Setup

3

4

5: Masses on springs

Problem analysis

Let `k` be the spring constant, and the bar be massless.

```
l0 = Sqrt[1.5 m ^2 + 2 m ^2]
l = Sqrt[2 m ^2 + (1.5 m + d) ^2]
```

2.5 m

$$\sqrt{\left(d + 1.5\ m\right)^2 + 4\ m^2}$$

```
F = k (l - l0);
Fy = F (1.5 m + d) / l;
```

Total force (weight) of one block

```
Fy /. d -> 2 m
```

$k \left(1.32939\ m\right)$

6: Wing

(a)

Find length of wing

In[8]:= `L[x_] = 200 Sqrt[1 - x^2 / 17];`
`Solve[L[x] == 0, x]`

Out[9]= $\left\{\left\{x \rightarrow -\sqrt{17}\right\}, \left\{x \rightarrow \sqrt{17}\right\}\right\}$

Set up forces and radii

In[10]:= `Fg = {0, -1600, 0}` ☐ N ⋯ ✓ `;`

`Rg = {Sqrt[17] / 2, 0, 0}` ☐ m ⋯ ✓ `;`

`Flift = {0, 50 Sqrt[17] Pi, 0}` ☐ N ⋯ ✓ `;`

`Rlift = {4 Sqrt[17] / (3 Pi), 0, 0}` ☐ m ⋯ ✓ `;`

Solve newton's 2nd law equations

In[14]:= `N@Solve[{Flift + Fg + {0, Freact, 0} == 0,`
`{0, 0, Treact} + Rg × Fg + Rlift × Flift == 0}, {Freact, Treact}]`

Out[14]= $\left\{\left\{\text{Freact} \rightarrow 952.344 \, \text{N}, \text{Treact} \rightarrow 2165.15 \, \text{J}\right\}\right\}$

(b)

Problem analysis

Basic assumptions: There are no structural loads being carried as torques in the joints...
The strut is mounted *d* meters below the wing mount

Forces

`Freact = {Freactx, Freacty, 0};`
`Fstrut = Fstrutl * -Normalize[{2, d, 0}];`
`Rstrut = {2 m, 0, 0};`

General Solution

In[18]:= `sol = Solve[{Flift + Fg + Freact + Fstrut == 0, Rstrut × Fstrut + Rg × Fg + Rlift × Flift == 0},`
`{Freactx, Freacty, Fstrutl}][[1]];`

Final result:

In[19]:= `N@niceForm[Freact /. sol]`
`N@(Fstrutl /. sol)`

Out[19]//TraditionalForm=

$$\frac{\hat{i}\,(-2165.15 \, \text{N})}{d} + \hat{j}\,(-130.232 \, \text{N})$$

Out[20]=

$$\frac{\sqrt{4. + \text{Abs}[d]^2}\,\left(-1082.58 \, \text{N}\right)}{d}$$

Specific solution

Assuming that $d = 1\,\text{m}$ yields the following specific result:

In[21]:= `N@niceForm[Freact /. sol /. d → 1]`
`N@(Fstrutl /. sol /. d → 1)`

Out[21]//TraditionalForm=

$\hat{i}\,(-2165.15\,\text{N}) + \hat{j}\,(-130.232\,\text{N})$

Out[22]= $-2420.71\,\text{N}$

7: Floating block

Problem Analysis

This problem has the trivial case, solved here, where the block is a cube. Another (pair of) solutions exist, but calculating them is an exercise for another day.

Forces

`Flead = {0, -(m - (m/` lead (element) `[EntityProperty["Element", "Density"]]) *`

water (chemical) `[` density `]) * g, 0};`

`Fg = {0, -(10 cm * 10 cm * d * 20 kg/m³ * g), 0};`

`Fbuoyancy = {0, (10 cm * 10 cm * d *` water (chemical) `[` density `]/2 * g), 0};`

Solve Equations of Motion

`Solve[(Flead + Fg + Fbuoyancy == 0) /. d -> 10 cm, {m}]`

$\{\{m \to 526.422\ g\}\}$

8: Beam

Problem Analysis

Definitions:
- Origin is at left support
- x right, y up

Calculating equivalent forces

```
Module[{f},
 f[x_] := {0, -Piecewise[{{200, x < 4}, {200 (7 - x) / 3, x ≥ 4}}], 0};
 Fload = Integrate[f[x], {x, -3, 7}];
 Rload = Integrate[Norm@f[x] {x, 0, 0}, {x, -3, 7}] / Norm[Fload];
]
```

Setting up forces

```
Fpin = {0, Fpiny, 0};
Froller = {0, Frollery, 0};
Rroller = {7, 0, 0};
```
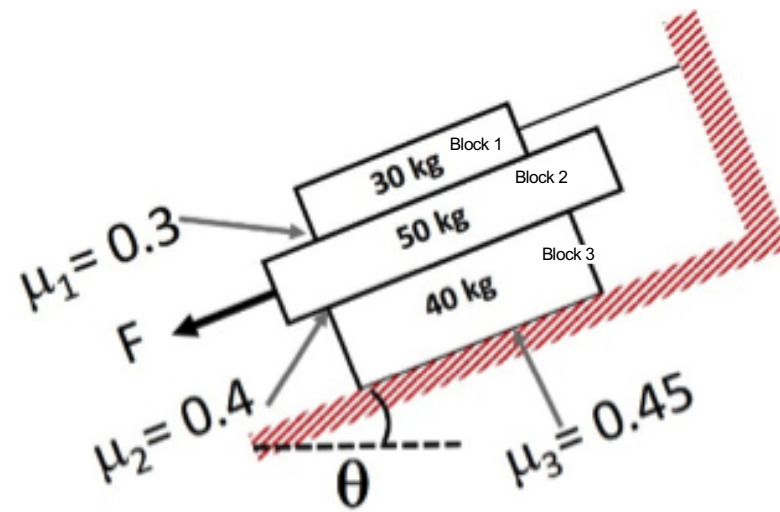
```
Fload + Fpin + Froller
```

$\{0, -1700 + \text{Fpiny} + \text{Frollery}, 0\}$

```
Rload × Fload + Rroller × Froller
```

$\{0, 0, -2200 + 7 \text{Frollery}\}$

Solving equations of motion

```
N@Solve[{Fload + Fpin + Froller == {0, 0, 0},
   Rload × Fload + Rroller × Froller == {0, 0, 0}}, {Fpiny, Frollery}]
```

$\{\{\text{Fpiny} \to 1385.71, \text{Frollery} \to 314.286\}\}$

9: Three blocks

Problem analysis

This problem is statically indeterminate because for a given load, the force can be distributed in many ways between the tension in the rope and the friction on the base. For the remainder of this problem, I assume those indeterminisms are resolved in the most favorable way for the blocks not slipping.

Figure out forces perpendicular to the force F

```
Clear[θ, F]

g = g;
Fn12[θ_] := 30 kg g Cos[θ];
Fn23[θ_] := Fn12[θ] + 50 kg g Cos[θ];
Fn3g[θ_] := Fn23[θ] + 40 kg g Cos[θ];
```

Consider the system containing only block 1

```
block1slips[θ_, F_] := - 30 kg g Sin[θ] ≥ 0.3 * Fn12[θ]
```

Consider the system containing only block 2

```
block2slips[θ_, F_] := F + 50 kg g Sin[θ] ≥ 0.3 * Fn12[θ] + 0.4 * Fn23[θ]
```

Consider the system containing blocks 2 and 3

```
blocks23slip[θ_, F_] := F + 90 kg g Sin[θ] + 50 kg g Sin[θ] ≥ 0.3 * Fn12[θ] + 0.45 * Fn3g[θ]
```

Plot the results

```
RegionPlot[Evaluate@{block1slips[θ, F], block2slips[θ, F], blocks23slip[θ, F]},
  {θ, -Pi/4, Pi/4}, {F, 0, 800 N}, PlotPoints → 5,
  PlotLegends → {"Block 1 slips", "Block 2 slips", "Blocks 2&3 slip"},
  AxesLabel → {"Angle(radians)", "Force(N)"}]
```
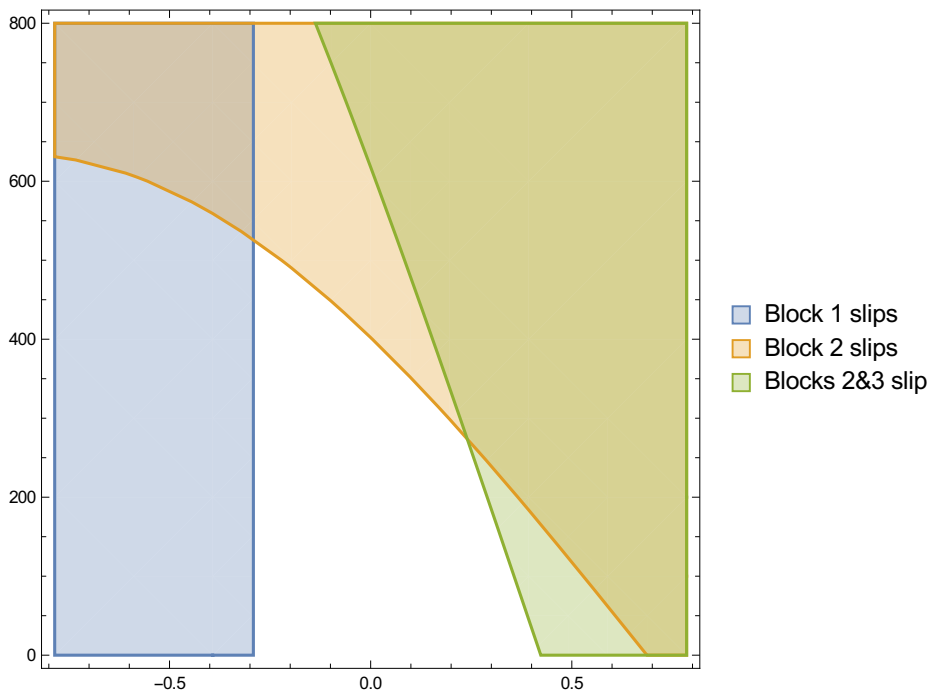

To clarify how this graph works: Each region represents the set of conditions under which the specified sets of joints would be satisfied with yielding, under the most optimal of the available statically indeterminate load distributions. In particular, the lowest color for a given coordinate is always the first to occur.

10: Static Indeterminism

1. Parked car

Because there are an infinite number of ways the wheels could be applying force toward each other with the parking brakes on, it is indeterministic.

2. Problem 9

As touched on before, if the string is slightly streachier than the frictional joints (a detail that lives outside the model), it will absorb less load, and the converse is true also.