1. Using computational tools
    a. A*v

    $$\begin{pmatrix} -3 \\ -7 \\ 4 \end{pmatrix}$$

    b. A(1:2,:)*v

    $$\begin{pmatrix} -3 \\ -7 \end{pmatrix}$$

    c. A(:, 2:4)*v

    Not possible, matrix A isn't wide enough to consider the 4th column

2. 2x2 identity matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

3. 3x3 deletion matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

4. 4x4 swap matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

5. (5-8) Scaling matrix

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

9. Asymmetric scaling

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

10. Rotation 1

The given matrix rotates 90° around the z axis (+y goes to +x)

11. Rotation 2

The given matrix rotates 90° around the x axis (+y goes to +z)

12. Rotation 3

The given matrix rotates 90° around the y axis (+z goes to +x)

13. General xy rotation matrix

$$\theta = 0 \implies \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \theta = \frac{\pi}{2} \implies \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

14. Using the rotation matrix

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

15. Proof time

$$\begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a\cos(\theta) - b\sin(\theta) \\ b\cos(\theta) + a\sin(\theta) \\ c \end{pmatrix}$$

$$Norm\begin{pmatrix} a\cos(\theta) - b\sin(\theta) \\ b\cos(\theta) + a\sin(\theta) \\ c \end{pmatrix} = \sqrt{(a\sin(\theta) + b\cos(\theta))^2 + (a\cos(\theta) - b\sin(\theta))^2 + c^2}$$

$$= \sqrt{a^2\sin^2(\theta) + a^2\cos^2(\theta) + b^2\sin^2(\theta) + b^2\cos^2(\theta) + c^2}$$

$$= \sqrt{a^2 + b^2 + c^2}$$

16. Explanation

This matrix represents a rotation by angle θ around the z axis.
Other rotation matrices below:

$$\text{Rotate around y} = \begin{pmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{pmatrix}$$

$$\text{Rotate around x} = \begin{pmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{pmatrix}$$

Matrices acting on Matrices

17. AB

$$\begin{pmatrix} -14 & 2 \\ -3 & -3 \end{pmatrix}$$

18. BA

$$\begin{pmatrix} -10 & 11 \\ 2 & -7 \end{pmatrix}$$

19. A*(B+C)

$$\begin{pmatrix} -16 & 12 \\ -12 & 3 \end{pmatrix}$$

20. AB+AC

$$\begin{pmatrix} -16 & 12 \\ -12 & 3 \end{pmatrix}$$

As expected, the outputs are equal.

21. A book

a.

Cover faces left

b.

Cover faces up. Matrix multiplication non-commutivity confirmed

c.

As expected, result is {0,-1,0}

d.

As expected, result is {0,0,1}

e.

Because matrix multiplication is communative, multiplication of transformation matrices early works as expected.

22. Inverse matrices

$$R = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$S = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$