Spanning Vectors

Eric Miller - QEA - April 10, 2016

Definitions

Independence

a) Yes, independent

b) Dependent

c) Dependent $\frac{c-a}{2} = b$

d) Dependent

e) Independent

Orthogonality

2.

```
{1, 2, 3}.{-3, 2, 1}
```

4

a) Not orthogonal

```
{1, 0, -3}.{3, 2, 1}
```

0

b) Orthogonal, not orthonormal

c) Not orthogonal (parallel)

3.

$$\begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix} . x == \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

```
LinearSolve[(1 1 1; 3 1 2; 1 2 2), (1; 2; 4)] // MatrixForm
```

$$\begin{pmatrix} -2 \\ -2 \\ 5 \end{pmatrix}$$

Spans

4.

These vectors span the $z = 0$ plane.

5.

These vectors span the entire $\mathbb{R}_2$ space.

6.

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} == 2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

True

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} == \frac{5}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

True

$$\text{Solve}\left[\text{RotationMatrix}[\theta].\{2, 3\} == \left\{\frac{5}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}\right]$$

$$\left\{\left\{\theta \to \text{ConditionalExpression}\left[-\frac{\pi}{4} + 2\,\pi\,C[1],\ C[1] \in \text{Integers}\right]\right\}\right\}$$

The two vectors are 45 ° rotated from each other.

Eigendecomposition

7.

Applying the eigenvector equation to each column of A demonstrates this.

8.

```
In[31]:= a = ( 3 / 2   -1 / 2 );
             ( -1 / 2   3 / 2 )
MatrixForm@a
Grid@Eigensystem@a

Q == MatrixForm@Transpose@Eigenvectors@a
Λ == MatrixForm@DiagonalMatrix@Eigenvalues@a
Q^-1 == MatrixForm@Inverse@Transpose@Eigenvectors@a
```

Out[32]//MatrixForm=

$$\begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix}$$

Out[33]=

2	1
{-1, 1}	{1, 1}

Out[34]= $Q == \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$

Out[35]= $\Lambda == \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

Out[36]= $\frac{1}{Q} == \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

9.

a.

Working out the multiplication, it becomes o

b.

Probably easiest to prove this inductively. Basically, all the $Q\,Q^{-1}$ pairs in the middle go away, leaving the middle power.

c.

Try multiplying the proposed rhs with the origional rhs. Everything goes away, leaving the identity matrix.