Quantum Harmonic Oscillator Model of Stock Return Distributions

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Abstract. Stochastic models have been used for decades to model stock prices. Recently, the advancement of quantum mechanics has led to novel research in using quantum models to model stock prices. This paper leverages the quantum harmonic oscillator to model the long term log stock return distributions of stock prices, which can be used for forecasting stock markets. A wave function represented by the square of the superposition of the eigenfunctions of the quantum harmonic oscillator is used to model past stock returns. A phase factor is applied to the wave function to predict future stock return distributions of that stock.

Introduction

Previous research regarding the quantum harmonic oscillator has only used it to model past stock return distributions. These research projects indicate that the quantum harmonic oscillator is effective for capturing the general shape of stock return distributions of the past. In previous research such as that done by Ahn et al. [1], it is shown that the quantum harmonic oscillator accurately captures the stock returns of 1-day, 5-day, and 20-day trading periods of the Financial Time Stock Exchange 100 index. Similarly, in previous research done by Bhatt and Gor [2], the quantum harmonic oscillator model accurately captured stock returns distributions of the Nifty Index of India. When compared with other classical models of modeling stock return distributions such as the Heston model, the quantum harmonic oscillator model deviated less from the empirical distributions of stock returns [1]. The fitted errors of the quantum harmonic oscillator model were smaller and better captured non-normal aspects of the stock return distribution compared to the classical models [1].

While the quantum harmonic oscillator model has been used to model stock returns, researchers have also used the quantum anharmonic oscillator to describe stock returns that are better represented by non-normal distributions due to extreme market fluctuations. In their research, Gao and Chen [3] used the anharmonic model to capture bimodal distributions of the Nikkei 225 index. Their results indicated that in cases where actual stock returns didn't follow a normal distribution, the anharmonic oscillator model was more accurate than the harmonic oscillator model [3]. However, this model assumes that the conditions when buying the stock are illiquid, which is not usually the case. Thus, this model is mainly useful for modeling securities that are highly difficult to trade, or for the majority of stocks during times of economic crisis, where buy and sell prices may differ significantly.

Using quantum oscillations instead of classical oscillations has become more popular over the past decade mainly as developments in quantum mechanics have shown multiple benefits to classical oscillations [4]. In terms of modeling for the stock market, quantum oscillations can provide multiple benefits. Quantum harmonic oscillators have a restoring force that pulls the system back to equilibrium, similar to how stock prices revert to their mean, as they exhibit reversion to an average value [4]. In quantum models, energy levels also only take on specific values, which can better capture the discrete nature of transactions and price changes in the stock market [4]. Stock prices change in discrete steps due to individual transactions and each trade can be seen as moving the stock price from one "energy level" to another [4]. Due to the unique attributes of the quantum harmonic oscillator model such as its energy levels, it is useful for modeling stock returns.

Unlike in previous papers, in this paper, the quantum harmonic oscillator is time-evolved to predict future stock return distributions. Additionally, this paper attempts to use the quantum model to model stock returns from a duration of only one year, unlike the duration of about a decade modeled by the previous papers. The purpose of this paper is to determine: How accurate is the quantum harmonic oscillator model when modeling stocks for shorter time durations? Is it feasible for the oscillation of wave functions over time to accurately represent the movement in stock return distributions over time?

Methods

First, stock data for S&P 500 and Nvidia were imported from yahoo finance through the yfinance package for python. This allowed for the collection of stock data through the usage of a stock ticker for any given duration of time in the past. After importing stock data and defining the time duration for the quantum harmonic oscillator model of the past, linspace was used to create an array for the data of stock returns. The log of the stock returns for the stocks was then computed to be used for graphs of empirical distributions of stock returns.

For this project, stock data was collected for S&P 500 and Nvidia 1-day returns, 5-day returns, and 20-day returns from January 1, 2023 to December 31, 2023. A total of 12 graphs were produced using Python, with a past quantum model graph and time-evolved quantum model graph for each of the six stock return distributions. For each graph, the y-axis is the probability, while the x-axis is the scaled log return. Log returns were scaled to ensure that the total probability under the quantum harmonic oscillator model was equivalent to one.

After importing stock data, the quantum harmonic oscillator model was defined in the same Python file through the usage of the schrodinger equation. The eigenfunctions and eigenenergies of the quantum harmonic oscillator were then computed and defined given in equation 1 and 2, where n is the number of the corresponding eigenenergy, ħ (reduced Plank's constant) = 6.626×10^{-34} joule-seconds, ω is angular frequency, m is mass, and H_n is the nth hermite polynomial.

$$E_n \text{ (nth eigenenergy)} = (n + \tfrac{1}{2})\hbar\omega \tag{1}$$

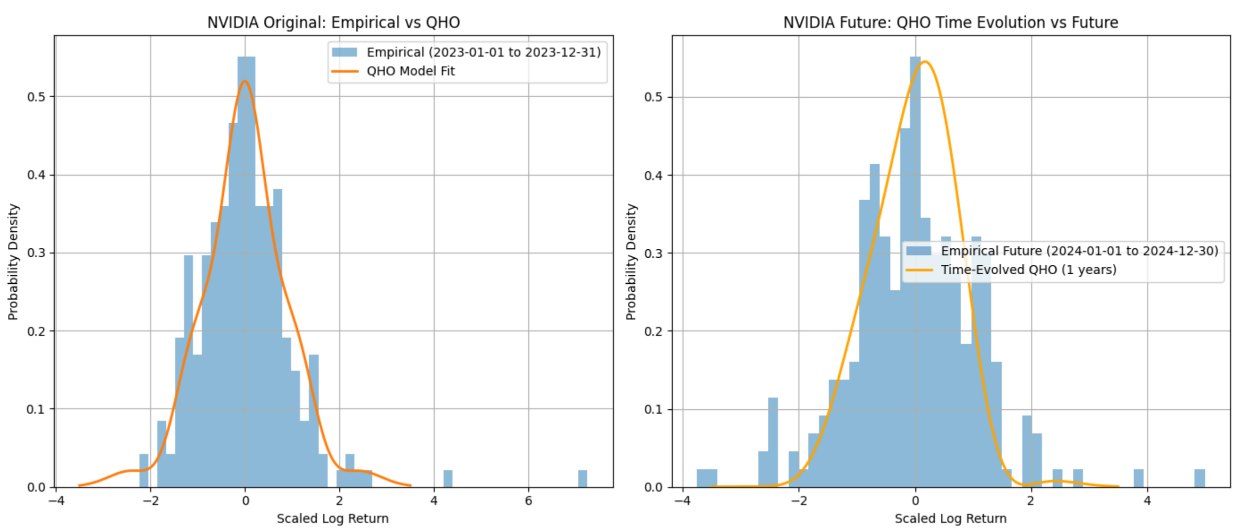
$$\psi_n(x) \text{ (nth eigenfunction)} = [1/(\sqrt{(2^n n!)})][(m\omega)/(\pi\hbar)]^{1/4}[e^{-[(m\omega x)^2]/(2\hbar)}]H_n(x\sqrt{[(m\omega)/\hbar]}) \quad (2)$$

Next, a superposition of the square of the first six (n = 0,1,2,3,4,5) eigenfunctions of the quantum harmonic oscillator model was fitted as a wave function to represent the empirical distributions for each of the past empirical stock return distributions that were used. This led to the creation of six graphs to represent 1-day returns for S&P 500 and Nvidia, 5-day returns for S&P 500 and Nvidia, and 20-day returns for S&P 500 and Nvidia during the year 2023. Each of these graphs showed the fitted quantum harmonic oscillator model curve compared with the actual empirical distributions of 2023 for S&P 500 and Nvidia.

After each of the past quantum harmonic oscillator models were created, a time-evolved quantum harmonic oscillator model was defined, with time units scaled to weeks. To do this, equation 3 was applied to the wavefunctions of the original quantum harmonic oscillator model, for each of the six models. In equation 3, c_n represents the proportion of the corresponding nth eigenfunction, $\psi_n(x)$ is the nth eigenfunction, $e^{-(iEnt)/\hbar}$ is the phase factor, and En is the nth eigenenergy. The time evolution was scaled to 52 weeks to represent a one year evolution of the wave function to represent stock returns of 2024. The time-evolved graphs were compared with the actual 2024 empirical distributions of stock returns.

$$\psi(x,t) = \sum c_n \psi_n(x) e^{-(iEnt)/\hbar} \quad (3)$$

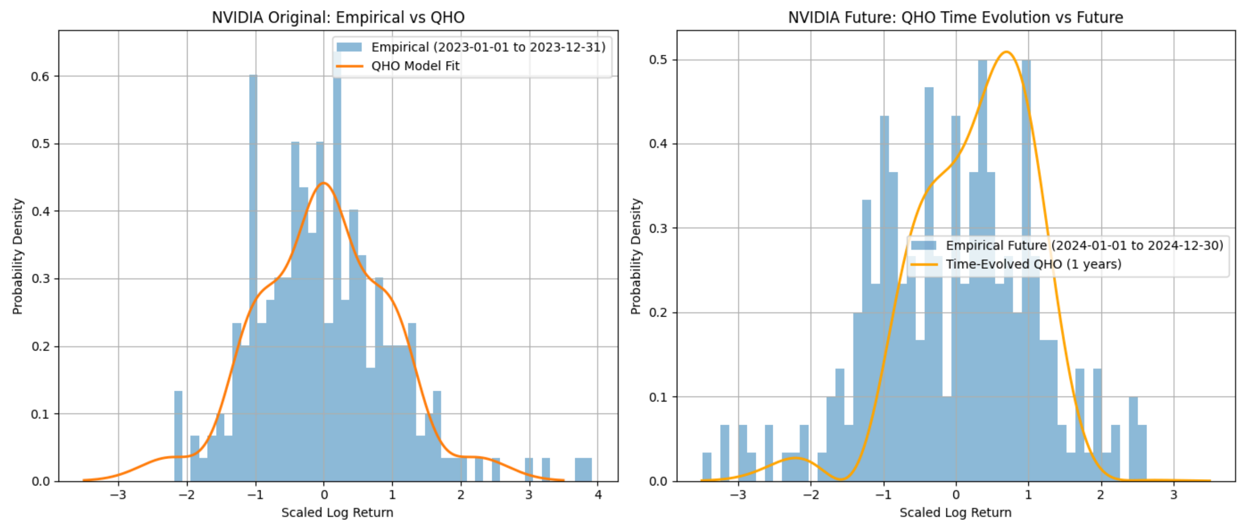
Results and Discussion



(a) (b)

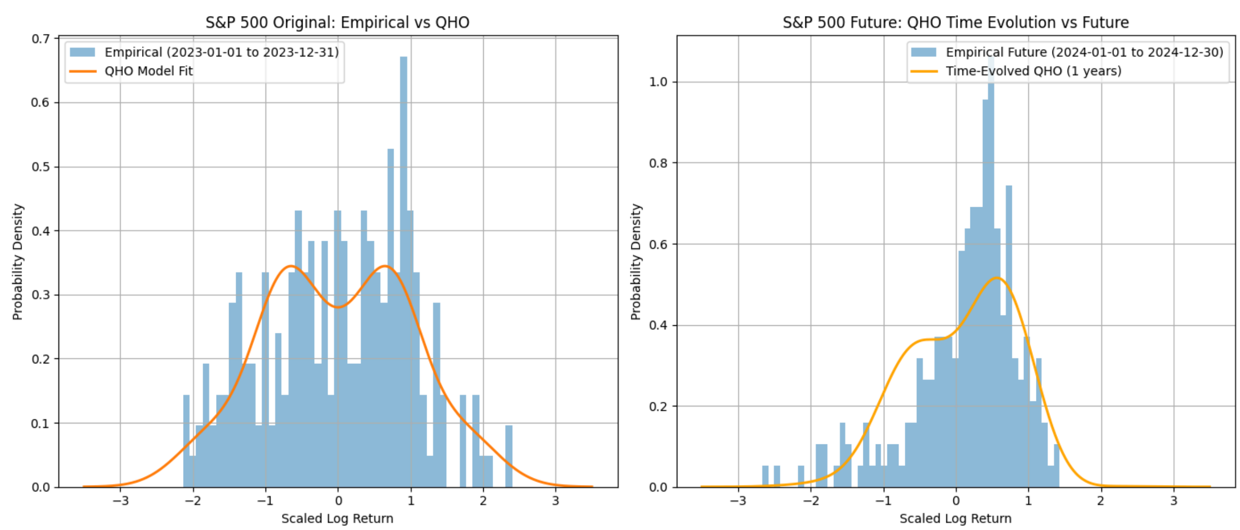
FIGURE 1. (a) Probability density graph of past quantum harmonic oscillator model wave function compared to 2023 Nvidia empirical stock return distribution for 1 day returns. (b) Probability density graph of time-evolved quantum harmonic oscillator model

wave function compared to 2024 Nvidia empirical stock return distribution for 1 day returns.



(a) (b)

FIGURE 2. (a) Probability density graph of past quantum harmonic oscillator model wave function compared to 2023 Nvidia empirical stock return distribution for 5 day returns. (b) Probability density graph of time-evolved quantum harmonic oscillator model wave function compared to 2024 Nvidia empirical stock return distribution for 5 day returns.



(a) (b)

FIGURE 3. (a) Probability density graph of past quantum harmonic oscillator model wave function compared to 2023 S&P 500 empirical stock return distribution for 20 day returns. (b)

Probability density graph of time-evolved quantum harmonic oscillator model wave function compared to 2024 S&P 500 empirical stock return distribution for 20 day returns.

While a total of 12 graphs were created, only six of them are displayed in the results to limit repetitiveness because there was little difference in the accuracy of the quantum models for different stocks; The above six graphs include three pairs of past quantum models and their time-evolved quantum model for 1-day, 5-day, and 20-day trading periods. For 1-day and 5-day returns, the graphs for Nvidia are displayed, while for 20-day returns, the graphs for S&P 500 are displayed.

Similar to previous research done by Ahn et al. [1] and Bhatt & Gor [2] where the quantum harmonic oscillator was used to model past stock returns, the quantum harmonic oscillator models in this paper for modeling stock returns also accurately captured the general shape of the actual distributions of the stocks in 2023. This indicates that the quantum harmonic oscillator model is also suitable for capturing past stock returns for shorter durations of time like one year instead of one decade. Additionally, there did not appear to be a difference in how accurate the model was for different trading-periods. This is also similar to the results of Ahn et al. [1], which included graphs for 1-day, 5-day, and 20-day trading periods showing that the quantum model worked similarly well for all three trading periods.

The time-evolved models also seemed to capture the overall shapes of the 'future' stock return distributions for 2024 in each of the three time-evolved graphs. Unlike the non-time-evolved models, which are symmetric due to the square of the eigenfunctions of the quantum harmonic oscillator being symmetric, the time-evolved models were not necessarily normal. During time-evolution, the oscillation of the wave function can lead to the wavefunction becoming non-symmetric, which can allow for the final result to capture if stock returns are more bearish or more bullish. In Fig. 1(b), the time-evolved model accurately captures the fact that the amount of bullish and bearish returns is pretty much equal. This indicates that day-trading during 2024 for Nvidia was likely to give zero gain or loss. In Fig. 2(b) and Fig. 3(b), the time-evolved model captures the higher concentration of bullish returns, for Nvidia 5-day returns and S&P 500 20-day returns, respectively. This indicates that longer-term trading of Nvidia and S&P 500 during 2024 was likely to lead to positive returns over the course of the year.

The main issue with these results is that it is difficult to gauge the true accuracy of the time-evolved quantum models because the model was not used for a large number of different stocks. Thus, any interpretations that were made from the few time-evolved models in this paper are potentially biased and not necessarily indicative of the time-evolved quantum harmonic oscillator for predicting future stock returns in general. The results of this paper should be extended by applying the time-evolved quantum model to a variety of different stocks while focusing on one trading period and time duration, and then running statistical tests to calculate the average errors of the time-evolved model for all of the stocks. This would provide a better estimation of how applicable the quantum harmonic oscillator would be for predicting future stock returns in general.

Conclusion

Similar to previous research, the Quantum Harmonic Oscillator model of past stock returns mostly aligned with the past empirical distributions. The main extension of this paper from previous papers was adding a time-evolved model. The time-evolved model for the most part, also seemed to match the "future" empirical distributions. Throughout the project, there was difficulty assessing the accuracy of the time-evolved model due to not having exact error graphs as well as not having previous papers to compare results with. Interpretations of how accurate the quantum model was were done from looking at the quantum model wave functions' deviation from the empirical distributions. This research could have been better with statistical tests to test exactly how much error the time-evolved model had. In the future, researchers should apply the time-evolved quantum model to a large variety and quantity of stocks, and then create statistical tests to determine the average magnitude of the error of the time-evolved model. They should then compare the time-evolved quantum model with other stochastic predictors of stock distributions. If the errors prove to be marginal, this would suggest the quantum harmonic oscillator model is useful for predicting future stock return distributions for durations of time that are at least about one year long.

References

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