Section 1: Buoyancy

Starting point

- Dry mass of hopper is required to equal 50g or more, for this purpose hitting exactly 50g should be possible.
- Density of air at sea level is $1.204\ \text{kg/m}^3$ (kilograms per cubic meter)
- Density of 100% pure hydrogen is $0.08375\ \text{kg/m}^3$ (kilograms per cubic meter)
- Density of 90% H_2, 10% O_2 is $0.08375\ \text{kg/m}^3$ (kilograms per cubic meter)

Assuming 90% effectiveness at separation during electrolysis:

$$Buoyancy =$$

$$\text{air density standard conditions } (20\,°\text{C}, 101.325\ \text{kPa}) - \left(0.9\ \text{hydrogen density standard conditions } (20\,°\text{C}, 101.325\ \text{kPa}) + 0.1\ \text{hydrogen density standard conditions } (20\,°\text{C}, 101.325\ \text{kPa})\right)$$

$$=$$

$$1.121\ \text{kg/m}^3 \text{ (kilograms per cubic meter)}$$

Balloon volume

The party balloons we are given have a nominal diameter of 12”. Assuming spheres, their combined volume is thus:

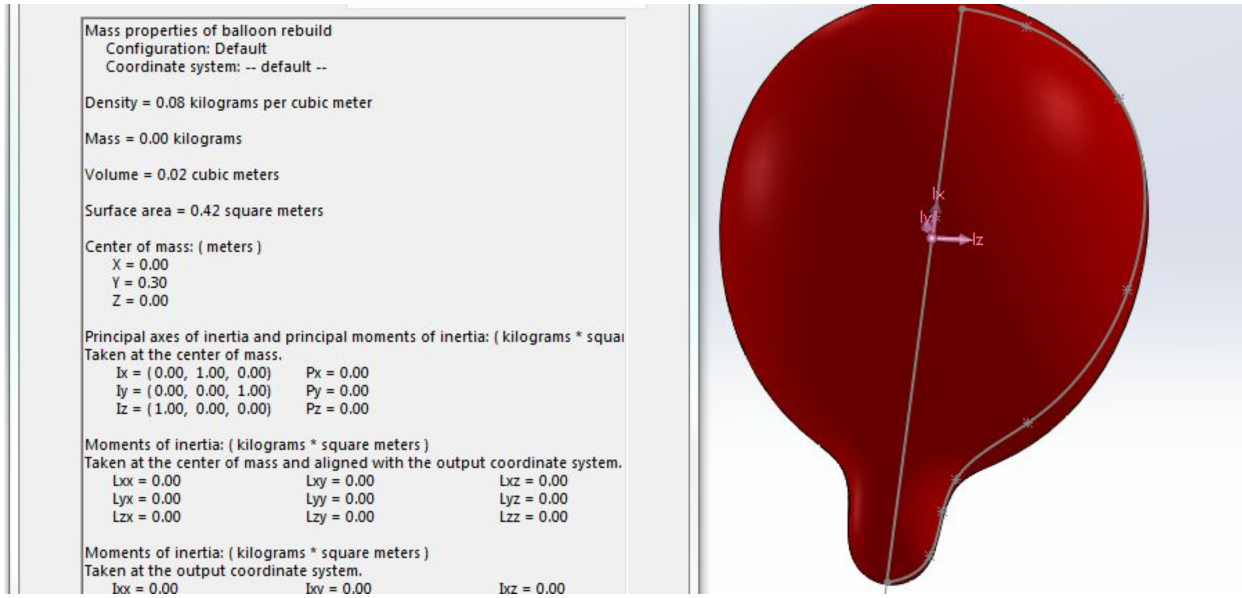
$$\frac{4}{3}\pi\,(6\ \text{inches})^3 \times 2 = 0.02965\ \text{m}^3$$

Were this the case,

$$\frac{4}{3}\pi\,(6\ \text{inches})^3 \times 2 \times 1.121\ \text{kg/m}^3 = 33.24\ \text{g}$$

which is well shy of the needed lift.

Fortunately: inflating, measuring and photographing a balloon, then tracing in SolidWorks reveals:



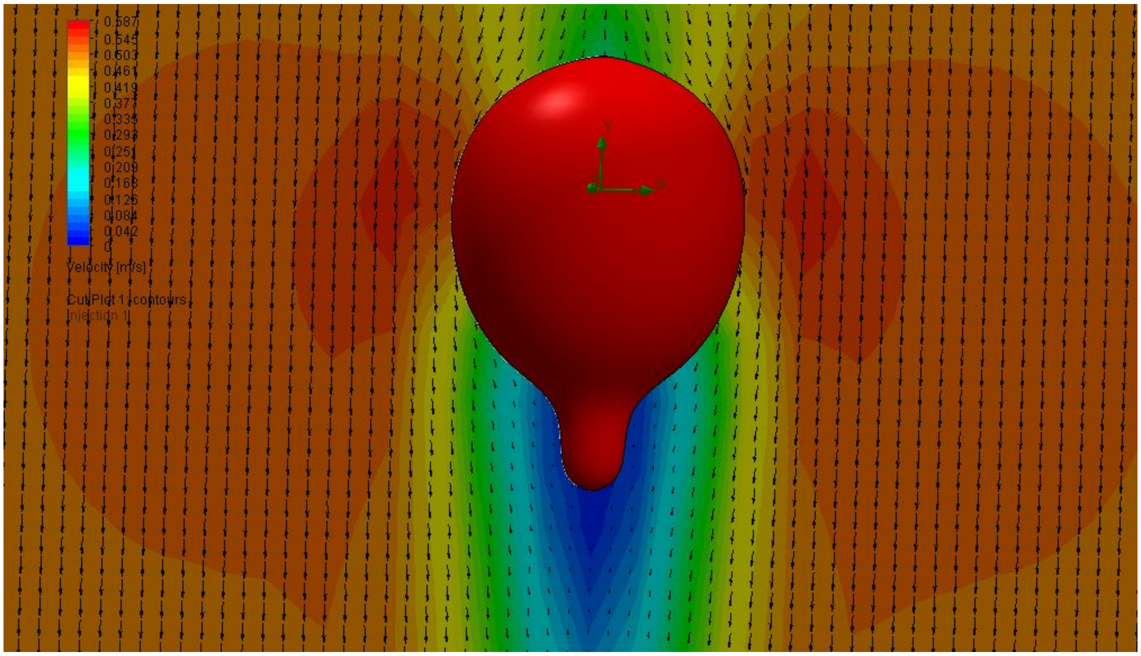
Volume of one balloon = 24607840.12 mm^3 = 0.024607840 m^3
Buoyancy of 2 balloons = 0.024607840 m^3 * 2 * 1.121 kilograms per cubic meter = **55.17 grams**

Note that it is probably possible to get significantly more volume inside by further over-inflating if needed.

Section 2: Aerodynamics

The obvious question is: how much vertical velocity does 5.17g of lift allow for? Note that this equates to about .05N of force.

Solidworks flow simulation results (velocity = 0.5 m/s):



Goal Name	Unit	Value	Averaged Value	Minimum Value	Maximum Value	Progress [%]	Use In Convergence	Delta	Criteria
GG Force (Y) 1	[N]	-0.004	-0.004	-0.004	-0.004	100	Yes	3.231e-005	6.490e-005

Calculations

Because aerodynamic drag is roughly proportional to the square of velocity:

$$0.004N * 2 = C * (0.5\frac{m}{s})^2$$

$$0.032 = C$$

Section 3: Ascent rate

As derived previously:

$$0.05N = (0.032) * {v_{terminal}}^2$$

Thus

$$v_{terminal} = 1.25\frac{m}{s}$$

Because the target height of the device is 300ft, estimated travel time is

$$\frac{300ft}{v_{terminal}} = 73.1s$$

Section 4: Comparison to simulation

The simulation data, pasted below, showed an estimated max height of 300 feet after 65 seconds of flight. The calculations here do not take into account gradual acceleration, and record more significant figures of accuracy, but are fundamentally based on the same aerodynamic calculations, and record a very similar height of 300 feet in 73.1 seconds. Stored mechanical energy is not particularly relevant to the calculations here, but there are other significant sources of error that make experimental testing necessary to figure out the correct timing to reach the height goal as accurately as possible. Firstly, the surface properties of the balloons aerodynamically are not necessarily modeled correctly, nor is the angle of attack modelled. Secondly the gondola itself has significant effect on the flight that is not properly accounted for in simulations or calculation. Overall, the damper effect used in the simulation matches expected results reasonably well.