Collision and Packet Loss Analysis in a LoRaWAN Network

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***Abstract*—Internet of things (IoT) is considered as the next technological revolution. Therefore, many solutions are developed either in free, *i.e.* ISM bands or in non free bands with the ultimate aim of affording connectivity over several kilometers. Based on this feature, in urban environment the density of IoT devices will be extremely high. In this paper we propose to analyze the collision and packet loss when LoRaWAN is considered. Based on the LoRaWAN features, we develop closed-form expressions of collision and packet loss probabilities. Simulation results confirm our theoretical developments. We also show that our theoretical expressions are more accurate than the Poisson distributed process to describe the collisions.**

I. Introduction

The Internet of things (IoT) deployment in free bands is mainly based on two communication technologies which are Sigfox [1] and LoRa [2]. Such technologies are called low power wide area network (LPWAN) and they share the same objectives, *i.e* the establishment of long range, low power and low data rate communications. Despite these similarities, they are technically and economically opposites. Indeed the Sigfox physical layer is based on ultra narrow band communication, whereas LoRa uses a spreading spectrum technique to exchange information. In this paper we focus our attention on LoRa in order to analyze the collision and packet loss when LoRaWAN is considered. Indeed, if we believe to the IoT success in our every day life, it is predictable to have several millions of IoT devices using such modulation techniques. Therefore, the coexistence of all these devices, when the communication is performed using the same range, will threat IoT communication with interferences.

The channel access in LoRaWAN class A is based on an ALOHA principle. For networks using an ALOHA type channel access, many packet collision studies have been done before. The considered system in this paper is different than the one used for classical analysis of packet collision in an un-slotted ALOHA based protocol [3]. Indeed, packet time on air and time between two successive transmissions depend on the IoT application. Thus, the model allowing to describe the collision effect based on Poisson distributed process (PDP) is not accurate enough. However, in our case this modeling is useful to define lower and upper bounds of the probability of success for a given spreading factor. In this paper we propose a more accurate and specific approach to predict the collision and packet loss. Thus, based on the LoRaWAN MAC mechanisms we develop closed-form expressions of the probability of collision and packet loss and our theoretical analyses are confirmed by the simulations.

The paper is organized as follows. In section II the LoRa modulation and the LoRaWAN MAC layer are introduced. In section III we develop the received signal model that we consider. Based on this model, section IV is devoted to the collision and packet loss theoritical analyse. Simulation results are given in section V.

II. LoRa and LoRaWAN

The following sections are dedicated to introduce the LoRa physical and MAC layers which is based on the patent [4]. For more details the reader is referred to [5]. It should be noted that the LoRa physical Layer is not published.

A. LoRa physical layer principle

LoRa is based on Chirp Spread Spectrum (CSS) modulation. CSS was proposed for the first time for communication systems by Winkler [6] and application to digital communication by Berni [7]. CSS is considered as a subcategory of Direct-Sequence Spread Spectrum. CSS is compliant with IoT network needs because it permits to come over the receiver's sensitivity issue and increase the communication range at the cost of a reduced spectral efficiency. The spectrum spreading in LoRa is achieved using a chirp signal that can be described by its instantaneous phase $\phi(t)$ or a specific time function $f_c(t)$. $f_c(t)$ is called the raw chirp that:

- increases linearly, for an up raw chirp, from an initial value $-\frac{B}{2}$ to a final value $\frac{B}{2}$,
- decreases linearly, for a down raw chirp, from an initial value $\frac{B}{2}$ to a final value $-\frac{B}{2}$,

where B stands for the ISM signal bandwidth used for the communication[1]. The raw chirp time duration is equal to the symbol period T_s. $f_c(t)$ is defined as follows:

$$f_c(t) = \pm \frac{B}{T_s} t \tag{1}$$

The relationship between the bandwidth and the symbol period is given by $T_s = \frac{2^{SF}}{B}$ where SF stands for the spreading factor exponent $SF \in [7, ..., 12]$. Let D_s be the symbol rate of the

[1] B depends on the used ISM band and can be chosen equal to 125, 250 or 500 kHz

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transmitted signal and D_b the bit rate, then $D_s = D_b/SF$. Longer range is achieved by varying the spreading factor, however to meet highly robust communication it is possible to vary the coding rate.

With LoRa, symbols are obtained from a binary combination of SF bits. Each symbol is associated to a unique chirp. The different chirps are orthogonal to each other in order to retrieve at the receiver the symbols without inter-symbol interference (IES). If we note M the set of symbols, the chirp associated to the symbol m, $m \in [0, M-1]$, is obtained by delaying the raw chirp $f_c(t)$ by $\tau_m = \frac{m}{B}$. The chirp outside $[-\frac{T_s}{2}, \frac{T_s}{2}]$ is cyclically shifted in the interval $[-\frac{T_s}{2}, -\frac{T_s}{2}+\tau_m]$. Thus, the chirp associated to the transmission of the m^{th} symbol is decomposed of 2 parts:

1) from $t \in [-\frac{T_s}{2}, -\frac{T_s}{2} + \tau_m[$, raw chirp (up ou down) advanced of $(T_s - \tau_m)$,
2) from $t \in [-\frac{T_s}{2} + \tau_m, \frac{T_s}{2}]$, raw chirp (up ou down) delayed of τ_m.

For an up chirp, we obtain:

$$f_c^m(t) = \frac{B}{T_s}(t - \tau_m) + B \qquad \text{for } t \in [-\frac{T_s}{2}, -\frac{T_s}{2} + \frac{m}{B}[$$
$$f_c^m(t) = \frac{B}{T_s}(t - \tau_m) \qquad \text{for } t \in [-\frac{T_s}{2} + \frac{m}{B}, \frac{T_s}{2}]$$

Thus, the expression of the baseband transmitted signal by the node n is given as follows:

$$r_n(t) = \sum_{k \in S_l} e^{j2\pi f_{c,k}(t-kT_s)(t-kT_s)+j\phi_0} \tag{2}$$

where $f_{c,k}(t)$ represents the transmitted chirp at time kT_s, S_l the set of transmitted symbols inside the packet p and ϕ_0 an initial phase. If we note K the S_l size, thus the silent time duration of the LoRa node will be at least equal to $\frac{KT_s}{d_c} - KT_s$. Thus, from $t + KT_s$ to $t + \frac{KT_s}{d_c}$ the node will be silent, where d_c is the duty cycle.

B. LoRaWAN: a LoRa Mac layer

LoRaWAN is an open standard developed by the LoRa Alliance. It's one of the possible MAC layer for the LoRa modulation and obviously the well known. The LoRaWAN specification defines 3 categories of nodes:

- Class A: a basic class of LoRa that is implemented in all LoRa chips. It allows bi-directional communications which is usually originally started by the node in an asynchronous way. The uplink transmission triggers two short downlink receive windows. The transmission slot is scheduled when needed by the node in a random time basis. According to LoRaWAN specifications, class A is an ALOHA based-protocol.
- Class B: this class is conceived to guarantee uplink and downlink separation. Nodes are synchronized using a beacon transmitted by the gateway. Thus, they can receive information from Internet without sending requests.
- Class C: the node has continuously open receive windows that are closed only while transmitting. Compared to A and B classes, C class consumes more energy to operate but it offers the lowest latency.

The packet time duration is called the time on air. This value depends on several parameters, such SF, B, the size of the payload, the coding rate, etc. After each uplink packet transmission, the node waits for a gateway acknowledgment (ACK) of the correct packet reception. LoRaWAN allows two possible acknowledgments on two different channels. The first is transmitted by the gateway with a constant delay of T_1 after the end of the uplink packet reception. The gateway uses the same channel than the preceding uplink for this ACK. The second ACK is transmitted on a different channel[2] than the uplink after a time $T_2 > T_1 + T_{on}^g$, where T_{on}^g is the time on air of the packet transmitted by the gateway. For more details on the LoRaWAN MAC structure see [6].

III. Received signal model

Based on the complex envelope definition of (2), we express in this section the model of the received signal that we use. We consider a system which is composed of one LoRa gateway and N LoRa nodes. The n^{th} node transmit P_n packets, and $n(p)$ represents the packet number p transmitted by the node n with a time on air T_{on}^n.

The bandwidth is narrow enough to make the assumption of flat fading propagation channel. We note h_n the channel coefficient associated to the node n. If $x(t)$ is the received signal at the LoRa gateway, we have:

$$x(t) = \sum_{n=1}^{N} h_n \sum_{p=1}^{P_n} r_n(t - T_p^n) \tag{3}$$

Let T_p^n be the beginning of the LoRa packet p. Based on this, we define:

$$\begin{aligned} T_p^n &= (p-1)(T_{on}^n + T_{off}^n) + \sum_{u=0}^{p} T_r(u) \\ &= (p-1)\frac{T_{on}^n}{d_c} + \sum_{u=0}^{p} T_r(u) \end{aligned} \tag{4}$$

where $T_r(u)$ is a stationary iid random process with entries uniformly distributed. $T_r(u)$ is used for modeling the different node constraints in terms of communications with the gateways. In the following sections we consider that $T_r(u)$ is uniformly distributed in the set $[T_{min}, T_{max}]$. This assumption is justified due to the huge number of IoT applications. T_{on}^n and T_{off}^n are the time on air and time off air, respectively.

IV. Collisions and packet loss at the LoRaWAN gateway

A. Probability of Collision at the LoRaWAN gateway

In the following we express the probability of collision between $n(p)$ and $n'(p')$, given that $n(p)$ represents the node of interest. If $C_{n'(p')}^{n(p)}$ denotes this collision, based on (4), the collision event is defined as follows:

$$C_{n'(p')}^{n(p)} = \{T_{p'}^{n'} \in \Omega_C = [T_p^n - T_{on}^{n'},\ T_p^n + T_{on}^n]\}$$

[2]Based on this we focus our attention only on T_1.

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A collision between between $n(p)$ and $n'(p')$ will happen if $T_{p'}^{n'} \in \Omega_C$. $\mathbb{P}\left(C_{n'(p')}^{n(p)}\right)$ represents the probability of this collision and we have:

$$\mathbb{P}(C_{n'(p')}^{n(p)}) = Pr\left(T_{p'}^{n'} \in \Omega_C\right) \tag{5}$$

The random variable $T_{p'}^{n'}$ is the summation of multiple uniform random variables (see (4)). For $p' >> 1$ we suppose, using the central limit theorem, that $T = \sum_{u=0}^{p'} T_r(u)$ is Gaussian distributed with a mean

$$\mu = p' \frac{T_{max} + T_{min}}{2} \tag{6}$$

and a variance

$$\sigma^2 = \frac{p'}{12}(T_{max} - T_{min})^2 \tag{7}$$

Under this assumption, $T_{p'}^{n'}$ is now a summation between two random variables T and $T_{on}^{n'}$ where

$$T \sim \mathcal{N}(\mu, \sigma^2) \tag{8}$$

and

$$T_{on}^{n'} \sim \mathcal{U}(T_{on}^{min}, T_{on}^{max}) \tag{9}$$

Indeed, due to the varity of IoT applications, the time on air $T_{on}^{n'}$ of the node n' can be considered uniformly distributed between the minimal time on air T_{on}^{min} and the maximal time on air T_{on}^{max}. Based on this, $\mathbb{P}(C_{n'(p')}^{n(p)})$ can be rewritten given $T_{on}^{n'}$ as

$$\mathbb{P}(C_{n'(p')}^{n(p)}) = \frac{1}{\Delta T_{on}} \int_{T_{on}^{min}}^{T_{on}^{max}} Pr\left(T_{p'}^{n'} \in \Omega_C \quad | \; T_{on}^{n'}\right) dT_{on}^{n'} \tag{10}$$

where $\Delta T_{on} = T_{on}^{max} - T_{on}^{min}$. With (8) we have

$$Pr\left(T_{p'}^{n'} \in \Omega_C | T_{on}^{n'}\right) = Q\left(\frac{T_p^n - T_{on}^{n'} - \mu'}{\sigma}\right) - Q\left(\frac{T_p^n + T_{on}^n - \mu'}{\sigma}\right)$$

where

$$\mu' = \mu + (p'-1)\frac{T_{on}^{n'}}{d_c} \tag{11}$$

given $Q(x) = (1 - Q(-x))$, (10) can be rewritten as follows:

$$\begin{aligned}\mathbb{P}(C_{n'(p')}^{n(p)}) = &\frac{1}{\Delta T_{on}} \int_{T_{on}^{min}}^{T_{on}^{max}} Q\left(\alpha T_{on}^{n'} + \beta\right) dT_{on}^{n'} \\ &- \frac{1}{\Delta T_{on}} \int_{T_{on}^{min}}^{T_{on}^{max}} Q\left(\gamma T_{on}^{n'} + \delta\right) dT_{on}^{n'}\end{aligned} \tag{12}$$

where

$$\alpha = \frac{p'-1}{\sigma d_c} \tag{13}$$

$$\beta = \frac{1}{\sigma}\left[\mu - T - T_{on}^n\left(1 + \frac{p-1}{d_c}\right)\right] \tag{14}$$

$$\gamma = \frac{1}{\sigma}\left(\frac{p'-1}{d_c} + 1\right) = \alpha + \frac{1}{\sigma} \tag{15}$$

$$\delta = \frac{1}{\sigma}\left[\mu - T - T_{on}^n \frac{p-1}{d_c}\right] = \beta + \frac{T_{on}^n}{\sigma} \tag{16}$$

with the two change of variables $y = \alpha T_{on}^{n'} + \beta$ and $z = \gamma T_{on}^{n'} + \delta$, (12) is rewritten as

$$\mathbb{P}(C_{n'(p')}^{n(p)}) = \frac{1}{\alpha \Delta T_{on}} \int_{y_1}^{y_2} Q\left(y\right) dy - \frac{1}{\gamma \Delta T_{on}} \int_{z_1}^{z_2} Q\left(z\right) dz \tag{17}$$

where

$$y_1 = \alpha T_{on}^{min} + \beta, \;\; y_2 = \alpha T_{on}^{max} + \beta$$
$$z_1 = \gamma T_{on}^{min} + \delta, \;\; z_2 = \gamma T_{on}^{max} + \delta$$

using:

$$\int_{x_1}^{x_2} Q(x)dx = \left[x_2 Q(x_2) - x_1 Q(x_1) + \frac{1}{\sqrt{2\pi}}\left(e^{-\frac{x_1^2}{2}} - e^{-\frac{x_2^2}{2}}\right)\right]$$

we get the probability of collision:

$$\mathbb{P}(C_{n'(p')}^{n(p)}) = \frac{1}{\alpha \Delta T_{on}} f(y_1, y_2) - \frac{1}{\gamma \Delta T_{on}} f(z_1, z_2) \tag{18}$$

$\mathbb{P}(C_{n'(p')}^{n(p)})$ represents the probability of collision between $n(p)$ and $n'(p')$. As we consider $n(p)$ as the node of interest, a finite set $\Omega_{n'}(p)$ of packets p' of the node n' have a non-null probability to be in collision with $n(p)$. We note $N_p^{n'}$ the number of packet of n' that can be in collision with $n(p)$. Thus, the more we increase p , the higher $N_p^{n'}$ will be. For $n(p)$, the events $C_{n'(p')}^{n(p)}$ with $p' \in \Omega_{n'}(p)$ are disjoint. If we note $C_{n'}^{n(p)}$ the event associated to the collision of $n(p)$ with n', we have

$$C_{n'}^{n(p)} = \bigcup_{p' \in \Omega_{n'}(p)} C_{n'(p')}^{n(p)} \tag{19}$$

Thus the probability of collision of $n(p)$ with n' is:

$$\mathbb{P}(C_{n'}^{n(p)}) = \sum_{p' \in \Omega_{n'}(p)} \mathbb{P}(C_{n'(p')}^{n(p)}) \tag{20}$$

Numerically, we observed that (20) is independent of p. This result is interesting because it demonstrates that (20) is not dependent on the packet number p, consequently it doesn't depend on the time. Logically $\mathbb{P}(C_{n'}^{n(p)})$ is a function of T_{on}^n, d_c and the $T_{on}^{n'}$ spreading in time (*i.e* the support of the distribution).

Based on (20) we express the probability of at least one collision in an environment composed of N independent nodes. Thus the probability of having at least one collision between n and the rest of the nodes (using the same SF) is given by:

$$\mathbb{P}_c^n(N) = 1 - \prod_{\substack{n'=1 \\ n' \neq n}}^{N} \left[1 - \mathbb{P}(C_{n'}^{n(p)})\right] \tag{21}$$

B. Packet loss at the LoRaWAN gateway

If no collision occurs on $n(p)$ we consider that the packet will be acknowledge by the gateway. In this case, after a fixed delay T_1 the LoRaWAN gateway will use the same bandwidth to transmit the ACK. We note T_{on}^g the time on air used by the gateway to answer and we suppose that it can be considered as a constant value. During T_{on}^g, the gateway is unable to receive a packet. Thus, a packet loss will occurs if a node send an

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information during the gateway ACK of $n(p)$. This means that the packet loss of one node depends on the success of another node. Our goal is to express the probability of packet loss of $n'(p')$ based on the success of $n(p)$. This will occur when:

$$T_{p'}^{n'} \in \ \Omega_L = \left[\max(T_p^n + T_{on}^n, \tau - T_{on}^{n'}), \ \tau + T_{on}^g\right] \quad (22)$$

where $\tau = T_p^n + T_{on}^n + T_1$, and $\max(x, y)$ refers to x if $x > y$ and y otherwise. Based on (22) 3 cases must be considered: case 1 when $T_1 < T_{on}^{min}$, case 2 when $T_1 > T_{on}^{max}$ and case 3 when $T_1 \in [T_{on}^{min}, T_{on}^{max}]$. We define the probability of packet loss of $n'(p')$ as $\mathbb{P}\left(L_{n'(p')}^{n(p)}\right)$, where $L_{n'(p')}^{n(p)}$ is the event associated to the $n'(p')$ loss given the success of $n(p)$. Using the same approach as the one developed in the previous section, we have :

- In case 1:

$$\mathbb{P}\left(L_{n'(p')}^{n(p)}\right) = \frac{1}{\Delta T_{on}} \int_{T_{on}^{min}}^{T_{on}^{max}} Pr\left(T_{p'}^{n'} \in \Omega_L^1 \ \mid T_{on}^{n'}\right) dT_{on}^{n'} \quad (23)$$

- In case 2:

$$\mathbb{P}\left(L_{n'(p')}^{n(p)}\right) = \frac{1}{\Delta T_{on}} \int_{T_{on}^{min}}^{T_{on}^{max}} Pr\left(T_{p'}^{n'} \in \Omega_L^2 \ \mid T_{on}^{n'}\right) dT_{on}^{n'} \quad (24)$$

- In case 3:

$$\mathbb{P}\left(L_{n'(p')}^{n(p)}\right) = \frac{0.5}{\Delta T_{on}^2} \int_{T_{on}^{min}}^{T_1} Pr\left(T_{p'}^{n'} \in \Omega_L^2 | T_{on}^{n'}\right) dT_{on}^{n'} \quad (25)$$

$$+ \frac{0.5}{\Delta T_{on}^1} \int_{T_1}^{T_{on}^{max}} Pr\left(T_{p'}^{n'} \in \Omega_L^1 | T_{on}^{n'}\right) dT_{on}^{n'} \quad (26)$$

where $\Omega_L^1 = [T_p^n + T_{on}^n, \tau + T_{on}^g]$, $\Omega_L^2 = [\tau - T_{on}^{n'}, \tau + T_{on}^g]$, $\Delta T_{on}^2 = T_1 - T_{on}^{min}$ and $\Delta T_{on}^1 = T_{on}^{max} - T_1$. Based on (8) we have:

$$Pr\left(T_{p'}^{n'} \in \Omega_L^1 | T_{on}^{n'}\right) = Q\left(\alpha T_{on}^{n'} + \frac{\mu - \tau - T_{on}^g}{\sigma}\right) - Q\left(\alpha T_{on}^{n'} + \beta\right) \quad (27)$$

and

$$Pr\left(T_{p'}^{n'} \in \Omega_L^2 | T_{on}^{n'}\right) = Q\left(\alpha T_{on}^{n'} - \frac{T_{on}^g}{\sigma}\right) - Q\left(\gamma T_{on}^{n'}\right) \quad (28)$$

Following the same manipulations as the ones from (10) to (18), we can express $\mathbb{P}\left(L_{n'(p')}^{n(p)}\right)$ for the 3 previous cases.

Regardless the value of T_1, we note $L_{n'}^{n(p)}$ the event associated to the packet loss of n' given the success of $n(p)$. Thus we have:

$$\mathbb{P}\left(L_{n'}^{n(p)}\right) = \sum_{p' \in \Omega_{n'}^l(p)} \mathbb{P}\left(L_{n'(p')}^{n(p)}\right) \quad (29)$$

where $\Omega_{n'}^l(p)$ is the set of $n'(p')$ that can be lost given the success of $n(p)$. Thus the probability of packet loss (*i.e* a collision with the ACK transmitted by the gateway to $n(p)$) given the success of $n(p)$ is:

$$\mathbb{P}_l^n(N) = \mathbb{P}_s^n(N) \left(1 - \prod_{\substack{n'=1 \\ n' \neq n}}^{N} \left[1 - \mathbb{P}\left(L_{n'}^{n(p)}\right)\right]\right) \quad (30)$$

where $\mathbb{P}_s^n(N) = 1 - \mathbb{P}_c^n(N)$ is the probability of success of $n(p)$ (*i.e* no collision).

V. Simulation results

We consider a system which is composed of one gateway that can establish connections with N nodes using the same spreading factor. We also consider that the signals transmitted by nodes using different SF are orthogonal. The nodes access the channel randomly in time. The bandwidth is fixed to B = 125kHz and $d_c = 1\%$. $T_{on}^{n'}$ is uniformly distributed. The support of his distribution begins from the minimum time on air to the maximum time on air. These values are reported in table I and obtained using the online LoRaWAN calculator. The minimum and maximum number of packet per day are fixed by the minimum and maximum channel time duration used, respectively. For our simulations we consider a minimum channel time use of $30s$ and a maximum of $864s$.

SF	Time on air min (ms)	Time on air max (ms)	Payload size (bytes) min	Payload size (bytes) max
12	761.86	3219.49	1	59
11	380.93	1740.8	1	59
10	190.46	935.94	1	59
9	95.23	992.26	1	123
8	47.62	987.65	1	230
7	23.81	567.55	1	230

Table I: LoRaWAN maximum and minimum time on air and associated number of bytes in the payload. Time on air values obtained from the LoRaWAN calculator.

First of all, we compare our theoretical results with simulations. Fig. 1 shows the evolution of $\mathbb{P}(C_{n'(p')}^{n(p)})$ when $p = 100$, $SF = 12$ for $T_{on}^n = 2s$ and $1s$. As it can be seen, our theoretical results are consistent with the simulations. Indeed, we verify the accuracy of the theoretical analysis, where both theoretical expression and simulation curves coincide. In fig. 2 we show the evolution of $\mathbb{P}_c^n(N)$. On the one hand we compare our theoretical results with the simulation when $T_{on}^n = 1s$ and $2s$. In both cases, simulation and theory are superimposed. On the other hand, we compare our results with a Poisson distributed process (PDP) generally use to describe collisions in such a network and defined as follows:

$$\mathbb{P}_{PDP}(N) = 1 - e^{-\frac{2T_{on}^n}{D_p} N}$$

where $D_p = \frac{T_{on}^n}{d_c} + \frac{T_{max} + T_{min}}{2}$.

As we can see when T_{on}^n is closed to the mean value of $T_{on}^{n'}$, our approach and the one based on PDP give the same results. However, when it's not the case the PDP based approach is not accurate to predict the collision. Indeed, when $T_{on}^n = 1s$ we can see a difference with our results. For example, for $\mathbb{P}_c^n(N) = 0.3$ our theoretical results and also the simulation show that 580 nodes can be managed by the gateway, whereas the PDP predict 820 nodes.

In fig. 3 we compare the probability of at least one collision for a payload size between 1 and 59 bytes, when 1000 nodes are considered. These sizes are available for each SF in LoRaWAN. The results can be grouped in three parts providing the same performance. The first is composed of $SF = 7$ and

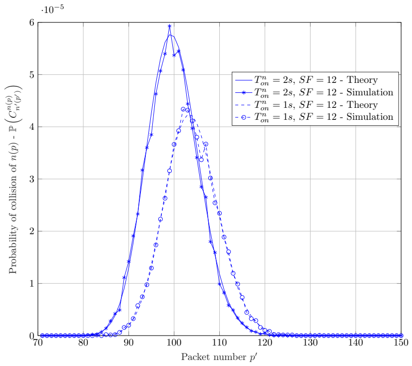

Figure 1: Probability of collision $\mathbb{P}\left(C^{n(p)}_{n'(p')}\right)$ when $p = 100$, $SF = 12$ and for $T^n_{on} = 1s$ and $2s$

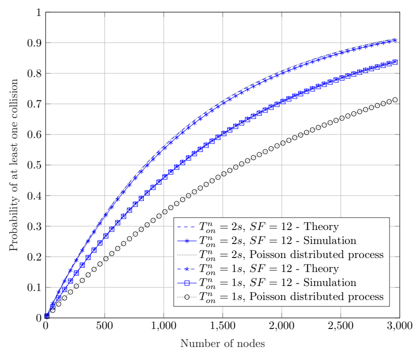

Figure 2: Probability of at least one collision for $SF = 12$, when $T^n_{on} = 1s$ and $2s$

8, the second of $SF = 9$ and the third of $SF = 10,\ 11,\ 12$. From the probability of collision point of view it's better to use $SF = 8$, $SF = 9$ or $SF = 12$. Indeed, $SF = 10$ and 11 give the same probability of collision than $SF = 12$ but they offer a lower sensitivity.

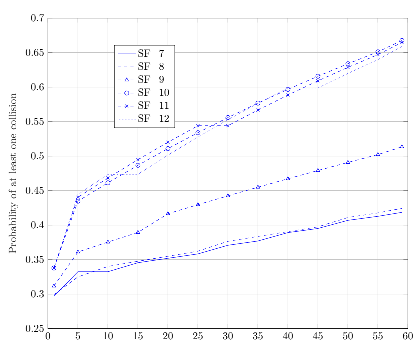

Figure 3: Probability of at least one collision for the different SF when 1000 nodes are considered

In fig. 4 and 5 we show the evolution of $\mathbb{P}^n_l(N)$ when $SF = 12$ and $SF = 7$, respectively. This allows to show the worst and the best case. For each SF, $T^g_{on} = T^{min}_{on}$ which corresponds to a ACK composed of 1 byte. We can see that the $\mathbb{P}^n_l(N)$ is bounded and pass through a maximum value. From the network point of view only the increasing part of the curves are interesting. Indeed the decreasing part corresponds to an important number of collision at the gateway.

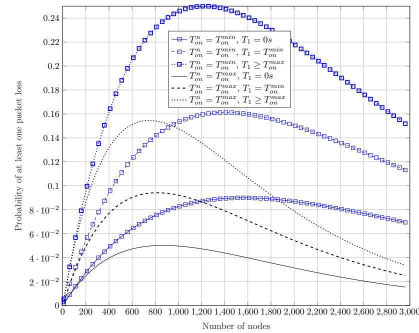

Figure 4: Probability of at least one packet loss, $SF = 12$

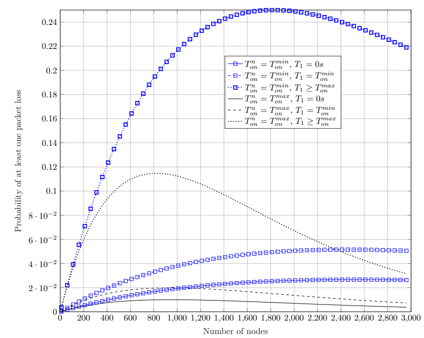

Figure 5: Probability of at least one packet loss, $SF = 7$

VI. Conclusion

In this paper we proposed an analysis of packet collision and loss when LoRaWAN is considered. Based on the LoRaWAN features, we developed theoretical expressions for both the collision and the packet loss. These developments have been confirmed by simulations results. We have also showed that our approach allows to more accurately describe the collision than the classical PDP approach. The perspectives of this work are numerous. We are currently working on the theoretical demonstration of the independence of $\mathbb{P}(C^{n(p)}_{n'})$ in p and the development of a software that can predict the interference level of each nodes based on their locations, duty-cycle, time on air, etc. Consider that the nodes with different SF can be non orthogonal is also an interesting perspective of this work.

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