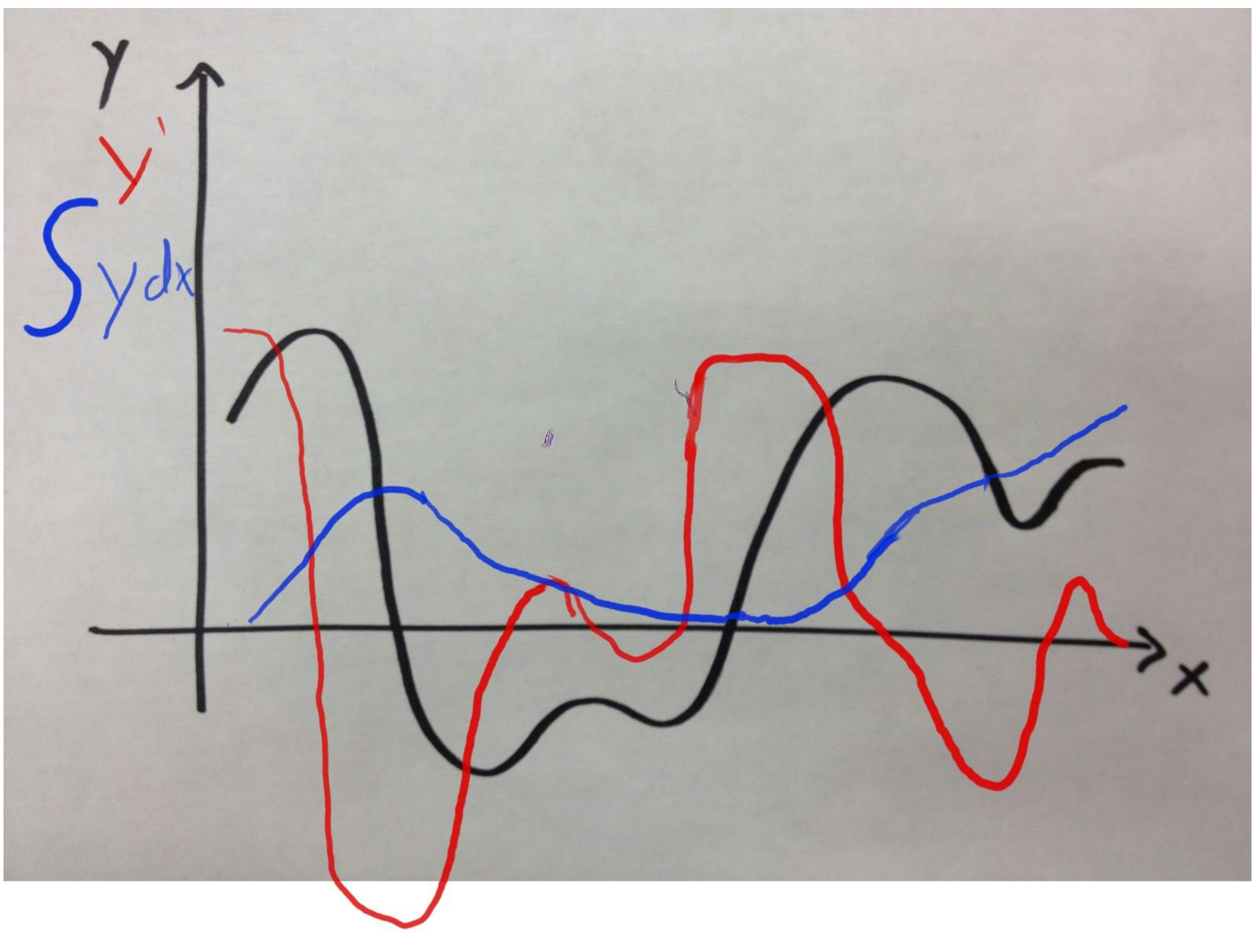
1. Create a table of the five fundamental functions , , , , and . List both their derivatives and their anti-derivatives. Include in your table at least one other example.
2. Consider the sketch of the function below. Now try to sketch the derivative and an anti-derivative.
3. Make a visual argument about why these properties are true.
4. Use your table of fundamental functions and the chain rule to determine the derivative of
5. Use your table of fundamental functions and the substitution rule to evaluate .
6. Use your table of fundamental functions and the product rule to determine the derivative
7. Use your table of fundamental functions and integration by parts to determine .
8. Recall some of the explicit functions you used to describe curves generated by taking sections through fruit, vegetables, or manufactured objects. Propose an integral that would determine the cross-sectional area of the section, and evaluate it.
9. Consider the family of functions defined by the power law, . Propose an integral that would determine the area enclosed on the top by , on the bottom by , on the left by , and on the right by the intersection of the top and bottom functions. Evaluate it and graph the enclosed area as a function of the parameter . Use a log-log plot and interpret the result.
10. Evaluate and for f.
11. Evaluate all four second-order derivatives of .
12. Review the article on partial derivative at http://mathworld.wolfram.com/PartialDerivative.html. Under what conditions on the function are the mixed partial derivatives equal?
13. How many second-order derivatives are there of ?
14. Sketch the regions of integration and compute the area of the enclosed region by evaluating the double integral.
15. Sketch the regions of integration and compute the area of the enclosed region by evaluating the double integral.
16. Consider the following region. Express the region as a union of simple regions, and compute the area using double integrals.
    - The hard way:
    - The easy way
17. Sketch the following regions of integration in the plane, and evaluate the double integral.
18. Visualize the following solids and compute their volume using a double integral.
    - Bounded by the planes , , , and .
    - Under the paraboloid and above the region bounded by and .
19. Visualize the solids defined by the limits of integration, and evaluate their volume.
20. Visualize the solid defined by the limits of integration, and figure out the five other triple integrals that are the same.
21. Visualize the solids defined by the limits of integration, and evaluate the triple integrals.
22. Find the total mass of the following plates and solids.
    - The plate bounded by the parabola and the x-axis; mass density .

1. Create a table of the five fundamental functions x^n, $\sin(x)$, $\cos(x)$, $\exp(x)$, and $\ln(x)$. List both their derivatives and their anti-derivatives. Include in your table at least one other example.

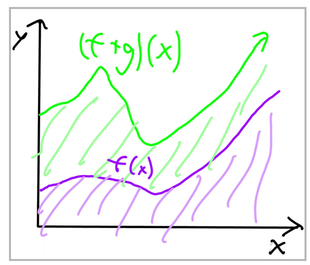
$f(x)$	$f'(x)$	$\int f(x)\,dx$
x^n	$n\,x^{n-1}$	$\frac{x^{n+1}}{n+1}$
$\sin(x)$	$\cos(x)$	$-\cos(x)$
$\cos(x)$	$-\sin(x)$	$\sin(x)$
e^x	e^x	e^x
$\log(x)$	$\frac{1}{x}$	$x\log(x) - x$

2. Consider the sketch of the function below. Now try to sketch the derivative and an anti-derivative.



3. Make a visual argument about why these properties are true.

1. Because slopes don't depend on vertical translation, we can consider the simplifying case where $f(x_0) = g(x_0) = 0$. In this case, $f'(x_0)$ is proportional to the value of $f(x_0 + d)$ for some infinitesimally small value of d. Because of the definition of function addition, $(f+g)(x_0+d) = f(x_0+d) + g(x_0+d)$, so the $(f+g)'(x_0) = f'(x_0) + g'(x_0)$.
2. Taking an image that shows a curve and a tangent line at some point, multiplying the function by a constant *c* is equivalent to scaling the entire image vertically. When that happens, the slope of the tangent line is also multiplied by *c*, and the tangent line remains tangent (because it's the same image).
3. Drawing the functions as shown reveals that the vertical distance between the green and purple lines at any given point is g(x). As a result, the green shaded area must be $\int g(x)\,dx$. Because the purple area is $\int f(x)\,dx$, their sum, the area under the green curve, must be $\int (f+g)(x)\,dx$.



4. Here, the same "scale the graph" approach works. Scaling the graph vertically by a factor of *c* both multiplies the graphed function by a factor of *c* and multiplies the area under that curve by the same factor.

4. Use your table of fundamental functions and the chain rule to determine the derivative of $(x^3-1)^{100}$.

$$300x^2(x^3-1)^{99}$$

5. Use your table of fundamental functions and the substitution rule to evaluate $\int_0^4 \sqrt{2x+1}\,dx$.

$$\frac{1}{3}(7\sqrt{7}+i)$$

6. Use your table of fundamental functions and the product rule to determine the derivative of $\sqrt{x}(1-x)$.

$$\frac{1-x}{2\sqrt{x}}-\sqrt{x}$$

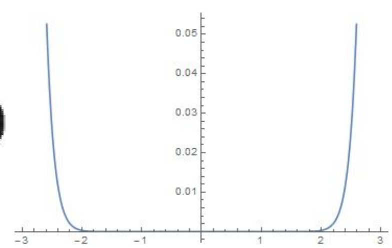
7. Use your table of fundamental functions and integration by parts to determine $\int_1^2 x\exp(-x)\,dx$.

$$\frac{3}{e^2}-\frac{2}{e}$$

8. Recall some of the explicit functions you used to describe curves generated by taking sections through fruit, vegetables, or manufactured objects. Propose an integral that would determine the cross-sectional area of the section, and evaluate it.

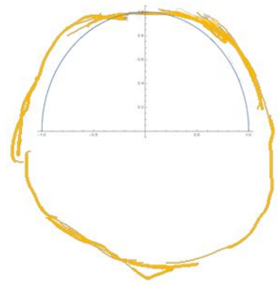
Soda can

$$2*\left(\frac{x}{3}\right)^{20}$$



$$\int_{-3}^{3}\left(2-2\left(\frac{x}{3}\right)^{20}\right)dx$$

Apple



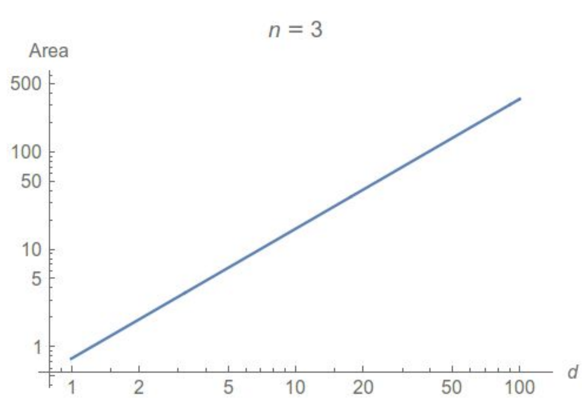
$$\sqrt{1-x^2}$$

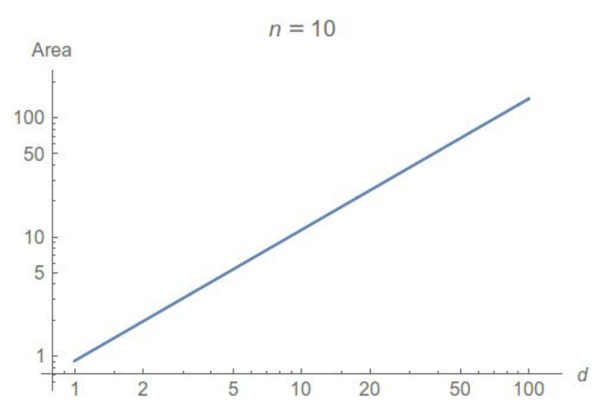
$$\int_{-1}^{1}\sqrt{1-x^2}\,dx$$

9. Consider the family of functions defined by the power law, $y=x^n, n=1,2,3,...$. Propose an integral that would determine the area enclosed on the top by $y=d$, on the bottom by $y=x^n$, on the left by $x=0$, and on the right by the intersection of the top and bottom functions. Evaluate it and graph the enclosed area as a function of the parameter d. Use a log-log plot and interpret the result.

Intersection point = $\left(x=d^{1/n}, y=d\right)$

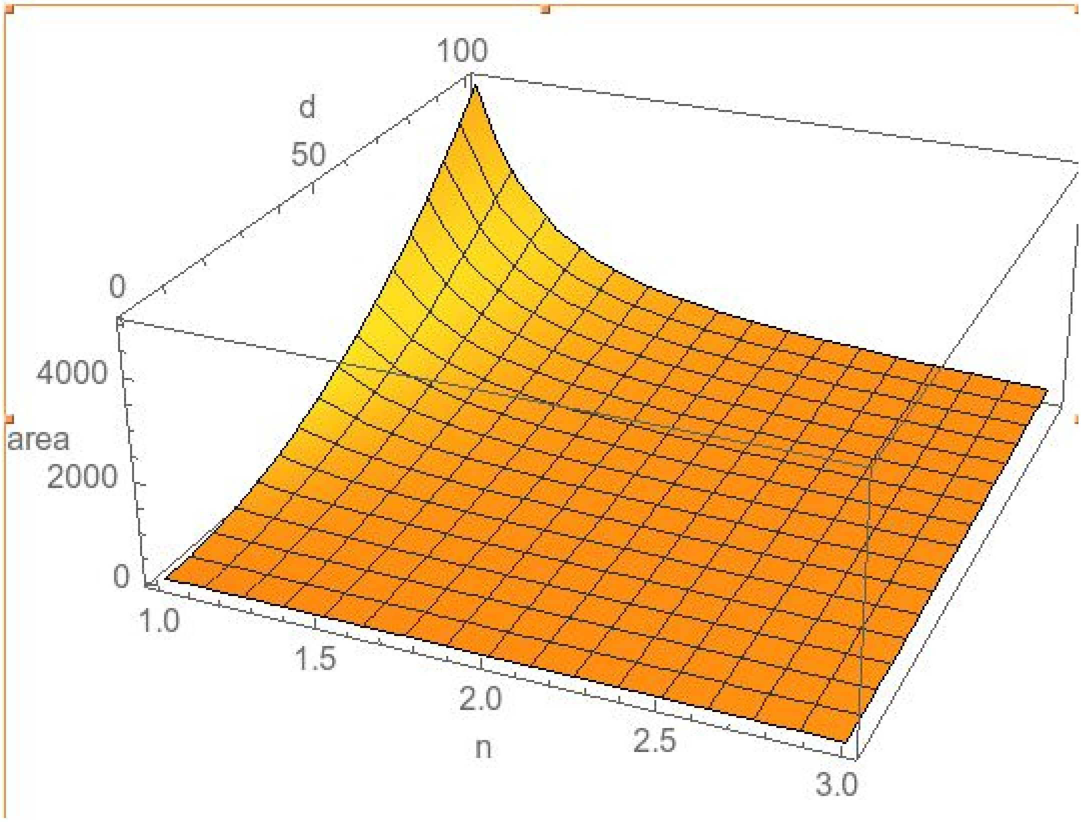
Area = $\int_0^{d^{1/n}}(d-x^n)\,dx \quad \rightarrow \quad \frac{n\times d^{\frac{1}{n}+1}}{n+1}$ (Mathematica)





This isn't surprising as, with *n* held constant, the expression above evaluates to a power of *d*.

Just for fun, a 3D plot (no log scale, sorry)



10. Evaluate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for f$(x,y) = x^2\sin(xy^2)$.

$$\frac{\partial f}{\partial x} = x^2y^2Cos[xy^2] + 2xSin[xy^2]$$

$$\frac{\partial f}{\partial y} = 2x^3y\cos(xy^2)$$

11. Evaluate all four second-order derivatives of $f(x,y) = x^2\sin(xy^2)$.

$$\frac{\partial^2 f}{\partial x^2} = -x^2y^4\sin(xy^2) + 2\sin(xy^2) + 4xy^2\cos(xy^2)$$

$$\frac{\partial^2 f}{\partial x\partial y} = 6x^2y\cos(xy^2) - 2x^3y^3\sin(xy^2)$$

$$\frac{\partial^2 f}{\partial y\partial x} = 6x^2y\cos(xy^2) - 2x^3y^3\sin(xy^2)$$

$$\frac{\partial^2 f}{\partial y^2} = 2x^3 \cos(xy^2) - 4x^4 y^2 \sin(xy^2)$$

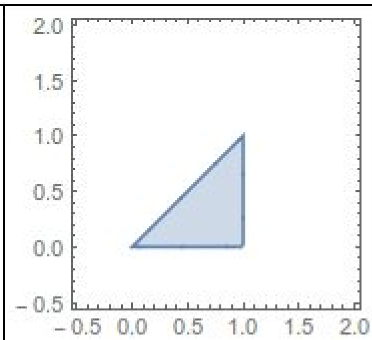
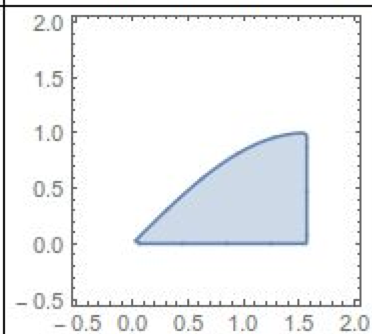
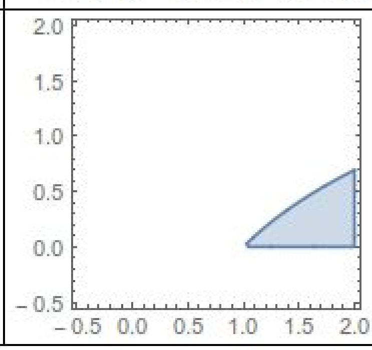
12. Review the article on partial derivative at http://mathworld.wolfram.com/PartialDerivative.html. Under what conditions on the function f are the mixed partial derivatives equal?

f, and its first-order derivatives, and its mixed second-order derivatives must exist and be continuous in the vicinity of the differentiation point.
I'm still not exactly satisfied with this, because I don't understand why the example on the page of a not-satisfying function doesn't satisfy this rule.

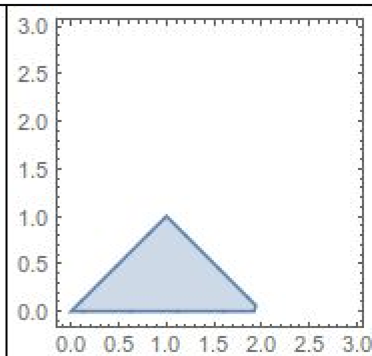
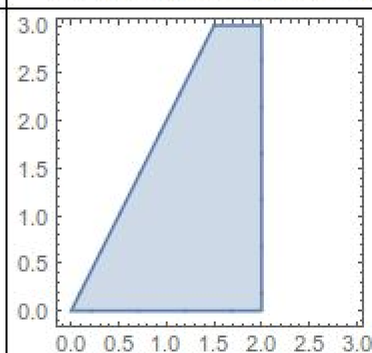
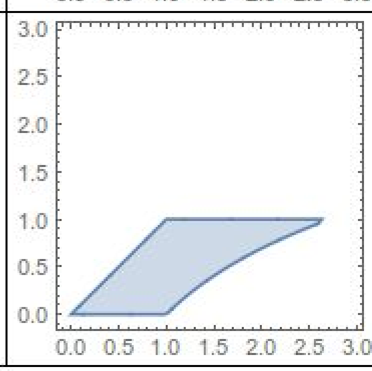
13. How many second-order derivatives are there of $f(x,y,z)$?

9 (if *f* is "nice", only 6 are unique)

14. Sketch the regions of integration and compute the area of the enclosed region by evaluating the double integral.

$x \geq 0 \wedge x \leq 1 \wedge y \geq 0 \wedge y \leq x$		$\frac{1}{2}$
$x \geq 0 \bigwedge x \leq \frac{\pi}{2} \bigwedge y \geq 0 \bigwedge y \leq \sin(x)$		1
$x \geq 1 \wedge x \leq 2 \wedge y \geq 0 \wedge y \leq \log(x)$		$\log(4) - 1$

15. Sketch the regions of integration and compute the area of the enclosed region by evaluating the double integral.

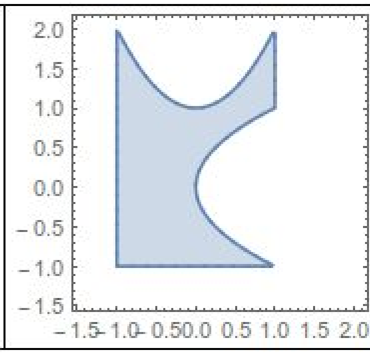
$x \geq y \wedge x \leq 2 - y \wedge y \geq 0 \wedge y \leq 1$		1
$x \geq \frac{y}{2} \wedge x \leq 2 \wedge y \geq 0 \wedge y \leq 4$		4
$x \geq y \wedge x \leq e^y \wedge y \geq 0 \wedge y \leq 1$		$\frac{1}{2}(2e - 3)$

16. Consider the following region. Express the region as a union of simple regions, and compute the area using double integrals.

The hard way:

$$\int_{-1}^{1}\int_{1}^{x2+1} dydx + \int_{-1}^{1}\int_{-1}^{y2} dxdy = \frac{10}{3}$$

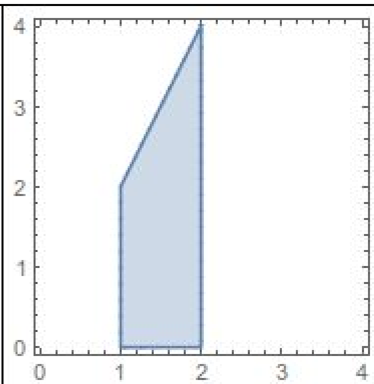
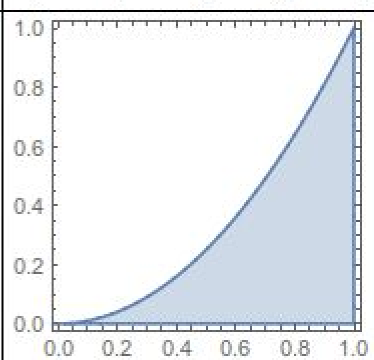
The easy way

$x \geq -1 \land x \leq 1 \land y \geq -1 \land y \leq x^2 + 1 \land x < y^2$		$\frac{10}{3}$

17. Sketch the following regions of integration in the plane, and evaluate the double integral.

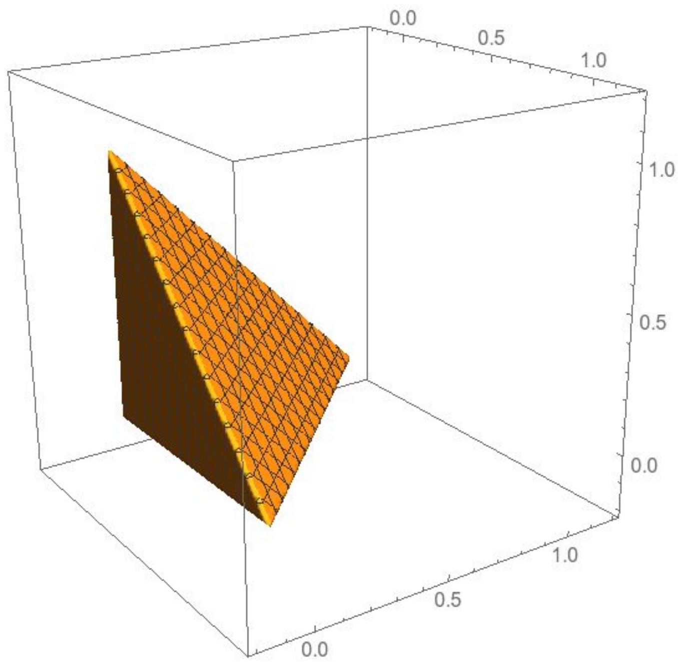
a) $\int\int_D \frac{4y}{x^3+2}\, dA, D=\{(x,y)|1\leq x\leq 2,0\leq y\leq 2x\}$

b) $\int\int_D x\cos y\; dA,$ \; D is bounded by $y=0,y=x^2,x=1$

$\frac{4y}{x^3+2}$		$\frac{8}{3}\log\left(\frac{10}{3}\right)$	3.21059
$x\cos(y)$		$\sin^2\left(\frac{1}{2}\right)$	0.229849

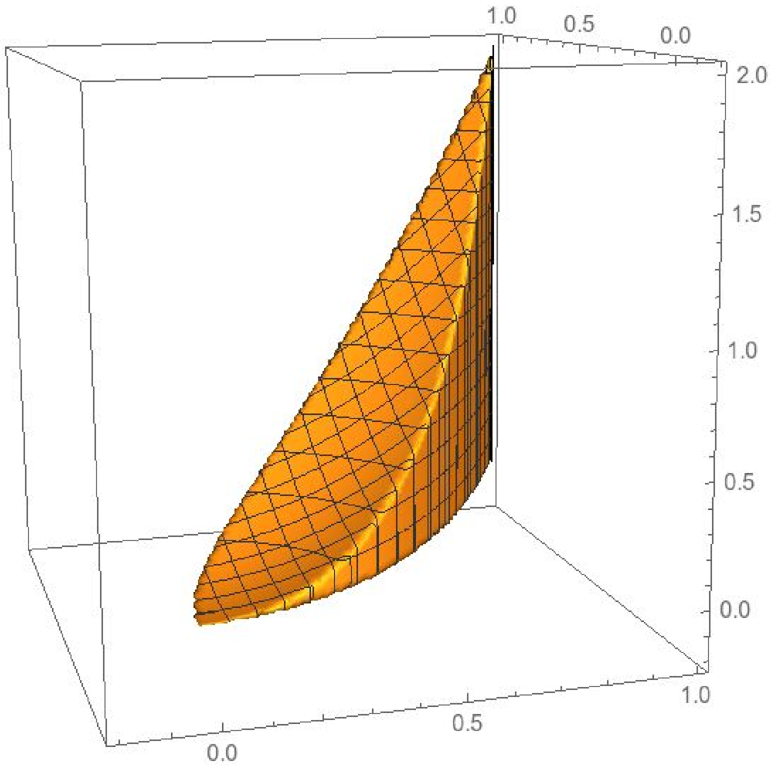
18. Visualize the following solids and compute their volume using a double integral.

a) Bounded by the planes $x=0$, $y=0$, $z=0$, and $x+y+z=1$.



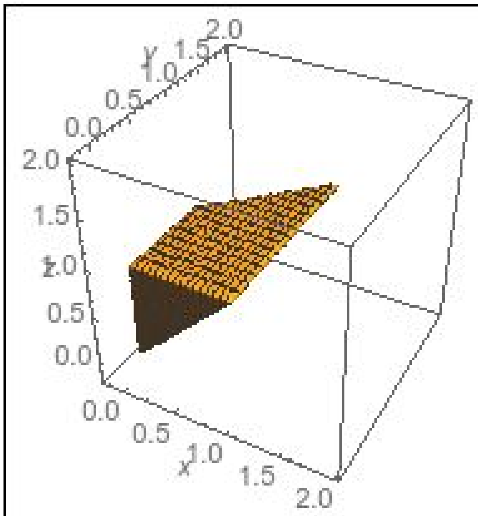
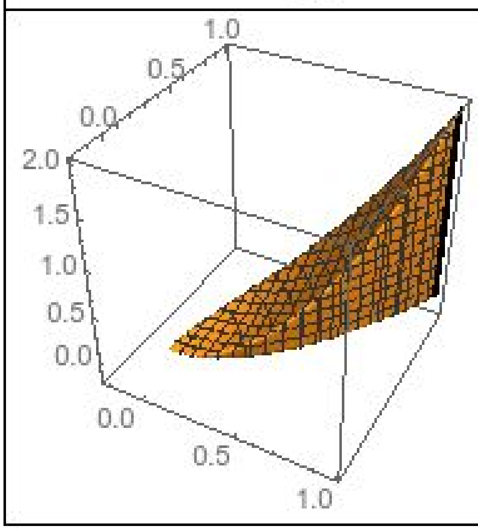
$$\frac{1}{6}$$

b) Under the paraboloid $z = x^2 + y^2$ and above the region bounded by $y = x^2$ and $x = y^2$.



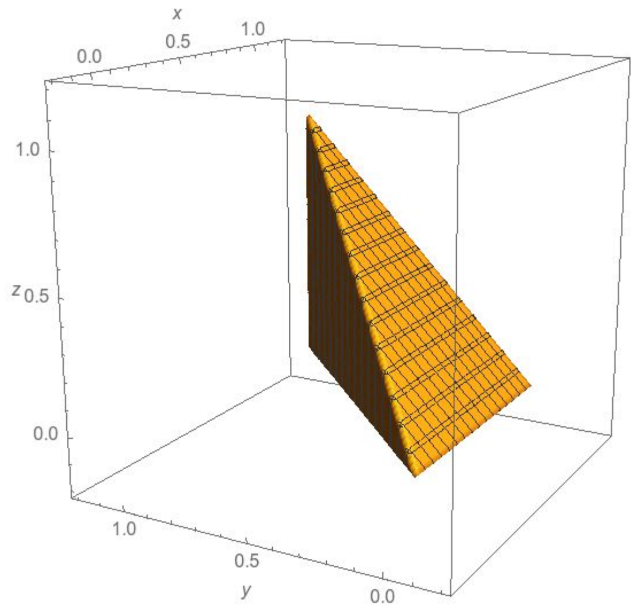
$$\frac{6}{35}$$

19. Visualize the solids defined by the limits of integration, and evaluate their volume.

	$\frac{1}{2}$
	$\frac{6}{35}$

20. Visualize the solid defined by the limits of integration, and figure out the five other triple integrals that are the same.

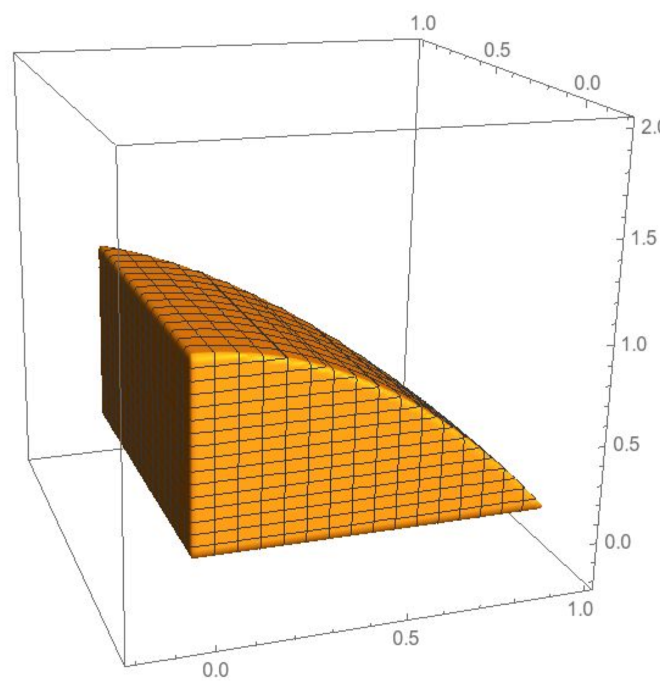
I was previously confident about this, but I think some cases may be incorrect. Checking in Mathematica may be justified.



a) $\int_0^1\int_y^1\int_0^y dzdxdy$

1. $\int_0^1\int_0^x\int_0^x dz\,dy\,dx$
2. $\int_0^1\int_0^x\int_0^x dy\,dz\,dx$
3. $\int_0^1\int_0^z\int_z^1 dx\,dy\,dz$
4. $\int_0^1\int_z^1\int_0^z dy\,dx\,dz$
5. $\int_0^1\int_0^y\int_y^1 dx\,dz\,dy$

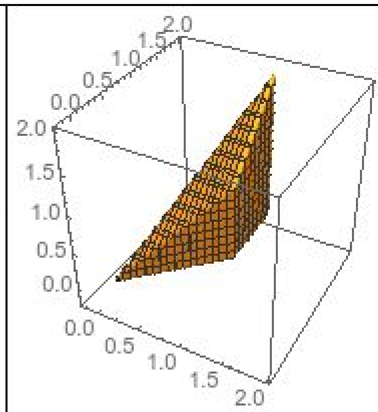
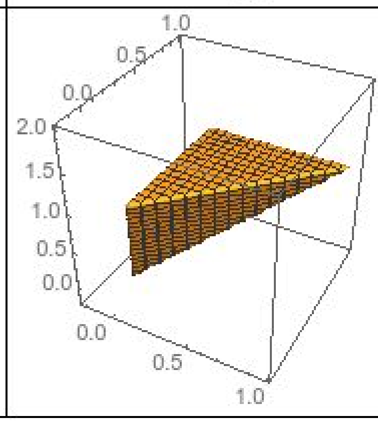
I call shenanigans on 20(b). Because integration limits cannot access variables undefined until later, I think solving this problem for the dx dz dy case may require using a piecewise function in the limit of an integral, which is *not OK*. I am interested to see what solutions the teaching team came up with.



b) $\int_0^1\int_0^{1-x2}\int_0^{1-x} dydzdx$

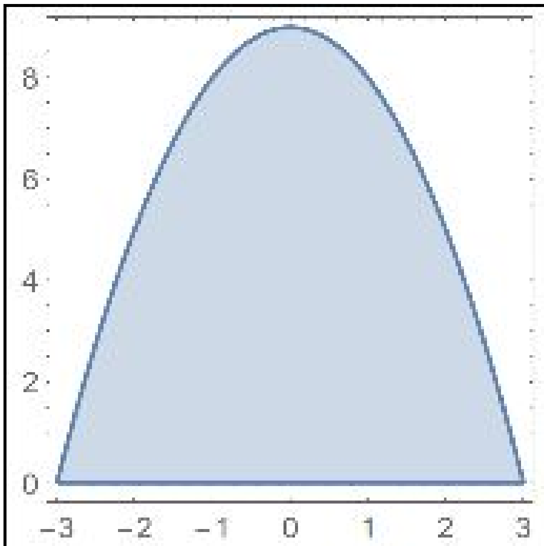
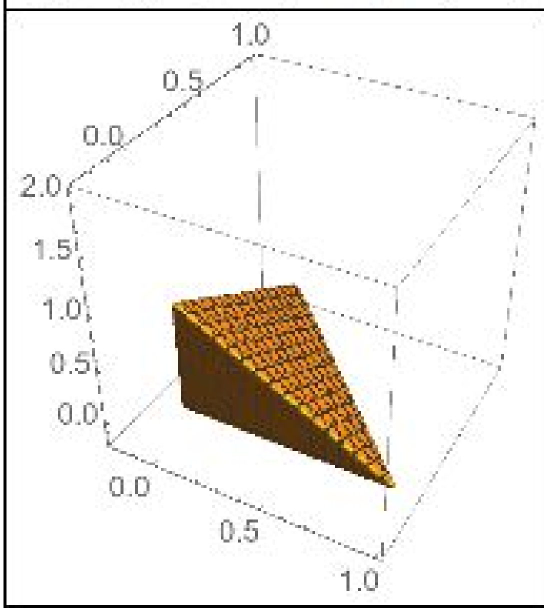
1. $\int_0^1\int_0^{1-x}\int_0^{1-x2} dz\,dy\,dx$
2. $\int_0^1\int_0^z\int_0^{\sqrt{1-z}} dx\,dy\,dz$
3.
4.
5.

21. Visualize the solids defined by the limits of integration, and evaluate the triple integrals.

$2\,x\,y\,z$		$\frac{5}{8}$
$e^{-y^2}\,z$		$\frac{1}{4\,e}$

22. Find the total mass of the following plates and solids.

a) The plate bounded by the parabola $y=9-x^2$ and the x-axis; mass density $\rho(x,y)=y$.

	$\frac{648}{5}$
	$\frac{1}{24}$