You may work with others to figure out how to do questions, and you are welcome to look for answers in the book, online, by talking to someone who had the course before, etc. However, you must write the answers on your own. You must also show your work (you may, of course, quote any result from the book).

1. Consider the two linear functions $h\colon \mathbb{R}^3 \to \mathcal{P}_2$ and $g\colon \mathcal{P}_2 \to \mathcal{M}_{2\times 2}$ given as here.

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \mapsto (a+b)x^2 + (2a+2b)x + c \qquad px^2 + qx + r \mapsto \begin{pmatrix} p & p-2q \\ q & 0 \end{pmatrix}$$

   Use these bases for the spaces.

$$B = \langle \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \rangle \quad C = \langle 1+x, 1-x, x^2 \rangle \quad D = \langle \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 3 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix} \rangle$$

   (a) Give the formula composition map $g \circ f\colon \mathbb{R}^3 \to \mathcal{M}_{2\times 2}$ directly from the above definition.

   (b) Represent f and g with respect to the appropriate bases.

   (c) Represent $g \circ f$ with resepct to the appropriate bases.

   (d) Check that the two matrices from the second part multiply to the matrix from the third part.

2. Use these matrices.

$$A = \begin{pmatrix} 1 & 3 & -1 \\ 0 & 1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 2 \\ -1 & 0 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & -2 & 3 \end{pmatrix} \quad D = \begin{pmatrix} 0 & 0 \\ 4 & 0 \end{pmatrix}$$

   (a) Find $-3A$ and $2B - 5D$, or state "not defined."

   (b) Which matrix products are defined?

   (c) Compute AB and AC, or state "not defined."

3. Show how to use matrix multiplication to bring this matrix to echelon form.

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 3 & 1 & -1 \\ 7 & 11 & 4 & -3 \end{pmatrix}$$