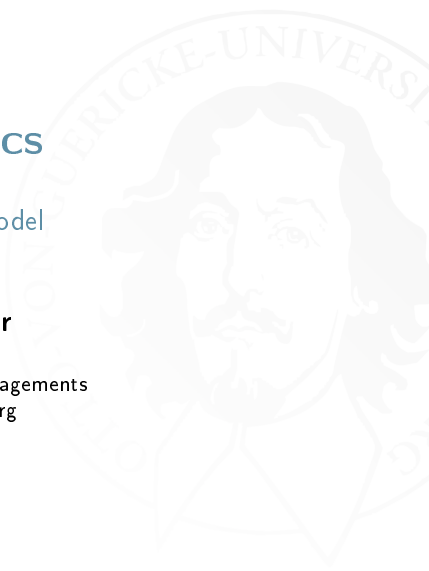


Predictive Analytics

Exercise 4: Multinomial Logit Model

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Quebec Heating System Choice

- ▶ Observations: 2,897 households located in Quebec region (CAN)
- ▶ Data collected 1989 by Hydro-Quebec (Energy Authority)
- ▶ Alternatives: combination of space and water heating system

1=gg gas / gas

2=ge gas / electricity

3=deo dual energy / oil

4=dee dual energy / electricity

5=oo oil / oil

6=oe oil / electricity

7=ee electricity / electricity

8=we wood / electricity

9=wee wood-electricity / electricity

dual energy: alternate between oil and electricity dependent on outside temperature

Choice Frequencies

Space heating	Water heating			
	Natural gas	Oil	Electricity	Total
Natural gas	27		9	36
Dual energy		72	201	273
Oil		12	20	32
Electricity			2351	2351
Wood			124	124
Wood-electricity			81	81
Total	27	84	2786	2897

The Data: quebec.dat

Variable	Description	Values
sector	Area of household location	Rural=1, Small urban=2, Urban=3, High Density=4
hdd	Heating degree days	in thousand
choice	Choice indicator	1=gg, 2=ge, ...
conv_year	Conversion year (1976-1989)	0 = unknown, 1=before 1976, 2=1976-1980, 3=1981-1982, 4=1983-1985, 5=1986, 6=1987 7=1988, 8=1989
house_type	Type of house	2=detached, 3=semi detached, 4=row of three or more, 13=house with multiple apartments with their own entrances
constr_year	Construction year (1920-1989)	1=before 1921, 2=1921-1945, 3=1946-1960, 4=1961-1965, 5=1966-1970, 6=1971-1975, 7=1976-1980, 8=1981-1985, 9=1986, 10=1987, 11=1988, 12=1989
nb_rooms	Number of rooms in house	
nb_persons	Number of persons in household	
own_rent	Owner or renter	1=owner, 2=renter
surface	Surface	thousand square feet
age	Age of household head	divided by 10 (years)
income	Household income	\$10,000
op_cost.X	Annual operating cost of alternative X	\$1,000
fix_cost.X	Annual fixed cost of alternative X	\$1,000
cost_incX	Interaction variable of alternative X expressed as annual fixed cost multiplied by income	
avail.X	Boolean variable indicating the availability of alternative X	

Descriptive Statistics

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
obs	2,897	1,449.000	836.436	1	725	2,173	2,897
sector	2,897	2.018	1.322	1	1	4	4
hdd	2,897	4.830	0.491	4.359	4.435	5.026	7.000
choice	2,897	6.705	1.236	1	7	7	9
conv_year	2,897	1.849	2.858	0	0	5	8
house_type	2,897	2.866	2.761	2	2	2	13
constr_year	2,897	8.260	3.064	1	6	10	12
nb_rooms	2,897	6.201	1.890	1	5	7	18
nb_pers	2,897	3.130	1.203	1	2	4	9
own_rent	2,897	1.092	0.289	1	1	1	2
surface	2,897	1.675	0.731	0.300	1.250	2.250	3.250
age	2,897	4.153	1.341	2.100	3.200	4.800	8.400
income	2,897	4.418	1.981	1.250	2.750	5.500	7.500
op_cost.1	2,897	0.240	0.520	0.000	0.000	0.000	2.969
op_cost.2	2,897	0.246	0.532	0.000	0.000	0.000	3.008
op_cost.3	2,897	1.613	0.383	0.494	1.351	1.846	3.700
op_cost.4	2,897	1.507	0.355	0.460	1.267	1.721	3.472
op_cost.5	2,897	1.871	0.447	0.558	1.567	2.143	4.364
op_cost.6	2,897	1.791	0.423	0.527	1.508	2.043	4.162
op_cost.7	2,897	1.449	0.338	0.453	1.223	1.655	3.245
op_cost.8	2,897	0.951	0.231	0.352	0.786	1.082	2.118
op_cost.9	2,897	1.325	0.309	0.428	1.116	1.514	2.934

Descriptive Statistics ctd.

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
fix_cost.1	2,897	0.165	0.345	0.000	0.000	0.000	1.401
fix_cost.2	2,897	0.162	0.339	0.000	0.000	0.000	1.386
fix_cost.3	2,897	1.215	0.111	0.294	1.201	1.228	1.618
fix_cost.4	2,897	0.994	0.109	0.138	0.988	0.995	1.386
fix_cost.5	2,897	0.968	0.025	0.644	0.956	0.982	1.140
fix_cost.6	2,897	0.746	0.015	0.431	0.743	0.749	0.908
fix_cost.7	2,897	0.281	0.213	0.065	0.145	0.299	0.913
fix_cost.8	2,897	0.586	0.010	0.563	0.582	0.589	0.933
fix_cost.9	2,897	0.680	0.015	0.437	0.677	0.684	0.940
cost_inc1	2,897	0.738	1.739	0.000	0.000	0.000	10.505
cost_inc2	2,897	0.726	1.712	0.000	0.000	0.000	10.394
cost_inc3	2,897	5.398	2.535	0.439	3.371	6.753	12.136
cost_inc4	2,897	4.413	2.093	0.173	2.732	5.475	10.394
cost_inc5	2,897	4.283	1.937	1.056	2.700	5.399	8.551
cost_inc6	2,897	3.298	1.484	0.790	2.061	4.122	6.809
cost_inc7	2,897	1.308	1.349	0.098	0.514	1.431	6.846
cost_inc8	2,897	2.589	1.165	0.712	1.620	3.239	5.133
cost_inc9	2,897	3.009	1.357	0.765	1.880	3.761	6.846
avail.1	2,897	0.187	0.390	0	0	0	1
avail.2	2,897	0.187	0.390	0	0	0	1
avail.3	2,897	1.000	0.000	1	1	1	1
avail.4	2,897	1.000	0.000	1	1	1	1
avail.5	2,897	1.000	0.000	1	1	1	1
avail.6	2,897	1.000	0.000	1	1	1	1
avail.7	2,897	1.000	0.000	1	1	1	1
avail.8	2,897	1.000	0.000	1	1	1	1
avail.9	2,897	1.000	0.000	1	1	1	1

Challenge Questions

- 1 Baseline model with only operational and fixed cost
- 2 Compute the predicted choice probabilities for the first observation by hand and use R to control your results.
- 3 Add income as an interaction variable with fixed cost to your model. How does the model improve?
- 4 Compute direct price elasticities for fixed and operational cost.
 - 1 Plot a histogram of the direct price elasticities for the fixed cost of the electric-electric alternative.
 - 2 Plot the direct price elasticities for the fixed cost of the electric-electric alternative versus its cost.
- 5 Compute cross price elasticities for oil-oil alternative.
- 6 Test whether the IIA property holds for our model?
- 7 Compute the willingness to pay (wtp) through higher installation (i.e., fixed) cost for a one-dollar reduction in operating costs. What does the value tell you?
- 8 Plot the wtp against the household income. Interpret the results.

Discount Rate and Life Cycle Cost

PV present value of the system

L life time of the system

LC life cycle cost: sum of installation cost and the present value of operating costs

r discount rate

$$PV = \sum_{t=1}^L \frac{\text{op_cost} \cdot X}{(1+r)^t}$$

- ▶ $L \rightarrow +\infty$ then $PV = \text{op_cost} \cdot X / r$
- ▶ If the life time of the system is sufficiently long, a one-dollar reduction in $\text{op_cost} \cdot X$ reduces the present value of future operating costs by $1/r$
- ▶ $\text{wtp} = 1/r$, i.e., rational trade-off between installation (fixed) cost and operating cost
- ▶ Hence $V = \alpha LC = \alpha(\text{fix_cost} \cdot X + \text{op_cost} \cdot X / r) = \alpha \text{fix_cost} \cdot X + (\alpha / r) \text{op_cost} \cdot X$

Challenge Questions ctd

- 1 Compute α , (α/r) , and r
- 2 What is the discount rate for a household with an annual disposable income of \$32,500?
- 3 Assume a price increase for operation cost of gas by 15%. Predict the market shares of the alternatives under this price scenario.
- 4 Play around with the other variables in the data set and come up with your own model specification. Which one do you prefer? Why?