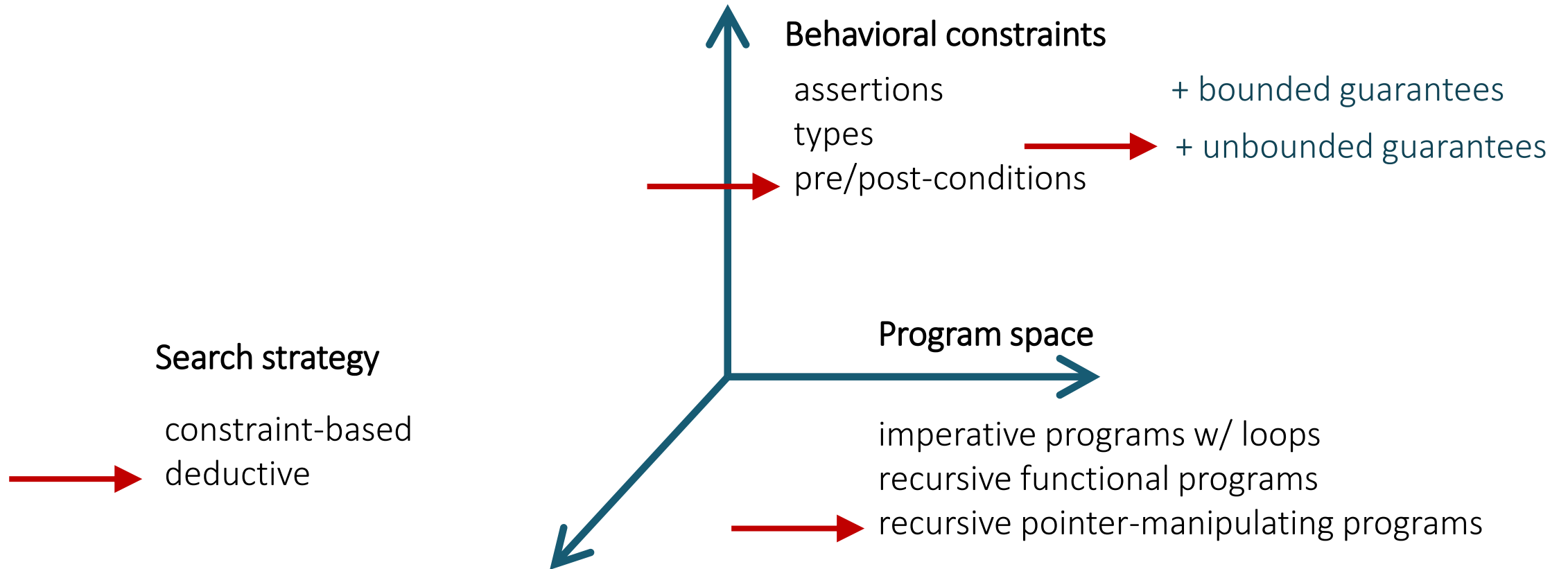


# Lecture 14

## Separation Logic and Deductive Synthesis

# Today

---



# Program synthesis

---

specification



code



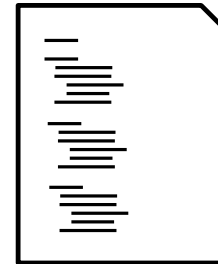
# Program synthesis

---

specification



C code?

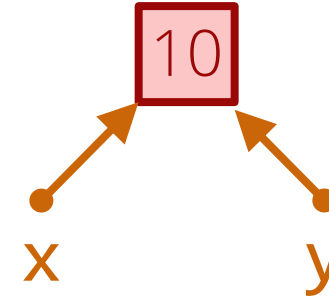


- ☹ verbose
- ☹ unstructured
- ☹ pointers

# The trouble with pointers

---

Can we naively apply Hoare logic to programs with pointers?

$$\{*x = 10 \wedge *y = 10\}$$
$$\Rightarrow$$
$$\{*x + 5 = 15 \wedge *y - 5 = 5\}$$
$$*x = *x + 5;$$
$$\{*x = 15 \wedge *y - 5 = 5\}$$
$$*y = *y - 5;$$
$$\{*x = 15 \wedge *y = 5\}$$


# SuSLik

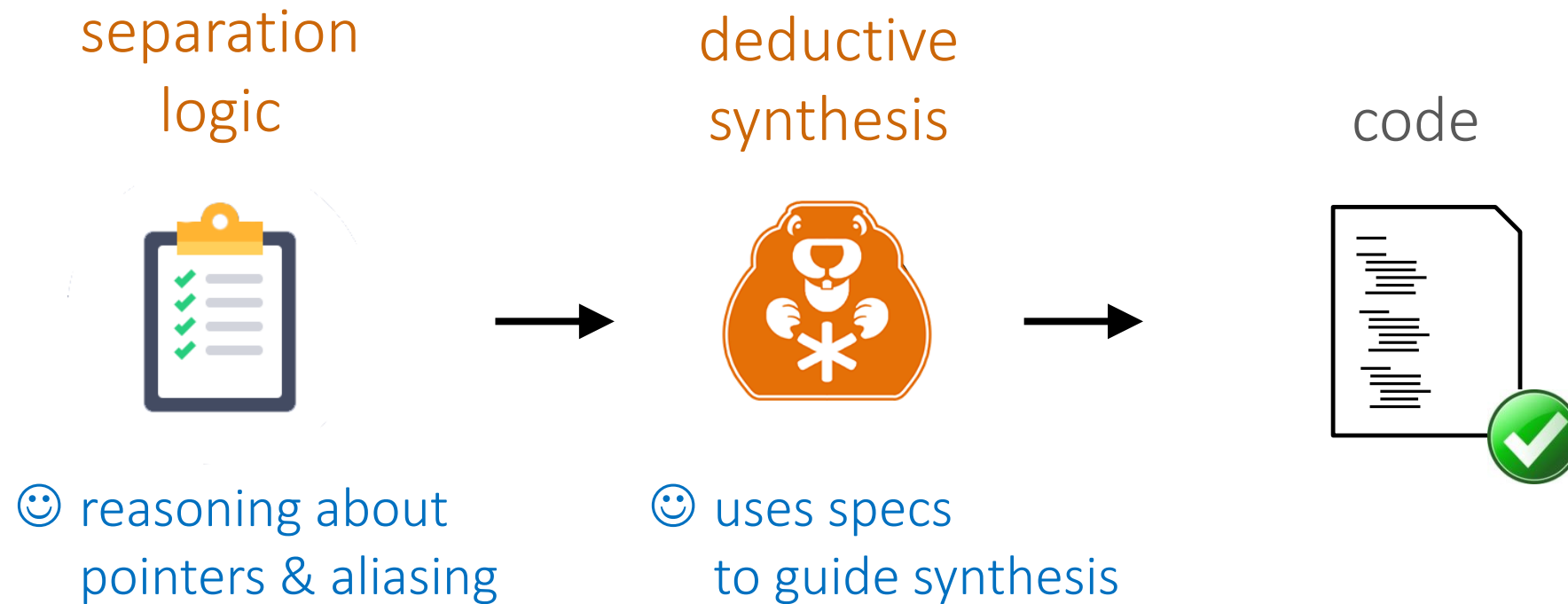
---



Synthesis Using Separation Logik

# The SuSLik approach

---



# Outline

---

example: swap

a taste of SuSLik

separation logic

specifying pointer-manipulating programs

deductive synthesis

from SL specifications to programs



# Outline

---

example: swap

separation logic

deductive synthesis

# Example: swap

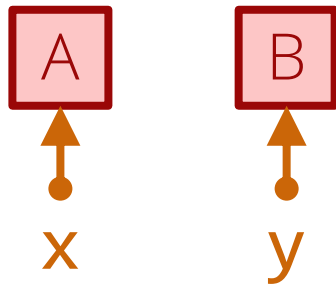
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Swap values of two *distinct* pointers

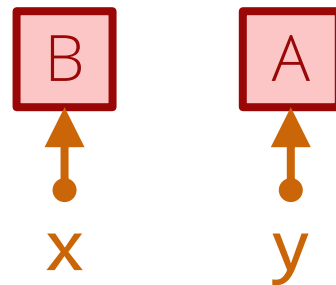
```
void swap(loc x, loc y)
```

# Example: swap

start state:



end state:



in separation logic:

$\{ x \mapsto A * y \mapsto B \}$  precondition

separately

**void swap(loc x, loc y)**

$\{ x \mapsto B * y \mapsto A \}$  postcondition

ghost variables

# Demo: swap

---

Swap values of two *distinct* pointers

```
void swap(loc x, loc y)
```

$$\{x \mapsto A * y \mapsto B\}$$

??

$$\{x \mapsto B * y \mapsto A\}$$

**let** a1 = \*x;

{ x  $\mapsto$  a1 \* y  $\mapsto$  B }

??

{ x  $\mapsto$  B \* y  $\mapsto$  a1 }

**let** a1 = \*x;

**let** b1 = \*y;

{ x  $\mapsto$  a1 \* y  $\mapsto$  b1 }

??

{ x  $\mapsto$  b1 \* y  $\mapsto$  a1 }

**let** a1 = \*x;

**let** b1 = \*y;

\*x = b1;

{ x  $\mapsto$  b1 \* y  $\mapsto$  b1 }

??

{ x  $\mapsto$  b1 \* y  $\mapsto$  a1 }



**let** a1 = \*x;

**let** b1 = \*y;

\*x = b1;

\*y = a1;

{ x  $\mapsto$  b1 \* y  $\mapsto$  a1 }

??

{ x  $\mapsto$  b1 \* y  $\mapsto$  a1 }

same



The diagram consists of two identical lines of text, each enclosed in curly braces and containing the expression 'x ↦ b1 \* y ↦ a1'. From the right side of each line, an orange arrow points towards the word 'same', which is positioned to the right of the space between the two lines. Above the word 'same', there are two red question marks '??'.

```
let a1 = *x;
```

```
let b1 = *y;
```

```
*x = b1;
```

```
*y = a1;
```



```
void swap(loc x, loc y) {  
    let a1 = *x;  
    let b1 = *y;  
    *x = b1;  
    *y = a1;  
}
```

# Outline

---

example: swap

separation logic

deductive synthesis

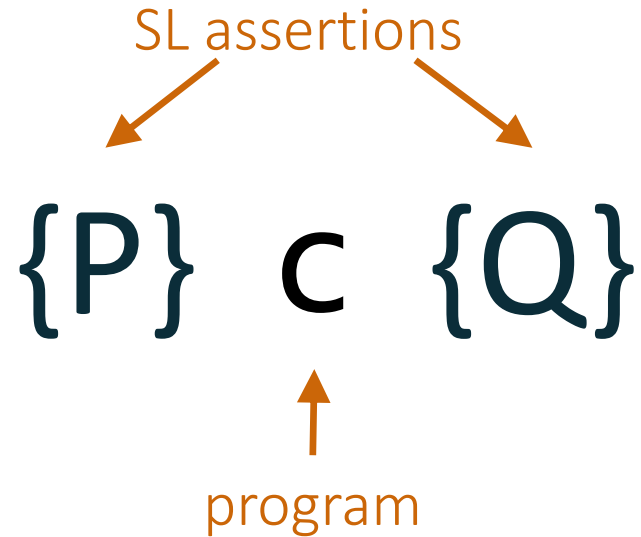
# Separation logic (SL)

---

Hoare logic  
“about the heap”

# Separation logic (SL)

---



starting in a state that satisfies  $P$   
program  $c$  will execute **without memory errors**,  
and upon its termination the state will satisfy  $Q$

# Outline

---

example: swap

separation logic

2.1. programs

2.2. assertions

2.3. specifying data transformations

deductive synthesis

# Separation logic (SL)

---

$\{P\} \quad c \quad \{Q\}$

↑

program



# Programs

---

do nothing

**skip**

# Programs

---

do nothing

read from heap

**skip**  
**let**  $y = *(x + n)$

offset (natural number)

variables

# Programs

---

do nothing

read from heap

write to heap

**skip**

**let**  $y = *(x + n)$

$*(x + n) = e$   expression  
(arithmetic, boolean)

# Programs

---

do nothing

**skip**

read from heap

**let**  $y = *(x + n)$

write to heap

$*(x + n) = e$

allocate block

**let**  $y = \text{malloc}(n)$

# Programs

---

do nothing

**skip**

read from heap

**let**  $y = *(x + n)$

write to heap

$*(x + n) = e$

allocate block

**let**  $y = \text{malloc}(n)$

free block

**free**( $x$ )

# Programs

---

do nothing

**skip**

read from heap

**let**  $y = *(x + n)$

write to heap

$*(x + n) = e$

allocate block

**let**  $y = \mathbf{malloc}(n)$

free block

**free**( $x$ )

procedure call

$p(e_1, \dots, e_n)$

# Programs

---

do nothing

**skip**

read from heap

**let**  $y = *(x + n)$

write to heap

$*(x + n) = e$

allocate block

**let**  $y = \text{malloc}(n)$

free block

**free**( $x$ )

procedure call

$p(e_1, \dots, e_n)$

assignment

only heap is mutable, not stack variables!

# Programs

---

do nothing

**skip**

read from heap

**let**  $y = *(x + n)$

write to heap

$*(x + n) = e$

allocate block

**let**  $y = \mathbf{malloc}(n)$

free block

**free**( $x$ )

procedure call

$p(e_1, \dots, e_n)$

sequential composition

$c_1 ; c_2$

conditional

**if** ( $e$ )  $\{c_1\}$  else  $\{c_2\}$



# Outline

---

example: swap

separation logic

2.1. programs

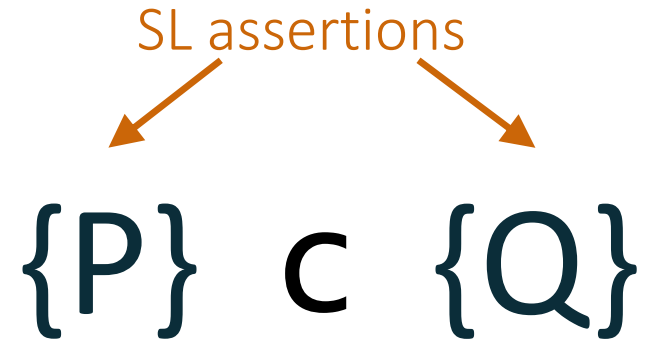
2.2. assertions

2.3. specifying data transformations

deductive synthesis

# Separation logic (SL)

---



# SL assertions

---

empty heap

{ emp }

# SL assertions

---

empty heap                       $\{ \text{emp} \}$

singleton heap                  $\{ y \mapsto 5 \}$



# SL assertions

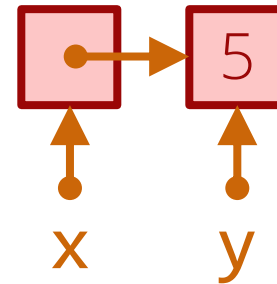
---

empty heap       $\{ \text{emp} \}$

singleton heap       $\{ y \mapsto 5 \}$

separating  
conjunction       $\{ x \mapsto y * y \mapsto 5 \}$

                                 ↙      ↘  
                                 heaplets



# SL assertions

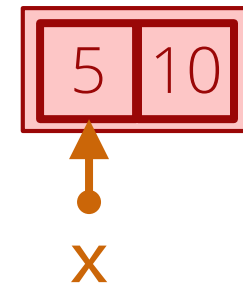
---

empty heap                     $\{ \text{emp} \}$

singleton heap                 $\{ y \mapsto 5 \}$

separating  
conjunction                    $\{ x \mapsto y * y \mapsto 5 \}$

memory block                  $\{ [x, 2] * x \mapsto 5 * (x + 1) \mapsto 10 \}$



# SL assertions

---

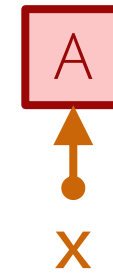
empty heap             $\{ \text{emp} \}$

singleton heap         $\{ y \mapsto 5 \}$

separating  
conjunction             $\{ x \mapsto y * y \mapsto 5 \}$

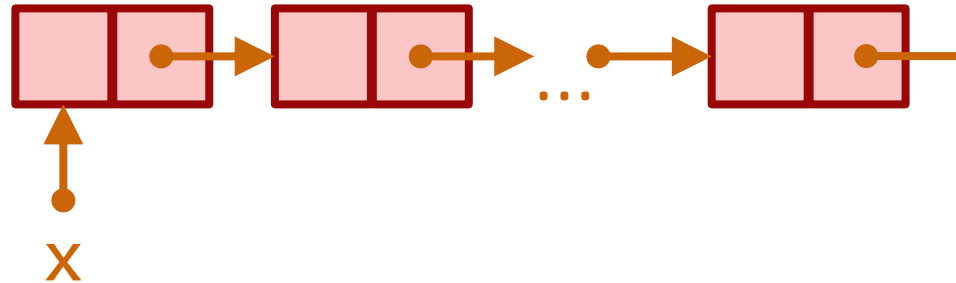
memory block          $\{ [x, 2] * x \mapsto 5 * (x + 1) \mapsto 10 \}$

+ pure formula         $\{ A > 5 ; x \mapsto A \}$



# SL assertions: linked structures

---





# SL assertions: linked structures

---

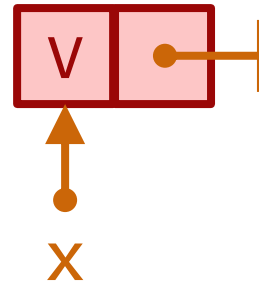
linked list      $\{ x = 0 ; \text{emp} \}$



# SL assertions: linked structures

---

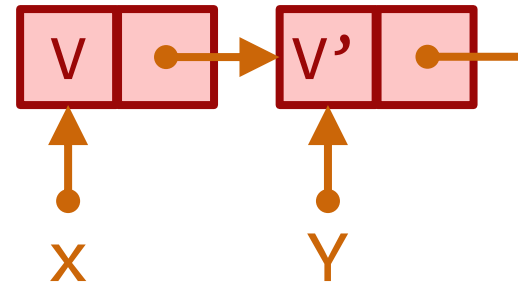
linked list     $\{ [x, 2] * x \mapsto V * (x + 1) \mapsto 0 \}$



# SL assertions: linked structures

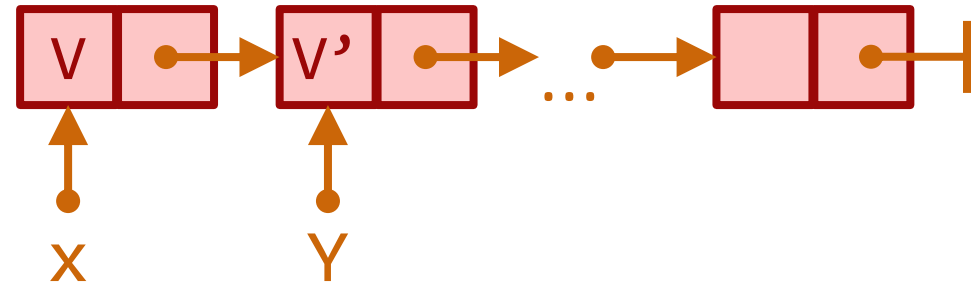
---

linked list     $\{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y *$   
                   $[Y, 2] * Y \mapsto V' * (Y + 1) \mapsto 0$   
                   $\}$



# SL assertions: linked structures

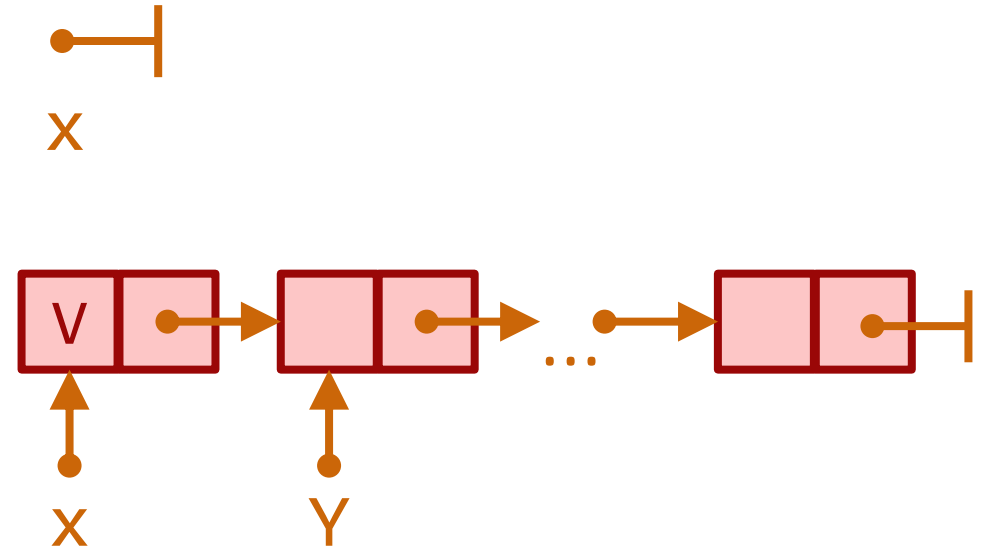
linked list     $\{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y *$   
                   $[Y, 2] * Y \mapsto V' * (Y + 1) \mapsto Y' *$   
                   $\dots$   
                   $\}$



inductive predicates to the rescue!

# The linked list predicate

```
predicate list (loc x) {  
  |  $x = 0 \Rightarrow \{ \text{emp} \}$   
  |  $x \neq 0 \Rightarrow \{ [x, 2]$   
      *  $x \mapsto V$   
      *  $(x + 1) \mapsto Y$   
      * list(Y)  
  }  
}
```



# Outline

---

example: swap

separation logic

2.1. programs

2.2. assertions

2.3. specifying data transformations

deductive synthesis

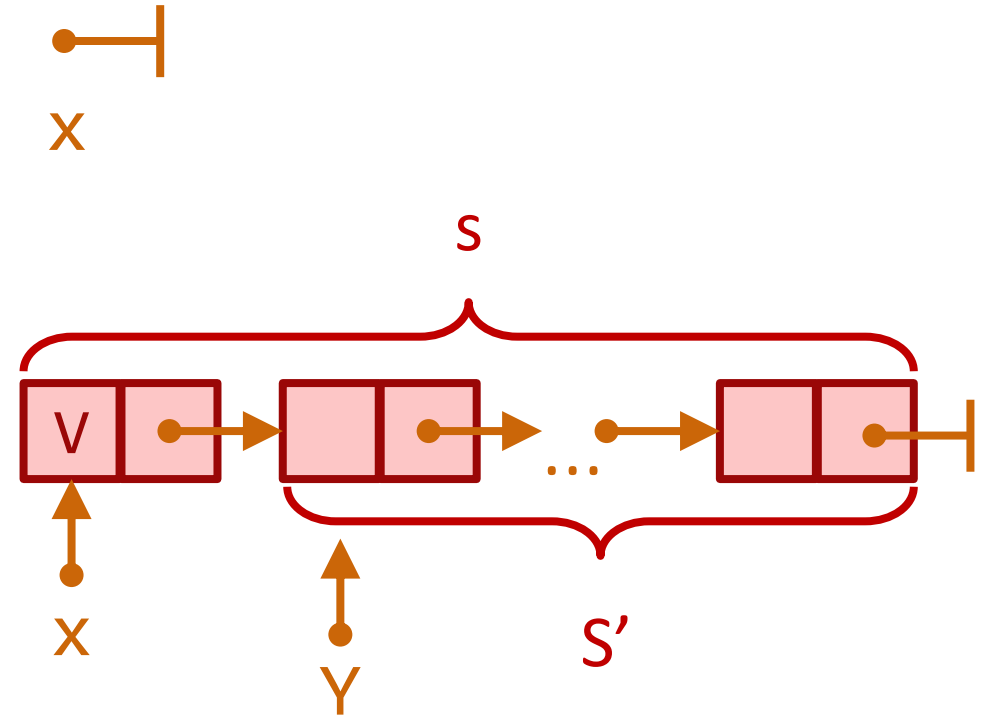
# Demo: dispose a list

---

```
void dispose(loc x)
{ list(x) }
{ emp }
```

# Linked list with elements

```
predicate list (loc x, set s) {  
  |  $x = 0 \Rightarrow \{ s = \emptyset ; \text{emp} \}$   
  |  $x \neq 0 \Rightarrow \{ s = \{V\} + S' ;$   
     $[x, 2]$   
     $* x \mapsto V * (x + 1) \mapsto Y$   
     $* \text{list}(Y, S') \}$   
}
```





# Demo: copy a list

---

```
void copy(loc x, loc r)
```

```
{ list(x, S) * r  $\mapsto$  _ }
```

```
{ list(x, S) * r  $\mapsto$  Y * list(Y, S) }
```



return location

# Outline

---

example: swap

the logic

deductive synthesis

# Deductive synthesis

---

synthesis as proof search

# this talk

---

example: swap

the logic

## deductive synthesis

3.1. proof system

3.2. proof search

# transforming entailment

---

$$P \rightsquigarrow Q \mid c$$

a state that satisfies  $P$   
can be transformed into a state that satisfies  $Q$   
using a program  $c$

# Synthetic separation logic (SSL)

---

proof system for  
transforming entailment

$\{\text{emp}\} \not\Rightarrow \{\text{emp}\} \mid ??$

(Emp)

$\{\text{emp}\} \rightsquigarrow \{\text{emp}\} \mid \mathbf{skip}$



(Frame)

$$\{ P \} \rightsquigarrow \{ Q \} \mid c$$

---

$$\{ P * R \} \rightsquigarrow \{ Q * R \} \mid ??$$

(Write)

$$\{ x \mapsto e * P \} \rightsquigarrow \{ x \mapsto e * Q \} \mid c$$

---

$$\{ x \mapsto \_ * P \} \rightsquigarrow \{ x \mapsto e * Q \} \mid ??$$

(Read)

$$[y/A]\{ x \mapsto A * P \} \rightsquigarrow [y/A]\{ Q \} \mid c$$

---

$$\{ x \mapsto A * P \} \rightsquigarrow \{ Q \} \mid ??$$

# SSL: basic rules

---

(Emp)

$$\{\text{emp}\} \rightsquigarrow \{\text{emp}\} \mid \text{skip}$$

(Frame)

$$\frac{\{P\} \rightsquigarrow \{Q\} \mid c}{\{P * R\} \rightsquigarrow \{Q * R\} \mid c}$$

(Read)

$$\frac{[y/A]\{x \mapsto A * P\} \rightsquigarrow [y/A]\{Q\} \mid c}{\{x \mapsto A * P\} \rightsquigarrow \{Q\} \mid \text{let } y = *x; c}$$

(Write)

$$\frac{\{x \mapsto e * P\} \rightsquigarrow \{x \mapsto e * Q\} \mid c}{\{x \mapsto \_ * P\} \rightsquigarrow \{x \mapsto e * Q\} \mid *x = e; c}$$

# Example: swap

$$\{ x \mapsto A * y \mapsto B \} \rightsquigarrow \{ x \mapsto B * y \mapsto A \} \mid \quad ??$$

$$\{ x \mapsto A * y \mapsto B \} \rightsquigarrow \{ x \mapsto B * y \mapsto A \} \mid \quad ??$$

$\{ x \mapsto a1 * y \mapsto B \} \rightsquigarrow \{ x \mapsto B * y \mapsto a1 \} \mid \quad ??$

---

$\{ x \mapsto A * y \mapsto B \} \rightsquigarrow \{ x \mapsto B * y \mapsto A \} \mid \quad \text{let } a1 = *x; ??$  (Read)

$$\{x \mapsto a1 * y \mapsto b1\} \rightsquigarrow \{x \mapsto b1 * y \mapsto a1\} \mid ??$$

(Read)

---


$$\{x \mapsto a1 * y \mapsto B\} \rightsquigarrow \{x \mapsto B * y \mapsto a1\} \mid \text{let } b1 = *y; ??$$

(Read)

---


$$\{x \mapsto A * y \mapsto B\} \rightsquigarrow \{x \mapsto B * y \mapsto A\} \mid \text{let } a1 = *x; ??$$



$$\{x \mapsto b1 * y \mapsto b1\} \rightsquigarrow \{x \mapsto b1 * y \mapsto a1\} \mid ??$$

---

(Write)

$$\{x \mapsto a1 * y \mapsto b1\} \rightsquigarrow \{x \mapsto b1 * y \mapsto a1\} \mid *x = b1; ??$$

---

(Read)

$$\{x \mapsto a1 * y \mapsto B\} \rightsquigarrow \{x \mapsto B * y \mapsto a1\} \mid \text{let } b1 = *y; ??$$

---

(Read)

$$\{x \mapsto A * y \mapsto B\} \rightsquigarrow \{x \mapsto B * y \mapsto A\} \mid \text{let } a1 = *x; ??$$

$$\{y \mapsto b1\} \rightsquigarrow \{y \mapsto a1\} \mid ??$$

(Frame)

$$\{x \mapsto b1 * y \mapsto b1\} \rightsquigarrow \{x \mapsto b1 * y \mapsto a1\} \mid ??$$

(Write)

$$\{x \mapsto a1 * y \mapsto b1\} \rightsquigarrow \{x \mapsto b1 * y \mapsto a1\} \mid *x = b1; ??$$

(Read)

$$\{x \mapsto a1 * y \mapsto B\} \rightsquigarrow \{x \mapsto B * y \mapsto a1\} \mid \text{let } b1 = *y; ??$$

(Read)

$$\{x \mapsto A * y \mapsto B\} \rightsquigarrow \{x \mapsto B * y \mapsto A\} \mid \text{let } a1 = *x; ??$$

$$\{ y \mapsto a1 \} \rightsquigarrow \{ y \mapsto a1 \} \mid ??$$

---


$$\{ y \mapsto b1 \} \rightsquigarrow \{ y \mapsto a1 \} \mid *y = a1; ??$$

---


$$\{ x \mapsto b1 * y \mapsto b1 \} \rightsquigarrow \{ x \mapsto b1 * y \mapsto a1 \} \mid ??$$

---


$$\{ x \mapsto a1 * y \mapsto b1 \} \rightsquigarrow \{ x \mapsto b1 * y \mapsto a1 \} \mid *x = b1; ??$$

---


$$\{ x \mapsto a1 * y \mapsto B \} \rightsquigarrow \{ x \mapsto B * y \mapsto a1 \} \mid \text{let } b1 = *y; ??$$

---


$$\{ x \mapsto A * y \mapsto B \} \rightsquigarrow \{ x \mapsto B * y \mapsto A \} \mid \text{let } a1 = *x; ??$$

$$\begin{array}{c}
\{ \text{emp} \} \rightsquigarrow \{ \text{emp} \} \mid ?? \\
\hline
\{ y \mapsto a1 \} \rightsquigarrow \{ y \mapsto a1 \} \mid ?? \quad \text{(Frame)} \\
\hline
\{ y \mapsto b1 \} \rightsquigarrow \{ y \mapsto a1 \} \mid *y = a1; ?? \quad \text{(Write)} \\
\hline
\{ x \mapsto b1 * y \mapsto b1 \} \rightsquigarrow \{ x \mapsto b1 * y \mapsto a1 \} \mid ?? \quad \text{(Frame)} \\
\hline
\{ x \mapsto a1 * y \mapsto b1 \} \rightsquigarrow \{ x \mapsto b1 * y \mapsto a1 \} \mid *x = b1; ?? \quad \text{(Write)} \\
\hline
\{ x \mapsto a1 * y \mapsto B \} \rightsquigarrow \{ x \mapsto B * y \mapsto a1 \} \mid \text{let } b1 = *y; ?? \quad \text{(Read)} \\
\hline
\{ x \mapsto A * y \mapsto B \} \rightsquigarrow \{ x \mapsto B * y \mapsto A \} \mid \text{let } a1 = *x; ?? \quad \text{(Read)}
\end{array}$$

$$\begin{array}{c}
\frac{}{\{ \text{emp} \} \rightsquigarrow \{ \text{emp} \} \mid \boxed{\text{skip}}} \text{(Emp)} \\
\hline
\{ y \mapsto a1 \} \rightsquigarrow \{ y \mapsto a1 \} \mid ?? \text{(Frame)} \\
\hline
\frac{}{\{ y \mapsto b1 \} \rightsquigarrow \{ y \mapsto a1 \} \mid \boxed{*y = a1;}} \text{(Write)} \\
\hline
\{ x \mapsto b1 * y \mapsto b1 \} \rightsquigarrow \{ x \mapsto b1 * y \mapsto a1 \} \mid ?? \text{(Frame)} \\
\hline
\frac{}{\{ x \mapsto a1 * y \mapsto b1 \} \rightsquigarrow \{ x \mapsto b1 * y \mapsto a1 \} \mid \boxed{*x = b1;}} \text{(Write)} \\
\hline
\{ x \mapsto a1 * y \mapsto B \} \rightsquigarrow \{ x \mapsto B * y \mapsto a1 \} \mid \boxed{\text{let } b1 = *y;} ?? \text{(Read)} \\
\hline
\{ x \mapsto A * y \mapsto B \} \rightsquigarrow \{ x \mapsto B * y \mapsto A \} \mid \boxed{\text{let } a1 = *x;} ?? \text{(Read)}
\end{array}$$

$\{ x \mapsto A * y \mapsto B \}$

**let** a1 = \*x; **let** b1 = \*y; \*x = b1; \*y = a1; **skip**

$\{ x \mapsto B * y \mapsto A \}$

# Synthetic separation logic (SSL)

---

basic rules

(Emp), (Read), (Write), (Frame)

(Alloc), (Free)

pure reasoning and unification

inductive predicates and recursion

# Synthetic separation logic (SSL)

---

basic rules

(Emp), (Read), (Write), (Frame)

(Alloc), (Free)

pure reasoning and unification

inductive predicates and recursion



# Example: dispose a list

---

```
void dispose(loc x)
{ list(x) }
{ emp }
```

$\{ \text{list}^1(x) \}$  **void** dispose(**loc** x) { emp }

$\{ \text{list}^0(x) \}$

??

(Induction)

{ emp }

```

predicate list (loc x) {
  | x = 0 => { emp }
  | x ≠ 0 => { [x, 2] * x ↦ V * (x + 1) ↦ Y * list(Y) }
}

```

```

{ list1 (x) } void dispose(loc x) { emp }

```

{ list<sup>0</sup>(x) }

??

(Open)

{ emp }

```
predicate list (loc x) {
```

```
  |  $x = 0 \Rightarrow \{ \text{emp} \}$ 
```

```
  |  $x \neq 0 \Rightarrow \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * \text{list}(Y) \}$ 
```

```
}
```

```
  if (x == 0) {
```

```
    { x = 0 ; emp }
```

```
    ??
```

```
    { emp }
```

```
  } else {
```

```
    {  $x \neq 0 ; [x, 2] * x \mapsto V * (x + 1) \mapsto Y * \text{list}^1(Y)$  }
```

```
    ??
```

```
    { emp }
```

```
  }
```

```
{ list1 (x) } void dispose(loc x) { emp }
```

```
predicate list (loc x) {
```

```
  |  $x = 0 \Rightarrow \{ \text{emp} \}$ 
```

```
  |  $x \neq 0 \Rightarrow \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * \text{list}(Y) \}$ 
```

```
}
```

```
  if (x == 0) {
```

```
    { x = 0 ; emp }
```

```
    ?? (Emp)
```

```
    { emp }
```

```
  } else {
```

```
    {  $x \neq 0 ; [x, 2] * x \mapsto V * (x + 1) \mapsto Y * \text{list}^1(Y)$  }
```

```
    ??
```

```
    { emp }
```

```
  }
```

```
{ list1 (x) } void dispose(loc x) { emp }
```

```

predicate list (loc x) {
  |  $x = 0 \Rightarrow \{ \text{emp} \}$ 
  |  $x \neq 0 \Rightarrow \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * \text{list}(Y) \}$ 
}

```

```

if (x == 0) {

```

```

  { x = 0 ; emp }

```

```

    skip

```

```

  { emp }

```

```

} else {

```

```

  {  $x \neq 0 ; [x, 2] * x \mapsto V * (x + 1) \mapsto Y * \text{list}^1(Y)$  }

```

```

    ??

```

```

  { emp }

```

```

}

```

```

{  $\text{list}^1(x)$  } void dispose(loc x) { emp }

```

```

predicate list (loc x) {
  |  $x = 0 \Rightarrow \{ \text{emp} \}$ 
  |  $x \neq 0 \Rightarrow \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * \text{list}(Y) \}$ 
}

```

```

 $\{ \text{list}^1(x) \}$  void dispose(loc x) { emp }

```

```

if (x == 0) { skip } else {
  {  $x \neq 0 ; [x, 2] * x \mapsto V * (x + 1) \mapsto Y * \text{list}^1(Y)$  }
}

```

??

```

{ emp }
}

```

```

predicate list (loc x) {
  |  $x = 0 \Rightarrow \{ \text{emp} \}$ 
  |  $x \neq 0 \Rightarrow \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * \text{list}(Y) \}$ 
}

```

```

if (x == 0) { skip } else {
  {  $x \neq 0 ; [x, 2] * x \mapsto V * (x + 1) \mapsto Y * \text{list}^1(Y)$  }

```

??

```

{ emp }
}

```

```

{  $\text{list}^1(x)$  } void dispose(loc x) { emp }

```

(Read)



```

predicate list (loc x) {
  |  $x = 0 \Rightarrow \{ \text{emp} \}$ 
  |  $x \neq 0 \Rightarrow \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * \text{list}(Y) \}$ 
}

```

```

 $\{ \text{list}^1(x) \}$  void dispose(loc x) { emp }

```

```

if (x == 0) { skip } else {
  let y1 = *(x + 1);
  {  $x \neq 0 ; [x, 2] * x \mapsto V * (x + 1) \mapsto y1 * \text{list}^1(y1) \}$ 

  ??

  { emp }
}

```

```

predicate list (loc x) {
  | x = 0 => { emp }
  | x ≠ 0 => { [x, 2] * x ↦ V * (x + 1) ↦ Y * list(Y) }
}

```

```

if (x == 0) { skip } else {
  let y1 = *(x + 1);

```

```

{ x ≠ 0 ; [x, 2] * x ↦ V * (x + 1) ↦ y1 * list1(y1) }

```

??

(Free)

```

{ emp }
}

```

```

{ list1(x) } void dispose(loc x) { emp }

```

```
predicate list (loc x) {  
  |  $x = 0 \Rightarrow \{ \text{emp} \}$   
  |  $x \neq 0 \Rightarrow \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * \text{list}(Y) \}$   
}
```

```
if (x == 0) { skip } else {  
  let y1 = *(x + 1);  
  free x;  
  {  $x \neq 0 ; \text{list}^1(y1)$  }  
  ??  
  { emp }  
}
```

```
{  $\text{list}^1(x)$  } void dispose(loc x) { emp }
```

```

predicate list (loc x) {
  | x = 0 => { emp }
  | x ≠ 0 => { [x, 2] * x ↦ V * (x + 1) ↦ Y * list(Y) }
}

```

```

if (x == 0) { skip } else {
  let y1 = *(x + 1);
  free x;
  { x ≠ 0 ; list1(y1) }
  ??
  { emp }
}

```

(Call)

```

{ list1(x) } void dispose(loc x) { emp }

```

```

predicate list (loc x) {
  | x = 0 => { emp }
  | x ≠ 0 => { [x, 2] * x ↦ V * (x + 1) ↦ Y * list(Y) }
}

```

```

if (x == 0) { skip } else {
  let y1 = *(x + 1);
  free x;
  dispose(y1);
  { x ≠ 0 ; emp }
  ??
  { emp }
}

```

```

{ list1 (x) } void dispose(loc x) { emp }

```

```

predicate list (loc x) {
  | x = 0 => { emp }
  | x ≠ 0 => { [x, 2] * x ↦ V * (x + 1) ↦ Y * list(Y) }
}

```

```

if (x == 0) { skip } else {
  let y1 = *(x + 1);
  free x;
  dispose(y1);
  { x ≠ 0 ; emp }
  ??
  { emp }
}

```

(Emp)

```

{ list1 (x) } void dispose(loc x) { emp }

```

```
predicate list (loc x) {  
  |  $x = 0 \Rightarrow \{ \text{emp} \}$   
  |  $x \neq 0 \Rightarrow \{ [x, 2] * x \mapsto V * (x + 1) \mapsto Y * \text{list}(Y) \}$   
}
```

```
  if (x == 0) { skip } else {  
    let y1 = *(x + 1);  
    free x;  
    dispose(y1);  
    skip  
  
  }
```

```
{ list1 (x) } void dispose(loc x) { emp }
```

```
void dispose(loc x) {  
    if (x == 0) { }  
    else {  
        let y1 = *(x + 1);  
        free x;  
        dispose(y1)  
    }  
}
```



# Synthetic separation logic (SSL)

---

basic rules

(Emp), (Read), (Write), (Frame), (Alloc), (Free)

pure reasoning and unification

inductive predicates and recursion

(Open), (Close), (Induction), (Call)

# Outline

---

example: swap

the logic

deductive synthesis

3.1. proof system

3.2. proof search

# SuSLik

best-first and-or search in SSL  
+ optimizations

# Optimizations

---

invertible rules

early failure

multi-phase search

goal memoization

# Optimization: invertible rules

---

invertible rules do not restrict the set of derivable programs

idea: invertible rules need not be backtracked

(Read)

$$\frac{[y/A]\{ x \mapsto A * P \} \rightsquigarrow [y/A]\{ Q \} \mid c}{\{ x \mapsto A * P \} \rightsquigarrow \{ Q \} \mid \mathbf{let} \ y = *x; \ c}$$

# Optimization: early failure

---

idea: sometimes you know that a goal is unsatisfiable

(Post-Inconsistent)

$$\frac{\psi \neq \perp \quad \vdash \phi \wedge \psi \Rightarrow \perp \quad \{ \text{emp} \} \rightsquigarrow \{ \perp ; \text{emp} \} \mid c}{\{ \phi ; P \} \rightsquigarrow \{ \psi ; Q \} \mid c}$$

# Optimization: multi-phase search

---

unfolding phase: eliminates inductive predicates

(Open), (Close), (Call), (Unify) , (Frame)

block phase: eliminates blocks

(Alloc), (Free), (Unify), (Frame)

pointer phase: aligns all pointers

(Unify), (Frame)

pure phase: deals with the content of pointers

(Write), (Unify), (Pure)

**idea:** if a phase fails, don't start the next one

# SuSLik: contributions

---

Synthesis with unbounded guarantees for recursive heap-manipulating programs

Does not require sketches

Optimizations from proof search



# SuSLik: limitations

---

No auxiliary functions

Only structurally recursive programs

Requires the spec to be inductive

Does not support loops

- But we can turn recursion into loops (?)

Requires SL expertise

# SuSLik: questions

---

Behavioral constraints? Structural constraints? Search strategy?

- SL specs
- Set of components + built-in language constraints
- Deductive search

Enumerative vs deductive search

- Enumerative = enumerates programs
- Deductive = enumerates derivations
- Synquid can be seen as deductive search in the space of typing derivations

# SuSLik: questions

---

Why is Frame rule not invertible?

- What other rule can it prevent from applying?

$$\text{(Frame)} \quad \frac{\{ P \} \rightsquigarrow \{ Q \} \mid c}{\{ P * \textcolor{red}{R} \} \rightsquigarrow \{ Q * \textcolor{red}{R} \} \mid c}$$