Knowledge Representation & Symbolic Reasoning Assignment 1: Propositional Logic

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1 Modelling and reasoning in propositional logic

1.1 From natural language to logic

1. "There is a box of hagelslag on the table".

$$hagelsag\_on\_table \tag{1}$$

2. "It is logically equivalent to say that it is possible to stock a product and to say that there is a product on the table and that the shelf is not full."

$$can\_stock\_product \leftrightarrow (product\_on\_table \land \neg full\_shelf) \tag{2}$$

3. "If there is hagelslag on the table or yogurt on the table then there is a product on the table".

$$(hagelslag\_on\_table \lor yogurt\_on\_table) \to product\_on\_table \tag{3}$$

4. "If the shelf is full, then it is not possible to stock a product".

$$full\_shelf \to \neg can\_stock\_product \tag{4}$$

5. "It is logically equivalent to say that the shelf is full and that there is no room in the shelf".

$$full\_shelf \leftrightarrow \neg room\_in\_shelf \tag{5}$$

1.2

1. "There is a box of hagelslag on the table".

$$hagelsag\_on\_table \tag{6}$$

2. "It is logically equivalent to say that it is possible to stock a product and to say that there is a product on the table and that the shelf is not full."

$$can\_stock\_product \leftrightarrow (product\_on\_table \land \neg full\_shelf) \tag{7}$$

Biconditional Equivalence:

$$\begin{aligned} &can\_stock\_product \leftrightarrow (product\_on\_table \land \neg full\_shelf) \equiv \\ &(can\_stock\_product \to (product\_on\_table \land \neg full\_shelf))\land \\ &((product\_on\_table \land \neg full\_shelf) \to can\_stock\_product) \end{aligned} \tag{8}$$

Implication Equivalence:

$$\begin{aligned} &(can\_stock\_product \to (product\_on\_table \land \neg full\_shelf))\land \\ &((product\_on\_table \land \neg full\_shelf) \to can\_stock\_product) \equiv \\ &(\neg can\_stock\_product \lor (product\_on\_table \land \neg full\_shelf))\land \\ &(\neg(product\_on\_table \land \neg full\_shelf) \lor can\_stock\_product) \end{aligned} \tag{9}$$

Associativity, de Morgan's laws and double negation

$$\begin{aligned}
&(\neg can_stock_product \vee (product_on_table \wedge \neg full_shelf))\wedge \\
&(\neg(product_on_table \wedge \neg full_shelf) \vee can_stock_product) \;\equiv \\
&(\neg can_stock_product \vee product_on_table)\wedge \\
&(\neg can_stock_product \vee \neg full_shelf)\wedge \\
&(\neg product_on_table \vee full_shelf \vee can_stock_product)
\end{aligned} \tag{10}$$

3. "If there is hagelslag on the table or yogurt on the table then there is a product on the table".

$$(hagelslag\_on\_table \vee yogurt\_on\_table) \rightarrow product\_on\_table \tag{11}$$

Implication equivalence:

$$\begin{aligned}
&(hagelsag_on_table \vee yogurt_on_table) \rightarrow product_on_table \equiv \\
&(\neg(hagelsag_on_table \vee yogurt_on_table)) \vee product_on_table
\end{aligned} \tag{12}$$

De Morgan's Law:

$$\begin{aligned}
&(\neg(hagelsag_on_table \vee yogurt_on_table)) \vee product_on_table \equiv \\
&((\neg hagelsag_on_table) \wedge (\neg yogurt_on_table)) \vee product_on_table
\end{aligned} \tag{13}$$

Distributive Law:

$$\begin{aligned}
&((\neg hagelsag_on_table) \wedge (\neg yogurt_on_table)) \vee product_on_table \equiv \\
&((\neg hagelsag_on_table) \vee product_on_table) \wedge ((\neg yogurt_on_table) \vee product_on_table)
\end{aligned} \tag{14}$$

1.3

1.3.1 Modus Ponens to find that product_on_table based on knowledge base

Since we have that:

$$hagelsag\_on\_table \tag{15}$$

And we have that:

$$(hagelslag\_on\_table \vee yogurt\_on\_table) \rightarrow product\_on\_table \tag{16}$$

We have that:

$$\frac{hagelsag\_on\_table \qquad (hagelslag\_on\_table \vee yogurt\_on\_table) \rightarrow product\_on\_table}{product\_on\_table} \tag{17}$$

1.3.2 Modus Ponens based on knowledge base

$$full\_shelf \leftrightarrow \neg room\_in\_shelf \tag{18}$$

We can say using biconditional equivalance and negation that:

$$room\_in\_shelf \rightarrow \neg full\_shelf \tag{19}$$

Using modus Ponens again we get that

$$\frac{room\_in\_shelf \qquad (room\_in\_shelf \rightarrow \neg full\_shelf}{\neg full\_shelf} \tag{20}$$

1.3.3

From 1.3.1 we have that:

$$product\_on\_table \tag{21}$$

From 1.3.2 we have that:

$$\neg full\_shelf \tag{22}$$

Using modus ponens:

$$\frac{product\_on\_table \wedge \neg full\_shelf \qquad (product\_on\_table \wedge \neg full\_shelf \rightarrow can\_stock\_product}{can\_stock\_product} \tag{23}$$