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Robust adaptive control for vehicle active suspension systems with uncertain dynamics

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Abstract

This paper proposes a new robust adaptive law for adaptive control of vehicle active suspensions with unknown dynamics (e.g. non-linear springs and piece-wise dampers), where precise estimation of essential vehicle parameters (e.g. mass of vehicle body, mass moment of inertia for the pitch motions) may be achieved. This adaptive law is designed by introducing a novel leakage term with the parameter estimation error, such that exponential convergence of both the tracking error and parameter estimation error may be proved simultaneously. Appropriate comparisons with several traditional adaptive laws (e.g. gradient and σ-modification method) concerning the convergence and robustness are presented. The mitigation of the vertical and pitch displacements can be achieved with the proposed control to improve the ride comfort. The suspension space limitation and the tyre road holding are also studied. A dynamic simulator consisting of commercial vehicle simulation software Carsim® and Matlab® is built to validate the efficacy of the proposed control scheme and to illustrate the improved estimation performance with the new adaptive law.

Keywords

Active suspension control, adaptive control, parameter estimation, vehicle suspension systems

Introduction

The design of a vehicle suspension system is essential for improving the ride comfort, maintaining vehicle manoeuvrability and retaining the safety of passengers (Cao et al., 2011; Hrovat, 1997). Generic vehicle suspension systems include wishbone, spring and shock absorber (e.g. damper) to transmit and mitigate forces between the car body and the road, which may contribute to the passenger comfort and the ride safety (Sun et al., 2011; Zhang et al., 2016). However, it was noted that the widely used passive suspension systems that use the springs and dampers with fixed physical dynamics (e.g. stiffness and damper coefficients) may not be able both to eliminate the vibrations from versatile road bumps and to achieve satisfactory suspension performance under wider driving manoeuvres. This fact stimulated the recent developments of semi-active suspension and active suspension (Canale et al., 2006; Sankaranarayanan et al., 2008; Savaresi and Spelta, 2009; Sun et al., 2015; Tseng and Hrovat, 2015).

The principle of semi-active suspensions is to use the springs and dampers whose properties (e.g. stiffness and damper coefficients) may be adjusted corresponding to different vehicle driving scenarios (Fallah et al., 2012; Zapateiro et al., 2012). These varying characteristic coefficients may retain considerable improvements over passive suspension systems. However, specific springs and/or dampers (e.g. a magnetorheological, MR, damper) must be used in this case and the construction of such components with tuning coefficients may not be trivial. Moreover, the use of such elements in the suspension system may lead to potential difficulties; for example, MR dampers exhibit hysteresis dynamics, which are difficult to model.

On the other hand, active suspension has also attracted significant attention because in this framework extra actuators are placed between the car body and the wheel-axle, and operated in parallel to other suspension mechanisms (e.g. damper and spring; Yagiz and Hacioglu, 2008). Consequently, these actuators may be controlled appropriately to dissipate energy from the road disturbances, and thus enable the suspension systems to reduce the uninterrupted effectiveness of road roughness and the displacement of vehicles. It is recognized that active suspension imposes high energy demand and increased cost, and has not been widely used in commercial vehicles. However, it is believed that active suspension techniques will be gradually adopted by the industry due to its emerging potentials to improve the suspension performance. In fact, considerable work on active suspension has been carried out in the past few decades (Allotta et al., 2008; Hoque et al., 2006; Karimi, 2006; Readman et al., 2010; Waldron and Abdallah, 2007).

In spite of hardware configuration, another crucial issue in the active suspension system designs is the control strategies,

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which should make the actuators pull down or push up the suspension motions, and also manage other suspension requirements. In Zuo et al. (2005), a model-reaching adaptive control was presented to achieve the ideal isolation of a sky-hook target. A robust control was designed (Leite and Peres, 2005) for active suspension of a quarter-car model by means of the state feedback gains. In Onat et al. (2009), a LPV gain-scheduling controller was proposed for a quarter-vehicle active suspension system. In Du and Zhang (2007), an H_∞ control was designed for active suspension systems that are subject to actuator time delay. In Akbari and Lohmann (2010), a linear quadratic Gaussian (LQG) control was used to obtain a trade-off between the conflicting suspension requirements. However, in most aforementioned results, the studied vehicle suspension systems are assumed to be with linear dynamics or precisely known.

It is known that the uncertainties and/or modelling errors are usually unavoidable in practical vehicle systems; for example, with the change of the number of passengers or the payload, the spring and damper may have non-linear or piece-wise behaviours, which cause notable system variations. In the past few decades, adaptive control (Ioannou and Sun, 1996; Slotine and Li, 1991) has been developed as an efficient technique to address the parameter uncertainties and even non-linearities in the control systems (Huang et al., 2015; Na et al., 2013b, 2014; Ren et al., 2009; Wang and Er, 2015; Zheng and Park, 2015). Considering this, adaptive control may also be used in the vehicle active suspension systems. Adaptive sliding-mode control scheme has been adopted in several studies (Chin and Lum, 2011; Chin and Wheeler, 2013; Huang and Chen, 2006) to address uncertainties. A non-linear adaptive neural network (NN) control was designed (Zapateiro et al., 2009) based on the backstepping approach. To address other suspension requirements further, an adaptive backstepping control was proposed in Sun et al. (2013), where the suspension spaces, dynamic tyre loads and actuator saturations are all considered. It is known that the non-linear stiffening spring and piece-wise damper dynamics are all assumed to be known in Sun et al. (2013), where only the vehicle body mass and the mass moment of inertia for the pitch motion are online updated in terms of the gradient method, such that the estimates may not converge to their true values.

In most available adaptive control methods, adaptive laws are designed by minimizing the tracking error via the gradient descent strategies (Ioannou and Sun, 1996; Slotine and Li, 1991). A well-known shortcoming of this framework is the potential bursting phenomenon, e.g. the estimated parameters may be unbounded when the system is subject to disturbances (Ioannou and Sun, 1996). The subsequent robust modifications (e.g. e-modification and σ-modification) or projection method (Slotine and Li, 1991), however, cannot guarantee the parameter estimation convergence due to the introduced forgetting factors. According to the certainty equivalence principle (Slotine and Li, 1991), it is possible to achieve better control response by proposing new adaptive laws, which may drive the estimated parameters to their true values.

Motivated by this and our recent work (Na et al., 2011, 2013a), which originally presented a novel parameter estimation framework and several adaptive law designs, we will propose a novel robust adaptive control for vehicle active suspension systems with unknown non-linear springs and piece-wise dampers. Thus, an improved adaptive law based on Na et al. (2011, 2013a) is suggested. This adaptive law is designed by developing a new leakage term with the parameter estimation error, which is obtained by applying filter operations on the tracking error dynamics. In this case, we may prove that the estimated parameters converge to their true values under the persistent excitation (PE) condition as Na et al. (2015). Appropriate comparisons between the novel adaptive law with several classical adaptive laws (e.g. gradient and σ-modification) are theoretically examined to address their convergence and robustness. Moreover, NNs are used to compensate online for the unknown dynamics of non-linear stiffening springs and piece-wise dampers. Some essential parameters (e.g. vehicle mass and inertia of pitch motion) may be precisely online estimated as well as the NN weights. Several other suspension performance requirements concerning the ride comfort, road holding and mechanical constraints are also studied. Simulations based on a half-car system in a dynamic simulator built with a professional vehicle simulation software Carsim® and Matlab® are provided to validate the efficacy of the proposed methods. The proposed control schemes illustrate the improved estimation and control performance.

The main contributions of this paper may be summarized as follows:

1) A robust adaptive control is proposed to study the active suspension control for vehicle systems, in which the unknown dynamics of non-linear springs and piece-wise dampers may be compensated by using augmented NNs. This control may also address the road holding and other suspension requirements.
2) A new adaptive law with a novel leakage term of the parameter estimation error is incorporated into adaptive control, such that both the suspension control and parameter estimation may be achieved simultaneously. Moreover, extensive comparisons with traditional adaptive laws are also provided.

The remainder of this paper is organized as follows: the problem statement is given in the next section. The design of adaptive control with a novel adaptive law for vertical displacement and the comparisons with other classical adaptations are derived, and adaptive control for the pitch motions and suspension performance analysis are presented. Simulations are provided and conclusions are finally given.

Problem formulation

In this paper, a half-car system with unknown non-linearities is studied, shown in Figure 1. All variables used in Figure 1 are defined in Table 1. The objective of this paper is to present a novel adaptive control, which provides appropriate control inputs u_1, u_2 to the active suspension systems, such that the vertical displacement y_c is mitigated and other suspension requirements (e.g. road holding and suspension space limitation) may be retained.

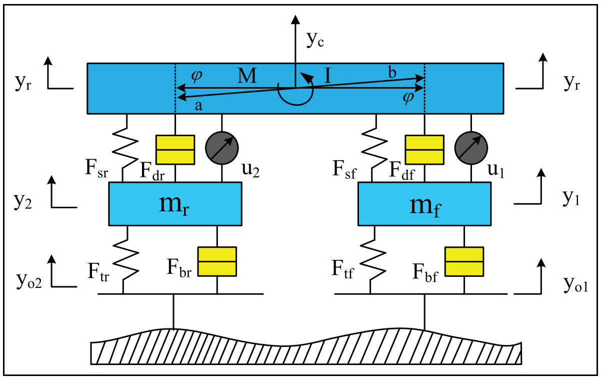


Figure 1. Schematic of active suspension control for half-car systems.

It should be noted that various control strategies have been proposed to address the active suspension problem, e.g. robust control (Du and Zhang, 2007), optimal control (Akbari and Lohmann, 2010) and adaptive control (Sun et al., 2013). However, the main contribution of this paper is to provide an alternative adaptive law, which may be incorporated into the standard adaptive control framework such that the online estimation of unknown model parameters and the suspension control may be guaranteed simultaneously. Moreover, the non-linear and unknown forces of the dampers and springs are considered in this paper. Thus, the proposed method may cover more realistic cases compared with other results (e.g. Sun et al., 2013), where the forces of the springs and dampers are all assumed to be linear and/or precisely known. It should be noticed that the forces produced by the springs, dampers and tyres are related to the motions of the sprung mass, unsprung mass and tyres, respectively.

We first present mathematical model of the active suspension system in Figure 1 as (Sun et al., 2013):

$$\begin{cases} M\ddot{y}_c + F_{df} + F_{dr} + F_{sf} + F_{sr} = u_y \\ I\ddot{\varphi} + a(F_{df} + F_{sf}) - b(F_{dr} + F_{sr}) = u_\varphi \\ m_f\ddot{y}_1 - F_{sf} - F_{df} + F_{tf} + F_{bf} = -u_1 \\ m_r\ddot{y}_2 - F_{sr} - F_{dr} + F_{tr} + F_{br} = -u_2 \end{cases} \tag{1}$$

where $u_y = u_1 + u_2$ and $u_\varphi = au_1 - bu_2$ are the lumped control actions, which are obtained in terms of the proposed control methods. Then, the real control actions u_1 and u_2 for the actuators may be calculated based on the output of controllers u_y and u_φ as

$$u_1 = \frac{bu_y + u_\varphi}{a+b}, u_2 = \frac{au_y - u_\varphi}{a+b} \tag{2}$$

Define the state variables as $x_1 = y_c, x_2 = \dot{y}_c, x_3 = \varphi, x_4 = \dot{\varphi}, x_5 = y_1, x_6 = \dot{y}_1, x_7 = y_2, x_8 = \dot{y}_2$, then system (1) is represented as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{M}(-F_{df} - F_{dr} - F_{sf} - F_{sr} + u_y) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{1}{I}[-a(F_{df} + F_{sf}) + b(F_{dr} + F_{sr}) + u_\varphi] \end{cases} \tag{3}$$

$$\begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = \frac{1}{m_f}(F_{sf} + F_{df} - F_{tf} - F_{bf} - u_1) \\ \dot{x}_7 = x_8 \\ \dot{x}_8 = \frac{1}{m_r}(F_{sr} + F_{dr} - F_{tr} - F_{br} - u_2) \end{cases} \tag{4}$$

where $\theta_1 = 1/M$ and $\theta_2 = 1/I$ are the uncertain parameters depending on the payloads or passengers. In this paper, the parameters θ_1 and θ_2 will be precisely estimated, which may help to facilitate other control and fault diagnosis purpose.

In realistic vehicles, the requirements for active suspension systems include not only eliminating vibrations from the road bumps but also considering the following aspects, which have been well recognized in the field (Cao et al., 2011; Ma and Chen, 2011; Sun et al., 2013):

1) *Ride comfort*. This means that the vertical and pitch motions x_1, x_3 of the car body need to be mitigated. This may be achieved by manipulating the actuators via u_1,u_2 to isolate the vehicle body from the road-induced shocks to enhance the comfort to passengers.
2) *Road holding*. The firm uninterrupted contact of wheels to the road should be firmly ensured, i.e. the dynamic tyre loads should not be too large for both the front and rear wheels, i.e.

$$|Z_f| = |F_{tf} + F_{bf}| < F_f, |Z_r| = |F_{tr} + F_{br}| < F_r \tag{5}$$

where the static tyre loads F_f and F_r are calculated by

$$F_f = \frac{Mgb + m_f g(a+b)}{a+b}, F_r = \frac{Mga + m_r g(a+b)}{a+b} \tag{6}$$

3) *Suspension movement limitation*. The suspension space should not exceed the allowable maximum values due

Table 1. Notation and physical meaning of parameters.

Variable	Physical meaning	Variable	Physical meaning
M	Mass of vehicle body	y_c	Vertical displacement
I	Inertia for pitch motion	φ	Pitch angle
m_f, m_r	Unsprung masses	y_1, y_2	Displacement of unsprung masses
F_{df}, F_{dr}	Forces of dampers	y_{o1}, y_{o2}	Road disturbance inputs
F_{sr}, F_{sf}	Forces of springs	a, b	Distances of suspensions to Centre mass of vehicle
F_{tr}, F_{tf}	Elasticity forces of the tires	u_1, u_2	Active suspension control input
F_{br}, F_{bf}	Damping forces of the tires		

to the restricted mechanical structures. The front and rear suspension spaces

$$\Delta y_f = y_c + a\sin\varphi - y_1, \Delta y_r = y_c - b\sin\varphi - y_2 \quad (7)$$

should be bounded by the maximum suspension deflections $\Delta y_{f\,\max}$ and $\Delta y_{r\,\max}$ as

$$\left|\Delta y_f\right| \le \Delta y_{f\,\max}, \left|\Delta y_r\right| \le \Delta y_{r\,\max} \quad (8)$$

Note the above performance indices have been well established and widely used in the literature for vehicle suspension control (e.g. Cao et al., 2011; Ma and Chen, 2011; Sun et al., 2013).

Adaptive suspension control design for vertical displacement

In this section, we propose an adaptive control to regulate x_1 and x_2 of system (3) and develop a novel adaptive law based on the parameter estimation error to retain the precise parameter estimation.

Adaptive control design

To address the suspension of vertical displacement x_1 in (3) by designing control u_y, we define a filtered error variable as:

$$s_1 = [\Lambda, 1][x_1, x_2]^T \quad (9)$$

where $\Lambda > 0$ is a positive constant. Then, based on Slotine and Li (1991), we know that x_1, x_2 are bounded if s_1 is bounded. Specifically, as shown in Slotine and Li (1991), the relationship between x_1 and s_1 may be reformulated as $\dot{x}_1 = -\Lambda x_1 + s_1$. Then we may solve this equation and verify $|x_1| \le |s_1|/\Lambda$, which further implies $|x_2| = |\dot{x}_1| \le |\Lambda x_1| + |s_1| \le 2|s_1|$.

Then we obtain the time derivative of s_1 along (3) as:

$$\begin{aligned} \dot{s}_1 &= \Lambda x_2 + \theta_1(-F_{df} - F_{dr} - F_{sf} - F_{sr} + u_y) \\ &= \Lambda x_2 + {w_1}^T\phi_1(Z_1) + \theta_1 u_y + \varepsilon_1 \\ &= \Lambda x_2 + {W_1}^T\Phi_1 + \varepsilon_1 \end{aligned} \quad (10)$$

where an NN is used to address the unknown dynamics over a compact set Ω as $T_1(Z_1) = \theta_1(-F_{df} - F_{dr} - F_{sf} - F_{sr}) = {w_1}^T\phi_1(Z_1) + \varepsilon_1, \forall Z_1 \in R^L$, where $w_1 = [w_{11}, w_{12}, \ldots, w_{1L}]^T \in R^L$ is the ideal NN weight vector, $\phi_1(Z_1) = [\phi_{11}, \phi_{12}, \ldots, \phi_{1L}]^T \in R^L$ is the regressor and $\varepsilon_1 \in R$ is the approximation error. It is known that $\|w_1\| \le w_{1N}, |\varepsilon_1| \le \varepsilon_{1N}$ with w_{1N} and ε_{1N} being positive constants. Note a typical linearly parameterized NN as in Na et al. (2013b, 2014) and Ren et al. (2009) is used in this paper. This NN has lower computational costs than other NNs (e.g. RBF NN and multilayer NN) and may be easily augmented to estimate the vehicle parameter θ_1.

For this purpose, we define $W_1 = [w_1^T, \theta_1]^T$ and $\Phi_1 = [\phi_1^T, u_y]^T$ as the augmented parameter vector and regressor vector, respectively. Then, we denote $\hat{W}_1 = [\hat{w}_1^T, \hat{\theta}_1]^T$ as the estimation of W_1 and design the control u_y as

$$u_y = \frac{1}{\hat{\theta}_1}[-{\hat{w}_1}^T\phi_1(Z_1) - k_1 s_1 - \Lambda x_2] \quad (11)$$

where $k_1 > 0$ is the feedback gain, $\hat{\theta}_1$ and $\hat{w}_1$ are the estimations of θ_1 and w_1, which will be updated online based on the adaptive law (15) to be designed.

Remark 1. The main idea of this paper is to propose a novel adaptive law (in comparison with classical schemes) to update $\hat{\theta}_1$ and $\hat{w}_1$ simultaneously, which will converge to their true values θ_1 and w_1. Moreover, it is worth noting that the mass θ_1 is unknown and the non-linear forces $F_{df}, F_{dr}, F_{sf}, F_{sr}$ of the springs and dampers are unknown, which may be lumped as $T_1(Z_1)$. Thus, the proposed method in this paper may cover more realistic cases compared with other results (e.g. Sun et al., 2013), where the forces of the springs and dampers are all assumed to be linear and/or precisely known.

Novel adaptive law

To design a new adaptive law to obtain $\hat{W}_1$ with guaranteed convergence, we define the variables $s_{1f}, \Phi_{1f}, x_{2f}$ by applying filter operations on system (10) as:

$$\begin{cases} k\dot{s}_{1f} + s_{1f} = s_1, & s_{1f}(0) = 0 \\ k\dot{\Phi}_{1f} + \Phi_{1f} = \Phi_1, & \Phi_{1f}(0) = 0 \\ k\dot{x}_{2f} + x_{2f} = x_2, & x_{2f}(0) = 0 \end{cases} \quad (12)$$

where $k > 0$ is a scalar filter parameter.

Moreover, we define the auxiliary matrix P_1 and vector Q_1 as

$$\begin{cases} \dot{P}_1 = -lP_1 + \Phi_{1f}\Phi_{1f}^T, & P_1(0) = 0 \\ \dot{Q}_1 = -lQ_1 + \Phi_{1f}[(s_1 - s_{1f})/k - \Lambda x_{2f}], & Q_1(0) = 0 \end{cases} \quad (13)$$

where $l > 0$ is a design parameter. It should be noted that the purpose of above filter operations (i.e. Equations (12) and (13)) is to obtain another auxiliary vector, which contains parameter estimation error for designing the adaptive law.

For this purpose, another auxiliary vector H_1 may be obtained based on P_1, Q_1 in (13) as:

$$H_1 = P_1\hat{W}_1 - Q_1 \quad (14)$$

where $\hat{W}_1$ is the estimation of W_1.

Now, the adaptive law for updating $\hat{W}_1$ is given by:

$$\dot{\hat{W}}_1 = \Gamma_1 s_1 \Phi_1 - \Gamma_1\sigma H_1 \quad (15)$$

with $\Gamma_1 > 0$ a constant learning gain and $\sigma > 0$ a positive constant.

The control structure with the proposed adaptive law for the vertical displacement may be found in Figure 2.

Remark 2. For adaptive law (15), $s_1\Phi_1$ is the gradient term used to guarantee the boundedness of the tracking error s_1; the second term σH_1 based on the variable H_1 in (14) is taken as a new leakage term, which contains the information of the error $\tilde{W}_1 = W_1 - \hat{W}_1$ between the unknown parameter W_1

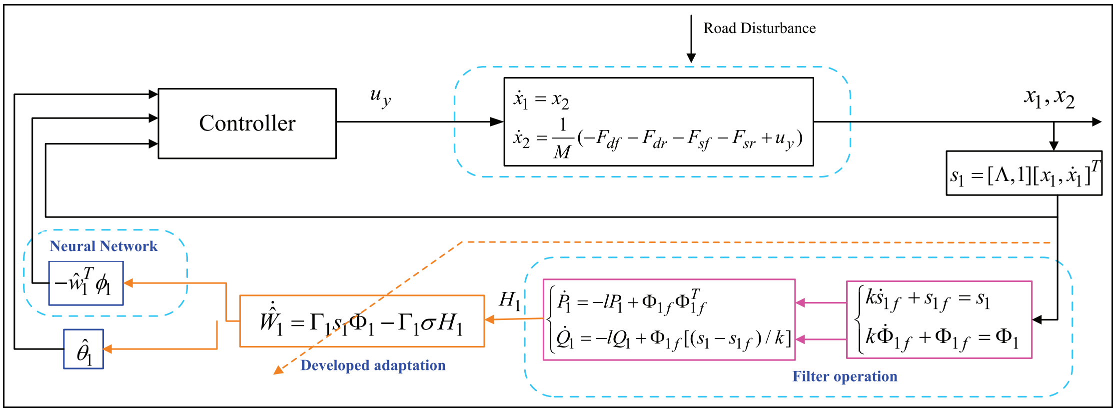


Figure 2. Diagram of adaptive control for vertical displacement.

and its estimation $\hat{W}_1$. As shown in Na et al. (2015) and the following stability analysis, the use of this term in the adaptive law (15) will lead to a quadratic term of $\tilde{W}_1$ in the Lyapunov analysis, such that faster error convergence and accurate estimation performance may be proved.

Remark 3. The novel leakage term σH_1 introduced in (15) is different from σ-modification and e-modification (Ioannou and Sun, 1996), which have been used to guarantee the boundedness of $\hat{W}_1$ and avoid the bursting phenomenon in the gradient scheme (Ioannou and Sun, 1996). Comparisons with these classical adaptations will be detailed later.

Stability analysis

To summarize the main result of this section, we first give the following Lemmas:

Lemma 1. *The vector H_1 defined in (14) can be reformulated as $H_1 = -P_1\tilde{W}_1 + \Delta_1$, where $\Delta_1 = -\int_0^t e^{-l(t-r)}\Phi_{1f}(r)\varepsilon_{1f}(r)dr$ with ε_{1f} being the filtered version of the NN error ε_1 in terms of $k\dot{\varepsilon}_{1f} + \varepsilon_{1f} = \varepsilon_1$, and thus Δ_1 is bounded by $\|\Delta_1\| < \varepsilon_{1Nf}$ for a constant $\varepsilon_{1Nf} > 0$.*

Proof. The proof of Lemma 1 may be conducted by solving the ordinary matrix differential equation (13) and applying the filter operation (12) on both sides of (10). We refer to Na et al. (2015) for a similar proof. Due to the limited space, we do not repeat it again in this paper.

Lemma 2 *(Na et al., 2011, 2013a). If the augmented regressor Φ_1 defined in (10) is PE (Ioannou and Sun, 1996), then the matrix P_1 defined in (13) is positive definite, i.e. its minimum eigenvalue fulfils $\lambda_{\min}(P_1) > \sigma_1 > 0$ with σ_1 being a positive constant.*

Proof. We refer to Na et al. (2011, 2013a) for detailed proof.
Now, the main results of this section are summarized as follows:

Theorem 1. *Consider the vertical displacement x_1 of system (3) with unknown dynamics, we design the adaptive control (11) with adaptive law (15), if the regressor vector Φ_1 in (10) is PE, then the control error s_1 and estimation error $\tilde{W}_1$ converge to a small set around zero, and all signals in the closed-loop are bounded.*

Proof. We first substitute the control (11) into (10), such that the closed-loop error dynamics are given as

$$\dot{s}_1 = -k_1 s_1 + \tilde{W}_1{}^T \Phi_1 + \varepsilon_1 \tag{16}$$

Select a Lyapunov function as

$$V_1 = \frac{1}{2}s_1{}^2 + \frac{1}{2}\tilde{W}_1{}^T \Gamma_1{}^{-1}\tilde{W}_1 \tag{17}$$

According to Lemma 2, if the regressor vector Φ_1 is PE, the condition $\lambda_{\min}(P_1) > \sigma_1 > 0$ holds. Then $\dot{V}_1$ may be computed from (15) and (16) as

$$\begin{aligned}\dot{V}_1 &= s_1(-k_1 s_1 + \tilde{W}_1{}^T\Phi_1 + \varepsilon_1) + \tilde{W}_1{}^T(-s_1\Phi_1 + \sigma H_1)\\ &= -k_1 s_1{}^2 + s_1\varepsilon_1 - \sigma\tilde{W}_1{}^T P_1 \tilde{W}_1 + \sigma\tilde{W}_1{}^T\Delta_1\end{aligned} \tag{18}$$

By applying Young's inequality $ab \le a^2\eta/2 + b^2/2\eta$ for $\eta > 0$ on (18), one may have

$$\begin{aligned}\dot{V}_1 \le &-\left(k_1 - \frac{1}{2\eta}\right)s_1{}^2 - \sigma\left(\sigma_1 - \frac{1}{2\eta}\right)\|\tilde{W}_1\|^2 + \frac{\eta}{2}\varepsilon_{1N}^2\\ &+ \frac{\sigma\eta}{2}\varepsilon_{1Nf}^2 \le -\tilde{\mu}_1 V_1 + \gamma_1\end{aligned} \tag{19}$$

where $\tilde{\mu}_1 = \min\{2(k_1 - 1/2\eta), 2\sigma(\sigma_1 - 1/2\eta)/\lambda_{\max}(\Gamma_1{}^{-1})\}$ and $\gamma_1 = \eta(\varepsilon_{1N}^2 + \sigma\varepsilon_{1Nf}^2)/2$ are positive constants for $k_1 > 1/2\eta$ and $\eta > 1/2\sigma_1$. According to the Lyapunov Theorem (Ioannou and Sun, 1996), we know s_1 and $\tilde{W}_1$ are uniformly ultimately bounded. Moreover, we obtain from (19) that $V_1(t) \le (V_1(0) - \gamma_1/\tilde{\mu}_1)e^{-\tilde{\mu}_1 t} + \gamma_1/\tilde{\mu}_1$, so that s_1, $\tilde{W}_1$ converge to a compact set defined by $\Omega_1 = \{\tilde{W}_1, s_1 | \|\tilde{W}_1\| \le \sqrt{2\gamma_1/\tilde{\mu}_1\lambda_{\min}(\Gamma_1{}^{-1})}, |s_1| \le \sqrt{2\gamma_1/\tilde{\mu}_1}\}$, whose size depends on the bound of NN approximation error ε_{1N}, the excitation level σ_1 and the control parameters Λ, k_1, σ and Γ_1. In this case, one may verify from (9) that x_1 and x_2 converge to a small residual set around zero, i.e. $|x_1| \le |s_1|/\Lambda \le \sqrt{2\gamma_1/\tilde{\mu}_1}/\Lambda$ and $|x_2| \le 2|s_1| \le 2\sqrt{2\gamma_1/\tilde{\mu}_1}$. Consequently, the active control action u_y in (11) is also bounded because $s_1, x_2, \hat{\theta}_1$ and $\hat{w}_1{}^T\phi_1(Z_1)$ are all bounded.

Comparison with other adaptations

One major contribution of this paper is to develop the adaptive law (15) with a new leakage term σH_1, which may achieve improved parameter estimation performance over some well-known adaptive laws. The aim of this subsection is to compare the convergence and robustness of the proposed adaptive law (15) with the gradient method and σ-modification scheme. For this purpose, three different adaptive laws and their error dynamics will be summarized:

Gradient method (Ioannou and Sun, 1996). The adaptive law is solely driven by the tracking error s_1 as

$$\dot{\hat{W}}_1 = \Gamma_1 s_1 \Phi_1 \tag{20}$$

Then the estimation error of (20) is given by:

$$\dot{\tilde{W}}_1 = -\Gamma_1 s_1 \Phi_1 \tag{21}$$

The gradient method (20) suffers from the potential bursting phenomenon in the estimated parameter $\hat{W}_1$ (Ioannou and Sun, 1996) and the poor robustness to uncertainties and disturbances. In fact, from (21), the boundedness of $\tilde{W}_1$ cannot be claimed even when the control error s_1 is bounded and the convergence of $\tilde{W}_1$ to zero does not hold even when the control error s_1 converges to zero.

σ-modification (Ioannou and Sun, 1996). To address the boundedness of the estimated parameter $\hat{W}_1$, a modification term is added in (20) to create the σ-modification method:

$$\dot{\hat{W}}_1 = \Gamma_1 s_1 \Phi_1 - \Gamma_1 \sigma \hat{W}_1 \tag{22}$$

where $\sigma > 0$ is a positive constant, and the estimation error may be obtained as:

$$\dot{\tilde{W}}_1 = -\Gamma_1 s_1 \Phi_1 + \Gamma_1 \sigma \hat{W}_1 = -\Gamma_1 \sigma \tilde{W}_1 - \Gamma_1 s_1 \Phi_1 + \Gamma_1 \sigma W_1 \tag{23}$$

It is shown in (23) that a forgetting factor $\Gamma_1 \sigma \tilde{W}_1$ is included in (23), which comes from the leakage term $\sigma \hat{W}_1$ in (22). Thus, one may verify that the error dynamics $\tilde{W}_1$ in (23) is bounded-input–bounded-output (BIBO), i.e. if the tracking error s_1 and unknown parameter W_1 are bounded, the estimation error $\tilde{W}_1$ is bounded. Consequently, the estimated parameter $\hat{W}_1$ is also bounded, and thus the σ-modification (22) has guaranteed robustness (Ioannou and Sun, 1996).

On the other hand, the involved damping term $\sigma \hat{W}_1$ in (23) makes the estimated parameter $\hat{W}_1$ stay in a neighbourhood of a pre-selected value rather than converge to their true value W_1. This is natural because the transfer function of (23) may be represented as $\tilde{W}_1 = \frac{1}{p+\Gamma_1\sigma}(-\Gamma_1 s_1 \Phi_1 + \Gamma_1 \sigma W_1)$ (p is the Laplace operation), which implies that the ultimate bound of $\tilde{W}_1$ depends on the unknown parameter W_1 and s_1. Consequently, $\tilde{W}_1$ cannot converge to zero even when s_1 is zero.

Proposed method. In the adaptive law (15), a new leakage term $\sigma P_1 \tilde{W}_1$ is used so that the estimation error dynamics are given by:

$$\dot{\tilde{W}}_1 = -\sigma \Gamma_1 P_1 \tilde{W}_1 - \Gamma_1 s_1 \Phi_1 + \sigma \Gamma_1 \Delta_1 \tag{24}$$

A forgetting factor $\sigma P_1 \tilde{W}_1$ is also included in (24), thus the estimation error $\tilde{W}_1$ is also BIBO stable, i.e. $\tilde{W}_1$ is bounded if the tracking error s_1 and NN error ε_1 (and thus Δ_1) are bounded. In this sense, the robustness of the adaptive law (15) is comparable with σ-modification (22).

However, in contrary to σ-modification (22), the use of the new leakage term σH_1 with $\tilde{W}_1$ in (15) may drive the estimated parameter $\hat{W}_1$ towards its true value W_1 even in the presence of NN error ε_1 (and thus Δ_1). This may be further shown by the fact that $\tilde{W}_1$ in (24) may be represented as $\tilde{W}_1 = \frac{1}{p+\Gamma_1\sigma P}(-\Gamma_1 s_1 \Phi_1 + \Gamma_1 \sigma \Delta_1)$, which indicates that the bound of $\tilde{W}_1$ depends on the tracking error s_1 and NN error ε_1, but not on the unknown parameter W_1. Thus, if s_1 and ε_1 are sufficiently small, $\tilde{W}_1$ will be small, the convergence of $\hat{W}_1$ to W_1 may be guaranteed. In this case, the proposed adaptive law (15) may obtain better estimation performance than σ-modification (22).

Adaptive suspension control for pitch motion

The active suspension control u_φ for the pitch motion in (3) will be considered in this section. Note although we present the control design for the pitch motion in a separated section for the ease of analysis, this control is implemented with the designed control for the vertical motion simultaneously to calculate the realistic control actions in (2). The dynamics of x_3, x_4 are with a similar form as that of x_1, x_2 in (3); thus, the control design of u_φ follows a similar procedure as shown in the above section, which may be briefly given as

$$\begin{aligned} s_2 &= [\Lambda, 1][x_3, x_4]^T \\ \dot{s}_2 &= \Lambda x_4 + T_2(Z_2) + \theta_2 u_\varphi \\ u_\varphi &= \frac{1}{\hat{\theta}_2}[-\hat{w}_2{}^T\phi_2(Z_2) - k_2 s_2 - \Lambda x_4] \\ \dot{\hat{W}}_2 &= \Gamma_2 s_2 \Phi_2 - \Gamma_2 \sigma H_2 \end{aligned} \tag{25}$$

where $T_2(Z_2) = \theta_2(-a(F_{df} + F_{sf}) + b(F_{dr} + F_{sr})) = w_2{}^T\phi_2(Z_2) + \varepsilon_2$ is the lumped non-linearities approximated by an NN $\hat{w}_2^T\phi_2(Z_2)$, $W_2 = [w_2^T, \theta_2]^T$ is the augmented parameter to be estimated, $\Phi_2 = [\phi_2^T, u_\varphi]^T$ is the augmented regressor and $\hat{W}_2 = [\hat{w}_2^T, \hat{\theta}_2]^T$ is the estimation of W_2; $k_2 > 0$ is the feedback gain, $\Gamma_2 > 0$ and $\sigma > 0$ are the learning gains. The leakage variable H_2 is given by $H_2 = P_2\hat{W}_2 - Q_2$ in terms of the filtered variables P_2, Q_2, which may be obtained similar to their counterparts P_1, Q_1 and H_1 given in (12)–(14).

Similarly to Theorem 1, we summarize the stability of pitch motion (3) with control (25) as:

Theorem 2. *Consider the pitch motion x_3 of system (3) with unknown dynamics, we design the adaptive control (25), if the regressor vector Φ_2 is PE, then the control error s_2 and estimation error $\tilde{W}_2$ converge to a small set around zero, and all signals in the closed-loop are bounded.*

Proof. The proof is similar to that of Theorem 1 by using the Lyapunov function $V_2 = \frac{1}{2}s_2{}^2 + \frac{1}{2}\tilde{W}_2{}^T\Gamma_2{}^{-1}\tilde{W}_2$. Thus, the detailed mathematical derivations are omitted. However, for the ease of further analysis, we provide the bounds for x_3 and x_4 as $|x_3| \le \sqrt{2\gamma_2/\tilde{\mu}_2}/\Lambda$, $|x_4| \le 2\sqrt{2\gamma_2/\tilde{\mu}_2}$, where $\tilde{\mu}_2 = \min\{2(k_2 - 1/2\eta), 2\sigma(\sigma_2 - 1/2\eta)/\lambda_{\max}(\Gamma_2{}^{-1})\}$ and $\gamma_2 = \eta(\varepsilon_{2N}^2 + \sigma\varepsilon_{2Nf}^2)/2$ with $|\varepsilon_2| \le \varepsilon_{2N}$ are all positive constants depending on the size of ε_2, σ_2 and the control parameters Λ, k_2, σ and Γ_2.

Suspension performance analysis

In the previous analysis, the stability of system (3) and the convergence of x_1 and x_3 to a small set around zero have been obtained in terms of the proposed controls u_y and u_φ, i.e. the primary suspension objective to mitigate the vertical displacement and pitch motion is achieved. In the following, we will show that another two suspension requirements (5) and (8) may be retained by appropriately selecting parameters.

For this purpose, the elasticity and damping forces F_{tf}, F_{tr}, F_{bf} and F_{br} of the tyres are set as in Sun et al. (2013) as $F_{tf} = k_{f2}(y_1 - y_{o1}), F_{tr} = k_{r2}(y_2 - y_{o2}), F_{bf} = b_{f2}(\dot{y}_1 - \dot{y}_{o1})$, $F_{br} = b_{r2}(\dot{y}_2 - \dot{y}_{o2})$, where k_{f2}, k_{r2}, b_{f2} and b_{r2} are the stiffness and damping coefficients of the tyres, respectively.

We first prove the boundedness of x_5, x_6, x_7, x_8 in (4). Without loss of generality, we will discuss the case with bounded NN approximation errors, and thus the vertical motions x_1, x_2 and the pitch motions x_3, x_4 converge to a small set around zero as indicated in Theorems 1 and 2. Substituting (2) into (4), one may obtain the following dynamics

$$\dot{x} = Ax + By_0 + \omega \tag{26}$$

where

$$x = \begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_{f2}}{m_f} & -\frac{b_{f2}}{m_f} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{k_{r2}}{m_r} & -\frac{b_{r2}}{m_r} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{k_{f2}}{m_f} & \frac{b_{f2}}{m_f} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{k_{r2}}{m_r} & \frac{b_{r2}}{m_r} \end{bmatrix}, y_o = \begin{bmatrix} y_{o1} \\ \dot{y}_{o1} \\ y_{o2} \\ \dot{y}_{o2} \end{bmatrix}, \omega = \begin{bmatrix} 0 \\ -\frac{b\varpi_1 + \varpi_2}{m_f(a+b)} \\ 0 \\ -\frac{a\varpi_1 - \varpi_2}{m_r(a+b)} \end{bmatrix} \tag{27}$$

where $\varpi_1 = \left(\varepsilon_1/\theta_1 + \left\|\tilde{W}_1\right\|/\theta_1 + k_1 s_1/\theta_1 + \Lambda x_2/\theta\right)$ and $\varpi_2 = \left(\varepsilon_2/\theta_2 + \left\|\tilde{W}_2\right\|/\theta_2 + k_2 s_2/\theta_2 + \Lambda x_4/\theta_2\right)$ denote the effects of the residual errors. Thus, ω in (26) is also bounded by $\|\omega\| \le \varpi$ for positive constant $\varpi > 0$.

Clearly the matrix A defined in (26) is stable and there exist positive matrices $P > 0, Q > 0$, which fulfil the Lyapunov equation $A^T P + AP = -Q$. We select the Lyapunov function as $V = x^T Px$ and calculate its derivative along (26) as:

$$\begin{aligned} \dot{V} &= x^T(A^T P + AP)x + 2x^T PBy_0 + 2x^T P\omega \\ &\le -x^T Qx + \frac{1}{\eta_1}x^T PBB^T Px + \eta_1 y_o{}^T y_o \\ &+ \frac{1}{\eta_2}x^T Px + \eta_2\omega^T P\omega \\ &\le -\left[\lambda_{\min}(Q) - \frac{1}{\eta_1}\lambda_{\max}(PBB^T P) - \frac{1}{\eta_2}\lambda_{\max}(P)\right] \\ &\quad \|x\|^2 + \eta_1 y_o{}^T y_o + \eta_2\lambda_{\max}(P)\omega^T\omega \\ &\le -\alpha_1 V + \beta_1 \end{aligned} \tag{28}$$

where $\alpha_1 = (\lambda_{\min}(Q) - \lambda_{\max}(PBB^T P)/\eta_1 - \lambda_{\max}(P))/\lambda_{\min}(P)$ and $\beta_1 = \eta_1\lambda_{\max}(P)y_{o\max} + \eta_2\lambda_{\max}(P)\omega_{\max}$ with $y_o{}^T y_o \le y_{o\max}$ and $\omega^T\omega \le \omega_{\max}$ are all positive constants for appropriately designed parameters fulfilling $\eta_1 > \lambda_{\max}(PBB^T P)/\lambda_{\min}(Q)$, $\eta_2 > \lambda_{\max}(P)/\lambda_{\min}(Q)$.

Integrating both sides of (28), we have $V(t) \le (V(0) - \beta_1/\alpha_1)e^{-\alpha_1 t} + \beta_1/\alpha_1$, which further implies that the states $x_i, i = 5, 6, 7, 8$ of (4) are all bounded by:

$$|x_i(t)| \le \sqrt{(V(0) + \beta_1/\alpha_1)/\lambda_{\min}(P)}, i = 5, 6, 7, 8 \tag{29}$$

Then we may address the performance requirement of the road holding (5), where the bounds of the tyre loads may be calculated as:

$$\begin{aligned} |Z_f| &\le (k_{f2} + b_{f2})\sqrt{(V(0) + \beta_1/\alpha_1)/\lambda_{\min}(P)} + k_{f2}\|y_{o1}\|_\infty \\ &\quad + b_{f2}\|\dot{y}_{o1}\|_\infty \\ |Z_r| &\le (k_{r2} + b_{r2})\sqrt{(V(0) + \beta_1/\alpha_1)/\lambda_{\min}(P)} + k_{r2}\|y_{o2}\|_\infty \\ &\quad + b_{r2}\|\dot{y}_{o2}\|_\infty \end{aligned}.$$

If we appropriately tune the initial conditions and design parameters η_1, η_2 and P, then $V(0)$ and α_1, β_1 may be tuned such that the road holding requirement $|Z_f| = |F_{tf} + F_{bf}| < F_f, |Z_r| = |F_{tr} + F_{br}| < F_r$ is guaranteed. Although the exact tyre stiffness and damping coefficients k_{f2}, k_{r2}, b_{f2}, b_{r2} may be unknown, their upper bounds may be determined by their physical properties, and they may be used to check the above condition.

Finally, the suspension space limitation (8) may be obtained as:

$$\begin{aligned} |\Delta y_f| &< |x_1| + a|\sin x_3| + |x_5| \le |x_1| + a|x_3| + |x_5| \le \sqrt{2\gamma_1/\tilde{\mu}_1}/\Lambda \\ &\quad + a\sqrt{2\gamma_2/\tilde{\mu}_2}/\Lambda + \sqrt{(V(0) + \beta_1/\alpha_1)/\lambda_{\min}(P)} \\ |\Delta y_r| &\le |x_1| + b|\sin x_3| + |x_7| \le |x_1| + b|x_3| + |x_7| \le \sqrt{2\gamma_1/\tilde{\mu}_1}/\Lambda \\ &\quad + b\sqrt{2\gamma_2/\tilde{\mu}_2}/\Lambda + \sqrt{(V(0) + \beta_1/\alpha_1)/\lambda_{\min}(P)} \end{aligned} \tag{30}$$

If we tune the initial conditions and design parameters η_1, η_2, P and Λ, k_1, k_2, σ, Γ_1, Γ_2 appropriately, then we may

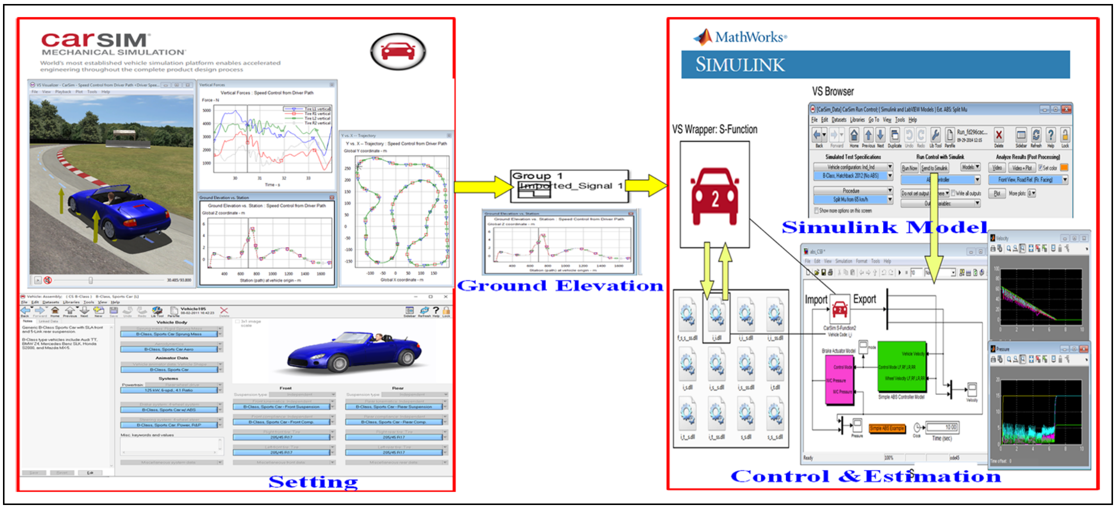


Figure 3. Schematic of the proposed dynamic simulation system.

guarantee that $|\Delta y_f| \le \Delta y_{f\,\max}, |\Delta y_r| \le \Delta y_{r\,\max}$ holds, and the mechanical movement limitation (8) is fulfilled.

Simulations

This section will present simulations under various road conditions to exemplify the efficacy of the designed controllers and the novel adaptive laws. For this purpose, a dynamics simulator was constructed based on a commercial vehicle simulation software Carsim® (Version 8.1) and Matlab® Simulink (2013), which may simulate realistic vehicle running conditions (e.g. vehicle parameters and road manoeuvres). The schematic of the overall simulation system is shown in Figure 3, where the realistic vehicle parameters, driving manoeuvres and the associated ground elevations (e.g. bump, bounce sine sweep road and bump random road) are produced by Carsim®, and the control schemes are implemented in the Simulink® to test the performance of active suspension systems. Thus, it should be noted that the simulation scenarios to be presented may represent very realistic situations of practical vehicle operations based on the built dynamic simulator, because the ground elevations from Carsim® were generated by the experimental data.

In the simulations, the unknown forces produced by the non-linear stiffening springs, piece-wise dampers and the tyre obeys (Sun et al., 2013) are given by:

$$F_{sf} = k_{f1}\Delta y_f + k_{nf1}\Delta y_f{}^3, F_{sr} = k_{r1}\Delta y_r + k_{nr1}\Delta y_r{}^3, \\ F_{df} = b_{e1}\Delta\dot{y}_f, F_{dr} = b_{e2}\Delta\dot{y}_r \tag{31}$$

where k_{f1}, k_{r1} and k_{nf1}, k_{nr1} are the stiffness coefficients, b_{ei}, $i = 1, 2$ are the damping coefficients for the extension and compression movements. k_{f2}, k_{r2}, b_{f2}, and b_{r2} are the stiffness and damping coefficients of the tyres.

The parameters of the half-car model shown in Figure 1 are listed in Table 2, which are based on a realistic vehicle prototype embedded in the professional vehicle simulation software Carsim®.

Table 2. Parameters for half-car model.

Parameter	Value	Parameter	Value
M	1590 kg	k_{r2}	140,000 N/m
I	750 kgm^2	b_{f2}	1500 Ns/m
m_f	120 kg	b_e	1500 Ns/m
m_r	150 kg	b_{r2}	2000 Ns/m
k_{f1}	15,000 N/m	b_c	1200 Ns/m
k_{r1}	15,000 N/m	a	1.18 m
k_{nf1}	1000 N/m	b	1.77 m
k_{nr1}	1000 N/m	V	40 km/h
k_{f2}	200,000 N/m	h	3.5 cm
l_b	40 cm		

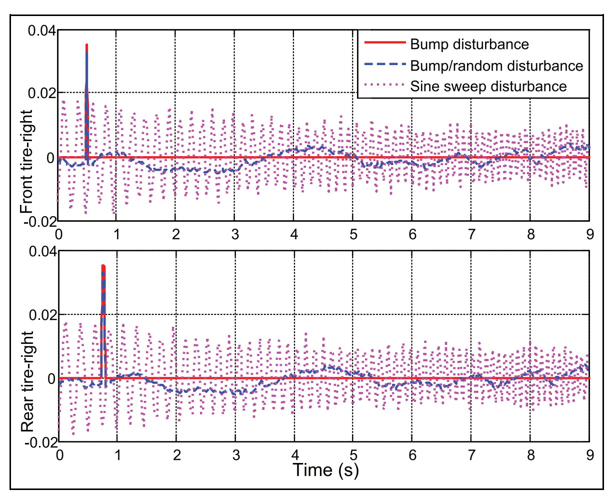


Figure 4. Ground road elevation.

In the simulation, the performance requirements for vehicle active suspension system are exemplified with the initial values

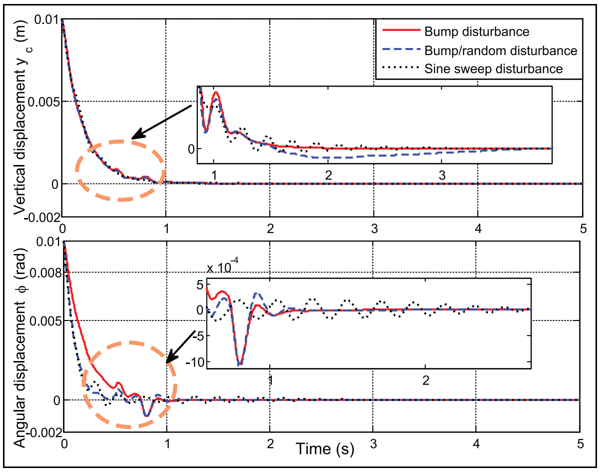


Figure 5. Vertical and pitch motions with different roads.

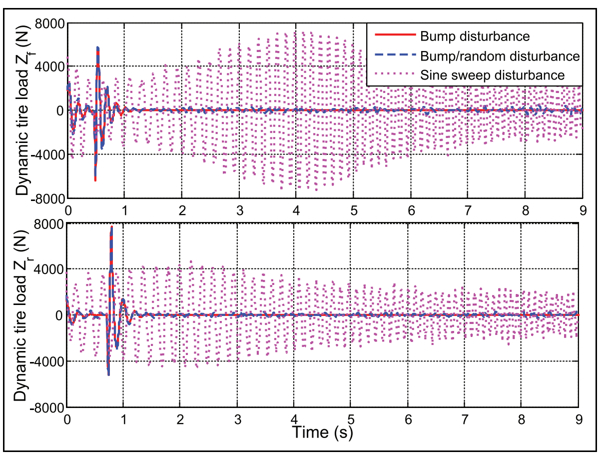


Figure 6. Dynamic tyre load of wheels.

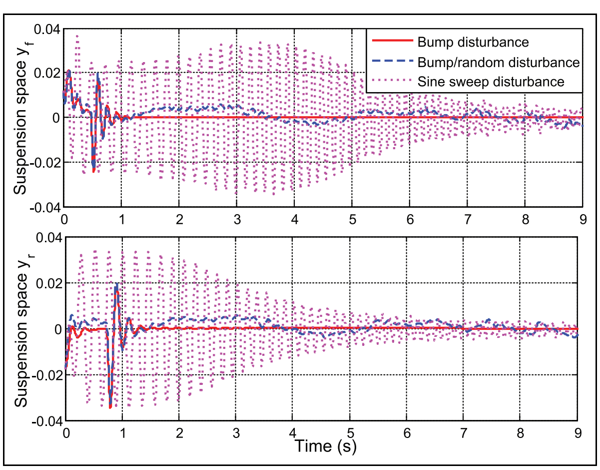
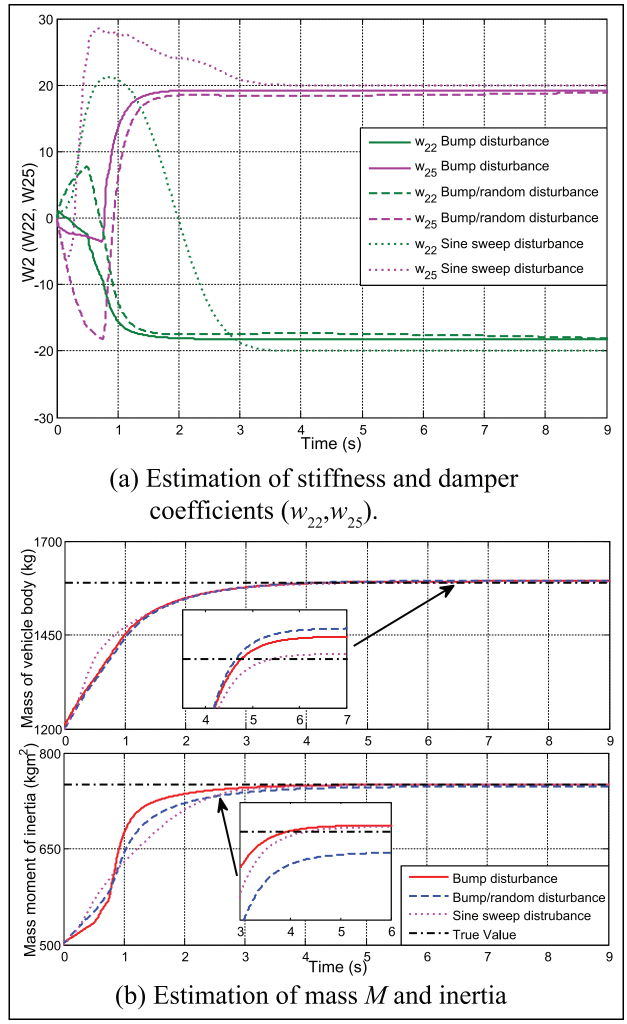


Figure 7. Suspension movements of front and rear wheels.

$x_i(0) = 0.01, i = 1, 3, 5, 7, \theta_1(0) = 1/1200, \theta_2(0) = 1/500$. The following three riding road conditions are simulated:



(a) Estimation of stiffness and damper coefficients (w_{22}, w_{25}).

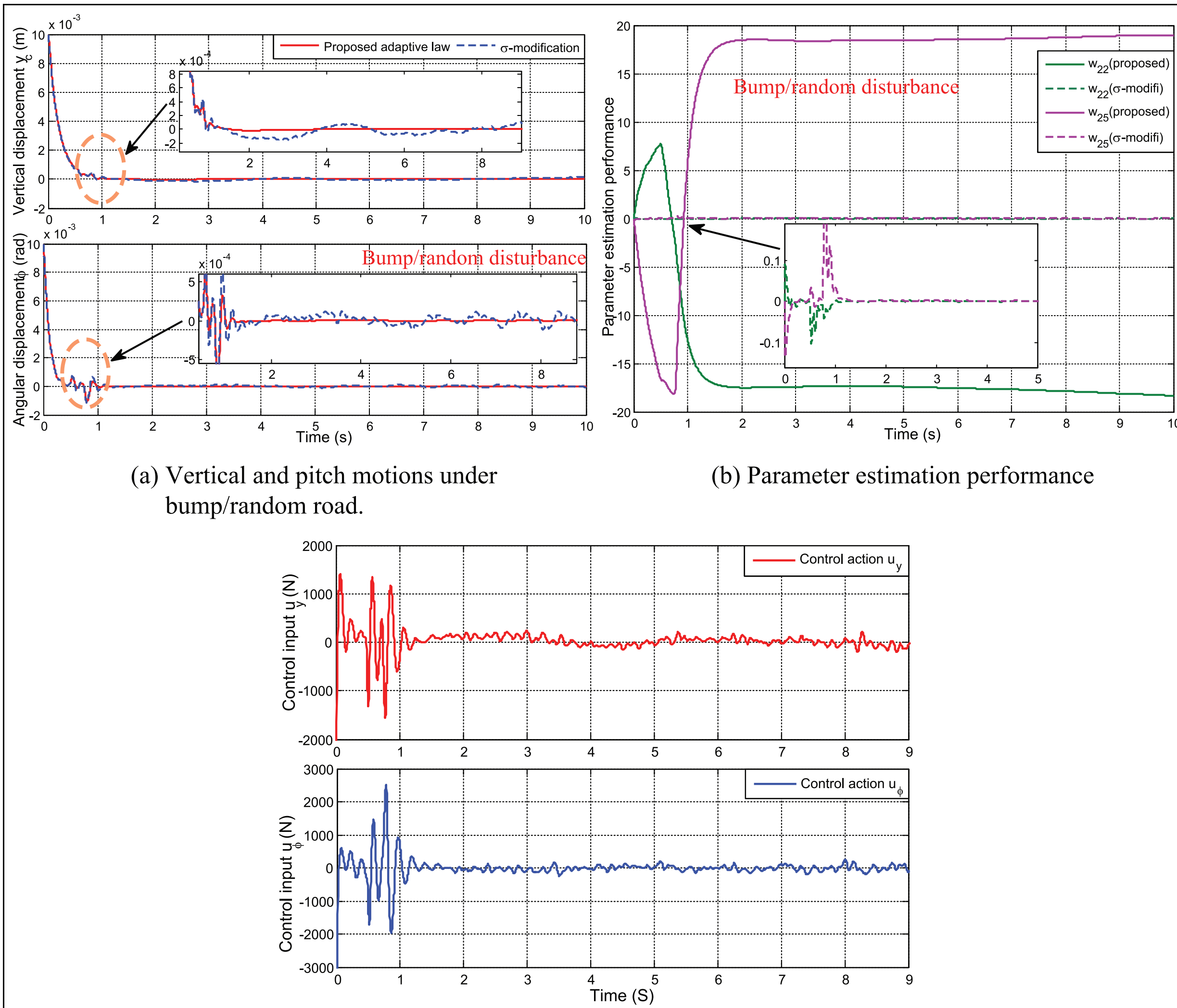
(b) Estimation of mass M and inertia

Figure 8. Parameter estimation performance under different ride roads conditions.

a) Bounce sine sweep road;
b) Bump (very sharp 3.5 cm high, 40 cm long) road;
c) Bump (very sharp 3.5 cm high, 40 cm long) road and straight line with random roughness profile.

Figure 4 illustrates the ground elevation profiles of the three test roads. It is worth noting that these road surfaces are typical profiles embedded in Carsim® for verifying the suspension performance. Thus, we are interested to test the capability of the active suspension system with the proposed control approaches under these different ride roads conditions.

In spite of suspension performance, we also validate the parameter estimation performance of the adaptive laws (15) and (25) with the novel leakage terms $\sigma H_1, \sigma H_2$. For this purpose, based on the spring and damper dynamics, we may set the elements of the regressor vector as $\phi_{11} = (x_2 + 1.2x_4 \cos(x_3) - x_6)$, $\phi_{12} = (x_2 - 1.5x_4 \cos(x_3) - x_8)$, $\phi_{13} = (x_1 + 1.2 \sin(x_3) - x_5)$, $\phi_{14} = (x_1 + 1.2 \sin(x_3) - x_5)^3$, $\phi_{15} = (x_1 - 1.5 \sin(x_3) - x_7)$, $\phi_{16} = (x_1 - 1.5 \sin(x_3) - x_7)^3$ and $\phi_{21} = 1.2(x_2 + 1.2x_4 \cos(x_3) - x_6), \phi_{22} = 1.2(x_1 +$



(a) Vertical and pitch motions under bump/random road.

(b) Parameter estimation performance

(c) Response of active suspension control

Figure 9. Control and estimation performance of σ-modification method.

$1.2\sin(x_3) - x_5)$, $\phi_{23} = 1.2(x_1 + 1.2\sin(x_3) - x_5)^3$, $\phi_{24} = 1.5(x_2 - x_4\cos(x_3) - x_8)$, $\phi_{25} = 1.5(x_1 - 1.5\sin(x_3) - x_7)$, $\phi_{26} = 1.5(x_1 - 1.5\sin(x_3) - x_7)^3$, so that the NN weights are $w_{11} = w_{12} = -b_e/M = -1500/1590$, $w_{13} = -k_{f1}/M = w_{15} = -k_{r1}/M = -15000/1590$, $w_{14} = -k_{nf1}/M = w_{16} = -k_{nr1}/M = -1000/1590$ and $w_{21} = -b_{e1}/I = -1500/750$, $w_{22} = -k_{f1}/I = -15000/750$, $w_{23} = -k_{nf1}/I = -1000/750$, $w_{24} = b_{e2}/I = 1500/750$, $w_{25} = k_{r1}/I = 15000/750$ and $w_{26} = k_{nr1}/I = 1000/750$. Other simulation parameters used in the proposed adaptive controls and adaptive laws are tuned in terms of the trial-and-error method as $k_1 = k_2 = 150, \Lambda_1 = 5, \Lambda_2 = 10, k = 0.001, l = 1, \sigma_1 = \sigma_2 = 5$ and $\Gamma_1 = diag([35, 50, 1.8e4, 1.2e6, 1e4, 17e6, 6e-9])$, $\Gamma_2 = diag([16, 4e4, 2.7e4, 8, 8e3, 2.7e4, 3.6e-7])$.

Figures 5–7 depict the simulation results. The time responses of the vertical and pitch motions are presented in Figure 5. One may find from Figure 5 that the proposed method may mitigate the vertical and pitch displacements effectively under all riding road conditions. Therefore, the major objective of vehicle active suspensions (e.g. isolate the vehicle body from the road disturbance) is fulfilled, which may enhance the riding comfort to passengers. Moreover, the firm uninterrupted contact of the wheels to the road should be guaranteed due to ride safety considerations, i.e. the condition (6) should be fulfilled for all $t > 0$. According to the static tyre load bounds given in (6), we know that the dynamic tyre loads should be constrained within a specified boundary, i.e. $F_f < 10{,}748$ N and $F_r < 7852$ N. It is shown in Figure 6 that the peaks of the tyre loads for two wheels are all limited within the specified bounds, which implies that the second performance requirement is fulfilled. Finally, the mechanical structure limitations imposed on the suspension motion spaces must be preserved. Figure 7 provides the suspension working space, where the suspension space constraint (8) with the proposed controllers is all satisfied by $\Delta y_{f\,\max} = \Delta y_{r\,\max} = 0.1m$. Consequently, all the suspension

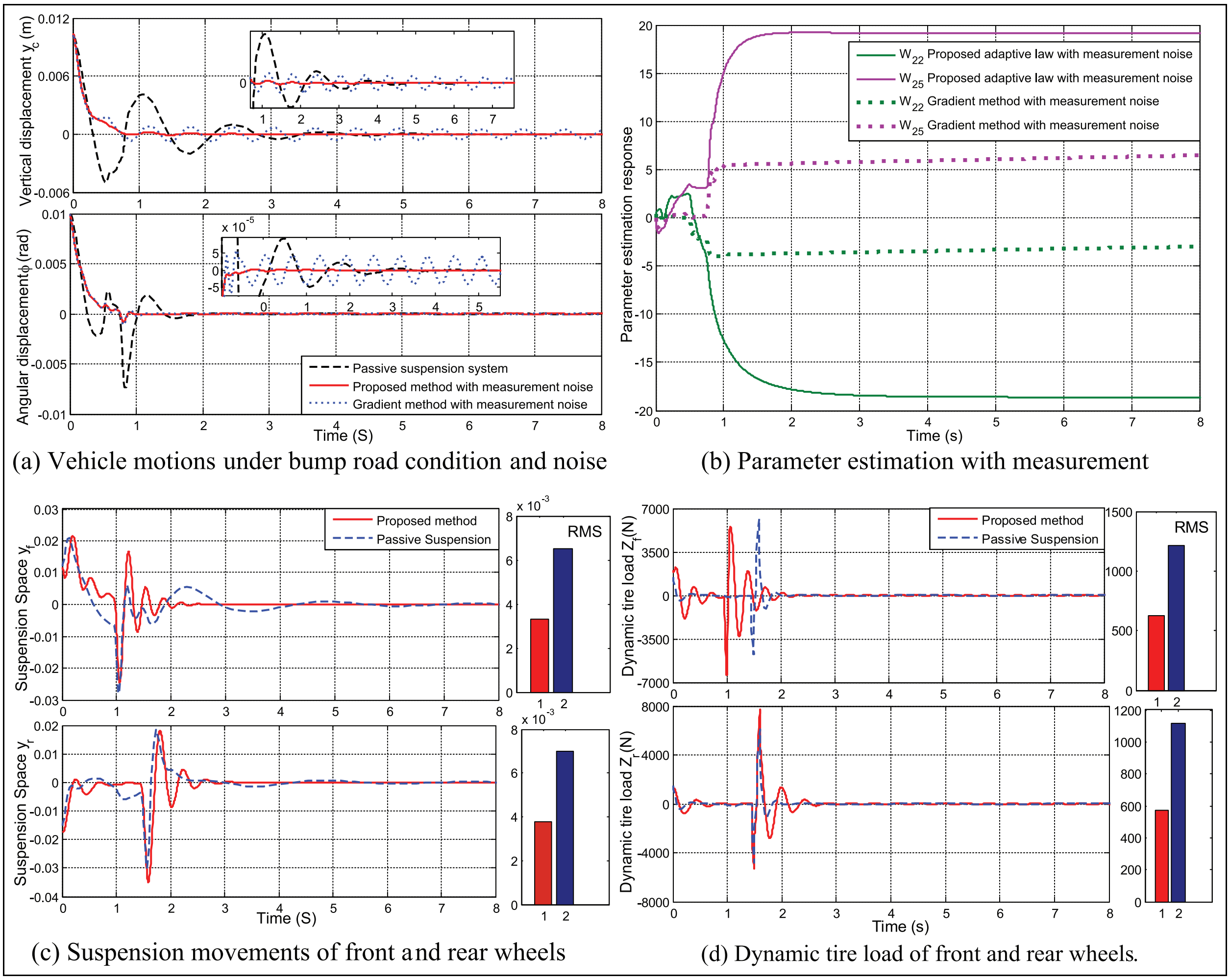


Figure 10. Comparison with passive suspension scheme under bump road condition and measurement noise.

requirements for vehicle active suspensions are achieved with the proposed adaptive control methods.

On the other hand, to show the necessity for using the newly developed adaptive laws for precise parameter estimation, some of the estimated vehicle parameters (e.g. the stiffness and damper coefficients w_{22}, w_{25} and the mass M and inertia I) are presented in Figure 8. It may be found that the proposed adaptive laws (15) and (25) may drive the estimated parameters to a neighbour around their true values in all three different driving road conditions. This may be attributed to the use of the new leakage terms $\sigma H_1, \sigma H_2$ in the adaptive laws. In this sense, the proposed control scheme may guarantee both the control convergence and the parameter estimation simultaneously.

To show the efficacy of the new leakage terms further and validate the theoretical analysis shown in the 'Comparison with other adaptations' subsection, comparative simulations with the σ-modification method are also provided in Figure 9, where both the control response and parameter estimations are all illustrated. One may conclude from Figure 9 that the estimated parameters with the σ-modification method converge to small values due to the introduced forgetting factors, whereas the new adaptive laws (15) and (25) may provide precise satisfactory estimation performance. This improved estimation response may further lead to better control response than σ-modification scheme, because the estimated parameters are used in the control. Furthermore, the profiles of the active control actions u_y and u_φ are plotted in Figure 9(c). One may find from Figure 9(c) that the forces requested by the proposed adaptive controllers are bounded under bump road conditions.

Moreover, in order to confirm the efficiency of the proposed method for active suspension system over the passive suspension system, comparative simulations are given in Figure 10, where a high-frequency sinusoid signal is also used as the measurement noise in the vertical velocity. One may find from Figure 10(a) that the proposed control has lower peak compared with that of the passive suspension, which implies that active suspension control may isolate the road perturbation effectively. On the other hand, it is found in Figure 10(a) that better control response may be achieved with the proposed adaptive algorithm compared with the gradient method even in the presence of the measurement noise. This is reasonable because precise parameter estimation response may be retained in this case (Figure 10b), whereas the estimated parameters with the gradient algorithms are not convergent. This result also validates the improved robustness of the new adaptive law in comparison with the gradient method. Figures 10(c) and 10(d) provide the suspension movements and dynamic tyre loads of the front and rear tyre, respectively. It is shown that both the proposed control and passive suspension may satisfy the road holding and suspension movement limitation requirement; however, the improved response may be observed in these figures and illustrated in terms of the performance index root mean square (RMS). Consequently, it is clear that the proposed adaptive control for vehicle active suspension systems achieves better ride comfort (i.e. shorter settling time).

These aforementioned simulation results illustrate how the proposed adaptive control method for vehicle active suspension system provides better riding comfort, road holding and suspension movement performance. Moreover, it is also verified that the suggested novel leakage terms in the adaptive law may achieve more accurate estimation performance of unknown system parameters.

Conclusions

This paper presents a new robust adaptive law, which may provide better estimation performance of unknown parameters in terms of convergence and robustness than some classical adaptations (e.g. gradient and σ-modification). This may be achieved by introducing a new leakage term containing the parameter estimation error into the adaptive law. This new adaptive algorithm is incorporated into an adaptive control designed for vehicle active suspension systems with unknown non-linear sprung and piecewise damper dynamics. An augmented NN is used to address the unknown non-linearities, and precise estimation of essential vehicle parameters (e.g. vehicle mass and inertia) and NN weights may be guaranteed. Moreover, several suspension requirements considering the ride comfort and vehicle safety are also investigated. Comparative simulations based on a half-car model embedded in a dynamic simulator built in Carsim® and Matlab® are conducted. It is found that better parameter estimation response may be obtained, which may further enhance the overall suspension performance. Future work will focus on the practical validation of the proposed active suspension control design in terms of experiments.

Declaration of Conflicting Interest

The authors declare that there is no conflict of interest.

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References

Akbari A and Lohmann B (2010) Output feedback H_∞/GH_2 preview control of active vehicle suspensions: a comparison study of LQG preview. *Vehicle System Dynamics* 48(12): 1475–1494.

Allotta B, Pugi L and Bartolini F (2008) Design and experimental results of an active suspension system for a high-speed pantograph. *IEEE/ASME Transactions on Mechatronics* 13(5): 548–557.

Canale M, Milanese M and Novara C (2006) Semi-active suspension control using 'fast' model-predictive techniques. *IEEE Transactions on Control Systems Technology* 14(6): 1034–1046.

Cao D, Song X and Ahmadian M (2011) Editors's perspectives: road vehicle suspension design, dynamics, and control. *Vehicle System Dynamics* 49(1–2): 3–28.

Chin C and Lum S (2011) Rapid modeling and control systems prototyping of a marine robotic vehicle with model uncertainties using xPC target system. *Ocean Engineering* 38(17): 2128–2141.

Chin CS and Wheeler C (2013) Sliding-mode control of an electromagnetic actuated conveyance system using contactless sensing. *IEEE Transactions on Industrial Electronics* 60(11): 5315–5324.

Du H and Zhang N (2007) H control of active vehicle suspensions with actuator time delay. *Journal of Sound and Vibration* 301(1): 236–252.

Fallah M, Bhat RB and Xie WF (2012) Optimized control of semiactive suspension systems using H robust control theory and current signal estimation. *IEEE/ASME Transactions on Mechatronics* 17(4): 767–778.

Hoque ME, Takasaki M, Ishino Y, et al. (2006) Development of a three-axis active vibration isolator using zero-power control. *IEEE/ASME Transactions on Mechatronics* 11(4): 462–470.

Hrovat D (1997) Survey of advanced suspension developments and related optimal control applications. *Automatica* 33(10): 1781–1817.

Huang J-T, Van Hung T and Tseng M-L (2015) Smooth switching robust adaptive control for omnidirectional mobile robots. *IEEE Transactions on Control Systems Technology* 23(5): 1986–1993.

Huang S-J and Chen H-Y (2006) Adaptive sliding controller with self-tuning fuzzy compensation for vehicle suspension control. *Mechatronics* 16(10): 607–622.

Ioannou PA and Sun J (1996) *Robust Adaptive Control*. Piscataway, NJ: Courier Dover Publications.

Karimi HR (2006) Optimal vibration control of vehicle engine-body system using Haar functions. *International Journal of Control Automation and Systems* 4(6): 714.

Leite VJS and Peres PLD (2005) Pole location control design of an active suspension system with uncertain parameters. *Vehicle System Dynamics* 43(8): 561–579.

Ma M-M and Chen H (2011) Disturbance attenuation control of active suspension with non-linear actuator dynamics. *IET Control Theory & Applications* 5(1): 112–122.

Na J, Chen Q, Ren X, et al. (2014) Adaptive prescribed performance motion control of servo mechanisms with friction compensation. *IEEE Transactions on Industrial Electronics* 61(1): 486–494.

Na J, Herrmann G, Ren X, et al. (2011) Robust adaptive finite-time parameter estimation and control of nonlinear systems. In: *2011 IEEE International Symposium on Intelligent Control (ISIC)*. Washington, DC: IEEE, pp. 1014–1019.

Na J, Mahyuddin MN, Herrmann G, et al. (2013a) Robust adaptive finite-time parameter estimation for linearly parameterized nonlinear systems. In: *2013 32nd Chinese Control Conference (CCC)*. Washington, DC: IEEE, pp. 1735–1741.

Na J, Mahyuddin MN, Herrmann G, et al. (2015) Robust adaptive finite-time parameter estimation and control for robotic systems. *International Journal of Robust and Nonlinear Control* 25(16): 3045–3071.

Na J, Ren X and Zheng D (2013b) Adaptive control for nonlinear pure-feedback systems with high-order sliding mode observer. *IEEE Transactions on Neural Networks and Learning Systems* 24(3): 370–382.

Onat C, Kucukdemiral I, Sivrioglu S, et al. (2009) LPV gain-scheduling controller design for a non-linear quarter-vehicle active suspension system. *Transactions of the Institute of Measurement and Control* 31(1): 71–95.

Readman MC, Corless M, Villegas C, et al. (2010) Adaptive Williams filters for active vehicle suspensions. *Transactions of the Institute of Measurement and Control* 32(6): 660–676.

Ren X, Lewis FL and Zhang J (2009) Neural network compensation control for mechanical systems with disturbances. *Automatica* 45(5): 1221–1226.

Sankaranarayanan V, Emekli ME, Gilvenc BA, et al. (2008) Semiactive suspension control of a light commercial vehicle. *IEEE/ASME Transactions on Mechatronics* 13(5): 598–604.

Savaresi SM and Spelta C (2009) A single-sensor control strategy for semi-active suspensions. *IEEE Transactions on Control Systems Technology* 17(1): 143–152.

Slotine J-JE and Li W (1991) *Applied Nonlinear Control*. Englewood Cliffs, NJ: Prentice-Hall.

Sun W, Gao H and Kaynak O (2011) Finite frequency control for vehicle active suspension systems. *IEEE Transactions on Control Systems Technology* 19(2): 416–422.

Sun W, Gao H and Kaynak O (2013) Adaptive backstepping control for active suspension systems with hard constraints. *IEEE/ASME Transactions on Mechatronics* 18(3): 1072–1079.

Sun W, Gao H and Kaynak O (2015) Vibration isolation for active suspensions with performance constraints and actuator saturation. *IEEE/ASME Transactions on Mechatronics* 20(2): 675–683.

Tseng HE and Hrovat D (2015) State of the art survey: active and semi-active suspension control. *Vehicle System Dynamics* 53(7): 1034–1062.

Waldron KJ and Abdallah ME (2007) An optimal traction control scheme for off-road operation of robotic vehicles. *IEEE/ASME Transactions on, Mechatronics* 12(2): 126–133.

Wang N and Er MJ (2015) Self-constructing adaptive robust fuzzy neural tracking control of surface vehicles with uncertainties and unknown disturbances. *IEEE Transactions on Control Systems Technology* 23(3): 991–1002.

Yagiz N and Hacioglu Y (2008) Backstepping control of a vehicle with active suspensions. *Control Engineering Practice* 16(12): 1457–1467.

Zapateiro M, Luo N, Karimi HR, et al. (2009) Vibration control of a class of semiactive suspension system using neural network and backstepping techniques. *Mechanical Systems and Signal Processing* 23(6): 1946–1953.

Zapateiro M, Pozo F, Karimi HR, et al. (2012) Semiactive control methodologies for suspension control with magnetorheological dampers. *IEEE/ASME Transactions on Mechatronics* 17(2): 370–380.

Zhang C, Chen Q and Qiu J (2016) Robust H∞ filtering for vehicle sideslip angle estimation with sampled-data measurements. *Transactions of the Institute of Measurement and Control*: 01423312 15627001.

Zheng B-C and Park JH (2015) Adaptive integral sliding mode control with bounded L2 gain performance of uncertain quantised control systems. *IET Control Theory & Applications* 9(15): 2273–2282.

Zuo L, Slotine JJ and Nayfeh SA (2005) Model reaching adaptive control for vibration isolation. *IEEE Transactions on Control Systems Technology* 13(4): 611–617.