AR_S}$$

$$\therefore\left(R_{out}+AR_S\right)\left(V_{in}+C_F R_S sV_{out}\right)=\left(1+C_F R_S s\right)\left(V_{in}R_{out}-V_{out}R_S\right)$$

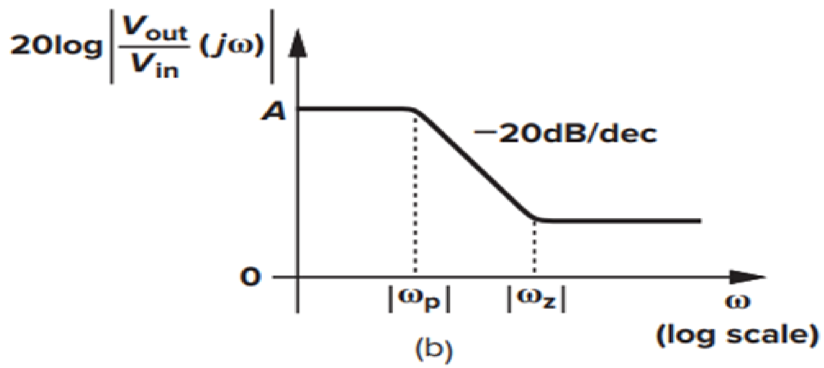
两边除以$R_S V_{in}$，整理得$H(s)=\frac{V_{out}}{V_{in}}(s)=\frac{C_F R_{out}s-A}{[(A+1)R_S+R_{out}]C_F s+1}$

例6.4 求传递函数（续）

$$\frac{V_{out}}{V_{in}}(s)=\frac{C_F R_{out}s-A}{[(A+1)R_S+R_{out}]C_F s+1}=-A\frac{1-\dfrac{s}{\omega_z}}{1+\dfrac{s}{\omega_p}}\text{，1极点1零点}$$

$$\text{（正）零点}\omega_Z=\frac{A}{C_F R_{out}}\text{，（负）极点}\omega_p=\frac{1}{[(A+1)R_S+R_{out}]C_F}$$

$$1\pm\text{时间常数}\times s=1\pm\frac{s}{\text{零极点角频率}}$$

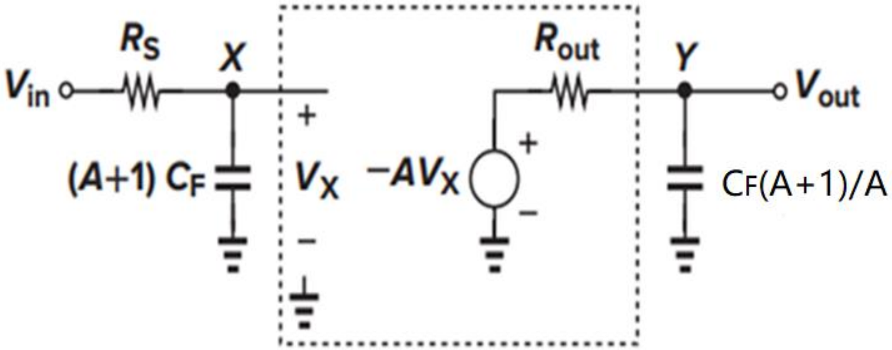


物理意义：s=jω

零点ω_Z表示该频率处幅度比低频增大$\sqrt{2}$，即+3dB

极点ω_p表示该频率处幅度比低频降低$\sqrt{2}$，即−3dB，相位延迟45°

例6.4（续）密勒定律方法近似求传递函数



$$\frac{V_X}{V_{in}} = \frac{\dfrac{1}{(1+A)C_F s}}{\dfrac{1}{(1+A)C_F s} + R_S} = \frac{1}{(1+A)R_S C_F s + 1}$$

$$\frac{V_{out}}{V_X}(s) = -A\frac{\dfrac{A}{(1+A)C_F s}}{\dfrac{A}{(1+A)C_F s} + R_{out}} = \frac{-A}{1 + \dfrac{(1+A)R_{out}C_F s}{A}}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{V_X}{V_{in}}\frac{V_{out}}{V_X} = \frac{-A}{[(1+A)R_S C_F s + 1](1 + \dfrac{(1+A)R_{out}C_F s}{A})}$$

密勒定律计算的传递函数（2极点，无零点）明显有小错误！

例6.4（续）密勒定律求传递函数的问题

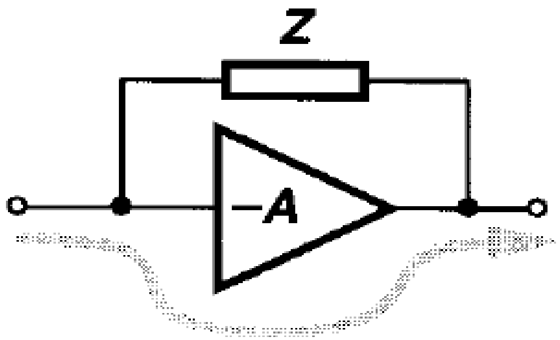
解电路方程得到 $\frac{V_{out}}{V_{in}}(s) = \frac{C_F R_{out} s - A}{[(A+1)R_S + R_{out}]C_F s + 1} = -A\frac{1-\frac{s}{\omega_z}}{1+\frac{s}{\omega_p}}$

密勒定律得到 $\frac{V_{out}}{V_{in}}(s) = \frac{-A}{[(1+A)R_S C_F s + 1](1 + \frac{(1+A)R_{out}C_F s}{A})}$

$= \frac{-A}{(1+\frac{s}{\omega_{p1}})(1+\frac{s}{\omega_{p2}})}$ 丢失零点，增加虚假极点

Rout较小时（相比ARs），两者基本相同。
密勒定律常用于手工估算输入极点（或阻抗）

总结：Miller theorem局限性



Main Signal Path

（1）一般手工计算使用的**Vy/Vx**是低频增益，与高频不符。故称密勒近似。

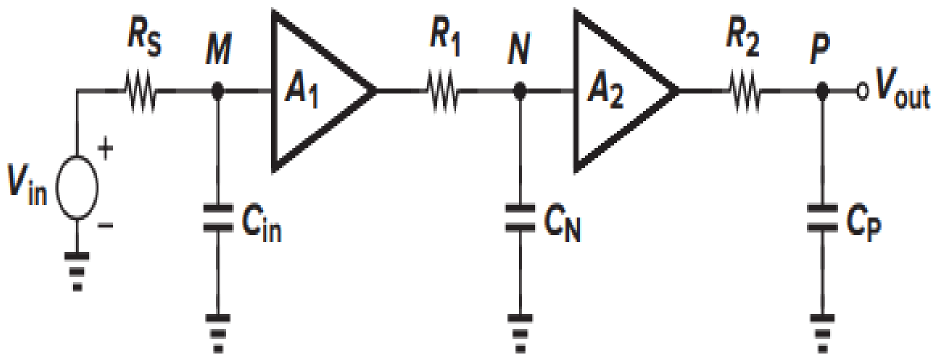
（2）应用米勒定律，输入阻抗计算正确，但输出阻抗以及增益计算近似正确的前提是：输入与输出反相。

（3）应用米勒效应进行电路手工简化计算的局限性：

1. 丢掉传递函数零点。所谓零点是指该频率下两条支路输出之和为“0”，实为 $1 \pm j$

2. 可能会多出虚假极点（一般会因频率高而忽略其影响）

6.1.2 极点与结点的关联（前向结构）



极点：传递函数分母="0"的频率，

结点或节点：信号通道电路连接点。

复频域$s = \sigma + j\omega$　（$\sigma > 0$发散）

$s = j\omega$得到稳态频率响应

图中：A为低频增益。

Rs为前级输出阻抗，R1和R2分别为放大器A1和A2的输出阻抗。

$$\frac{V_{out}}{V_{in}}(s) = \frac{\frac{1}{C_{in}s}}{\frac{1}{C_{in}s}+R_s} * A_1 * \frac{\frac{1}{C_N s}}{\frac{1}{C_N s}+R_1} * A_2 * \frac{\frac{1}{C_P s}}{\frac{1}{C_P s}+R_2} = \frac{A_1}{1+R_s C_{in} s} \bullet \frac{A_2}{1+R_1 C_N s} \bullet \frac{1}{1+R_2 C_P s}$$

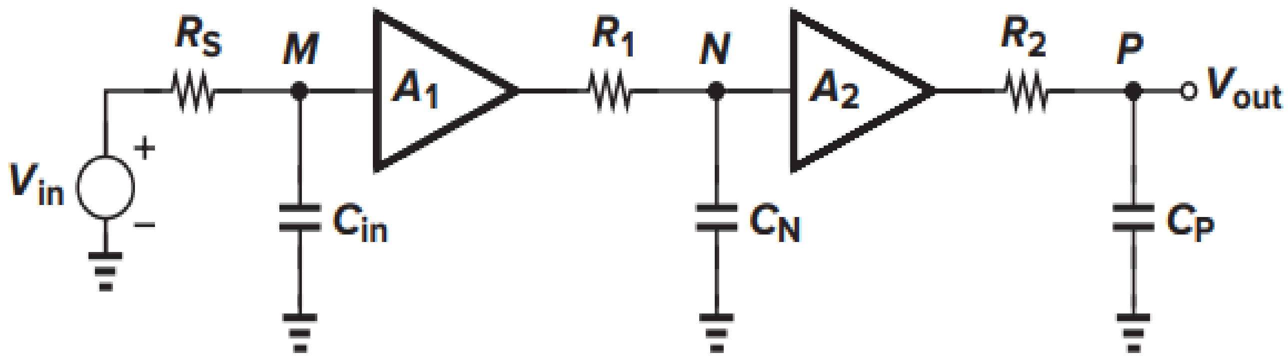
$$= \frac{A_1}{1+s/\omega_{in}} \bullet \frac{A_2}{1+s/\omega_1} \bullet \frac{1}{1+s/\omega_2} = \frac{\text{低频增益}}{\text{极点因子}\left(1+\frac{s}{\omega_p}\right)}$$

$\omega_j = \frac{1}{\tau_j} = \frac{1}{R_j C_j}$ ，　τ_j称为j节点的时间常数，s的系数

极点（角频率）表示比低频时下降3dB

前向结构电路节点的时间常数与极点

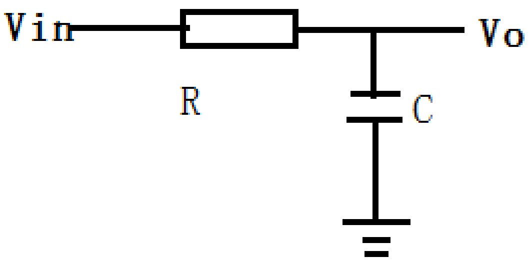
- 在前向结构放大器电路中，每个结点j的时间常数（极点）：
  =节点到地总电阻*节点到地总电容，
  其倒数对应各极点的角频率。
  即信号通道上每个结点阻容值乘积（时间常数）之倒数贡献一个极点。



- 在仅有电容反馈情况下，可采用密勒近似计算方法（求输入阻抗和输入节点关联的极点）。上述结论可用于手工计算能等效为前向结构的负反馈放大器极点。但如有RC反馈回路，则结论不成立。

电路节点的时间常数与极点

（1）



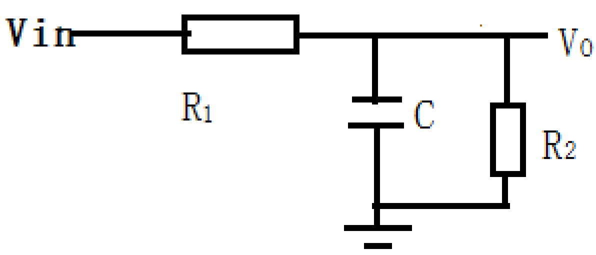
记 $\overset{*}{C} = 1/sC$

$$A = \frac{V_o}{V_{in}} = \frac{1/sC}{R + 1/sC} \times \frac{R}{R} = \frac{1/sC \,||\, R}{R}$$

$$= \frac{\overset{*}{C} \,||\, R}{R} \quad （记\ \overset{*}{C} = 1/sC） = \frac{1}{1 + sCR}$$

时间常数 $\tau = RC$， s 的系数

（2）



$$A = \frac{V_o}{V_{in}} = \frac{\overset{*}{C} \,||\, R_2}{R_1 + \overset{*}{C} \,||\, R_2} \times \frac{R_1}{R_1} = \frac{\overset{*}{C} \,||\, R_2 \,||\, R_1}{R_1} \times \frac{R_2 \,||\, R_1}{R_2 \,||\, R_1}$$

$$= \frac{1}{1 + sC(R_1 \,||\, R_2)} \times \frac{R_2 \,||\, R_1}{R_1} = \frac{1}{1 + sC(R_1 \,||\, R_2)} \times \frac{R_2}{R_1 + R_2}$$

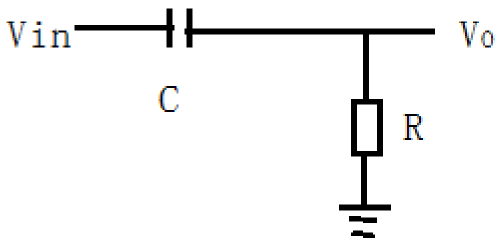
Vo节点到地总电阻为R1||R2

低频 $A_0 = \frac{R_2}{R_1 + R_2}$

极点时间常数=节点到地总电阻*总电容

电路节点的时间常数与零极点

（3）

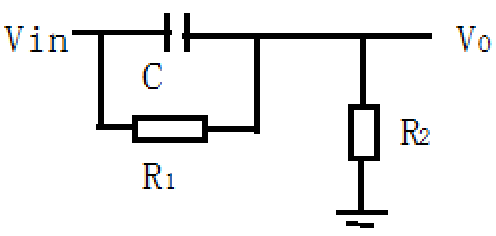


$$A=\frac{V_o}{V_{in}}=\frac{R}{R+1/sC}=\frac{sCR}{1+sCR}$$（真零点）

极点（分母=0时角频率）结论仍然正确，高频时$A_\infty=1$

（4）

记 $\overset{*}{C}=1/sC$



$$A=\frac{V_o}{V_{in}}=\frac{R_2}{R_2+\overset{*}{C}\,||\,R_1}\times\frac{\overset{*}{C}\,||\,R_1}{\overset{*}{C}\,||\,R_1}$$

$$=\frac{\overset{*}{C}\,||\,R_1\,||\,R_2}{\overset{*}{C}\,||\,R_1}\times\frac{R_1\,||\,R_2}{R_1\,||\,R_2}\times\frac{R_1}{R_1}$$

$$=\frac{1+sCR_1}{1+sC(R_1\,||\,R_2)}\times\frac{R_1\,||\,R_2}{R_1}$$

$$=\frac{1+sCR_1}{1+sC(R_1\,||\,R_2)}\times\frac{R_2}{R_1+R_2}$$

输入输出RC 2条通路，有零点

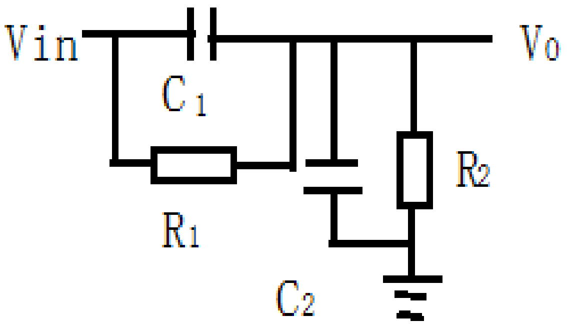
重复利用 $\frac{\overset{*}{C}\,||\,R}{R}=\frac{1}{1+sCR}$

低频 $A_0=\frac{R_2}{R_1+R_2}$，

极高频 $A_\infty=\frac{R_1}{(R_1\,||\,R_2)}\times\frac{R_2}{R_1+R_2}=1$

零点表示输入至输出有R_1和C两条通路

（5）



记 $\overset{*}{C} = 1/sC$

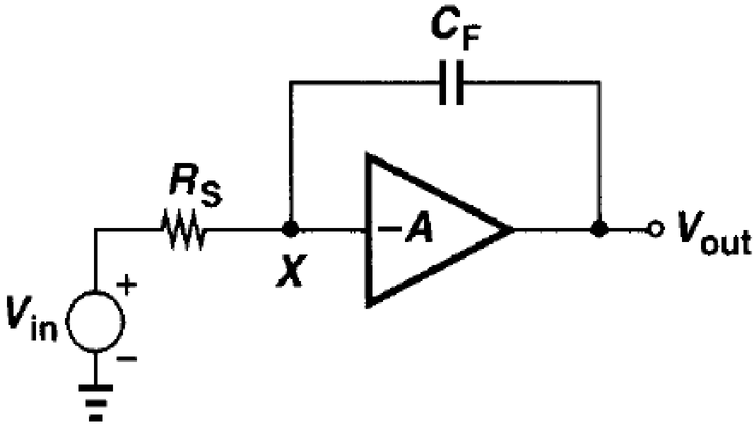
- 极点时间常数 =节点到地总电阻*节点到地总电容
- 零点时间常数=输入输出之间的R1*C1，两条支路电流相等，令Vo=0

$$A = \frac{V_o}{V_{in}} = \frac{\overset{*}{C_2} \,||\, R_2}{\overset{*}{C_1} \,||\, R_1 + \overset{*}{C_2} \,||\, R_2} \times \frac{\overset{*}{C_1} \,||\, R_1}{\overset{*}{C_1} \,||\, R_1} = \frac{(\overset{*}{C_2} \,||\, R_2) \,||\, (\overset{*}{C_1} \,||\, R_1)}{\overset{*}{C_1} \,||\, R_1} \times \frac{R_2 \,||\, R_1}{R_2 \,||\, R_1}$$

$$= \frac{1}{1 + s\,(C_1 + C_2)(R_1 \,||\, R_2)} \times \frac{R_2 \,||\, R_1}{\overset{*}{C_1} \,||\, R_1} \times \frac{R_1}{R_1} = \frac{1 + sC_1R_1}{1 + s\,(C_1 + C_2)(R_1 \,||\, R_2)} \times \frac{R_2}{R_1 + R_2}$$

低频 $A_0 = \frac{R_2}{R_1 + R_2}$，极高频 $A_\infty = \frac{C_1R_1}{(C_1 + C_2)(R_1 \,||\, R_2)} \times \frac{R_2}{R_1 + R_2} = \frac{C_1}{C_1 + C_2}$

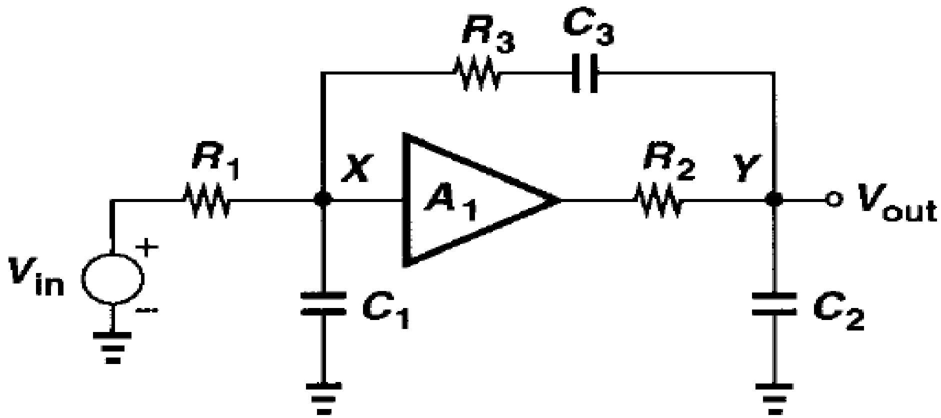
结点与极点关联的电路局限性



输入极点ω_{in}

$$=\omega_{px}=\frac{1}{R_S(1+A)C_F} \quad （弧度/秒，rad/s）$$

适用于前馈电路，或单一性质反馈器件



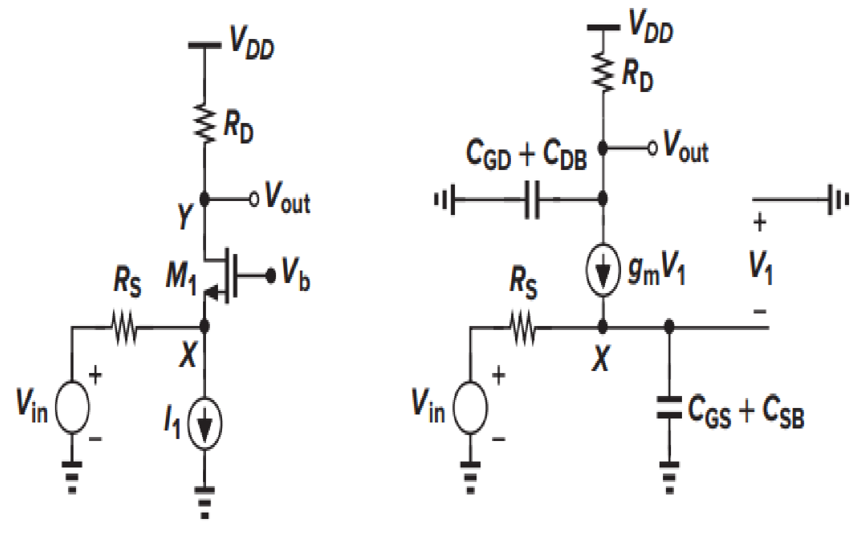
- X超过1个极点。
- 每个结点贡献一个极点的结论不成立

Example 6.5

Neglecting channel-length modulation. Calculate the transfer function of common-gate stage shown in fig.6.12

$$\omega_{in} = \omega_X = \frac{1}{(C_{GS} + C_{SB})\left(R_S \,||\, \frac{1}{g_m + g_{mb}}\right)}$$

$$\omega_{out} = \frac{1}{(C_{DG} + C_{DB})R_D}$$



(a) (b)

$$H(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)} = \frac{\frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S}}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)}$$

仅线性时不变系统

MOS管的极限频率

特征频率 f_T ： 来源于三极管截止频率 f_T 的定义，

即输出电流=输入电流时的频率。MOS设计时可用于参考。

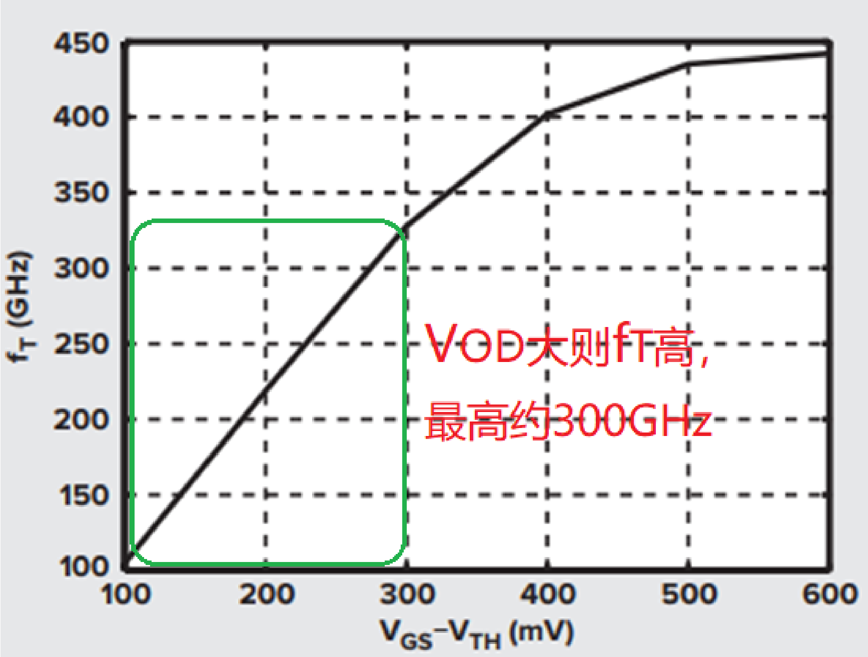
令MOS的 $I_{in} = I_{out}$， 即 $\omega_T C_{GS} V_{in} = g_m V_{in} \Rightarrow f_T = \dfrac{g_m}{2\pi C_{GS}}$

由工艺最小尺寸MOS, 得到工艺特征频率

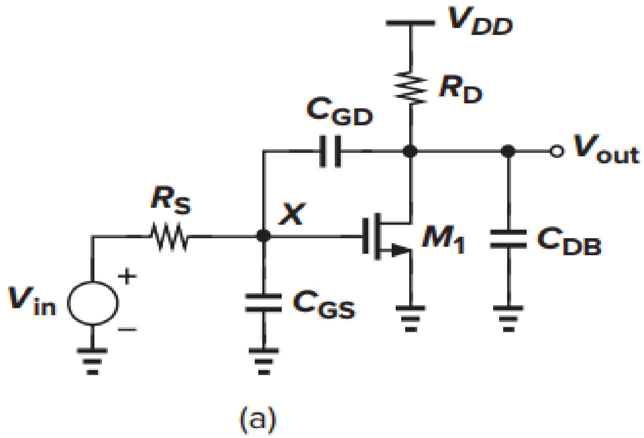
$$f_T = \frac{g_m}{2\pi C_{GS}} \propto \frac{V_{GS} - V_{TH}}{C_{GS}}$$

$$\text{或} \propto \sqrt{\frac{I_D}{WL}}$$

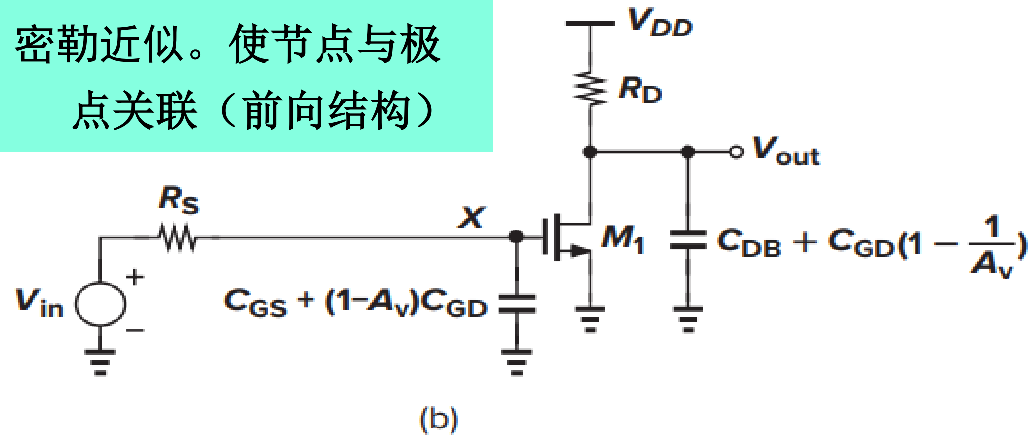
低电压时（VOD小）fT减小



6.2 共源级的频率特性



密勒近似。使节点与极点关联（前向结构）



低频增益 $A_0 \approx -g_m R_D$　　　输出节点：$C_{DB} + (1 - A_V^{-1})C_{GD} \approx C_{DB} + C_{GD}$

$$\frac{V_{GS}}{V_{in}} = \frac{\dfrac{1}{s[C_{GS} + (1+g_m R_D)C_{GD}]}}{R_S + \dfrac{1}{s[C_{GS} + (1+g_m R_D)C_{GD}]}} = \frac{1}{1+sR_S[C_{GS} + (1+g_m R_D)C_{GD}]} = \frac{1}{1+\dfrac{s}{\omega_{in}}}$$

V_{out}变小：
$$\frac{g_m\left(R_D \,||\, \dfrac{1}{s(C_{DB}+C_{GD})}\right)}{g_m R_D} = \frac{1}{1+sR_D(C_{DB}+C_{GD})} = \frac{1}{1+\dfrac{s}{\omega_{out}}}$$

采用密勒近似得到的传输函数估算

$$\frac{V_{out}}{V_{in}}(s)=\frac{-g_m R_D}{(1+\frac{s}{\omega_{in}})(1+\frac{s}{\omega_{out}})}$$

Miller定律截断了输入输出前向通路,导致零点丢失，仅用作极点的近似计算.

极点的物理意义：

（1）带宽：最低频极点（主极点）一般为3dB带宽截止频率。

（2）与负反馈系统稳定性密切相关（也与零点相关）。

多极点（级联）电路的总带宽变窄：

低通（高频截止）滤波器：$\frac{1}{\omega_{total}^2}\approx\frac{1}{\omega_1^2}+\frac{1}{\omega_2^2}+\ldots+\frac{1}{\omega_n^2}$

直接分析Obtain the exact transfer function

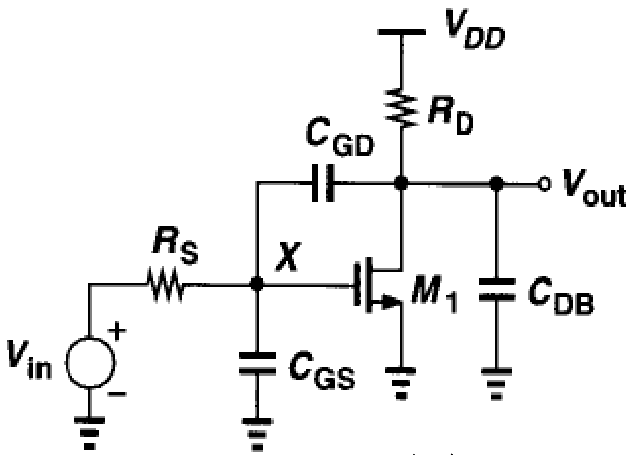


图6.13

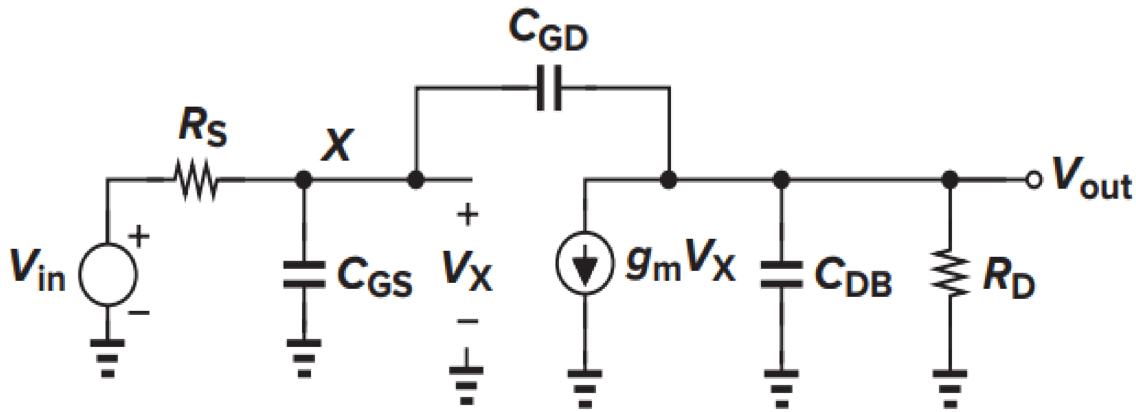


图6.15

$$\frac{V_X - V_{in}}{R_S} + V_X C_{GS} s + (V_X - V_{out}) C_{GD} s = \frac{-V_{in}}{R_S} - V_{out} C_{GD} s + V_X\left(\frac{1}{R_S} + C_{GS} s + C_{GD} s\right) = 0$$

$$(V_{out} - V_X) C_{GD} s + g_m V_X + V_{out}\left(\frac{1}{R_D} + C_{DB} s\right) = 0 \quad (6.27)$$

$$\Rightarrow \quad V_X = -\frac{\frac{1}{R_D} + C_{DB} s + C_{GD} s}{g_m - C_{GD} s} V_{out}$$

Obtain the exact transfer function (cont.)

$$\frac{-V_{in}}{R_S} - V_{out}C_{GD}s - \frac{\frac{1}{R_D} + C_{DB}s + C_{GD}s}{g_m - C_{GD}s} V_{out}\left(\frac{1}{R_S} + C_{GS}s + C_{GD}s\right) = 0$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{-1/R_S}{C_{GD}s + \dfrac{\frac{1}{R_D} + C_{DB}s + C_{GD}s}{g_m - C_{GD}s}\left(\frac{1}{R_S} + C_{GS}s + C_{GD}s\right)}$$

$$= \frac{-(g_m - C_{GD}s)R_D}{R_S R_D C_{GD}s(g_m - C_{GD}s) + [1 + R_D(C_{DB} + C_{GD})s][1 + R_S(C_{GS} + C_{GD})s]}$$

$$= \frac{-(g_m - C_{GD}s)R_D}{R_S R_D \xi s^2 + [R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{DB} + C_{GD})]s + 1}$$

$\xi = C_{GS}C_{DB} + C_{GS}C_{GD} + C_{GD}C_{DB}$ 3电容两两乘积之和，仅2电容独立

（6.30）

Obtain the exact transfer function (cont.)

$$\frac{V_{out}}{V_{in}}(s)$$

$$= \frac{-(g_m - C_{GD}s)R_D}{R_S R_D \xi s^2 + [R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{DB}+C_{GD})]s + 1}$$

$$= \frac{-g_m R_D\left(1-\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)} = A_0 \frac{1-\frac{s}{\omega_z}}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)} \quad (6.30)$$

2极点，1零点

设主极点ω_{p1} << 次极点ω_{p2}

$$\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right) = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)s + 1 \approx \frac{s^2}{\omega_{p1}\omega_{p2}} + \frac{s}{\omega_{p1}} + 1$$

直观方法（密勒定律）可简单估算极点

直接分析得到：$\omega_{p1} \approx \dfrac{1}{\mathrm{R_S}(1+g_m R_D)C_{GD} + \mathrm{R_S} C_{GS} + R_D(C_{DB}+C_{GD})}$

密勒近似得到：$\omega_{in} \approx \dfrac{1}{\mathrm{R_S}[(1+g_m R_D)C_{GD} + C_{GS}]}$

输入阻抗与输出有关时，节点与极点难以关联

直接分析得到：$\omega_{p2} = \dfrac{1}{\omega_{p1}\mathrm{R_S} R_D \xi}$

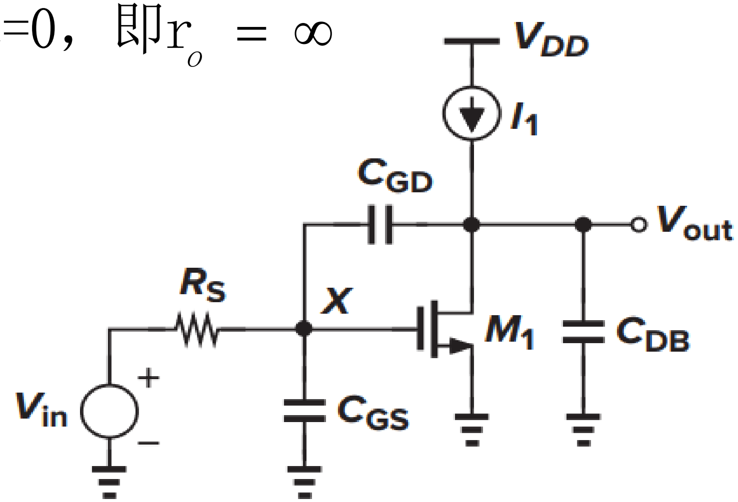
$$\approx \frac{\mathrm{R_S}(1+g_m R_D)C_{GD} + \mathrm{R_S}C_{GS} + R_D(C_{DB}+C_{GD})}{\mathrm{R_S}R_D(C_{GS}C_{DB} + C_{GS}C_{GD} + C_{GD}C_{DB})} \qquad (C_{GD} << C_{GS})$$

若$\mathrm{R_S}C_{GS}$很大或R_D很小（负载阻抗小）时$\omega_{p2} \Rightarrow \dfrac{1}{\mathrm{R_D}(C_{GD}+C_{DB}]}$

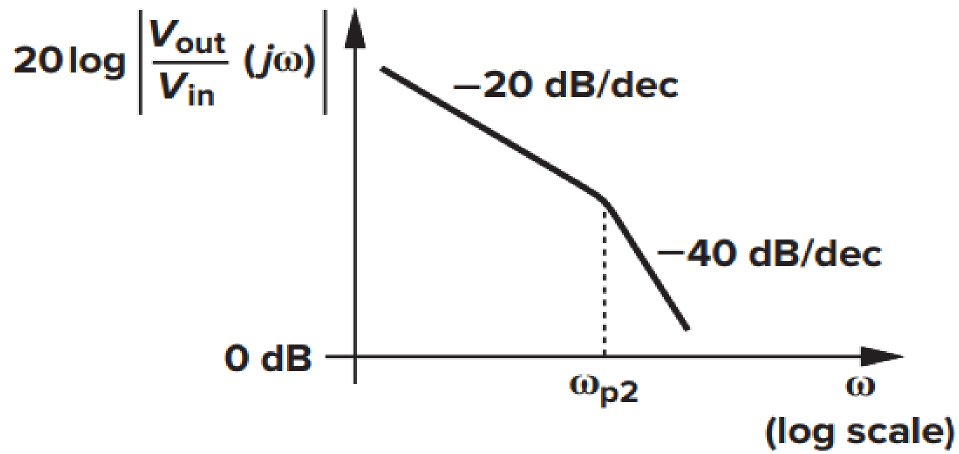
密勒近似得到：$\omega_{out} \approx \dfrac{1}{\mathrm{R_D}(C_{GD}+C_{DB}]}$ 次极点估算误差较大

例6.7 计算传递函数

设λ=0，即$r_o = \infty$



(a)



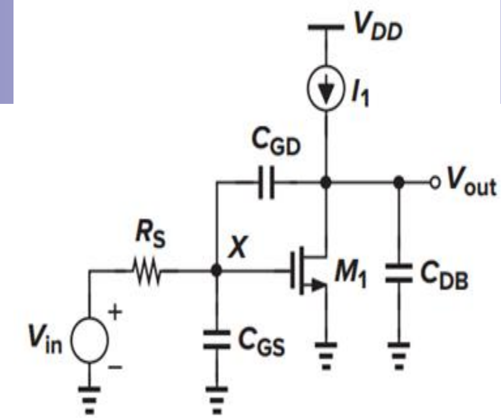
(b)

由（6. 30）代入R_D=∞，∴ $\dfrac{V_{out}}{V_{in}}(s)$

$$= \frac{-(g_m - C_{GD}s)R_D}{R_S R_D \xi s^2 + [R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{DB} + C_{GD})]s + 1}$$

$$= \frac{-(g_m - C_{GD}s)}{R_S \xi s^2 + [R_S g_m C_{GD} + (C_{DB} + C_{GD})]s} = \frac{-(g_m - C_{GD}s)}{s[R_S \xi s + g_m R_S C_{GD} + (C_{DB} + C_{GD})]}$$

例6.7 计算传递函数（续）



$$\frac{V_{out}}{V_{in}}(s)=\frac{-(g_m-C_{GD}s)}{s[R_S\xi s+g_mR_SC_{GD}+(C_{DB}+C_{GD})]}$$

$$=\frac{-g_m\left(1-\frac{s}{g_m/C_{GD}}\right)}{s\left[1+\frac{s}{\frac{g_mR_SC_{GD}+(C_{DB}+C_{GD})}{R_S\xi}}\right][g_mR_SC_{GD}+(C_{DB}+C_{GD})]}$$

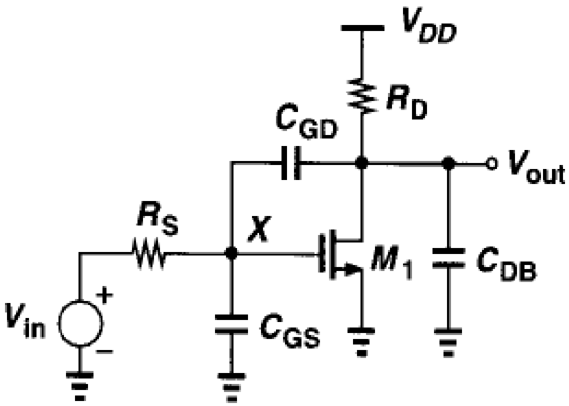
$$\omega_{p2}=\frac{g_mR_SC_{GD}+(C_{DB}+C_{GD})}{R_S\xi}=\frac{(1+g_mR_S)C_{GD}+C_{DB}}{R_S(C_{GS}C_{DB}+C_{GS}C_{GD}+C_{GD}C_{DB})}$$

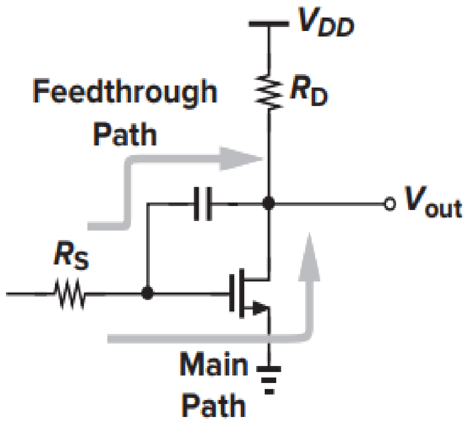
若C_{DB}很大(假设，实际不可能)，$\omega_{p2}\approx\frac{1}{R_S(C_{GS}+C_{GD})}=\omega_X=\omega_{in}$

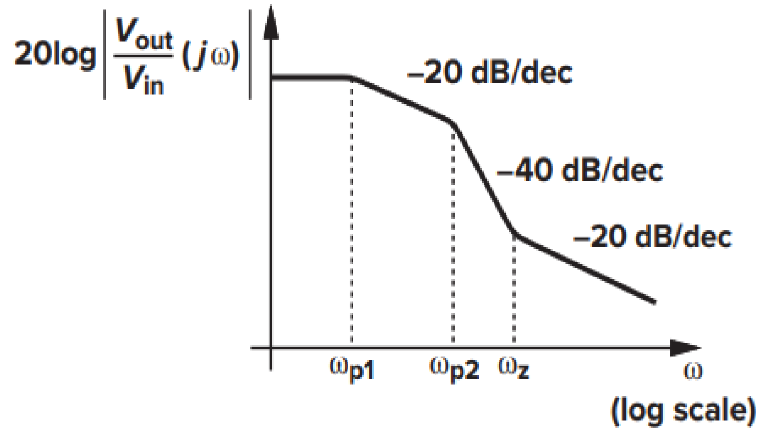
$s=0$时直流增益无穷大，是由输出节点的理想电流源产生。

C_{DB}很大使得高频时输出近似为0，$C_{GS}+C_{GD}$并联，密勒效应消失

零点（s域）的产生



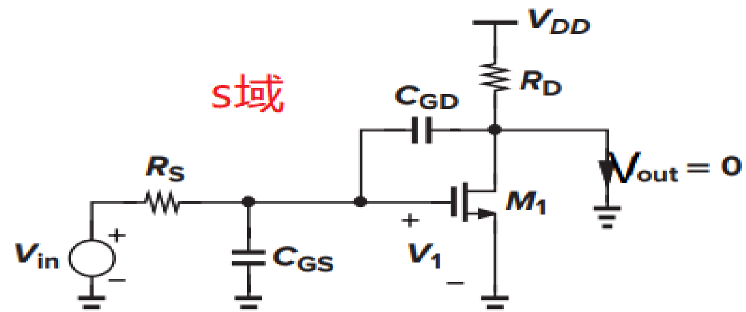




$$\frac{V_{out}}{V_{in}}(s) = \frac{-(g_m - C_{GD}s)R_D}{R_S R_D \xi s^2 + [R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{DB}+C_{GD})]s + 1}$$

$$令\ \frac{V_{out}}{V_{in}}(s) = 0，则\ (g_m - C_{GD}s) = g_m\left(1 - \frac{s}{s_Z}\right) = 0,\ s_Z = \frac{g_m}{C_{GD}}$$

$g_m - C_{GD}s = 0$可理解为MOS 输出接地（交流短路）时 V_{in}通过g_m 和C_{GD} 的电流相等(输出汇合点相减,-的来源)

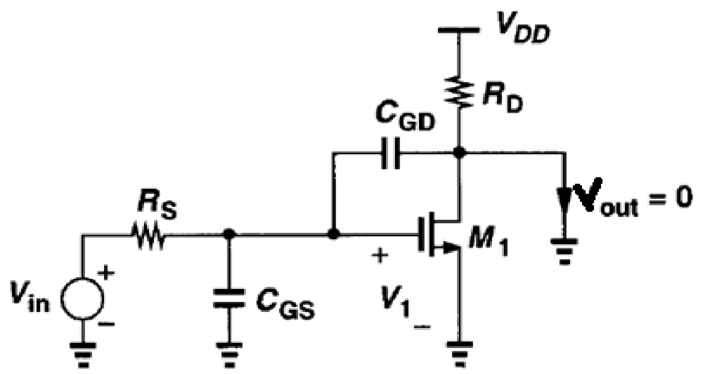


Calculation of the zero

稳态响应s=jω, $g_m - C_{GD}s = g_m - j\omega C_{GD} = g_m\left(1 - j\frac{\omega}{g_m/C_{GD}}\right) = 0$

$\omega=\omega_Z=g_m/C_{GD}$ 时，上式=$g_m(1-j)$实部（阻性）和虚部（容性）电流相等

大于零点频率后，在输出负载上，由输入输出之间路径传输的容性虚信号将超过阻性实信号。



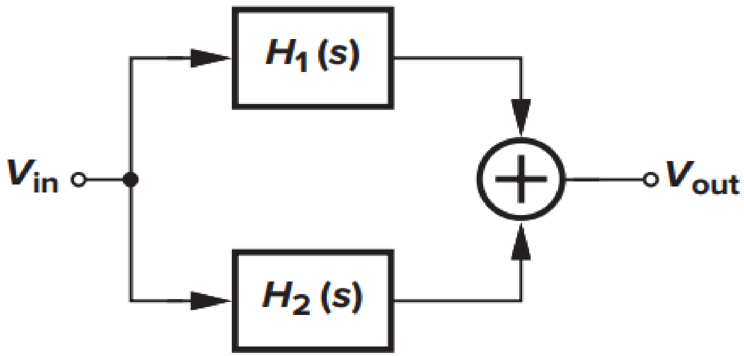
零点计算：令输出短路Vout=0，
得到阻容两支路相同分量：

$$V_1 C_{GD}s_Z = g_m V_1, \Rightarrow s_Z = \frac{g_m}{C_{GD}}$$

s_Z很大，即零点频率很高

零点意义：输入输出有阻容两条支路，在零点频率时传输的电流幅度相等，即阻容两条支路阻抗的模相同。零点频率上总输出电流的模增大 $|1 \pm j| = \sqrt{2}$倍

两个一阶低通电路并联会产生零点



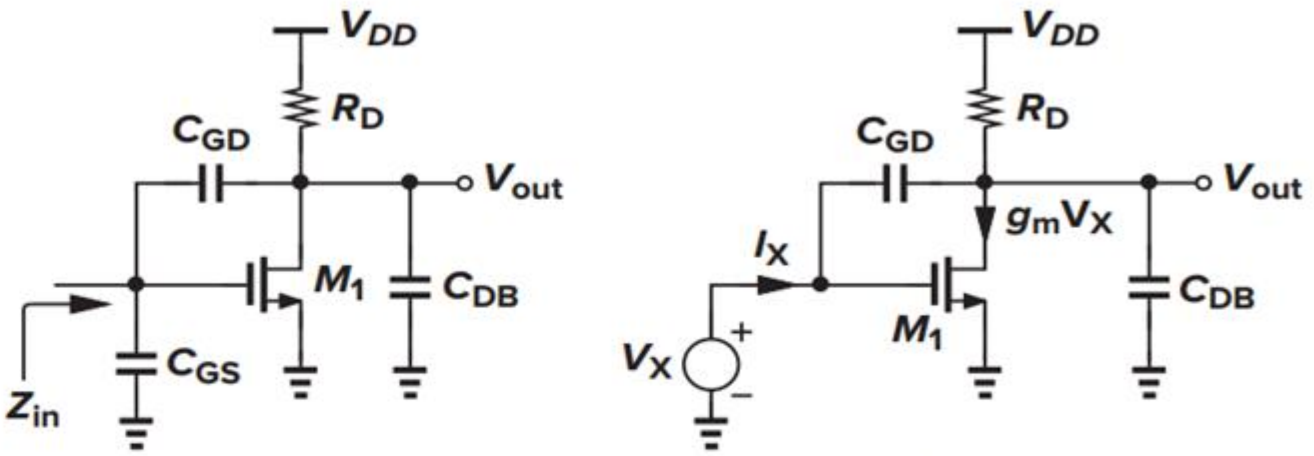
$$H_1(s)=\frac{A_1}{1+\dfrac{s}{\omega_{p1}}},\quad H_2(s)=\frac{A_2}{1+\dfrac{s}{\omega_{p2}}}$$

$$\frac{V_{out}}{V_{in}}(s)=H_1(s)+H_2(s)=\frac{A_1}{1+\dfrac{s}{\omega_{p1}}}+\frac{A_2}{1+\dfrac{s}{\omega_{p2}}}$$

$$=\frac{\left(\dfrac{A_1}{\omega_{p2}}+\dfrac{A_2}{\omega_{p1}}\right)s+A_1+A_2}{\left(1+\dfrac{s}{\omega_{p1}}\right)\left(1+\dfrac{s}{\omega_{p2}}\right)}=(A_1+A_2)\frac{1+\dfrac{s}{\omega_Z}}{\left(1+\dfrac{s}{\omega_{p1}}\right)\left(1+\dfrac{s}{\omega_{p2}}\right)}$$

式中 $$\omega_Z=\frac{A_1+A_2}{\dfrac{A_1}{\omega_{p2}}+\dfrac{A_2}{\omega_{p1}}}=\omega_{p1}\frac{1+A_1/A_2}{1+\dfrac{A_1\omega_{p1}}{A_2\omega_{p2}}}>\omega_{p1}$$

Calculation of input impedance of CS stage



(a) (b)

精确计算技巧：
先不计CGS

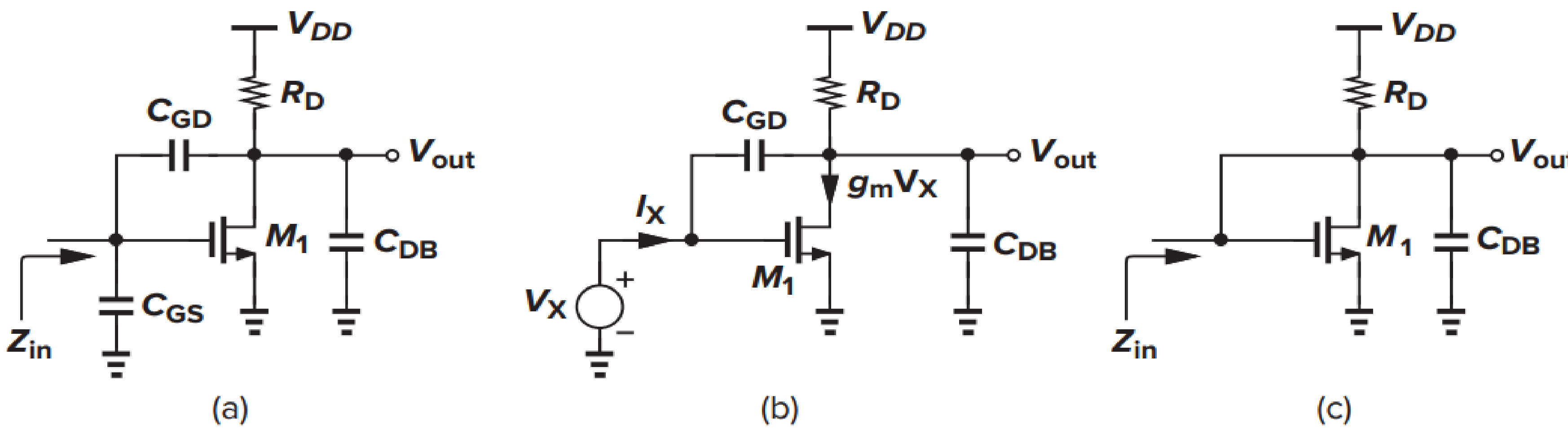
低频时，增益约为$-g_m R_D$

密勒近似 $Z_{in}=\dfrac{V_X}{I_X}=\dfrac{1}{[C_{GS}+(1+g_m R_D)C_{GD}]s}$ 较准

但在高频时，增益与输出节点有关，密勒近似不准确。
精确计算：因C_{GS}接地，可先不计，后并联上即可。

$$(I_X-g_m V_X)\left(R_D \,||\, \frac{1}{C_{DB}s}\right)+\frac{I_X}{C_{GD}s}=V_X$$

Calculation of input impedance (cont.)



$$\Rightarrow (I_X - g_m V_X)\frac{R_D}{1+R_D C_{DB}s} + \frac{I_X}{C_{GD}s} = V_X$$

C_{GD}很大，没意义

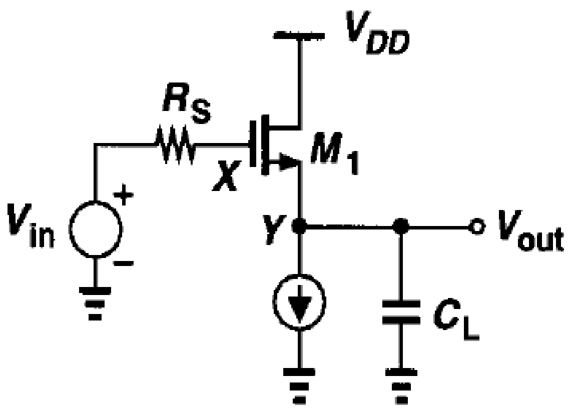
$$Z_{in} = \frac{V_X}{I_X} \,||\, \frac{1}{C_{GS}s} = \frac{1 + R_D(C_{GD} + C_{DB})s}{C_{GD}s(1 + g_m R_D + R_D C_{DB}s)} \,||\, \frac{1}{C_{GS}s}$$

低频（s小）时$R_D(C_{GD} + C_{DB})s << 1$, 且$1 + g_m R_D >> R_D C_{DB}s$,

则$Z_{in} \approx \dfrac{1}{C_{GD}s(1 + g_m R_D)} \,||\, \dfrac{1}{C_{GS}s}$

6.3 Source Followers源跟随器的频率特性

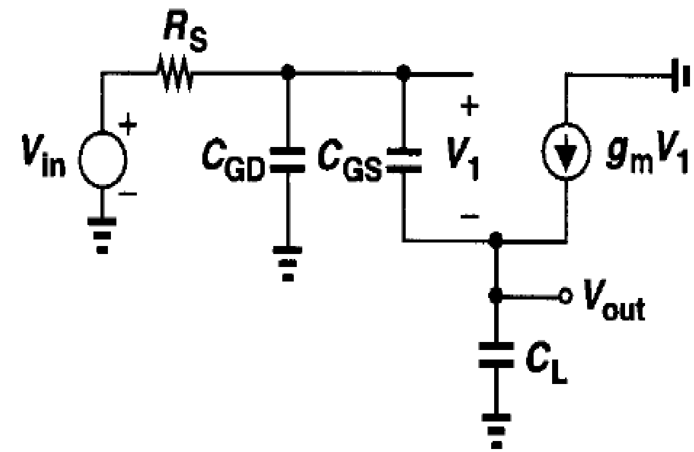
用于缓冲或电平移位



SF低频增益:

$$A_v = \frac{g_m R_S}{1 + (g_m + g_{mb}) R_S}$$

上式中Rs是MOS源极电阻,不是左图信号源电阻



理想电流源电阻无穷大

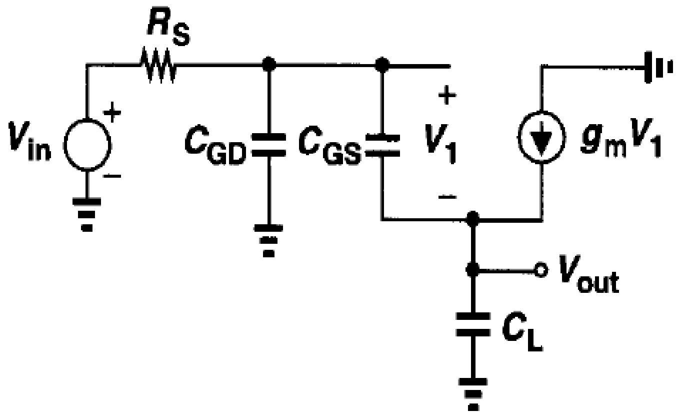
X和Y点通过CGS有很强的相互作用，极点无法与相应的单独结点进行关联； 且SF为同向输入输出，因此不适用密勒定律计算输出阻抗。

$$V_1 C_{GS} s + g_m V_1 = V_{out} C_L s,$$

$$\Rightarrow V_1 = \frac{C_L s}{C_{GS} s + g_m} V_{out}$$

暂不计MOS管的二阶效应，主要是1/gmb

Source Followers (cont.)



$$V_{in} = R_S[V_1C_{GS}s + (V_1 + V_{out})C_{GD}s] + V_1 + V_{out}$$

2个极点1个零点，
注意零点极性

$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS}s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_mR_SC_{GD} + C_L + C_{GS})s + g_m}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{1 + \frac{C_{GS}}{g_m}s}{\frac{R_S}{g_m}\xi s^2 + (R_SC_{GD} + \frac{C_L + C_{GS}}{g_m})s + 1} = \frac{1+\frac{s}{\omega_z}}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$

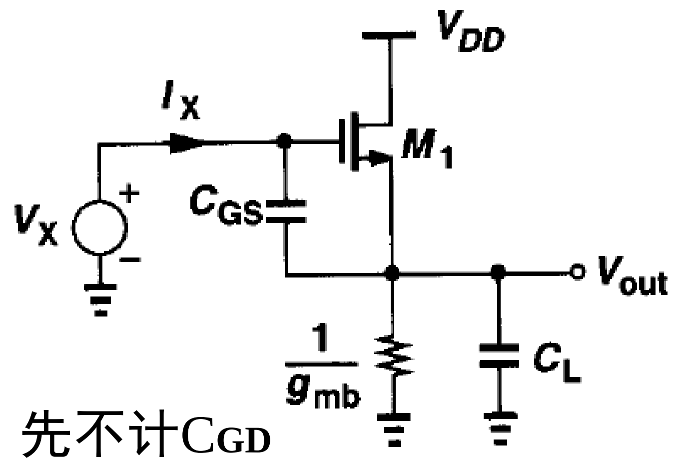
乘以低频增益

式中$\xi = C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L$，$\omega_{p1} \approx \frac{g_m}{g_mR_SC_{GD} + C_L + C_{GS}}$，$\omega_Z = \frac{g_m}{C_{GS}}$

Input impedance of SF

$$V_X = \frac{I_X}{C_{GS}s} + (I_X + \frac{g_m I_X}{C_{GS}s}) \times (\frac{1}{g_{mb}} \| \frac{1}{C_L s})$$

$$Z_{in} = \frac{1}{C_{GS}s} + (1 + \frac{g_m}{C_{GS}s}) \times \frac{1}{g_{mb} + C_L s}, \quad (6.58)$$



先不计CGD

低频时：$Z_{in} \approx \frac{1}{C_{GS}s} + \frac{g_m}{C_{GS}s} \times \frac{1}{g_{mb}} = \frac{g_m + g_{mb}}{sC_{GS}g_{mb}}$

$$= \frac{1}{sC_{GS}\frac{g_{mb}}{g_m + g_{mb}}}$$

相当于使C_{GS}大大减小，自举

低频$A_v = \frac{g_m}{g_m + g_{mb}}$

密勒近似输入电容

$$= (1-A_v)C_{GS} = C_{GS}\frac{g_{mb}}{g_m + g_{mb}}$$

总输入电容（低频）$= C_{GS}\frac{g_{mb}}{g_m + g_{mb}} \| C_{GD}$

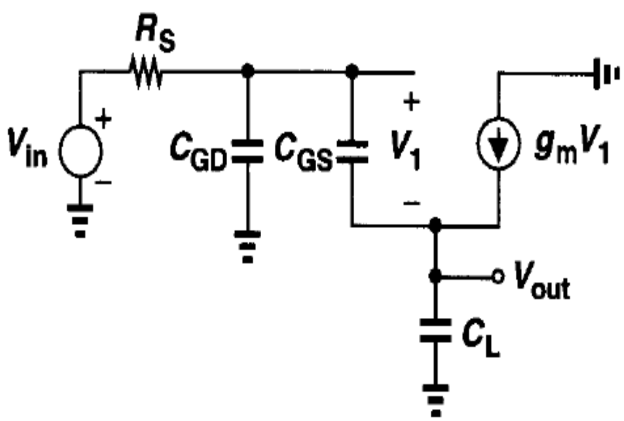
高频时：$Z_{in} = \frac{V_X}{I_X} \approx \frac{1}{C_{GS}s} + (1 + \frac{g_m}{C_{GS}s}) \times \frac{1}{sC_L} = \frac{1}{sC_{GS}} + \frac{1}{sC_L} + \frac{g_m}{C_{GS}C_L s^2}$ 负阻

例6.11 CL=0 源跟随器的传输函数

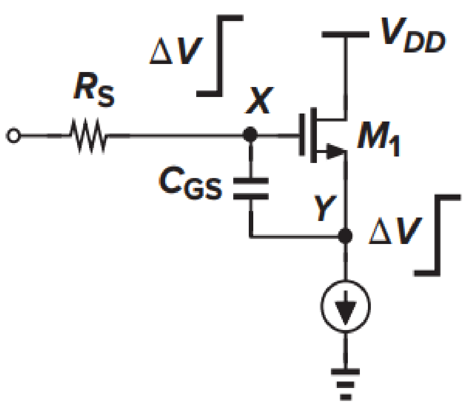
$$\frac{V_{out}}{V_{in}}(s)=\frac{1+\frac{C_{GS}}{g_m}s}{\frac{R_S}{g_m}\xi s^2+(R_S C_{GD}+\frac{C_L+C_{GS}}{g_m})s+1}$$

式中 $\xi=C_{GS}C_{GD}$

$$=\frac{1+\frac{C_{GS}}{g_m}s}{\frac{C_{GS}}{g_m}\times R_S C_{GD}s^2+(R_S C_{GD}+\frac{C_{GS}}{g_m})s+1}=\frac{1}{1+R_S C_{GD}s}$$



栅极到源极电压增益为1，CGS上电压不变，电流为0，相当于消失，既不贡献极点也不贡献零点。CGS被源跟随器“自举”，称为**自举**电容



源跟随器高频输入阻抗中负阻分量的讨论

高频时：$Z_{in} = \dfrac{V_X}{I_X} \approx \dfrac{1}{C_{GS}s} + (1 + \dfrac{g_m}{C_{GS}s}) \times \dfrac{1}{sC_L} = \dfrac{1}{sC_{GS}} + \dfrac{1}{sC_L} + \dfrac{g_m}{C_{GS}C_L s^2}$ 负阻

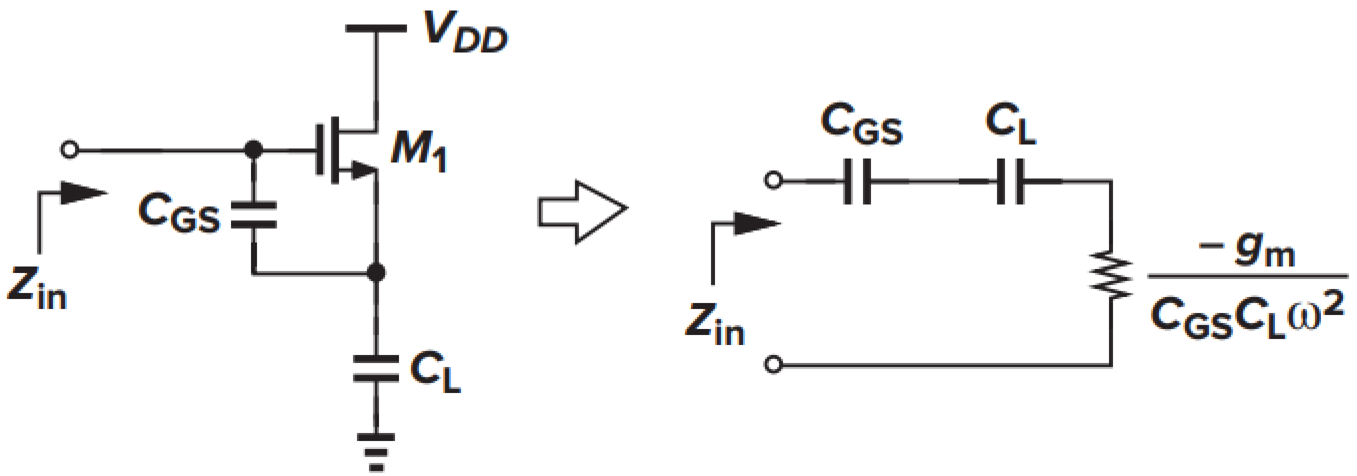
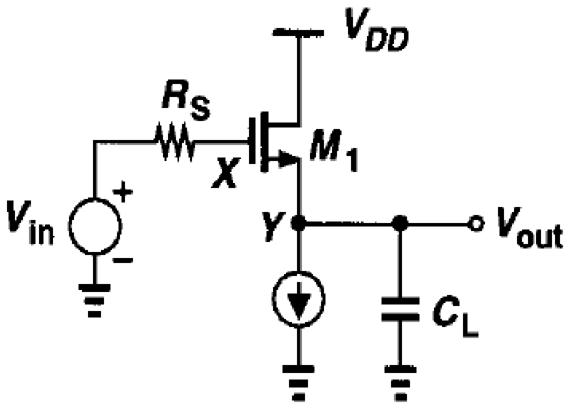
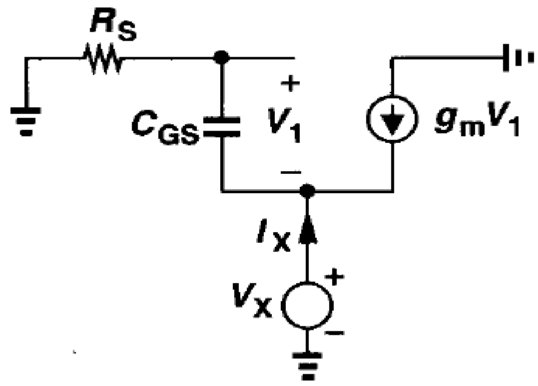


Figure 6.26 Negative resistance seen at the input of a source follower.

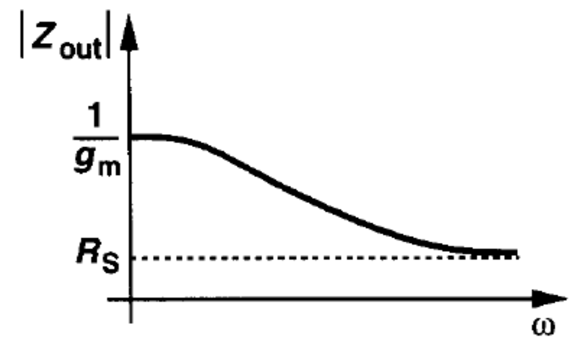
负阻在反馈电路中实质上相当于正反馈。原则上在放大电路中可能引起不稳定，可应用产生振荡器电路，或用于提高电路速度（带宽）。

Output impedance of SF

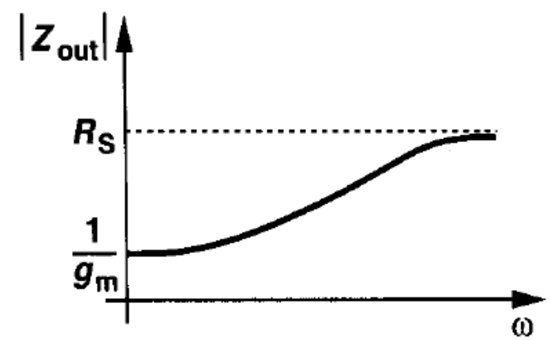




(a)



(b)



(c)

体效应产生的等效电阻1/gmb、ro、CSB位于输出到地, 均先不计,最后并联，以简化计算

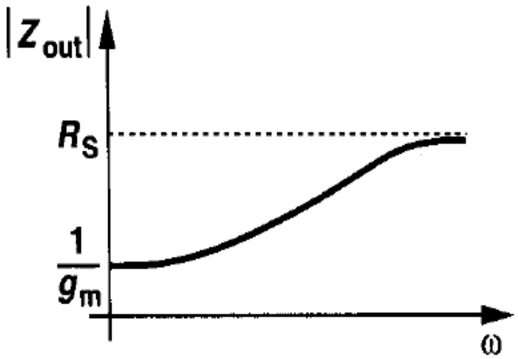
设可忽略C_{GD}， $V_1 C_{GS} s + g_m V_1 = -I_X$

$$V_1 + V_X = -V_1 C_{GS} s R_S$$

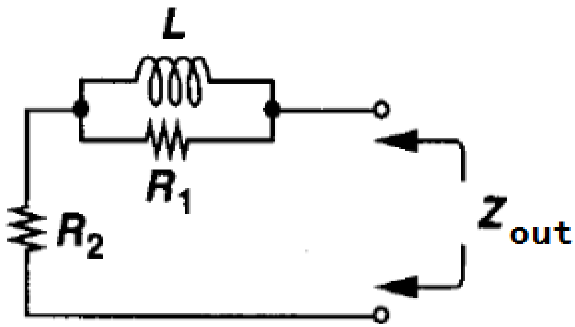
$$\therefore Z_{out} = \frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{g_m + C_{GS} s}$$

$s = 0$ 时$Z_{out} = \dfrac{1}{g_m}$， $s = \infty$时$Z_{out} = R_S$

Output impedance of SF (cont.)



(c)



$$Z_{out} = R_2 + sL \,||\, R_1$$
$$= \frac{1}{g_m} + sL \,||\, (R_S - \frac{1}{g_m})$$
$$= \frac{R_S C_{GS} s + 1}{g_m + C_{GS} s}$$

$$R_2 = \frac{1}{g_m},\ R_S = R_2 + R_1$$

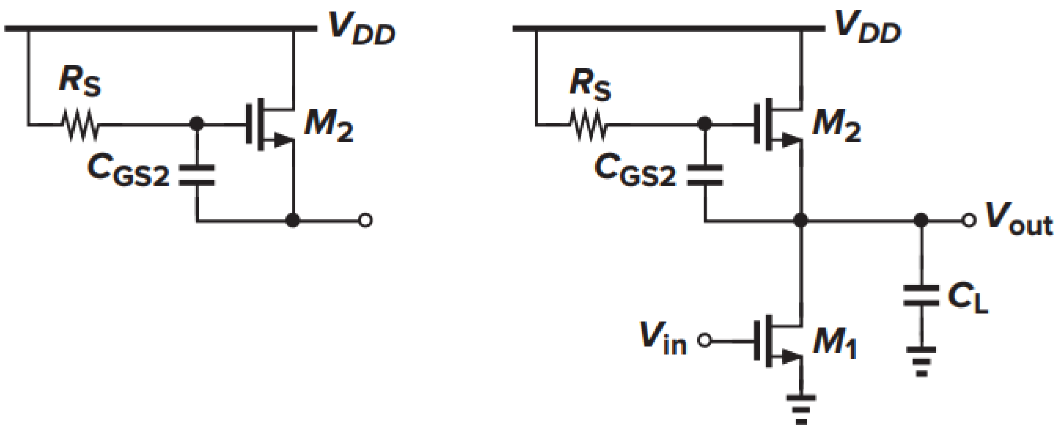
$$Z_{out} - R_2 = Z_{out} - \frac{1}{g_m} = \frac{R_S C_{GS} s + 1}{g_m + C_{GS} s} - \frac{1}{g_m} = \frac{C_{GS} s (R_S - \frac{1}{g_m})}{g_m + C_{GS} s}$$

$$\frac{1}{Z_{out} - \frac{1}{g_m}} = \frac{g_m + C_{GS} s}{C_{GS} s (R_S - \frac{1}{g_m})} = \frac{1}{R_S - \frac{1}{g_m}} + \frac{g_m}{C_{GS} s (R_S - \frac{1}{g_m})} = \frac{1}{R_1} + \frac{1}{sL}$$

$\therefore L = \frac{C_{GS}}{g_m}(R_S - \frac{1}{g_m})$ 有源电感，Q值很低

令 $\frac{\partial L}{\partial g_m} = 0 \Rightarrow \frac{1}{g_m} = \frac{R_S}{2}$ 时 $L_{max} = \frac{C_{GS} R_S^2}{4}$，$C_{GS}$ 可以是外接电容

有源电感



(a) (b)

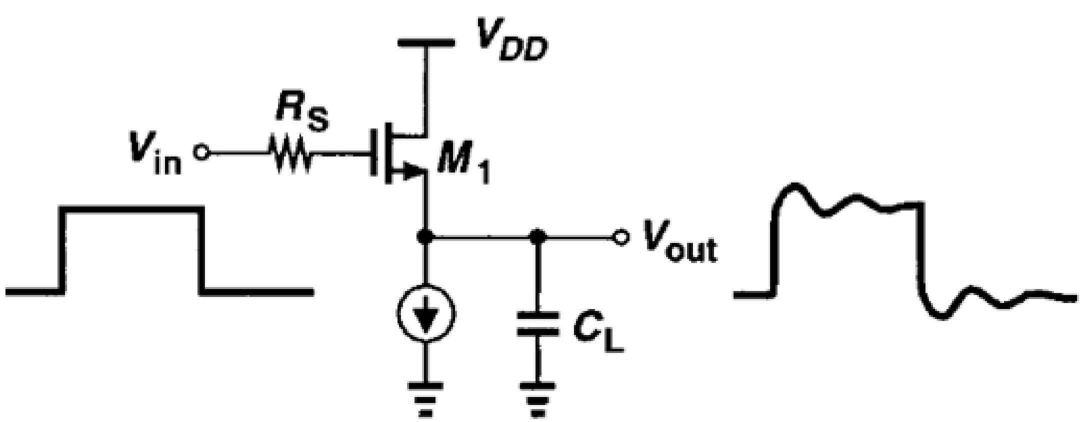
$$L=\frac{C_{GS2}}{g_{m2}}\left(R_S-\frac{1}{g_{m2}}\right)$$

Q值很低。

提升高频阻抗。

以减小输出电压余度（V_{GS2} ）为代价。

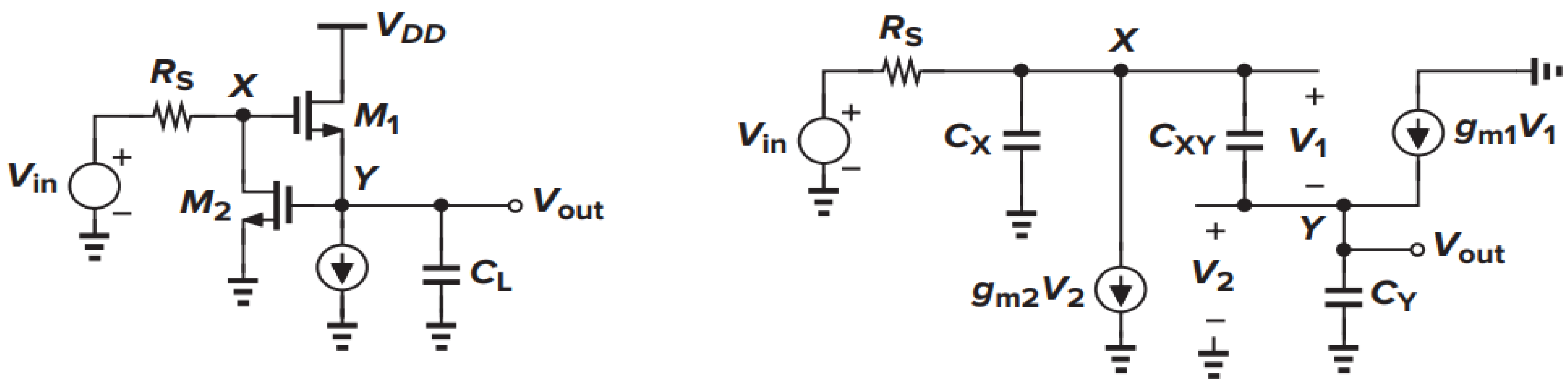
可用于高速接口均衡电路、超宽带放大器中提高带宽。



有源电感使源跟随器阶跃响应信号的输出边沿出现振荡或过冲

Example 6.13* 计算传递函数

Neglecting channel-length modulation and body effect, calculate the transfer function of the circuit shown in Fig. 6.27(a).



(a) (b)

$$C_X = C_{GD1} + C_{DB2},\ C_{XY} = C_{GS1} + C_{GD2},$$

$$C_Y = C_{SB1} + C_{GS2} + C_L,$$

X点电流：

$$(V_1 + V_{out})C_X s + g_{m2}V_{out} + V_1 C_{XY} s = \frac{V_{in} - V_1 - V_{out}}{R_S}$$

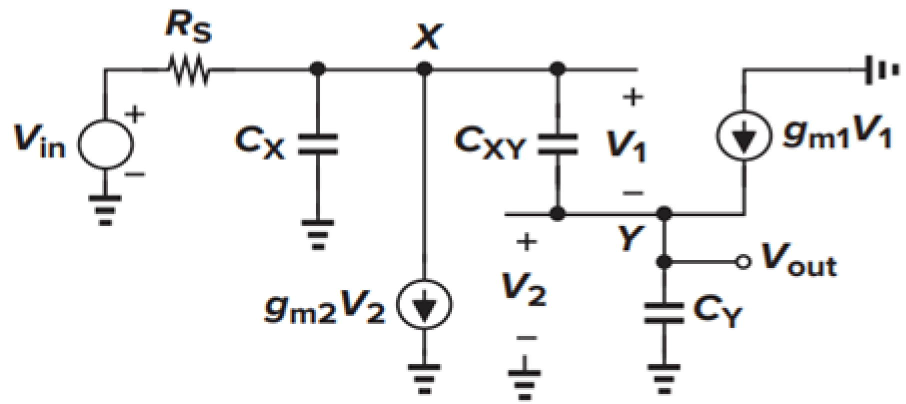
Example 6.13 (cont.)

Vout 电流：

$$V_1 C_{XY} s + g_{m1} V_1 = V_{out} C_Y s$$

hence $V_1 = V_{out} C_Y s/(C_{XY} s + g_{m1})$

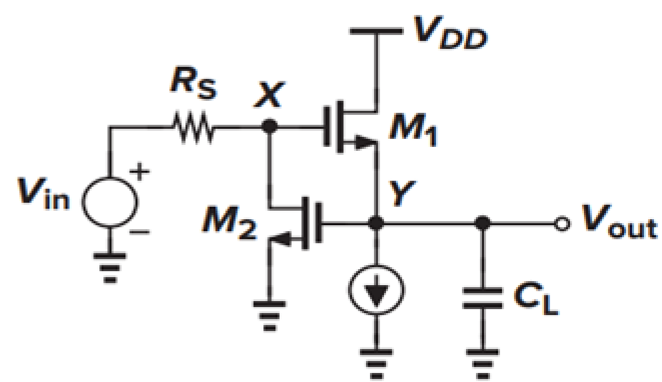
$$V_2 = V_{out}.$$



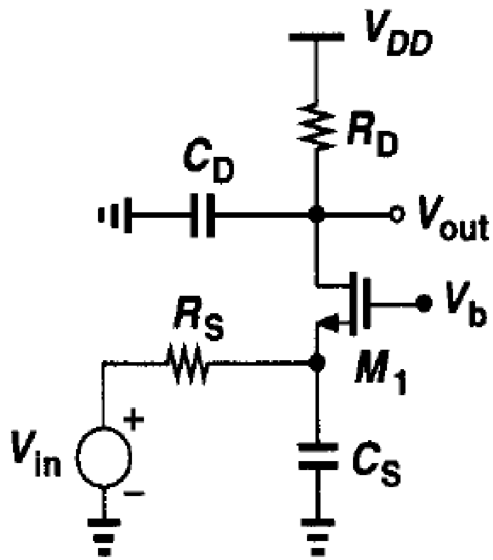
$$\frac{V_{out}}{V_{in}}(s) = \frac{g_{m1} + C_{XY} s}{R_S \xi s^2 + [C_Y + g_{m1} R_S C_X + (1 + g_{m2} R_S) C_{XY}] s + g_{m1}(1 + g_{m2} R_S)}$$

where $\xi = C_X C_Y + C_X C_{XY} + C_Y C_{XY}$

利用增加M2改变零极点设计。
本例实质上是负反馈电路，
增益太小，基本上无实际用途



6.4 Common-gate stage共栅级的频率特性



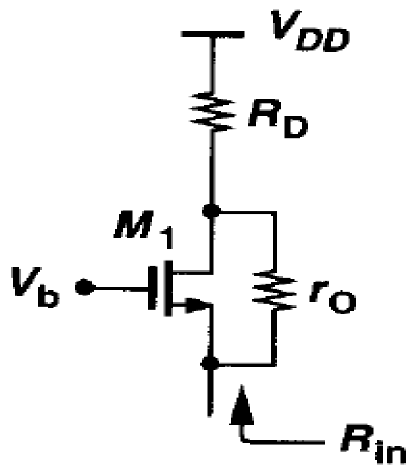
(忽略沟道长度调制效应)

$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S} \frac{1}{\left(1 + \dfrac{C_S}{g_m + g_{mb} + R_S^{-1}} s\right)(1 + R_D C_D s)}$$

沟道长度调制不能忽略时

$\lambda \neq 0$

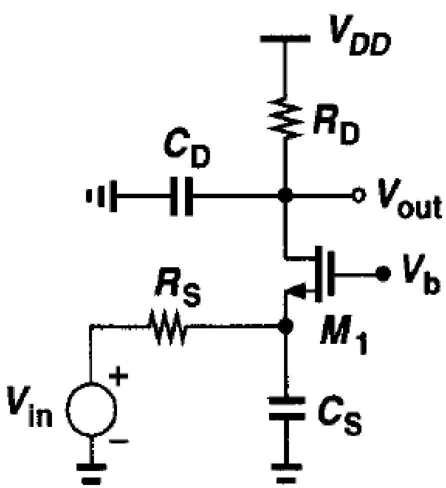
第三章



(a)

$$\frac{V_X}{I_X} = \frac{R_D + r_O}{1 + (g_m + g_{mb})r_O}$$

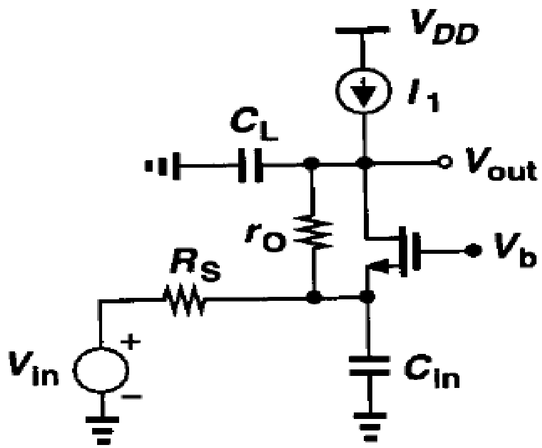
$$\approx \frac{R_D}{(g_m + g_{mb})r_O} + \frac{1}{g_m + g_{mb}},$$



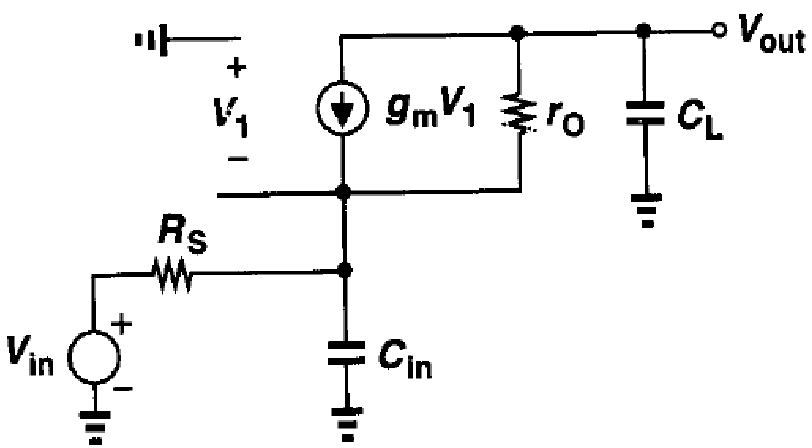
$$Z_{in} \approx \frac{Z_L}{(g_m + g_{mb})r_O} + \frac{1}{g_m + g_{mb}}$$

where $Z_L = R_D \| [1/(C_D s)]$

Example 6.15：计算传递函数和输入阻抗



(a)



(b)

$$(-V_{out}C_L s + V_1 C_{in} s)R_S + V_{in} = -V_1$$

$$V_1 = -\frac{-V_{out}C_L s R_S + V_{in}}{1 + C_{in} R_S s}$$

$$r_O(-V_{out}C_L s - g_m V_1) - V_1 = V_{out}$$

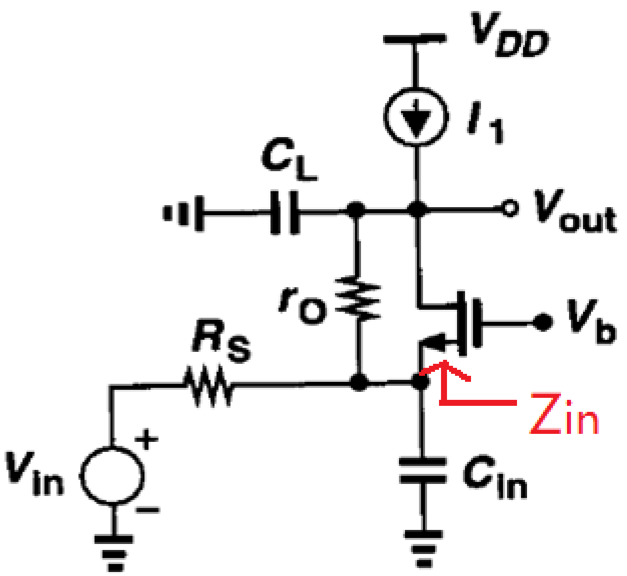
Example 6.15 (cont.)

$$\frac{V_{out}}{V_{in}}(s) = \frac{1 + g_m r_O}{r_O C_L C_{in} R_S s^2 + [r_O C_L + C_{in} R_S + (1 + g_m r_O) C_L R_S] s + 1}$$

body effect can be included by simply replacing g_m with $g_m + g_{mb}$.

$$Z_{in} \approx \frac{Z_L}{(g_m + g_{mb}) r_O} + \frac{1}{g_m + g_{mb}}$$

$$Z_{in} = \frac{1}{g_m + g_{mb}} + \frac{1}{C_L s} \cdot \frac{1}{(g_m + g_{mb}) r_O}$$



高频：
s较大时

$$\omega_{p,in} = \frac{1}{\left(R_S \middle\| \frac{1}{g_m + g_{mb}}\right) C_{in}}$$

CL使高频增益降低，抑制了密勒效应

6.5 Cascode 频率特性

$$A\text{到}X\text{的增益} \approx \frac{-g_{m1}}{g_{m2}+g_{mb2}}$$

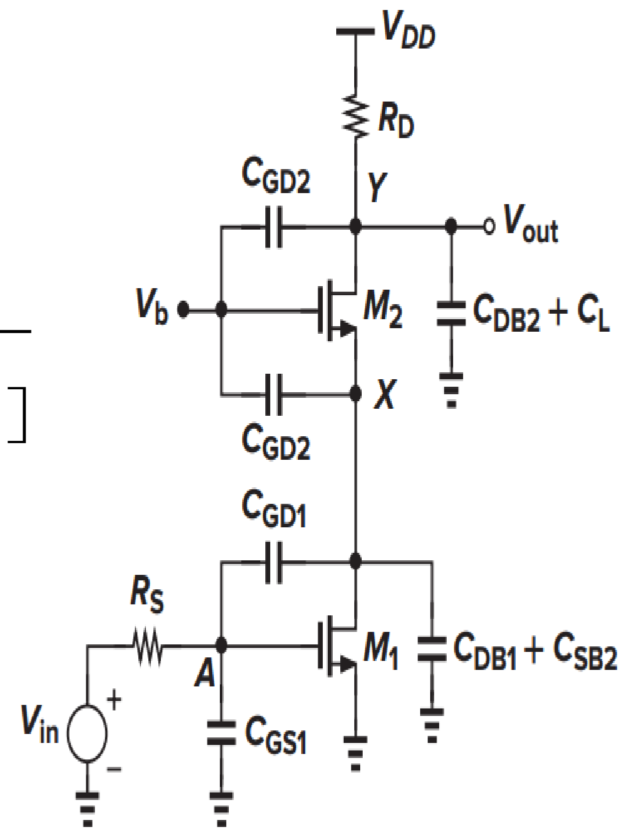
$$（M_1\text{与}M_2\text{相同时}）\approx -1$$

$$\omega_{p,A} \approx \frac{1}{R_S\left[C_{GS1}+\left(1+\dfrac{g_{m1}}{g_{m2}+g_{mb2}}\right)C_{GD1}\right]}$$

A到X的增益小（~1），米勒效应电容小。
主极点大，适合高频。

$$\omega_{p,X} \approx \frac{g_{m2}+g_{mb2}}{2C_{GD1}+C_{DB1}+C_{SB2}+C_{GS2}}$$

$$\omega_{p,Y} = \frac{1}{R_D(C_{GD2}+C_{DB2}+C_L)}$$

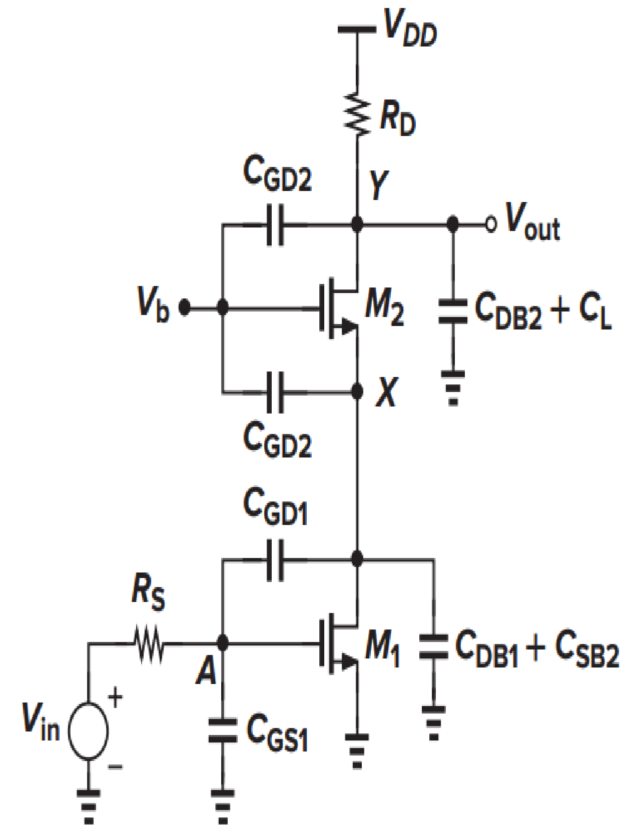


Cascode Stage (cont.)

$$\frac{V_{out}}{V_{in}}(s)$$

$$= \frac{A_0}{\left(1+\frac{S}{\omega_{p,A}}\right)\left(1+\frac{S}{\omega_{p,Y}}\right)\left(1+\frac{S}{\omega_{p,X}}\right)}$$

$$\approx \frac{-g_{m1}R_D}{\left(1+\frac{S}{\omega_{p,A}}\right)\left(1+\frac{S}{\omega_{p,Y}}\right)\left(1+\frac{S}{\omega_{p,X}}\right)}$$



例6.16 即使RD很大，X极点并不受影响

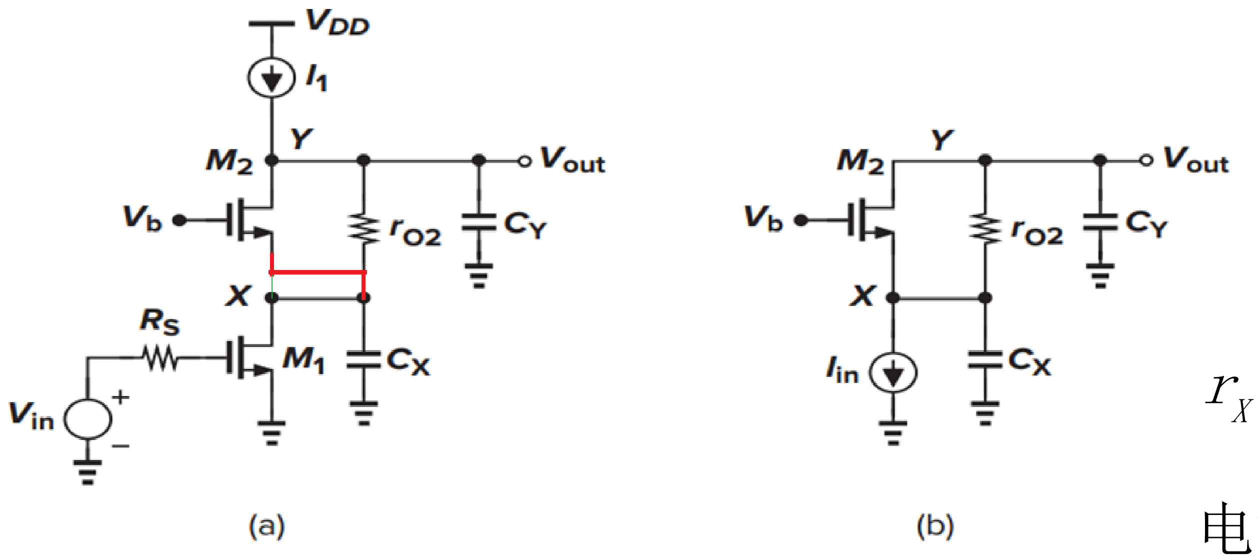


Figure 6.35 Simplified model of a cascode stage.

RD为电流源，X向M2源级看的低频输入阻抗rx很大。X点关联的极点会变低吗?

$$r_X = \frac{R_D + r_{o2}}{1 + (g_{m2} + g_{mb2})r_{o2}}$$

电流源I_1的$R_D = \infty$

Cx电流为反向CY电流减去M1的ID1=Iin

仅关注X点，故可设C_{GS1}=0

$$V_{out} = V_{CX} + r_{o2}(-I_{CY} - I_{D2}) = V_{CX} + r_{o2}(-I_{CY} + g_{m2}V_{CX})$$
$$= V_{CX}(1 + g_{m2}r_{o2}) - r_{o2}I_{CY}$$
$$= \frac{-V_{out}C_Y s - I_{D1}}{C_X s}(1 + g_{m2}r_{o2}) - r_{o2}V_{out}C_Y s$$

例 6.16 （续）RD不影响X极点

$$V_{out} = \frac{-V_{out}C_Y s - I_{in}}{C_X s}(1+g_{m2}r_{o2}) - r_{o2}V_{out}C_Y s$$

$$\frac{V_{out}}{I_{in}} = -\frac{\dfrac{1+g_{m2}r_{o2}}{C_X s}}{1+\dfrac{C_Y}{C_X}(1+g_{m2}r_{o2})+r_{o2}C_Y s}$$

一般有$C_Y > C_X$：
$$\frac{V_{out}}{I_{in}} \approx -\frac{\dfrac{g_{m2}}{C_X s}}{\dfrac{C_Y}{C_X}g_{m2}+C_Y s}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{V_{out}}{I_{in}/g_{m1}} \approx -\frac{g_{m1}g_{m2}}{s(C_Y g_{m2}+C_X C_Y s)} = -\frac{g_{m1}}{sC_Y\left(1+\dfrac{s}{\dfrac{g_{m2}}{C_X}}\right)}$$

原因：负载实为$(R_D=\infty)\,||\,\dfrac{1}{C_Y s} \approx \dfrac{1}{C_Y s}$，与大$R_D$无关

6.6 Differential pair差动对的频率特性

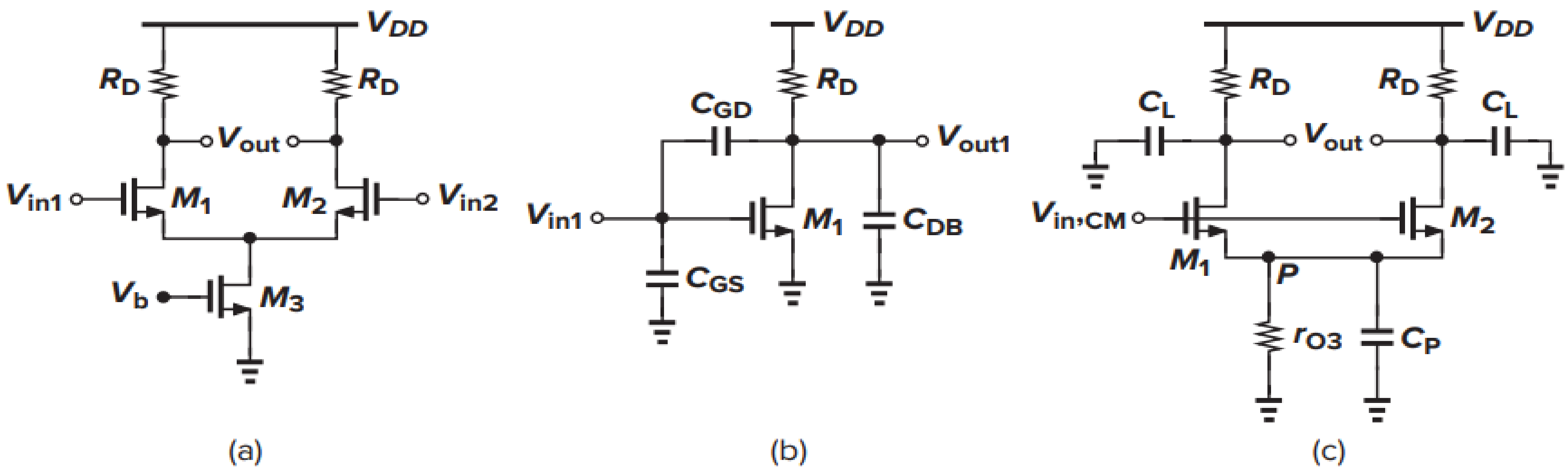


Figure 6.36 (a) Differential pair; (b) half-circuit equivalent; (c) equivalent circuit for common-mode inputs.

6.6.1 无源和电流源负载差动对，匹配电路，低频：

差动增益$A_{vd} = -g_m R_D$ （输出与输入同侧）

$$共模增益A_{v,CM} = \frac{\partial \mathrm{V}_{\mathrm{out}}}{\partial \mathrm{V}_{\mathrm{in,\,CM}}} = -\frac{R_D/2}{\frac{1}{2g_m} + r_{O3}}$$

高频时

$$R_D \Rightarrow R_D \| \frac{1}{C_L s},$$

$$r_{O3} \Rightarrow r_{O3} \| \frac{1}{C_P s}$$

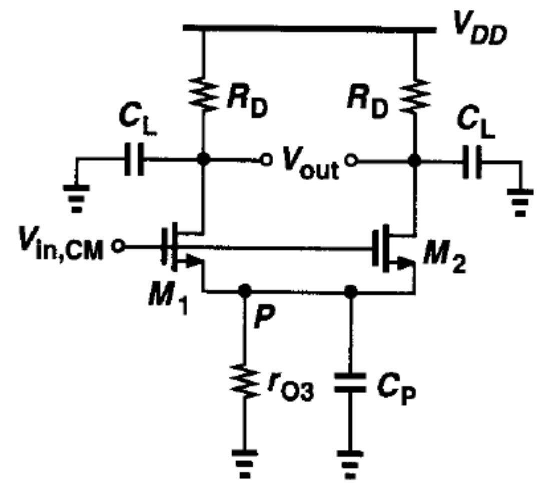
$$共模抑制比CMRR = \frac{A_{vd}}{A_{v,CM}} = 2g_m\left(\frac{1}{2g_m} + r_{O3}\right) = 1 + 2g_m r_{O3}$$

输入管失配的影响：

仅考虑跨导失配$\Delta g_m = g_{m1} - g_{m2}$，低频时：

$$A_{DM} = -\frac{R_D}{2}\frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}r_{O3}}{1 + (g_{m1} + g_{m2})r_{O3}}$$

$$A_{CM-DM} = -\frac{\Delta g_m R_D}{(g_{m1} + g_{m2})r_{o3} + 1}$$



(c)

高频时考虑寄生电容。

$$C_p \approx C_{GD3} + C_{DB3} + C_{GS1} + C_{SB1} + C_{GS2} + C_{SB2}$$

低频$R_D \Rightarrow$ 高频$R_D || \frac{1}{C_L s}$，$r_{O3} \Rightarrow r_{O3} || \frac{1}{C_P s}$

$$A_{DM} = -\frac{R_D || \frac{1}{C_L s}}{2}\frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}(r_{O3} || \frac{1}{C_P s})}{1 + (g_{m1} + g_{m2})(r_{O3} || \frac{1}{C_P s})}$$

CM信号（噪声）的影响在于电路失配将CM转换成了DM信号!

CMRR：输入管失配的影响（续）

$$A_{DM} = -\frac{R_D}{2(1+R_D C_L s)} \frac{(g_{m1}+g_{m2})(1+r_{03}C_P s)+4g_{m1}g_{m2}r_{03}}{1+r_{03}C_P s+(g_{m1}+g_{m2})r_{03}}$$

$$= -\frac{R_D(g_{m1}+g_{m2}+4g_{m1}g_{m2}r_{03})}{1+(g_{m1}+g_{m2})r_{03}} \times \frac{1+\dfrac{(g_{m1}+g_{m2})r_{03}C_P s}{g_{m1}+g_{m2}+4g_{m1}g_{m2}r_{03}}}{2(1+R_D C_L s)\left[1+\dfrac{r_{03}C_P s}{1+(g_{m1}+g_{m2})r_{03}}\right]}$$

$$A_{CM-DM} = \frac{\Delta g_m\left(R_D \,||\, \dfrac{1}{C_L s}\right)}{(g_{m1}+g_{m2})\left(r_{o3} \,||\, \dfrac{1}{C_P s}\right)+1} = \frac{\Delta g_m \dfrac{R_D}{R_D C_L s+1}}{(g_{m1}+g_{m2})\dfrac{r_{o3}}{r_{o3}C_P s+1}+1}$$

$$= \frac{\Delta g_m R_D(r_{o3}C_P s+1)}{(R_D C_L s+1)\left[(g_{m1}+g_{m2})r_{o3}+r_{o3}C_P s+1\right]}$$

$$= \frac{\Delta g_m R_D}{(g_{m1}+g_{m2})r_{o3}+1} \times \frac{1+r_{o3}C_P s}{(1+R_D C_L s)\left[1+\dfrac{r_{o3}C_P s}{(g_{m1}+g_{m2})r_{o3}+1}\right]}$$

CMRR：输入管失配的影响（续）

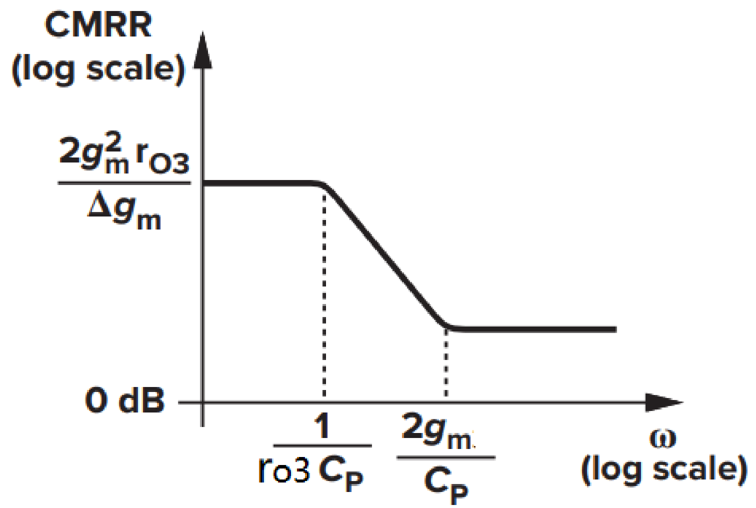
记$g_{m1}+g_{m2}=2g_m$，

共模抑制比$CMRR=\dfrac{A_{DM}}{A_{CM-DM}}=\dfrac{g_m+2g_{m1}g_{m2}r_{O3}+g_m r_{O3}C_P s}{\Delta g_m\times(1+r_{o3}C_P s)}$

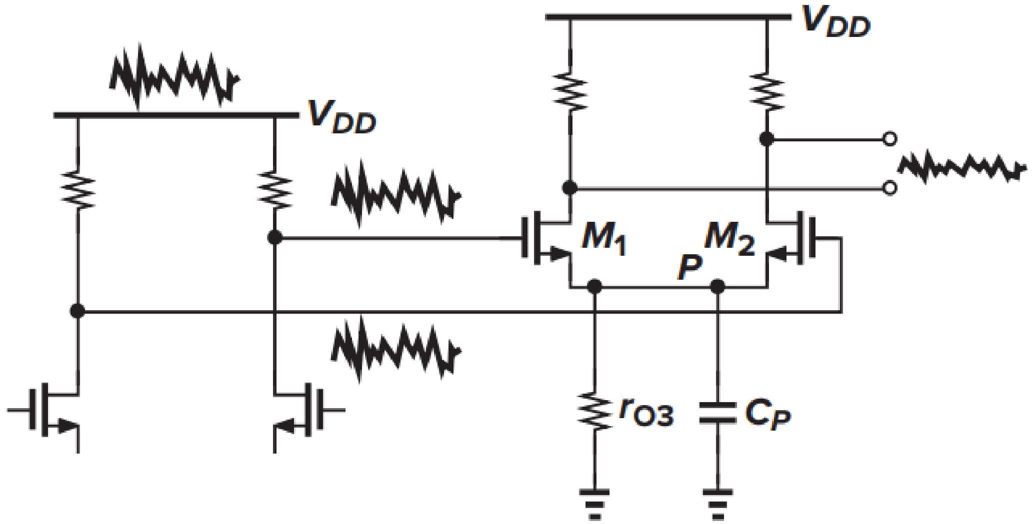
$$=\frac{(g_m+2g_{m1}g_{m2}r_{O3})\left(1+\dfrac{g_m r_{O3}C_P}{g_m+2g_{m1}g_{m2}r_{O3}}s\right)}{\Delta g_m\times(1+r_{o3}C_P s)}$$

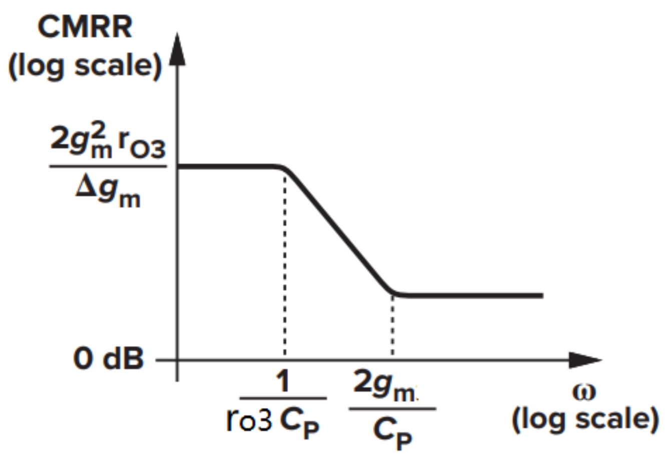
若$g_{m1}g_{m2}\approx g_{\mathrm{m}}^2$

$$\frac{A_{DM}}{A_{CM-DM}}=\frac{2g_m^2 r_{O3}\left(1+\dfrac{C_P}{2g_m}s\right)}{\Delta g_m\times(1+r_{o3}C_P s)}$$



差动对中高频电源噪声的影响





$$C_P \propto W_3$$

当$\omega > \dfrac{1}{r_{o3}C_P}$时，CMRR显著降低。

电流源负载差动对

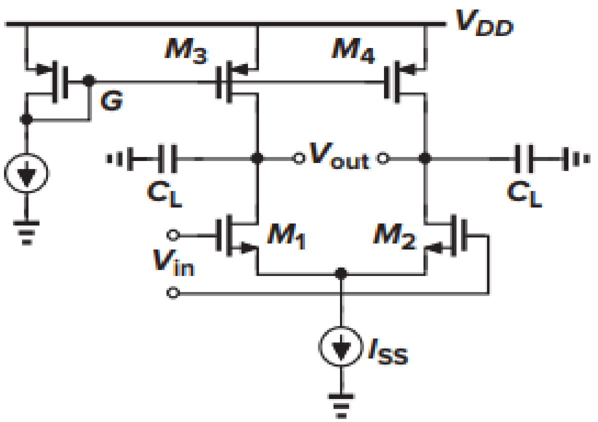


图6.39(a) 电流源负载差动对

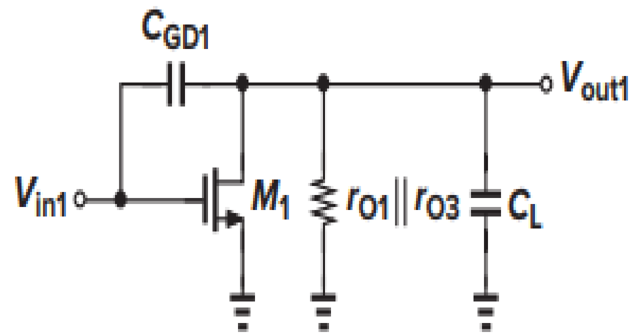


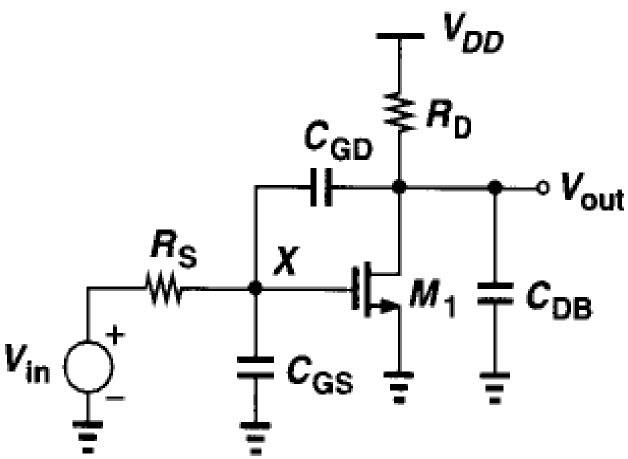
图6.39(c) 半边等效电路

不计 C_{GS} 是由于未给前级信号源电阻 R_S(下图)

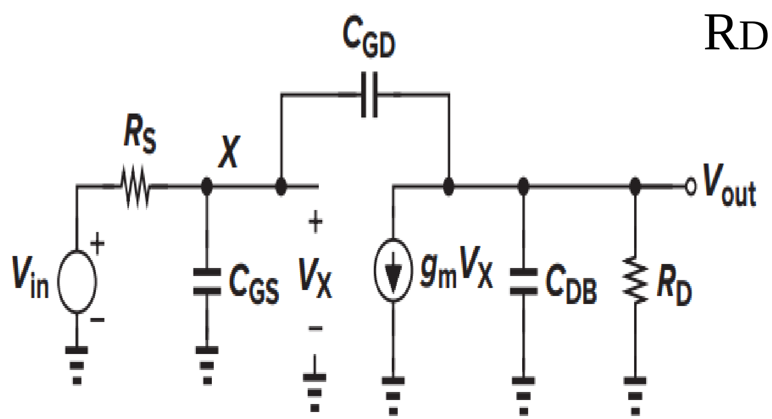
上下图中，

$C_L=C_{DB1}+C_{DB3}$，

$R_D=r_{o1}||r_{o3}$



对比：图6.13 高频单端CS



对比：图6.15 高频单端CS等效电路

电流源负载差动对（续）

$$\frac{V_{out}}{V_{in}}(s)=\frac{-(g_m-C_{GD}s)R_D}{R_S R_D \xi s^2+[R_S(1+g_m R_D)C_{GD}+R_S C_{GS}+R_D(C_{DB}+C_{GD})]s+1}$$

$\xi=C_{GS}C_{DB}+C_{GS}C_{GD}+C_{GD}C_{DB}$ 式（6.30）对应单端CS 高频等效电路

图6.39差分对，在（6.30）式中

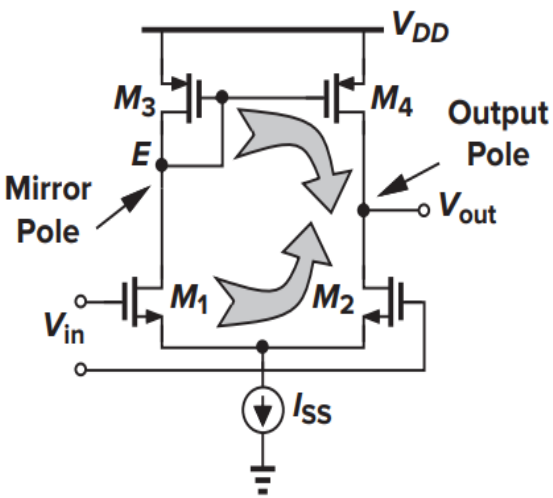
令$R_S=0$, $R_D=r_{o1}\,||\,r_{o3}$, $C_{DB}=C_L$(实为$C_{DB1}+C_{GD3}+C_{DB3}$)

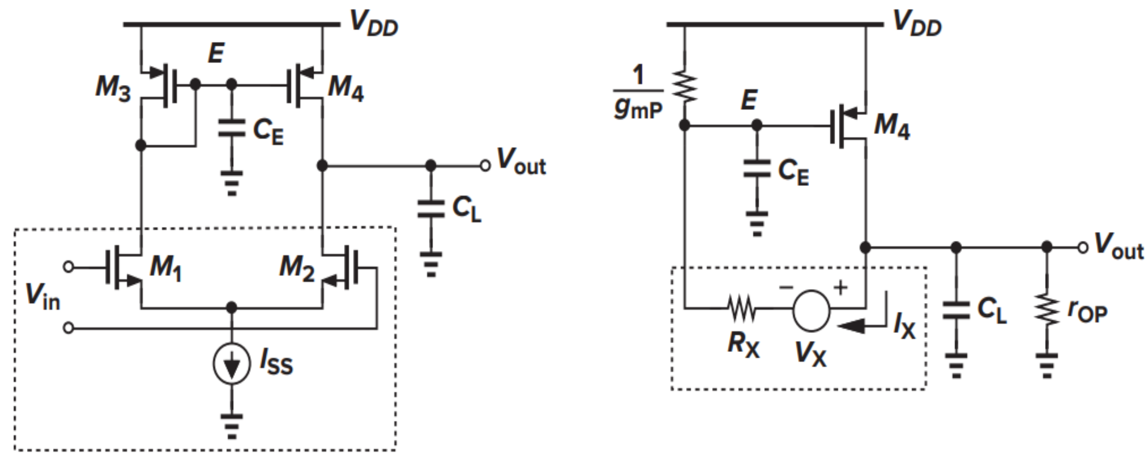
$$\frac{V_{out}}{V_{in}}(s)=\frac{-(g_{m1}-C_{GD1}s)(r_{o1}\,||\,r_{o3})}{(r_{o1}\,||\,r_{o3})(C_L+C_{GD1})s+1}$$

$$=-g_{m1}(r_{o1}\,||\,r_{o3})\frac{1-\dfrac{C_{GD1}s}{g_{m1}}}{1+(r_{o1}\,||\,r_{o3})(C_L+C_{GD1})s}$$

极点$\omega_p=\dfrac{1}{(r_{o1}\,||\,r_{o3})(C_L+C_{GD1})}$，（正）零点$\omega_Z=\dfrac{g_{m1}}{C_{GD1}}$

6.6.2 电流镜负载差动对的频率特性





(a) (b)

电流镜负载放大器不是标准的全差分电路；不能利用半边电路获得传递函数。

两条信号通路，节点E 关联极点$\omega_{pE} \approx \dfrac{g_{m3}}{C_E}$

$C_E = C_{GS3} + C_{GS4} + C_{DB3} + C_{DB1} + C_{GD1}$密勒效应（小)+$C_{GD4}$密勒效应（大）

$C_L \approx$ 真实负载$C_L' + C_{DB4} + C_{DB2} + C_{GD4} + C_{GD2}$

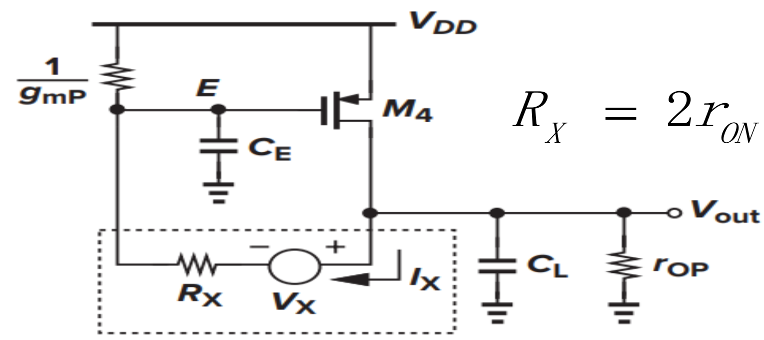
记$r_{OP} = r_{04} = r_{03}$，$r_{ON} = r_{01} = r_{02}$，$g_{mP} = g_{m3}=g_{m4}$

电流镜负载差动对的频率特性(续)

$$V_X = g_{mN} r_{ON} V_{in}$$

$$I_X = V_E (g_{mP} + sC_E)$$

$$-V_E g_{m4} - I_X = V_{out}(sC_L + \frac{1}{r_{OP}})$$



$$R_X = 2r_{ON}$$

$$V_E = (V_{out} - V_X)\frac{\frac{1}{g_{mP}} \,||\, \frac{1}{C_E s}}{R_X + \frac{1}{g_{mP}} \,||\, \frac{1}{C_E s}} = (V_{out} - V_X)\frac{\frac{1}{g_{mP} + C_E s}}{R_X + \frac{1}{g_{mP} + C_E s}}$$

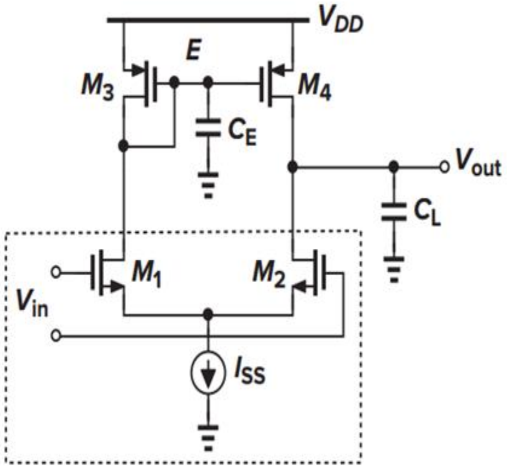
$$\frac{V_{out}}{V_{in}} = \frac{g_{mN} r_{ON}(2g_{mP} + C_E s) r_{OP}}{2r_{OP} r_{ON} C_E C_L s^2 + [(2r_{ON} + r_{OP})C_E + r_{OP}(1 + 2g_{mP} r_{ON})C_L]s + 2g_{mP}(r_{ON} + r_{OP})}$$

$$= \frac{g_{mN} r_{ON} r_{OP}}{r_{ON} + r_{OP}} \times \frac{1 + \frac{C_E s}{2g_{mP}}}{(1 + \frac{s}{\omega_{p1}})(1 + \frac{s}{\omega_{p2}})} = g_{mN}(r_{ON} \,||\, r_{OP})\frac{1 + \frac{s}{\omega_Z}}{(1 + \frac{s}{\omega_{p1}})(1 + \frac{s}{\omega_{p2}})}$$

电流镜负载差动对的频率特性(续)

$$\omega_{p1} \approx \frac{2g_{mP}(r_{ON}+r_{OP})}{(2r_{ON}+r_{OP})\ C_E + r_{OP}(1+2g_{mP}r_{ON})C_L} \approx \frac{2g_{mP}(r_{ON}+r_{OP})}{r_{OP}(1+2g_{mP}r_{ON})C_L}$$

$$\approx \frac{r_{ON}+r_{OP}}{r_{OP}r_{ON}C_L} = \frac{1}{(r_{OP}\ ||\ r_{ON})C_L}$$ 主极点(时间常数最大)与输出节点关联



$$\because \frac{1}{\omega_{p1}\omega_{p2}} \approx \frac{r_{ON}r_{OP}C_EC_L}{g_{mP}(r_{ON}+r_{OP})} = \frac{C_E}{g_{mP}} \times (r_{ON}\ ||\ r_{OP})C_L$$

$$\therefore \omega_{p2} \approx \frac{g_{mP}}{C_E}$$ 镜像极点（次极点）与电流镜节点E关联

零点 $\omega_Z = \frac{2g_{mP}}{C_E} = 2\omega_{p2}$ 两条信号路径

$$\therefore \frac{V_{out}}{V_{in}}(s) = g_{mN}(r_{ON}\ ||\ r_{OP})\frac{1+\frac{s}{\omega_Z}}{(1+\frac{s}{\omega_{p1}})(1+\frac{s}{\omega_{p2}})} = A_0\frac{1+\frac{s}{2\omega_{p2}}}{(1+\frac{s}{\omega_{p1}})(1+\frac{s}{\omega_{p2}})}$$

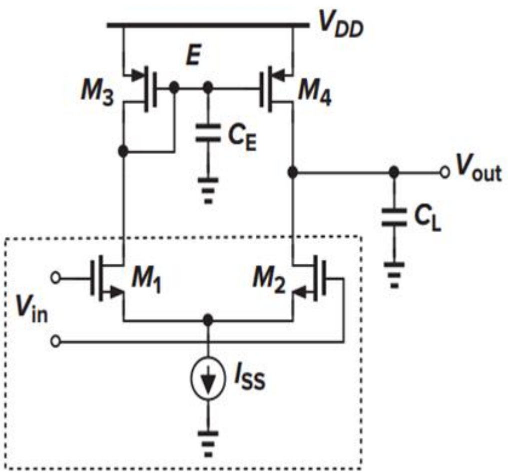
电流镜负载差动对的频率特性(续)

$$\therefore \frac{V_{out}}{V_{in}}(s)=A_0\frac{1+\frac{s}{2\omega_{p2}}}{(1+\frac{s}{\omega_{p1}})(1+\frac{s}{\omega_{p2}})}=A_0\frac{1}{1+\frac{s}{\omega_{p1}}}\times\frac{\frac{1}{2}(2+\frac{s}{\omega_{p2}})}{1+\frac{s}{\omega_{p2}}}$$

$$=\frac{A_0}{2}\frac{1}{1+\frac{s}{\omega_{p1}}}(1+\frac{1}{1+\frac{s}{\omega_{p2}}})=\frac{A_0}{2}\frac{1}{1+\frac{s}{\omega_{p1}}}+\frac{A_0}{2}\frac{1}{(1+\frac{s}{\omega_{p1}})}\times\frac{1}{(1+\frac{s}{\omega_{p2}})}$$

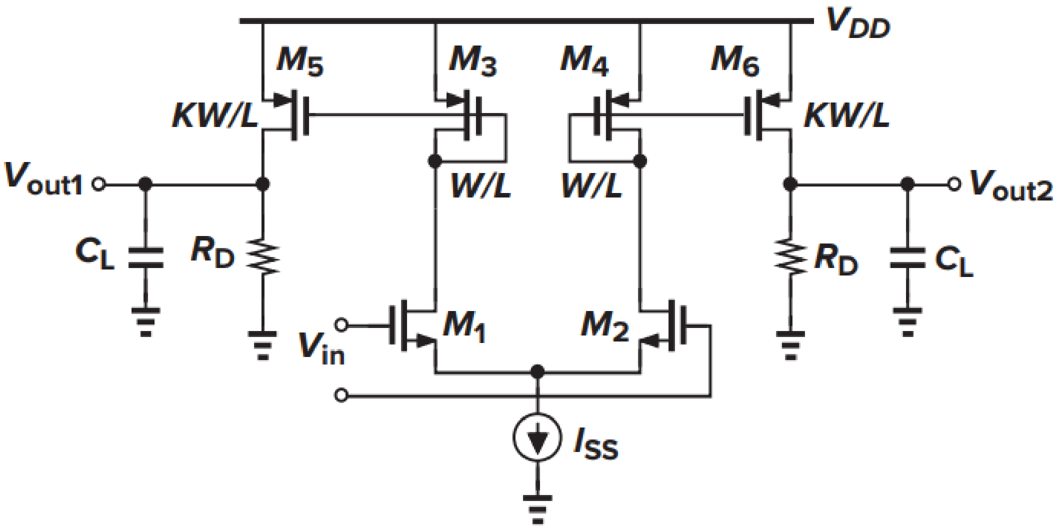
快通路（带宽大）信号+慢通路（带宽小）信号

左平面零点（信号相加）的作用：
电路输出由快通路(M2)和
慢通路(M1、M3和M4)并联组成。

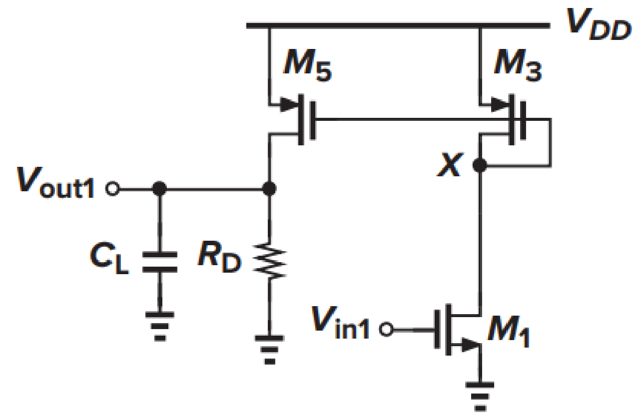


电流镜负载差动对具有镜向极点（缺点）。

例 6.17 估算传递函数



(a)

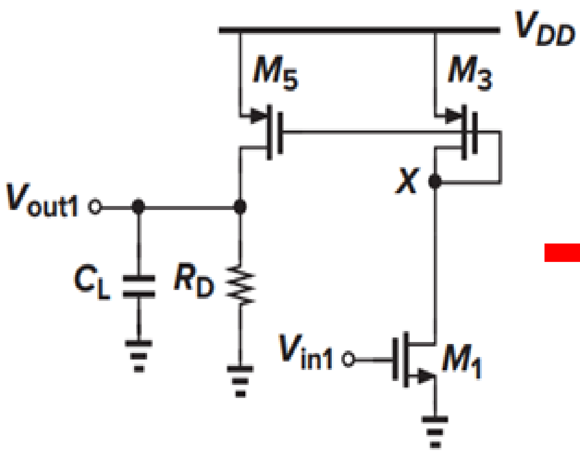


(b)

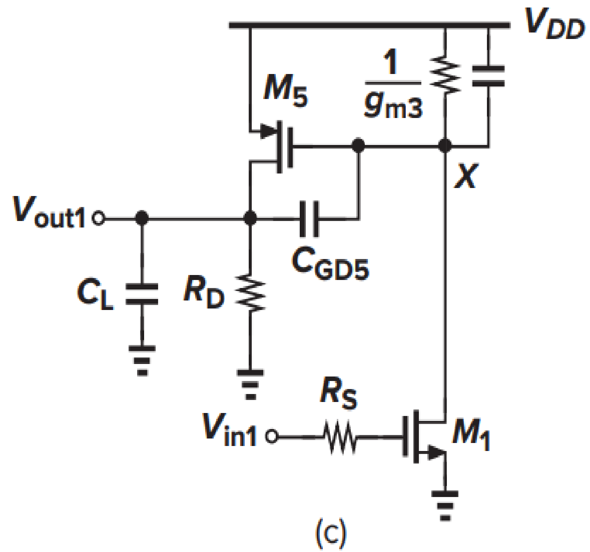
低频增益 $A_v=g_{m1}KR_D$

半边电路

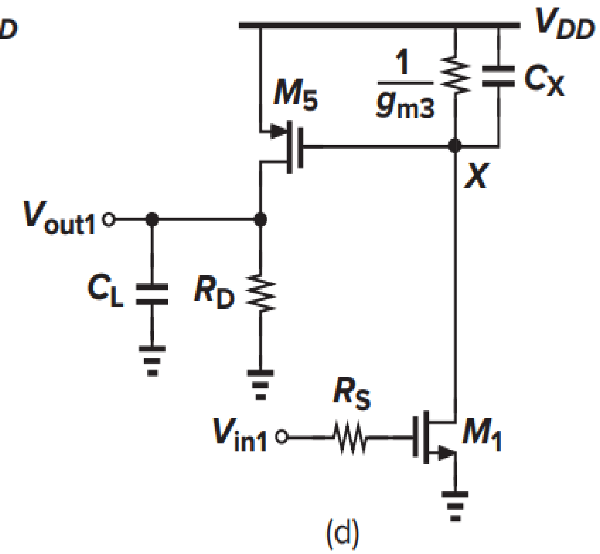
例 6.17 求传递函数（续）



半边电路



高频等效半边电路

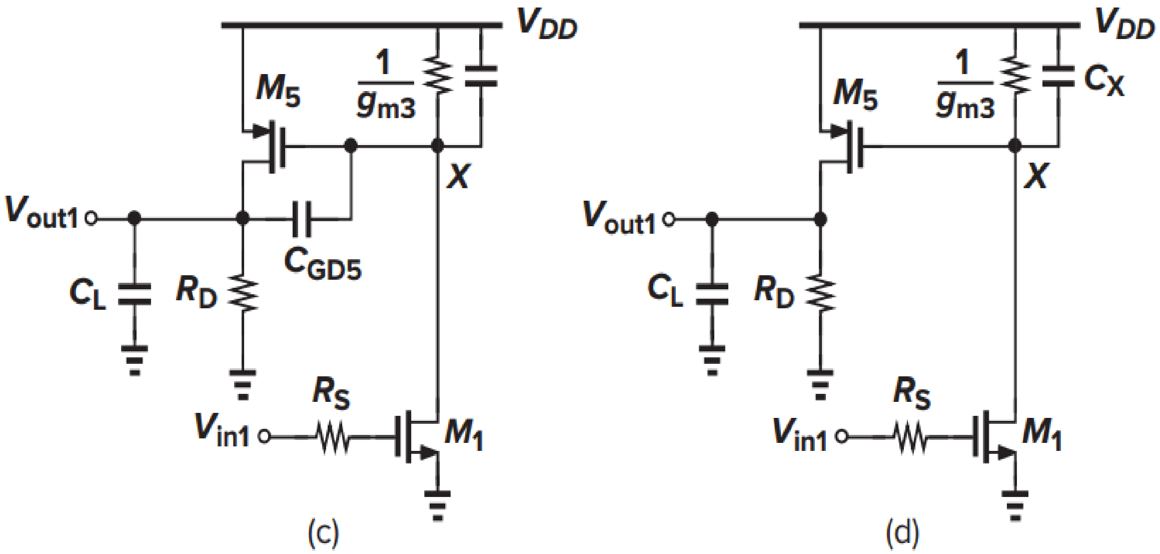


用密勒定律处理CGD5

$$C_X \approx C_{GS3} + C_{DB3} + C_{GS5} + C_{GD5}(1+g_{m5}R_D) + C_{DB1} + C_{GD1}\left(1+\frac{g_{m3}}{g_{m1}}\right)$$

C_L较大而C_{GD5}很小，$C_L||C_{GD5}$（密勒效应）$\approx C_L$

Example 6.17 (cont.) transfer function



$$\frac{V_{out}}{V_{in}} = \frac{V_{out1}}{V_{in1}}$$

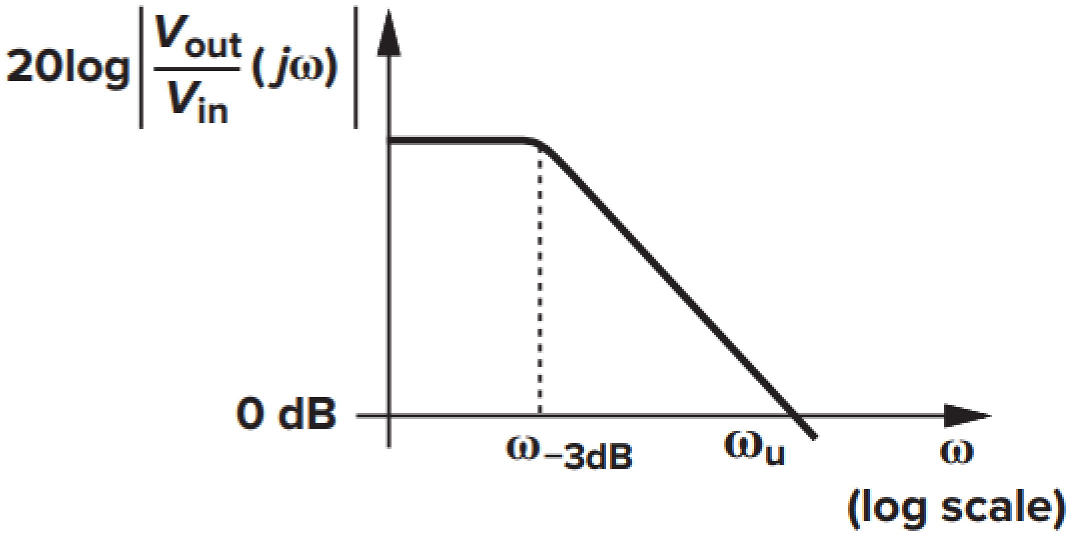
$$= \frac{V_{GS1}}{V_{in1}} \times \frac{V_X}{V_{GS1}} \times \frac{V_{out1}}{V_X}$$

$$\frac{V_{out}}{V_{in}} \approx A_0 \frac{1}{\left(1+\frac{s}{\omega_{in}}\right)\left(1+\frac{s}{\omega_X}\right)\left(1+\frac{s}{\omega_{out}}\right)}, \quad 式中 A_0 \approx g_{m1} K R_D$$

$$\omega_{in} = \frac{1}{R_S\left[C_{GS1} + C_{GD1}\left(1+\frac{g_{m1}}{g_{m3}}\right)\right]}, \quad \omega_X = \frac{g_{m3}}{C_X}, \quad \omega_{out} = \frac{1}{R_D C_L}$$

忽略了CGD1和CGD5引起的高频零点gm/CGD

6.7 增益-带宽的折中



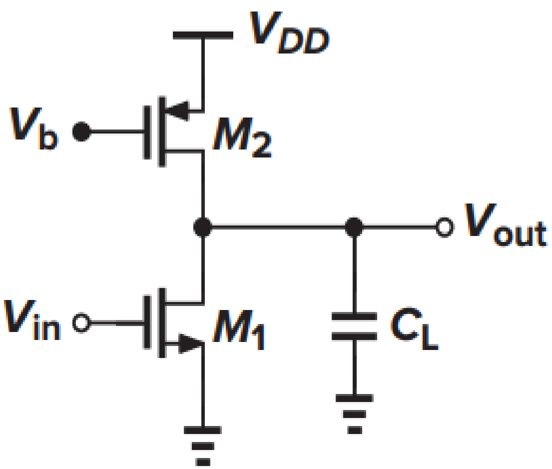
低通电路中的$-3dB$ 带宽$\omega_{-3dB}(\text{rad}/s)=2\pi f_{3dB}$ ，

f_{3dB}简称$3dB$ 带宽（Hz）；ω_{-3dB}一般由主极点（最低频率）确定。

单位增益带宽：由$\omega_u(\text{rad}/s)=2\pi f_u$，$\Rightarrow f_u=\dfrac{\omega_u}{2\pi}\quad(Hz)$

增益带宽积GBW= |低频增益|*3dB带宽（Hz）=A0*f3dB

6.7.1 单极点电路



$$\frac{V_{out}}{V_{in}}(s) = -g_{m1}\left(r_{o1} \,||\, r_{o2} \,||\, \frac{1}{sC_L}\right)$$

$$= -g_{m1}\frac{(r_{o1} \,||\, r_{o2})\dfrac{1}{C_L s}}{(r_{o1} \,||\, r_{o2}) + \dfrac{1}{C_L s}} = \frac{-g_{m1}(r_{o1} \,||\, r_{o2})}{1 + (r_{o1} \,||\, r_{o2})C_L s}$$

$$= \frac{A_0}{1 + \dfrac{s}{\omega_p}}$$

$$\text{增益带宽积}GBW = |A_0|\frac{\omega_p}{2\pi} = |A_0| f_{3dB}$$

$$= g_{m1}(r_{o1} \,||\, r_{o2})\frac{1}{2\pi(r_{o1} \,||\, r_{o2})C_L} = \frac{g_{m1}}{2\pi C_L}$$ 与输出电阻无关！

例：GBW

设$g_{m1}=\dfrac{1}{100\Omega}$，$C_L=50fF$

$$GBW=\frac{g_{m1}}{2\pi C_L}=\frac{1}{100\times 2\pi\times 50\times 10^{-15}}\approx 31.83\ GHz$$

单极点系统的增益-带宽积约为单位增益带宽。

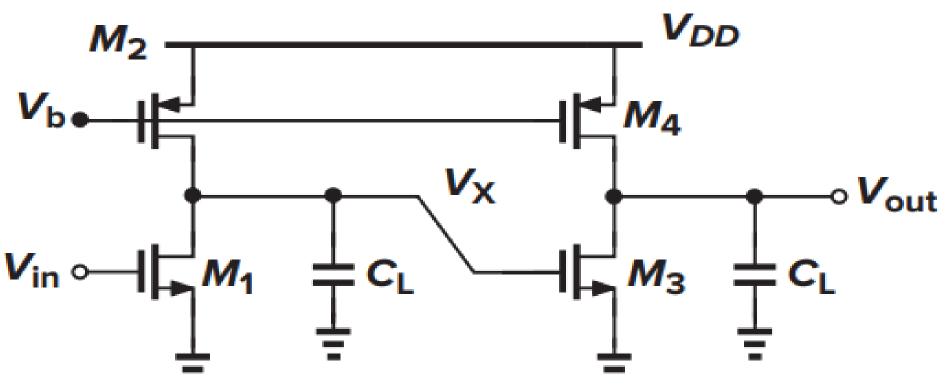
单位增益角频率ω_u，令$\left|\dfrac{V_{out}}{V_{in}}(s)\right|=\left|\dfrac{A_0}{1+\dfrac{s}{\omega_p}}\right|_{s=j\omega_u}=\dfrac{|A_0|}{\sqrt{1+\left(\dfrac{\omega_u}{\omega_p}\right)^2}}=1$

$\omega_u=\omega_p\sqrt{A_0^2-1}\approx A_0\omega_p=A_0\omega_{-3dB}$，即$A_0 f_{3dB}=f_u=GBW$

6.7.2 多个电路级联

级联电路用于增大增益和GBW；但必然形成多极点或多重极点，电路总带宽减小， 多极点反馈系统稳定性变差。

（1） 相同电路级联形成多重极点



$$\frac{V_{out}}{V_{in}}(s) = \frac{A_0^2}{\left(1+\frac{s}{\omega_p}\right)^2}$$

A_0和ω_p是单个CS放大器的低频增益和带宽。

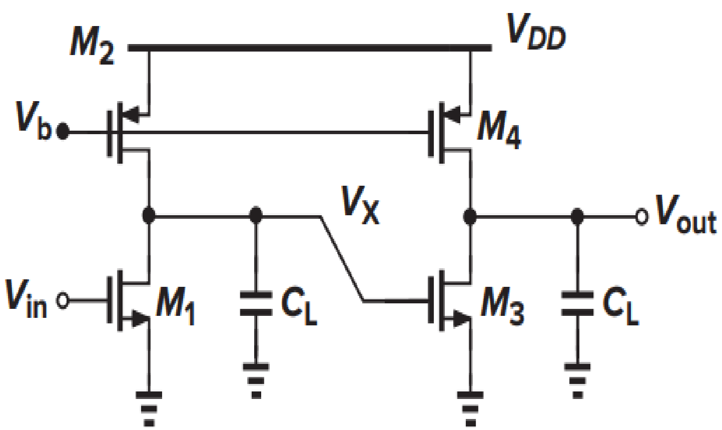
$$\left|\frac{V_{out}}{V_{in}}(s = j\omega_{-3dB})\right| = \frac{A_0^2}{1+\left(\frac{\omega_{-3dB}}{\omega_p}\right)^2} = \frac{A_0^2}{\sqrt{2}}$$

多重极点电路（续）

2重极点电路： $1+\left(\dfrac{\omega_{-3dB}}{\omega_p}\right)^2=\sqrt{2}$

$\Rightarrow\ \omega_{-3dB}=\omega_p\sqrt{\sqrt{2}-1}=0.643\omega_p$

带宽减小.



$$GBW=A_0^2\omega_p\sqrt{\sqrt{2}-1}=0.643A_0^2\omega_p=0.643A_0\times A_0\omega_p$$

N个相同电路级联形成N重极点，

总带宽为$\omega_{N,-3dB}=\omega_p\sqrt{\sqrt[N]{2}-1}$

不同电路级联形成的多极点电路

$$\frac{V_{out}}{V_{in}}(s) = \frac{A_0}{(1+\frac{s}{\omega_{p1}})(1+\frac{s}{\omega_{p2}})\ldots(1+\frac{s}{\omega_{pN}})}$$

$$\sqrt{1+\left(\frac{\omega_{-3dB}}{\omega_{p1}}\right)^2} \times \sqrt{1+\left(\frac{\omega_{-3dB}}{\omega_{p2}}\right)^2} \times \ldots \sqrt{1+\left(\frac{\omega_{-3dB}}{\omega_{pN}}\right)^2} = \sqrt{2}$$

$\because \quad \omega_{-3dB}$一定小于$\omega_{pi}$

$$\therefore \quad \left(\frac{\omega_{-3dB}}{\omega_{p1}}\right)^2 + \left(\frac{\omega_{-3dB}}{\omega_{p2}}\right)^2 + \ldots \left(\frac{\omega_{-3dB}}{\omega_{pN}}\right)^2 \approx 1$$

$$\omega_{-3dB} \approx \frac{1}{\sqrt{\left(\frac{1}{\omega_{p1}}\right)^2 + \left(\frac{1}{\omega_{p2}}\right)^2 + \ldots \left(\frac{1}{\omega_{pN}}\right)^2}}$$

本章知识要点

- 密勒效应减小输入阻抗；
- 输入输出反向时，密勒定律可用于近似计算输出阻抗以及传递函数，但会丢失零点和增加可忽略极点；
- 只有前向（馈）结构电路的极点与节点关联；
- 极点是信号路径上电容使得高频比低频信号幅度衰减3dB的（角）频率；
- 零点是输入输出之间两条支路的阻性和容性信号相等时的（角）频率，总幅度增加3dB；零点有正有负；
- 传递函数可用低频增益和零极点构成；一般来说主极点是3dB带宽；
- 共源放大器输入和输出阻抗高频时降低；
- 源极跟随器输出阻抗可能形成有源电感，但Q值很低；
- Cascode结构由于密勒效应小，适合高速或高频电路；
- 差分放大电路共模抑制高频时变差，尾电流源尺寸不宜很大；
- 电流镜负载差动放大器与电流源负载差动放大器相比，具有镜像极点，频率特性稍慢，镜像极点对稳定性有影响；
- 单极点电路的增益带宽积与单位增益带宽的关系。