Lecture 7
Constraint-based search

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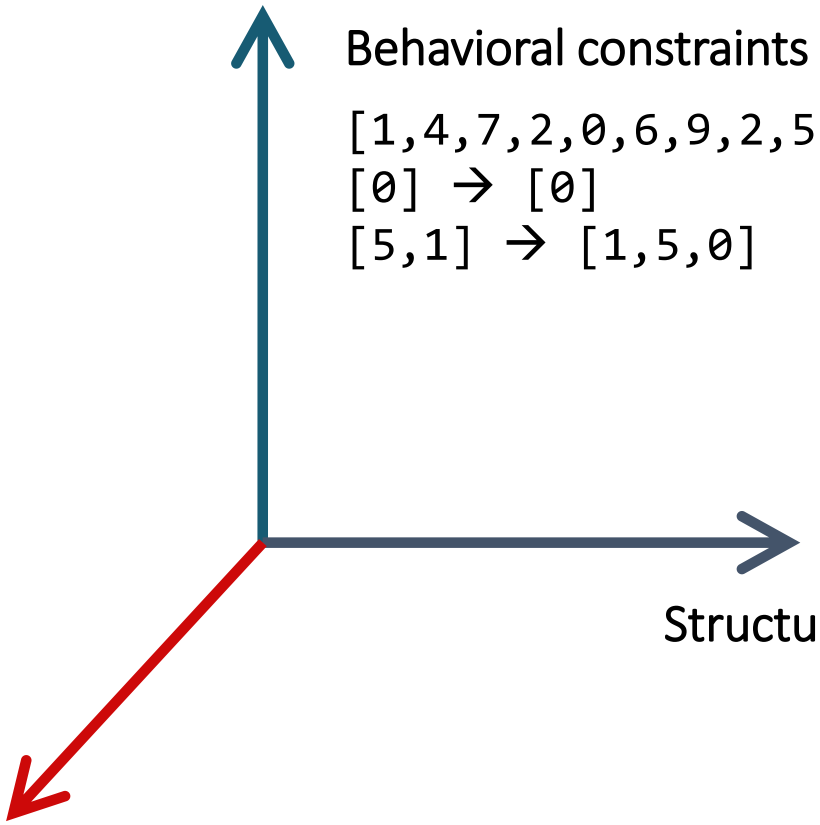
The problem statement



Behavioral constraints = examples

```
[1,4,7,2,0,6,9,2,5]  →  [1,2,4,7,0]
[0] → [0]
[5,1] → [1,5,0]
```

Structural constraints = we'll see...

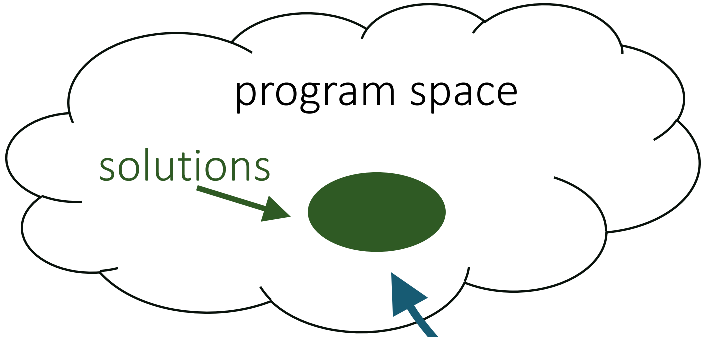
Search strategy?

Enumerative
Stochastic
Representation-based
Constraint-based

Constraint-based search

Idea: encode the synthesis problem as a SAT/SMT problem and let a solver deal with it

What is an encoding?



parameter space

$$C = \{c_1, \dots, c_n\}$$

$$\{C \mid \mathrm{wf}(C)\}$$

well-formedness constraint

$$\{C \mid \mathrm{wf}(C) \wedge \mathrm{sem}(C)\}$$

semantic correctness constraint

$$\mathrm{sem}(C) = \bigwedge_{\langle i,o\rangle \in E} \varphi(C, i, o)$$

C denotes a program that gives output *o* on input *i*

How to define an encoding

Define the parameter space $C = \{c_1, \dots, c_n\}$

- `encode : Prog → C`
- `decode : C → Prog` (might not be defined for all C)

Define a formula $\mathrm{wf}(c_1, \dots, c_n)$

- that holds iff `decode[C]` is a "well-formed" program

Define a formula $\varphi(c_1, \dots, c_n, i, o)$

- that holds iff `(decode[C])(i) = o`

Constraint-based search

```
constraint-based (wf, φ, E = [i → o]) {
  match SAT(wf(C) ∧ ⋀⟨i,o⟩∈E φ(C, i, o)) with
    Unsat -> return "No solution"
    Model C* -> return decode[C*]
}
```

Find a satisfying assignment for c_1, ..., c_n (i and o are fixed)

SAT encoding: example

program space

`{x,  x & 1}`

$\mathrm{wf}(c) \equiv \top$

`x` is a two-bit word
$(x = x_h x_l)$

`E = [11 → 01]`

parameter space

$C = \{c\colon \mathrm{Bool}\}$

```
decode[0] → x
decode[1] → x & 1
```

$$\varphi(c, i, o) \equiv (\neg c \;\Rightarrow\; o_h = i_h \wedge o_l = i_l)$$
$$\wedge\;(c \;\Rightarrow\; o_h = 0 \wedge o_l = i_l)$$

$$\mathrm{SAT}(\varphi(c, 11, 01))$$

$$\mathrm{SAT}((\neg c \;\Rightarrow\; 0 = 1 \wedge 1 = 1) \wedge (c \;\Rightarrow\; 0 = 0 \wedge 1 = 1)) \xrightarrow{\text{SAT solver}} \texttt{Model \{c→1\}}$$

`return` `decode[1]` i.e. `x & 1`

SMT encoding: example

program space

`x + N | x * N`

N is an in integer literal
x is an integer

`E = [2 → 9]`

parameter space

$$C = \{c_{op}: \mathrm{Bool}, c_N: \mathrm{Int}\}$$

```
decode[0,N] → x + N
decode[1,N] → x * N
```

$$\mathrm{wf}(c) \equiv \top$$

$$\varphi(c_{op}, c_N, i, o) \equiv (\neg c_{op} \Rightarrow o = i + c_N) \wedge (c_{op} \Rightarrow o = i * c_N)$$

$$\mathrm{SAT}(\varphi(c, 2, 9))$$

$$\mathrm{SAT}((\neg c_{op} \Rightarrow 9 = 2 + c_N) \wedge (c_{op} \Rightarrow 9 = 2 * c_N))$$

SMT solver →

Model $\{c_{op} \to 0, c_N \to 7\}$

`return decode[0,7]` i.e. `x + 7`

What is a good encoding?

Sound

- if $\mathrm{wf}(C) \land \mathrm{sem}(C)$ then `decode[C]` is a solution

Complete

- if `decode[C]` is a solution then $\mathrm{wf}(C) \land \mathrm{sem}(C)$

Small parameter space

- avoid symmetries

Solver-friendly

- decidable logic, compact constraint

DSL limitations

Program space can be parameterized with a finite set of parameters

- Counterexample:

```
L ::= sort(L)  |  L[N..N]
    |  L + L  |  [N]  |  x
N ::= find(L,N)  |  0
```

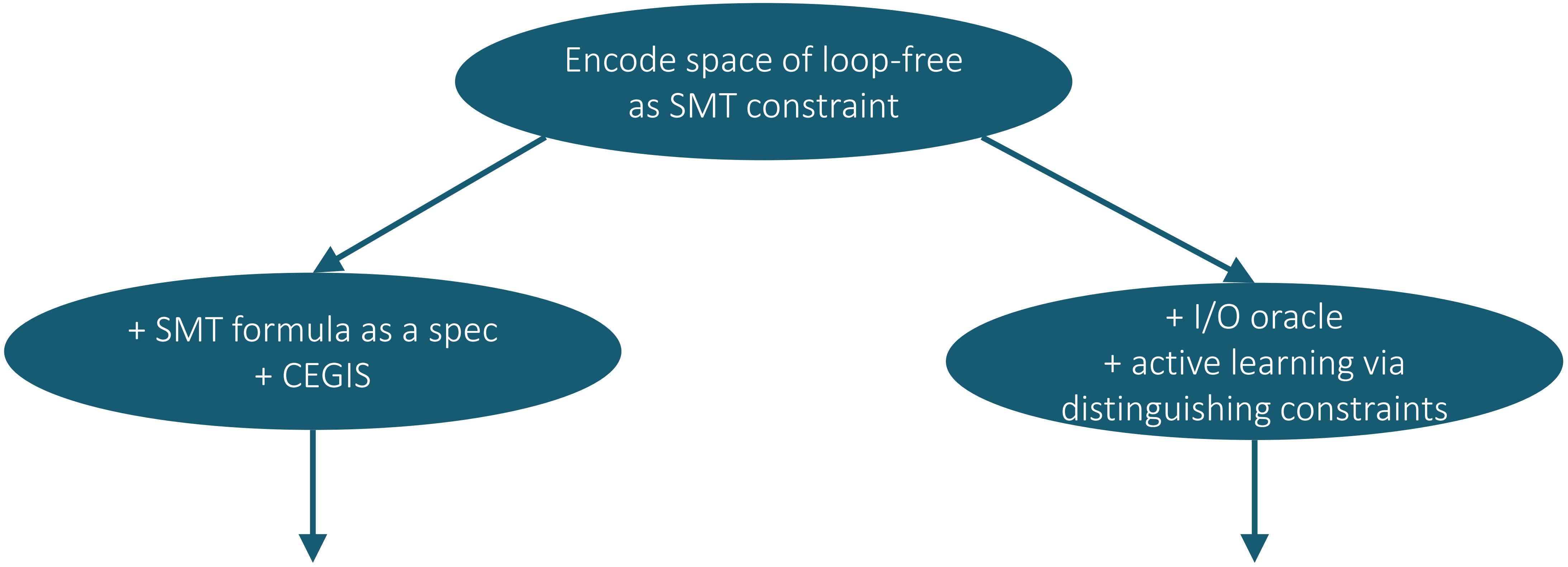
- Workaround

```
L0 ::= x    L1 ::= sort(L0)  |  L0[N0..N0]
N0 ::= 0         |  L0 + L0  |  [N0]  |  L0
            N1 ::= find(L0,N0)  |  N0
```

Program semantics $\varphi(C, i, o)$ is expressible as a (decidable) SAT/SMT formula

- Counterexample

Brahma



[Gulwani, Jha, Tiwari, Venkatesan. PLDI’11]

[Jha, Gulwani, Seshia, Tiwari. ICSE’10]

Brahma encoding

[Jha et al. ‘10]

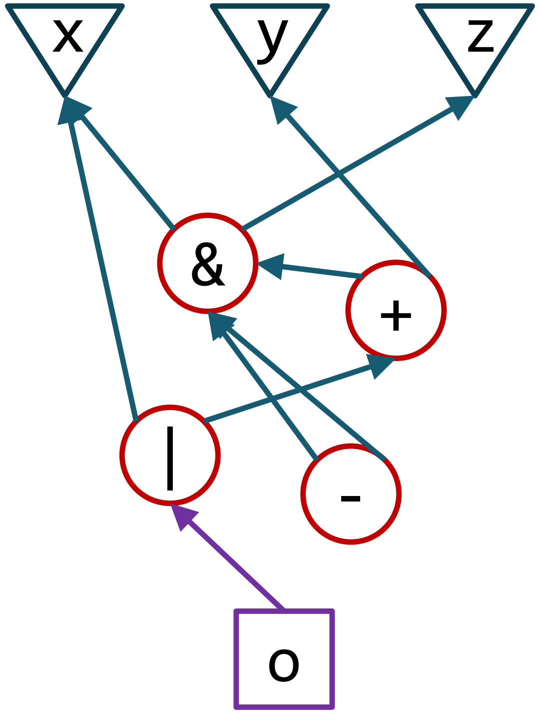
program space = DAG

I

inputs

F

components

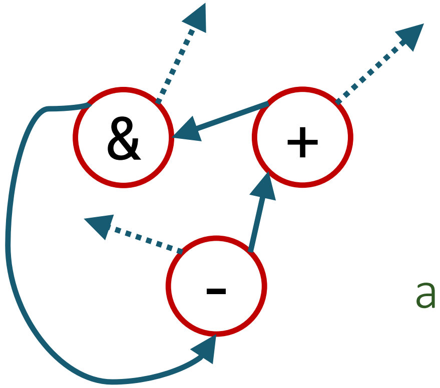


$N = I \cup F$ nodes

parameter space

$$C = \{c_o : N\} \cup \bigcup_{f \in F} \{c_1^f, c_2^f : N\}$$

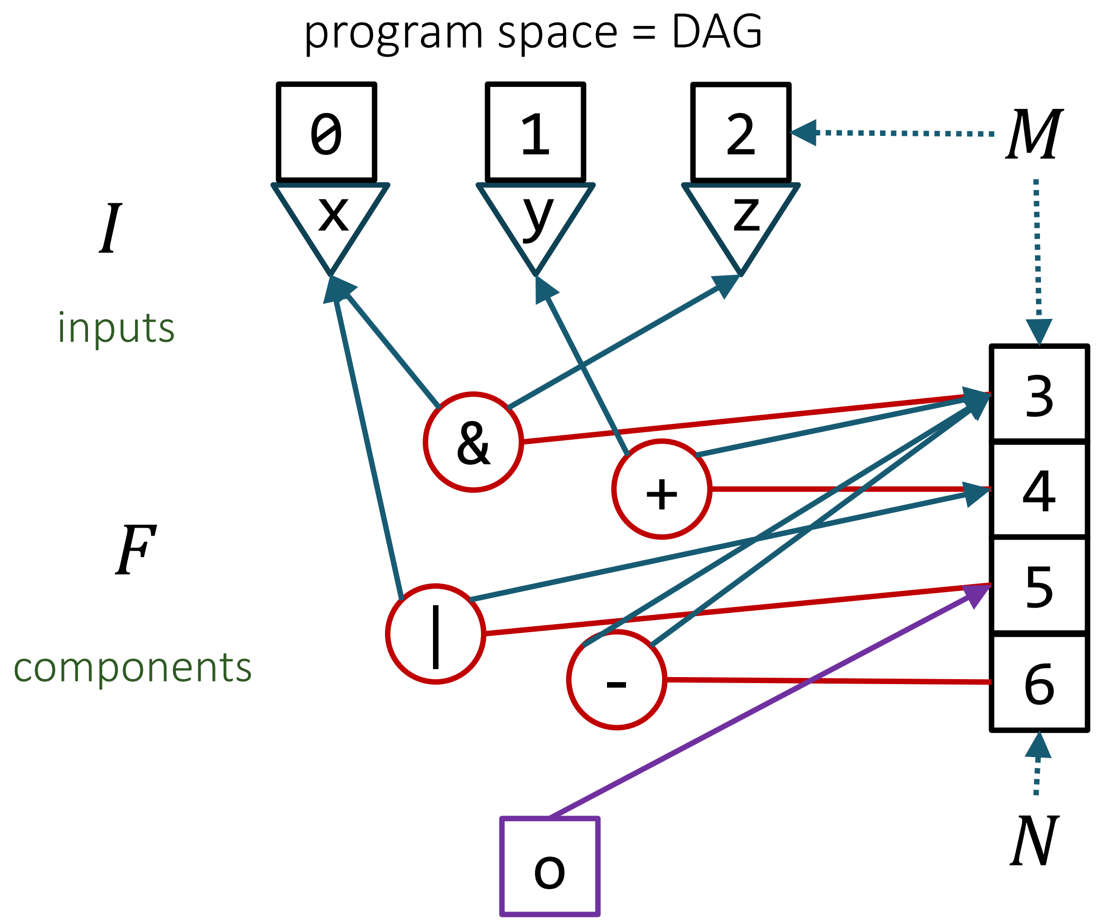
$\mathrm{wf}(C) \equiv ?$



hard to encode acyclicity!

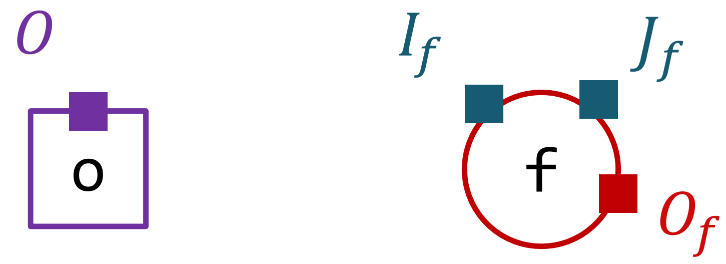
Brahma encoding: take 2

[Jha et al. '10]



parameter space

$$C = \{c_O : \mathrm{Int}\} \cup \bigcup_{f \in F} \{c_{O_f}, c_{I_f}, c_{J_f} : \mathrm{Int}\}$$

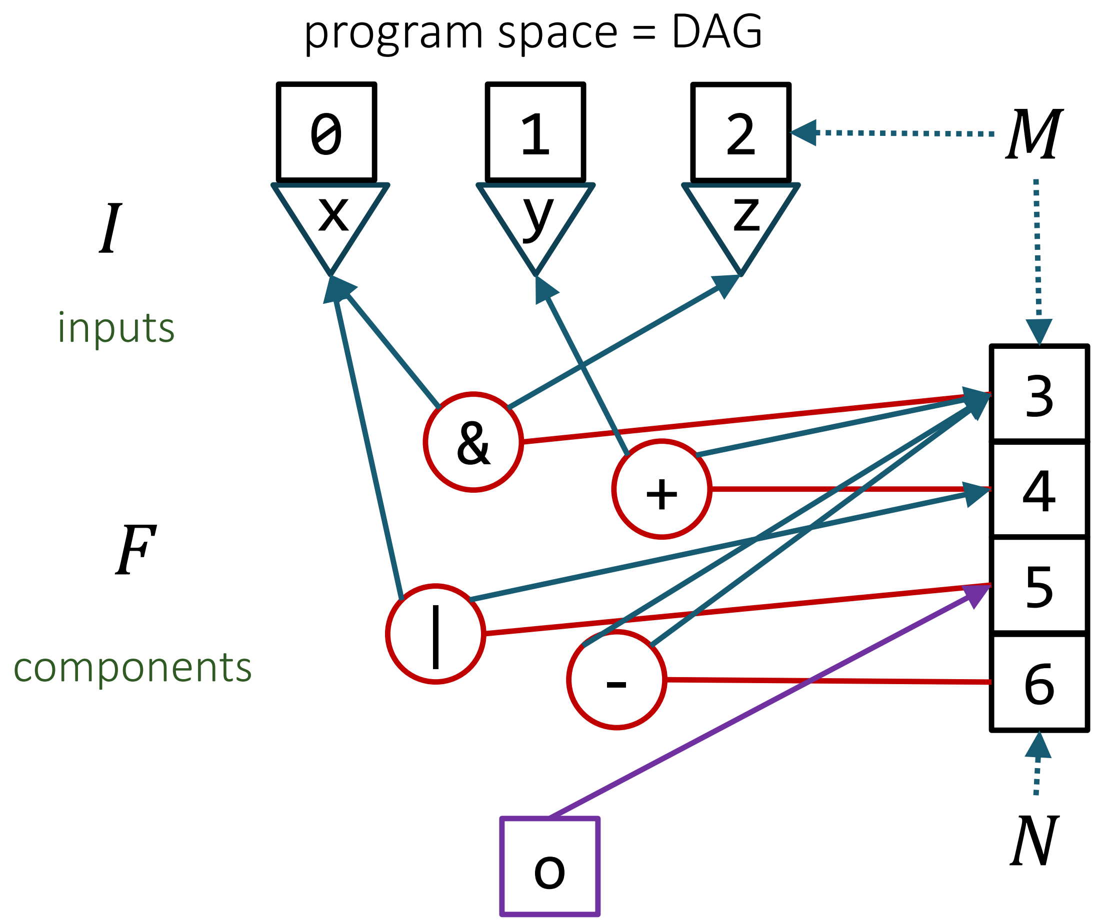


$$\mathrm{wf}(C) \equiv c_O \in M \ \wedge \bigwedge_{f \in F} c_{O_f} \in N \wedge c_{I/J_f} \in M$$

$$\wedge \bigwedge_{f,g \in F, f \not\equiv g} c_{O_f} \neq c_{O_g} \wedge \bigwedge_{f \in F} c_{I/J_f} < c_{O_f}$$

Brahma encoding: take 2

[Jha et al. '10]



parameter space

$$C = \{c_o : \text{Int}\} \cup \bigcup_{f \in F} \{c_{O_f}, c_{I_f}, c_{J_f} : \text{Int}\}$$

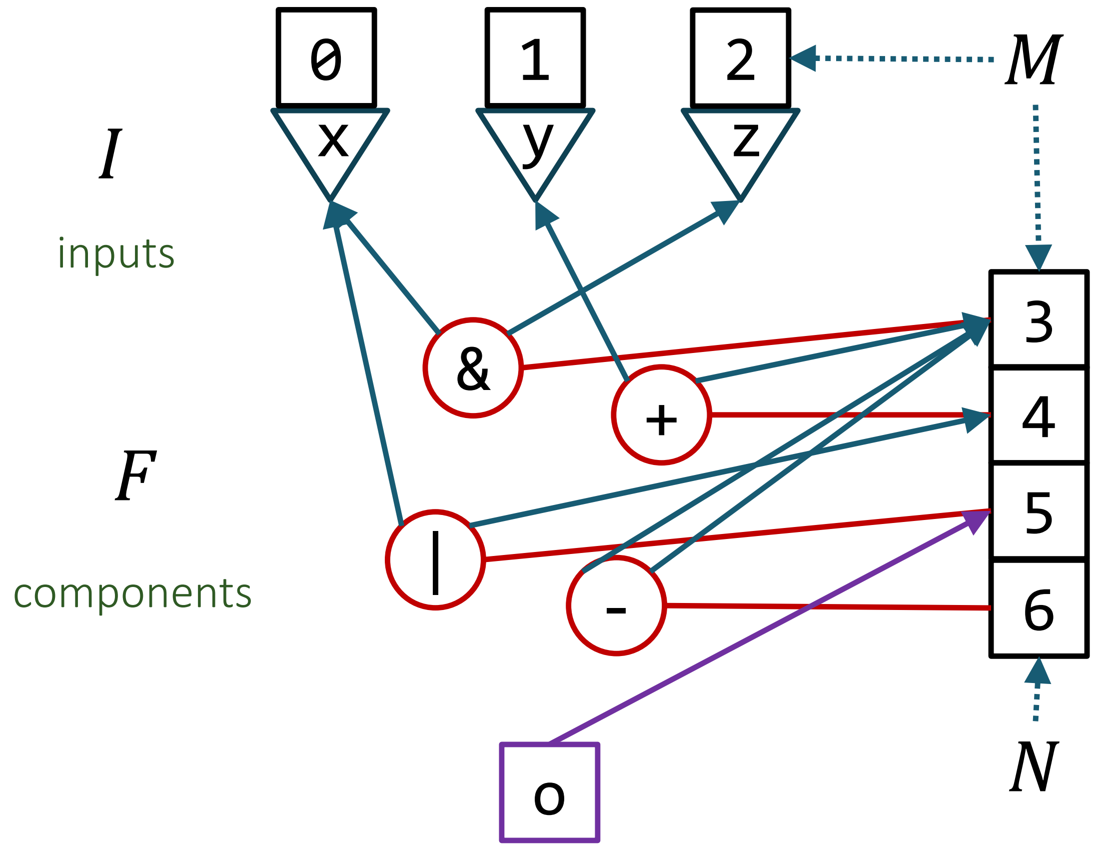
$$P = \bigcup_{f \in F} \{I_f, J_f\} \qquad R = \bigcup_{f \in F} \{O_f\}$$

$$\varphi(C, I, o) \equiv \exists P, R. \bigwedge_{f \in F} O_f = F(I_f, J_f)$$

$$\wedge \bigwedge_{x \in P \cup R \cup I \cup \{O\}} c_x = c_y \Rightarrow x = y$$

Limit #components to K?

program space = DAG



parameter space

$$C = \{c_O : \text{Int}\} \cup \bigcup_{f \in F} \{c_{O_f}, c_{I_f}, c_{J_f} : \text{Int}\}$$

$$\text{wf}(C) \equiv c_O \in M \ \wedge \bigwedge_{f \in F} c_{O_f} \in N \wedge c_{I/J_f} \in M$$

$$\wedge \cdots$$

$$\text{wf}'(C) \equiv \text{wf}(C) \wedge c_O < |I| + K$$

Comparison of search strategies