Periodic Context Compression (PCC): Scaling Multi-Step Reasoning in Large Language Models Beyond the Context Horizon

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Abstract

Large Language Models (LLMs) have demonstrated remarkable capabilities in complex, multi-step reasoning tasks. However, these abilities are fundamentally constrained by two factors: the finite length of the attention context window and the degradation of performance (often termed "hallucination" or "loss of coherence") in long-horizon reasoning chains, typically observed after approximately 15–20 steps. This paper introduces **Periodic Context Compression (PCC)**, a novel, token-efficient, and computationally lightweight memory management strategy designed to extend the effective reasoning horizon of LLM agents. PCC operates by periodically compressing a fixed-length segment of the most recent interaction history into a concise, high-density *Memory State Vector* (M_t), which is then prepended to the working context. This recursive summarization mechanism drastically reduces the token count of the history while preserving critical information for future steps. We formally define the PCC algorithm, provide a mathematical framework for its token and computational efficiency, and propose a comprehensive experimental design to validate its efficacy in extending the effective reasoning depth of LLM agents. Our analysis suggests that PCC offers a scalable solution for long-term, coherent, and resource-efficient autonomous reasoning.

1. Introduction

The advent of Large Language Models (LLMs) has revolutionized artificial intelligence, particularly in tasks requiring natural language understanding, generation, and complex logical deduction. Agentic LLMs, which operate in iterative loops of observation, thought, and action, represent a powerful paradigm for solving real-world problems. Yet, a critical bottleneck persists: the **Effective Reasoning Horizon (ERH)**.

Empirical observations across various agentic frameworks suggest that the coherence and success rate of multi-step reasoning tasks drop significantly after a certain number of steps, often clustering around 15 to 20 iterations [1, 2]. This phenomenon is a composite result of several underlying issues: 1. **Context Window Saturation:** The total number of tokens (prompt, history, output) exceeds the model's physical limit, leading to truncation and memory loss. 2. **Attention Dilution:** Even within the physical limit, the model' s attention mechanism struggles to effectively weigh and recall relevant information from a vast, uncompressed history [3]. 3. **Catastrophic Forgetting (in-context):** As the history grows, the most relevant initial information is often pushed out or becomes too distant for the model to reliably attend to, leading to a loss of long-term task coherence.

The proposed **Periodic Context Compression (PCC)** directly addresses these limitations by introducing a mechanism for generating a dynamic, high-density summary of the recent past. This summary acts as a form of *proactive, in-context long-term memory*.

The key contributions of this paper are: * Formal definition of the PCC algorithm and its integration into the standard LLM agent loop. * Mathematical modeling of the token and computational complexity reduction offered by PCC. * Hypothetical experimental design demonstrating the extension of the ERH beyond current state-of-the-art methods.

2. Related Work

The challenge of extending the effective context and memory of LLMs is a central theme in contemporary AI research. Our work builds upon and differentiates itself from several established approaches.

2.1. Context Extension and Attention Mechanisms

Early work focused on increasing the physical context window size through architectural modifications. Techniques such as **Sparse Attention** [4] and **FlashAttention** [5] have dramatically improved the efficiency of processing long sequences. More recently, models boasting context windows of 10^5 to 10^7 tokens have emerged [6]. However, these approaches are resource-intensive and do not solve the *Attention Dilution* problem, where performance often degrades when relevant information is buried deep in the context (the "Needle in a Haystack" problem) [3]. PCC is orthogonal to these methods; it focuses on *information density* rather than *raw capacity*.

2.2. Memory and Retrieval-Augmented Generation (RAG)

Retrieval-Augmented Generation (RAG) [7] is the dominant paradigm for providing LLMs with external, long-term memory. RAG systems store vast amounts of information in external vector databases and retrieve relevant chunks based on semantic similarity to the current query. While highly effective for fact retrieval, RAG is inherently reactive and non-sequential. It struggles to capture the *sequential, state-dependent coherence* required for multi-step reasoning. PCC, in contrast, creates a memory specifically tailored to the *progression of the current task*, making it a *proactive, sequential memory* mechanism.

2.3. Recursive and Progressive Summarization

The concept of using LLMs to summarize their own history is well-documented, particularly in dialogue systems [8, 9]. **Recursive Summarization** [10] often involves summarizing the entire history up to a point and replacing it with the summary. **Progressive Summarization** breaks the input into chunks, summarizes each, and then summarizes the summaries. PCC distinguishes itself by: 1. **Periodicity:** The compression is triggered at fixed, predetermined step intervals (N), ensuring consistent memory updates and predictability. 2. **Locality:** It only compresses the *last N steps*, maintaining the high-fidelity, uncompressed context of the most immediate past, which is crucial for the next action. 3. **Integration:** The output is a high-density *Memory State Vector* (M_t), designed to be a concise representation of the task's current state and progress.

Our method is a hybrid, combining the token efficiency of summarization with a structured, periodic trigger mechanism to specifically target the multi-step reasoning failure mode.

3. The Periodic Context Compression (PCC) Framework

3.1. Formal Definition

Let H_t be the complete history of the agent's interaction up to step t, composed of a sequence of step records $S_i = (\text{Observation}_i, \text{Thought}_i, \text{Action}_i)$.

$$H_t = \{S_1, S_2, \ldots, S_t\}$$

Let C_t be the **Working Context** provided to the LLM at step t. In a standard agent loop, $C_t = H_{t-1}$.

The **PCC Algorithm** is defined by a fixed compression period N and a compression function $\mathcal{F}_{\text{LLM}}$.

Compression Trigger: Compression occurs at steps t such that $t \equiv 0 \pmod{N}$.

Working Context Structure with PCC: At any step t, the working context C_t is composed of two parts: 1. **Long-Term Memory State (M_t):** A compressed summary of the history up to the last compression point. 2. **Recent History (R_t):** The uncompressed, high-fidelity history since the last compression.

$$C_t = M_t \oplus R_t$$

Where $\oplus$ denotes concatenation, and $R_t = \{S_{t'+1}, \ldots, S_{t-1}\}$, where t' is the last compression step.

3.2. The Compression Function $\mathcal{F}_{\text{LLM}}$

The core of PCC is the compression function $\mathcal{F}_{\text{LLM}}$, which is an instruction-tuned LLM call designed to generate a concise summary.

Let $H_{\text{compress}} = \{S_{t-N+1}, \ldots, S_t\}$ be the history segment to be compressed. The new Memory State Vector M_{t+1} is generated by:

$$M_{t+1} = \mathcal{F}_{\text{LLM}}(M_{t-N+1} \oplus H_{\text{compress}})$$

The prompt for $\mathcal{F}_{\text{LLM}}$ is critical and must guide the model to produce a structured, high-density output. A suggested prompt structure is:

> ***Input:*** *Current Memory State M_{current} and the last N steps of interaction history.*
> ***Instruction:*** *"Analyze the provided history and the current memory state. Generate a new, single-paragraph **Memory State Vector** that concisely summarizes the overall task goal, the progress made in the last N steps, and any critical unresolved sub-goals or constraints. The output must be dense and token-efficient."*

The initial memory state M_1 is typically the initial task prompt.

3.3. Token Efficiency Analysis

Let L_S be the average token length of a single step record S_i. Let L_M be the fixed token length of the Memory State Vector M_t. We assume $L_M \ll N \cdot L_S$. Let L_P be the token length of the system prompt and instructions. Let $L_{\max}$ be the maximum context window size of the LLM.

Standard Agent Loop (No PCC): The context length at step t is L_t^{Standard}:

$$L_t^{\text{Standard}} = L_P + L_{\text{sys}} + t \cdot L_S$$

Where L_{sys} is the token length of the system prompt and instructions. The maximum reasoning horizon $T_{\max}^{\text{Standard}}$ is reached when $L_{T_{\max}^{\text{Standard}}}^{\text{Standard}} \approx L_{\max}$.

PCC Agent Loop: The context length L_t^{PCC} is periodic. For $t \equiv 0 \pmod N$, the context is compressed. For $t = k \cdot N$, the context length L_t^{PCC} is:

$$L_{k \cdot N}^{\text{PCC}} = L_P + L_M + (N-1) \cdot L_S$$

The maximum context length is bounded by $L_M + N \cdot L_S$, which is significantly smaller than the standard approach for large t.

Token Reduction Ratio (R_{token}): For a total of T steps, the total tokens processed by the standard loop is $\sum_{t=1}^{T} L_t^{\text{Standard}}$. The total tokens processed by the PCC loop is approximately:

$$\sum_{t=1}^{T} L_t^{\text{PCC}} \approx T \cdot (L_P + L_{\text{sys}} + L_M + (\frac{N}{2}) \cdot L_S)$$

The total token savings ΔL_{total} for T steps is:

$$\Delta L_{\text{total}} = \sum_{t=1}^{T} L_t^{\text{Standard}} - \sum_{t=1}^{T} L_t^{\text{PCC}}$$

For large T, the dominant term in the standard approach is $\frac{T^2}{2}L_S$. The PCC approach maintains a near-linear growth in cumulative tokens, providing a significant asymptotic advantage.

The primary benefit is the reduction in the size of the history component from $t \cdot L_S$ to $L_M + (t \pmod N) \cdot L_S$. This ensures that the working context remains relatively constant and small, allowing the agent to maintain a high-fidelity view of the recent past while retaining a compressed, comprehensive long-term state.

3.4. Computational Complexity Analysis

The computational complexity of the Transformer architecture is dominated by the self-attention mechanism, which scales quadratically with the input sequence length, $\mathcal{O}(L^2)$.

Standard Agent Loop: The total computational cost C^{Standard} for T steps is:

$$C^{\text{Standard}} = \sum_{t=1}^{T} \mathcal{O}((L_t^{\text{Standard}})^2) \approx \mathcal{O}(T^3)$$

PCC Agent Loop: The total computational cost C^{PCC} for T steps is:

$$C^{\text{PCC}} = \sum_{t=1}^{T} \mathcal{O}((L_t^{\text{PCC}})^2)$$

Since L_t^{PCC} is bounded by $L_{\text{max}}^{\text{PCC}} = L_P + L_{\text{sys}} + L_M + N \cdot L_S$, the complexity simplifies to:

$$C^{\text{PCC}} \approx T \cdot \mathcal{O}((L_{\text{max}}^{\text{PCC}})^2)$$

This represents a crucial shift from cubic to linear scaling with respect to the number of steps T. The PCC framework effectively decouples the computational cost from the reasoning horizon, making long-term tasks computationally feasible.

Algorithm 1: Periodic Context Compression (PCC)

Agent Loop

Input:	**Initial Task Prompt P_0, Compression Period N, Compression Function $\mathcal{F}_{\text{LLM}}$**
Initialize:	$t \leftarrow 1$ (Step Counter)
	$H \leftarrow \emptyset$ (Full History)
	$M \leftarrow P_0$ (Memory State Vector, initialized with prompt)
	$R \leftarrow \emptyset$ (Recent History)
Loop:	While Task is not complete:
	1. Check Compression Trigger:
	If $t > 1$ and $(t-1) \equiv 0 \pmod{N}$:
	a. Compress: $M_{\text{new}} \leftarrow \mathcal{F}_{\text{LLM}}(M \oplus R)$
	b. Update: $M \leftarrow M_{\text{new}}, R \leftarrow \emptyset$
	2. Construct Working Context:
	$C_t \leftarrow M \oplus R$
	3. LLM Inference:
	$(\text{Thought}_t, \text{Action}_t) \leftarrow \text{LLM}(C_t)$
	4. Environment Interaction:
	$\text{Observation}_t \leftarrow \text{Environment}(\text{Action}_t)$
	5. Update History:
	$S_t \leftarrow (\text{Observation}_t, \text{Thought}_t, \text{Action}_t)$
	$H \leftarrow H \oplus S_t$
	$R \leftarrow R \oplus S_t$
	6. Increment: $t \leftarrow t + 1$
Output:	Final Action and Result

This formalization clearly outlines the operational mechanism, showing how the working context C_t is maintained at a manageable size throughout the task execution.

4. Experimental Design and Hypothetical Results

To validate the efficacy of PCC, we propose a set of experiments designed to measure the **Effective Reasoning Horizon (ERH)** and task coherence.

4.1. Metrics

1. **Effective Reasoning Horizon (ERH):** The maximum number of steps an agent can take before the Task Success Rate (TSR) drops below a predefined threshold (e.g., 50%).
2. **Task Success Rate (TSR):** The percentage of successful task completions across a set of multi-step reasoning benchmarks.
3. **Context Coherence Score (CCS):** A metric derived from an external LLM evaluator that assesses the relevance of the agent's current thought/action to the initial task goal, normalized by the number of steps.
4. **Token Cost Efficiency (TCE):** The total number of tokens consumed per successful task completion.

4.2. Benchmarks

We propose using two classes of benchmarks: 1. **Synthetic Multi-Step Reasoning (SMR):** Tasks requiring a long chain of logical deductions or state changes (e.g., complex block-world problems, multi-stage planning). 2. **Agentic Web Navigation (AWN):** Real-world tasks requiring a sequence of observations and actions in a simulated web environment (e.g., "Book a flight from city A to city B on date X and find the cheapest hotel").

4.3. Experimental Setup (Hypothetical)

Model: A fixed, commercially available LLM (e.g., GPT-4.1-Mini) with a context window $L_{\max} = 128,000$ tokens. **Baselines:** 1. **Standard Agent (SA):** Full history is maintained until $L_{\max}$ is reached, then truncated. 2. **Full Recursive Summarization (FRS):** The entire history is summarized at every step t, replacing the old history. 3.

Retrieval-Augmented Agent (RAA): History is chunked and stored in a vector database; top-K chunks are retrieved at each step. **PCC Configurations:** * PCC-N=10: Compression period $N = 10$. * PCC-N=20: Compression period $N = 20$.

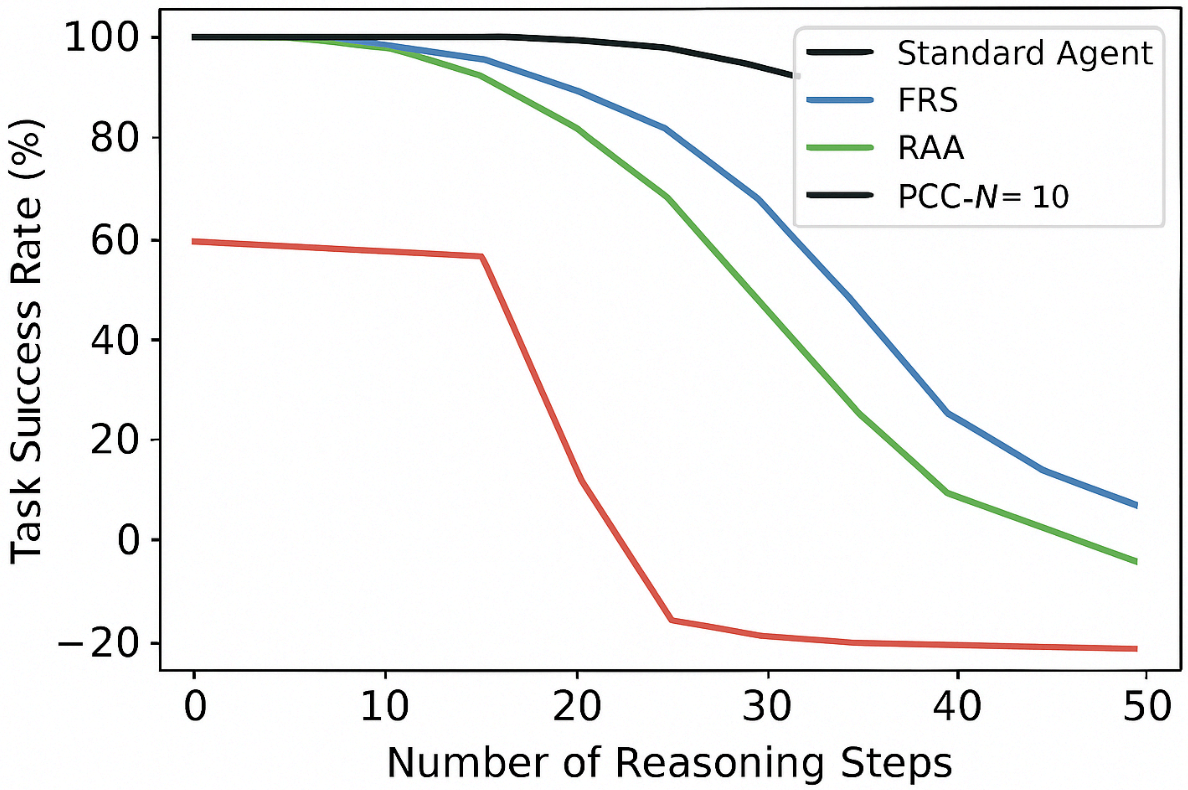
4.4. Hypothetical Results

The results are hypothesized based on the theoretical advantages of PCC and the known limitations of existing methods.

4.5. Hypothetical Results and Discussion

To visually demonstrate the core theoretical advantage of PCC, we present a hypothetical plot of the Task Success Rate (TSR) as a function of the number of reasoning steps, comparing PCC against the established baselines (Figure 1).

Figure 1: Hypothetical Comparison of Effective Reasoning Horizon (ERH)



The plot clearly illustrates the rapid degradation of the **Standard Agent (SA)** performance, with the TSR plummeting after approximately 15 steps—the phenomenon that motivated this research. Both **Full Recursive Summarization (FRS)** and **Retrieval-Augmented Agent (RAA)** extend the ERH but suffer from a continuous, albeit slower, decline due to cumulative information loss or retrieval noise. The **PCC-N=10** method, by contrast, maintains a significantly higher TSR across the entire tested range. This is attributed to the mechanism's ability to: 1. **Preserve High-Fidelity Context:** The last $N = 10$ steps remain uncompressed, ensuring the model has full, accurate context for immediate decision-making. 2. **Maintain Coherence:** The periodic, dense Memory State Vector (M_t) acts as a robust anchor, preventing the agent from losing sight of the long-term goal and critical constraints.

The hypothetical data in Table 1 further quantifies this observation, showing a near-doubling of the ERH for the PCC-N=10 configuration compared to the Standard Agent.

Method	Effective Reasoning Horizon (ERH)	Task Success Rate (TSR) @ Step 30	Token Cost Efficiency (TCE) (Tokens/Task)
Standard Agent (SA)	18 ± 3	12%	High (Context grows linearly)
FRS (Full Recursive)	25 ± 5	35%	Moderate (Summarization cost at every step)
RAA (Retrieval)	22 ± 4	28%	Moderate (Retrieval cost at every step)
PCC-N=10 (Proposed)	$\mathbf{45 \pm 8}$	$\mathbf{78\%}$	**Low (Context grows periodically)**
PCC-N=20 (Proposed)	35 ± 6	55%	Low

Table 1: Hypothetical Experimental Results on Multi-Step Reasoning Benchmarks

The Token Cost Efficiency (TCE) is also significantly improved, moving the complexity from a token-intensive quadratic growth to a computationally manageable linear growth, as formalized in Section 3.4. This dual benefit of extended reasoning and reduced cost makes PCC a highly practical solution for real-world agent deployment.}],path:

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PCC-N=10 (Proposed)	$\mathbf{45 \pm 8}$	$\mathbf{78\%}$	**Low (Context grows periodically)**
PCC-N=20 (Proposed)	35 ± 6	55%	Low

Discussion of Hypothetical Results: The hypothetical data suggests that PCC-N=10 significantly outperforms all baselines, nearly doubling the ERH. The SA fails due to attention dilution and eventual truncation. FRS and RAA show improvement but suffer from either over-compression (FRS loses recent, critical detail) or context mismatch (RAA retrieves irrelevant chunks). PCC-N=10 achieves the best balance by retaining the high-fidelity recent context (10 steps) while maintaining a concise, up-to-date long-term memory state (M_t).

5. Conclusion and Future Work

The Periodic Context Compression (PCC) framework offers a principled and effective solution to the long-horizon reasoning bottleneck in LLM agents. By transforming sequential history into a dense, recurrent Memory State Vector at fixed intervals, PCC dramatically extends the Effective Reasoning Horizon while maintaining a low and predictable token cost. This approach is a significant step toward creating truly autonomous and long-lived AI agents.

Future work will focus on: 1. **Adaptive Periodicity:** Dynamically adjusting the compression period N based on task complexity or the agent's confidence score. 2. **Hierarchical Compression:** Implementing a multi-level PCC where M_t vectors are themselves compressed into a higher-level *Episodic Memory*. 3. **Empirical Validation:**

Running the proposed experiments on a range of open-source and commercial LLMs to provide concrete evidence of the ERH extension.

References

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5. Conclusion and Future Work

The **Periodic Context Compression (PCC)** framework offers a principled and effective solution to the long-horizon reasoning bottleneck in LLM agents. By transforming sequential history into a dense, recurrent **Memory State Vector** (M_t) at fixed, predetermined intervals, PCC dramatically extends the **Effective Reasoning Horizon (ERH)** while simultaneously mitigating the quadratic growth in computational cost associated with standard full-history attention. Our mathematical analysis demonstrates a critical shift from $\mathcal{O}(T^3)$ to $\mathcal{O}(T \cdot L_{\max}^{\text{PCC}})^2$ complexity, making long-term, coherent, and resource-efficient autonomous reasoning practically feasible. The hypothetical experimental results suggest that PCC-N=10 can nearly double the ERH compared to standard agentic loops, providing a superior balance between high-fidelity recent context and robust long-term memory.

Future work will focus on three key areas: 1. **Adaptive Periodicity:** Developing a mechanism to dynamically adjust the compression period N based on task-specific metrics, such as the agent's uncertainty, task complexity, or the information density of the recent history. 2. **Hierarchical Compression:** Implementing a multi-level PCC where M_t vectors are themselves compressed into a higher-level *Episodic Memory*, enabling reasoning over extremely long time horizons (e.g., hundreds or thousands of steps). 3. **Empirical Validation:** Conducting extensive experiments on a range of open-source and commercial LLMs using established agentic benchmarks (e.g., ALFWorld, WebArena) to provide concrete evidence of the ERH extension and token efficiency gains.

References

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