Vectors and Vector Operations: Cartesian

1. Dot and Cross: What does the dot product of two vectors tell you about the two vectors? What about the cross product?

2. Geometrical Vectors: The diagram shows three vectors, , , and . All three are in the plane of the page; their magnitudes are (respectively) 2, 1, and 3. For each operation below, either draw the results of the identified operations, or (if appropriate) give a best guess as to the value, or (if appropriate) identify the operation as nonsense.

3. Cartesian Vectors: Let , , and . Find the results of identified operations, or (if appropriate) identify the operation as nonsense.

4. Vector Construction: and are two arbitrary, non-parallel vectors in three-dimensional space. Using them, construct the following vectors (i.e., find mathematical expressions for the specified vector in terms of the vectors and) :

   a. The vector , which has a length of 1, and points in the direction of .

   b. The vector , which has a length of 1, and is perpendicular to both and .

Forces: Ideas and Models

5. Consider a sprinter who is running (and accelerating) over a level surface. What fundamental forces are acting on the sprinter (or parts of the sprinter)? How do these translate into phenomenological forces? How large might these different forces be? What force is responsible for accelerating the runner forward?

6. "I am pushing a coffee cup across my desk. Since it is in motion, I know that the magnitude of the frictional force acting on the cup is given by , where is the coefficient of kinetic friction and is the normal force the table exerts on the cup." True or false? Discuss, and use a free body diagram to explain.

7. "I am pushing on a coffee cup that is sitting on my desk. Since it is not moving, I know that the magnitude of the frictional force acting on the cup is given by , where is the coefficient of static friction and is the normal force the table exerts on the cup." True or false? Discuss, and use a free body diagram to explain.

8. Your physicist friend tells you, "Newton's third law says that for every force there is an equal and opposite force. For example, when an object is sitting on the ground, the force exerted by gravity (), is exactly offset by the normal force ()." How do you respond to your friend's claim? Is she right? Wrong? Why?

9. Two forces are applied to an eye hook. They lie in the plane of the page with the orientation and magnitudes as shown. Determine the orientation and magnitude of the net or resultant force acting on the hook.

10. A man is pulling a rope running between points A and B with a force of 70 lbs. Describe this force as a vector in terms of components aligned with the x-y-z coordinate system shown.

Moments: Ideas and Calculating

11. A multi-section beam is attached to a wall. What is the moment about point created by the 800 N force action on point as shown?

12. Imagine that you are riding a bicycle (without cleats or toe clips). At what point during a pedaling cycle is the moment on the crank generated by your feet on the pedals greatest (assume you are calculating the moment about the center point of the crank)? When is it least? What is the direction of the moment? Think about this both in 2D and in 3D: how do your answers differ, and why?

13. (Think in 2D for this one): As you step on the pedal, the moment you apply to the crank is offset a moment produced by tension in the chain -- thus, the tension in the chain is proportional to the

moment you apply to the crank. This tension, in turn, applies a moment to a cog attached to the rear wheel. Which cog, or , will yield the highest moment on the rear wheel? What will the direction of this moment?

14. Why are multi-speed bikes desirable? Given the answer to above, it seems like a pretty simple design would yield very high moment on the rear wheel. Isn't this always desirable, since this means you can exert more force against the ground, and hence accelerate faster?

15. What might a 1000 N-m torque wrench look like? Why? Propose a nominal design (size, configuration, etc.).

Free Body Diagrams

16. Recall the multi-section beam in the previous section (question 15). Draw a free body diagram of multi-section beam in the previous section. Under what conditions would it be reasonable to neglect the weight of the beam? Why?

17. The picture at the right shows a person trying to remove a nut with a wrench. Draw a free body diagram for

a. The wrench only

b. The nut only

18. Consider a bike that is stationary with a rider standing on the pedals (see below). Draw a complete FBD for the crankset. Sketch your vectors such that your FBD appears to satisfy the equilibrium condition.

19. The diagram at right shows a front-wheel drive car on a hill. The car is in drive, but is not moving: the brakes are engaged.

a. System = the entire car.

b. System = a rear wheel, hub, and rotor of the car. Assume a disc brake, as shown at right. Note that when the brake is engaged, the caliper pinches the rotor between the brake pads.

c. System = a front wheel, hub, and rotor of the car. Assume a disc brake, as shown at right. Note that when the brake is engaged, the caliper pinches the rotor between the brake pads. Remember that it's a front-wheel drive car in drive!

20. A pin joint or pin connector is a mathematical idealization of the interaction between two connected bodies. The two bodies can rotate freely relative to one another about an axis passing through the pin joint. It does not allow relative motion between the bodies. Wheels rotating on axles or bearing are often modeled as pin joints. See the .pdf document on the website which describes pin joints and various other supports.

Find a real connection or support that can be modeled as a pin joint and take a photo of it. Draw a free body diagram of the pin between the two bodies. What idealizations must be made so that this representation can be made?

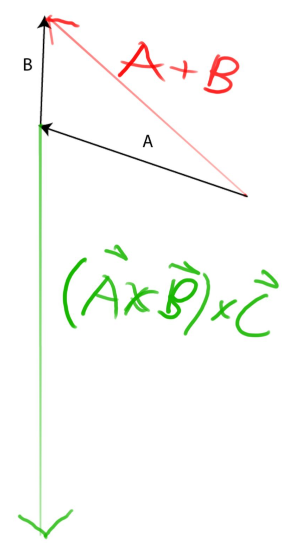
21. The diagram at the right shows a rope routed over a pulley wheel that is suspended from a bracket by a pin connector. Draw a free body diagram the wheel of the pulley.

Vectors and Vector Operations: Cartesian

1. **Dot and Cross:** What does the dot product of two vectors *tell you* about the two vectors? What about the cross product?

Dot: The dot product, representing |A|*|B|*Cos(θ), reflects the *parallellness* of the two vectors. Much more precisely, $\vec{A}\cdot\vec{b}=0 \leftrightarrow$ the vectors are perpendicular. For A and B as unit vectors, the dot product simplifies to Cos(the angle between them).
Cross: The cross product similarly represents the perpendicularity and plane of the vectors. For A and B as unit vectors, |AxB| = sin(angle) and AxB is a normal vector to the plane containing A and B. If A and B are not unit vectors, the magnitude is corrupted with the product of their magnitudes.

2. **Geometrical Vectors:** The diagram shows three vectors, $\vec{A}$, $\vec{B}$, and $\vec{C}$. All three are in the plane of the page; their magnitudes are (respectively) 2, 1, and 3. For each operation below, either draw the results of the identified operations, or (if appropriate) give a best guess as to the value, or (if appropriate) identify the operation as nonsense.

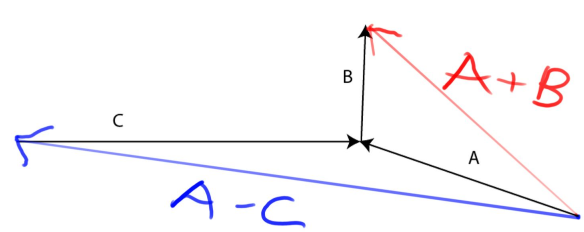


c. $\vec{A}\cdot\vec{B}$

$$2\cos\left(\frac{75\pi}{180}\right)\approx 0.517638$$

d. $(\vec{A}\times\vec{B})\times\vec{C}$

$$Magnitude \approx 23\sin\left(\frac{75\pi}{180}\right)\approx 5.79555$$



e. $(\vec{A}\cdot\vec{B})\times\vec{C}$

Nonsensical b/c Cross products are defined over two vectors, not a vector and a scalar.

3. **Cartesian Vectors:** Let $\vec{A}=3\hat{\imath}+4\hat{\jmath}$, $\vec{B}=\hat{\imath}-\hat{\jmath}$, and $\vec{C}=-5\hat{\jmath}$. Find the results of identified operations, or (if appropriate) identify the operation as nonsense.

a. $|\vec{A}+\vec{B}|$

5

b. $\vec{A}\times\vec{C}$

$-15\hat{k}$

c. $\vec{A}\cdot\vec{B}$

-1

d. $(\vec{A}\times\vec{B})\times\vec{C}$

$-35\hat{\imath}$

e. $(\vec{A}\times\vec{B})\cdot\vec{C}$

0

4. **Vector Construction:** $\vec{A}$ and $\vec{B}$ are two arbitrary, non-parallel vectors in three-dimensional space. Using them, construct the following vectors (i.e., find mathematical expressions for the specified vector in terms of the vectors $\vec{A}$ and $\vec{B}$) :

$$Let\,\|X\|=\sqrt{(X\cdot\hat{\imath})^2+(X\cdot\hat{\jmath})^2+(X\cdot\hat{k})^2}$$

a. The vector $\hat{A}$, which has a length of 1, and points in the direction of $\vec{A}$.

$\vec{A}/\|A\|$

b. The vector $\widehat{n}$, which has a length of 1, and is perpendicular to both $\vec{A}$ and $\vec{B}$.

$$\frac{A\times B}{\|A\times B\|} \quad \text{----OR----} \quad -\frac{A\times B}{\|A\times B\|}$$

Forces: Ideas and Models

5. Consider a sprinter who is running (and accelerating) over a level surface. What fundamental forces are acting on the sprinter (or parts of the sprinter)? How do these translate into phenomenological forces? How large might these different forces be? What force is responsible for accelerating the runner forward?

Gravity acts between the runner and the earth, pulling the runner down. This doesn't really need a phenomenological interpretations, other than the standard simplifications of ignoring the speed of light and approximating with the centers of mass of the objects, and the curvature of the earth.
Electromagnetic interactions act between the sprinter's shoes and the track, pushing the sprinter both up and sideways. These phenomenologically translate to a vertical constraint force and a horizontal static friction force, which is responsible for the runner's acceleration.
Electromagnetic forces act between the runner and the air, modelled as drag.
Strong forces act within the atoms of the runner's body, holding it together,
Weak forces... do whatever weak forces do. Ummm. Yeah.

6. "I am pushing a coffee cup across my desk. Since it is in motion, I know that the magnitude of the frictional force acting on the cup is given by $\mu_k N$, where μ_k is the coefficient of kinetic friction and N is the normal force the table exerts on the cup." True or false? Discuss, and use a free body diagram to explain.

False. Because the situation is not static, F_{push} can be significantly greater (or less than) F_f (friction) as long as the cup is accelerating proportionally to the force imbalance.

7. "I am pushing on a coffee cup that is sitting on my desk. Since it is not moving, I know that the magnitude of the frictional force acting on the cup is given by $\mu_s N$, where μ_s is the coefficient of static friction and N is the normal force the table exerts on the cup." True or false? Discuss, and use a free body diagram to explain.

False. Because static friction acts as a constraint force, its value is whatever is needed to match F_{push}, not necessarily always equal to its maximum possible magnitude.

8. Your physicist friend tells you, "Newton's third law says that for every force there is an equal and opposite force. For example, when an object is sitting on the ground, the force exerted by gravity ($-mg\hat{\jmath}$), is exactly offset by the normal force ($+mg\hat{\jmath}$)." How do you respond to your friend's claim? Is she right? Wrong? Why?

They are wrong. Newton's third law observes that every single force always acts on two objects in equal and opposite ways. Because the force of gravity and the normal force are two different forces acting on the same object, Newton's third law does not apply here. Instead, Newton's second law is applicable, which states that the forces are allowed to not be equal and opposite as long as their sum is proportional to the acceleration of the object. Because the object doesn't happen to be accelerating, the same result holds, but the source isn't Newton's 3rd Law.

9. Two forces are applied to an eye hook. They lie in the plane of the page with the orientation and magnitudes as shown. Determine the orientation and magnitude of the net or resultant force acting on the hook.

$$\hat{\imath}\,(-814.2\mathrm{N}) + \hat{\jmath}\,(93.22\mathrm{N})$$

10. A man is pulling a rope running between points A and B with a force of 70 lbs. Describe this force as a vector in terms of components aligned with the x-y-z coordinate system shown.

$$\hat{\imath}\,(44.71\mathrm{b}) + \hat{\jmath}\,(-4.471\mathrm{b}) + \hat{k}\,(-53.71\mathrm{b})$$

Moments: Ideas and Calculating

Something weird happened at the end of this video w.r.t. order mattering in cross products. In particular, he used the form of cross product that doesn't include the unit vector, and I don't know why. If he included that, it seems it would add another negative, restoring the equation to its original sign.

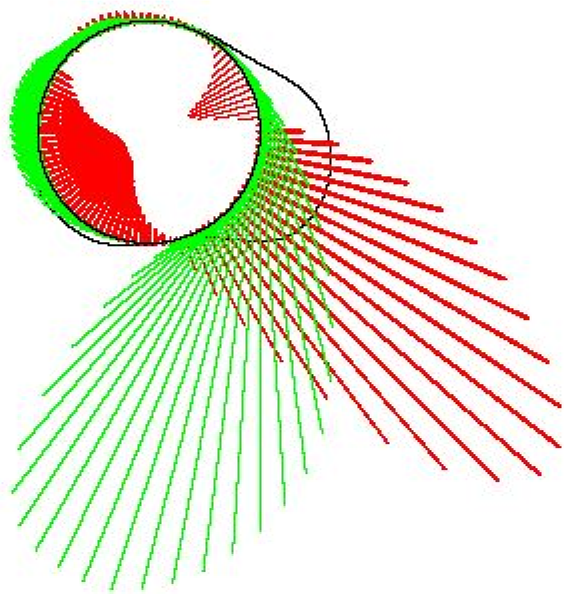
11. A multi-section beam is attached to a wall. What is the moment about point O created by the 800 N force action on point A as shown?

$$\hat{\imath}\,(-1261.0\mathrm{mN}) + \hat{\jmath}\,(0\mathrm{mN}) + \hat{k}\,(-1513.2\mathrm{mN})$$

12. Imagine that you are riding a bicycle (without cleats or toe clips). At what point during a pedaling cycle is the moment on the crank generated by your feet on the pedals greatest (assume you are calculating the moment about the center point of the crank)? When is it least? What is the direction of the moment? Think about this both in 2D and in 3D: how do your answers differ, and why?

This problem is sketchy. I know that you had some clear image in your head of what this should look like and what assumptions make this into a reasonably well-defined system, but you did a poor job of communicating those assumptions to me. For example, your "no cleats" ***looks*** **like it constrains the direction of the force, but doesn't actually do anything because a.) pedals rotate and b.) friction is a thing.**

I think the answer you are looking for here is that the moment is greatest when the pedals are horizontal, but the actual system is more dependent on the behavior of the cyclist than the motion of the crank. Thinking in 3D just puts the moment "out of the page", unless there is something else I'm not seeing.
Of course, the proper way to answer this question is to gather the actual data, shown to the right (source). This image shows that maximum tangential force (and thus moment) occurs approximately 25° after the crank is horizontal, which is substantially different than my intuition.



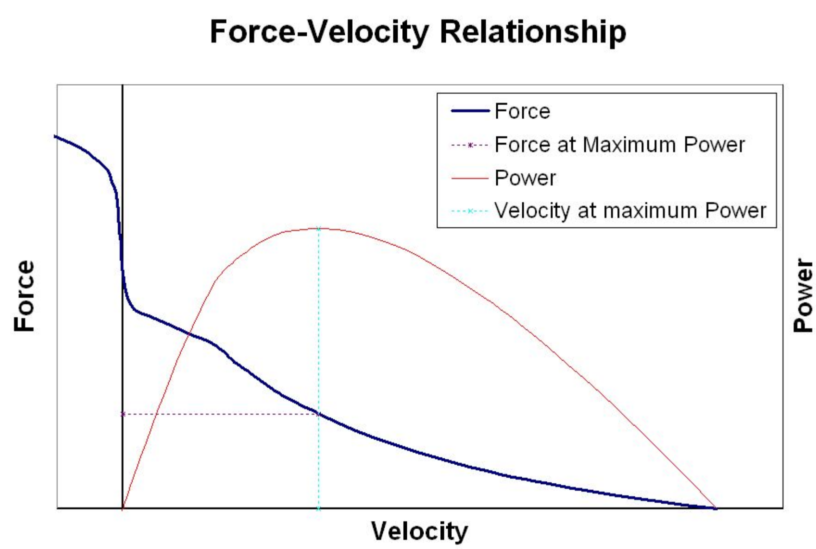
Out of curiosity, is there a nice way to represent 2d (scalar) rotation in vector math? Taking the cross product of two 2d vectors just doesn't work.

13. (Think in 2D for this one): As you step on the pedal, the moment you apply to the crank is offset a moment produced by tension in the chain -- thus, the tension in the chain is proportional to the moment you apply to the crank. This tension, in turn, applies a moment to a cog attached to the rear wheel. Which cog, A or B, will yield the highest moment on the rear wheel? What will the direction of this moment?

The larger cog, B, will result in a higher moment, but both will result in a moment that is parallel in 3D to the moment applied to the pedals by the feet. **In 2D, I wish I knew of a more mathematically rigorous way to describe it than "counterclockwise", but I don't.**

14. Why are multi-speed bikes desirable? Given the answer to above, it seems like a pretty simple design would yield very high moment on the rear wheel. Isn't this always desirable, since this means you can exert more force against the ground, and hence accelerate faster?

Were human muscles a constant-force source of energy, this would be undeniably true. Unfortunately, human muscles have a very non-flat force-velocity curve, as shown to the right. As such, the optimal bike design must endeavor to utilize human muscles as close to possible to the maximum power point indicated by the dotted blue line. [source Wikimedia]

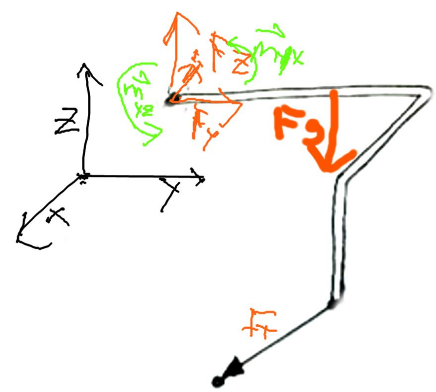


15. What might a 1000 N-m torque wrench look like? Why? Propose a nominal design (size, configuration, etc.).

It is (barely) possible for me to lift 50kg, which is approximately 500 Newtons of force. This implies that the wrench needs to be ~2m long, and definitely needs to support two-handed use. If I needed to do such a thing repeatedly, a rigidly mounted powered wrench is an appealing alternative.

Free Body Diagrams

16. Recall the multi-section beam in the previous section (question 15). Draw a free body diagram of multi-section beam in the previous section. Under what conditions would it be reasonable to neglect the weight of the beam? Why?

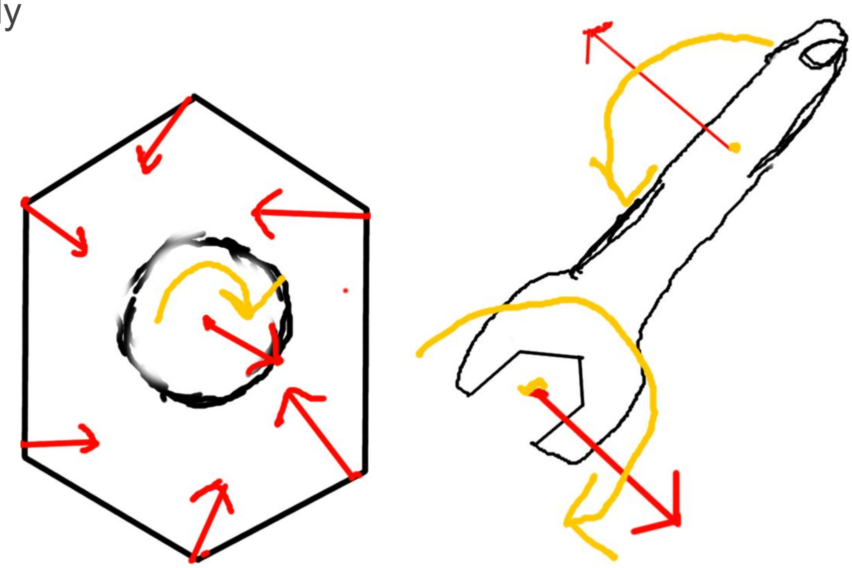


Because the two primary forces driving the movement of the beam are the mass of the beam and the force of the rope, the weight is reasonable to ignore if it is significantly smaller than the force of the rope pull.

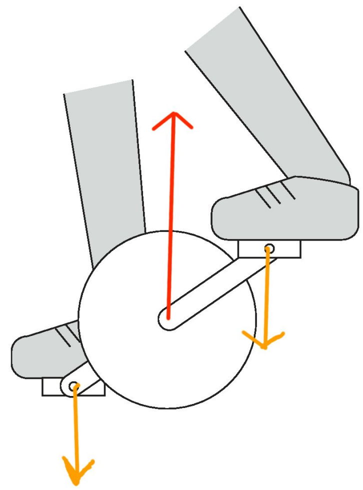
17. The picture at the right shows a person trying to remove a nut with a wrench. Draw a free body diagram for

a. The wrench only

b. The nut only

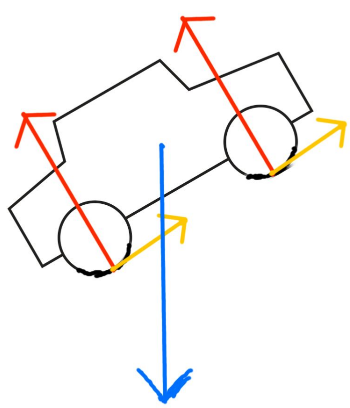


18. Consider a bike that is stationary with a rider standing on the pedals (see below). Draw a complete FBD for the crankset. Sketch your vectors such that your FBD appears to satisfy the equilibrium condition.

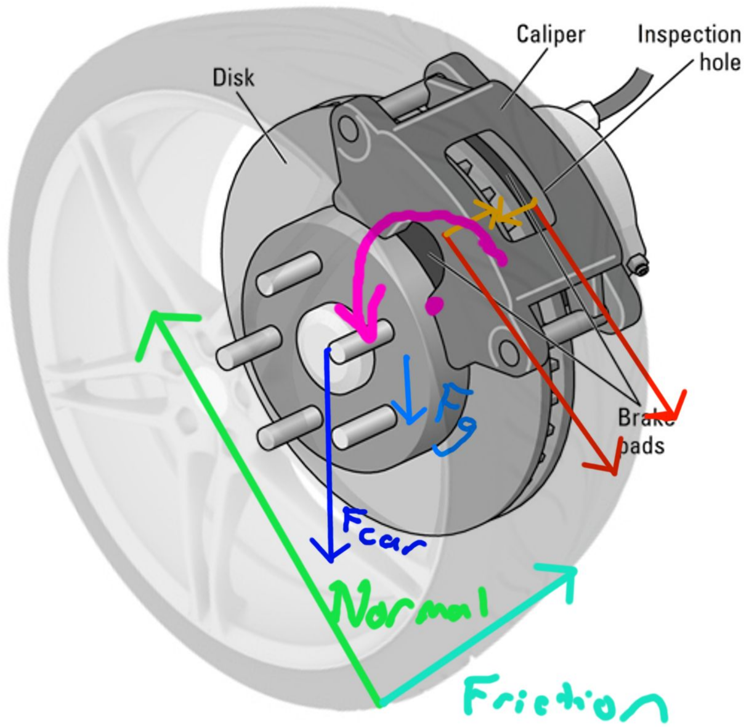


19. The diagram at right shows a front-wheel drive car on a hill. The car is in drive, but is not moving: the brakes are engaged.

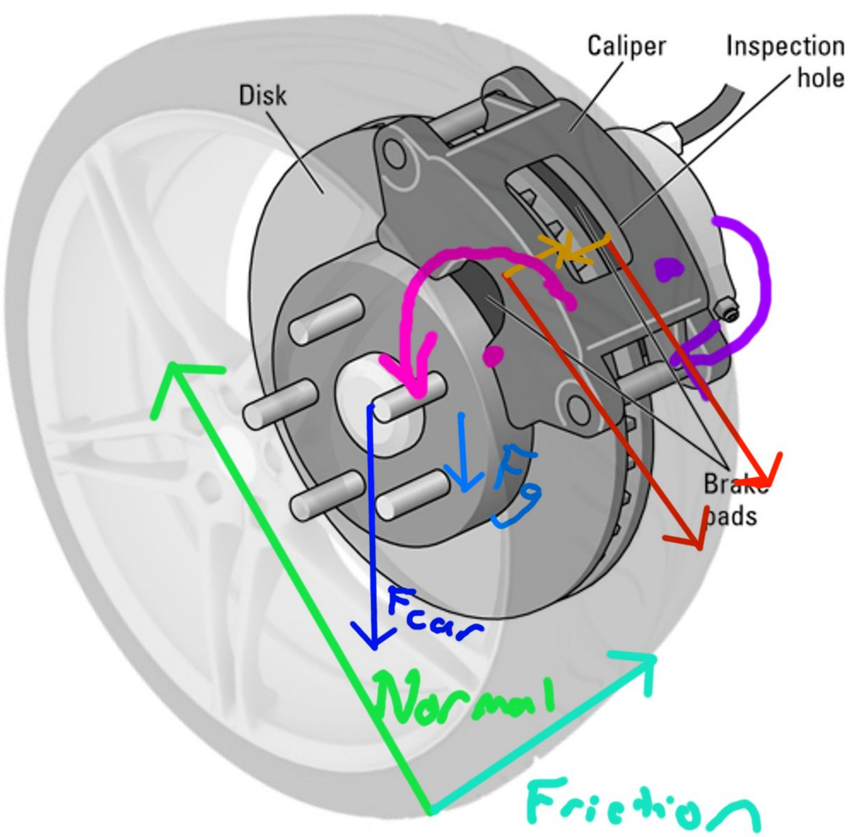
a. System = the entire car.



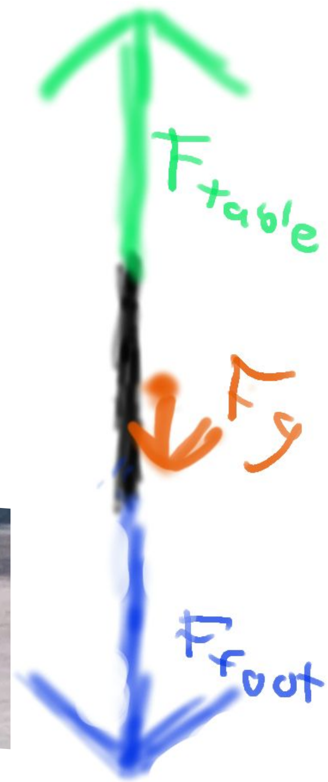
b. System = a rear wheel, hub, and rotor of the car. Assume a disc brake, as shown at right. Note that when the brake is engaged, the caliper pinches the rotor between the brake pads.



c. System = a front wheel, hub, and rotor of the car. Assume a disc brake, as shown at right. Note that when the brake is engaged, the caliper pinches the rotor between the brake pads. Remember that it's a front-wheel drive car in drive!



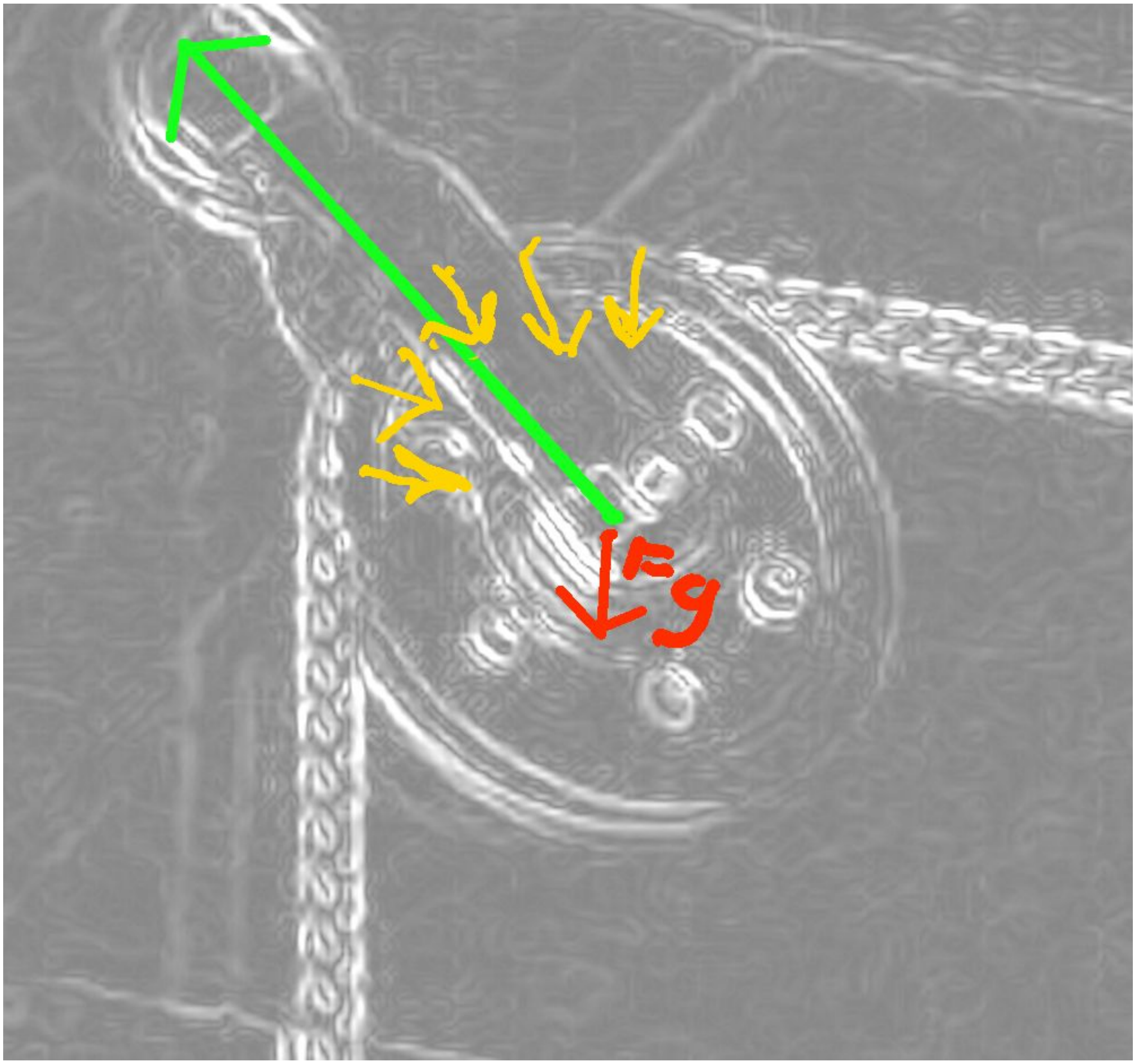
20. A pin joint or pin connector is a mathematical idealization of the interaction between two connected bodies. The two bodies can rotate freely relative to one another about an axis passing through the pin joint. It does not allow relative motion between the bodies. Wheels rotating on axles or bearing are often modeled as pin joints. See the .pdf document on the website which describes pin joints and various other supports. Find a real connection or support that can be modeled as a pin joint and take a photo of it. Draw a free body diagram of the pin between the two bodies. What idealizations must be made so that this representation can be made?



The foot of the table is a screw connection, but for small angles and large forces, it closely approximates a pin joint. Primary differentiators include the existance of friction, the “wobble” that results in some *very interesting* speed-dependent amplifying oscillations, and the fact that the screw acts as a constraint force that gradually allows the foot to move vertically under large amounts of rotation.

I am sorry this FBD is such a cop-out, as the FBD of a horizontal pin joint like the wheel of the car could be much more interesting. Admittedly, I did solve that situation in the last problem. I didn’t think ahead to realize how overly simple a vertical and balanced pin joint would be. I do think it’s kind of neat how screws are pin joints under small movement regimes.

21. The diagram at the right shows a rope routed over a pulley wheel that is suspended from a bracket by a pin connector. Draw a free body diagram of the wheel of the pulley.



Reflections

Material

I found the material of Interactions 1 to be useful in the sense of reviewing the details of stuff I haven't used in ~1.5 years, but none of it was really *new*. I liked thinking more concretely about distributed forces and FBDs, but nothing made me sit back and think "Wow, that's surprising". As such, while I think this was a useful exercise set, I'm not sure it was sufficiently useful to qualify as being equivalent to a whole week of learning in a normal class. At least it didn't take too long.

Time management

I finished most of this p-set Friday, with only a couple drawings left for Saturday. In reality, this is probably primarily due to the scaling-down of this p-set by Mark in response to the over-estimates that were Shapes I/II, but I like to think that this is my tool skills, fought for so hard in the last 1.5 weeks, finally paying off. Between Mathematica for the vector computation and GIMP for sketching, the overhead for this problem set felt much smaller than most assignments. Yay me!

Physics

I haven't done physics like this in a class since Mr. Bait, but Physics II was undeniably effective in preparing me for this. I'm glad I had that preparation, and hopefully we will quickly move on to stuff that is as effective at pushing me now as Physics II was Freshman year.