Code for Machine Discovery of Partial Differential Equations from Spatiotemporal Data

Installation

To run the codes in S3d, one should install the CVX toolbox previously for constructing and solving convex programs. The latest version of CVX toolbox can be downloaded here. Follow these steps to install it.

S3d consists of the following subfolders.

Subfolders	Description
Functions	Subfunctions for identifying PDEs
Examples	Identification of 8 simulated PDEs
Realdata	Identification of Complex Ginzburg-Landau Equation
SI-fig	Figures and codes for generating these figures in SI
M-fig	Figures and codes for generating these figures in main text
comparison	Comparing the ability of three different algorithms to identify PDEs from local data

Functions

Twelve subfunctions used for identifying PDEs are included in this directory. They are listed in the following table.

Function	Description
build_Theta	Building the candidate dictionaries
finitediff	Computing the derivative of given data by using the finite difference method
make_fft	Computing the derivative of given data by using Fourier

	spectral method to approximate derivative
make_input_fd	Computing the spatial derivatives in the dictionary functions by using the finite difference method
make_input_pade	Computing the spatial derivatives in the dictionary functions by using Pade method
make_input_poly	Computing the spatial derivatives in the dictionary functions by using the polynomial interpolation method
make_y_fd	Computing the time derivative on LHS of PDE by using the finite difference method
make_y_pade	Computing the time derivative on LHS of PDE by using Pade method
make_y_ploy	Computing the time derivative on LHS of PDE by using the polynomial interpolation method
pade_c4	Computing the derivative of given data by using the Pade method
POD	Calculating the POD basis vectors without subtracting the mean of the ensemble
tac_reconstruction	Solving a linear regression problem with L1 regularization term.

Examples

This directory contains eight subfolders, and each subfolder includes scripts for identifying a simulated PDE.

Subfolder	**Identified simulated PDE**
Fisher	Fisher's equation
FN	FitzHugh-Nagumo
KdV	Korteweg-de Vries
KG	Klein-Gordon

KS	Kuramoto Sivashinsky
NS	Navier Stokes
NLS	Nonlinear Schrödinger
SG	Klein-Gordon

In each subfolder, there are two scripts, main.m and main_noisy.m, and a data file. main.m uses clean data to identify the PDE, while main_noisy.m adds 1% Gaussian noise to the data. Users can run these two scripts directly.

Realdata

This directory includes codes for identifying the Complex Ginzburg-Landau Equation based on data acquired in physical experiments. More details can be found in:

Kolodner, P. Counterpropagating quasilinear wave packets in binary-fluid convection. *Physical review letters* **69,** 2519 (1992).

Kolodner, P., Slimani, S., Aubry, N. & Lima, R. Characterization of dispersive chaos and related states of binary-fluid convection. *Physica D: Nonlinear Phenomena*, **85** 165-224 (1995).

There are seven groups of data, as listed in this table.

Data name	**Corresponding ε/τ_0**
car06172cb/cal06172cb	0.00932
car06182cb/cal06182cb	0.00422
car06192cb/cal06192cb	0.00177
car06212cb/cal06212cb	0.00638
car06222cb/cal06222cb	0.01207
car06242cb/cal06242cb	0.01403
car06252cb/cal06252cb	0.01628

The special letter “cal” and “car” respectively represent the left-going waves and right-going ones. Seven bifurcation parameters ε scaled by the characteristic time τ_0

represent the same experiments but conducted with different bifurcation parameters.

Codes in these seven subfolders identify coefficients in the CGLE. **Note** that data used for identification are not given. They are available from

Kolodner, P., Glazier, J. A. & Williams, H. Dispersive chaos in one-dimensional traveling-wave convection. *Physical Review Letters* **65**, 1579 (1990).

Kolodner, P. Counterpropagating quasilinear wave packets in binary-fluid convection. *Physical review letters* **69,** 2519 (1992).

SI-fig

This directory contains figures and codes for generating these figures in Supplementary Information. Additionally, it involves codes for generating the simulated data used in **Examples**. Details can be found in comments at the beginning of each script.

M-fig

This directory contains figures and codes for generating these figures in main text. Details can be found in comments at the beginning of each script.

Comparison

This directory contains the codes for utilizing three different methods to identify NLS and QH equation from local data. The methods are S3d, PDE-FIND, and Douglas-Rachford.