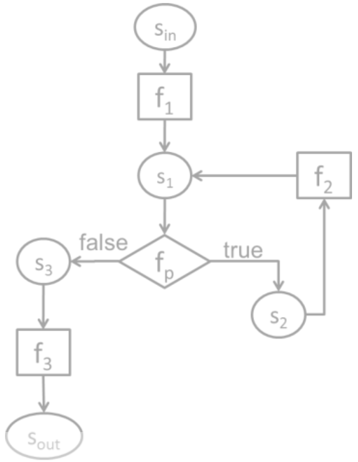
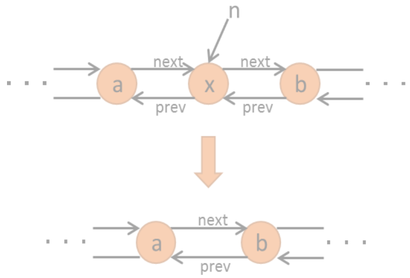
$\exists c \forall in \; Q(c, in)$

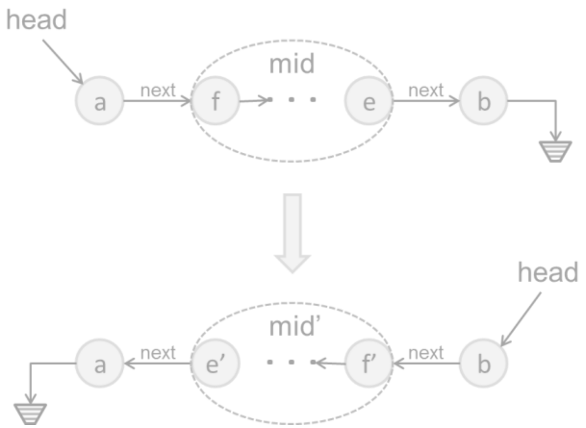
```
/* Average of x and y without using x+y (avoid overflow)*/
int avg(int x, int y){
  int t = expr({x/2, y/2, x%2, y%2, 2 }, {PLUS, DIV});
  assert t == (x+y)/2;
  return t;
}
```

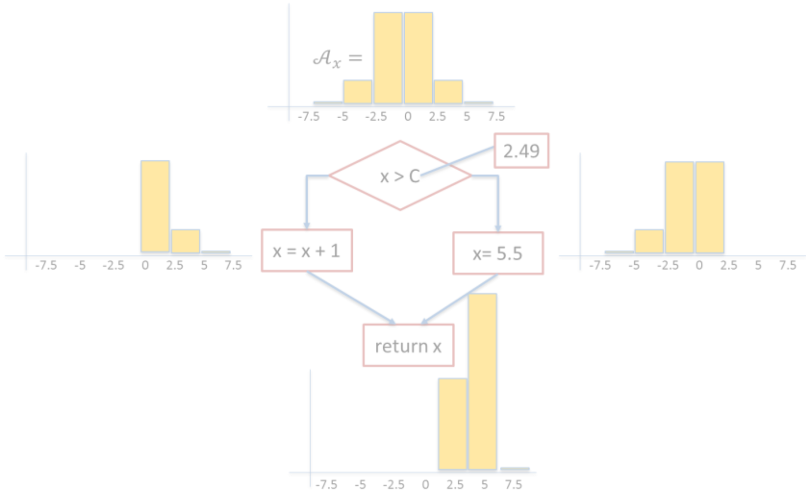





```
{
   s  = n.succ;
   p = n.pred;
   p.succ = s;
   s.pred = p;
}
```

Module I: Searching for Simple Programs





$\varphi(p)$

$Sk[c](in)$

Lecture 2
Syntax-Guided Synthesis and Enumerative Search

Nadia Polikarpova

Logistics

Shared Google folder

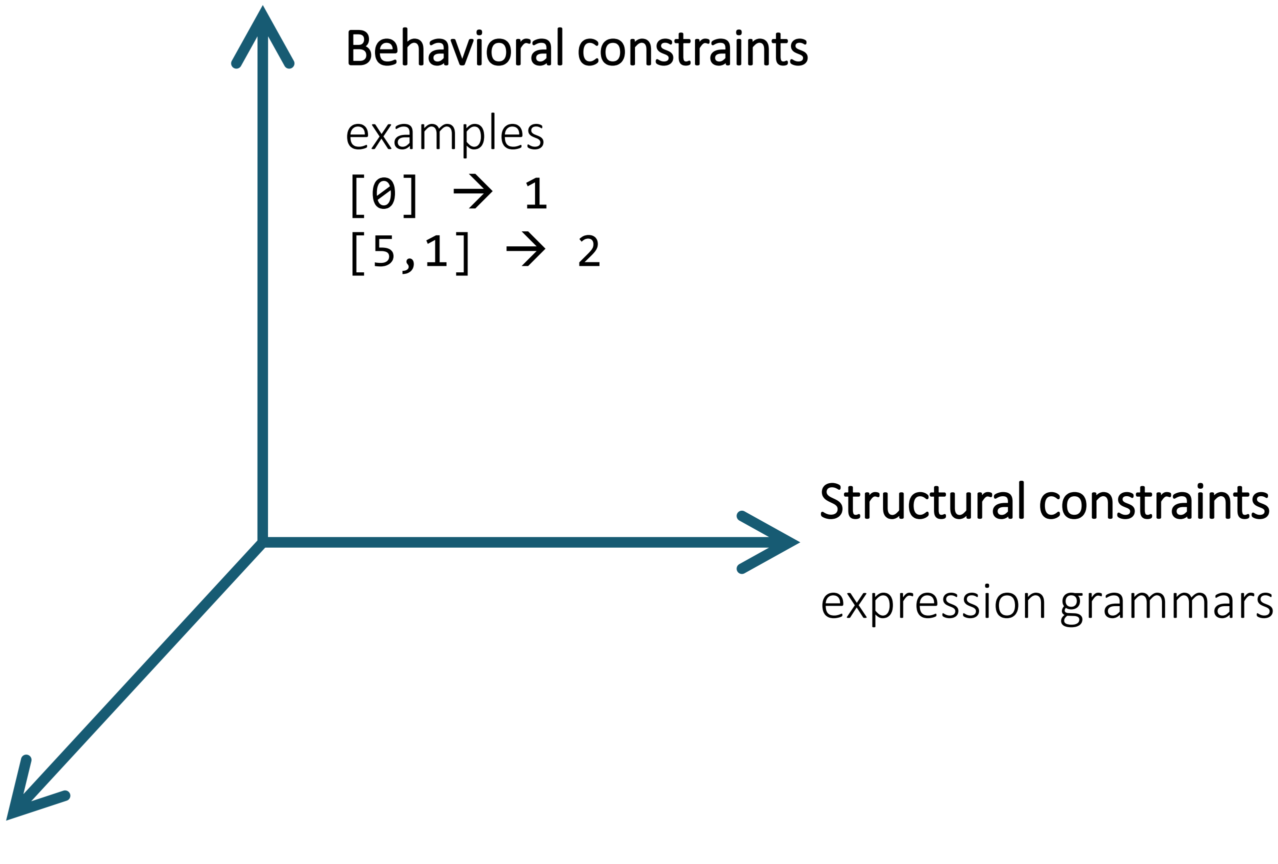
- Does everyone have access?
- Register your team by next Friday

EasyChair

- Does everyone have PC access?
- Submit review by Wednesday
- Paper discussion on Thursday

Other questions?

Week 1-2


Behavioral constraints
examples
[0] → 1
[5,1] → 2
Structural constraints
expression grammars
Search strategy
Enumeration

Today

Synthesis from examples: motivation and history

Syntax-guided synthesis

- expression grammars as structural constraints
- the SyGuS project

Enumerative search

- enumerating all programs generated by a grammar
- bottom-up vs top-down

Synthesis from examples

Synthesis from Examples

=

Programming by Example

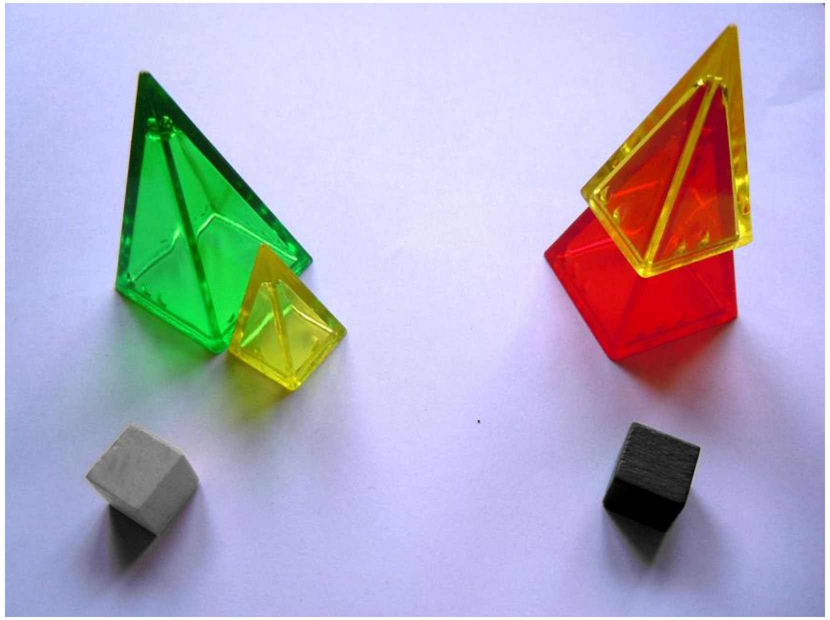
=

Inductive Synthesis

Inductive Programming

Inductive Learning

The Zendo game



This is called inductive learning!

The teacher makes up a secret rule

- e.g. all pieces must be grounded

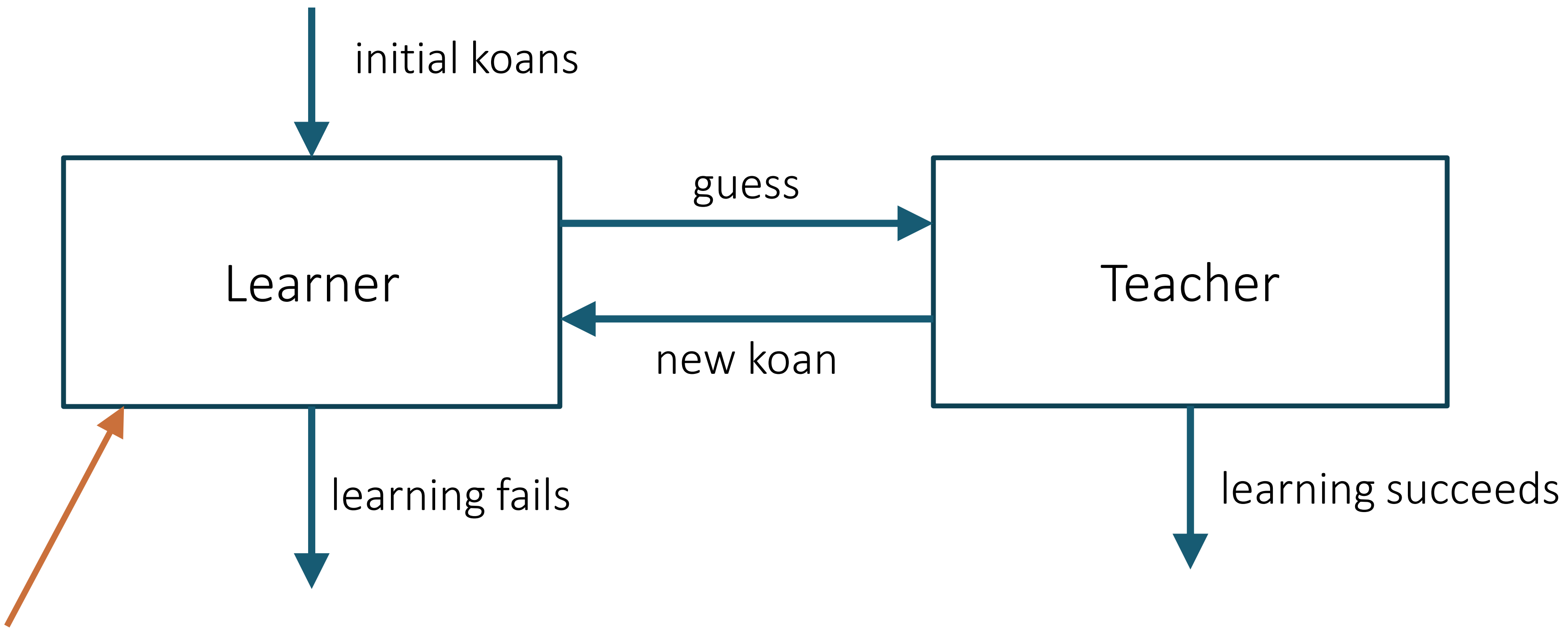
The teacher builds two koans (a positive and a negative)

Students take turns to build koans and ask the teacher to label them

A student can try to guess the rule

- if they are right, they win
- otherwise, the teacher builds a koan on which the two rules disagree

The Zendo game



1960s: humans are good at this…
can computers do this?

A little bit of history: inductive learning

MIT/LCS/TR-76

LEARNING STRUCTURAL DESCRIPTIONS FROM EXAMPLES

Patrick H. Winston

September 1970

Patrick Winston

Explored the question of generalizing from a set of observations

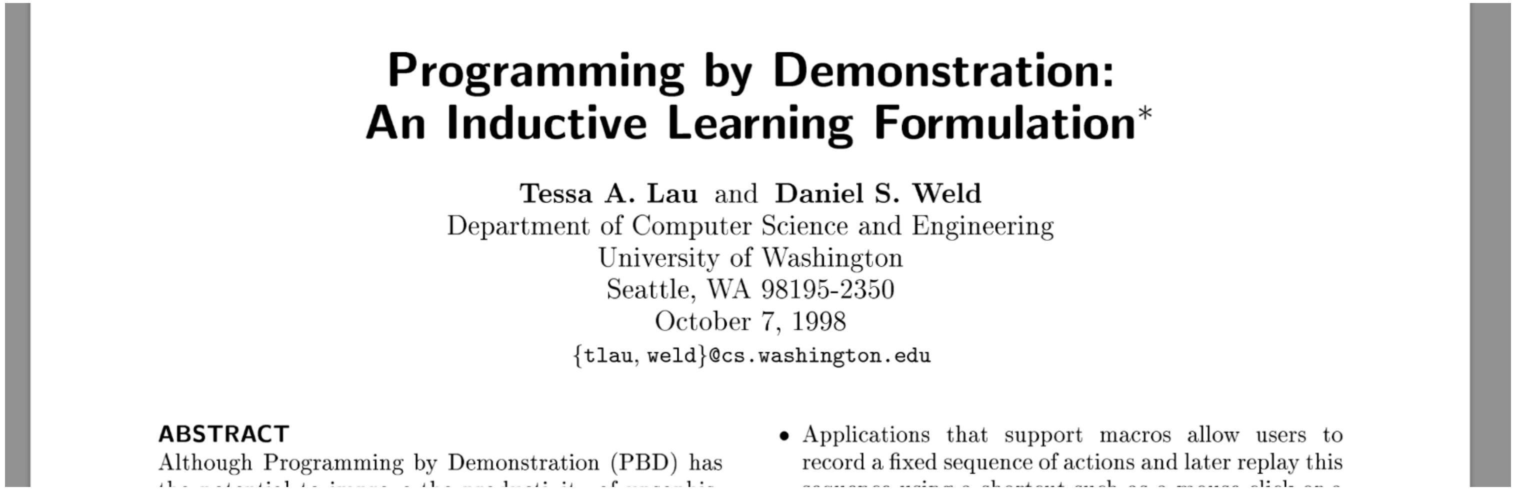
- Similar to Zendo

Became the foundation of machine learning

A little bit of history: PBE/PBD

1980s: searching a predefined list of programs

1990s (Tessa Lau): bring inductive learning techniques into PBE



Programming by Demonstration:
An Inductive Learning Formulation*

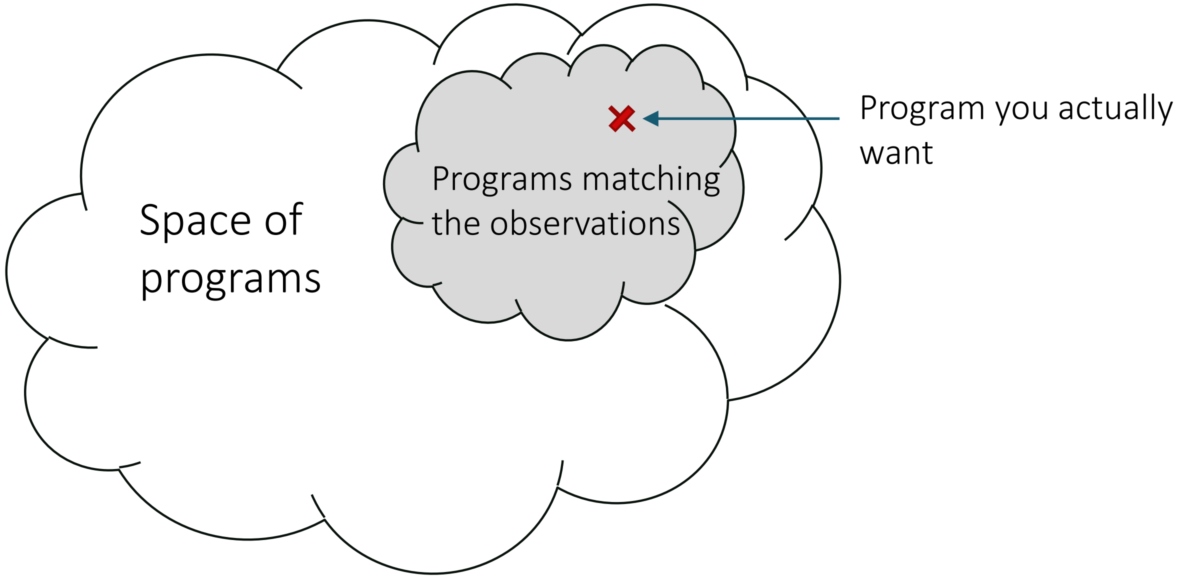
Tessa A. Lau and **Daniel S. Weld**
Department of Computer Science and Engineering
University of Washington
Seattle, WA 98195-2350
October 7, 1998
{tlau, weld}@cs.washington.edu

ABSTRACT
Although Programming by Demonstration (PBD) has

- Applications that support macros allow users to record a fixed sequence of actions and later replay this

Tessa Lau

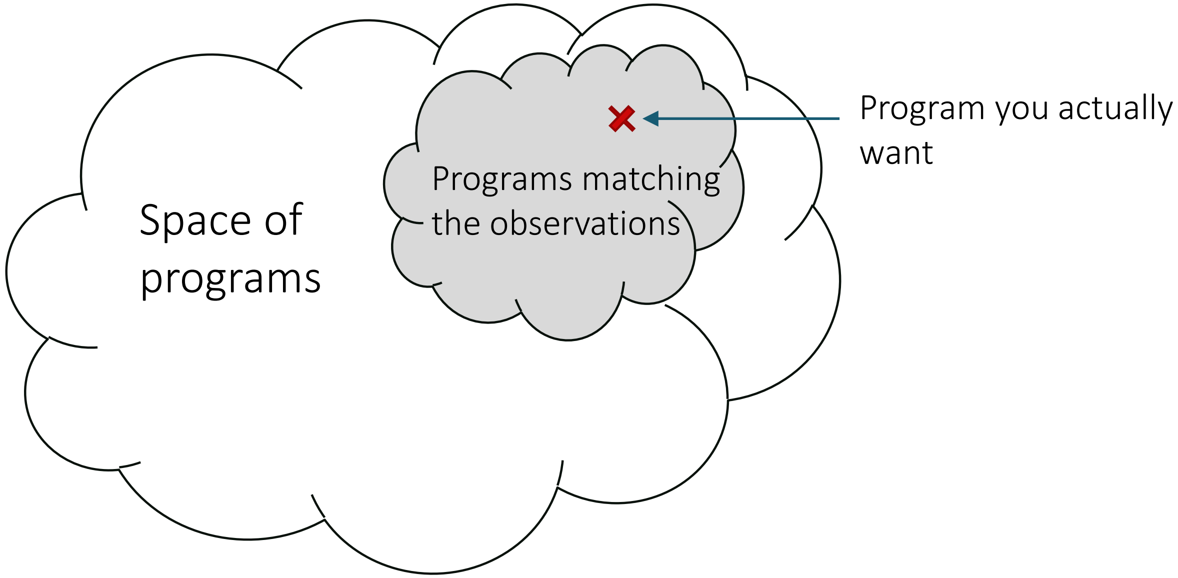
Key issues in inductive learning



(1) How do you find a program that matches the observations?

(2) How do you know it is the program you are looking for?

Key issues in inductive learning



Traditional ML emphasizes (2)

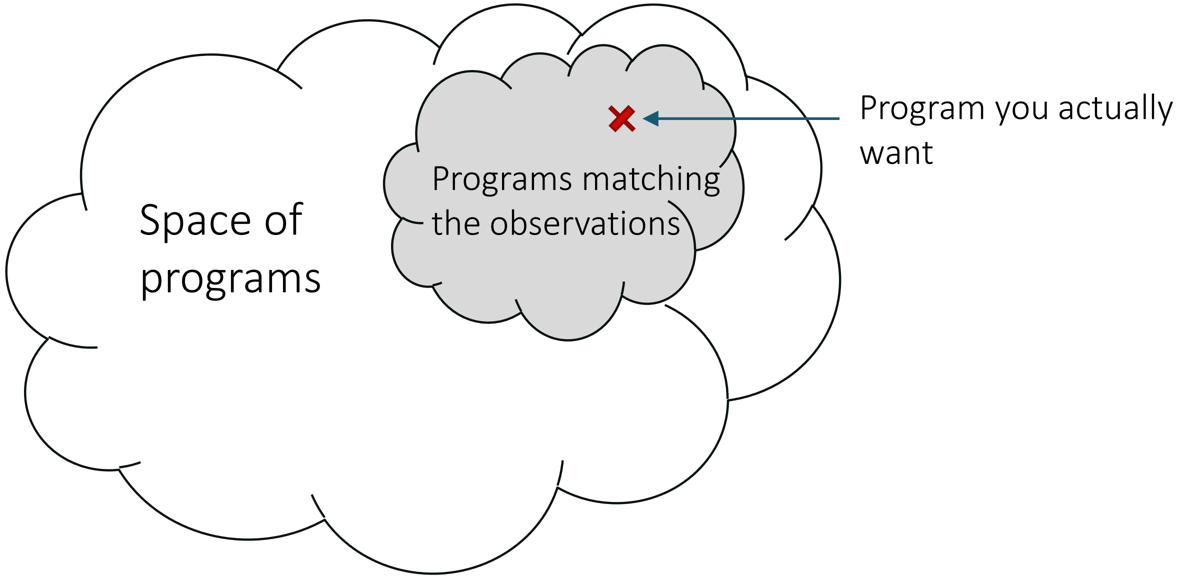
- Fix the space so that (1) is easy

So did a lot of PBD work

(1) How do you find a program that matches the observations?

(2) How do you know it is the program you are looking for?

The synthesis approach

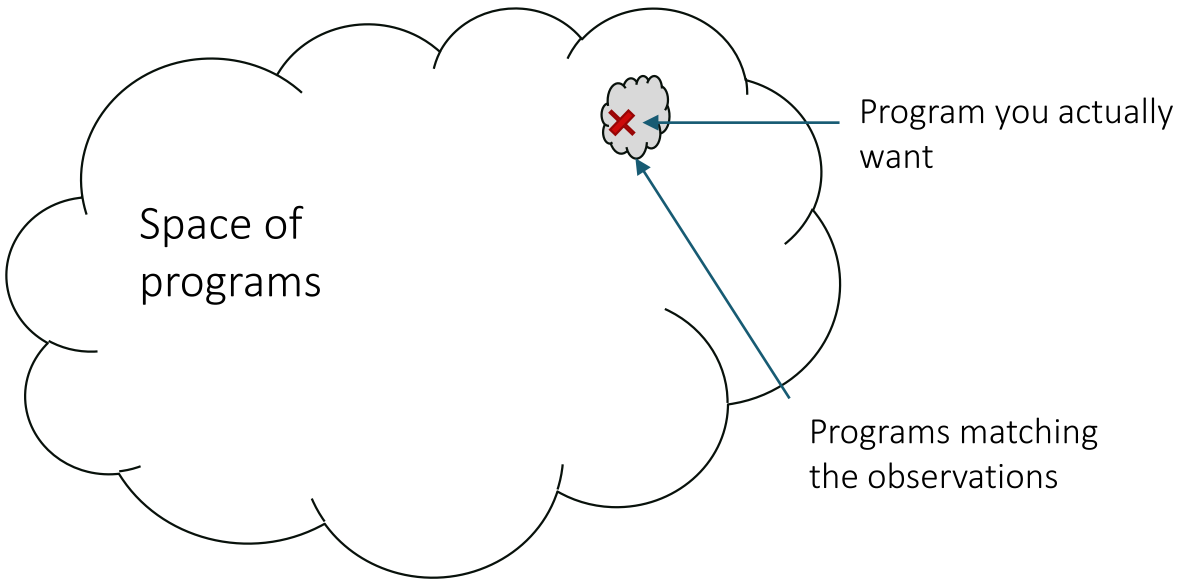


Modern emphasis

(1) How do you find a program that matches the observations?

(2) How do you know it is the program you are looking for?

The synthesis approach



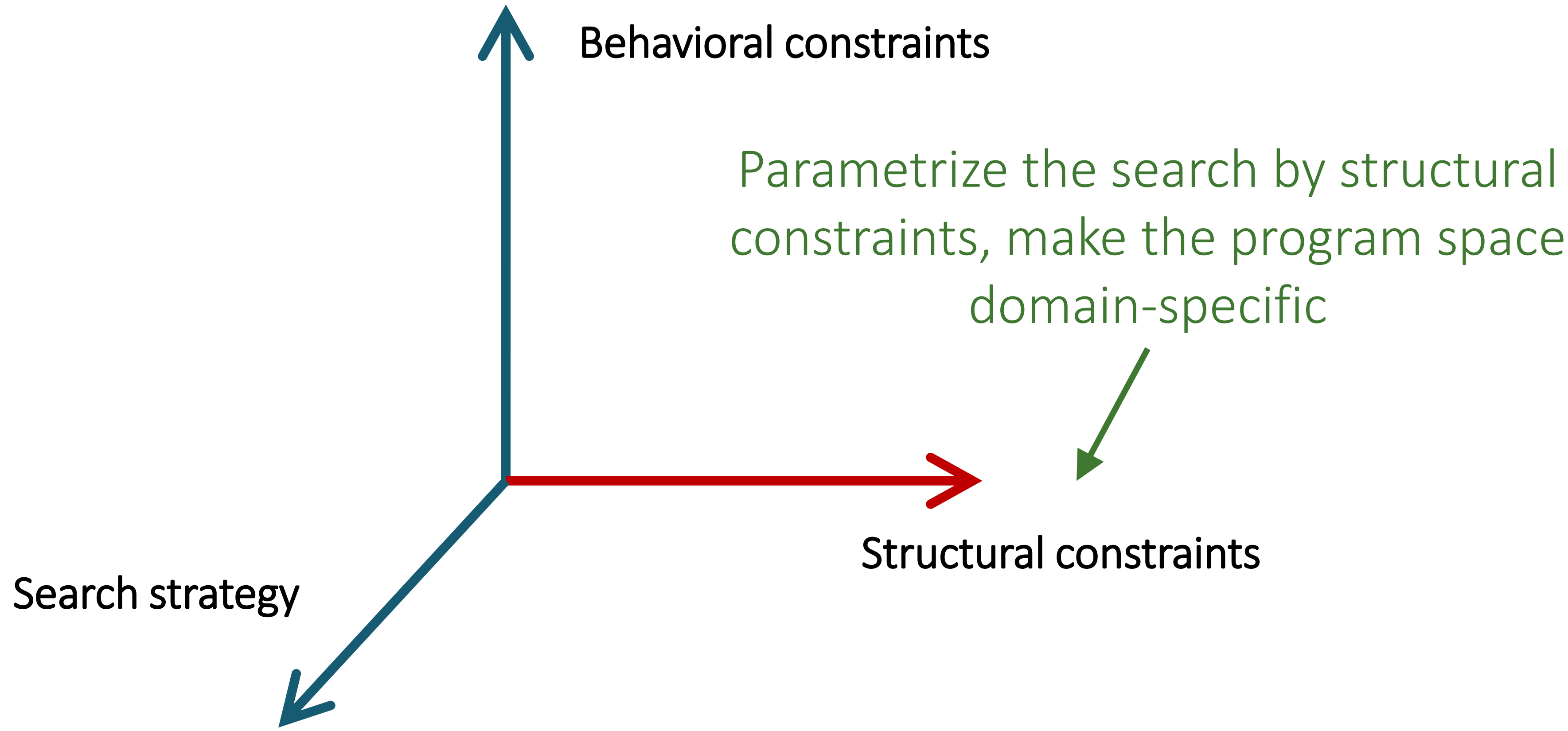
Modern emphasis

- If you can do really well with (1) you can win
- (2) is still important

(1) How do you find a program that matches the observations?

(2) How do you know it is the program you are looking for?

Key idea



Behavioral constraints
Parametrize the search by structural constraints, make the program space domain-specific
Structural constraints
Search strategy

Syntax-Guided Synthesis

Example

```
[1,4,7,2,0,6,9,2,5,0]  →  [1,2,4,7,0]

f(x) := sort(x[0..find(x, 0)]) + [0]

L ::= sort(L)    |
      L[N..N]    |
      L + L      |
      [N]        |
      x
N ::= find(L,N)  |
      0
```

Context-free grammars (CFGs)

starting nonterminal

terminals

nonterminals

```
L ::= sort(L)      |
      L[N..N]      |
      L + L        |
      [N]          |
      x
N ::= find(L,N)    |
      0
```

productions

Context-free grammars (CFGs)

nonterminals rules (productions)

terminals starting nonterminal

`<T, N, R, S>`

Sentential forms: $\{\alpha \in (N \cup T)^*\}$ `x N x`

Rewrites to: $\alpha A \beta \rightarrow \alpha \gamma \beta \;\equiv\; (A \rightarrow \gamma) \in R$ `x N x -> x find(L,N) x`

(Incomplete) terms/programs: $\{\alpha \in (N \cup T)^* | A \rightarrow^* \alpha\}$ `find(L+L,N)`

Ground programs: $\{\alpha \in T^* | A \rightarrow^* \alpha\}$ `find(x+x,0)`

= programs without holes, complete programs

Whole programs: $\{\alpha \in (N \cup T)^* | S \rightarrow^* \alpha\}$ `sort(L+L)`

= roughly, programs of the right type

CFGs as structural constraints

Space of programs
=
all ground, whole programs

```
L ::= sort(L)      |
      L[N..N]      |
      L + L        |
      [N]          |
      x
N ::= find(L,N)    |
      0
```

```
x    sort(x)    x + x    x[0..0]
...
x[0..find(x, 0)]
...
sort(x[0..find(x, 0)]) + [0]
...
```

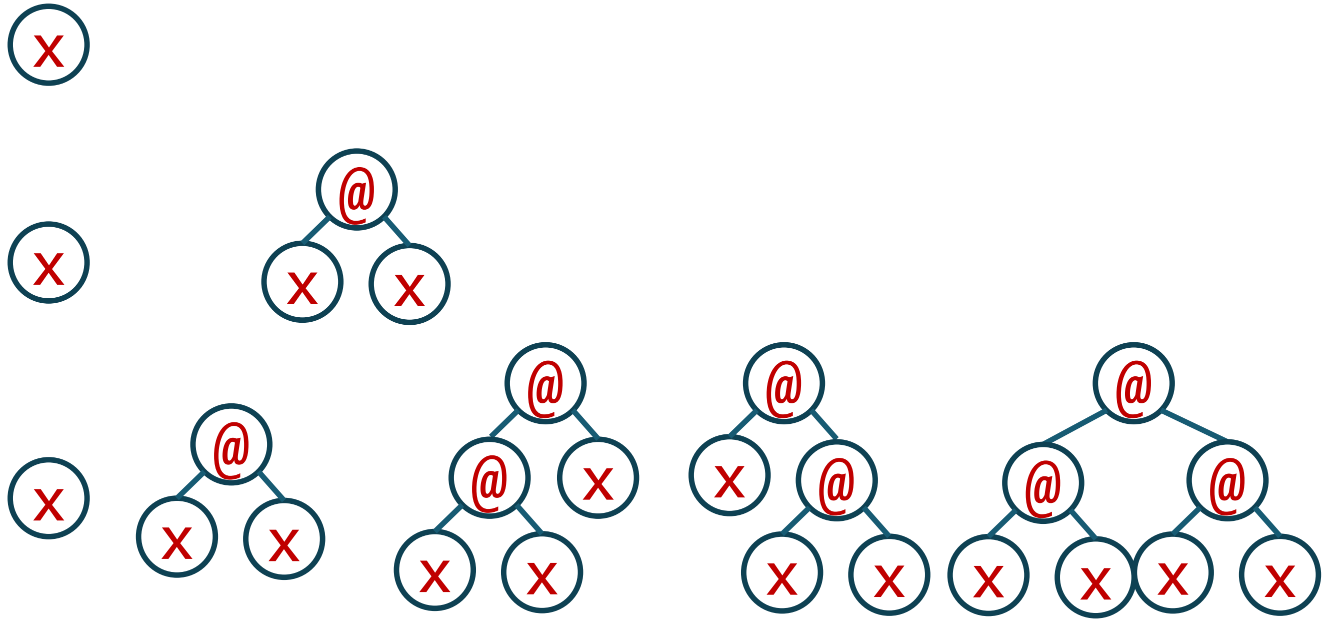
How big is the space?

E ::= x | E @ E

depth <= 0 — N(0) = 1

depth <= 1 — N(1) = 2

depth <= 2 — N(2) = 5



$$N(d) = 1 + N(d - 1)^2$$

How big is the space?

$$E ::= x \mid E \; @ \; E$$

$$N(d) = 1 + N(d - 1)^2 \qquad N(d) \sim c^{2^d} \qquad (c > 1)$$

N(1) = 1
N(2) = 2
N(3) = 5
N(4) = 26
N(5) = 677
N(6) = 458330
N(7) = 210066388901
N(8) = 44127887745906175987802
N(9) = 1947270476915296449559703445493848930452791205
N(10) = 3791862310265926082868235028027893277370233152247388584761734150717768254410341175325352026

How big is the space?

$$E ::= x_1 \mid \ldots \mid x_k \mid$$
$$E\ @_1\ E \mid \ldots \mid E\ @_m\ E$$

$$N(0) = k$$

$$N(d) = k + m * N(d - 1)^2$$

N(1) = 3
N(2) = 30
N(3) = 2703
N(4) = 21918630
N(5) = 1441279023230703
N(6) = 6231855668414547953818685622630
N(7) = 116508075215851596766492219468227024724121520304443212304350703

k = m = 3

The SyGuS project

[Alur et al. 2013]

https://sygus.org/

Goal: Unify different syntax-guided approaches

Collection of synthesis benchmarks + yearly competition

- 6 competitions since 2013
- consider writing a SyGuS solver for your project!

Common input format + supporting tools

- parser, baseline synthesizers

SyGuS problems

SyGuS problem = < theory, spec, grammar >

A “library” of types and function symbols

Example: Linear Integer Arithmetic (LIA)

```
True, False
0,1,2,...
∧, ∨, ¬, +, ≤, ite
```

CFG with terminals in the theory (+ input variables)

Example: Conditional LIA expressions w/o sums

```
E ::= x | ite C E E
C ::= E ≤ E | C ∧ C | ¬C
```

SyGuS problems

SyGuS problem = < theory, spec, grammar >

A first-order logic formula over the theory

Examples:

```
f(0, 1) = 1 ∧
f(1, 0) = 1 ∧
f(1, 1) = 1 ∧
f(2, 0) = 2
```

SyGuS demo

SyGuS problems

SyGuS problem = < theory, spec, grammar >

can inductive synthesis handle these?

A first-order logic formula over the theory

Examples:

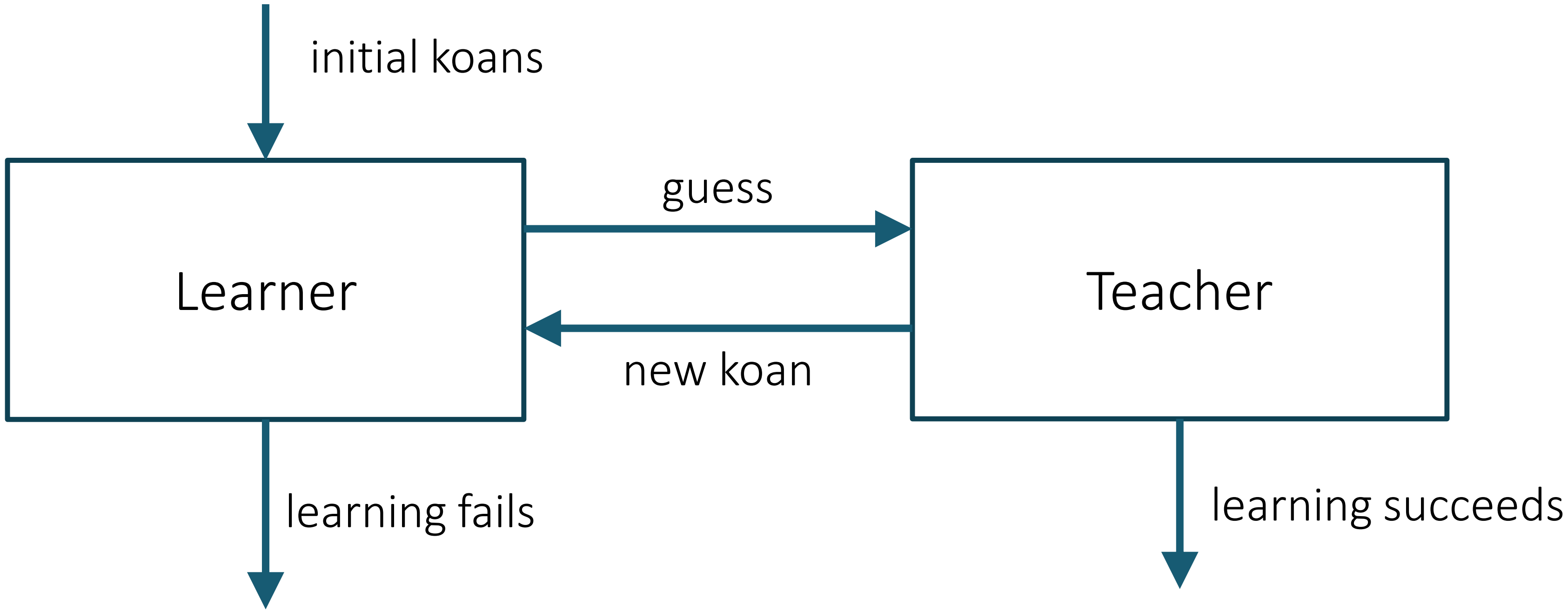
```
f(0, 1) = 1 ∧
f(1, 0) = 1 ∧
f(1, 1) = 1 ∧
f(2, 0) = 2
```

Formula with free variables:

```
x ≤ f(x, y) ∧
y ≤ f(x, y) ∧
(f(x, y) = x ∨ f(x, y) = y)
```

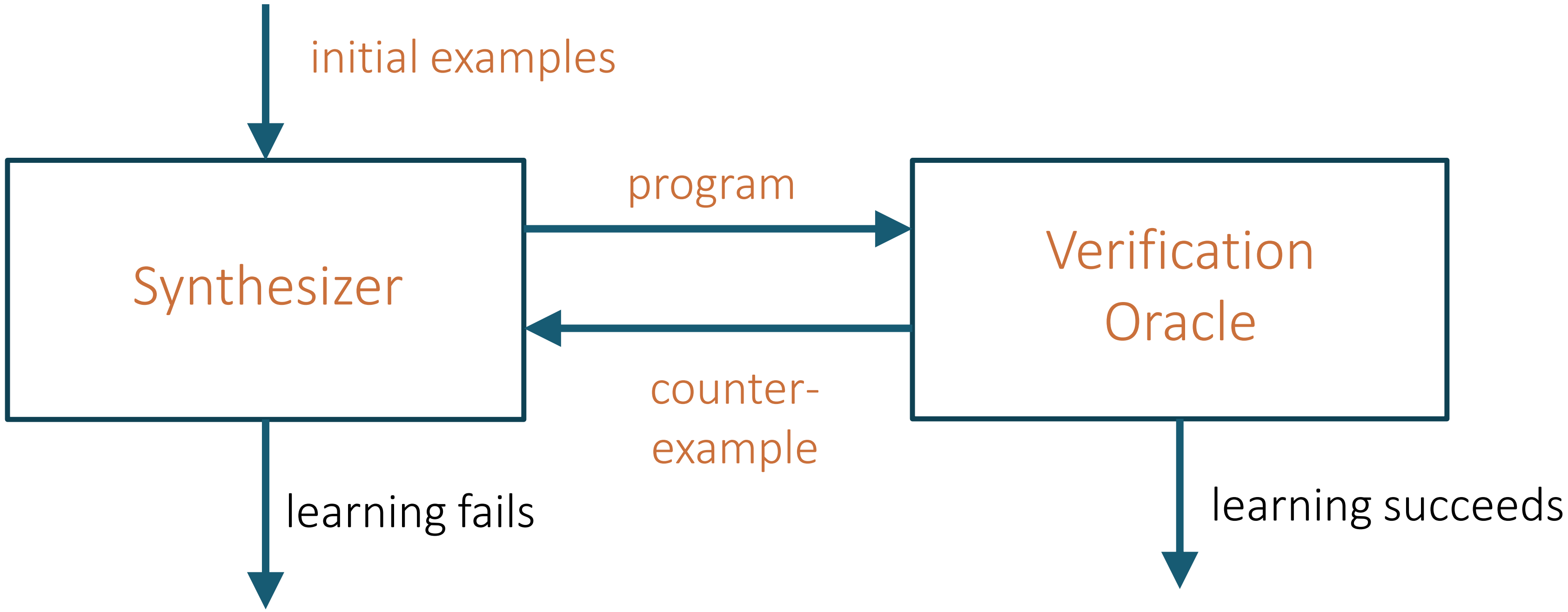
Counter-example guided inductive synthesis (CEGIS)

The Zendo of program synthesis

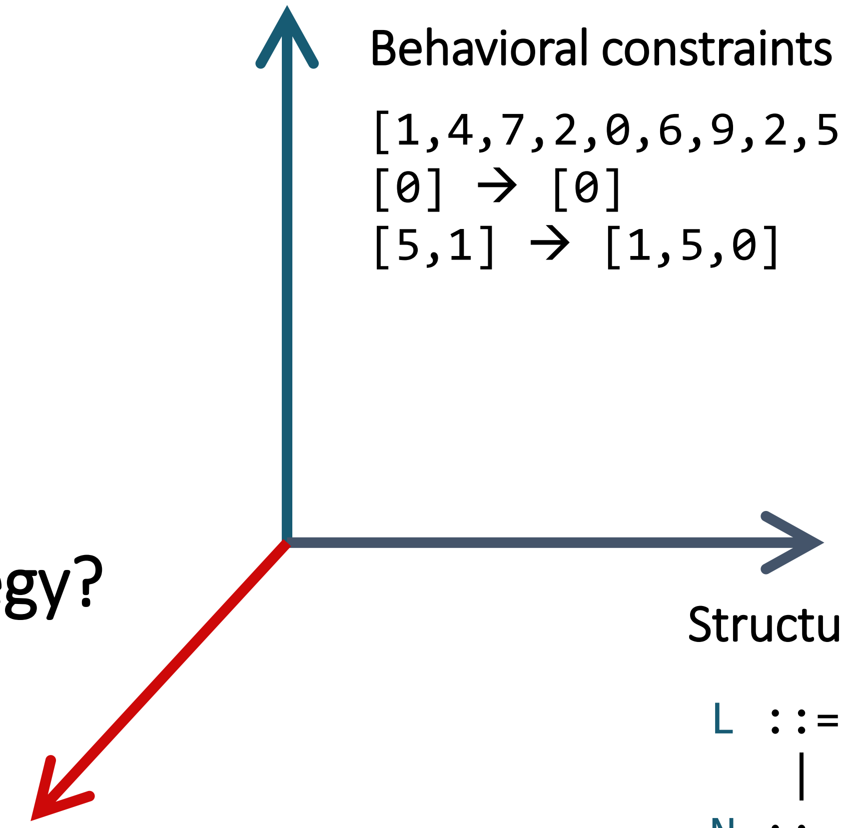


Counter-example guided inductive synthesis (CEGIS)

The Zendo of program synthesis



The problem statement



Behavioral constraints = examples

```
[1,4,7,2,0,6,9,2,5]  →  [1,2,4,7,0]
[0] → [0]
[5,1] → [1,5,0]
```

Search strategy?

Structural constraints = grammar

```
L ::= sort(L)  |  L[N..N]
    |  L + L  |  [N]  |  x
N ::= find(L,N)  |  0
```

Enumerative search

Enumerative search

=

Explicit / Exhaustive Search

Idea: Sample programs from the grammar one by one and test them on the examples

Challenge: How do we systematically enumerate all programs?

bottom-up vs top-down

Bottom-up enumeration

Start from terminals

Combine sub-programs into larger programs using productions

```
L ::= sort(L)      |
      L[N..N]      |
      L + L        |
      [N]          |
      x
N ::= find(L,N)    |
      0

[[1,4,0,6] → [1,4]]
```

Bottom-up enumeration

nonterminals rules (productions)

terminals starting nonterminal

```
bottom-up (<T, N, R, S>, [i → o]) {
  P := [t | t in T && t is nullary]
  while (true)
    forall (p in P)
      if (whole(p) && p([i]) = [o])
        return p;
    P += grow(P);
}

grow (P) {
  P’ := []
  forall (A ::= rhs in R)
    P’ += [rhs[B -> p] | p in P, B →* p]
  return P’;
}
```

Q: “Run” bottom-up on the board with:

```
L ::= sort(L)    |
      L[N..N]    |
      L + L      |
      [N]        |
      x
N ::= find(L,N)  |
      0
```

[[1,4,0,6] → [1,4]]

Bottom-up: example

Program bank P

iter 0: `x` `0`

iter 1: `sort(x)` `x[0..0]` `x + x` `[0]` `find(x,0)`

iter 2:
```
sort(sort(x))   sort(x[0..0])   sort(x + x)
sort([0])   x[0..find(x,0)]   x[find(x,0)..0]
x[find(x,0)..find(x,0)]   sort(x)[0..0]
x[0..0][0..0]   (x + x)[0..0]   [0][0..0]
x + (x + x)   x + [0]   sort(x) + x   x[0..0] + x
(x + x) + x   [0] + x   x + x[0..0]   x + sort(x)
...
```

```
L ::= sort(L)      |
      L[N..N]      |
      L + L        |
      [N]          |
      x
N ::= find(L,N)    |
      0
```

[[1,4,0,6] → [1,4]]

Top-down enumeration

Start from the start non-terminal

Expand remaining non-terminals using productions

```
L ::= L[N..N]    |
      x
N ::= find(L,N)  |
      0

[[1,4,0,6]  →  [1,4]]
```

Top-down enumeration

```
top-down(<T, N, R, S>, [i → o]) {
  P := [S]
  while (P != [])
    p := P.dequeue();
    if (ground(p) && p([i]) = [o])
      return p;
    P.enqueue(unroll(p));
}

unroll(p) {
  P' := []
  forall (A in p)
    forall (A ::= rhs in R)
      P' += p[A -> rhs]
  return P';
}
```

Q: "Run" top-down on the board with

```
L ::= L[N..N]    |
      x
N ::= find(L,N)  |
      0
```

```
[[1,4,0,6] → [1,4]]
```

Top-down: example

Worklist P

iter 0: `L`

iter 1: `x` `L[N..N]`

iter 2: `L[N..N]`

iter 3: `x[N..N]` `L[N..N][N..N]`

iter 4: `x[0..N]` `x[find(L,N)..N]` `L[N..N][N..N]`

iter 5: `x[0..0]` `x[0.. find(L,N)]` `x[find(L,N)..N]` …

iter 6: `x[0.. find(L,N)]` `x[find(L,N)..N]` … …

iter 7: `x[0.. find(x,N)]` `x[0.. find(L[N..N],N)]` … … …

iter 8: `x[0.. find(x,0)]` `x[0.. find(x,find(L,N))]` … … … …

iter 9:

```
L ::= L[N..N] |
      x
N ::= find(L,N) |
      0
```

`[[1,4,0,6]` → `[1,4]]`

Bottom-up vs top-down

Bottom-up | **Top-down**

Smaller to larger

- Has to explore between $3*10^9$ and 10^{23} programs to find `sort(x[0..find(x, 0)]) + [0]` (depth 6)

Bottom-up: Candidates are ground but might not be whole

- Can always run on inputs
- Cannot always relate to outputs

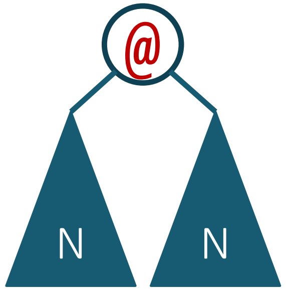
Top-down: Candidates are whole but might not be ground

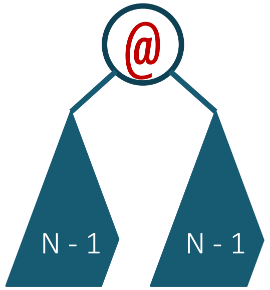
- Cannot always run on inputs
- Can always relate to outputs (?)

How to make it scale

Prune

Discard useless subprograms





$m * N^2$ $m * (N - 1)^2$

Prioritize

Explore more promising candidates first

```
P = { [0][N..N] ,
      x[N..N]  ,
      ... }
```

dequeue this first