You may work with others to figure out how to do questions, and you are welcome to look for answers in the book, online, by talking to someone who had the course before, etc. However, you must write the answers on your own. You must also show your work (you may, of course, quote any result from the book).

1. Find a basis for, and the dimension of, each space.

   (a)
   $$\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \in \mathbb{R}^4 \mid x - w + z = 0 \}$$

   (b) the set of 5×5 matrices whose only nonzero entries are on the diagonal (e.g., in entry $1,1$ and $2,2$, etc.)

   (c) $\{ a_0 + a_1 x + a_2 x^2 + a_3 x^3 \mid a_0 + a_1 = 0 \text{ and } a_2 - 2a_3 = 0 \} \subseteq \mathcal{P}_3$

2. Give a basis for the column space of this matrix. Give the matrix's rank.

   $$\begin{pmatrix} 1 & 3 & -1 & 2 \\ 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 4 \end{pmatrix}$$

3. Give a basis for the span of each set, in the natural vector space.

   (a) $\{ \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 12 \\ 6 \end{pmatrix} \}$

   (b) $\{ x + x^2, 2 - 2x, 7, 4 + 3x + 2x^2 \}$