Driven systems

In[1]:=
```
SetDirectory@NotebookDirectory[];
<< "../MMA library.m"
```

Problem 1: Solve DiffEQ

```
With[{context = "p1`"}, If[Context[] ≠ context, Begin[context]]];
Dynamic[Refresh[Context[], UpdateInterval → 1]]
```

p2`

```
eqn = y'[t] + 2 y[t] == x[t]
drive = x[t] == UnitStep[t]
initial = y[0] == 0
```

2 y[t] + y′[t] == x[t]

x[t] == UnitStep[t]

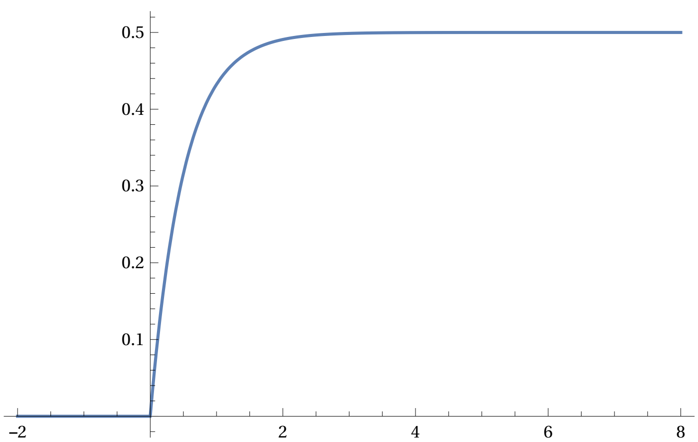
y[0] == 0

Approach 1: Full Mathematica

```
sol = Simplify@DSolveValue[{eqn, drive, initial}, y[t], t]
```

$\frac{1}{2} e^{-2t} \left(-1 + e^{2t}\right)$ UnitStep[t]

```
Plot[sol, {t, -2, 8.}]
```



Approach 2: Mathematica Laplace

```
lap = LaplaceTransform[eqn, t, s]
lap2 = Solve[lap, LaplaceTransform[y[t], t, s]][[1, 1, 2]]
lap3 = lap2 /. {y[0] → 0, x[t_] :> UnitStep[t]}
sol2 = InverseLaplaceTransform[lap3, s, t]
```

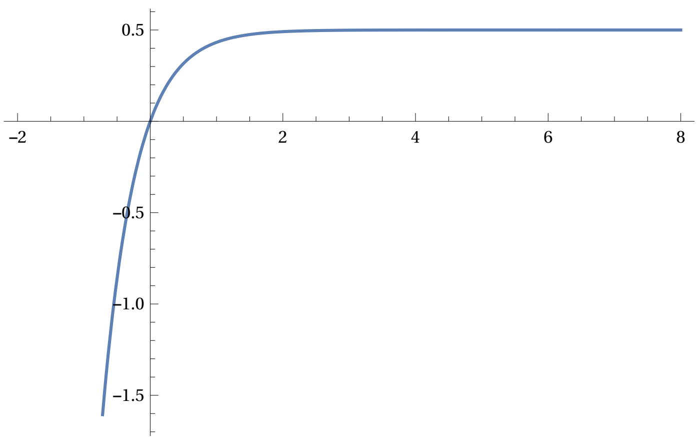
2 LaplaceTransform[y[t], t, s] + s LaplaceTransform[y[t], t, s] − y[0] == LaplaceTransform[x[t], t, s]

$$\frac{\text{LaplaceTransform}[x[t], t, s] + y[0]}{2 + s}$$

$$\frac{1}{s\,(2 + s)}$$

$$\frac{1}{2} - \frac{e^{-2t}}{2}$$

```
Plot[sol2, {t, -2., 8.}]
```




```
Simplify[sol == sol2]
```

$e^{-2t} == 1 \,||\, t \geq 0$

```
With[{context = "p1`"}, If[Context[] == context, End[]]];
Dynamic[Refresh[Context[], UpdateInterval → 1]]
```

p2`

Car Suspension

In[3]:=
```
With[{context = "p2`"}, If[Context[] ≠ context, Begin[context]]];
Dynamic[Refresh[Context[], UpdateInterval → 1]]
```

Out[3]= p2`

Setup

In[91]:=
```
eqn = m y''[t] + c y'[t] + k y[t] == c x'[t] + k x[t]
drive = {x[t_] :→ UnitStep[t]};
```

Out[91]= k y[t] + c y′[t] + m y″[t] == k x[t] + c x′[t]

In[112]:=
```
initial = {y[0] → 0, x[0] → 0, y'[0] → 0};
```

a) Laplace

In[113]:=
```
lap = LaplaceTransform[eqn, t, s];
(lap2 = Solve[lap /. initial /. drive, LaplaceTransform[y[t], t, s]]) //
 TraditionalForm
lap3 = lap2[[1, 1, 2]]
```

Out[114]//TraditionalForm=

$$\left\{\left\{\mathcal{L}_t[y(t)](s) \to \frac{c\,s+k}{s\left(c\,s+k+m\,s^2\right)}\right\}\right\}$$

Out[115]=

$$\frac{k+c\,s}{s\,\left(k+c\,s+m\,s^2\right)}$$

c) Solve it

Setup desired roots

In[37]:=
```
Roots[m s^2 + c s + k == 0, s]
```

Out[37]=

$$s == \frac{-c-\sqrt{c^2-4\,k\,m}}{2\,m}\ ||\ s == \frac{-c+\sqrt{c^2-4\,k\,m}}{2\,m}$$

In[38]:=

$$\text{roots} = \left\{r1 == \frac{-c-\sqrt{c^2-4\,k\,m}}{2\,m},\ r2 == \frac{-c+\sqrt{c^2-4\,k\,m}}{2\,m}\right\}$$

Out[38]=

$$\left\{r1 == \frac{-c-\sqrt{c^2-4\,k\,m}}{2\,m},\ r2 == \frac{-c+\sqrt{c^2-4\,k\,m}}{2\,m}\right\}$$

Approach 1: Pure Mathematica

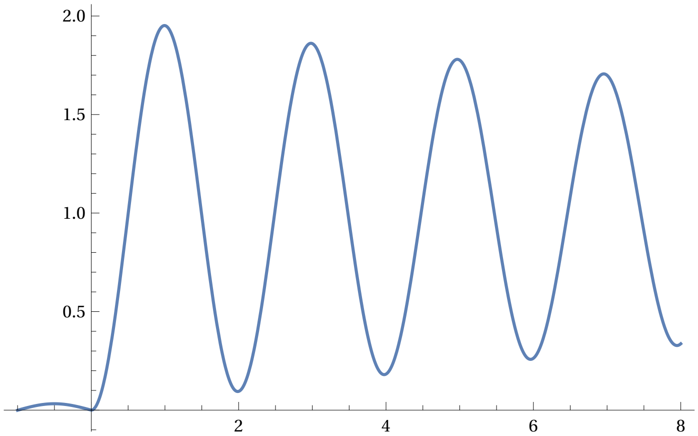
In[62]:=
```
initial2 = {y[0] == 0, y'[0] == 0};
```

In[76]:=
```
sol1 = Simplify@DSolveValue[{eqn, x[t] == UnitStep[t], initial2}, y[t], t]
```

Out[76]=

$$\frac{1}{2\sqrt{c^4-4\,c^2\,k\,m}}\,e^{-\frac{\left(c^2+\sqrt{c^4-4\,c^2\,k\,m}\right)t}{2\,c\,m}}\left(-2\,c^2\left(-1+e^{\frac{\sqrt{c^4-4\,c^2\,k\,m}\;t}{c\,m}}\right)-\left(-c^2\left(-1+e^{\frac{\sqrt{c^4-4\,c^2\,k\,m}\;t}{c\,m}}\right)+\left(1+e^{\frac{\sqrt{c^4-4\,c^2\,k\,m}\;t}{c\,m}}-2\,e^{\frac{\left(c^2+\sqrt{c^4-4\,c^2\,k\,m}\right)t}{2\,c\,m}}\right)\sqrt{c^4-4\,c^2\,k\,m}\right)\text{UnitStep}[t]\right)$$

In[173]:=
```
plot1 = makePlot[sol1, {m → 1000, c → 100, k → 10 000}]
```

Out[173]=


Approach 2: From Laplace

In[64]:= $\texttt{lap4} = \dfrac{k + c\, s}{s\,(s - a_1)\,(s - a_2)};$

$\texttt{sol2 = InverseLaplaceTransform[lap4, s, t]}$

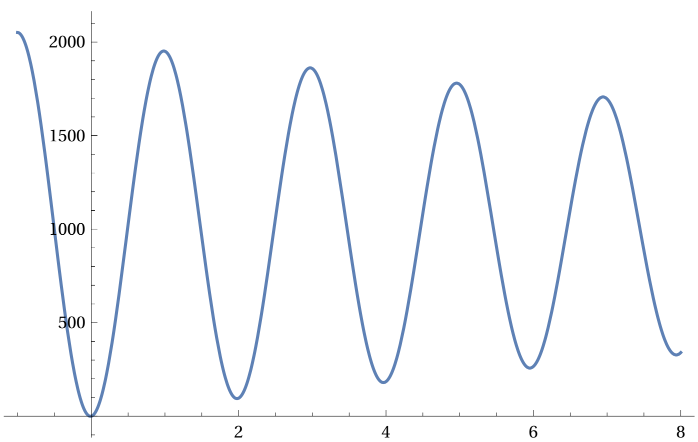
Out[65]= $$\frac{e^{t\, a_1}\,(k + c\, a_1)}{a_1\,(a_1 - a_2)} + \frac{k}{a_1\, a_2} + \frac{e^{t\, a_2}\,(-k - c\, a_2)}{(a_1 - a_2)\, a_2}$$

Plot it

In[168]:= $\texttt{makePlot[sol\_, params\_] := With}\Big[\Big\{\texttt{roots} = \Big\{a_1 \to \dfrac{-c - \sqrt{c^2 - 4\,k\,m}}{2\,m},\ a_2 \to \dfrac{-c + \sqrt{c^2 - 4\,k\,m}}{2\,m}\Big\}\Big\},$

$\quad \texttt{Plot[sol /. roots /. params, \{t, -1, 8\}]}\Big]$

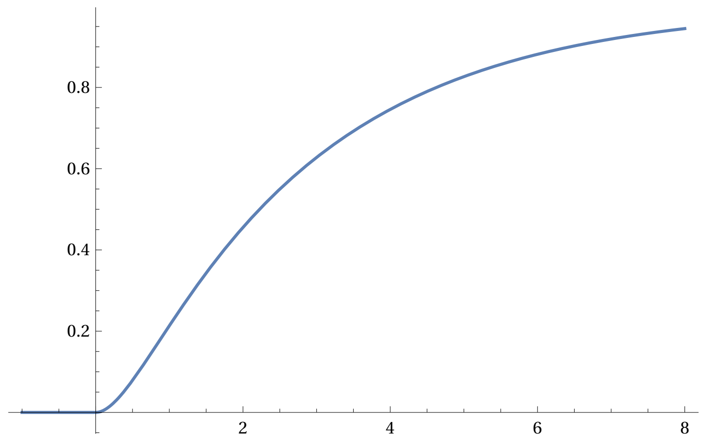
In[169]:=
```
makePlot[sol2, {m → 1000, c → 100, k → 10 000}]
```

Out[169]=


Play with it

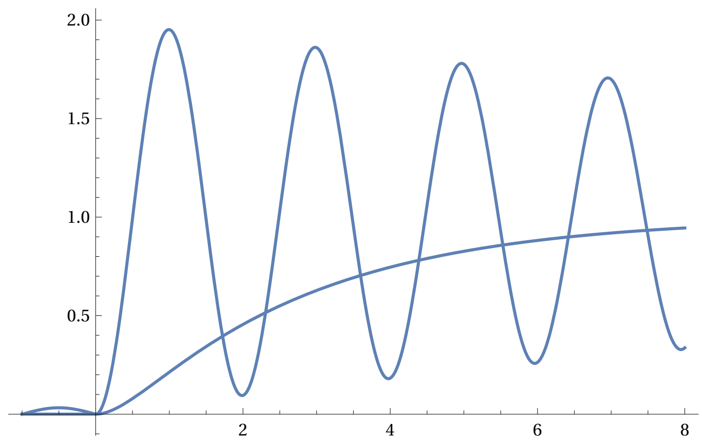
In[170]:=
```
plot2 = makePlot[sol1 * UnitStep[t], {m → 1000, c → 3000, k → 1000}]
```

Out[170]=



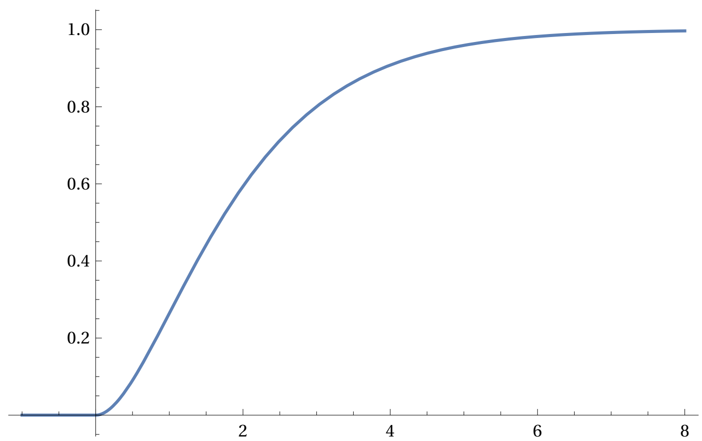
In[174]:=
```
Show[plot1, plot2]
```

Out[174]=



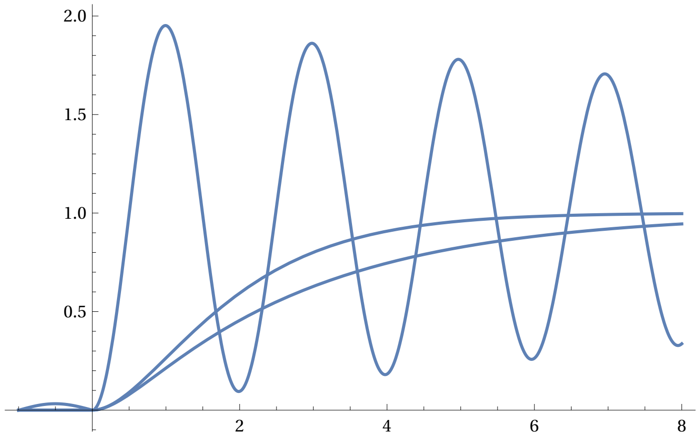
In[181]:=
```
plot3 = makePlot[sol1 * UnitStep[t], {m → 1000, c → 2001, k → 1000}]
```

Out[181]=



In[182]:=
```
Show[plot1, plot2, plot3]
```

Out[182]=




```
With[{context = "p2`"}, If[Context[] == context, End[]]];
Dynamic[Refresh[Context[], UpdateInterval → 1]]
```

Scratch Work

In[183]:=
```
exportNotebookPDF[]
```