Faculty of Computing,
Engineering & Technology

Digital modulation: ASK, FSK, PSK

Communications
COMMS (CE700038-2)

Alison L Carrington
C203
A.L.Carrington@staffs.ac.uk
www.fcet.staffs.ac.uk/alg1

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STAFFORDSHIRE UNIVERSITY

Overview

Digital modulation:
ASK, FSK, PSK

- Why is modulation is needed?
- What is modulation?
- Digital modulation & demodulation techniques
- Obtain SNR of system by:
  - Signal Power
  - Noise Power
- Determine the performance of different Digital Mod/Demod techniques with different amounts of noise

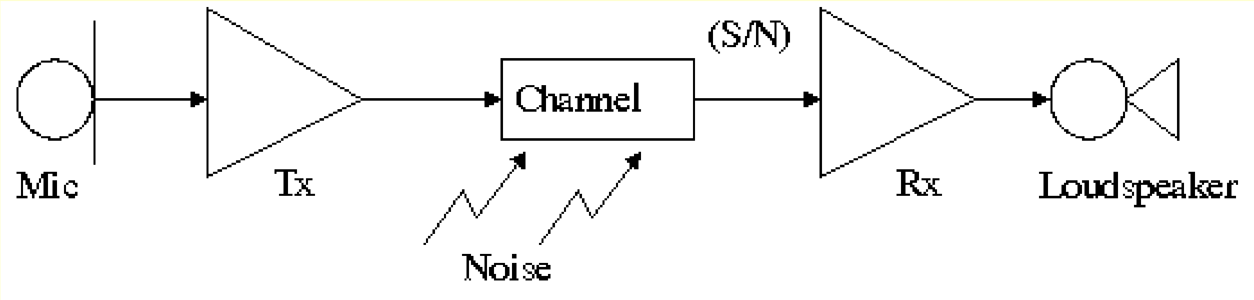
Introduction

Digital modulation: ASK, FSK, PSK

- Often send information in a digital form
  - As binary words, bits represented by 1’s & 0’s
  - Could be from a keyboard, characters encoded as 1’s & 0’s
  - Or a microphone (speech)
  - Or a temperature sensor
    - converted to binary form by A/D converter.
- Why do we need to modulate?

Why modulate?

- The purpose of a communication system is to transfer information from a source to a destination



- Problems arise in baseband transmissions:
  - Noise
    - external & circuit noise reduces the signal-to-noise (S/N) ratio at the receiver, (Rx) reducing the quality of the output.
  - Not able to fully utilise the available bandwidth:
    - telephone quality speech has a bandwidth $\simeq$ 3kHz
    - co-axial cable has a bandwidth of 100's of Mhz.

What is Modulation? #1

- A **message** signal containing **information** is used to control parameters of a **carrier** signal
  - i.e. the information is embedded onto the carrier
- The **message** can either be:
  - Analogue – *m*(*t*)
  - Digital – *d*(*t*), sequence of 1's and 0's
  - When message is
    - analogue = Analogue Modulation
    - digital = Digital Modulation
- The **carrier** could either a 'sinusoidal wave' or a 'pulse train'
- At the destination the **carrier+message** must be demodulated so that the **message** can be Rx'd

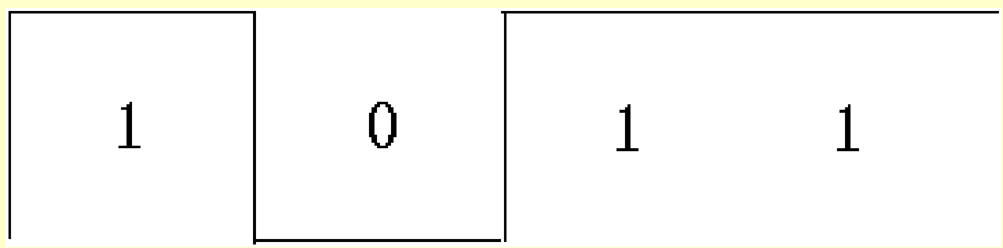
Digital modulation:
ASK, FSK, PSK

What is Modulation? #2

Digital modulation: ASK, FSK, PSK

- If the message $d(t)$ controls
  - amplitude = **AMPLITUDE SHIFT KEYING ASK**.
  - Frequency = **FREQUENCY SHIFT KEYING FSK**
  - Phase = **PHASE SHIFT KEYING PSK**.

- $d(t)$ is a binary or 2 level signal representing 1's and 0's

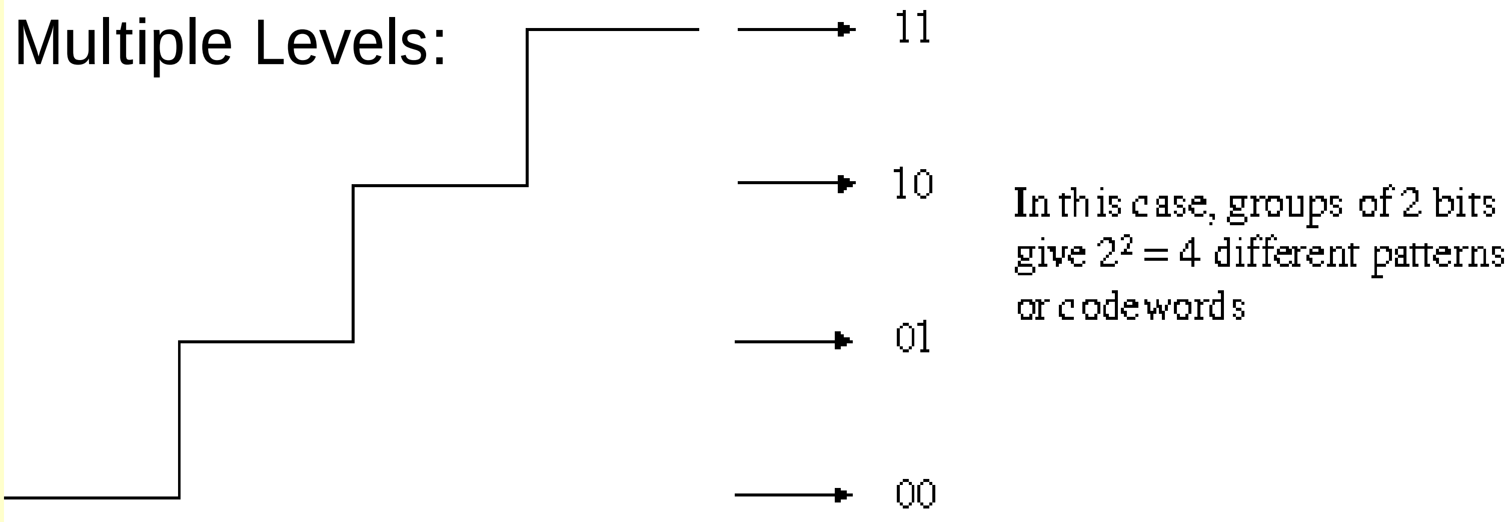


- This is binary or 2 level, *e.g.* Binary FSK, BFSK, BPSK, *etc.* or 2 level FSK, 2ASK, 2PSK *etc.*

What is Modulation? #3

Digital modulation:
ASK, FSK, PSK

- Multiple Levels:



- A message signal could be multi-level or *m* levels
  - each level represents a discrete pattern of 'information' bits. For example, $m = 4$ levels
- Why increase *M*?
  - As the number of levels increase the amount of information transmitted increases – but the probability of receiving an error also increases

Baud Rate #1

Digital modulation: ASK, FSK, PSK

- The baud rate of a data communications system is the number of symbols per second transferred.
- As a symbol may have > 2 states, it may represent > 1 binary bit (a binary bit always represents exactly two states).
- Therefore the baud rate may not equal the bit rate

Baud Rate #2

Digital modulation: ASK, FSK, PSK

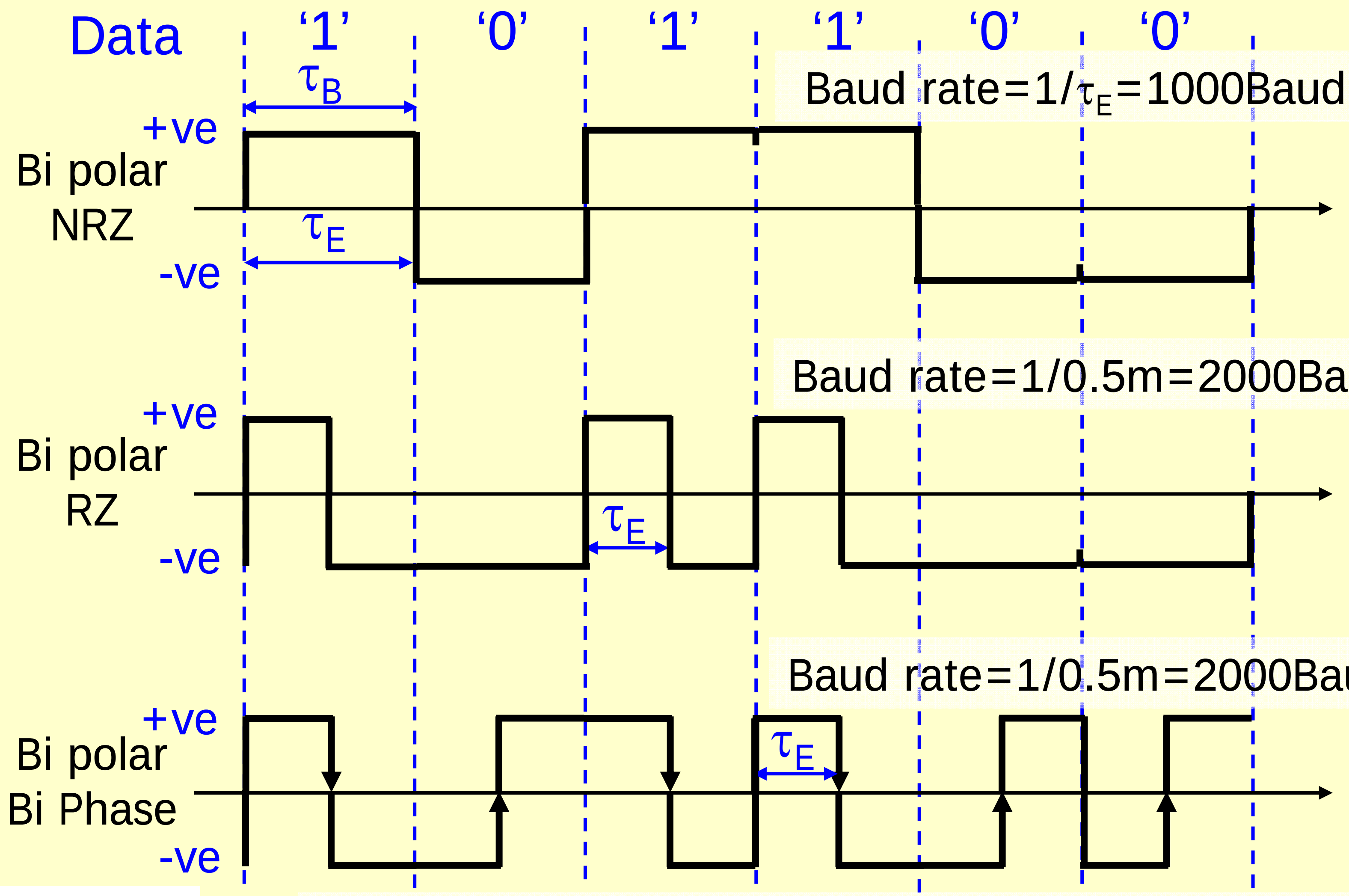
- A Bell 212A modem uses QPSK modulation
  - each symbol has one of four phase shifts (of 0(deg), 90(deg), 180(deg), or 270(deg))
  - two bits represent four states (00, 01, 10, and 11),
  - the modem transmits 1,200 bits/s of information, using a symbol rate of 600 baud.
- Usually the baud rate of a modem will not equal the bit rate and is of no interest to the end user--only the data rate, in bits per second.
- As modems transfer signals over a telephone line
  - the baud rate is limited to a maximum of 2400 baud.
  - This is a physical restriction of telephone lines
  - Higher data throughput achieved with 9600 or higher baud modems is accomplished by using sophisticated phase modulation, and data compression techniques

Baud Rate #3

- **Asymmetric Digital Subscriber Line** (**ADSL**) is used for Broadband Internet access
  - Allows faster data transmission over copper telephone lines than a conventional voiceband modem
  - By using the frequencies that are not used by a voice telephone call using Frequency Division Multiplexing (FDM).
- A splitter - or microfilter - allows a single telephone connection to be used for both ADSL service and voice calls simultaneously
  - As phone lines vary in quality and were not originally engineered with DSL in mind, it can generally only be used over short distances, typically less than 4km.

Baud Rate #4

Digital modulation: ASK, FSK, PSK



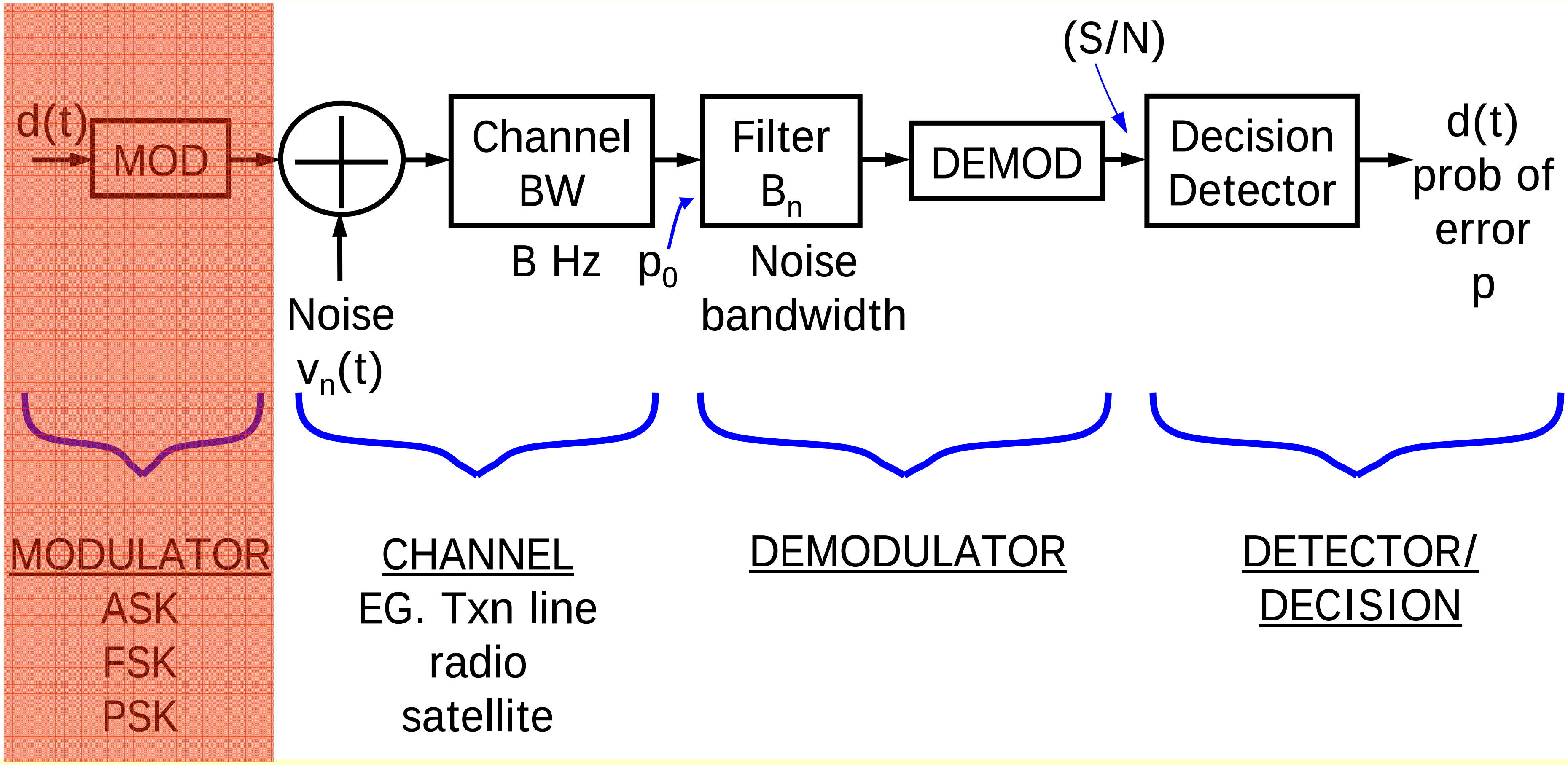
Bit rate=1/τ_B

Advantage of a transition in every bit – self checking

Baud rate=1/τ_E

e.g. if τ_B=1ms, Bit rate=1000bps

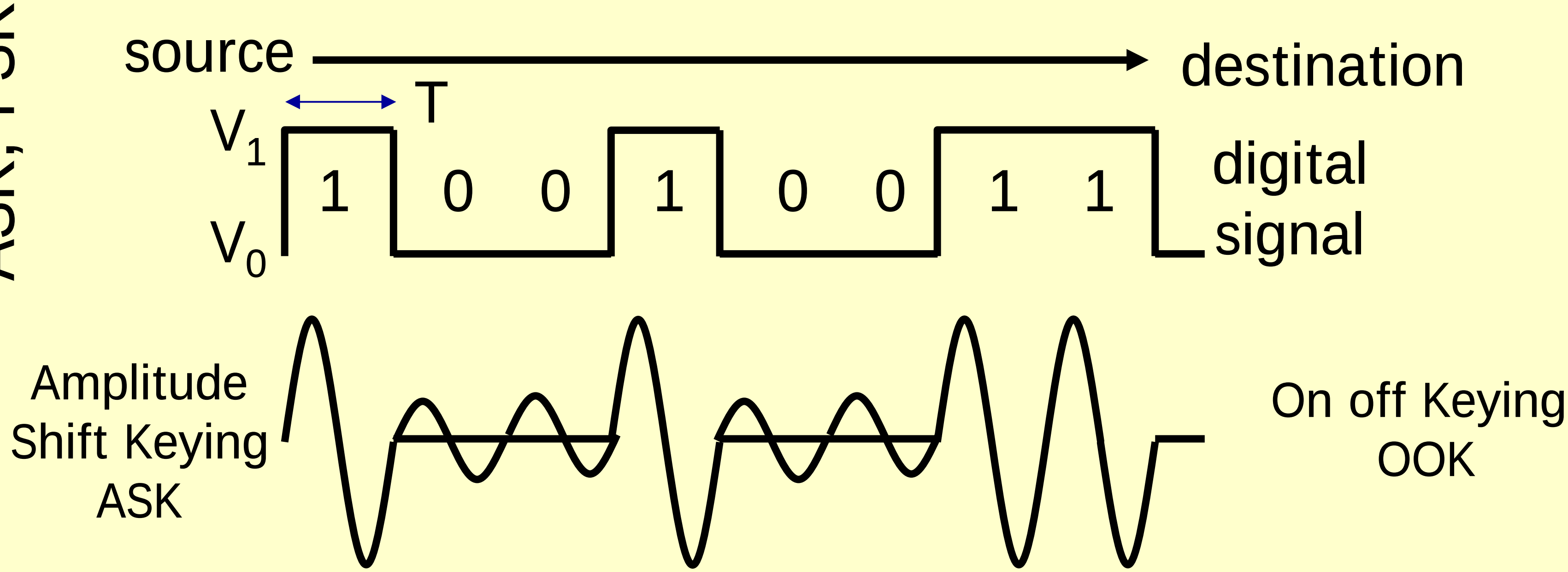
System Block Diagram



Amplitude Shift Keying, ASK

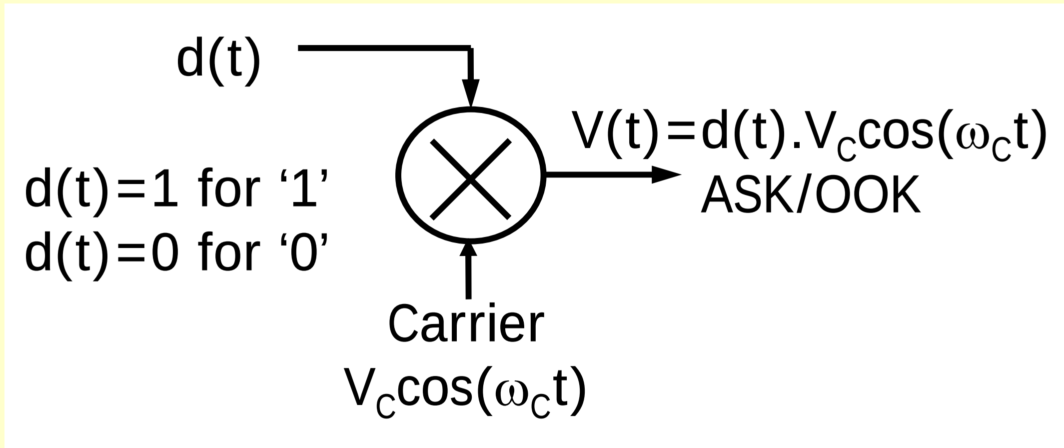
Digital modulation: ASK, FSK, PSK

- Different amplitudes of carrier
- Usually, one amplitude is ‘0’
  - i.e. presence and absence of carrier is used
- Susceptible to sudden gain changes
- Inefficient, up to 1200bps on voice grade lines
- Used over optical fiber – due to low error

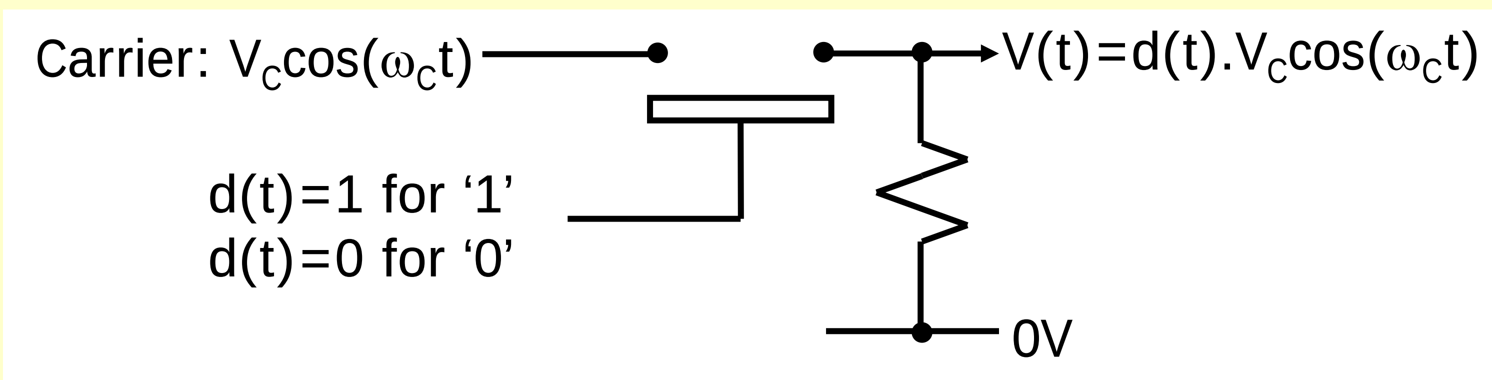


ASK Modulation of data sequence *d(t)*

- Amplitude Shift Keying
  - Analogue Multiplier

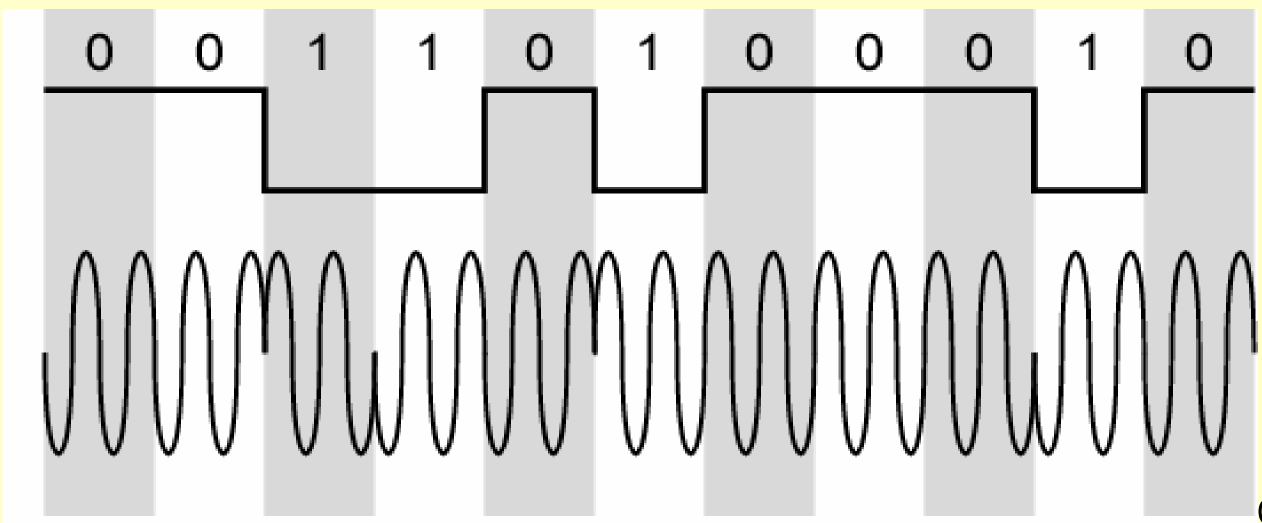


  - Analogue Gate (4066)



Phase Shift Keying, PSK

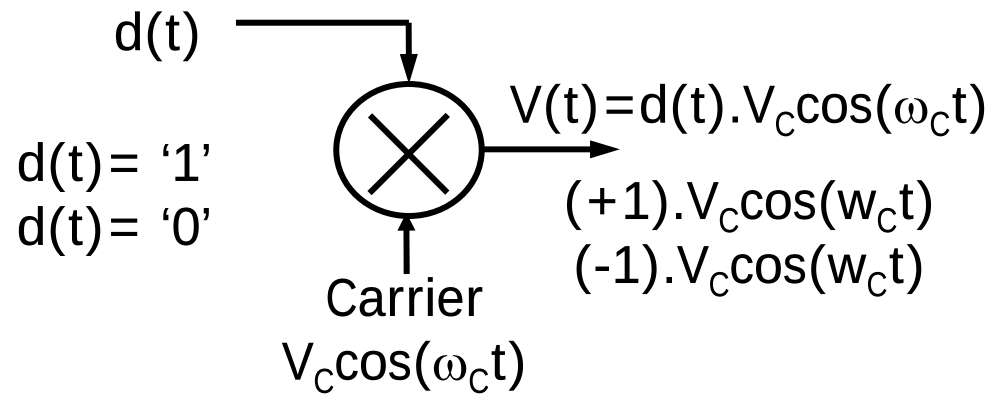
- Phase of carrier signal is shifted data
- Binary PSK: Two phases represent two binary digits
- Differential PSK: Phase shifted relative to previous transmission rather than some reference signal

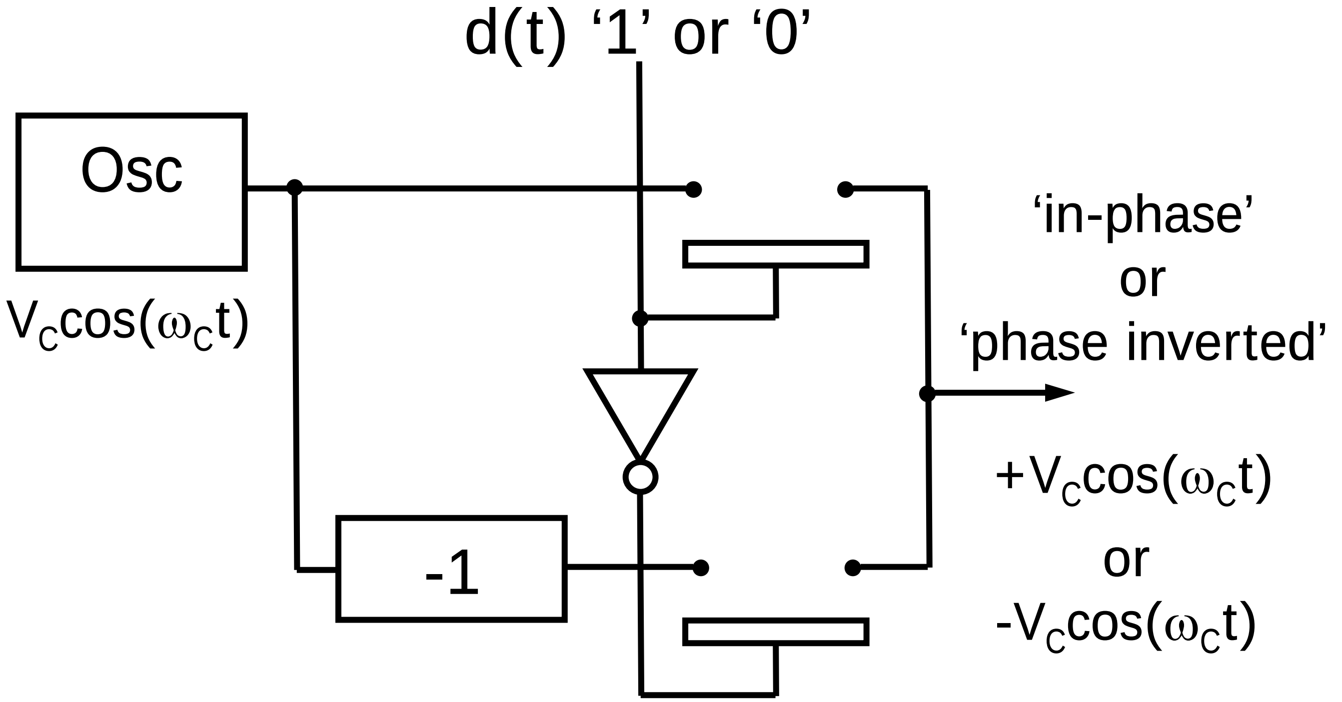


PSK Modulation of data sequence, *d(t)*

Digital modulation:
ASK, FSK, PSK

- Phase Shift Keying

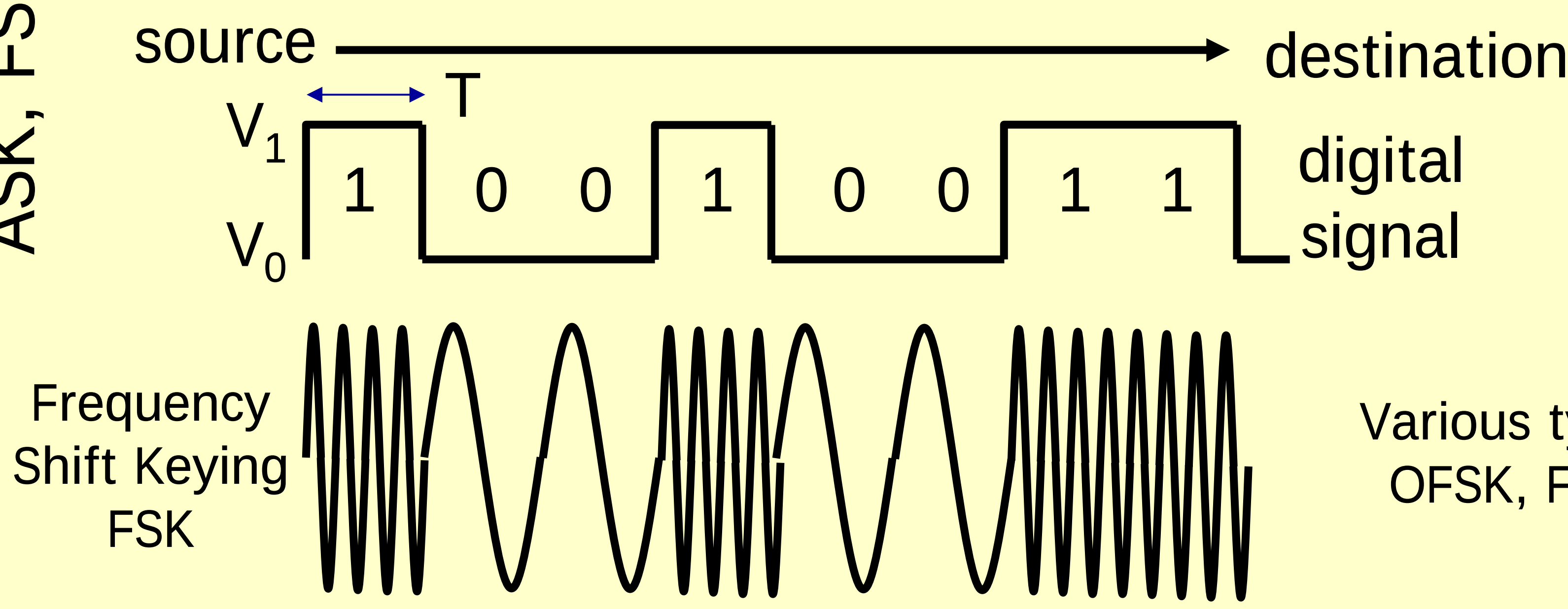




Frequency Shift Keying, FSK

Digital modulation: ASK, FSK, PSK

- Two binary values represented by two different frequencies (near carrier)
- Less susceptible to error than ASK
- Applications: Up to 1200bps on voice grade lines, High frequency radio, even higher frequency on LANs using co-ax

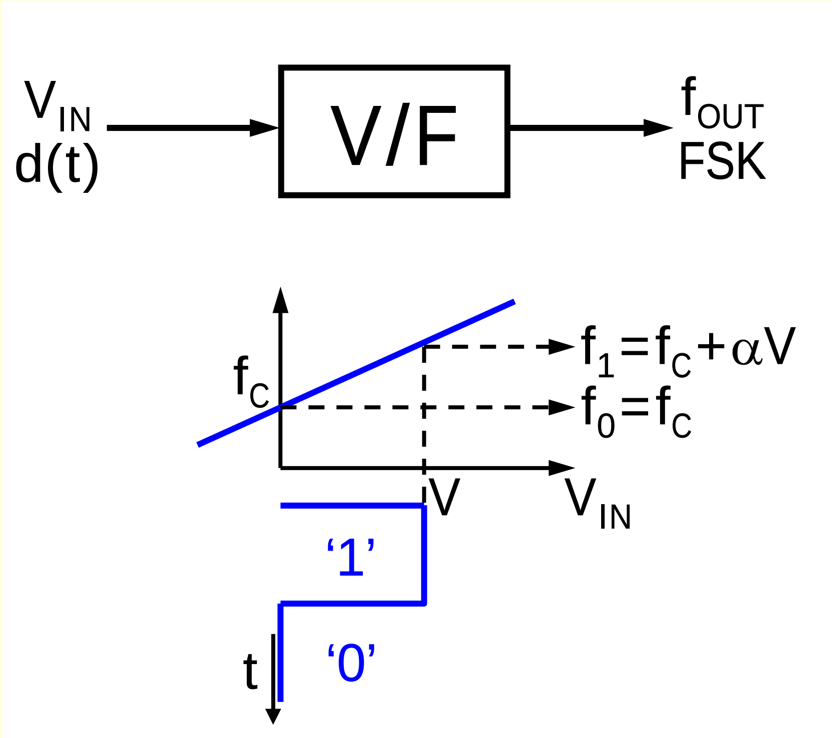


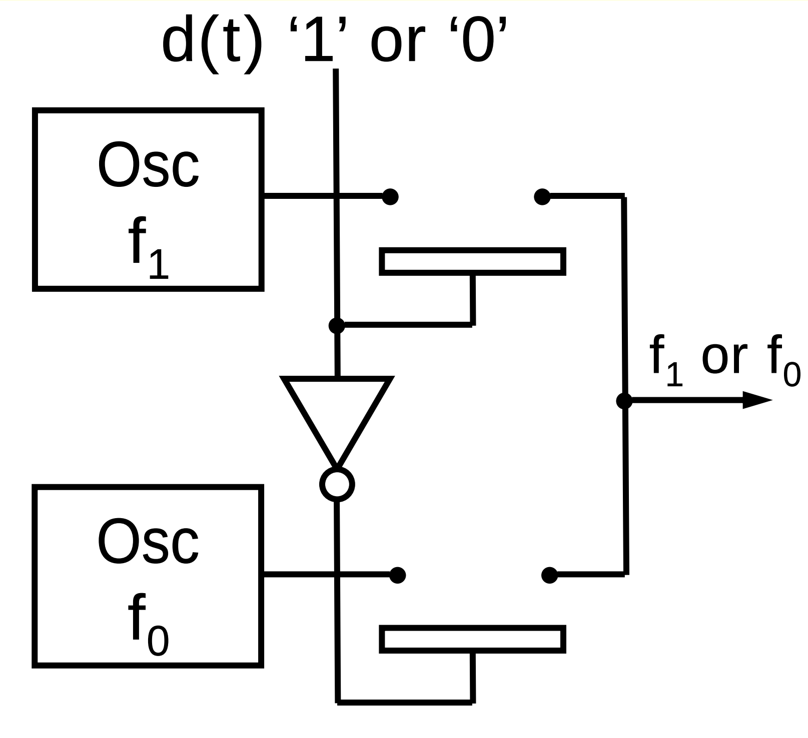
Various types: OFSK, FFSK

FSK Modulation of *d(t)*

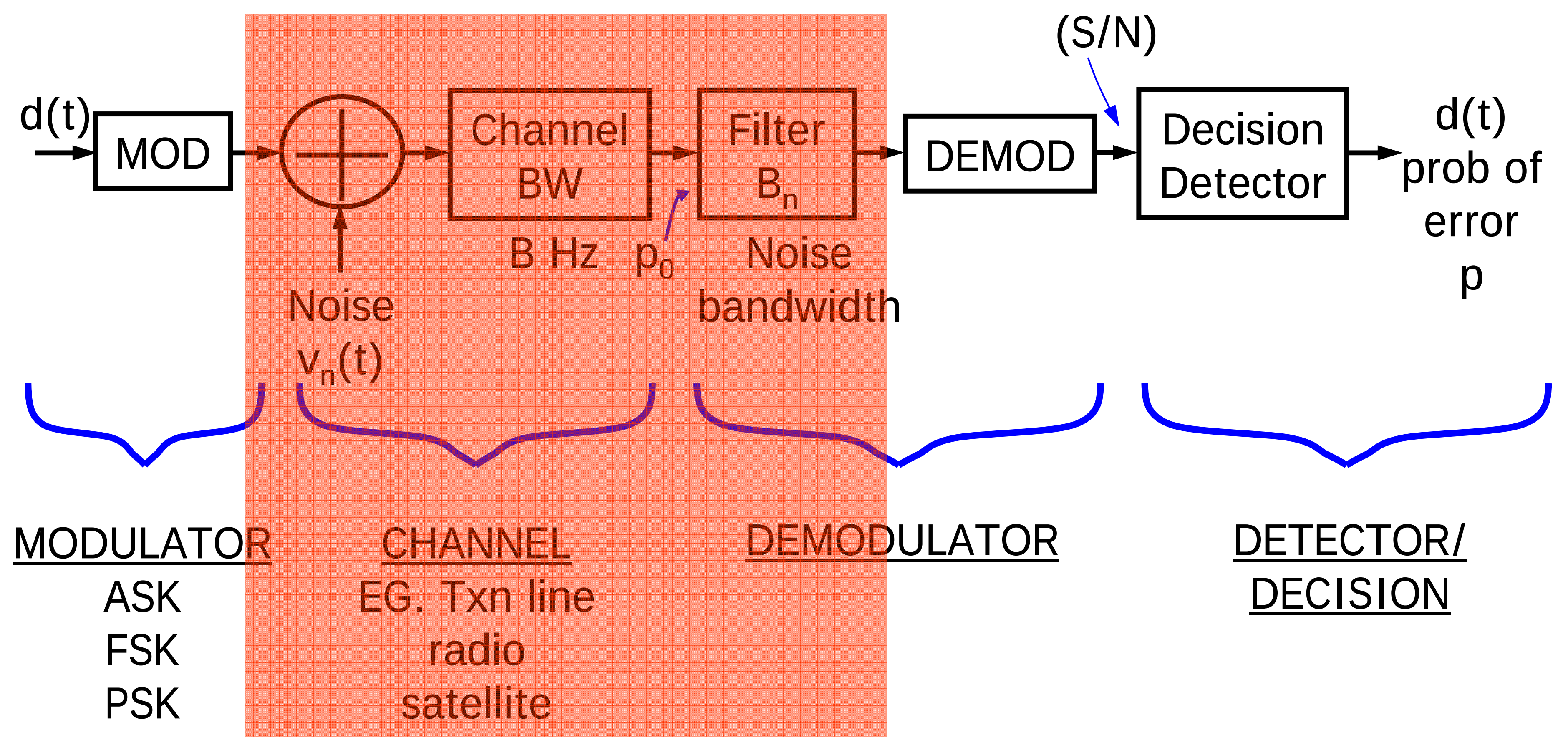
Digital modulation: ASK, FSK, PSK

- Frequency Shift Keying



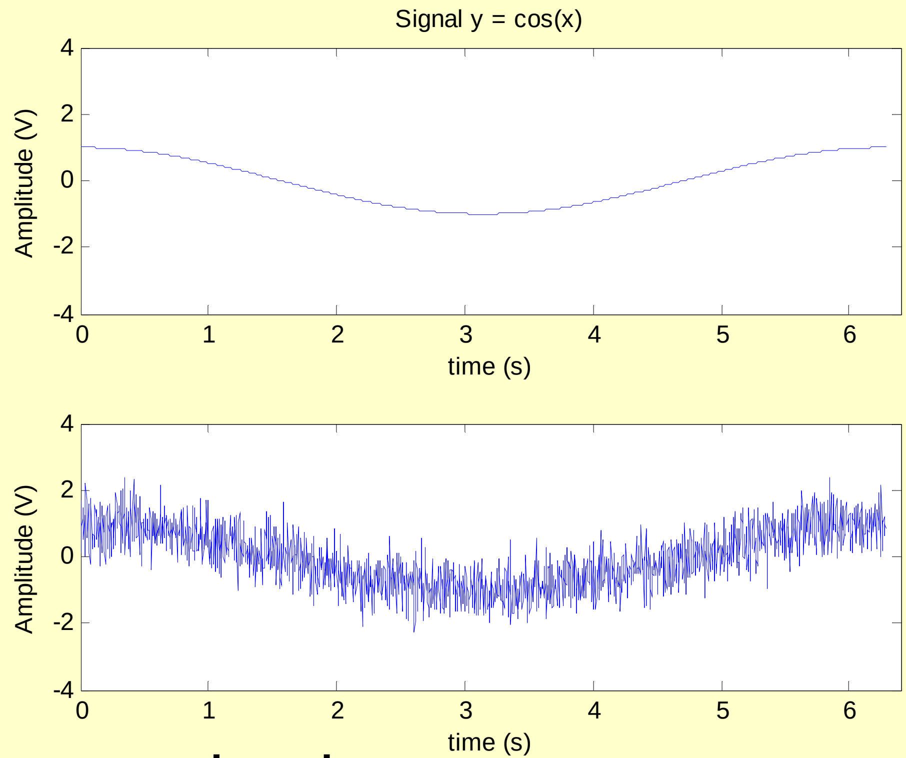


System Block Diagram – model for analysis



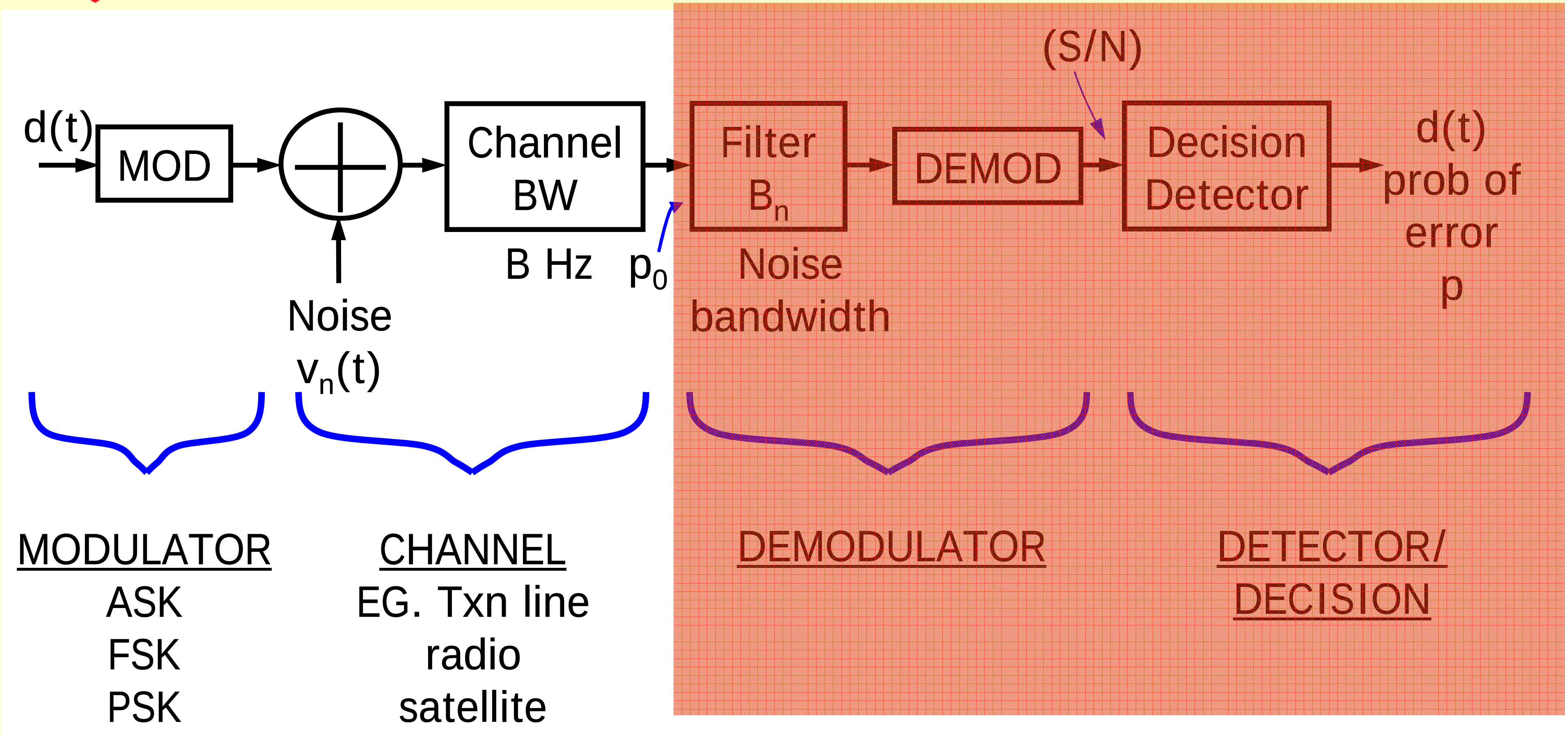
Channel - Impairments

- When signals are transmitted they are subject to noise



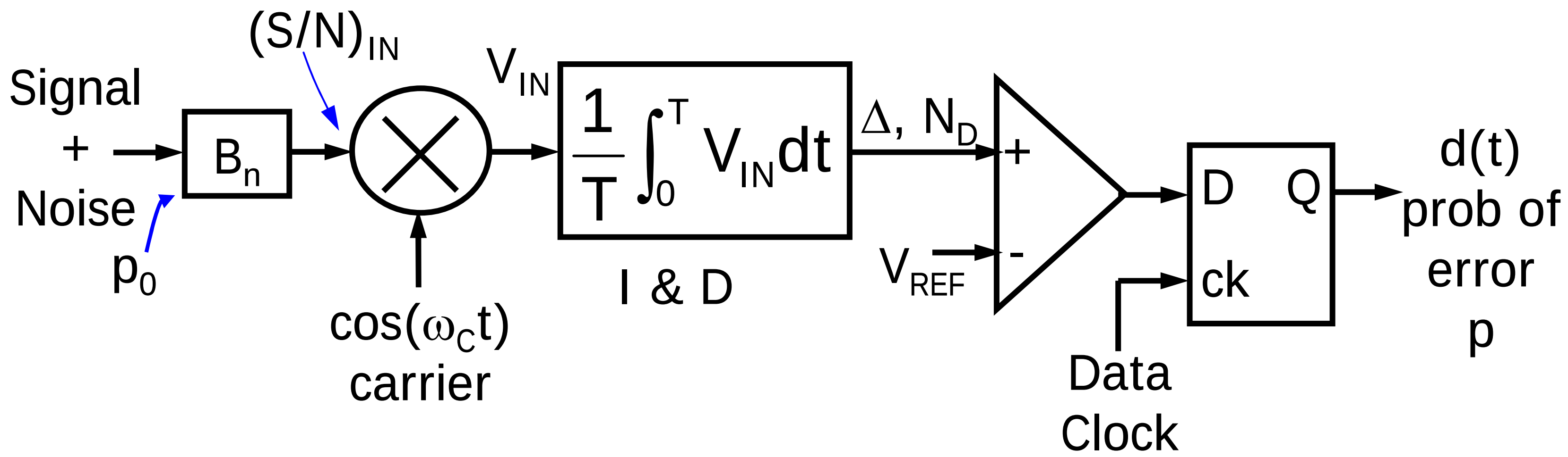
- Noise is assumed to be
  - ADDITIVE, WHITE, GAUSSIAN, AWGN – mean of zero
  - infinite bandwidth – hence a bandlimiting filter is required
  - p_0 is the noise power spectral density (Watts/Hertz)

System Block Diagram – model for analysis



ASK & PSK Demodulator

- Optimum Demodulator, based on ‘correlation’:
  - i.e. how similar is the received signal to one of the possible transmitted signals:

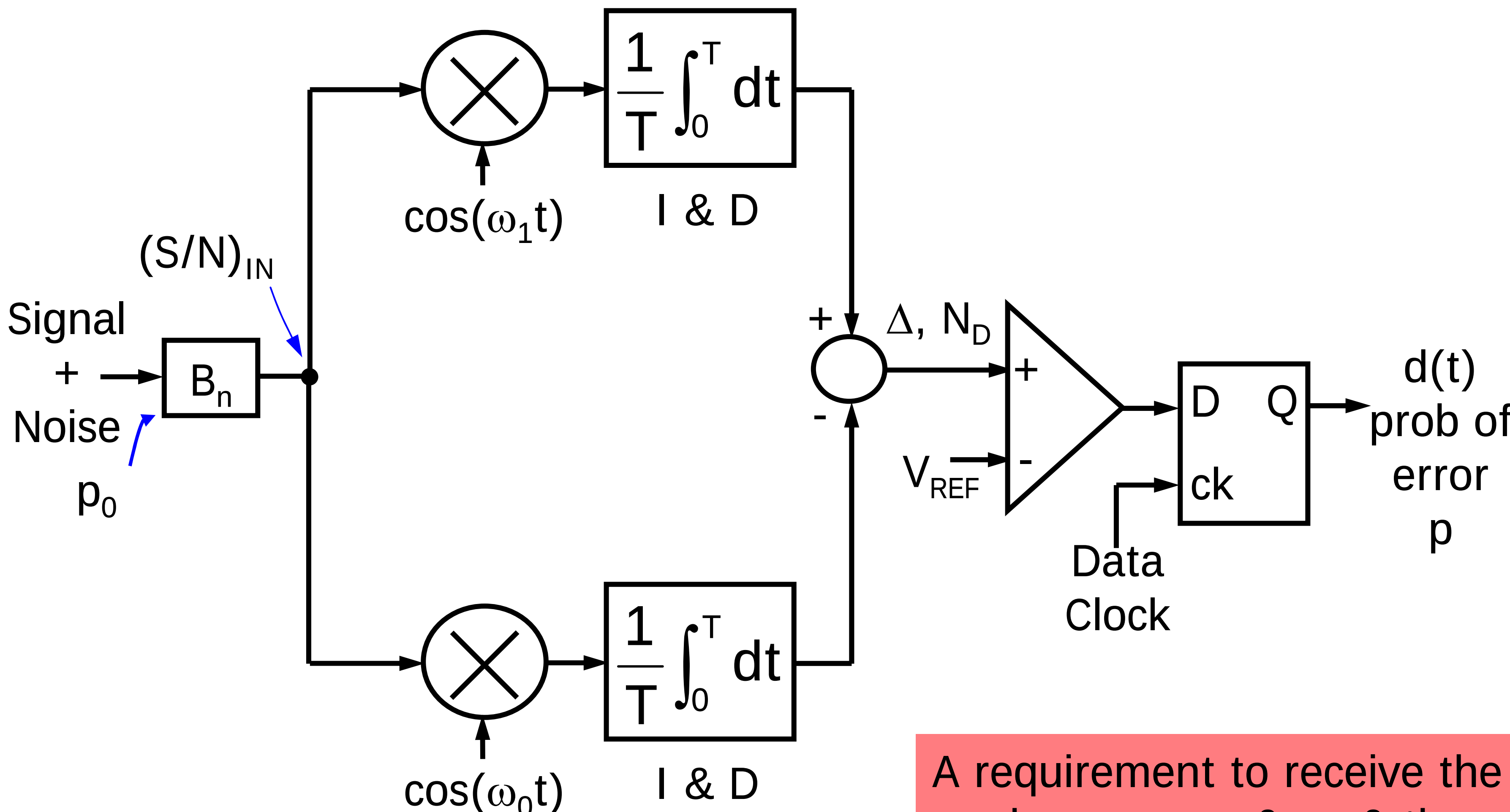


Integrate & Dump,
T is the bit duration

Digital modulation:

Demodulator – FSK

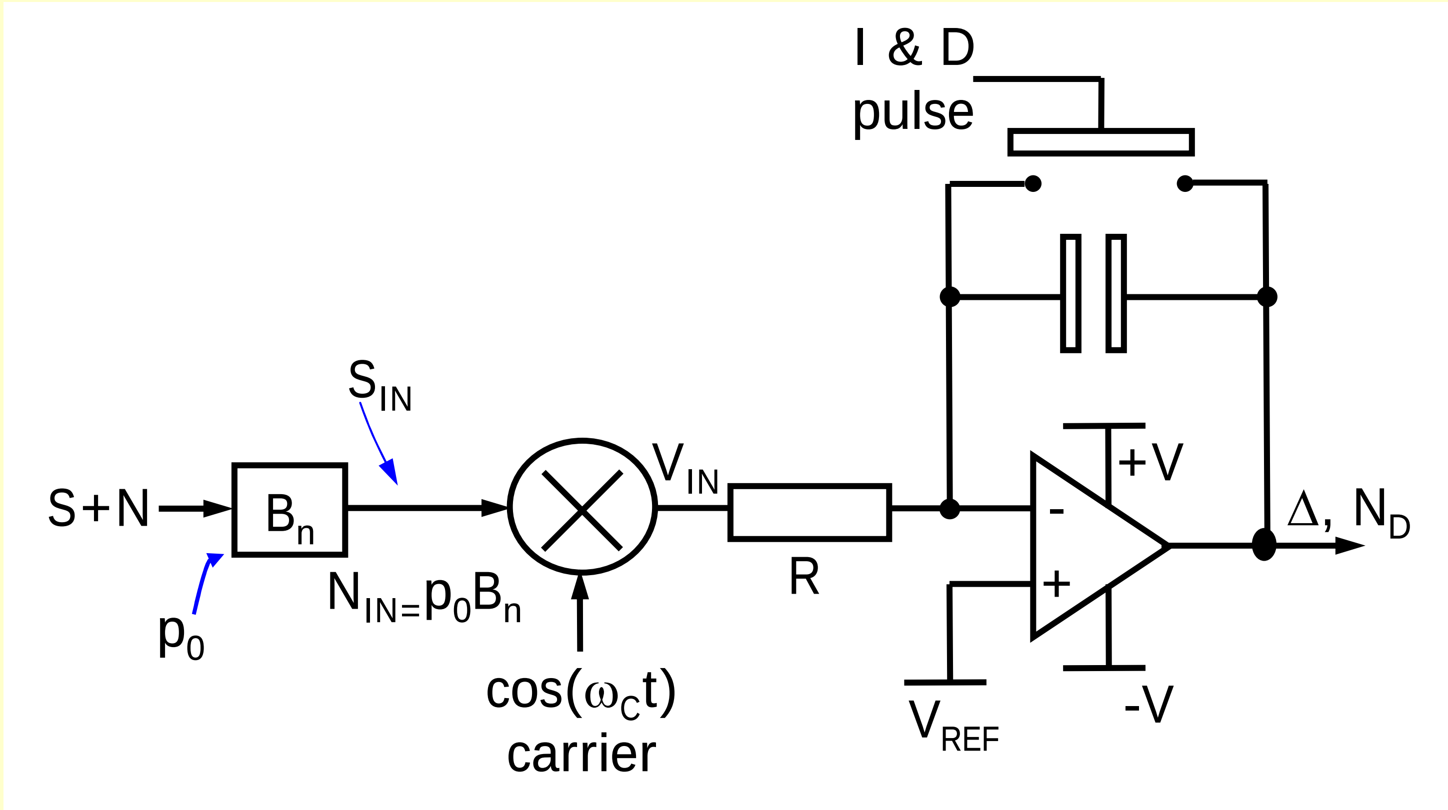
- Optimum Demodulator, based on ‘correlation’:



A requirement to receive the carrier ω_C or ω_1 & ω_0 & the ‘bit rate’ clock - DATA CLOCK

Demodulator – Circuit – A/P/FSK

Digital modulation:
ASK, FSK, PSK



- Input signal + bandlimited noise $N_{IN}=p_oB_N$ multiplied by a carrier
  - $\cos(\omega_c t)$ for ASK/PSK
  - $\cos(\omega_0 t)$ & $\cos(\omega_1 t)$ for FSK

PSK demodulation ex #1

Digital modulation: ASK, FSK, PSK

- If d(t) is a '1', input is $d(t)V_C\cos(\omega_C t)$

  $V_{IN}=V_1=d(t)V_C\cos(\omega_C t)*\cos(\omega_C t)+\text{Noise}$

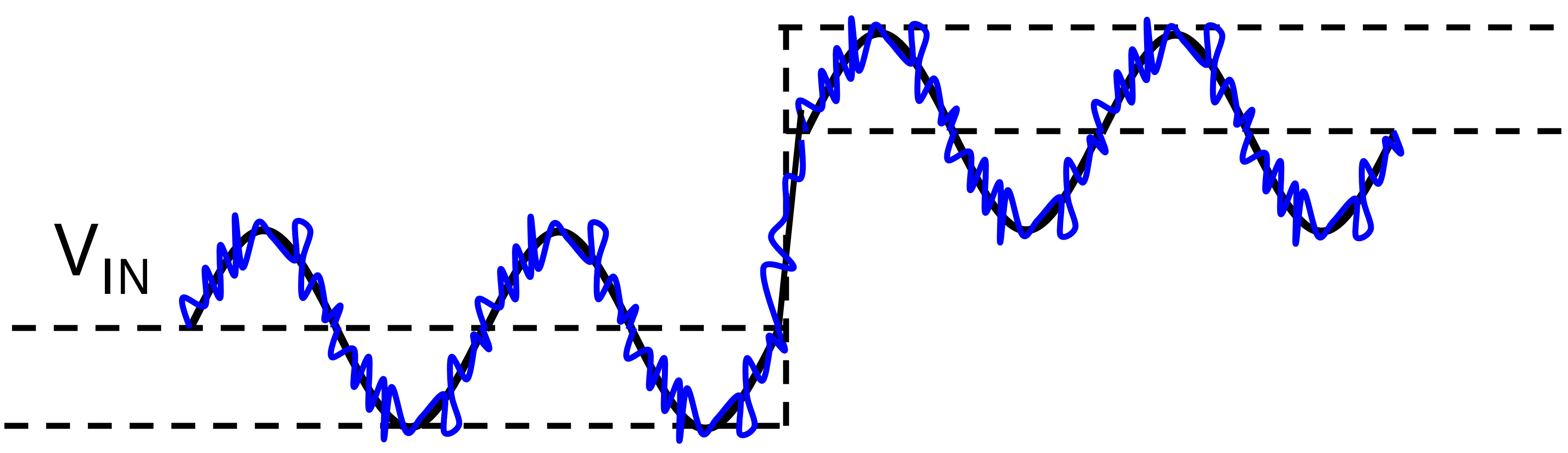
  $V_{IN}=d(t)V_C\cos^2(\omega_C t)+\text{Noise}$

$$V_{IN} = \frac{d(t)}{2}V_C + \frac{d(t)}{2}V_C cos(2\varpi_C t) + Noise$$

- If d(t) is a '0', input is $-d(t)V_C\cos(\omega_C t)$

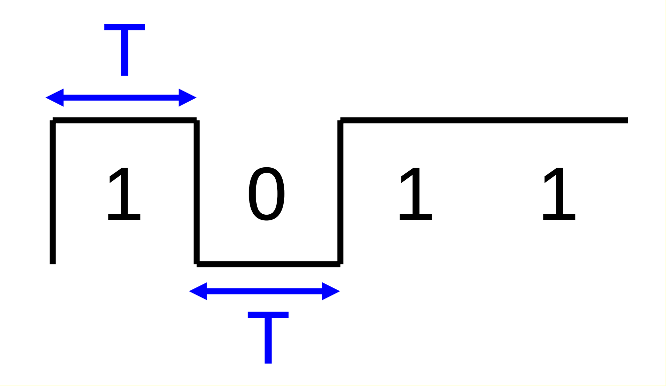
  $V_{IN}=-d(t)V_C\cos^2(\omega_C t)+\text{Noise}$

$$V_{IN} = -\frac{d(t)}{2}V_C - \frac{d(t)}{2}V_C cos(2\varpi_C t) + Noise$$



PSK demodulation ex #2

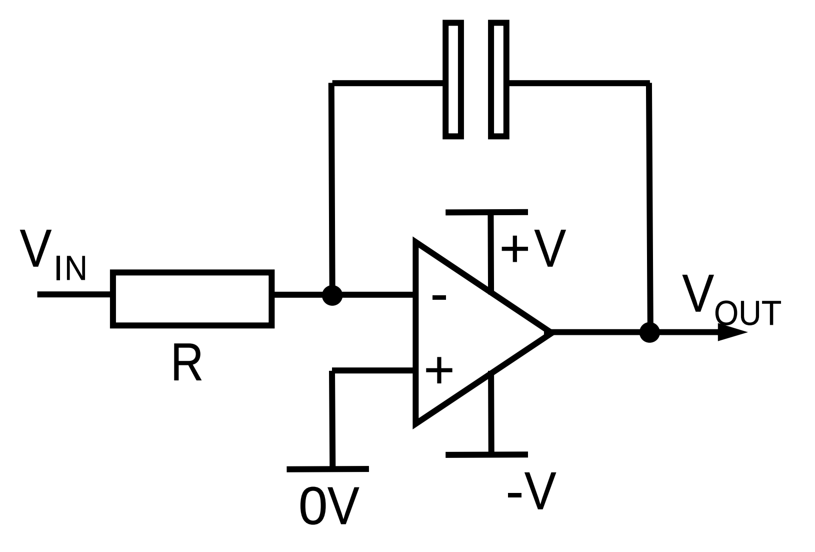
- The Integrate & Dump performs the ‘function’
  - $\frac{1}{T}\int_0^T V_{IN}\,dt$ where T is the duration of a bit



- The Integrator

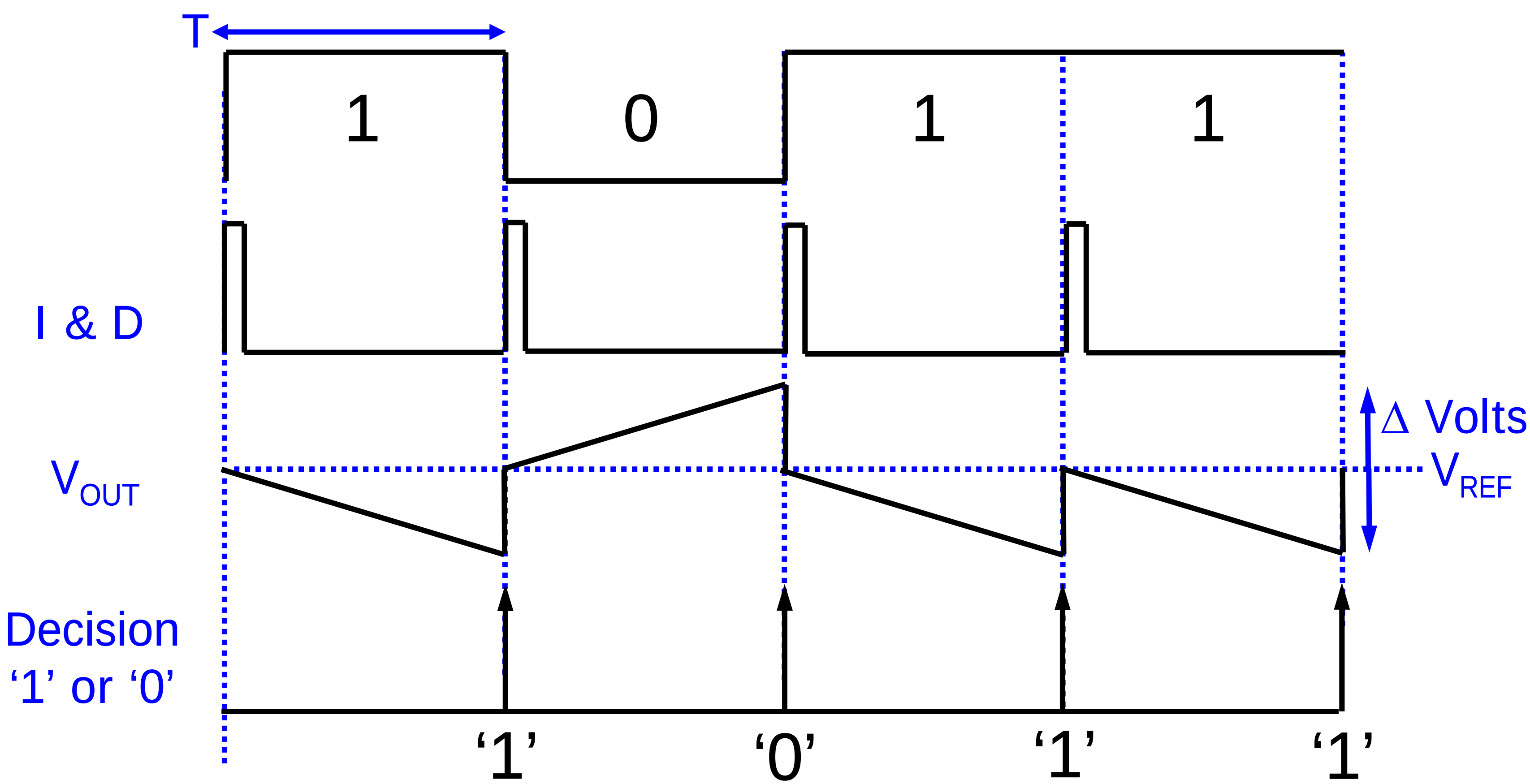
$$V_{OUT} = -\frac{1}{RC}\int_0^{RC} V_{IN}\,dt$$

  - Design RC=T



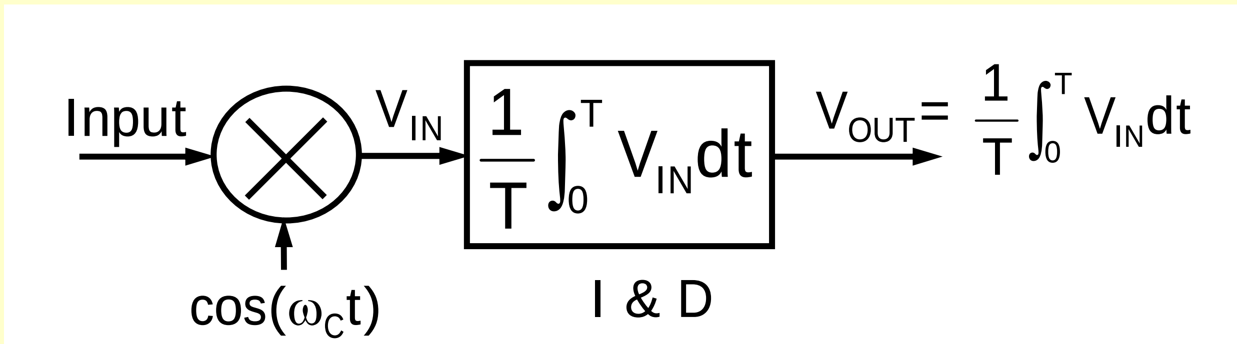
PSK demodulation ex #3

- I&D pulse shown, allows the integrator to integrate for a time T, after which the integrator is cleared or 'dumped' by shorting out C & the integration re-commences



PSK demodulation ex #4

- Integrate & Dump Output:

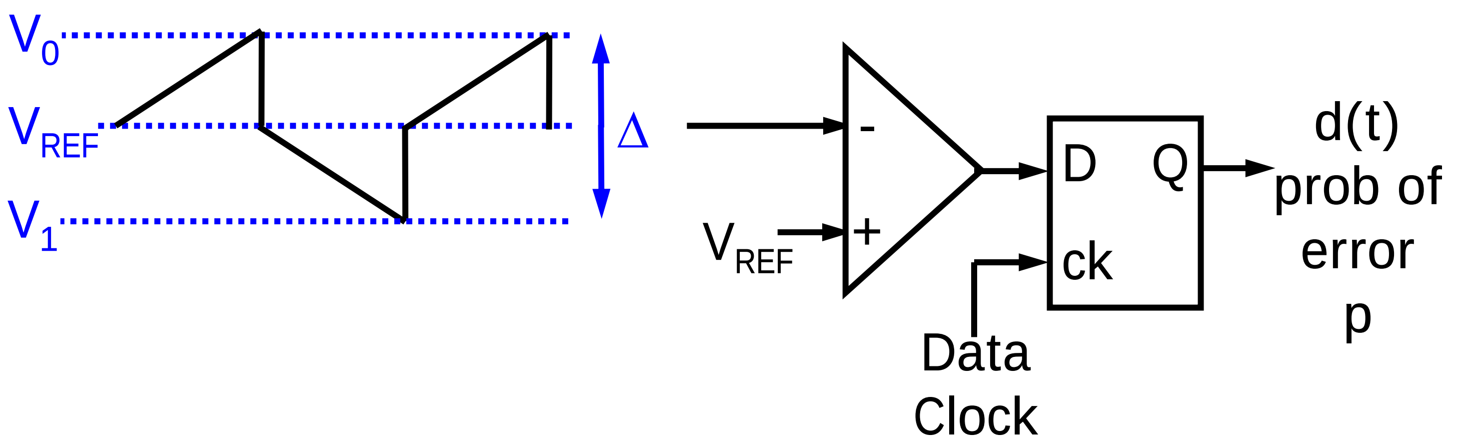


- Input = $d(t)V_C\cos(\omega_C t)$

$$V_{OUT} = \frac{1}{T}\int_0^T d(t).V_C \cos^2(\varpi_C t)dt$$

$$V_{OUT} = \frac{V_C d(t)}{2}$$

Digital modulation: ASK, FSK, PSK



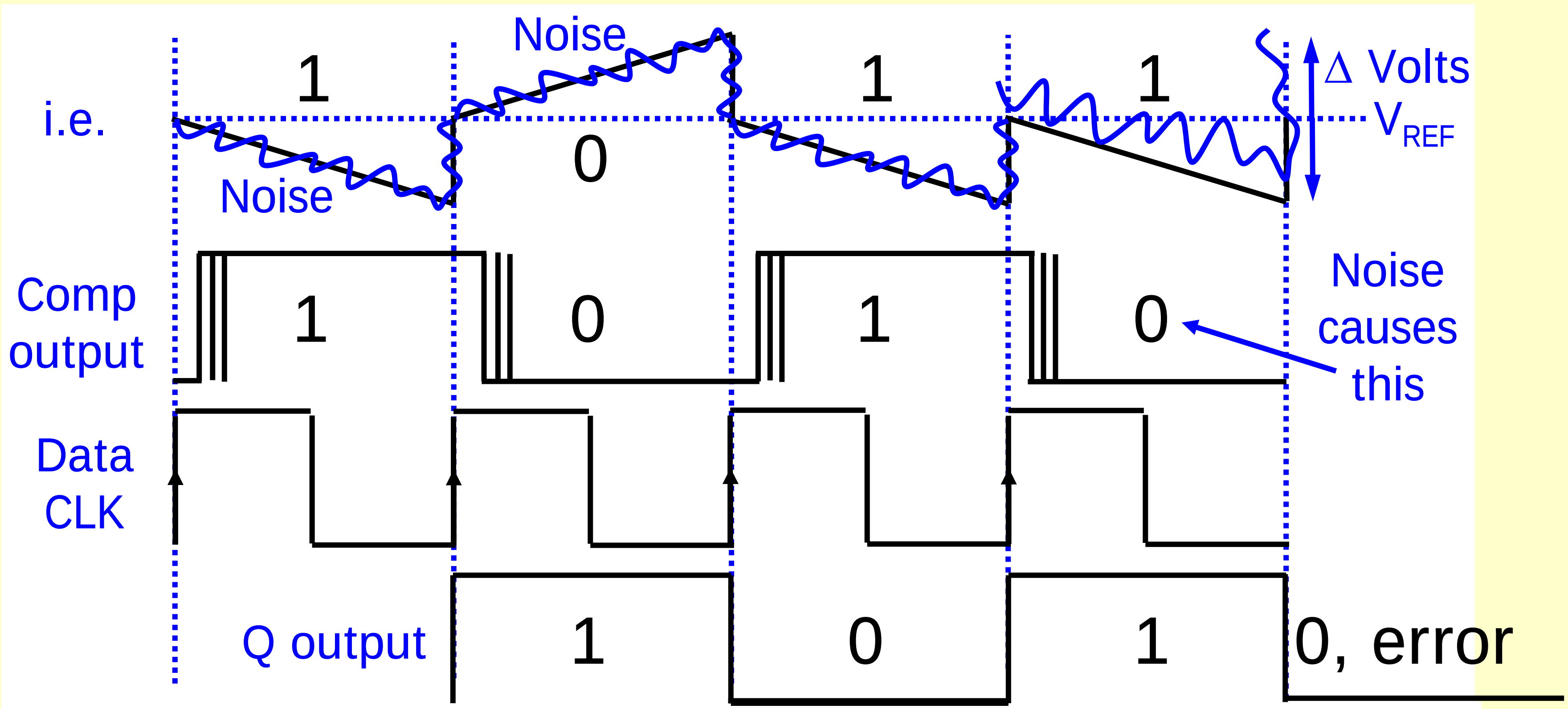
- V_0 is the voltage when a '0' was transmitted
- V_1 is the voltage when a '1' was transmitted
- The peak-to-peak input to the comparator is
  Change in Volts $\Delta = V_0 - V_1$
- This difference between a '1' & a '0'
- The reference V_{REF} is set midway between V_1 & V_0.

$$V_{REF} = V_0 - \frac{\Delta}{2} = V_1 + \frac{\Delta}{2} = \frac{V_1 + V_0}{2}$$

- A HALF LEVEL THRESHOLD DETECTOR
- If V_1=10V, V_0=5V: Δ=2.5 & V_{REF}=7.5V

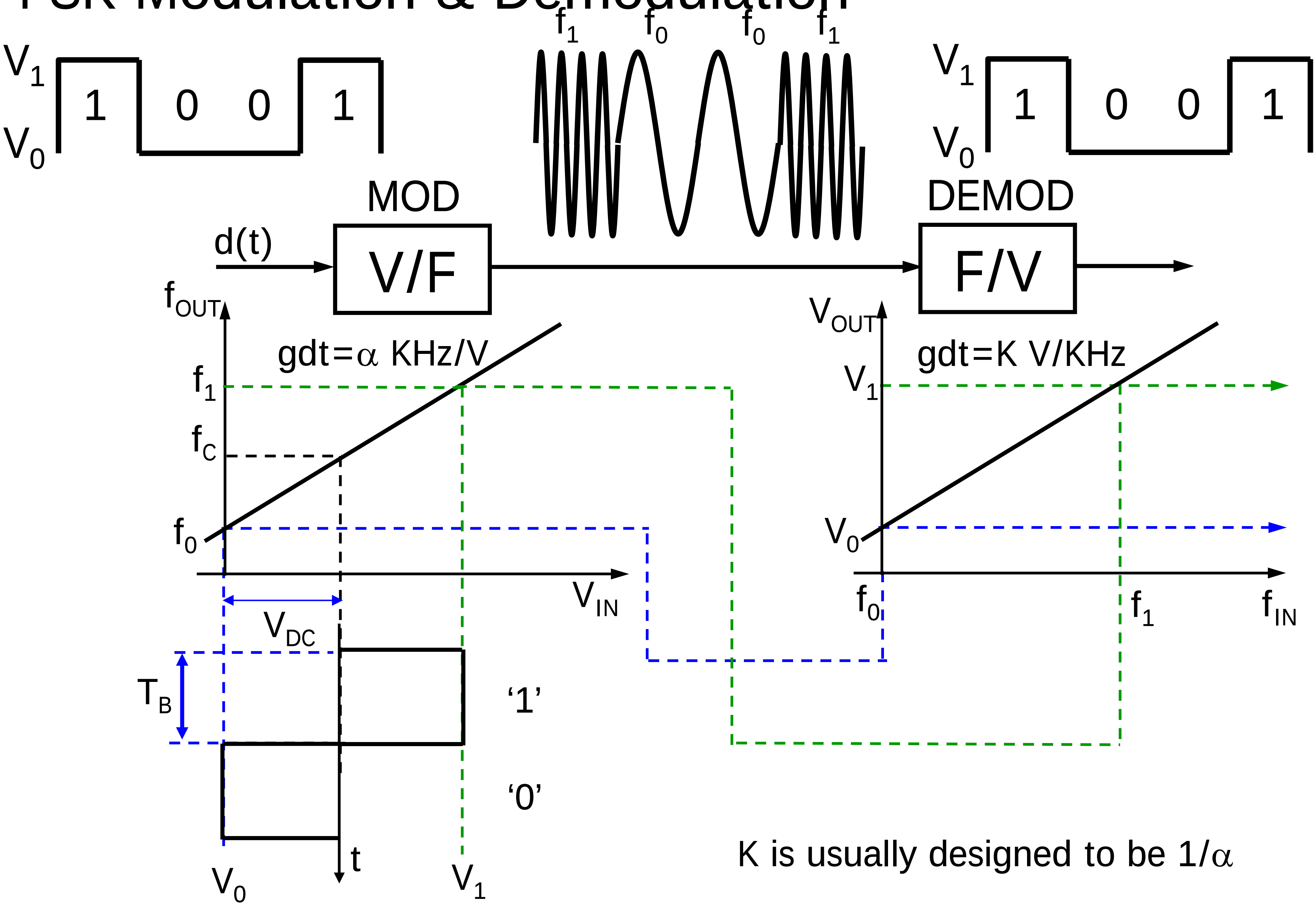
PSK demodulation ex #6

- Integration in presence of Noise
  - which is clocked at the Data Clock Rate
  - This makes a final decision if a ‘1’ or a ‘0’ is received



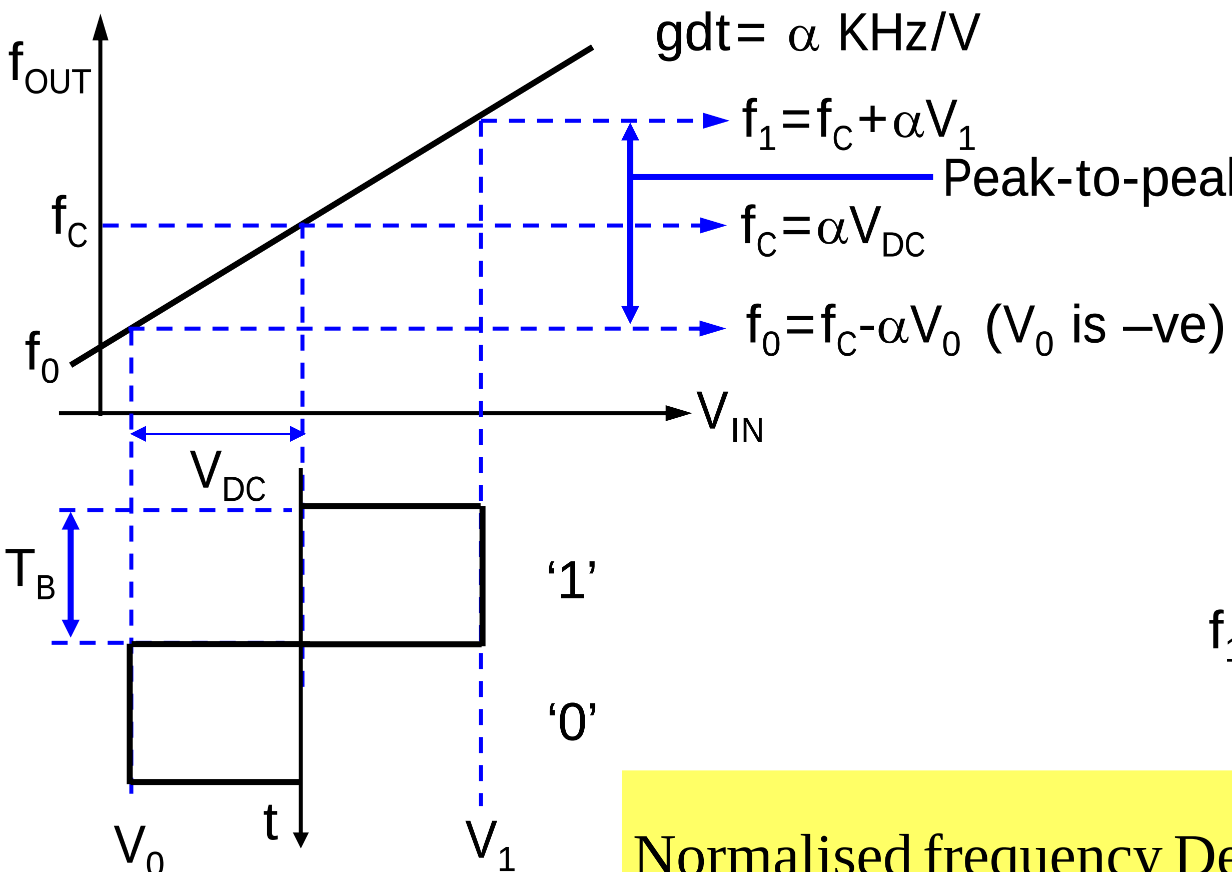
Noise causes the integrator to fluctuate & causes errors

FSK Modulation & Demodulation



K is usually designed to be $1/\alpha$

FSK Demodulation #2



Bit rate=Baud rate

$R_b=1/\tau_B$

$y=mx+c$

$f_{OUT}=\alpha V_{IN}+f_0$

$V_{IN}=V_{DC}+d(t)$

$V_{IN}=V_{DC}+V_1$ for 1's

$V_{IN}=V_{DC}+V_0$ for 0's

$f_1=\alpha V_{DC}+\alpha V_1+f_0$ for 1's

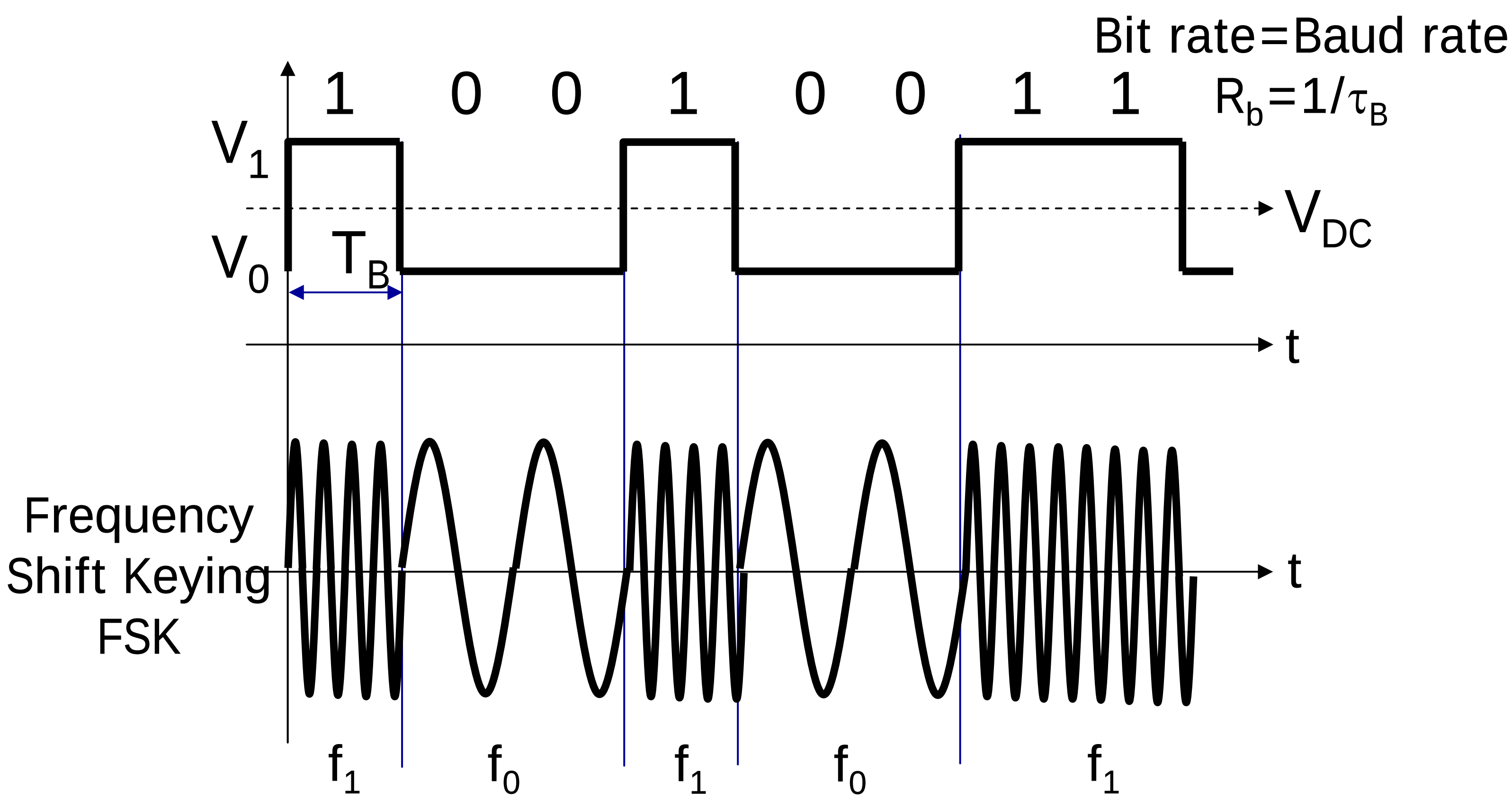
$f_C=\alpha V_{DC},\ T_C=1/f_C$

Normalised frequency Deviation ratio, $h=\dfrac{|f_1-f_0|}{R_b}$

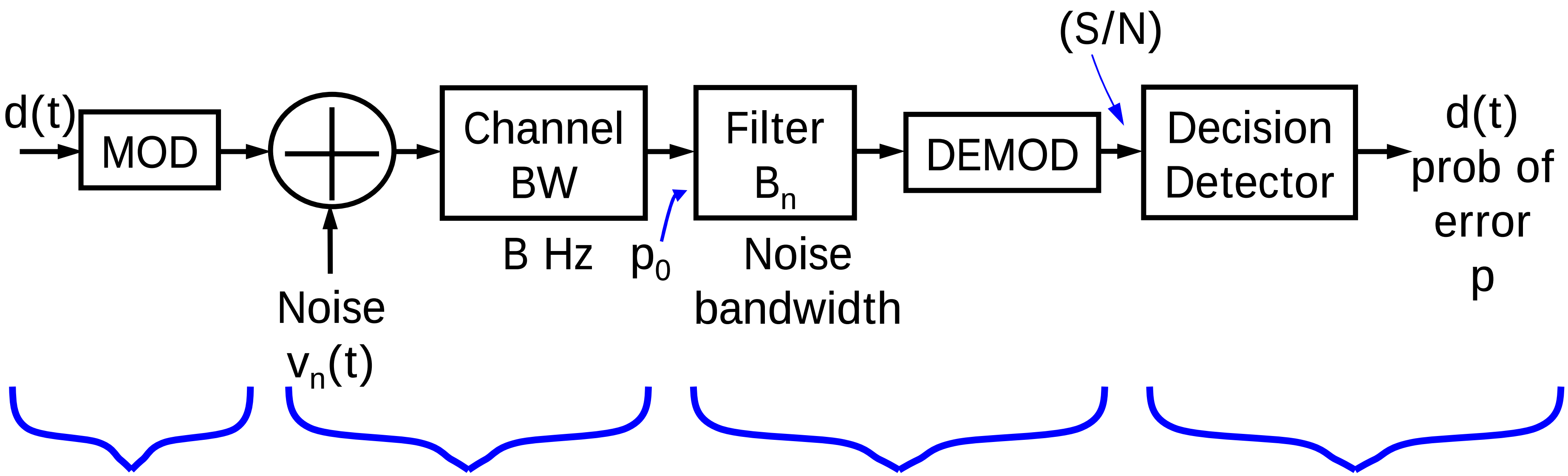
i.e. modulus of f_1-f_0

FSK Demodulation #3

- The spectrum of the FSK signal depends on h.
- Waveforms



System Block Diagram – model for analysis



MODULATOR
ASK
FSK
PSK

CHANNEL
EG. Txn line
radio
satellite

DEMODULATOR

DETECTOR/
DECISION

Performance of Digital Systems #1

Digital modulation:
ASK, FSK, PSK

- Performance of Digital Data Systems is
  - dependent on the bit error rate, b.e.r.*
  - i.e. the probability of a bit being in error

$$p = \frac{no.of\ errors, E}{Total\ no.of\ bits\ N}\ as\ N \to \infty$$

- *p* depends on several factors
  - The modulation type: ASK, FSK or PSK
  - The demodulation method
  - The noise in the 'system'
  - The signal-to-noise ratio

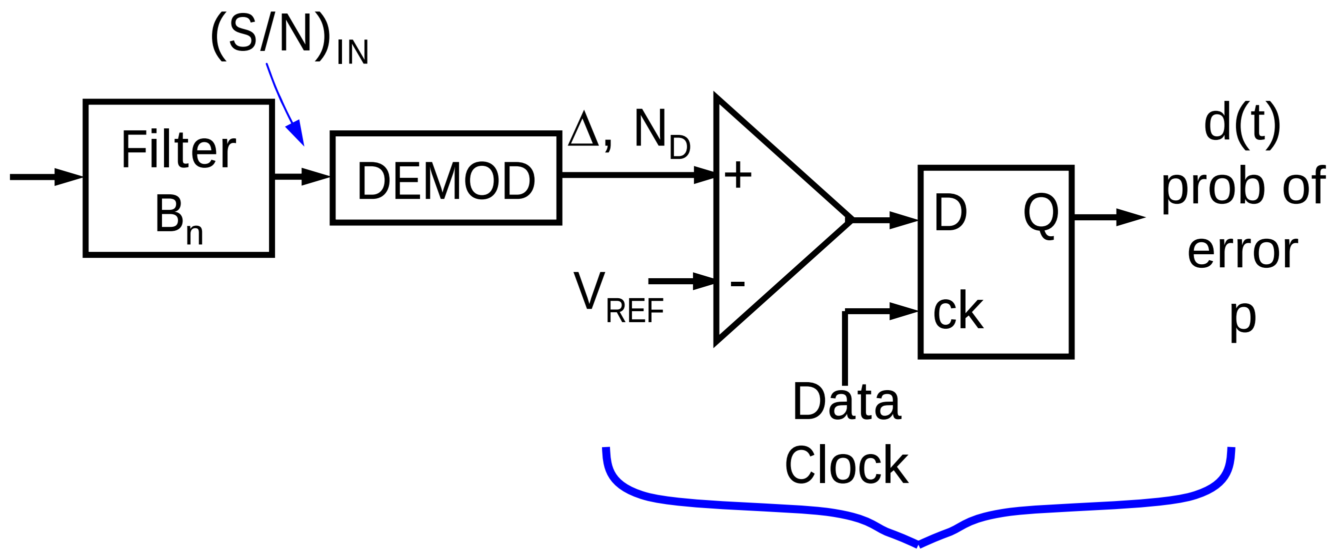
**p* is a prediction of how the system will perform
b.e.r. is a measure of how the system actually performs

Performance #2

- Obtain the received
  - Signal Power
  - Noise Power
- Calculate the SNR based on a particular modulation method
- Predict the theoretical performance using Error Function Tables

Performance #3

Digital modulation:
ASK, FSK, PSK



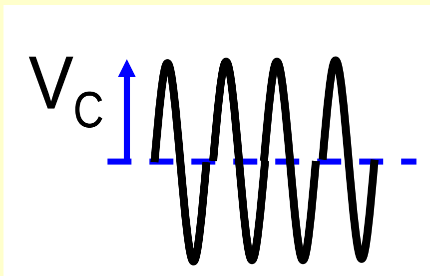
Find *Δ*, N_D in terms of S_{IN} & N_{IN}

p as a function of $(S/N)_{IN}$

Performance #4 - Signal Power, S_{IN}

Digital modulation:
ASK, FSK, PSK

- Usually assume an un-modulated carrier
  i.e. a signal=$V_C\cos(\omega_C t)$ - ALWAYS



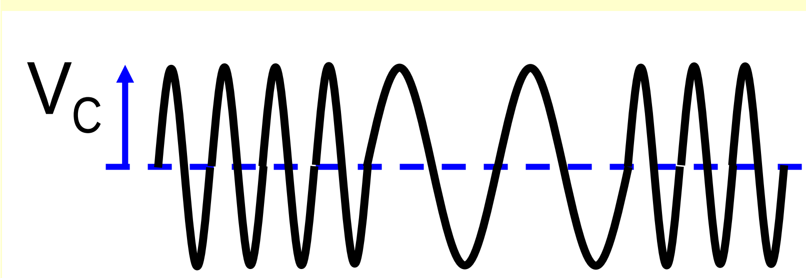
Normalised Average Power=$(V_{RMS})^2$

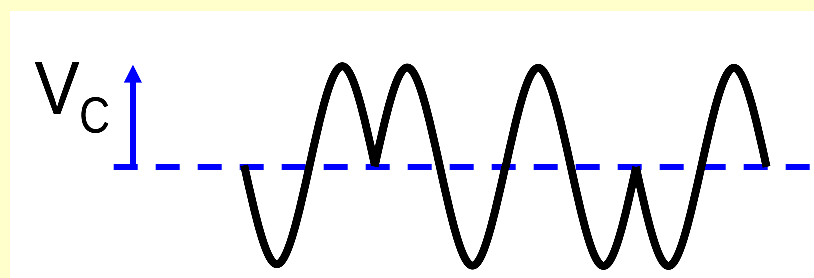
$$V_{RMS} = \frac{V_{peak}}{\sqrt{2}} = \frac{V_C}{\sqrt{2}}$$

Normalised Average Power=$(V_{RMS})^2 = \left(\frac{V_C}{\sqrt{2}}\right)^2 = \frac{V_C^{\,2}}{2} = S_{IN}$

$$S_{IN} = \frac{V_C^{\,2}}{2}\,\text{Watts}$$

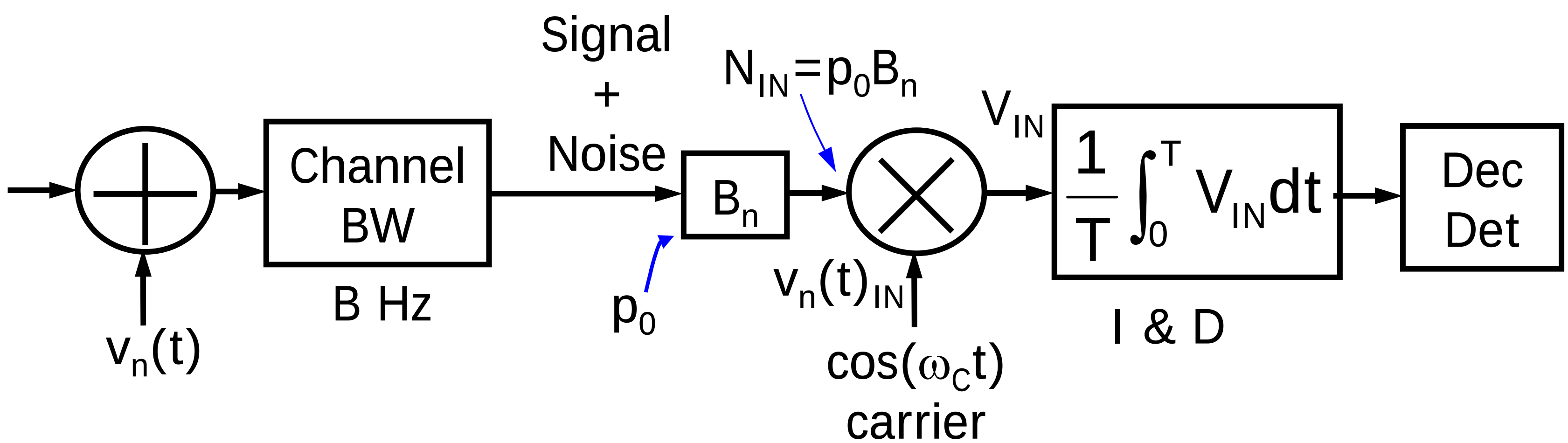
Similarly for FSK & PSK





Performance #4 - Noise Power, N_{IN}

Digital modulation:
ASK, FSK, PSK



'Peak' value of noise, $=\sqrt{2p_0B}$

Contrived to be equivalent to noise power

Normalised Average Noise Power, $\therefore N_{IN}=\left(\frac{N_{pk}}{\sqrt{2}}\right)^2$

Performance #5 – *SNR* & *p*

Digital modulation:
ASK, FSK, PSK

- S_{IN}/N_{IN}

$$S_{IN} = \frac{V_C^{\ 2}}{2} Watts$$

$$N_{IN} = \left(\frac{N_{pk}}{\sqrt{2}} \right)^2$$

$$\frac{S_{IN}}{N_{IN}} = \frac{V_C^{\ 2}}{2} / \frac{N_{pk}^{\ 2}}{2} = \frac{V_C^{\ 2}}{N_{pk}^{\ 2}}$$

$$\frac{S_{IN}}{N_{IN}} = \frac{V_C^{\ 2}}{N_{pk}^{\ 2}}$$

- Substitute into the correct equation to obtain *p*

$$\begin{array}{ll} B\text{-}ASK \ \ OOK & p = \frac{1}{2}\left(1 - erf\sqrt{\frac{S_{IN}}{4N_{IN}}}\right) \\ B-FSK & p = \frac{1}{2}\left(1 - erf\sqrt{\frac{S_{IN}}{2N_{IN}}}\right) \\ B-PSK \ \ PRK & p = \frac{1}{2}\left(1 - erf\sqrt{\frac{S_{IN}}{N_{IN}}}\right) \end{array}$$

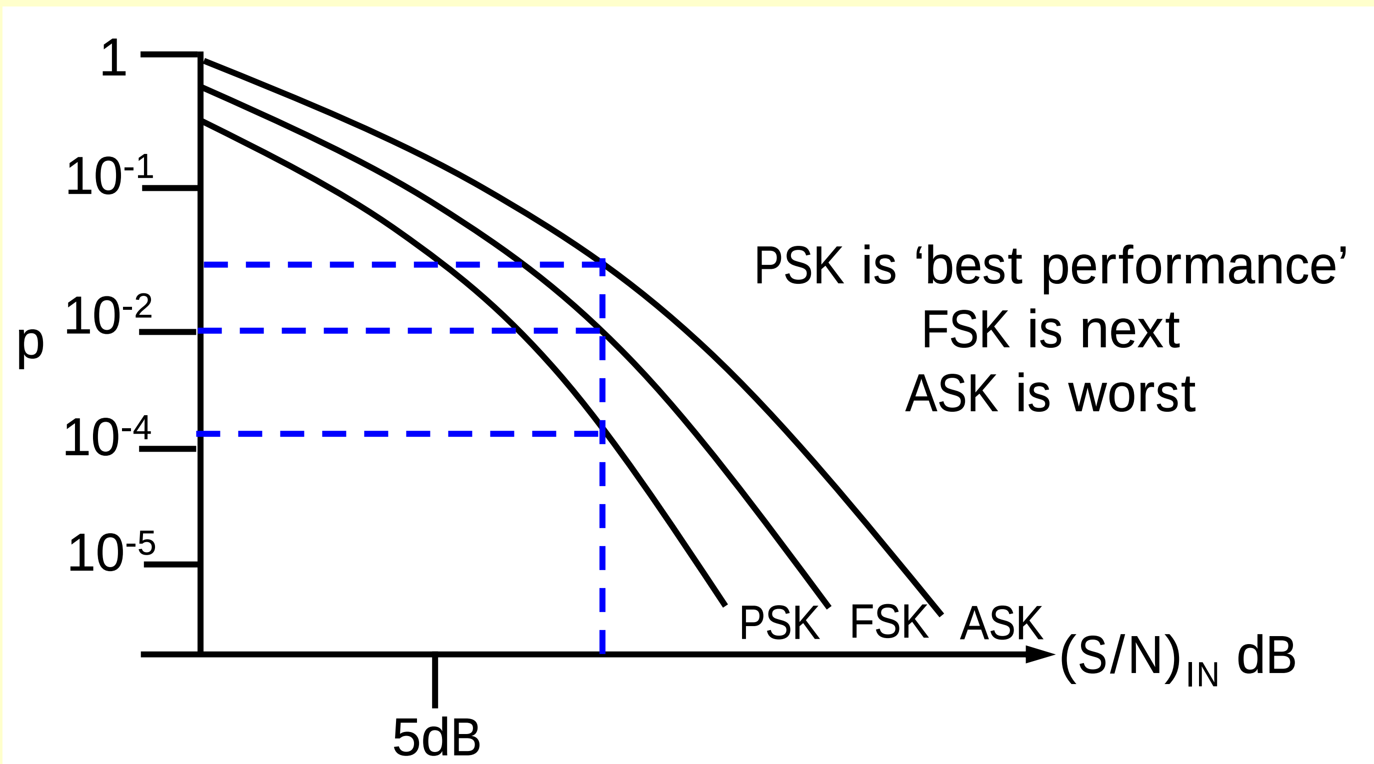
Performance #6

Digital modulation: ASK, FSK, PSK

B-ASK / OOK $$p = \frac{1}{2}\left(1 - erf\sqrt{\frac{S_{IN}}{4N_{IN}}}\right)$$

B–FSK $$p = \frac{1}{2}\left(1 - erf\sqrt{\frac{S_{IN}}{2N_{IN}}}\right)$$

B–PSK / PRK $$p = \frac{1}{2}\left(1 - erf\sqrt{\frac{S_{IN}}{N_{IN}}}\right)$$

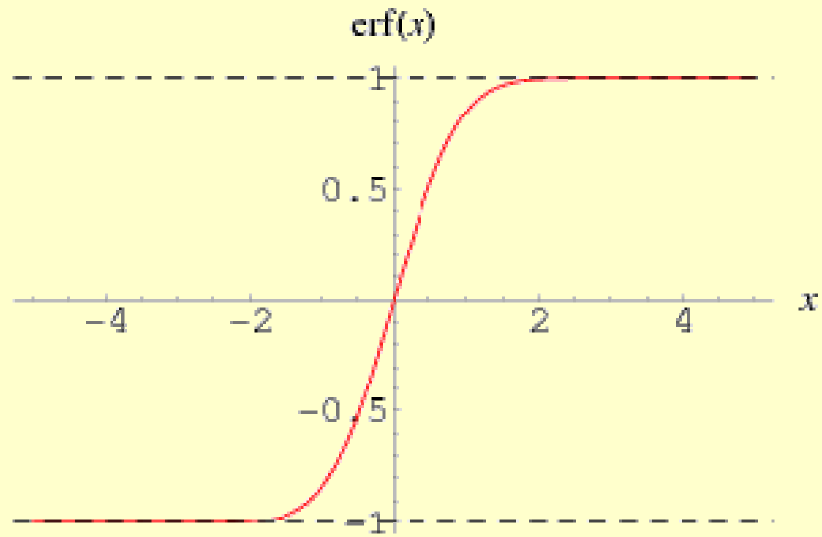


Error Function

The "error function" encountered in integrating the normal distribution (which is a normalized form of the Gaussian function).
It is an entire function defined by :

$$erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$$

$$erfc(z) = 1 - erf(z)$$



x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3
erf(x)	0.000	0.112	0.223	0.329	0.428	0.520	0.604	0.678	0.742	0.797	0.843	0.880	0.910	0.934
erfc(x)	1.000	0.888	0.777	0.671	0.572	0.480	0.396	0.322	0.258	0.203	0.157	0.120	0.090	0.066

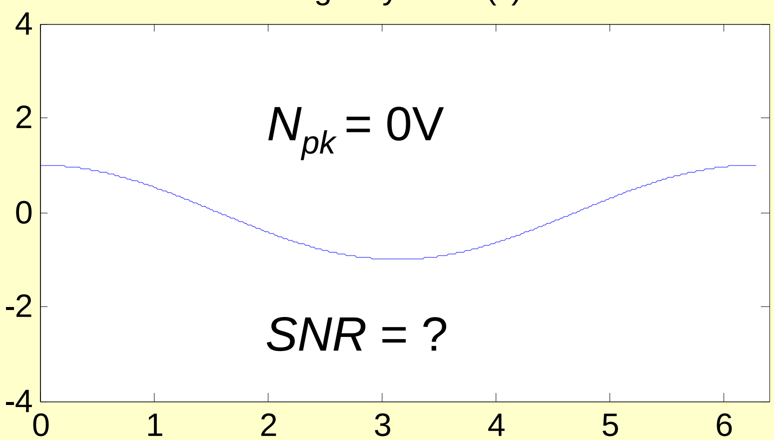
x	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7
erf(x)	0.952	0.966	0.976	0.984	0.989	0.993	0.995	0.997	0.998	0.999	0.999	1.000	1.000	1.000
erfc(x)	0.048	0.034	0.024	0.016	0.011	0.007	0.005	0.003	0.002	0.001	0.001	0.000	0.000	0.000

x	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5
erf(x)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
erfc(x)	7.50E-05	4.11E-05	2.21E-05	1.16E-05	6.03E-06	3.06E-06	1.52E-06	7.43E-07

Tutorial 1: find SNR_{dB} for each case

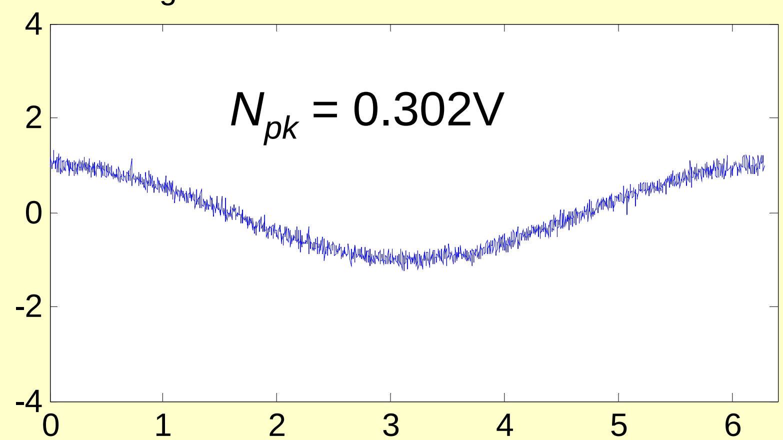
Case A

Signal y = cos(x)



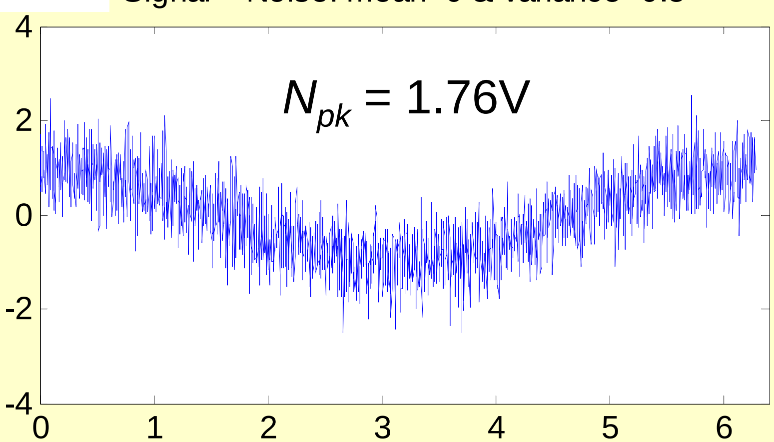
Case B

Signal + Noise: mean=0 & variance=0.1



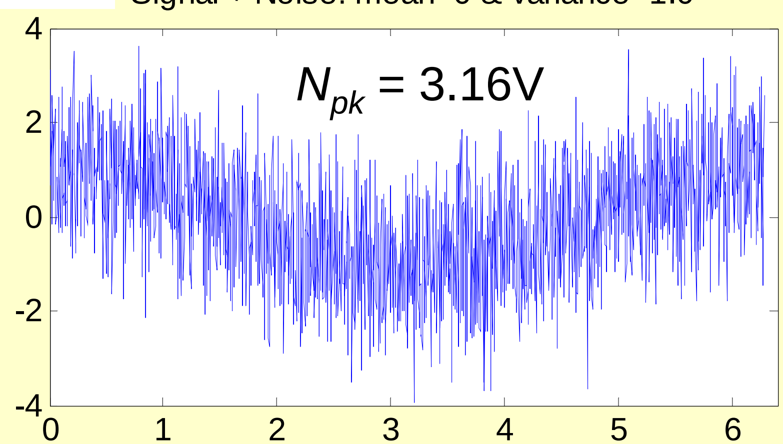
Case C

Signal + Noise: mean=0 & variance=0.5



Case D

Signal + Noise: mean=0 & variance=1.0



Solution 1

- SNR for case A:

$$\frac{S_{IN}}{N_{IN}} = \frac{V_C^{\ 2}}{N_{pk}^{\ \ 2}} = \frac{1}{0} = \infty$$

$$SNR_{dB} = 10\log_{10}\left(\frac{S_{in}}{N_{in}}\right)$$

$$SNR_{dB} = \underline{\infty dB}$$

- SNR for case B:

$$\frac{S_{IN}}{N_{IN}} = \frac{1}{(0.302)^2} = 10.96$$

$$SNR_{dB} = 10\log_{10}(10.96)$$

$$SNR_{dB} = \underline{10.40dB}$$

- SNR for case C: <u>-4.91dB</u>

- SNR for case D: <u>-10.00dB</u>

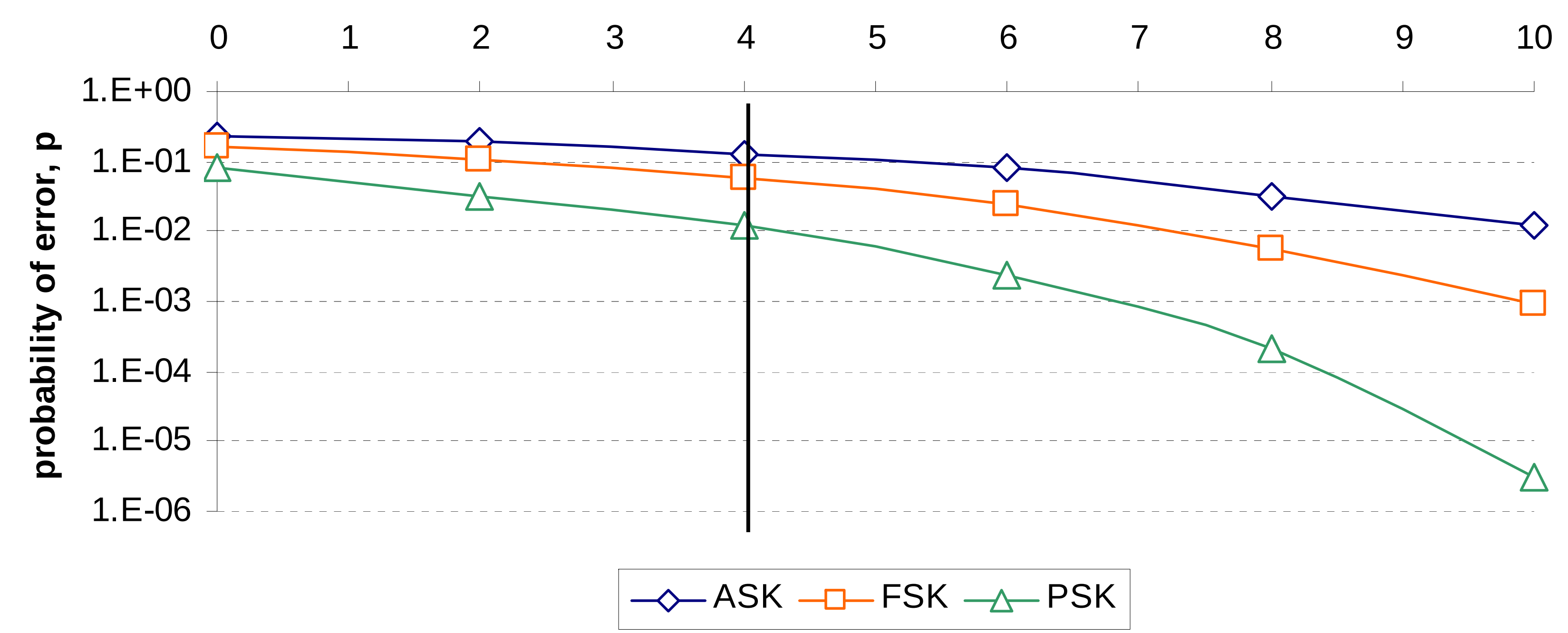
Tutorial 2

- With Reference to the following equations and error function tables, what is the performance of Binary ASK, FSK & PSK at 0, 4 & 8dB?
- Plot your results on logarithmic / linear paper
- Comment on the performance at 4dB, rank the modulation schemes and comment

Solution 2

Digital modulation: ASK, FSK, PSK

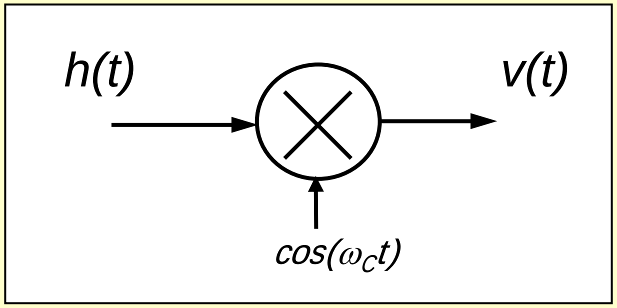
		0	4	8
SNR (dB)		0	4	8
SNR		1.000	2.512	6.310
ASK	SNR/4	0.250	0.628	1.577
	sqrt(SNR/4)	0.500	0.792	1.256
	1-erfc(SNR/4)	0.480	0.258	0.066
	ASK - p	0.240	0.129	0.033
FSK	SNR/2	0.500	1.256	3.155
	sqrt(SNRdB/2)	0.707	1.121	1.776
	1-erfc(sqrt(SNR/4))	0.322	0.120	0.011
	FSK - p	0.161	0.060	0.005
PSK	SNR	1.000	2.512	6.310
	sqrt(SNR)	1.000	1.585	2.512
	1-erfc(SNR)	0.157299	0.023652	0.000407
	PSK - p	0.08	0.01	2.03E-04
PSK cmp ASK		0.161	0.117	0.033
PSK cmp FSK		0.082	0.048	0.005



Tutorial 3

Digital modulation:
ASK, FSK, PSK

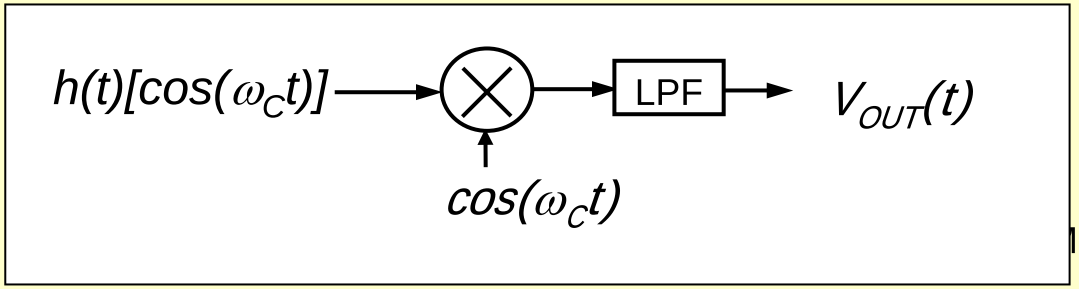
- Explain what is meant by 'Baud Rate'
- If data signal is represented as
  - *h(t)* = +V for 1's
  - = -V for 0's



i) What modulation scheme is this?

ii) Sketch the output waveform, *v(t)*

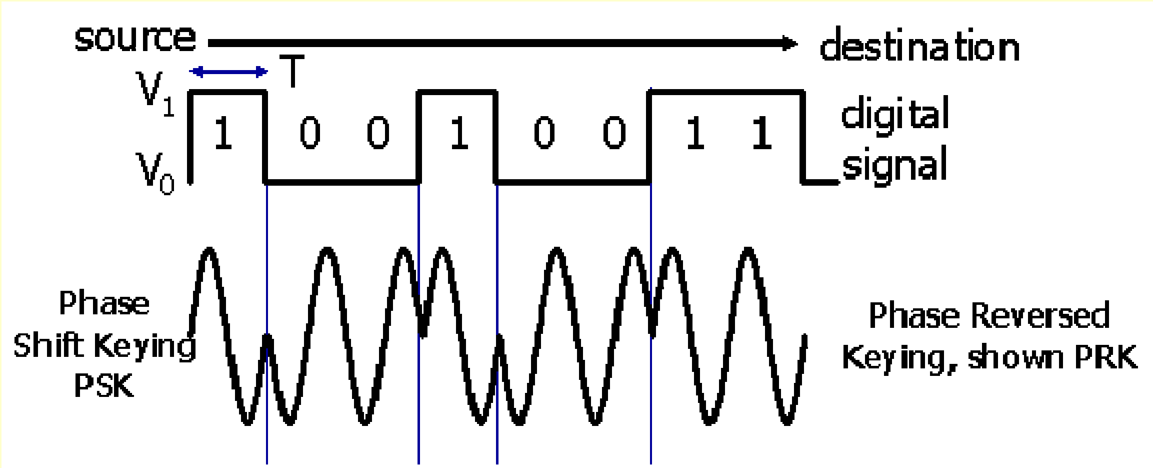
iii)Obtain $V_{out}(t)$ stating any assumptions



Solution 3

i) Phase reversed keying

ii)



iii) $V_{out}(t)$:

$$V_{OUT}(t) = h(t)\cos(\varpi_C t).\cos(\varpi_C t)$$

$$V_{OUT}(t) = \frac{h(t)}{2}\cos(0) + \underbrace{\frac{h(t)}{2}\cos(2\varpi_C t)}_{\text{use LPF to filter out this component}}$$

$$V_{OUT}(t) = \frac{h(t)}{2}$$