

```
In[212]:= SetDirectory[  
    "D:\\cygwin64\\home\\koust\\Code\\Trophic_Maelstorm\\simulations\\Analytics\\Rietkerk 3  
    Sp"]
```

```
Out[212]:= D:\\cygwin64\\home\\koust\\Code\\Trophic_Maelstorm\\simulations\\Analytics\\Rietkerk 3 Sp
```

Now evaluating numerically for different values of constants.

```

In[229]:= ClearAll[rw, c, K1, aij, hij, d, R,  $\xi$ , G*]
(*Constants*)
rw = 0.2 / 24.0;
c = 10000; d = 0.25 / 24.0;
K1 = 5;
aij = 3.6 * 10.0-6.08 * 20.0-0.37;
hij = 1;  $\xi$  = 0.25 / 24; (*  $\xi$  is Kappa from analytical notation*)
ej = 0.45; mj = 0.061609 * 20.0-0.25 / 8760.0;
ajm = 3.6 * 10.0-6.08 * 40.0-0.37; hjm = 1;
em = 0.85; mm = 0.061609 * 40.0-0.25 / 8760.0;
G* = mm / ((em - mm * hjm) * ajm);
(*Define the equation*)
equation[R_] := c * rw * K1 * (aij G* + d * (1 + aij * hij * V*)) * (1 + aij * hij * V*) ==
(c * R - d * V*) * (1 + aij * hij * V*) * ( $\xi$  * (1 + aij * hij * V*) - aij * G*) -
aij * V* * G* * ( $\xi$  * (1 + aij * hij * V*) - aij * G*)
(*Numerical Solution*)
numericalSolutions[R_] := NSolve[equation[R], V*]
analyticalSolutions[R_] := Solve[equation[R], V*]

Range = Range[0, 0.3, 0.01]
numericalResults = numericalSolutions /@ Range;

(*Display the Numerical Results*)
Grid[Join[{"R", "Numerical Solutions"}],
Transpose[{Range, numericalResults}]], Dividers → All]
analResults = analyticalSolutions /@ Range;
(*Display the Analytical Results*)
Grid[Join[{"R", "Analytical Solutions"}],
Transpose[{Range, analResults}]], Dividers → All]

anaResults = Flatten /@ Transpose[{Range, V* /. analResults}];
numResults = Flatten /@ Transpose[{Range, V* /. numericalResults}];

(*Export Analytical Solutions to CSV with headers*)
Export["Trial_Analytical_solutions.csv", anaResults,
"CSV", "TableHeadings" → {"R", "Soln1", "Soln2", "Soln3"}]

(*Export Numerical Solutions to CSV with headers*)
Export["Trial_Numerical_solutions.csv", numResults,
"CSV", "TableHeadings" → {"R", "Soln1", "Soln2", "Soln3"}]

```

... ClearAll: r_w is not a symbol or a string.

... **ClearAll:** K_1 is not a symbol or a string.

... **ClearAll:** a_{ij} is not a symbol or a string.

... **General:** Further output of ClearAll::ssym will be suppressed during this calculation.

Out[240]= {0., 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1, 0.11, 0.12, 0.13, 0.14, 0.15,
0.16, 0.17, 0.18, 0.19, 0.2, 0.21, 0.22, 0.23, 0.24, 0.25, 0.26, 0.27, 0.28, 0.29, 0.3}

Out[242]=

| R | Numerical Solutions |
|------|--|
| 0. | $\{\{V^* \rightarrow -1.01218 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow -40017.\}\}$ |
| 0.01 | $\{\{V^* \rightarrow -1.01217 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow -30420.9\}\}$ |
| 0.02 | $\{\{V^* \rightarrow -1.01217 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow -20824.7\}\}$ |
| 0.03 | $\{\{V^* \rightarrow -1.01216 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow -11228.4\}\}$ |
| 0.04 | $\{\{V^* \rightarrow -1.01216 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow -1632.05\}\}$ |
| 0.05 | $\{\{V^* \rightarrow -1.01216 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 7964.36\}\}$ |
| 0.06 | $\{\{V^* \rightarrow -1.01215 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 17560.8\}\}$ |
| 0.07 | $\{\{V^* \rightarrow -1.01215 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 27157.4\}\}$ |
| 0.08 | $\{\{V^* \rightarrow -1.01215 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 36754.\}\}$ |
| 0.09 | $\{\{V^* \rightarrow -1.01214 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 46350.7\}\}$ |
| 0.1 | $\{\{V^* \rightarrow -1.01214 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 55947.4\}\}$ |
| 0.11 | $\{\{V^* \rightarrow -1.01214 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 65544.2\}\}$ |
| 0.12 | $\{\{V^* \rightarrow -1.01213 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 75141.\}\}$ |
| 0.13 | $\{\{V^* \rightarrow -1.01213 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 84737.9\}\}$ |
| 0.14 | $\{\{V^* \rightarrow -1.01213 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 94334.9\}\}$ |
| 0.15 | $\{\{V^* \rightarrow -1.01212 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 103932.\}\}$ |
| 0.16 | $\{\{V^* \rightarrow -1.01212 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 113529.\}\}$ |
| 0.17 | $\{\{V^* \rightarrow -1.01212 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 123126.\}\}$ |
| 0.18 | $\{\{V^* \rightarrow -1.01211 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 132723.\}\}$ |
| 0.19 | $\{\{V^* \rightarrow -1.01211 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 142320.\}\}$ |
| 0.2 | $\{\{V^* \rightarrow -1.01211 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 151918.\}\}$ |
| 0.21 | $\{\{V^* \rightarrow -1.01211 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 161515.\}\}$ |
| 0.22 | $\{\{V^* \rightarrow -1.0121 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 171112.\}\}$ |
| 0.23 | $\{\{V^* \rightarrow -1.0121 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 180710.\}\}$ |
| 0.24 | $\{\{V^* \rightarrow -1.0121 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 190307.\}\}$ |
| 0.25 | $\{\{V^* \rightarrow -1.0121 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 199905.\}\}$ |
| 0.26 | $\{\{V^* \rightarrow -1.01209 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 209502.\}\}$ |
| 0.27 | $\{\{V^* \rightarrow -1.01209 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 219100.\}\}$ |
| 0.28 | $\{\{V^* \rightarrow -1.01209 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 228697.\}\}$ |
| 0.29 | $\{\{V^* \rightarrow -1.01209 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 238295.\}\}$ |
| 0.3 | $\{\{V^* \rightarrow -1.01208 \times 10^6\}, \{V^* \rightarrow -1.01133 \times 10^6\}, \{V^* \rightarrow 247892.\}\}$ |

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Out[244]=

Out[247]= Trial_Analytical_solutions.csv

Out[248]= Trial_Numerical_solutions.csv