Bset 3: The power of LTI

In[1]:=
```
SetDirectory@NotebookDirectory[];
<< "../MMA library.m"
```

Solar house

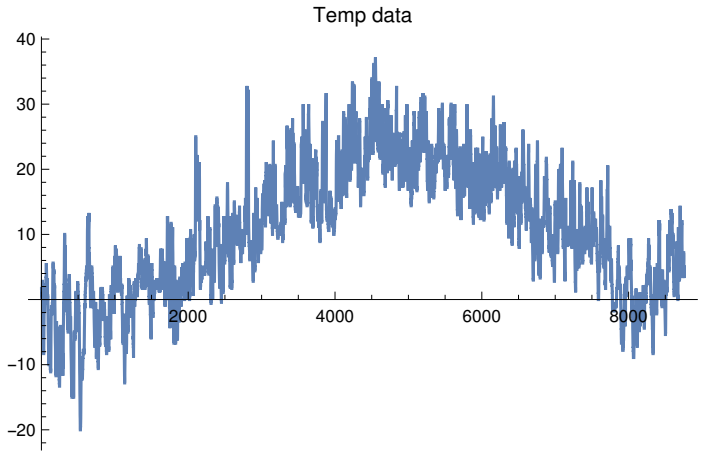
In[86]:=
```
With[{context = "p2`"}, If[Context[] ≠ context, Begin[context]]];
Dynamic[Refresh[Context[], UpdateInterval → 1]]
```

Out[86]= p2`

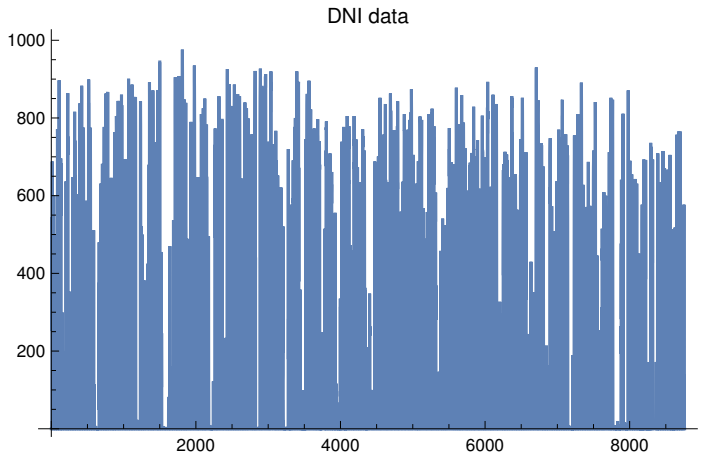
In[102]:=
```
Module[{tempdata = Import["provided files/BostonTempData.mat", "LabeledData"]},
 dni = ("dni" /. tempdata)[[All, 1]];
 hour = Round@("hour" /. tempdata)[[All, 1]];
 temp = ("temp" /. tempdata)[[All, 1]];]
```

In[9]:=
```
ListLinePlot[Transpose[{hour, temp}], PlotLabel → "Temp data"]
ListLinePlot[Transpose[{hour, dni}], PlotLabel → "DNI data", PlotRange → Full]
```

Out[9]=

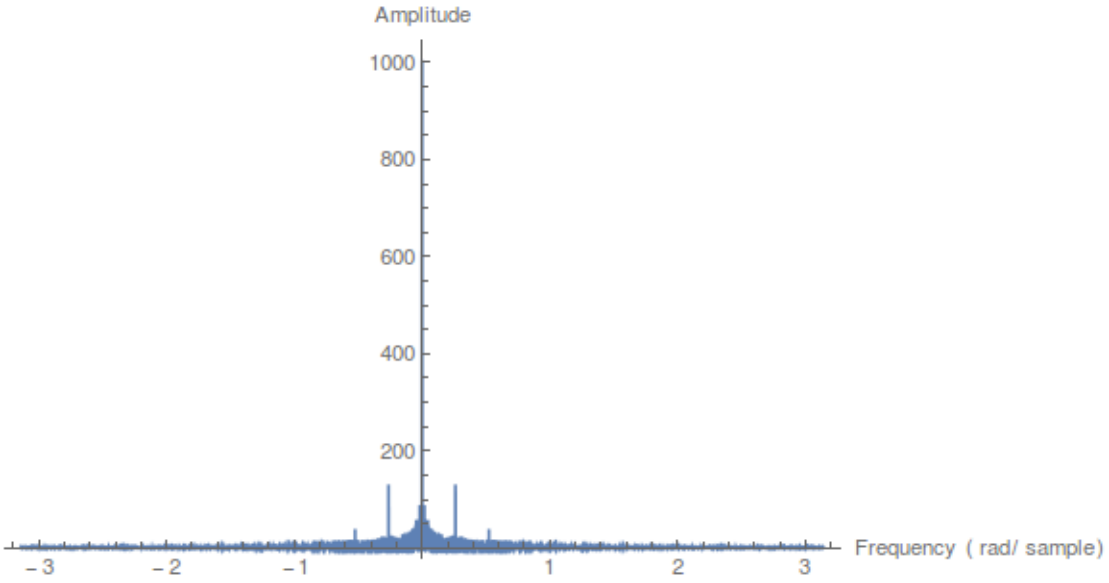


Out[10]=

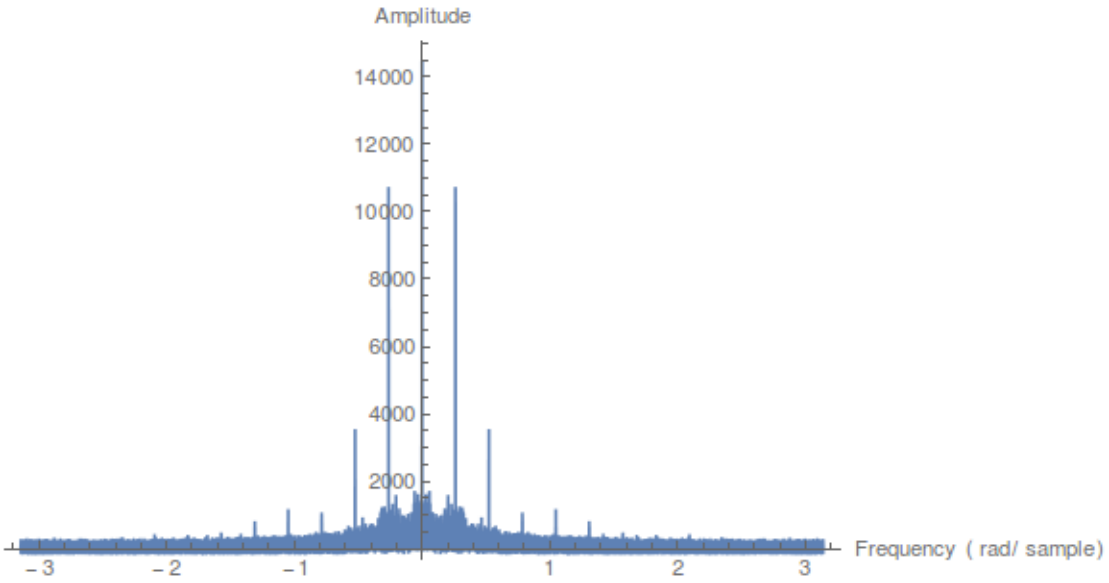


In[11]:=
```
plotFFT@temp
plotFFT@dni
```

Out[11]=



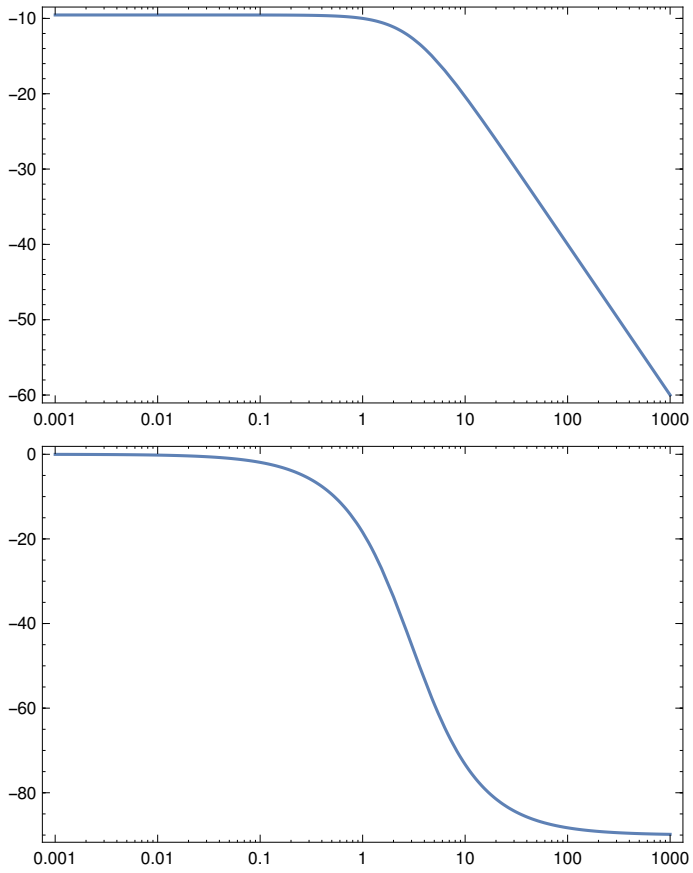
Out[12]=



Converting to Frequency Domain

In[13]:=
```
BodePlot[1 / (I ω + c) /. c → 3, {ω, 10^-3, 10^3}, ImageSize → Medium]
```

Out[13]=



In[91]:=
```
ch = 10^7;
kh = 1500;
```

Note that because of the units given, h[ω] is defined for ω in radians/second

In[93]:=
```
h[ω_] := 1 / (I ω + kh / ch)
```

In[94]:=
```
UnitSimplify[ 1 W / 1 J ]
```

Out[94]=
```
1 per second
```

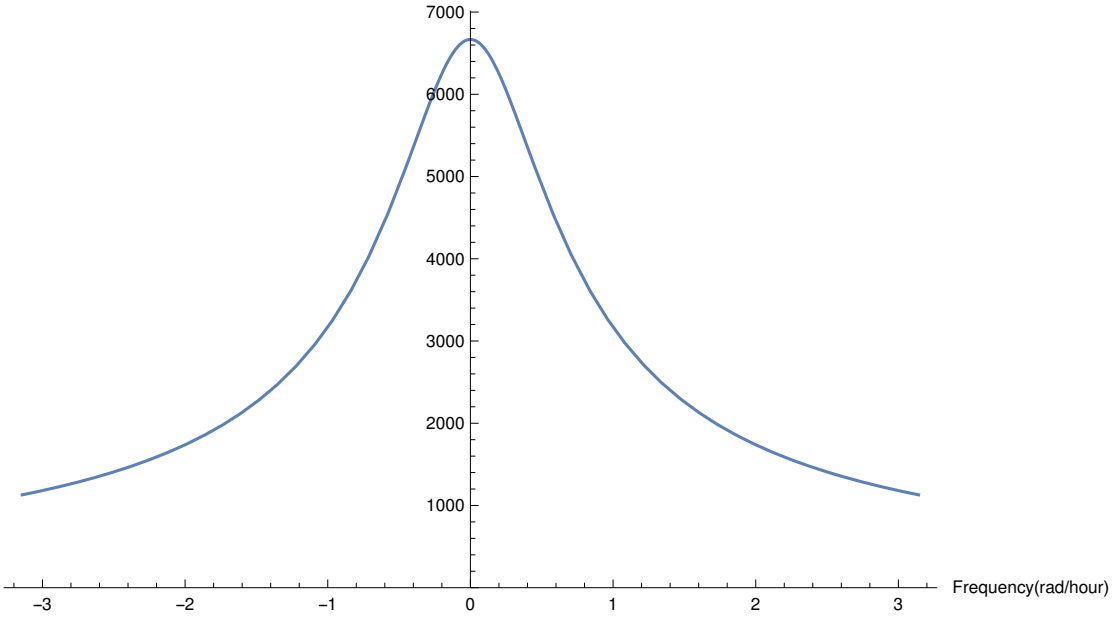
In[95]:=
```
N@h[0]
```

Out[95]=
```
6666.67
```

Transfer function plot (vs frequency in radians/hour)

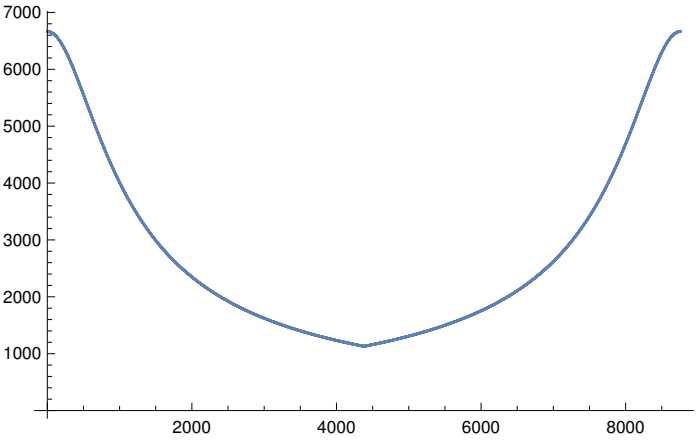
In[96]:=
```
scalar = UnitSimplify[1 s / 1 h];
Plot[Abs@h[ω * scalar], {ω, -Pi, Pi}, AxesLabel → {"Frequency(rad/hour)"},
 ImageSize → Large, PlotRange → {0, Automatic}]
```

Out[97]=



In[132]:=
```
hlist = RotateRight[h /@ (scalar * Subdivide[-Pi, Pi, Length@temp - 1]), Length@temp / 2];
ListPlot[Abs@hlist]
```

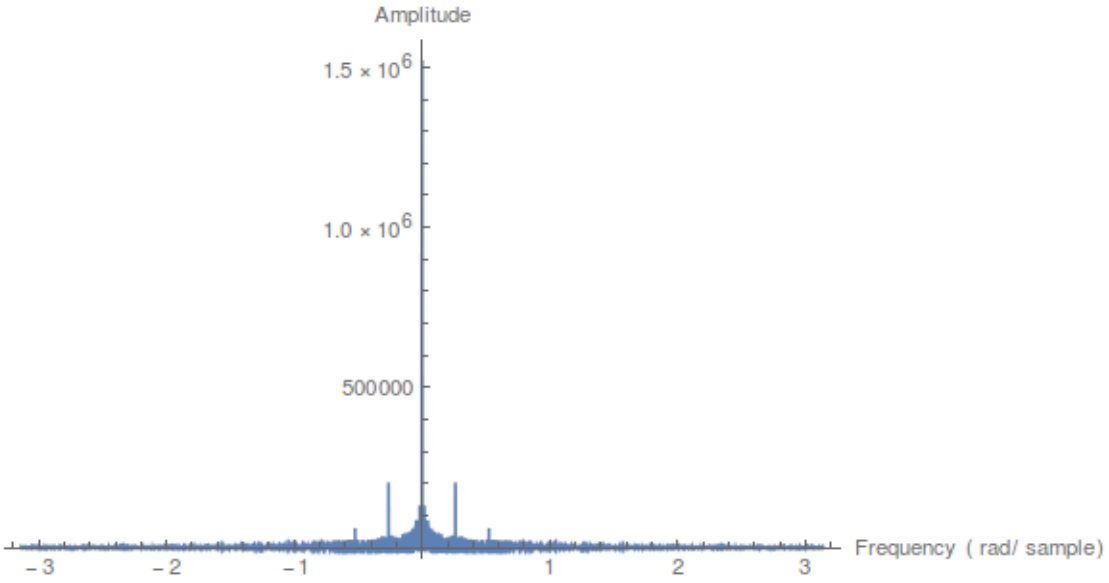
Out[133]=



Input data frequency domain plot

In[108]:=
```
x = dni + kh temp;
plotFFT@x
```

Out[109]=

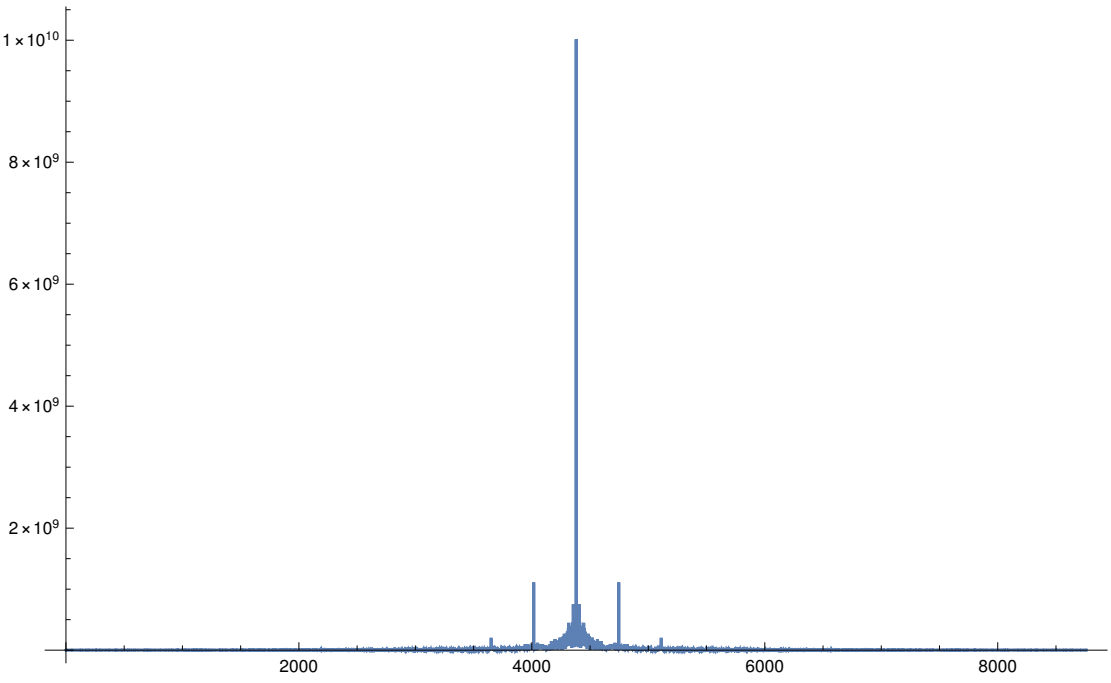


Compute and plot the product of the two

In[134]:=
```
f = Fourier[x];
Length@f
ftrans = hlist * f;
ListLinePlot[RotateRight[Abs@ftrans, Length@f / 2],
 PlotRange → Full, ImageSize → Large]
```

Out[135]= 8760

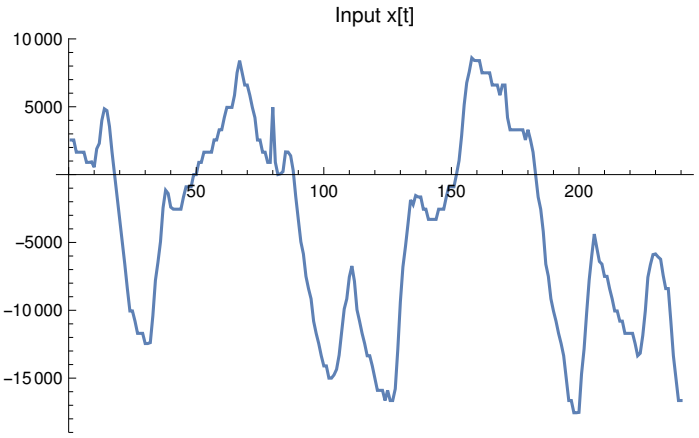
Out[137]=



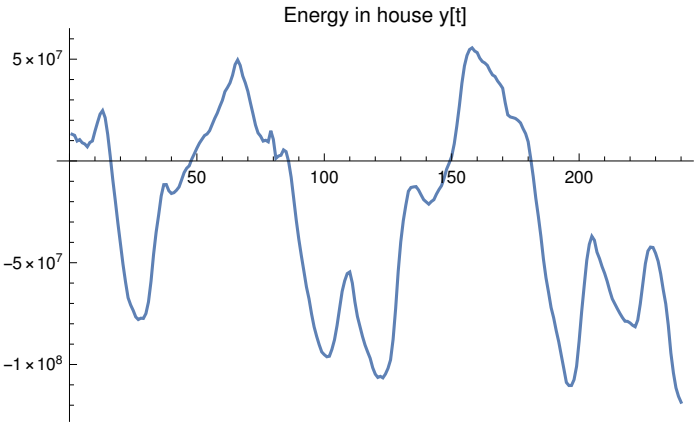
Convert back to time domain

In[141]:=
```
interiorEnergy = Re@InverseFourier@ftrans;
ListLinePlot[x[[1 ;; 24 * 10]], PlotLabel → "Input x[t]"]
ListLinePlot[interiorEnergy[[1 ;; 24 * 10]], PlotLabel → "Energy in house y[t]"]
```

Out[142]=



Out[143]=



In[27]:=
```
With[{context = "p2`"}, If[Context[] == context, End[], "Not in context"]]
```

Out[27]= p2`

Car problem

```
With[{context = "p34`"}, If[Context[] ≠ context, Begin[context]]];
Dynamic[Refresh[Context[], UpdateInterval → 1]]
```

p2`

Define differential equations for car motion

```
(de1 = m x''[t] == - k (x[t] - r[t]) - B D[x[t] - r[t], t]) // TraditionalForm
(de2 = r[t] == a Cos[2 Pi v t / L]) // TraditionalForm
```

$$m\,x''(t) = -k\,(x(t) - r(t)) - \mathrm{B}\,(x'(t) - r'(t))$$

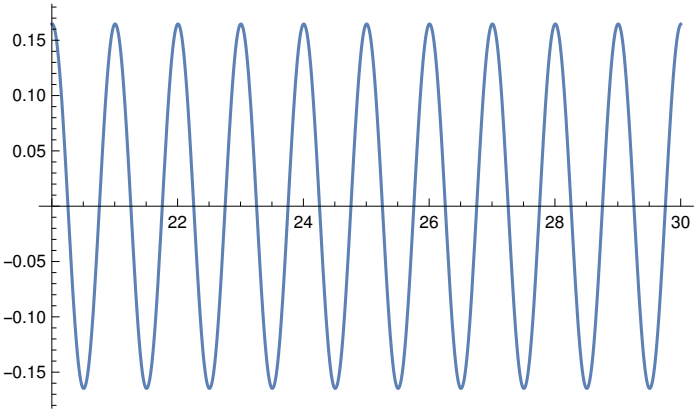
$$r(t) = a\cos\left(\frac{2\,\pi\,t\,v}{L}\right)$$

Setup values of declared variables

```
ClearAll@consts
consts[speed_] := <| a → 1/10, L → 10, v → speed, k → 10^5, B → 10^3, m -> 10^3|>
consts[] = consts[10];
```

Solve the Differential Equation for given values (Note, this is entirely symbolic)

```
sol = DSolveValue[{de1, de2, x'[0] == x[0] == 0} /. consts[], x[t], {t, 0, 30}];
Plot[sol /. t → x, {x, 20, 30}, PlotRange → Full]
fourier = FullSimplify@FourierTransform[sol, t, w]
```

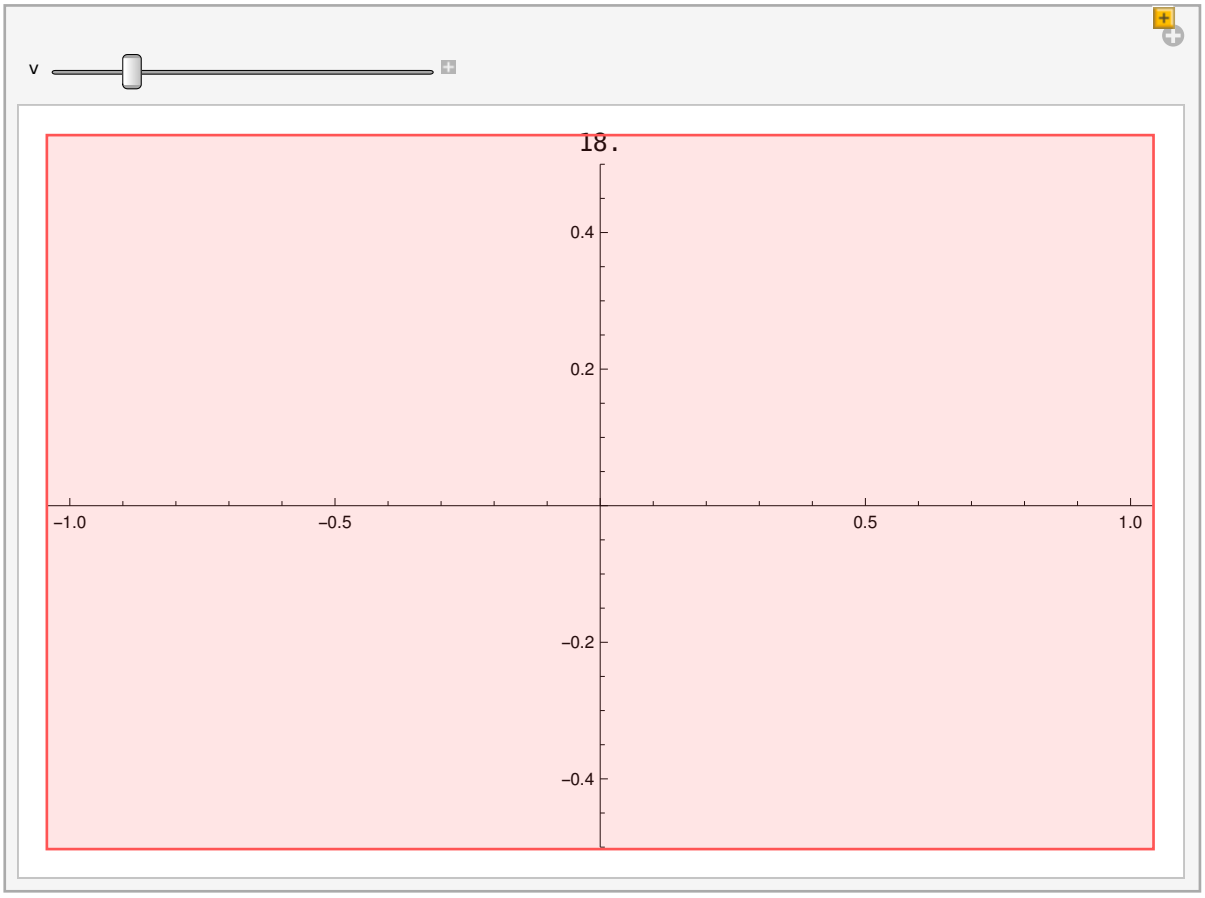


$$\frac{1}{3990\left(2500 - 199\,\pi^2 + 4\,\pi^4\right)}$$

$$\left(39\,501\sqrt{2}\;\pi^{5/2}\,\mathrm{DiracDelta}\left[-\mathrm{i} + \sqrt{399} - 2\,w\right] + 99\,\mathrm{i}\,\sqrt{798}\;\pi^{5/2}\,\mathrm{DiracDelta}\left[-\mathrm{i} + \sqrt{399} - 2\,w\right] - 8\,\mathrm{i}\,\sqrt{798}\;\pi^{9/2}\,\mathrm{DiracDelta}\left[-\mathrm{i} + \sqrt{399} - 2\,w\right] - 997\,500\sqrt{2\,\pi}\;\mathrm{DiracDelta}\left[-\mathrm{i} + \sqrt{399} - 2\,w\right] - 2500\,\mathrm{i}\,\sqrt{798\,\pi}\;\mathrm{DiracDelta}\left[-\mathrm{i} + \sqrt{399} - 2\,w\right] - \frac{39\,501\,\pi^{5/2}\,\mathrm{DiracDelta}[-2\,\pi + w]}{\sqrt{2}} + 399\,\mathrm{i}\,\sqrt{2}\;\pi^{7/2}\,\mathrm{DiracDelta}[-2\,\pi + w] + 498\,750\sqrt{2\,\pi}\;\mathrm{DiracDelta}[-2\,\pi + w] - \frac{39\,501\,\pi^{5/2}\,\mathrm{DiracDelta}[2\,\pi + w]}{\sqrt{2}} - 399\,\mathrm{i}\,\sqrt{2}\;\pi^{7/2}\,\mathrm{DiracDelta}[2\,\pi + w] + 498\,750\sqrt{2\,\pi}\;\mathrm{DiracDelta}[2\,\pi + w] + 39\,501\sqrt{2}\;\pi^{5/2}\,\mathrm{DiracDelta}\left[\mathrm{i} + \sqrt{399} + 2\,w\right] - 99\,\mathrm{i}\,\sqrt{798}\;\pi^{5/2}\,\mathrm{DiracDelta}\left[\mathrm{i} + \sqrt{399} + 2\,w\right] + 8\,\mathrm{i}\,\sqrt{798}\;\pi^{9/2}\,\mathrm{DiracDelta}\left[\mathrm{i} + \sqrt{399} + 2\,w\right] - 997\,500\sqrt{2\,\pi}\;\mathrm{DiracDelta}\left[\mathrm{i} + \sqrt{399} + 2\,w\right] + 2500\,\mathrm{i}\,\sqrt{798\,\pi}\;\mathrm{DiracDelta}\left[\mathrm{i} + \sqrt{399} + 2\,w\right]\right)$$

Play with the value of v (Note, this is numeric because it makes the manipulate[] faster.

```
Manipulate[
 sol = NDSolveValue[{de1, de2, x'[0] == x[0] == 0} /. consts[v], x[t], {t, 0, 30}];
 Grid[
  {{v}, {Plot[sol /. t → x, {x, 20, 30}, ImageSize → Large, PlotRange → .5 {-1, 1}]}}],
 {v,
  0,
  100}]
```



... NDSolveValue: No functions were specified for output from NDSolveValue.

... ConstantArray: Single or list of non–negative machine–sized integers expected at position 2 of ConstantArray[{r}, −∞].

... Take: Cannot take positions 1 through 2 in {False}.

... Transpose: The first two levels of {const e, i const t ω} cannot be transposed.

... Part: Part 2 of Transpose[{const e, i const t ω}] does not exist.

... Transpose: The first two levels of {0, 0} cannot be transposed.

... Part: Part 2 of Transpose[{0, 0}] does not exist.

... Set: Part 2 of Transpose[{const e, i const t ω}] does not exist.

... Rule: Rule called with 1 argument; 2 arguments are expected.

... Rule: Rule called with 1 argument; 2 arguments are expected.

Rule: Rule called with 1 argument; 2 arguments are expected.

General: Further output of Rule::argr will be suppressed during this calculation.

NDSolveValue: No functions were specified for output from NDSolveValue.

ConstantArray: Single or list of non–negative machine–sized integers expected at position 2 of ConstantArray[{r}, −∞].

Take: Cannot take positions 1 through 2 in {False}.

Transpose: The first two levels of {c e, ī c t ω} cannot be transposed.

Part: Part 2 of Transpose[{c e, ī c t ω}] does not exist.

Transpose: The first two levels of {0, 0} cannot be transposed.

Part: Part 2 of Transpose[{0, 0}] does not exist.

Set: Part 2 of Transpose[{c e, ī c t ω}] does not exist.

Rule: Rule called with 1 argument; 2 arguments are expected.

Rule: Rule called with 1 argument; 2 arguments are expected.

Rule: Rule called with 1 argument; 2 arguments are expected.

General: Further output of Rule::argr will be suppressed during this calculation.

ReplaceAll: {consts[18.]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

NDSolveValue: Equation or list of equations expected instead of ≪1≫ in the first argument ≪1≫.

NDSolveValue: 20.000204285714286` cannot be used as a variable.

ReplaceAll: {consts[18.]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

NDSolveValue: 20.000204285714286` cannot be used as a variable.

NDSolveValue: 20.204285918367347` cannot be used as a variable.

General: Further output of NDSolveValue::dsvar will be suppressed during this calculation.

ReplaceAll: {consts[18.]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General: Further output of ReplaceAll::reps will be suppressed during this calculation.

Problem 4: Frequency-domain analysis

de1 // TraditionalForm

$$m\,x''(t) = -k\,(x(t) - r(t)) - \mathrm{B}\,(x'(t) - r'(t))$$

Verify the transfer function provided for x[t]

```
Block[{r, x},
 r[t_] := E^(I * t * ω);
 x[t_] := (k + I ω B) / (k + I ω B - m ω^2) Exp[I ω t];
 de1 // TraditionalForm]
FullSimplify@%
```

$$-\frac{m\,\omega^2\,e^{i\,t\,\omega}\,(k+i\,\mathrm{B}\,\omega)}{i\,\mathrm{B}\,\omega+k-m\,\omega^2}=-k\left(\frac{e^{i\,t\,\omega}\,(k+i\,\mathrm{B}\,\omega)}{i\,\mathrm{B}\,\omega+k-m\,\omega^2}-e^{i\,t\,\omega}\right)-\mathrm{B}\left(\frac{i\,\omega\,e^{i\,t\,\omega}\,(k+i\,\mathrm{B}\,\omega)}{i\,\mathrm{B}\,\omega+k-m\,\omega^2}-i\,\omega\,e^{i\,t\,\omega}\right)$$

True

Generate a transfer function for x[t]-r[t]

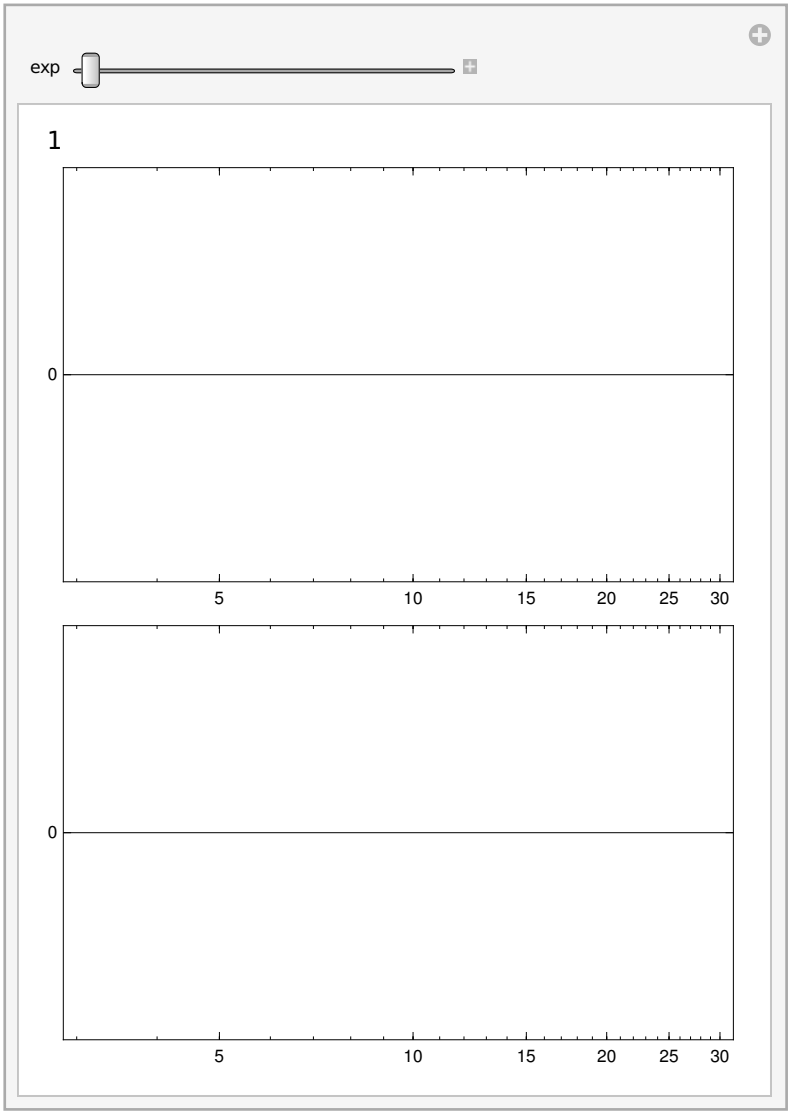
```
Block[{r, y},
 (*r[t_] := E^(I*t*ω);
 x[t_]:=c E^(I*t*ω);
 y[t_]:=x[t]-r[t];*)
 DSolveValue[de1, (x[t] - r[t]) / r[t], t]]
transferfunc3 = FullSimplify[% /. C[_] → 0]
ExportString[%, "TeXFragment"]
```

$$\frac{c\,e^{i\,t\,\omega}-\frac{c\,e^{-\frac{k\,t}{B}+\frac{t\,(k+i\,B\,\omega)}{B}}\,\left(k+\omega\,(i\,B-m\,\omega)\right)}{k+i\,B\,\omega}-e^{-\frac{k\,t}{B}}\,C[1]}{\frac{c\,e^{-\frac{k\,t}{B}+\frac{t\,(k+i\,B\,\omega)}{B}}\,\left(k+\omega\,(i\,B-m\,\omega)\right)}{k+i\,B\,\omega}+e^{-\frac{k\,t}{B}}\,C[1]}$$

$$\frac{m\,\omega^2}{k+i\,B\,\omega-m\,\omega^2}$$

```
\[\frac{m \omega ^2}{k+i B \omega -m \omega ^2}\]
```

```
Manipulate[β = 10^exp;
 Column@{β, BodePlot[(transferfunc3 /. B → β) /. consts[],
    {ω, 3, 30}, ImageSize → Medium, PlotRange → Full]}, {exp, 0, 5}]
```



ReplaceAll: {consts[]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll: {consts[]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

"

```
(transferfunc3 /. B → 10)
```

$$\frac{m\,\omega^2}{k + 10\,i\,\omega - m\,\omega^2}$$

```
With[{context = "p34`"}, If[Context[] == context, End[], "Not in context"]]
```

```
Not in context
```

Scratch work

In[144]:=
```
exportNotebookPDF[]
```

```
/home/eric/Documents/School/QEA2/Acoustic Modem/Bset 2/Mathematica scratch.pdf
```