Lecture 10
Bounded Constraint-Based Synthesis

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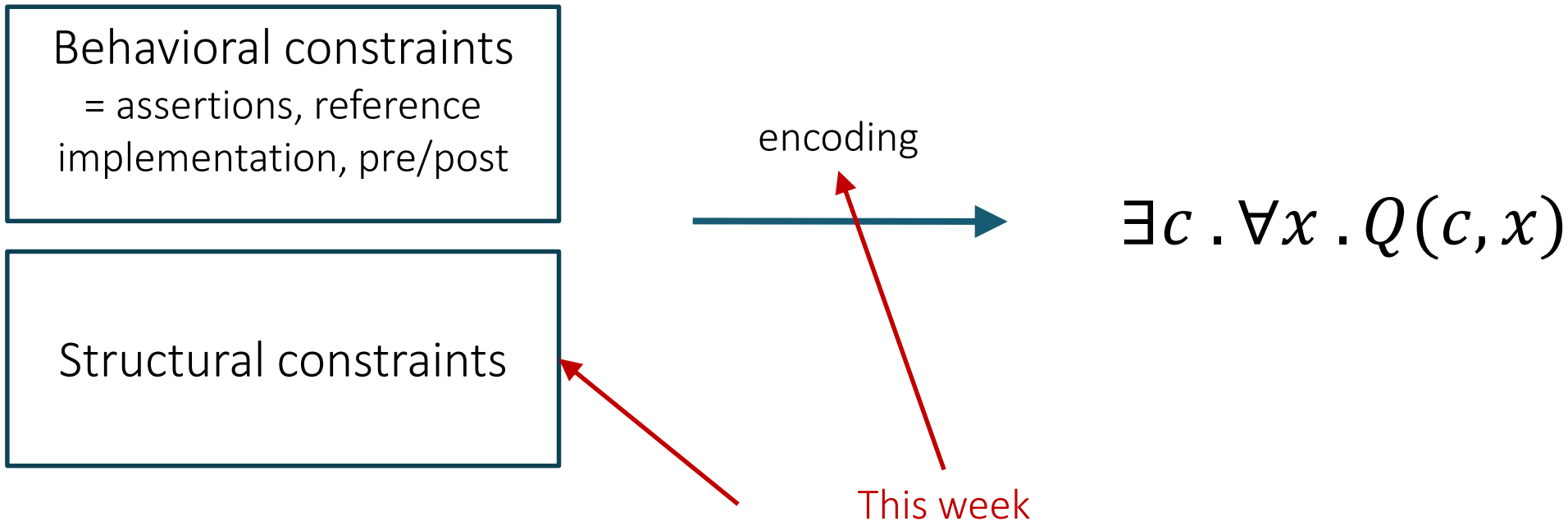
Constraint-based synthesis from specifications



$$\exists c \,.\, \forall x \,.\, Q(c, x)$$

Program sketching

Behavioral constraints
= assertions / reference implementation

Structural constraints
= sketches

symbolic execution →

$$\exists c \,.\, \forall x \,.\, Q(c, x)$$

Structural constraints in Sketch

Different constraints good for different problems

- CFGs
- Components
- Just figure out the constants

Idea: Allow the programmer to encode all kinds of constraints using... programs (duh!)

Language Design Strategy

Extend base language with one construct

Constant hole: **??**

```
int bar (int x)
{
    int t = x * ??;
    assert t == x + x;
    return t;
}
```

```
int bar (int x)
{
    int t = x * 2;
    assert t == x + x;
    return t;
}
```

Synthesizer replaces **??** with a natural number

Constant holes → sets of expressions

Expressions with `??` == sets of expressions

- linear expressions
- polynomials
- sets of variables

```
x*?? + y*??
x*x*?? + x*?? + ??
?? ? x : y
```

Example: swap without a temporary

Swap two integers without an extra temporary

```
void swap(ref int x, ref int y){
    x = ... // sum or difference of x and y
    y = ... // sum or difference of x and y
    x = ... // sum or difference of x and y
}

harness void main(int x, int y){
    int tx = x; int ty = y;
    swap(x, y);
    assert x==ty && y == tx;
}
```

Syntactic sugar

```
{| RegExp |}
```

RegExp supports choice '|' and optional '?'

- can be used arbitrarily within an expression
  - to select operands `{| (x | y | z) + 1 |}`
  - to select operators `{| x (+ | -) y |}`
  - to select fields `{| n(.prev | .next)? |}`
  - to select arguments `{| foo( x | y, z) |}`

Set must respect the type system

- all expressions in the set must type-check
- all must be of the same type

Complex program spaces

Idea: To build complex program spaces from simple program spaces, borrow abstraction devices from programming languages

Function: abstracts expressions

Generator: abstracts set of expressions

- Like a function with holes...
- ...but different invocations → different code

Example: swap without a temporary

```
generator int sign() {
    if ?? {return 1;} else {return -1;}
}

void swap(ref int x, ref int y){
    x = sign()*x + sign()*y;    ➔ 1 1
    y = sign()*x + sign()*y;    ➔ 1 -1
    x = sign()*x + sign()*y;    ➔ 1 -1
}

harness void main(int x, int y){
    int tx = x; int ty = y;
    swap(x, y);
    assert x==ty && y == tx;
}
```

Recursive generators

Can generators encode a CFG?

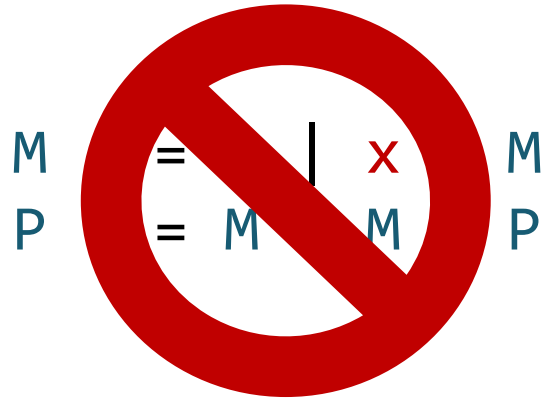
M ::= *n* | x * M
P ::= M | M + P

```
generator int mono(int x) {
    if (??) {return ??;}
    else {return x * mono(x);}
}

generator int poly(int x) {
    if (??) {return mono(x);}
    else {return mono(x) + poly(x);}
}
```

Recursive generators

What if monomial of every degree can occur at most once?




```
generator int mono(int x, int n) {
    if (n <= 0) {return ??;}
    else {return x * mono(x, n - 1);}
}

generator int poly(int x, int n) {
    if (n <= 0) {return mono(x,0);}
    else {return mono(x,n) + poly(x, n - 1);}
}
```

Encoding sketches

Behavioral constraints
= assertions / reference implementation

Structural constraints
= sketches

symbolic execution →

$$\exists c \,.\, \forall x \,.\, Q(c, x)$$

Semantics of a simple language

```
e  :=  n | x | e1 + e2
c  :=  x := e | c1 ; c2 | if e then c1 else c2 | while e do c
```

What does an expression mean?

- An expression reads the state and produces a value
- The state is modeled as a map σ from variables to values
- $\mathcal{A}[\![\cdot]\!] : e \to \Sigma \to \mathbb{Z}$

Ex:

- $\mathcal{A}[\![x]\!] = \lambda\sigma.\,\sigma[x]$
- $\mathcal{A}[\![n]\!] = \lambda\sigma.\,n$
- $\mathcal{A}[\![e_1 + e_2]\!] = \lambda\sigma.\,\mathcal{A}[\![e_1]\!]\sigma + \mathcal{A}[\![e_2]\!]\sigma$

Semantics of a simple language

```
e  :=  n | x | e1 + e2
c  :=  x := e | c1 ; c2 | if e then c1 else c2 | while e do c
```

What does a command mean?

- A command modifies the state
- $\mathcal{C}[\![\cdot]\!] : c \to \Sigma \to \Sigma$

Ex:

- $\mathcal{C}[\![x := e]\!] = \lambda\sigma.\sigma[x \to (\mathcal{A}[\![e]\!]\sigma)]$
- $\mathcal{C}[\![c_1; c_2]\!] = \lambda\sigma.\mathcal{C}[\![c_2\,]\!](\mathcal{C}[\![c_1\,]\!]\sigma)$
- $\mathcal{C}[\![\text{if } e \text{ then } c_1 \text{ else } c_2]\!] =$

$$\lambda\sigma.\lambda x.\mathcal{A}[\![e]\!]\sigma \;\; ? \;\; (\mathcal{C}[\![c_1\,]\!]\sigma)[x] \;\; : \;\; (\mathcal{C}[\![c_2\,]\!]\sigma)[x]$$

What about loops?

Semantics of a while loop

- Let $W = \mathcal{C}[\![while\ e\ do\ c]\!]$
- W satisfies the following equation:

$$(W\ \sigma)[x] = \mathcal{A}[\![e]\!]\sigma\ \ ?\ \ (W(\mathcal{C}[\![c]\!]\sigma))[x]\ \ :\ \sigma[x]$$

- One strategy: find a fixpoint (see next week)
- We'll settle for a simpler strategy: unroll k times and then give up

Symbolic execution: example

```
harness void main(int x, int u){
  int z = 0; int i = 0;
  int y = 2 * x;
  if (u > 0) {
    z = 2 * x;
  } else {
    while (i < 2) {
      z = z + x;
      i = i + 1;
    }
  }
  assert y == z;
}
```

Step 1: unroll with depth = 2

```
if (i < 2) {
  z = z + x;
  i = i + 1;
  if (i < 2) {
    z = z + x;
    i = i + 1;
    assert !(i < 2);
  }
}
```

Symbolic execution: example

```
harness void main(int x, int u){
  int z = 0; int i = 0;
  int y = 2 * x;
  if (u > 0) {
    z = 2 * x;
  } else {
    if (i < 2) {
      z = z + x;
      i = i + 1;
      if (i < 2) {
        z = z + x;
        i = i + 1;
        assert !(i < 2);
      }
    }
  }
  assert y == z;
}
```

$\sigma = \{x \to X, u \to U\}$

$\sigma = \{x \to X, u \to U, z \to 0, i \to 0, y \to 2X\}$

$U > 0$

$\sigma = \{x \to X, u \to U, z \to 2X, i \to 0, y \to 2X\}$

$0 < 2$

$\sigma = \{x \to X, u \to U, z \to X, i \to 1, y \to 2X\}$

$\sigma = \{x \to X, u \to U, z \to X + X, i \to 2, y \to 2X\}$

$\neg(2 > 2)$

$\sigma = \{x \to X, u \to U, z \to X + X, i \to 2, y \to 2X\}$

$\sigma = \{\ldots, z \to U > 0\ ?\ 2X : X + X, i \to U > 0\ ?\ 0 : 2, y \to 2X\}$

$2X = (U > 0\ ?\ 2X : X + X)$

Semantics of sketches

```
e  :=  n | x | e1 + e2 | ??
c  :=  x := e | c1 ; c2 | if e then c1 else c2 | while e do c
```

What does an expression mean?

- Like before, but values are "parameterized" by the valuation of the holes
- $\mathcal{A}[\![\cdot]\!] : e \to \Sigma \to (\mathbf{\Phi} \to \mathbb{Z})$

Ex:

- $\mathcal{A}[\![x]\!] = \lambda\sigma.\,\lambda\phi.\,\sigma[x]$
- $\mathcal{A}[\![??_i]\!] = \lambda\sigma.\,\lambda\phi.\,\phi[i]$
- $\mathcal{A}[\![e_1 + e_2]\!] = \lambda\sigma.\,\lambda\phi.\,\mathcal{A}[\![e_1]\!]\sigma\phi + \mathcal{A}[\![e_2]\!]\sigma\phi$

Symbolic Evaluation of Commands

Commands have two roles

- Modify the symbolic state
- Generate constraints

$$\mathcal{C}[\![\cdot]\!] : c \to \langle \Sigma, \Psi \rangle \to (\Sigma, \Psi)$$

Constraints on ϕ tables, e.g.
$\lambda\phi.\, \phi[1] > 3$

Symbolic Evaluation of Commands

Example: assignment and assertion

$$\mathcal{C}[\![x := e]\!]\langle\sigma, \psi\rangle = \langle\sigma[x \mapsto \mathcal{A}[\![e]\!]\sigma], \psi\rangle$$

$$\mathcal{C}[\![\textbf{assert}\ e]\!]\langle\sigma, \psi\rangle = \langle\sigma, \lambda\phi.\, \psi(\phi) \wedge \mathcal{A}[\![e]\!]\sigma\phi = 1\rangle$$

Symbolic Evaluation of Commands

Example: conditional

$$\mathcal{C}[\![\text{if } e \text{ then } c_1 \text{ else } c_2]\!]\langle\sigma,\psi\rangle = \left\langle \begin{array}{c} \lambda x.\, \mathcal{A}[\![e]\!]\sigma \;?\; \sigma_1[x] \;:\; \sigma_2[x], \\ \lambda\phi.\, \psi(\phi) \wedge \mathcal{A}[\![e]\!]\sigma \;?\; \psi_1(\phi) : \psi_2(\phi) \end{array} \right\rangle$$

where

$$\langle\sigma_1,\psi_1\rangle = \mathcal{C}\,[\![c_1]\!]\,\langle\sigma,\psi\rangle$$
$$\langle\sigma_2,\psi_2\rangle = \mathcal{C}\,[\![c_2]\!]\,\langle\sigma,\psi\rangle$$

Symbolic execution of sketches: example

```
harness void main(int x, int u){
  int z = 0; int i = 0;
  int y = ??₁ * x;
  if (u > 0) {
    z = 2 * x;
  } else {
    ...
  }
  assert y == z;
}
```

$U > 0$

$\sigma = \{x \to X, u \to U\}, \quad \psi = \top$

$\sigma = \{x \to X, u \to U, z \to 0, i \to 0, y \to \phi_1 * X\}, \quad \ldots$

$\sigma = \{x \to X, u \to U, z \to 2X, i \to 0, y \to \phi_1 * X\}$

$\sigma = \{x \to X, u \to U, z \to X + X, i \to 2, y \to \phi_1 * X\}$

$\sigma = \{\ldots, z \to U > 0\,?\,2X : X + X, i \to U > 0\,?\,0 : 2, y \to \phi_1 * X\}$

$\phi_1 * X = (U > 0\,?\,2X : X + X)$

$\sigma = \{\ldots\}, \quad \psi = (\phi_1 * X = (U > 0\,?\,2X : X + X))$

$\exists \phi_1 . \forall X\, U.\ \phi_1 * X = (U > 0\,?\,2X : X + X)$ —CEGIS→ $\{\phi_1 \mapsto 2\}$

Controls for generators

```
harness void main(int x, int y){
  z = mono(x)   +   mono(y);
  assert z == x + x + 3;
}

generator int mono(int x) {
    if (??₁) {return ??₂;}
    else {return x * mono(x);}
}
```

→

$$\sigma = \{z \to (\phi_1 \ ? \ \phi_2 : \ X * \phi_2) + \ (\phi_1 \ ? \ \phi_2 : Y * \phi_2)\}$$

We need to map different calls to **mono** to different controls!

Controls for generators: context

```
harness void main(int x, int y){
  z = mono[1](x,1) +  mono[2](y,2);
→ assert z == x + x + 3;
}

generator int mono(int x, context τ) {
    if (??τ_1) {return ??τ_2;}
    else {return x * mono[3](x, τ.3);}
}
```

$$\sigma = \{z \to (\phi_1^1 \; ? \; \phi_2^1 : \; X * \phi_2^{1.3}) + \; (\phi_1^2 \; ? \; \phi_2^2 : \; X * \phi_2^{2.3})\}$$

$$\{\phi_1^1 \mapsto 0, \phi_2^{1.3} \mapsto 2, \phi_1^2 \mapsto 1, \phi_2^{1.3} \mapsto 3\}$$

Sketch: contributions

Expressing structural and behavioral constraints as programs

- the only primitive extension is an integer hole ??
- why is it important to keep extensions minimal?

CEGIS

- became extremely popular; now used in most constraint-based synthesizers

Sketch: limitations

Everything is bounded

- loops are unrolled
- integers are bounded
- are any of the above easily fixable?

Unclear if sketching is a good user interaction model

- but: as search gets better, less user input is required

CEGIS relies on the Bounded Observation Hypothesis

- what about other techniques?

Sketches hard to debug

- what about other techniques?

Sketch: questions

Behavioral constraints? structural constraints? search strategy?

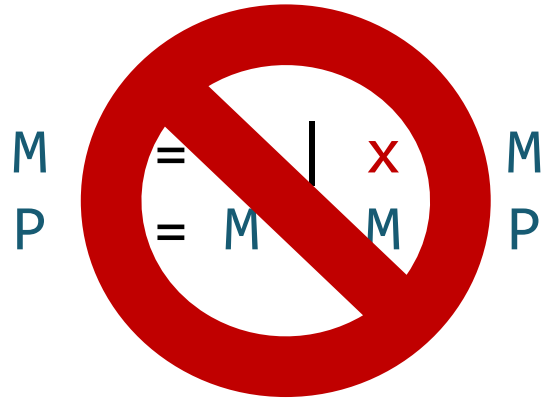
- assertions / reference implementation
- sketches
- constraint-based (CEGIS + SAT)

Sketches vs CFGs? Brahma's components?

- A generator can encode a multiset of components (although it's not very straightforward)
- Can a generator encode a CFG?

Recursive generators

What if monomial of every degree can occur at most once?




```
generator int mono(int x, int n) {
    if (n <= 0) {return ??;}
    else {return x * mono(x, n - 1);}
}

generator int poly(int x, int n) {
    if (n <= 0) {return mono(x,0);}
    else {return mono(x,n) + poly(x, n - 1);}
}
```

Generators are more expressive than CFGs!

- but unbounded generators cannot be encoded into constraints
- need to bound unrolling depth
- bounded generators less expressive than CFGs (but more convenient)

CEGIS: the worst case

Satisfiable constraint $\exists c.\, \forall x.\, Q(c, x)$ that violates the Bounded Observation Hypothesis

$$Q(c, x) \equiv (x = 000 \Rightarrow c = 111) \wedge \cdots \wedge (x = 111 \Rightarrow c = 000)$$

$$Q(c, x) \equiv x \oplus 101 = x \oplus c$$

$$Q(c, x) \equiv (c \,\&\, x \neq 000) \vee (x = 0)$$

$$Q(c, x) \equiv x \leq c$$

does not violate BOH,
but will require 2^N iterations with worst-case counterexamples

$$Q(c, x) \equiv c \neq x \vee c = 0$$

violates BOH:
no small set of counter-examples exists

Synthesis frameworks

[Torlak,Bodik ‘13]

Sketch

- Programming language + constant holes + generators + assertions
- Can use generator libraries to build a “synthesis DSL”

Rosette: takes it a step further

- Racket (metaprogramming!) + symbolic variables + solver queries
- Can define full-fledged SDSLs (Solver-aided DSLs)

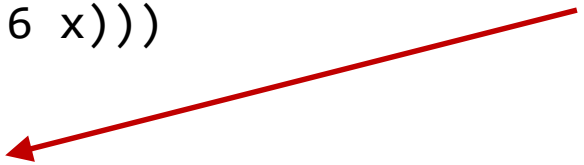
Rosette: example

```
(require rosette/lib/synthax)

(define (poly x)
 (+ (* x x x x) (* 6 x x x) (* 11 x x) (* 6 x)))

(define (factored x)
 (* (+ x (??)) (+ x 1) (+ x (??)) (+ x (??))))

(define (same p f x)
 (assert (= (p x) (f x))))
```




```
> (define-symbolic i integer?)

> (define binding
    (synthesize #:forall (list i)
                #:guarantee (same poly factored i)))
> (print-forms binding)
'(define (factored x) (* (+ x 0) (+ x 1) (+ x 2) (+ x 3)))
```