

You may work with others to figure out how to do questions, and you are welcome to look for answers in the book, online, by talking to someone who had the course before, etc. However, you must write the answers on your own. You must also show your work (you may, of course, quote any result from the book).

1. Assume that each matrix represents a map $h: \mathbb{R}^m \rightarrow \mathbb{R}^n$ with respect to the standard bases. In each case, (i) state m and n (ii) find $\mathcal{R}(h)$ and $\text{rank}(h)$ (iii) find $\mathcal{N}(h)$ and $\text{nullity}(h)$, and (iv) state whether the map is onto and whether it is one-to-one.

(a) $\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 3 \\ 2 & 3 & 4 \\ -2 & -1 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix}$

2. Verify that the map $h: \mathbb{R}^m \rightarrow \mathbb{R}^n$ represented by this matrix with respect to the standard bases

$$\begin{pmatrix} 2 & 1 & 0 \\ 3 & 1 & 1 \\ 7 & 2 & 1 \end{pmatrix}$$

is an isomorphism.

3. For these matrices

$$A = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 4 & -2 \\ 0 & 0 & 0 \\ 1 & -1 & 5 \end{pmatrix} \quad C = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

- (a) Find, or state “not defined”: $5A$, $6B$, $7C$.
- (b) Find, or state “not defined”: $A + B$, $B + C$.