Foreword/premise

The first four exercise sets I completed for QEA were works of art. Beautifully typeset with Mathematica, I tried to find a way to make every problem interesting, even those I (honestly) didn't have much to learn from. Viewed one way, that approach is a great way to challenge myself. Viewed another, it is simply an experiment to see how *long* I can possibly spend working on this set of problems without falling over from boredom. Those experiments were quite successful, so it seems only fair to run the obvious mirror experiment now. My personal learning goal for this problem set is therefore:

"How little time can I spend working on this problem set without falling over from unsatisfied perfectionism?"

This mentality lasted until about problem ~6, at which point the available time was extended and I switched back to my normal mode. Experiment failed.

Statics

1. Cables

(a)

Something like $\sqrt{2}/2 * W_{block}$

(b)

Wider mounts = more tension

(c)

Strain gauge, pressure sensor in cylinder, ruler (cable stretches)

Actually downloading a datasheet is boring. Sorry, it's for the experiment.

2. hanging mass

$$\mu_k >= m_2 / m_1$$

3. cable mass

IF $6\mu_s^2+\mu_s>1$ ...

$\frac{1}{2}\sqrt{5}gm(cm)$ (tipping)

OTHERWISE

$$\frac{5\sqrt{5}mg\mu_s}{2(2\mu_s+1)}(cm)$$

4. block on ramp

(a)

$$\mu_{1s}>=tan(\theta)$$
$$\mu_{2s}\in R$$

(b)

Straight up, somewhere to the left of the center of the ramp, between the centroid of the triangle and the center of the block.

Its magnitude is $g(m_1+m_2)$

(c)

The force on the wedge from the ground has horizontal component $g\mu_{k1}\mathrm{m}_1\sin(\theta)\cos(\theta)+g\mathrm{m}_1\cos^2(\theta)+g\mathrm{m}_2$ and vertical component $g\mu_{k1}\mathrm{m}_1\cos^2(\theta)-g\mathrm{m}_1\sin(\theta)\cos(\theta)$. It is located slightly to the left of where it was before the block started sliding. I know how to calculate the position (solve the moment equation around the tip of the ramp) and actually working that out isn't worth my time.

Setup

3

4

5: Masses on springs

Problem analysis

Let k be the spring constant, and the bar be massless.

```
l0 = Sqrt[1.5 m ^2 + 2 m ^2]
l = Sqrt[2 m ^2 + (1.5 m + d) ^2]
```

2.5 m

$$\sqrt{(d + 1.5\,m)^2 + 4\,m^2}$$

```
F = k (l - l0);
```

F_y = F (1.5 m + d) / l;

Total force (weight) of one block

F_y /. d -> 2 m

k (1.32939 m)

6: Wing

(a)

Find length of wing

```
In[8]:= L[x_] = 200 Sqrt[1 - x^2 / 17];
        Solve[L[x] == 0, x]
```

Out[9]= $\{\{x \to -\sqrt{17}\}, \{x \to \sqrt{17}\}\}$

Set up forces and radii

```
In[10]:= Fg = {0, -1600, 0} N ;

Rg = {Sqrt[17] / 2, 0, 0} m ;

Flift = {0, 50 Sqrt[17] Pi, 0} N ;

Rlift = {4 Sqrt[17] / (3 Pi), 0, 0} m ;
```

Solve newton's 2nd law equations

```
In[14]:= N@Solve[{Flift + Fg + {0, Freact, 0} == 0,
    {0, 0, Treact} + Rg × Fg + Rlift × Flift == 0}, {Freact, Treact}]

Out[14]= {{Freact → 952.344 N , Treact → 2165.15 J }}
```

(b)

Problem analysis

Basic assumptions: There are no structural loads being carried as torques in the joints...
The strut is mounted *d* meters below the wing mount

Forces

```
Freact = {Freactx, Freacty, 0};
Fstrut = Fstrutl * -Normalize[{2, d, 0}];
Rstrut = {2 m , 0, 0};
```

General Solution

```
In[18]:= sol = Solve[{Flift + Fg + Freact + Fstrut == 0, Rstrut × Fstrut + Rg × Fg + Rlift × Flift == 0},
     {Freactx, Freacty, Fstrutl}][[1]];
```

Final result:

```
In[19]:= N@niceForm[Freact /. sol]
         N@(Fstrutl /. sol)
```

Out[19]//TraditionalForm=

$$\frac{\hat{i}\,(-2165.15\text{ N})}{d} + \hat{j}\,(-130.232\text{ N})$$

Out[20]=

$$\frac{\sqrt{4. + \text{Abs}[d]^2}\,\left(-1082.58\text{ N}\right)}{d}$$

Specific solution

Assuming that $d = 1\,\text{m}$ yields the following specific result:

In[21]:=
```
N@niceForm[Freact /. sol /. d → 1]
N@(Fstrutl /. sol /. d → 1)
```

Out[21]//TraditionalForm=

$$\hat{\imath}\,(-2165.15\text{ N}) + \hat{\jmath}\,(-130.232\text{ N})$$

Out[22]= -2420.71 N

7: Floating block

Problem Analysis

This problem has the trivial case, solved here, where the block is a cube. Another (pair of) solutions exist, but calculating them is an exercise for another day.

Forces

```
Flead = {0, -(m - (m / lead (element) [EntityProperty["Element", "Density"]]) *
      water (chemical) [density]) * g , 0};
Fg = {0, -(10 cm * 10 cm * d * 20 kg/m^3 * g), 0};
Fbuoyancy = {0, (10 cm * 10 cm * d * water (chemical) [density] / 2 * g), 0};
```

Solve Equations of Motion

```
Solve[(Flead + Fg + Fbuoyancy == 0) /. d -> 10 cm , {m}]
{{m → 526.422 g }}
```

8: Beam

Problem Analysis

Definitions:
- Origin is at left support
- x right, y up

Calculating equivalent forces

```
Module[{f},
 f[x_] := {0, -Piecewise[{{200, x < 4}, {200 (7 - x) / 3, x ≥ 4}}], 0};
 Fload = Integrate[f[x], {x, -3, 7}];
 Rload = Integrate[Norm@f[x] { x, 0, 0}, {x, -3, 7}] / Norm[Fload];
]
```

Setting up forces

```
Fpin = {0, Fpiny, 0};
Froller = {0, Frollery, 0};
Rroller = {7, 0, 0};
```

```
Fload + Fpin + Froller
```

```
{0, -1700 + Fpiny + Frollery, 0}
```

```
Rload × Fload + Rroller × Froller
```

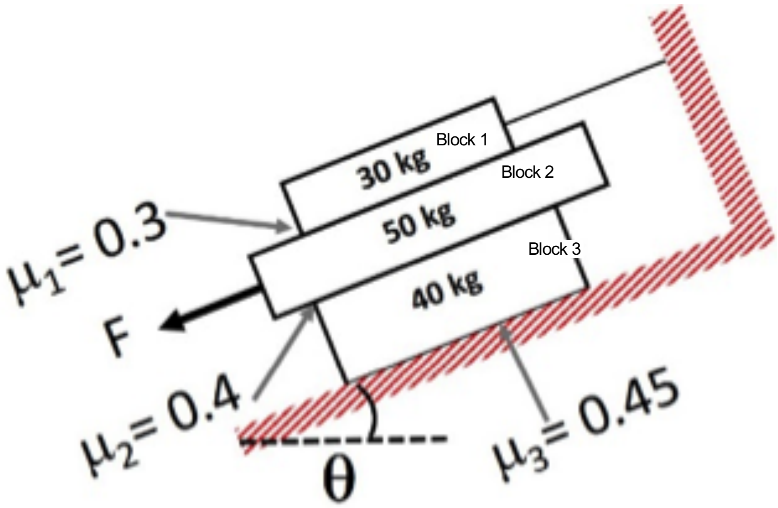
```
{0, 0, -2200 + 7 Frollery}
```

Solving equations of motion

```
N@Solve[{Fload + Fpin + Froller == {0, 0, 0},
   Rload × Fload + Rroller × Froller == {0, 0, 0}}, {Fpiny, Frollery}]
```

```
{{Fpiny → 1385.71, Frollery → 314.286}}
```

9: Three blocks



Problem analysis

This problem is statically indeterminate because for a given load, the force can be distributed in many ways between the tension in the rope and the friction on the base. For the remainder of this problem, I assume those indeterminisms are resolved in the most favorable way for the blocks not slipping.

Figure out forces perpendicular to the force F

```
Clear[θ, F]
```

```
g = g;
Fn12[θ_] := 30 kg g Cos[θ];
Fn23[θ_] := Fn12[θ] + 50 kg g Cos[θ];
Fn3g[θ_] := Fn23[θ] + 40 kg g Cos[θ];
```

Consider the system containing only block 1

```
block1slips[θ_, F_] := - 30 kg g Sin[θ] ≥ 0.3 * Fn12[θ]
```

Consider the system containing only block 2

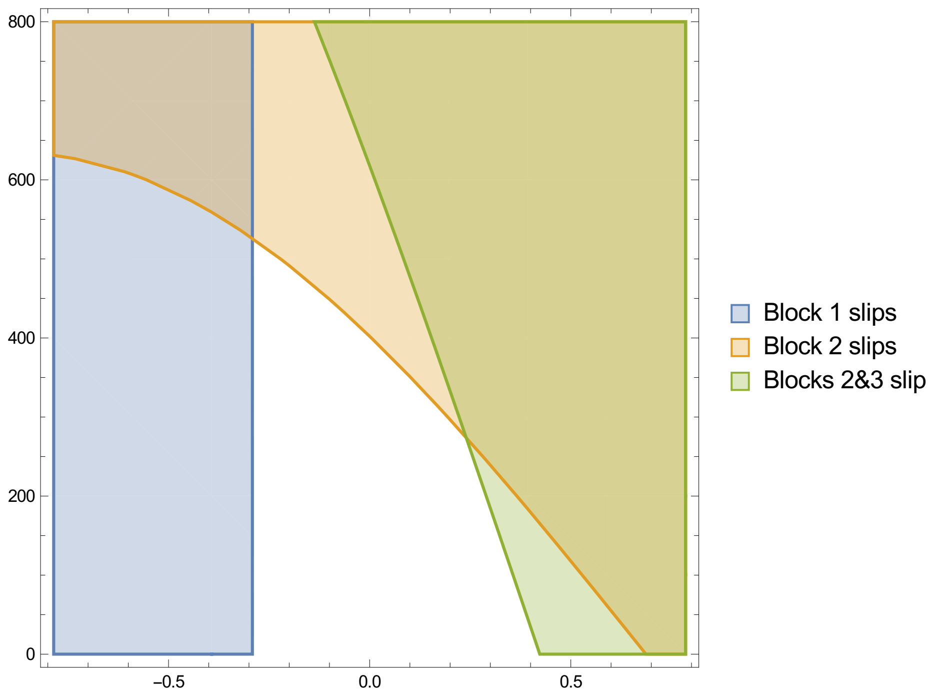
```
block2slips[θ_, F_] := F + 50 kg g Sin[θ] ≥ 0.3 * Fn12[θ] + 0.4 * Fn23[θ]
```

Consider the system containing blocks 2 and 3

```
blocks23slip[θ_, F_] := F + 90 kg g Sin[θ] + 50 kg g Sin[θ] ≥ 0.3 * Fn12[θ] + 0.45 * Fn3g[θ]
```

Plot the results

```
RegionPlot[Evaluate@{block1slips[θ, F], block2slips[θ, F], blocks23slip[θ, F]},
 {θ, -Pi/4, Pi/4}, {F, 0, 800 N}, PlotPoints → 5,
 PlotLegends → {"Block 1 slips", "Block 2 slips", "Blocks 2&3 slip"},
 AxesLabel → {"Angle(radians)", "Force(N)"}]
```



To clarify how this graph works: Each region represents the set of conditions under which the specified sets of joints would be satisfied with yielding, under the most optimal of the available statically indeterminate load distributions. In particular, the lowest color for a given coordinate is always the first to occur.

10: Static Indeterminism

1. Parked car

Because there are an infinite number of ways the wheels could be applying force toward each other with the parking brakes on, it is indeterministic.

2. Problem 9

As touched on before, if the string is slightly streachier than the frictional joints (a detail that lives outside the model), it will absorb less load, and the converse is true also.