Module 3: Integrative Problems on Governing Equations

Due Thursday, Oct. 20, 2016

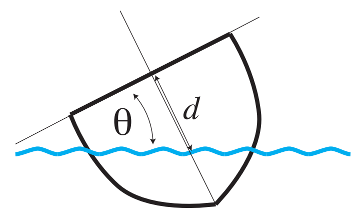
These are *integrative* problems. We would like you to be very intentional about taking a step-by-step approach to solving them, so that you are explicitly thinking about each of the steps we talked about in class on Monday.

Oh, no! Not a Boat!

You may recall from a little while ago the idea of a boat's center of buoyancy, displacement, etc. Yeah, it's rusty, we know.

Well, it's time to dust off that rust. We'd like you to develop a model for how a boat rocks and/or bobs up and down in the water.

For the sake of making life a little more specific, here's some information that has already been determined for you by experimentation:



1. The water displaced by the boat depends only on the location of the waterline:
$$V(d) = A(2-d)^2$$
where $A = 5$ meters, and d is the distance from the deck to the water as measured along the center of the boat. Note that $0 < d < 2$: when $d < 0$, the boat is completely submerged, and when $d = 2$, the boat is not in the water at all.

2. The location of the center of buoyancy moves according to both θ and d: the horizontal distance measured from the center of mass to the center of buoyancy is given by
$$RA(d,\theta) = C(2-d)sin(2\theta)$$
where $C = 0.5$.

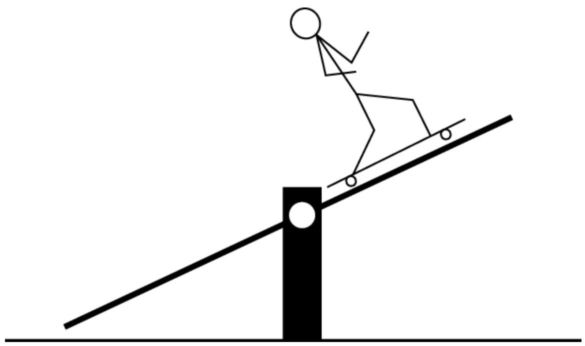
3. The boat floats with a natural waterline of $d = d_0 = 1$ meter.

Develop three different mathematical models (sets of governing equations) for the boat: one that allows you to make predictions about how the boat might bob up and down if it were staying flat in the water; one that makes predictions about how if might rock back and forth if it were not bobbing up and down, and one that can do both things at the same time.

If you feel so inclined, try simulating this – but that's in the bonus category. We mostly just want you to be confident about the governing equations that you derive.

Skateboards Again??

The reading includes a pretty darn complete derivation of the equations of motion for the skateboard on the rotating ramp. But let's say you wanted to be able to put a motor on the skateboard, so that the rider can speed up (or slow down) as they see appropriate. Develop the governing equations for this situation.



What?!? Pendulums are actually useful?

Why doesn't a Segway tip over (forward) when someone rides it leaning forward?

1. First, generate a simple equation of motion whose solution can answer a more basic question-if the Segway is not moving, will you tip forward when leaning forward? Start out by considering motion to lie in a vertical plane (gravity acts downward in the veritcal plane) and you, the rider, to be represented by a single point mass, m. Doesn't that sound like a pendulum-an inverted pendulum?

2. Now allow the pivot point (base of the Segway) to move forwards or backwards in a specific, known manner. Modify the equation of motion to capture this behavior.

3. Now consider the base of the Segway to be a rigid mass, M, that can move forward/backward as an unknown function of time (*e.g.*, $x(t)$). You should also assume that there is a motor fixed to the base which drives the wheels. The rider can specify (*i.e.*, input) the torque (moment) generated by the motor. Write equations of motion that govern the behavior of M and m.