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Seminar Paper

# Crew Scheduling

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February 17th, 2023

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# Abstract

This paper aims to explore various crew scheduling problems, models, and methodologies. It provides a comprehensive overview of the evolution of crew scheduling approaches over time, including the introduction of different approaches and their recent advancements. Additionally, the paper presents a column generation approach based experiment which address the bus driver (Crew) scheduling problem using a dataset comprising fictitious information on 50 daily bus trips. The experiment employs two optimization models to optimize the function, tracking the total time spent driving, including the current shift of the driver. Implementing these models can contribute to ensuring drivers' well-being, safety and providing high-quality transportation services to passengers.

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# 1. Introduction

As the society continues to develop, complex problems and the need for automation have become more prevalent, especially in public transportation. Planning to meet employee needs and comply with regulations and resource limitations has become increasingly difficult due to the sheer amount of information and constraints involved. Therefore, it is crucial to explore and apply effective new planning methods.

One of the most fundamental aspects of traffic is crew scheduling. Crew scheduling is a critical part of transporting operations involving allocating crew members to flights, trains, or buses to minimize costs while ensuring maximum efficiency and safety. However, this task can be extremely complex due to various variables, constraints, and requirements, making a powerful and efficient crew planning algorithm essential in the aviation industry.

A well-designed crew scheduling algorithm can yield substantial benefits, including improved operational efficiency, cost reduction, and better service quality. Optimizing crew schedules ensure that crew members are utilized efficiently, reducing the likelihood of delays, cancellations, or other operational interruptions. This, in turn, enhances the customer experience and boosts operating profitability.

Developing a crew planning algorithm is also a thrilling opportunity to advance public transportation operations research. By leveraging sophisticated optimization techniques and machine learning algorithms, researchers can create more complex and efficient solutions, improving the accuracy and efficiency of crew scheduling. These techniques enable the development of algorithms capable of handling even the most complex scheduling problems with ease.

In this paper, we summarized the development process of crew scheduling studies and the popular algorithms being used to solve this optimization problem. Besides, we use Google OR-Tools, which is an open-source software suite for optimization, which includes tools for solving combinatorial optimization problems such as driver scheduling and to solve a specific crew scheduling problem for bus operation.



## 2. Literature Review

### 2.1 General Overview

Crew scheduling involves organizing the duties and shifts of employees, drivers, and staff. Crew Scheduling Problem (CSP) involves assigning duties to crew members so they can cover all transport activities during a specified time period. A crew schedule is designed to optimize the distribution of tasks and shifts to drivers and staff. While minimizing costs and maximizing productivity, factors such as availability, skill set, and labor laws are taken into account [Heil et al. \[2020\]](#). In order for a transport system to be efficient, several elements must be in place, including technology, government rules, and planning processes. As a result of this complex interaction, decisions making becomes intractable. A crew scheduling problem involves creating a feasible crew schedule given a defined plan and collection of trips, log-in and log-out times, types of duties, and sequence of trips that cover the schedule while meeting work rules constraints. Combined, they form the crew schedule. During crew scheduling, we strive to maximize the effectiveness of human personnel along with complying with any work restrictions. Typically, crew scheduling aims at minimising crew costs, minimizing the crew number required, and optimizing crew efficiency. [Chen and Niu \[2012\]](#). Those researches are highly application-oriented across all industries [Heil et al. \[2020\]](#). Some common research approaches include integer programming, linear programming, heuristics, simulation, and multi-objective optimization. A commonly used approach in the airline industry is column generation which was more often used for research and problem solving from 1990-2000 because it gave good quality results [Jütte \[2012\]](#). By using the combination of interior-point algorithm, two steepest edge simplex, and standard LP-based B and B to achieve better results than standard linear programming techniques. Some researchers for solving linear programming relaxation introduced dual coordinate search, which was applied in the CARMEN system in some major airlines.

### 2.2 History

The foundations of transportation crew scheduling issues may be traced back to 1950-1960 [Arabeyre et al. \[1969\]](#). In the 1980s, computational power advancements



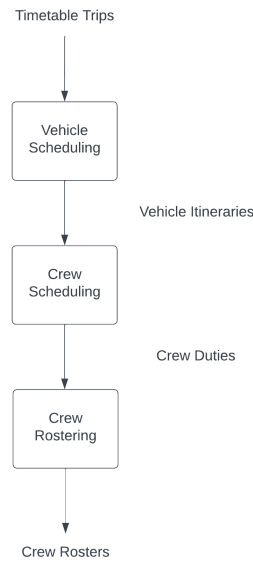
gave it increased momentum Carraresi and Gallo [1984]. Mainly airline and bus transportation research activities during this time period. Research activities on crew scheduling started to gain importance in the railway industry during the 1990s due to technology, government policy, planning processes, and control strategies, and these interactions lead to intractable decision-making challenges. A number of practical applications with promising results have been developed in recent years, mainly for airline and bus transportation. Interest in Operational Research techniques began to rise as a result of potential cost savings and increased computing power, allowing railway operators to solve more complex crew scheduling problems. European and international railway privatization even led to more cost-efficient resource use. A basic crew scheduling problem is when crews are assigned to duties with fixed starting and ending times to ensure they do not exceed a total time limit. Transportation industry CSPs share a lot of similarities in principle. Crew scheduling studies are often focused on a single application due to its particular features and challenges.

### 2.3 Urban Bus Crew Scheduling

In public bus transportation, a depot is a place where vehicles are stored, maintained, and managed. It is possible for a depot to have its own capacity and fleet type. Vehicle scheduling problems can be divided into single-depot and multiple-depot types, depending on the number of available depots. When a vehicle leaves the depot, a crew must be responsible for it. The CSP is responsible for creating crew workdays or responsibilities. The VSP solution addresses the CSP by considering vehicle itineraries. It is often the case that a vehicle's operation cannot be fully handled by a single crew. It is not possible for the change the Crews that are responsible for vehicles, whenever you like. The change is only gonna be possible on the relief points. A trip is the modified unit in the VSP and it is a task in the CSP. Tasks are really the minimum level of work that a crew can undertake Boyer et al. [2018]. An urban transit line's scheduling task is to coordinate all shifts with different duties in one day. A crew's requirements are influenced by their working hours, residence, and living habits. Therefore, how to organize crew schedules such that personnel resources be used effectively while meeting a set of requirements laid down by the labor regulations has an extremely significant theoretical and practical significance Chen and Niu [2012]. Optimizing crew scheduling for a circle bus line as LP problem reduces the total idle time while improving efficiency.

### 2.4 Planning Levels

Decisions about crew personnel can also be categorized according to different planning stages. The three levels can be described as strategic planning, tactical planning, and operational planning Caprara et al. [2007]. Long-term problem decisions are done in Strategic planning. On the top level, our goal is to satisfy end-user needs while staying within predetermined financial constraints. In addition, we describe the bus lines that travel throughout the city. Furthermore, the selection of crew depot sites, including opening and closing choices, is critical at strategic level Simões et al. [2021]. We decide the frequency with which bus routes must be traversed and the timetable during tactical planning. The timetable lists the daily journeys



**Figure 2.1:** Planning Levels

that the public transportation operator must make. The start and end, time, and location are given for each trip.<sup>z</sup> A vehicle fleet operating routine and crew duties is planned in operational planning [Simões et al. \[2021\]](#). The planning process of the public bus transport system is highly complex and normally decomposed into several subproblems. Fig. 2.1 shows that we have a VSP that consists of setting up daily operating routines for a fleet of vehicles that ensures all timetable trips are executed, costs are minimized, and all operational constraints are met. For each vehicle in the fleet, there must be a crew (driver). As a result, CSP is focused on determining duties of the crew members. When addressing the CSP, our objective is to minimize number of drivers, number of working hours and number of driving hours of the driver while ensuring that all governmental, labor and operational regulations are fulfilled. Tasks in operational planning involve crews and are often separated into two stages: crew scheduling and crew rostering. The first creates anonymous tasks that cover all journeys for a specified time-period, such as a single work day. Following that, the tasks are integrated into weekly, or sometimes monthly, sequences that are then assigned to specific crew members; this is known as crew rostering.



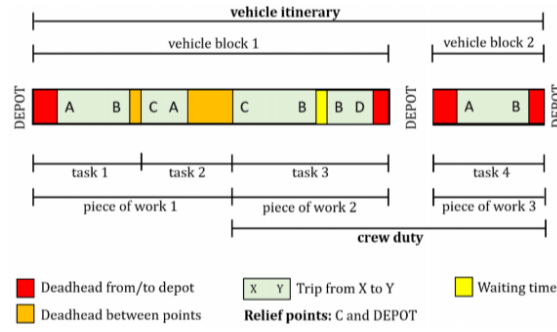
## 3. Existing Models And Methodologies

### 3.1 Models

The model may be classified into three types based on its goal: set covering, set partitioning, and multi-objective optimization programs. Integer linear programming can be used to solve CSP on the basis of a set covering problems [Smith and Wren \[1988\]](#). The set covering issue seeks to identify the smallest range of sets that cover all of the components if provided with a list of elements, it is an NP-hard problem. [Darby-Dowman and Mitra \[1985\]](#) developed a computer-aided bus crew scheduling system called extended set partitioning model. Using dynamic programming, [Beasley and Cao \[1998\]](#) created a lower limit in crew scheduling. A set partitioning challenge helps determine how to separate entries in one set into sub - sets. All elements in the primary set need to be stored in a single partition. Using metaheuristics, genetic algorithms, and tabu search techniques, the model tackled multiobjective crew scheduling [Lourenço et al. \[2001\]](#). A concept of integrated technique for solving a VSP and a CSP on a single bus route has also been proposed.

### 3.2 Methods

Crew scheduling methods include run-cutting heuristics, HASTUS, and set-covering [Desrochers and Soumis \[1989\]](#). HHeuristics are run in two phases: Firstly, heuristic divides the vehicle blocks into parts and afterwards merges two or three parts to generate a build day that is achievable. Secondly, the heuristic attempts to reduce, cost of the schedule by swapping parts between workdays or adjusting the cutting of a vehicle block into parts to get a reduced price. Three phases are involved in the Haustus method of scheduling. Using an LP formulation that relaxes integral constraints and physical feasibility, an approximate schedule is constructed in the first phase. The second phase consists of cutting the vehicle books into pieces and then using a matching algorithm to combine them into workdays. Thirdly, each vehicle block is cut into pieces again and local improvements are made. The set



**Figure 3.1:** Crew Duty and Vehicle Itinerary

covering problem consists of set partitioning problems in which the inequalities are replaced by equalities, to choose the workdays belonging to the schedule. With the set partitioning problem, over-covering tasks are not allowed [Desrochers and Soumis \[1989\]](#).

### 3.3 Two Integrated Approaches

In terms of integrated approaches, there are two types: partial integrations involving sequential problem-solving; and complete integrations requiring simultaneous decision-making for the integrated problems [Ibarra-Rojas et al. \[2015\]](#). It is more difficult to define and solve integrated approaches because they include all degrees of freedom. VSP and CSP are fundamentally network flow problems and set covering or partition problems. Furthermore, in addition to CG methods and LR techniques, Set covering/partition formulations can also be used to develop a fully integrated framework for VSP and CSP. For achieving better results MDVSP can be embedded with crew scheduling problems, dealing with these two problems simultaneously allows us to cater better scheduling for both problems at a same time. Column generation methods are also designed for integrated methods with customizable scheduling [Ibarra-Rojas et al. \[2015\]](#). Researchers realize the intractable nature of complex situations and present a meta-heuristic strategy for achieving higher-quality solutions in a shorter amount of time. [Steinzen et al. \[2007\]](#). [Leone et al. \[2011\]](#) Taking several hard constraints into account, like a restricted distribution across, a finite number of pauses, upper limits for idle time (when the crew member is not working and also not on a break), as well as limited workhours (the maximum hours which a crew personnel is allowed to work as per rules and regulations), integration of MDVSP and CSP is achievable.

## 4. Crew Scheduling Problem

### 4.1 Bus Driver Scheduling Problem

Crew Scheduling defines a generic set of duties covering a set of vehicle blocks with planned routes to minimize the costs of duties, which includes number of drivers, working time, driving time etc., while complying with work laws and regulations. To tackle this challenge, it would be required to identify the beginning and finishing timings of each driver's duty. Usual DSP limits include a restricted duty span and rest for drivers, a limited amount of constant work without pauses, and duty start times [and Rousseau \[1995\]](#). In the DSP, many objective functions are taken into account, such as reducing the cost of duties, reducing idle times, minimization of penalties for constraint violations, minimization of uncovered duties, and minimizing disruption costs. Often, the potential area of the DSP is limited by answers to previous subproblems of transit network problem. This is because the DSP is one of the last problems in TNP to be solved sequentially. The DSP has this characteristic, which makes it difficult to find a feasible solution that satisfies all its constraints. Usually, the DSP is formulated in terms of partitioning and covering sets. All possible duties must be fed into the model in these types of formulations. All trips should be covered by the chosen duties at a minimum cost. These formulations are intractable to exhaustive enumeration methods because of the large number of variables involved. In addition to reducing the number of duties, CG approaches are also used for solving these formulations, and dual bounds can be obtained through heuristics [Boschetti et al. \[2004\]](#) for use in Branch and Bound algorithms.

### 4.2 Column Generation Approach

Column generation approaches in terms of Crew scheduling problems are often incapable by the need to solve sub-problems [Ibarra-Rojas et al. \[2015\]](#). Using column generation, a problem is segregated in two steps: First, a set covering problem and secondly, a subproblem. A practical schedule is chosen from the set covering the problem. New feasible workdays are suggested for improving the set covering problem as a result of the subproblem. Transit crew scheduling can be optimized

with this approach. Some researchers based on labor regulations constraints, under a Branch and Bound architecture, suggested developing a set of valid shifts by using column generation. This will enable us to choose the most productive group of crews from the available shift options. However, it does not guarantee that the proposed approach will produce an optimal result. [Chen and Shen \[2013\]](#) defined an algorithm that generates a set of possible efficient shifts (columns), reducing the computational time of column generation. [Boschetti et al. \[2004\]](#) extend this method to multi-depot DSPs. The proposed CG approach can be implemented more efficiently through the implementation of Lagrangian Relaxation and column generation. [Portugal et al. \[2009\]](#) Despite simplifying some specific aspects of real problems and business situations using traditional set partition and set covering models, it is hard to implement such models in automated planning methodologies because results that are generated have to be enhanced/revised manually so that they can meet the needs of real situations. [Steinzen et al. \[2009\]](#) consider the homogeneity of drivers' schedules. The formulation set was solved in two stages using CG and LR:- After resolving the formulations of linear relaxation, localized branching techniques are used to produce integer results that preserve the optimum drivers' costs while enhancing the consistency of overall schedules. Utilizing both real-world and artificially produced situations, this approach is tested. Based on the results of numerical simulations, low increments in operational costs could be achieved by improving the homogeneity of drivers' schedules.

### 4.3 Heuristics

Heuristic algorithms are suggested since huge instances of Driver Scheduling Problems are unmanageable and methods may take a long time to analyze. [Zhao \[2006\]](#) divides the problem in two parts, labeling them as "day" and "night" challenges. Every problem is resolved heterogeneously afterward they are merged to obtain a desired result with respect to the whole day. [Kecskeméti and Bilics \[2013\]](#) compare three solution methodologies: Column Generation, Evolutionary Algorithm (EA), and a hybrid of CG and EA. The solution of CG shows the best quality results, but on the other hand, EA and hybrid algorithm solution run time is better. Hybrid algorithm execution time is more than EA, but it is reasonable and shows better quality results. Make the assumption that drivers are up to the tasks assigned to them during specified hours of the day. [Chen and Shen \[2013\]](#) discuss the DSP where drivers' lunch breaks must fall inside a certain time range of the day, the breaks are hard constraints which make this interaction more complex. Implementation of heuristic in this manner generates feasible results even for a large number of instances. Several solutions based on different parameter values may be required to choose best optimal solution for (TN) transit network operations [Ibarra-Rojas et al. \[2015\]](#). An approach based on greedy solutions is proposed in two steps. Initially, uncertain shifts are produced, which only include traveling activities of drivers and trips. Second, the shifts are constructed, each of which includes all of the mandatory operations, and the downtime are filled appropriately with idle operations. To tackle DSP in airline transportation, merge recovery and robust methodologies. Minimizing the impact of disruptions in the original schedule on future actions is the objective. Comparing the deterministic approach with computational experiments, fewer flight

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changes are required. When a disruption occurs, [Ibarra-Rojas et al. \[2015\]](#) examine the decision to reschedule based on constraints, such as driver coverage, break times, continuous shifts, and work time bounds.





## 5. Algorithms and Experiments

### 5.1 Dataset description

In our experiment, we applied column generation to a data set consisting of fictitious information about 50 bus trips a day, including shift id, starting and ending time of each shift, and the duration of each shift. Sample data is presented in the table 5.1 below. The average time of each bus trip is 47.1 minutes, the shortest is 10 minutes, and the longest is 90 minutes. The earliest bus in the morning starts at 5:18, and the last one in the evening ends at 20:48. The converted start, end minutes, and duration of each shift are used in our experiment.

### 5.2 Experiment design

Two distinct models were employed to optimize the function and address the bus driver scheduling predicament, as detailed in 5.1. The first model is to focus on finding the minimum amount of drivers, regardless of their working hours. In the second model, based on the result of the first model, a maximum number of drivers will be established as non-optional, and the goal will be to minimize the total working hours of these drivers.

The driver scheduling conditions are a set of rules that need to be met in order to ensure that drivers follow a safe and legal work schedule. The first condition states that the driving time per driver should not exceed 9 hours, which is a legal limit set by transportation authorities to ensure driver safety and avoid accidents caused by fatigue. The second condition states that the working time per driver should not exceed 12 hours, which includes both driving and non-driving time. The third condition is a soft constraint that specifies that the minimum working time per driver should be 6.5 hours, which is a threshold that, if not met, may result in penalties or additional costs. The fourth condition requires a 30-minute break after every 4 hours of driving time to ensure that drivers can rest and avoid fatigue. The fifth condition specifies that a 10-minute preparation time is necessary before the start of the first shift to allow drivers to check their vehicles and ensure their safety. The

**Table 5.1:** Sample timetable of shift duties in one day of bus system

Shift ID	Start time	End time	Start minute	End minute	Duration in minutes
0	5:18	6:00	318	360	42
1	5:26	6:08	326	368	42
2	5:40	5:56	340	356	16
3	6:06	6:51	366	411	45
4	6:40	7:52	400	472	72
5	6:42	7:13	402	433	31
6	6:48	8:15	408	495	87
7	6:59	8:07	419	487	68
8	7:20	7:36	440	456	16
...	...	...	...	...	...
49	20:05	20:48	1205	1248	43

sixth condition requires a 15-minute cleaning time after the last shift to maintain the cleanliness and safety of the vehicle. Finally, the seventh condition requires a 2-minute waiting time after each shift for passenger boarding and alighting, which is necessary to ensure a safe and efficient operation of the transportation service. By following these conditions, driver scheduling can help to ensure that drivers are well-rested, safe, and capable of delivering transportation services of superior quality to passengers.

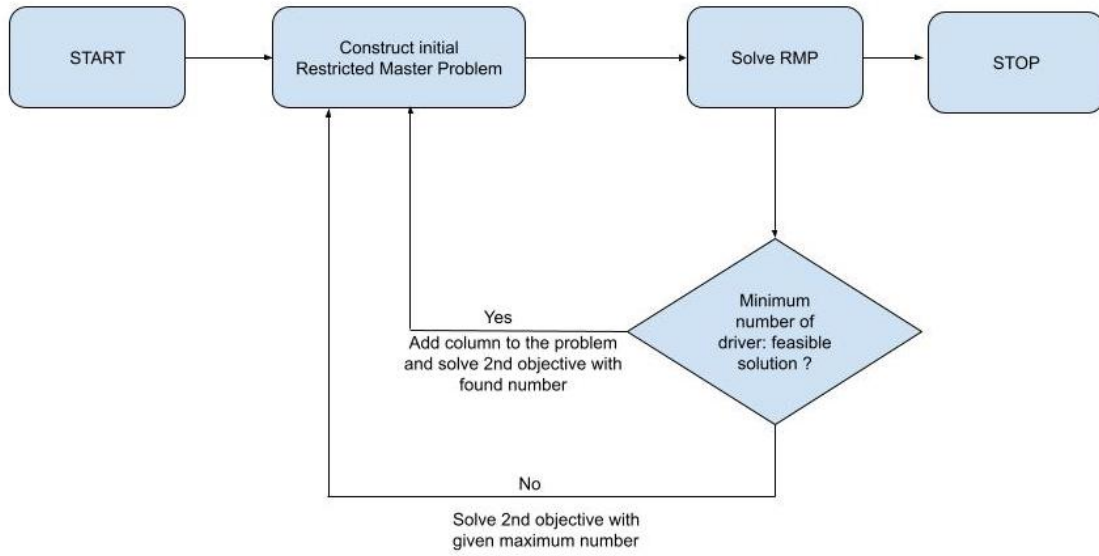
In this study, we define a node as the pairing of a driver and their respective shift. To evaluate potential solutions, we will analyze the aggregate duration of driving time for each node, which encompasses both the current shift's duration and the accumulated driving time since the driver's most recent 30-minute break.

Regarding special arcs for the beginning and ending working time of each driver, they have the following characters:

- From the original point to shift, the arc sets the started time and accumulates the first shift
- From the shift to the end arc sets the ending time and updates the driving time variable

For the normal arcs between two shifts, we added the duration of the shift to the total driving time of each driver in the data matrix. However, if the time between two shifts is more than 30 minutes, the accumulated driving time is reset. Otherwise, the duration of the shift is added to accrued driving time since the last break.

The nodes are a combination of each shift and each driver. Node contains information of the total driving time of that driver at that shift, the status of active performed driving at that shift of mentioned driver, and time spent driving since the last break. In the last shift of the driver, we established the completion time of that driver and determined their driving time. An inactive node is a shift not served by a specific driver. We set their driving time to 0 and created a looping arc on the node. With



**Figure 5.1:** The flow of column generation algorithm

the active node, we added a 10-minute setup time and a 15-minute cleaning time to establish each node's upper and lower boundary. If the interval between the preceding shift's termination and the current shift's commencement exceeds 30 minutes, we examine the uninterrupted driving that has been restarted.

### 5.3 Formulation proposes

Variables for the model:

- $start\_time^d$ : the start-time of bus driver  $d$  with  $d \in \text{set of drivers } D$
- $end\_time^d$ : the end-time of bus driver  $d$  with  $d \in D$
- $driving\_time^d$ : the driving time of bus driver  $d$  with  $d \in D$
- $total\_driving_s^d$ : total duration of driving performed by a driver  $d$  at shift  $s$  with  $\forall d \in D, s \in \text{set of shifts } S$
- $no\_break\_driving_s^d$ : no break driving performed by a driver  $d$  at shift  $s$  with  $\forall d \in \text{set of driver } D, s \in S$

Decisive variables

- $active_s^d \in \{0, 1\}$  decide whether driver  $d$  work on shift  $s$  or not
- $source\_shift_s^d \in \{0, 1\}$  decide whether the shift  $s$  is the source of driver  $d$
- $final\_shift_s^d \in \{0, 1\}$  decide whether the shift  $s$  is the end of driver  $d$
- $working^d \in \{0, 1\}$  decide whether the driver work on mentioned day

Objective:

Minimize  $working^d$  and Minimize  $working\_time^d * working^d$

Subject to these constraints:

1.  $start\_time^d = (shift\_start\_minute_s - setup\_time) * source\_shift_s^d$   
 $\forall d \in D, s \in S$
2.  $start\_time^d \leq start\_time_s^d - setup\_time$  if  $active_s^d = 1, \forall d \in D, s \in S$
3.  $end\_time^d = shift\_end\_minute_s + cleanup\_time$  if  $source\_shift_s^d = 1 \forall d \in D, s \in S$
4.  $end\_time^d \geq end\_time_s^d + cleanup\_time$  if  $active_s^d = 1, \forall d \in D, s \in S$
5.  $total\_driving^d = duration_s * active_s^d \forall d \in D$
6.  $total\_driving_s^d = duration_s$  if  $source\_shift_s^d = 1, \forall d \in D, s \in S$
7.  $no\_break\_driving_s^d = duration_s$  if  $source\_shift_s^d = 1, \forall d \in D, s \in S$
8.  $driving\_time^d = total\_driving_s^d$  if  $end\_shift_s^d$
9.  $total\_driving_s^d = 0$  if  $active_s^d = 0, \forall d \in D, s \in S$
10.  $no\_break\_driving_s^d = 0$  if  $active_s^d = 0, \forall d \in D, s \in S$
11.  $working\_time^d = end\_time^d - start\_time^d$
12.  $working\_time^d = min\_working\_time$  if  $working^d = 0$   
 $\geq min\_working\_time$  if  $working^d = 1$
13.  $total\_driving_s^d \leq 540$  (9 hours)  $\forall d \in D$
14.  $390 \leq working\_time_s^d \leq 720$  (12 hours)  $\forall d \in D$
15.  $0 \leq no\_break\_driving_s^d \leq 240 \forall d \in D, s \in S$
16.  $total\_driving^d, no\_break\_driving_s^d, driving\_time^d, total\_driving_s^d, no\_break\_driving_s^d, working\_time^d \geq 0 \forall d \in D, s \in S$

## 5.4 Experimental results and discussion

We find the optimal solution for crew scheduling by using Google's Ortool packages to solve the optimal solution for the mix integer linear programming problem as proposed formular. We also inherited some suggested code from source code of Ortool to solve the problem. A feasible solution was found for the minimum number of drivers to complete all shifts for the day which is 8 drivers. After 8 drivers were identified, a similar model was run to arrange shifts to obtain the lowest total working hours and satisfy the given conditions to ensure drivers' health and compliance the labor regulation.

The results of the model are presented in the table 5.2 below. Accordingly, the minimum total driving time of the driver is 200 minutes or 3 hours 20 minutes.

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Maximum total driving time is 425 minutes or 7 hours 5 minutes. The average driving time of each driver is 4 hours 54 minutes. Meanwhile, the total working time of 8 drivers is from 6 hours 31 minutes to 11 hours 59 minutes. The mean duration of work per driver is 9 hours and 16 minutes. The preceding outcome indicates the fulfillment of all the conditions enunciated in the aforementioned section. This substantiates the efficacy of the developed model in effectively scheduling bus drivers to their respective work shifts.

**Table 5.2:** Bus drivers scheduling results

	Begin	Break	End
Driver 1: total driving time= 242 working time = 451	shift 0: 05:18 - 06:00	** break**	shift 5: 06:42 - 07:13 shift 8: 07:20 - 07:36 shift 10: 07:50 - 08:55 shift 18: 09:23 - 09:49 shift 20: 09:57 - 10:20 **Break** shift 24: 11:45 - 12:24
Driver 2: total driving time = 394 working time = 719	shift 1: 05:26 - 06:08 **break** shift 4: 06:40 - 07:52 shift 11: 08:00 - 09:05 shift 19: 09:30 - 09:40 shift 21: 10:09 - 11:03	** break**	shift 28: 14:03 - 14:50 shift 32: 15:03 - 15:50 shift 36: 15:58 - 16:45 shift 40: 16:50 - 17:00
Driver 3: total driving time = 271 working time = 465	shift 2: 05:40 - 05:56 shift 3: 06:06 - 06:51 shift 7: 06:59 - 08:07 shift 13: 08:11 - 09:41	** break**	shift 23: 11:00 - 11:10 **break** shift 25: 12:18 - 13:00
Driver 4: total driving time = 200 working time = 665	shift 9: 07:35 - 08:22 shift 15: 08:35 - 08:45 **break** shift 22: 10:20 - 10:30	** break**	shift 31: 14:48 - 15:35 **break** shift 39: 16:36 - 17:21 shift 45: 17:34 - 18:15
Driver 5: total driving time = 425 working time = 707	shift 6: 06:48 - 08:15 shift 14: 08:28 - 08:50 shift 17: 09:03 - 10:28 **break** shift 26: 13:18 - 14:44	** break**	shift 33: 15:28 - 16:54 shift 42: 17:01 - 17:13 shift 44: 17:23 - 18:10
Driver 6: total driving time = 267 working time = 656	shift 12: 08:00 - 08:35 shift 16: 08:40 - 08:50 **break** shift 27: 13:53 - 14:49	** break**	shift 34: 15:38 - 16:25 shift 38: 16:28 - 17:15 shift 43: 17:19 - 18:31
Driver 7: total driving time = 284 working time = 403	shift 30: 14:30 - 15:41	** break**	shift 41: 16:54 - 18:20 shift 47: 18:34 - 19:58 shift 49 : 20:05 - 20:48
Driver 8: total driving time = 272 working time = 391	shift 29: 14:28 - 15:15 shift 35: 15:40 - 15:56 shift 37: 16:04 - 17:30	** break**	shift 46 : 18:04 - 19:29 shift 48 : 19:56 - 20:34

## 6. Conclusion

The domain of crew scheduling has a long-standing history of development, with novel methods yielding numerous benefits in practical applications. However, the inherent complexity of real-world crew scheduling problems far surpasses those addressed in the scope of this research. Nevertheless, the proliferation of artificial intelligence and machine learning has spurred the emergence of new approaches and techniques for more efficiently and effectively solving such intricate scheduling problems. This includes advanced optimization algorithms capable of processing vast amounts of data, detecting patterns, and making predictions to optimize crew schedules.

Moreover, the advent of novel technologies like autonomous vehicles, intelligent transportation systems, and on-demand transportation services is poised to catalyze considerable changes in the public transportation sector, necessitating innovative and pioneering methods for crew scheduling. Furthermore, the ongoing research and development in the field of transportation systems and operations is expected to drive novel advancements in crew scheduling, encompassing data analytics, simulation modeling, and other cutting-edge tools for generating more accurate and efficient crew schedules.

In essence, the future of solving crew scheduling problems in public transportation seems optimistic, as it is anticipated to bring new breakthroughs and solutions that will augment the efficiency, safety, and quality of public transportation services, benefiting both passengers and crew members.





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We herewith assure that we wrote the present paper independently, that the thesis has not been partially or fully submitted as graded academic work and that I have used no other means than the ones indicated. We have indicated all parts of the work in which sources are used according to their wording or to their meaning.

Magdeburg, 17th February 2023