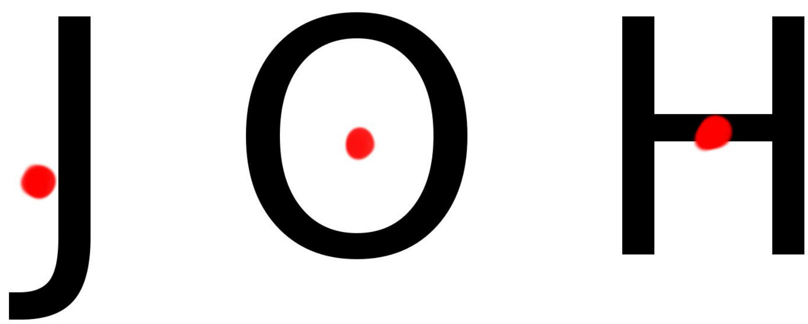
Thinking about distributed forces

1. In 1968, Dick Fosbury won a high jump gold medal at the Summer Olympic Games using his revolutionary new technique appropriately called the ``Fosbury Flop.'' The complete biomechanics is quite involved but do some research and then discuss what role the position the center of mass plays in the jump.

Because the Fosbury Flop involves arching your back to place your center of mass outside the extents of your body, it allows the jumper to physically pass over the pole even though their center of mass was never higher than it. This reduces the takeoff energy required for high jumps.

2. The diagram below shows three different shapes cut out of a metal plate. Mark the approximate location of the COM on each.



3. Locate the center of mass of the plate and the solid from Shapes II: Derivatives and Integrals in Multiple Dimensions 22(a) and 22(b).

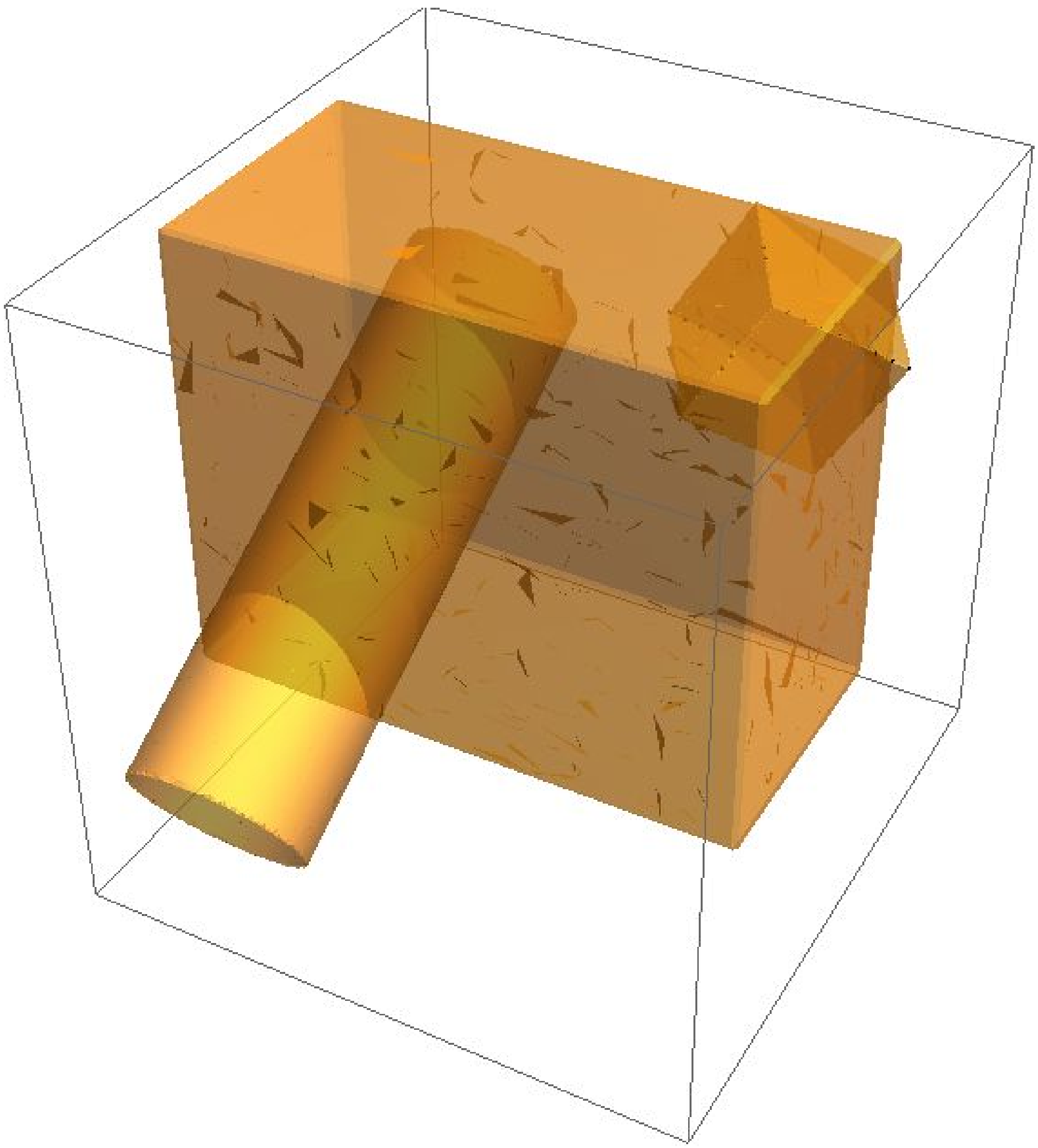
22 a) $\frac{36\,\hat{j}}{7}$

22 b) $\frac{\hat{i}}{5}+\frac{2\,\hat{j}}{5}+\frac{\hat{k}}{5}$

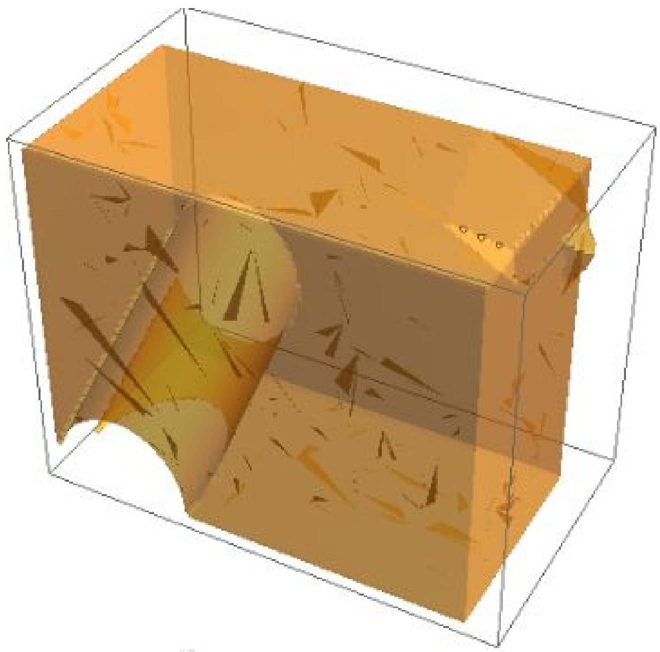
4. Define two rectangular blocks and a cylinder. Combine them in some way to make your own object. Determine the center of mass of your object analytically (by integrals and/or tables). Verify your answer using SolidWorks.

I could create a simple region here that I can figure out how to symbolically integrate, but I'm currently going through the process of transitioning from "math=skill" to "math=tool", so I'm going to take this chance to make a region I don't know how to integrate, and let Mathematica figure out the integration. Sue me.

Regions



Combined region



Volume=

$$Volume = -\frac{128\left(-11915701765390059783744289725 00\sqrt{2}+37143667279192900598753009184\pi\sqrt{3}-16508296568530178043890226304\sqrt{3}+26264539011254179398539294880\pi\sqrt{6}-11673128449446301954906353280\sqrt{6}-16851347041807654097826534509 25\right)}{9\left(\sqrt{2}-2\right)^2\left(\sqrt{2}+1\right)^6\left(\sqrt{2}+2\right)^4\left(2\sqrt{2}+3\right)^5\left(3\sqrt{2}+4\right)^5\left(7\sqrt{2}+10\right)^2\left(12\sqrt{2}+17\right)^5\left(41\sqrt{2}+58\right)^3}$$

$$\approx 651.143$$

Distributed Forces: Conceptual

1. A child places a toy submarine into a tub of water. The sub has a mass of 500 grams, and is has a total volume of 1 liter (i.e., when fully submerged it displaces a liter of water). What could you put inside the sub to make it sink to the bottom of the tub? Explain your answer(s).

a. One liter of water.

Were the child to put one liter of water inside the sub, it would definitely sink because its total mass (1500g) would exceed its buoyancy (1000g). Unfortunately, fitting 1L of water inside a container with external volume 1L is near-impossible, so I wish the child luck.

b. 500 grams of water.

500 grams of water (or anything else) would make the submarine almost neutrally buoyant. Which side of the line it fell on would depend on a few subtle effects. If the tub of water is more than a few degrees away from 4℃, then the sub (which masses 1000 grams) would sink, because the lowered density of the surrounding water would cause the displaced liquid to weigh slightly less than 1000 grams. Concretely, at 20℃, water weighs 0.9982 g/cm^3 , so the submarine would be 1.8g heavier than the water it displaces. On the other hand, if the tub is exactly 4℃, then the sub would float because the decrease in mass associated with dispelling air to add the water would make it (marginally) float. Somewhere in between, the sub would gradually sink until the pressure-induced increase in ambient water density caused it to reach equilibrium.

c. 700 grams of water

It sinks.

d. You don't need to put anything in. It already displaces more water than it weighs so it's already on its way down.

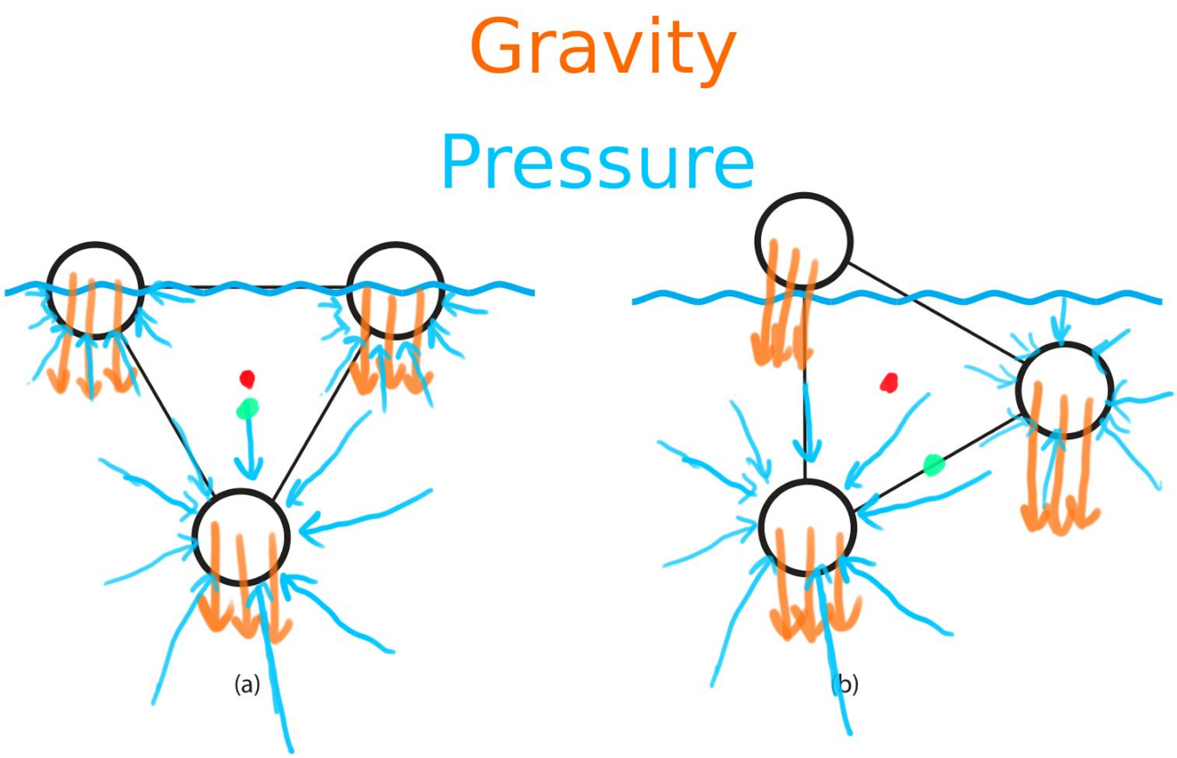
Nope, displacement == buoyancy, so if displacement>weight, the sub floats.

2. Homer was out fishing. To make room for his catch, he threw his anchor overboard. The anchor sunk to the bottom of the sea. Did the sea level a) rise, b) fall, or c) stay the same after he dropped it in?

It fell. Because the boat is at equilibrium, the amount of water displaced because of the anchor must be the volume of water with equivalent mass to the anchor. Because the anchor is more dense than water, this volume is significantly larger than the volume of the anchor itself. This means the amount the boat rises (lowering the sea level) is significantly more than the amount the anchor displaces (raising the sea level), leading to a net drop.

3. An autonomous underwater vehicle (AUV) can be modeled as three, hollow cylinders in the arrangement shown (front view) below...

a. There are two distributed forces acting on the AUV: gravity and buoyancy. Sketch the distribution of these forces using about 10-20 arrows for gravity, and 10-20 arrows for buoyancy. Do this for both cases.



d. Comment on what both the net force and the net moment acting on the AUV are in both cases.

Case 1	Case 2
Net force: 0 Net moment: 0	Net force: *slightly* down Net moment: Out of page

Distributed Forces: Computational

4. The diagram below shows the distributed lift force acting on a wing. The lift force varies as a function of position along the wing according to...

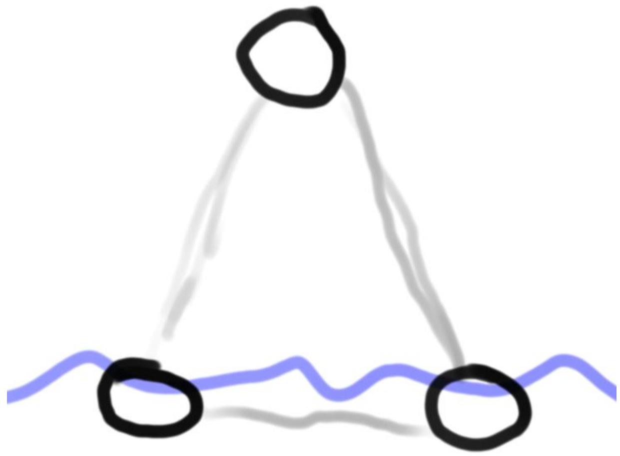
$$Magnitude = 50\sqrt{17}\pi$$
$$Location{:}x = \frac{4\sqrt{17}}{3\pi}$$

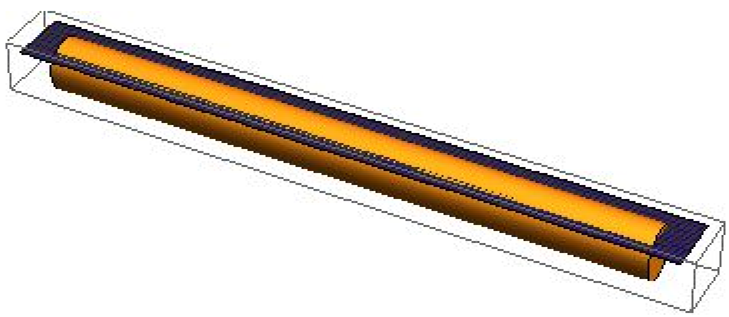
5. Remember the AUV in the problem above? Well, let's make it a bit more real. We'll make the cylinders out of aluminum, 2 m in length with a wall thickness of 1 cm (both on the walls and on the caps) and inner diameter of 15 cm. Again, assume that the struts have negligible weight and volume.

a. Determine the location of the center of buoyancy when the AUV is not loaded.

See section 5 in the Mathematica document (lots of calculations happened here)
The center of buoyancy is located straight between the two lower tubes, $0.55834cm$ below their axis.

b. Sketch how the AUV will float when it is not loaded.





c. What is the maximum load the AUV can carry without sinking?

$$51.8952 kg$$

6. Consider an ABS plastic body that is a solid, half-cylinder (length = 1 m, radius = 15 cm) with matching half-hemispherical (radius = 15 cm) ends.

a. Determine the location of the center of mass of the body analytically.

$\frac{5(160+9\pi)}{48\pi}$ below the COM

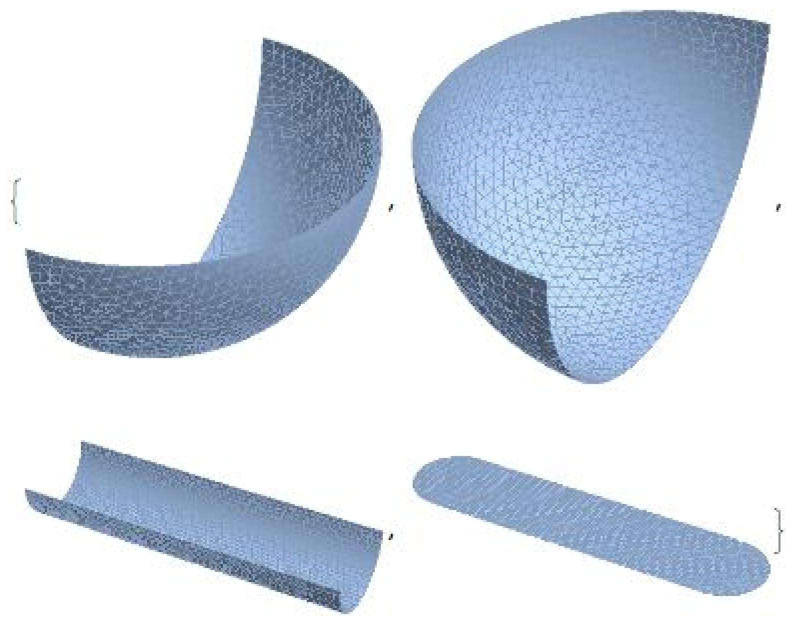
b. Verify your answer using Solidworks. Locate the center of buoyancy, assuming that the waterline goes to the top surface.

I don't see a personal learning goal in replicating this using SolidWorks. It simply isn't the right tool for these calculations.

c. Now consider the body to be a steel, thin-walled shell with the outer dimensions as above and wall thickness = 0.1cm. Where is the shell's center of mass?

This problem ended my honeymoon period with Mathematica. Congratulate yourselves, it needed to happen and this finally did it. For example, it should be fairly easy to define the region here as the RegionBoundary[] of the region above, but Mathematica simply chokes and spins for unacceptably long periods of time. Eventually, I figured out a way to make it easy enough for to finish quickly, but the frustration taught me the value of hand computation on occasion (don't tell, it's embarrassing). I did manage to eventually finish the problem computationally to repair my self-image.

Consider the shell to be a 2D region embedded in 3D space and defined as the union of the following 4 regions:



Note that this representation simply treats the 1mm thick wall as "thin" and ignores the question of where the thickness lies.

Defining the origin to lie at the center of the flat plane, the COM then becomes:

$$-\frac{15(40+3\pi)\hat{k}}{40+29\pi} \approx -5.65474\,\hat{k}$$

d. Where is the center of buoyancy?

In the same location as the COM was for the original plastic body,

$\frac{5(160+9\pi)}{48\pi}$ below the COM

e. Will it float in water?

Yes, total mass is $7.72773kg$, whereas the mass of displaced water is $42.4115kg$

f. How will the centers of mass and buoyancy change if several steel ball bearings (radius = 0.2cm) are placed inside the shell?

The introduction of several ball bearings would not change the center of buoyancy, but would change the center of mass in a difficult to predict way that depends on the angle at which it is floating. With enough ball bearings, they would tend to congregate at one end, making the object float vertically rather than horizontally.

Reflections

The good

- Finally, this problem set corrected my irrational love of Mathematica. The problems here became symbolically challenging enough that I was forced to think about exactly how to split it up myself, something that's been needing to happen for a couple weeks. Thank you. I've been pushing the boundaries to see how far they go, and now I know.
- This problem set didn't take me tooo long. I still had time to do some optimization work toward the final project.

The bad

- While I did a reasonable job budgeting my time, that ultimately did not result in me getting the head start on the next problem set that I was hoping for.

The ugly

- Thinking back, it's hard to enumerate what I learned in the process of working on this problem set. Most of the formal material was things I've learned before, and while the exercises were *challenging*, that doesn't necessarily mean they were *enlightening*. I need to think more before finalizing this statement, but thinking about QEA in general, I think there is too much work and too little learning happening.

Project Plan

The rest of this assignment was at least meant to be short enough that you would have time to do some stuff for the project. At a minimum, you should come to class on Thursday with a plan for your project that addresses the following questions:

a. What is your proposed schedule?
b. What do you see as “doable” tasks right now? Identify which parts of the project seem tractable, and discuss how you will tackle them.
c. What do you see as the major roadblocks (conceptually or technically) right now? Make a list of questions that you have.

In addition to a project plan and decomposition, we would love to see you make some concrete progress before class – but only if that's tractable given time constraints.