

## *Module 1, B-set o: Communications Breakdown and Review*

*Everything you should remember from ModSim, iSIM, and  
QEA I but don't..*

*September 11, 2016*

*Goals:*

By the end of this building block you should be able to:

1. Apply question deconstruction to an engineering system.
2. Understand satellite communications systems at a base level.  
(Don't worry too much about the math yet! That will come later.)
3. Articulate and apply key concepts and quantitative analysis skills from ModSim, iSIM, and QEA I.
4. Describe what a *difference equation* is.
5. Demonstrate a bad acoustic modem.

*Overview and Orientation:*

This Building Block has four chunks. The biggest chunk you to spend some serious time reviewing stuff from QEA last semester, from ModSim, and from iSIM. Expect to spend some time on this. We'd like you to spend some time doing a deconstruction of communications in a satellite context: what are the big ideas that one needs to learn in order to understand and create a communication system? There's a shortish section that introduces some notation and ideas around *difference equations*. And finally, we'd like you to finish up your bad modem from our kickoff day.

The purpose of this BSet is not to have an in-depth understanding of satellite communications, or to have every concept from previous classes crystal clear in your heads, but rather to get things kick-started a bit.

As with all BSets, there's a lot of stuff here. Ask for help early and often.

*Communication Breakdown ( 1.5 hours)*

The first thing we'd like you to do is spend a bit of time trying to just understand the "space" of communicating with space, by poking

around the interwebs and doing a question deconstruction. Start with the question of "what do I need to know in order to *build* and *understand* the operation of a satellite communication system?" Using the power of the internet, try to answer this question by identifying sub-questions and associated concepts. A great place to begin is the cubesat Wikipedia page.

Spend at least 30 minutes creating a concept map of a satellite communications system. As a reminder, the process of concept mapping goes as follows:

1. Write the key term at the top or in the center of a piece of butcher paper. Circle it, since you don't know it.
2. Research it, and identify terms that are immediately associated with it. Write them down and connect them with lines.
3. Label the lines to indicate what you understand the connection to be.
4. Circle new terms you don't understand, and break these down too.

Once you've created your map, identify 8-10 key underlying concepts that you think you will need to understand in order to be able to make sense out of how satellite communication works. By "underlying concepts," we mean more fundamental ideas as opposed to stuff that is specific to satellites: for example, "frequency" as opposed to "Ka Band". Write them down somewhere. You will discuss these next class with your table.

*Review of ModSim, iSIM, and QEA I (12 hours)*

So you've taken a number of classes at this point that we're going to be building on. The key ones for the work we'll do this semester are ModSim, iSIM, and QEA I. The following sections are meant to help you review these courses. Don't pretend to do the review. Find old notes, class websites, homeworks, projects, etc. Spend some time reviewing the material from each course, and then proceed to the following pages...

### ModSim Ideas

Here are some key ideas from ModSim:

- Modeling is a fundamental process for engineering: it allows one to predict the behavior of a physical system (either existing or proposed) in order to explain behaviors, to make predictions about future behaviors, and to make decisions about designs.
- The modeling process involves a number of steps: *abstraction*, *implementation*, *validation*, and *application*.
- Modeling is an iterative process.
- The type of model we build depends on the type of work we want to do with it.
- Abstraction involves deciding what type of model you will create, what effects you will include, and where your system boundaries are.
- Lots of models can be built on the principle of conservation of quantities like number, mass, energy, etc. by keeping track of how much stuff (stocks) there is, and how the stuff moves (flows).
- Many stock and flow models can be formulated as systems of differential equations.
- When you are implementing models based on differential equations, you can either approach the implementation numerically (by computer) or analytically (by hand) Numerical approaches can be used to find solutions in virtually any situation; analytical approaches provide insight, but are applicable only sometimes.
- Most engineering courses develop models that can be solved by hand. We have not done this, and have instead emphasized the use of numerical tools, such as ode45.
- Whilst mechanical systems can be formulated using stocks and flows, it is more usual to use free-body diagrams and Newton's laws to develop the model.

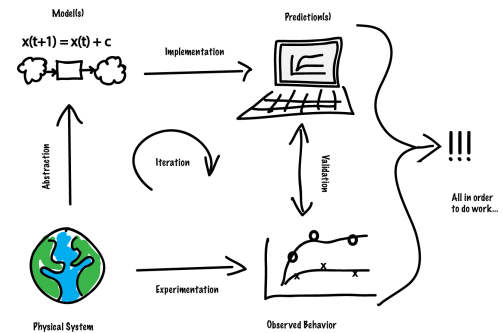


Figure 1: You've seen this before.

*ModSim Questions*

1. Walden Pond in Concord, MA is a glacial kettle-hole lake with no surface inlet or outlet — it is a flow-through lake. The mean depth is 12.9 meters. Water enters the lake at a rate of roughly 2.1 meters per year due to precipitation (45 %) and seepage from the ground water aquifer (55 %) on the eastern side. Water exits the lake at a rate of roughly 2.1 meters per year due to evaporation (26 %) and seepage to the ground water aquifer (74 %) on the western side. Build a model in order to determine how long it will take Walden Pond to dry out if precipitation is consistently 25% below normal.
2. Your team has decided to do a ModSim project on designing parachutes for HALO jumps (high altitude, low open). Propose the following:
  - (a) A possible “punchline graph” that you might use on your final poster.
  - (b) A first pass model, in the form of key frames, a set of appropriate free body diagrams, and a set of differential equations.
3. You’ve developed a model that has the following differential equations associated:

$$\begin{aligned}\dot{x} &= v \\ \dot{v} &= -\alpha x - \beta v^2\end{aligned}$$

where  $v$  is the velocity of the thing you are simulating,  $x$  is the position, and  $\alpha$  and  $\beta$  are physical constants.

- (a) What are the units of  $\alpha$  and  $\beta$ ?
- (b) Assume that (in whatever unit system you’ve chosen)  $\alpha = 1$  and  $\beta = 1$ ,  $x(0) = 1$ , and  $v(0) = 1$ . Using `ode45`, simulate the position and velocity of the object for an appropriate length of time.
4. Build a mathematical model in order to determine how long it will take freshly-brewed, piping-hot tea to cool to a comfortable drinking temperature when poured into a typical ceramic tea cup that is at room temperature.

*iSIM Ideas*

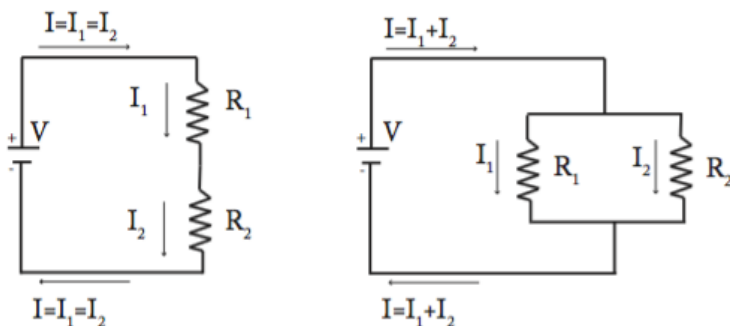
Here are some key ideas from iSIM:

- You can collect data from the world by measuring voltages and currents produced by sensors.
- Measuring these quantities requires signal conditioning, through the use of amplifiers and filters.
- One approach to the construction of amplifiers and filters is the use of op-amps, resistors, and capacitors.
- Resistors are characterized by Ohm's law,  $V = IR$ : the voltage drop across a resistor is equal to the current through the resistor times the resistance value. In a water flow analogy, a resistor is like a pipe, and higher resistance is equivalent to a narrower pipe.
- Capacitors are characterized by  $I = C \frac{dV}{dt}$ : the rate of increase of the voltage drop across a capacitor is proportional to the current flowing into the capacitor. In a water flow analogy, a capacitor is like a cylindrical tank, where the capacitance of the tank is analogous to its cross-sectional area.
- Operational amplifiers can nominally be thought of as allowing no current into the input terminals, and no voltage drop across the input terminals. The output voltage and output current take whatever values are necessary to satisfy these conditions.
- Application of Kirchhoff's laws allow one to generate a system of differential equations that characterize the currents and voltages in a circuit.
- You can represent a sinusoidal voltage or current with a complex exponential.
- The concept of complex impedance, combined with Kirchhoff's laws, allows one to generate a system of algebraic questions that characterize the currents and voltages in a circuit.
- One can use a Bode plot to express how a filter or amplifier behaves for a range of different input frequencies. The Bode plot tells you the "gain" (ratio of output signal amplitude to input signal amplitude) and the "phase" (phase difference between the output signal and input signal) over a range of frequencies.

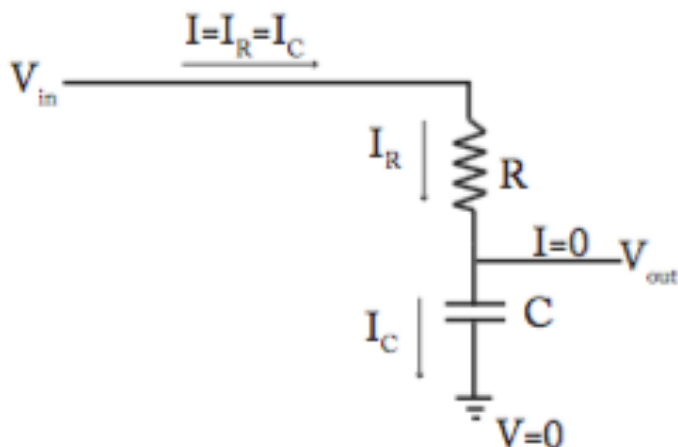
That is, so long as you don't ask the op-amp to provide a voltage greater than the supply voltage or an output current in excess of the op-amp's capacity.

*iSIM Questions*

1. Determine the current flowing in the following two circuits assuming that the power supply is supplying a steady 12 V and that the resistors are  $R_1 = 10k\Omega$  and  $R_2 = 20k\Omega$  respectively.



2. Determine the time-dependent output voltage of the following circuit if all voltages are initially set to 1 V and then suddenly the input voltage is pulled down to 0 V. Assume that  $R = 10k\Omega$  and  $C = 1\mu F$ .



3. Reduce the following complex number to the form  $a + bj$  ( $a$  and  $b$  are the real and imaginary parts respectively) and also to the form  $A \exp(j\theta)$  ( $A$  and  $\theta$  are the amplitude and phase respectively).

$$\frac{(1 + j)(3 + j)(-2 - j)}{(j)(3 + 4j)(5 + j)}$$

4. Make a plot of the complex plane showing the values (in the plane) of  $z(t) = e^{jt} - .3e^{j3t} + .2e^{j5t}$  for  $t = 0, t = \pi/4, t = \pi/2, t = 3\pi/4$ , and  $t = \pi$ . Then create a plot as a function of time of  $\text{Re}(z(t))$  for  $0 < t < 6\pi$ , and explain why it looks as it does.
5. Design a low-pass RC filter with a cut-off frequency of  $1\text{kHz}$ . Draw the circuit, and make a Bode plot (amplitude and phase) of its behavior.
6. You need to isolate a signal of frequency  $1\text{Hz}$  from a background that includes both a DC offset and high frequency noise. Design a circuit that will do this and explain its operation.



*Review of QEA I*

Good news. Rather than asking you to solve a number of problems using concepts from QEA I (which you of course took more recently than ModSim or iSIM), we're going to ask you something different.

1. Please create a bullet list of "big ideas" from QEA I that are analogous to the lists we generated above for ModSim and iSIM. Note that these bullet lists are lists of coherent sentences that identify ideas, and that relate to each other. They are not just bullet lists of content. *In other words, if you have a bullet that says only "eigenstuff", you're doing it wrong.*
2. Once you've created your list of sentences, generate *one* good question and an accompanying solution that gets at some subset of these ideas. Please go for something a bit substantial: a question like "evaluate the indefinite double integral  $\int \int xy \, dx \, dy$ " would be weak indeed.

## Difference Equations (2 hours)

This semester we will be working with difference equations as well as differential equations. You saw some examples of these in ModSim, but we didn't spend a lot of time on these formally. So we're going to ask you to spend a little while reading about and trying out some difference equation ideas...

A difference equation is an equation that recursively defines a sequence of numbers, once one or more initial numbers are given. Other terms for difference equation include **recurrence relation** or **map**. There are specific technical definitions of each of these, but we will use them somewhat interchangeably.

A simple example is probably the easiest way to highlight what a difference equation is. The difference equation

$$x_{n+1} = x_n + 3, \quad n = 0, 1, 2, \dots$$

will generate a sequence of numbers  $x_0, x_1, x_2, \dots$  where the next number is the previous number + 3. To start the sequence we need to provide a starting value or initial condition. If we choose  $x_0 = 5$ , then we generate the sequence 5, 8, 11, 14, 17, ... More generally, the  $n$ th term in the sequence is simply

$$x_n = x_0 + 3n$$

The difference equation

$$x_{n+1} = 2x_n, \quad n = 0, 1, 2, \dots$$

will generate a sequence where the next number is 2 times the previous number. If we choose  $x_0 = 3$  then we generate the sequence 3, 6, 12, 24, ... More generally, the  $n$ th term in this sequence is determined by the expression

$$x_n = 2^n x_0$$

1. Determine the formula for the  $n$ th term in the sequence generated by the difference equation

$$x_{n+1} = 2x_n + 3$$

In the first few examples, the next number in the sequence depends on the previous number. Difference equations might also depend on several previous numbers. For example, the difference equation

$$x_{n+2} = x_{n+1} + x_n, \quad n = 0, 1, 2, \dots$$

with the initial conditions  $x_0 = 1$  and  $x_1 = 1$  will generate the Fibonacci sequence 1, 1, 2, 3, 5, 8, ... Wouldn't it be cool if we could

find a formula for the  $n$ th term in the Fibonacci sequence? We can do precisely that, and one way to do it is to use eigenvalues and eigenvectors!

The key here is to create an equivalent problem using vectors. Let's define

$$\mathbf{w}_0 = \begin{pmatrix} x_1 \\ x_0 \end{pmatrix}, \mathbf{w}_1 = \begin{pmatrix} x_2 \\ x_1 \end{pmatrix}, \mathbf{w}_2 = \begin{pmatrix} x_3 \\ x_2 \end{pmatrix}, \dots$$

The governing difference equation can be written in terms of  $\mathbf{w}$  as

$$\mathbf{w}_{n+1} = A\mathbf{w}_n, \quad n = 0, 1, 2, \dots$$

where  $A$  is the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

2. Find the eigenvalues  $(\lambda_1, \lambda_2)$  and eigenvectors  $(\mathbf{v}_1, \mathbf{v}_2)$  of  $A$ .
3. Prove that the solution to the difference equation is

$$\mathbf{w}_n = c_1 \lambda_1^n \mathbf{v}_1 + c_2 \lambda_2^n \mathbf{v}_2$$

4. Determine  $c_1$  and  $c_2$  by using your eigenvectors and the initial conditions  $x_0 = 1$  and  $x_1 = 1$ .
5. Use your formula for Fibonacci numbers to determine the 100th number in the sequence.

### *Finish Your Bad Modem (4 hours)*

Your final piece of work for this building block is to try to finish the (bad) acoustic modem that you started on in class.

Some advice: “Modem” stands for modulator - demodulator. The key to getting your acoustic modem to work is going to be (1) being able to *modulate* your signal on the transmitter end (i.e., turn it into a signal that has appropriate frequency content to allow it to be sent acoustically and then (2) to *demodulate* your signal on the receiver end (i.e., turn the signal that is in the acoustic frequency range back into something you can interpret). There are a number of different strategies commonly used for modulating and demodulating, including amplitude modulation, frequency modulation, and phase modulation.

You’ll find that your work will be easier if you take some time to be explicit about what modulation and demodulation scheme you are using. It might be really simple, but if you’re at least clear about it your life will be easier.

A second issue that is likely to make life challenging is *handshaking*. Somehow your receiver computer needs to know when to start listening and when to stop. You might want to think about this problem as distinct from knowing how to interpret a signal once it’s been received.

This is not a final project. It won’t be perfect; if you don’t get handshaking to work that’s OK, etc. We are expecting you to spend about 4 hours out of class on this, so if you find yourself spending significantly more than that, you’re probably working too hard. It’s meant to get you thinking about the issues.