# Code for Machine Discovery of Partial Differential Equations from Spatiotemporal Data

## Installation

To run the codes in S3d, one should install the CVX toolbox previously for constructing and solving convex programs. The latest version of CVX toolbox can be downloaded here. Follow these steps to install it.

**S3d consists of the following subfolders.**

| Subfolders | Description |
|---|---|
| Functions | Subfunctions for identifying PDEs |
| Examples | Identification of 8 simulated PDEs |
| Realdata | Identification of Complex Ginzburg-Landau Equation |
| SI-fig | Figures and codes for generating these figures in SI |
| M-fig | Figures and codes for generating these figures in main text |

## Functions

Twelve subfunctions used for identifying PDEs are included in this directory. They are listed in the following table.

| Function | Description |
|---|---|
| build_Theta | Building the candidate dictionaries |
| finitediff | Computing the derivative of given data by using the finite difference method |
| make_fft | Computing the derivative of given data by using Fourier spectral method to approximate derivative |
| make_input_fd | Computing the spatial derivatives in the dictionary functions |

| | |
|---|---|
| | by using the finite difference method |
| make_input_pade | Computing the spatial derivatives in the dictionary functions by using Pade method |
| make_input_poly | Computing the spatial derivatives in the dictionary functions by using the polynomial interpolation method |
| make_y_fd | Computing the time derivative on LHS of PDE by using the finite difference method |
| make_y_pade | Computing the time derivative on LHS of PDE by using Pade method |
| make_y_ploy | Computing the time derivative on LHS of PDE by using the polynomial interpolation method |
| pade_c4 | Computing the derivative of given data by using the Pade method |
| POD | Calculating the POD basis vectors without subtracting the mean of the ensemble |
| tac_reconstruction | Solving a linear regression problem with L1 regularization term. |

# Examples

This directory contains eight subfolders, and each subfolder includes scripts for identifying a simulated PDE.

| **Subfolder** | **Identified simulated PDE** |
|---|---|
| Fisher | Fisher's equation |
| FN | FitzHugh-Nagumo |
| KdV | Korteweg-de Vries |
| KG | Klein-Gordon |
| KS | Kuramoto Sivashinsky |
| NS | Navier Stokes |

| | |
|---|---|
| NLS | Nonlinear Schrödinger |
| SG | Klein-Gordon |

In each subfolder, there are two scripts, main.m and main_noisy.m, and a data file. main.m uses clean data to identify the PDE, while main_noisy.m adds 1% Gaussian noise to the data. Users can run these two scripts directly.

# Realdata

This directory includes codes for identifying the Complex Ginzburg-Landau Equation based on data acquired in physical experiments. More details can be found in:

Kolodner, P. Counterpropagating quasilinear wave packets in binary-fluid convection. *Physical review letters* **69,** 2519 (1992).

Kolodner, P., Slimani, S., Aubry, N. & Lima, R. Characterization of dispersive chaos and related states of binary-fluid convection. *Physica D: Nonlinear Phenomena*, **85** 165-224 (1995).

There are seven groups of data, as listed in this table.

| **Data name** | **Corresponding $\varepsilon/\tau_0$** |
|---|---|
| car06172cb/cal06172cb | 0.00932 |
| car06182cb/cal06182cb | 0.00422 |
| car06192cb/cal06192cb | 0.00177 |
| car06212cb/cal06212cb | 0.00638 |
| car06222cb/cal06222cb | 0.01207 |
| car06242cb/cal06242cb | 0.01403 |
| car06252cb/cal06252cb | 0.01628 |

The special letter "cal" and "car" respectively represent the left-going waves and right-going ones. Seven bifurcation parameters $\varepsilon$ scaled by the characteristic time $\tau_0$ represent the same experiments but conducted with different bifurcation parameters.

Codes in these seven subfolders identify coefficients in the CGLE. **Note** that data used for identification are not given. They are available from

Kolodner, P., Glazier, J. A. & Williams, H. Dispersive chaos in one-dimensional traveling-wave convection. *Physical Review Letters* **65**, 1579 (1990).

Kolodner, P. Counterpropagating quasilinear wave packets in binary-fluid convection. *Physical review letters* **69,** 2519 (1992).

# SI-fig

This directory contains figures and codes for generating these figures in Supplementary Information. Additionally, it involves codes for generating the simulated data used in **Examples**. Details can be found in comments at the beginning of each script.

# M-fig

This directory contains figures and codes for generating these figures in main text. Details can be found in comments at the beginning of each script.