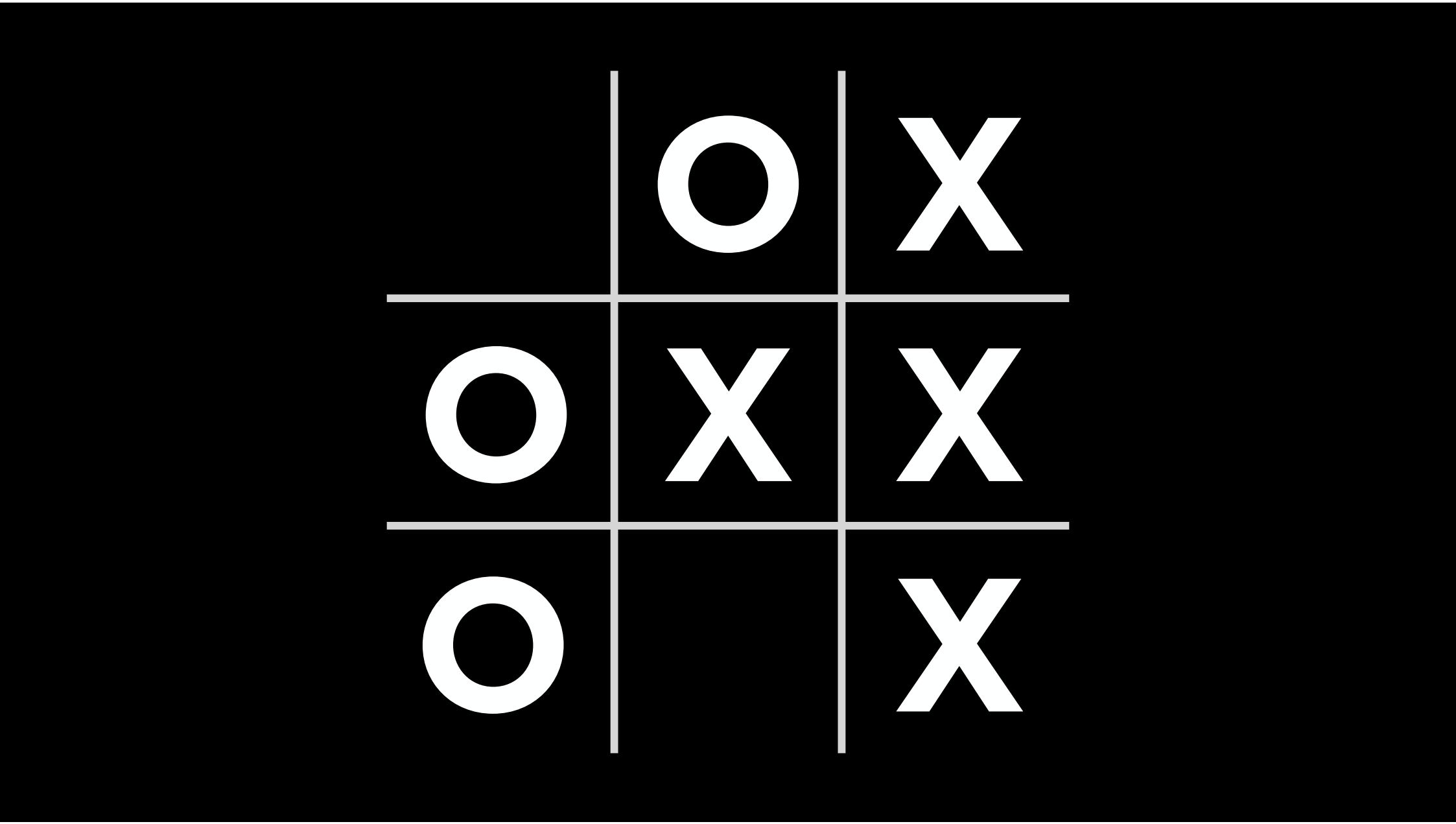
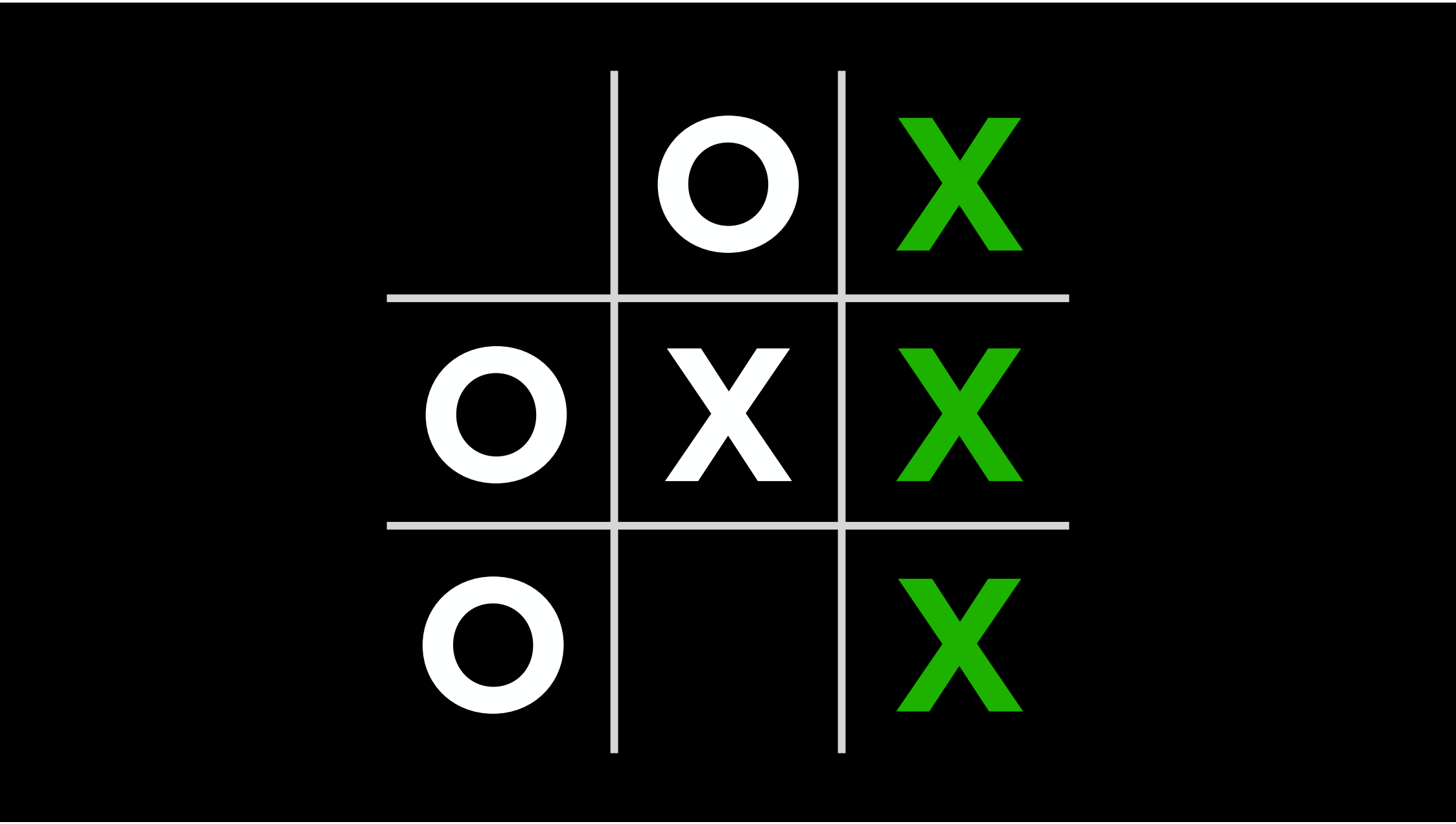
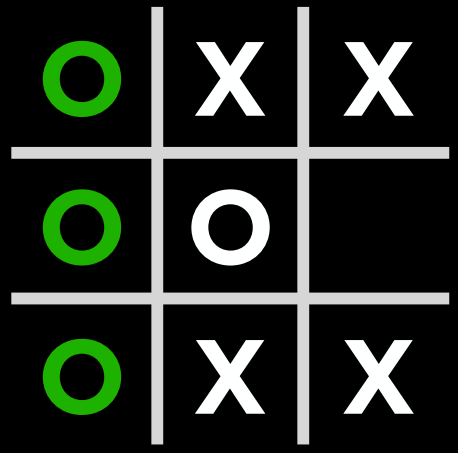
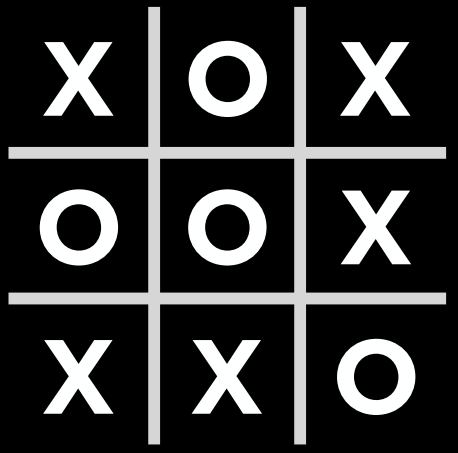

O X
O X X
O X


O X
O X X
O X

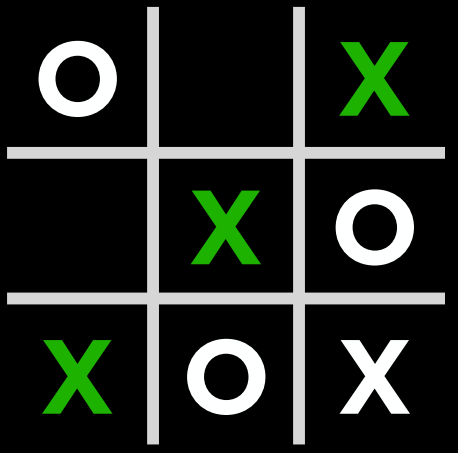
Minimax



-1



0



1

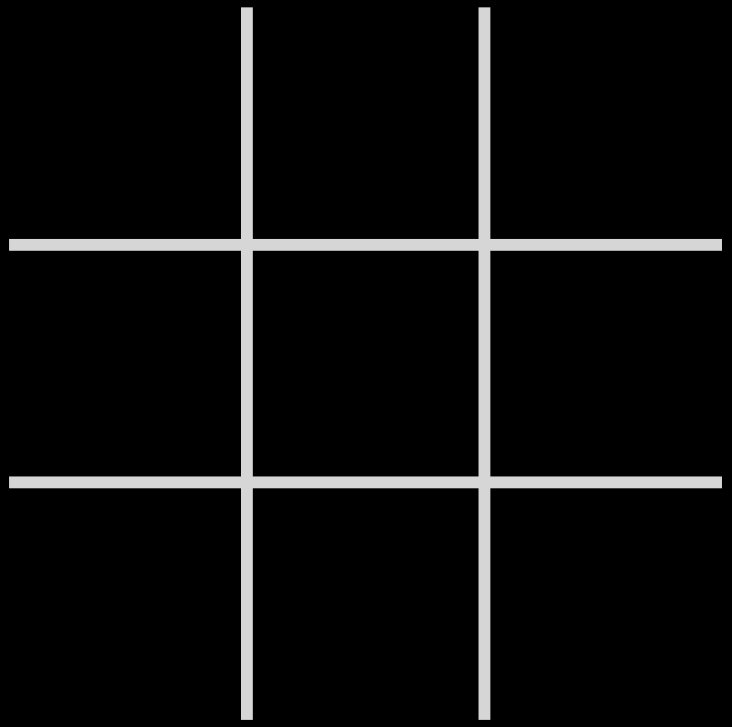
Minimax

- **MAX** (X) aims to maximize score.
- **MIN** (O) aims to minimize score.

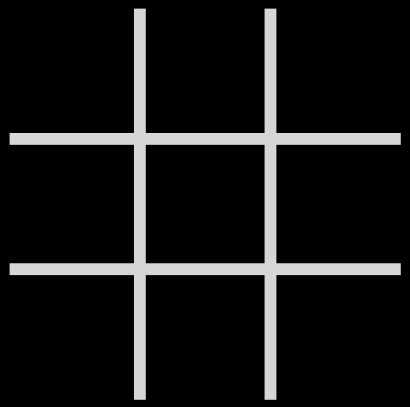
Game

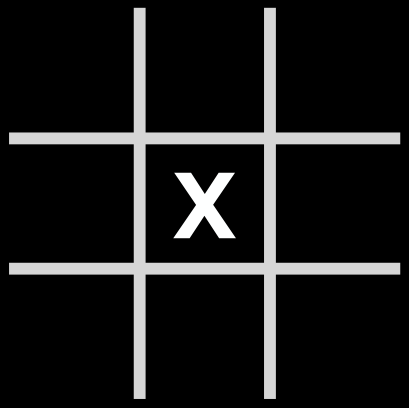
- S_0 : initial state
- PLAYER(s) : returns which player to move in state s
- ACTIONS(s) : returns legal moves in state s
- RESULT(s, a) : returns state after action a taken in state s
- TERMINAL(s) : checks if state s is a terminal state
- UTILITY(s) : final numerical value for terminal state s

Initial State



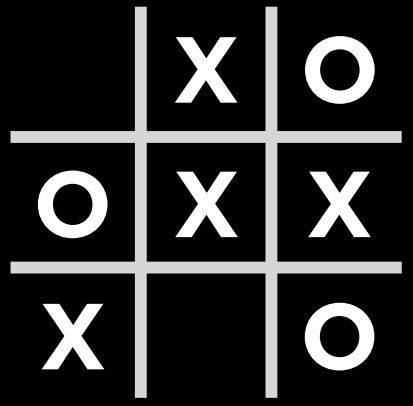
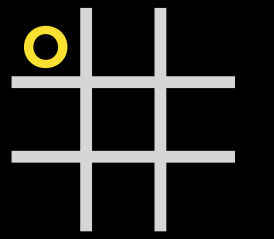
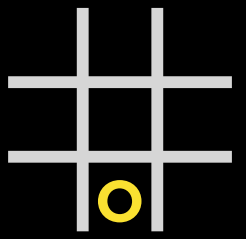
PLAYER(*s*)

PLAYER() = X

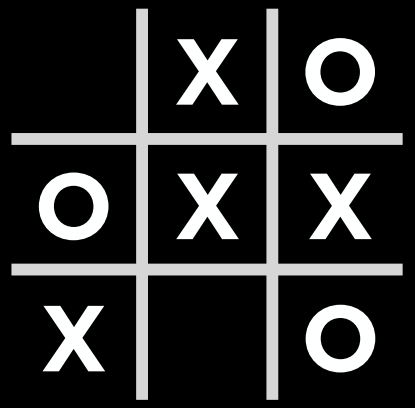
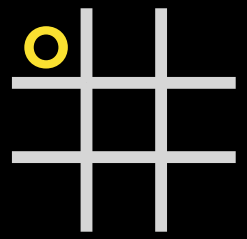
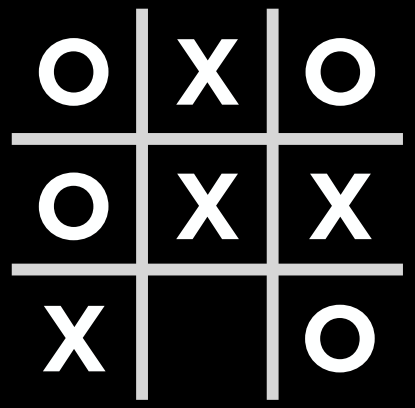
PLAYER(

) = O

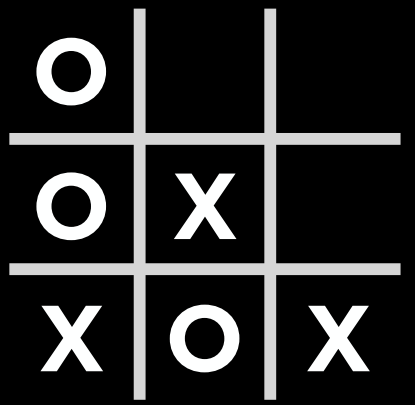
ACTIONS(s)

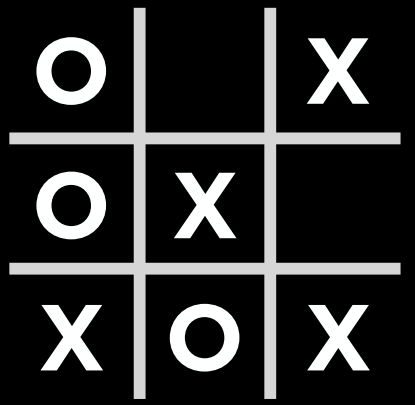
ACTIONS() = {  ,  }

RESULT(*s*, *a*)

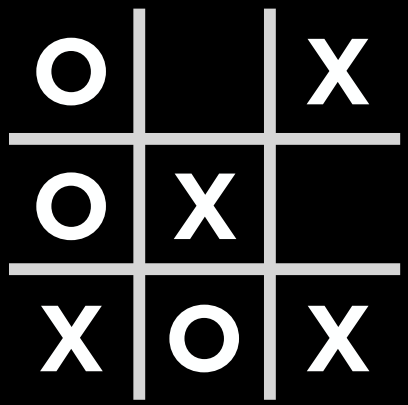
RESULT( , ) = 

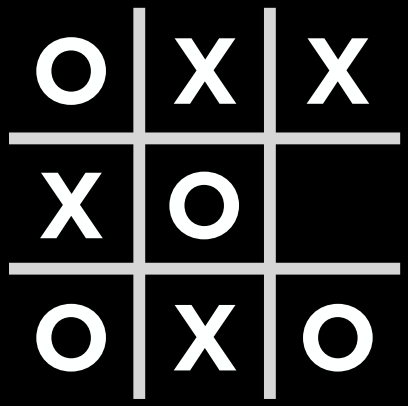
TERMINAL(s)

TERMINAL() = false

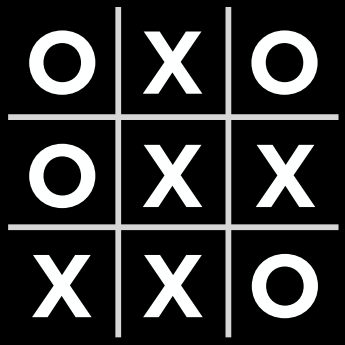
TERMINAL() = true

UTILITY(s)

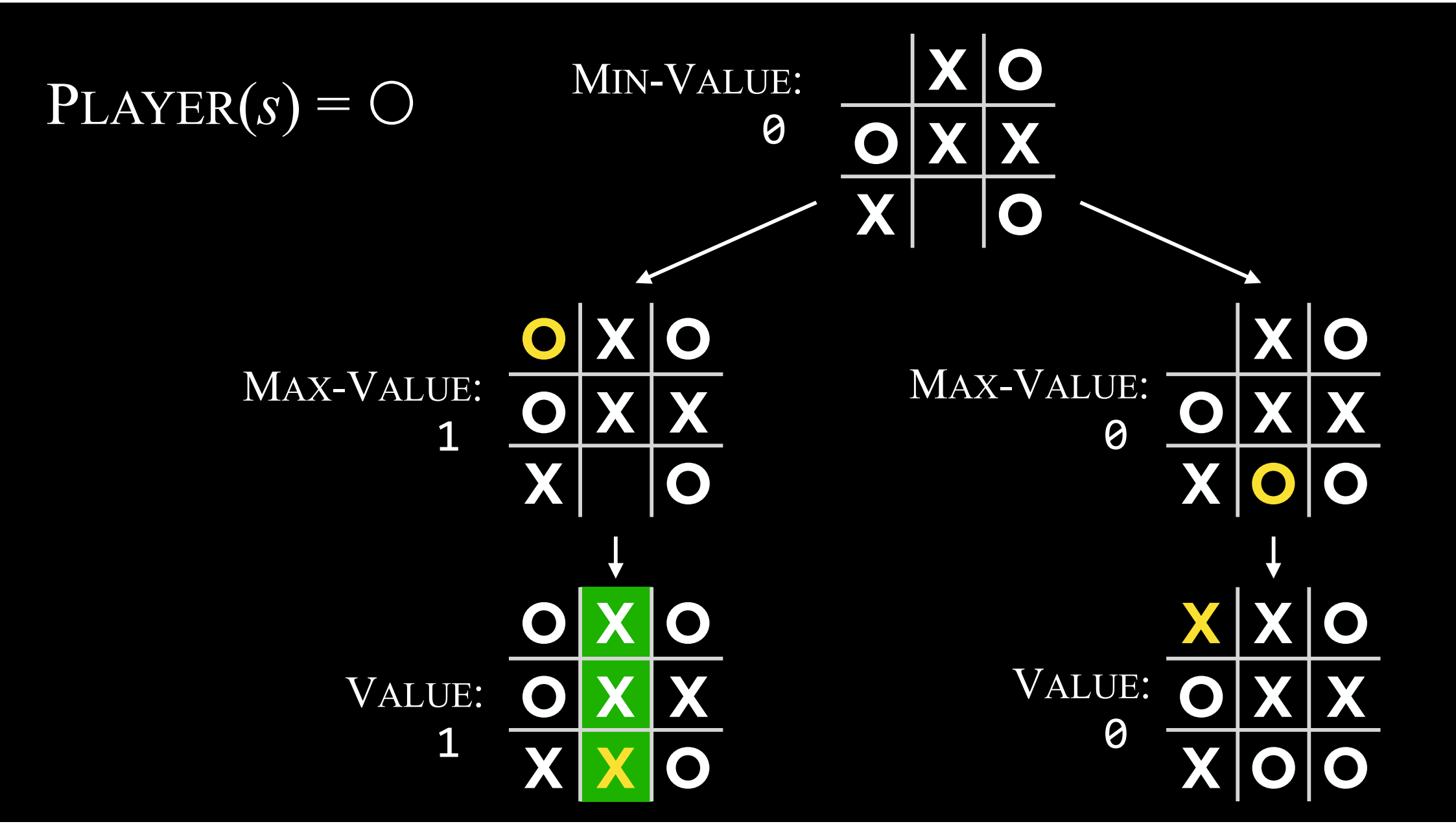
UTILITY() = 1

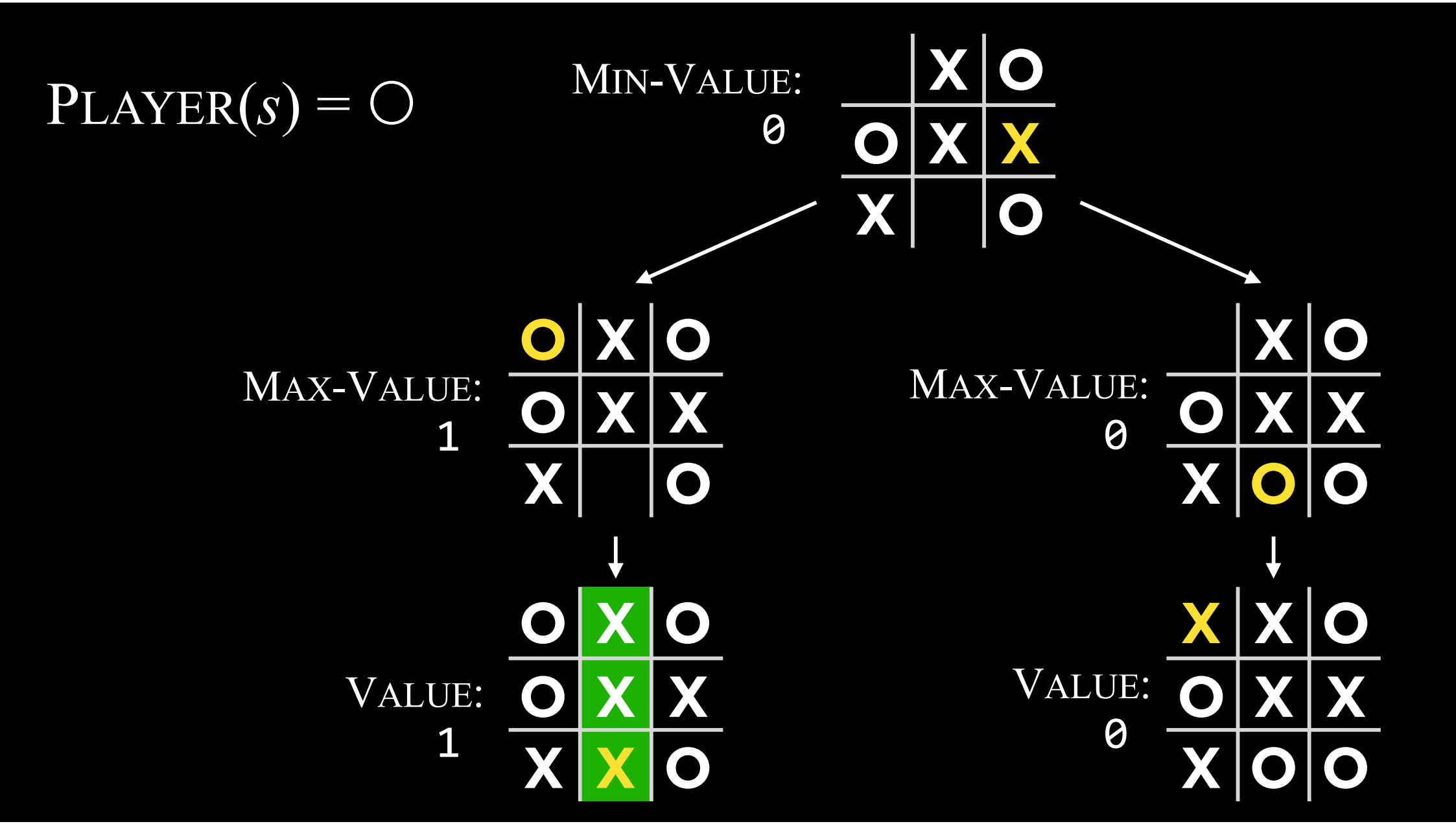


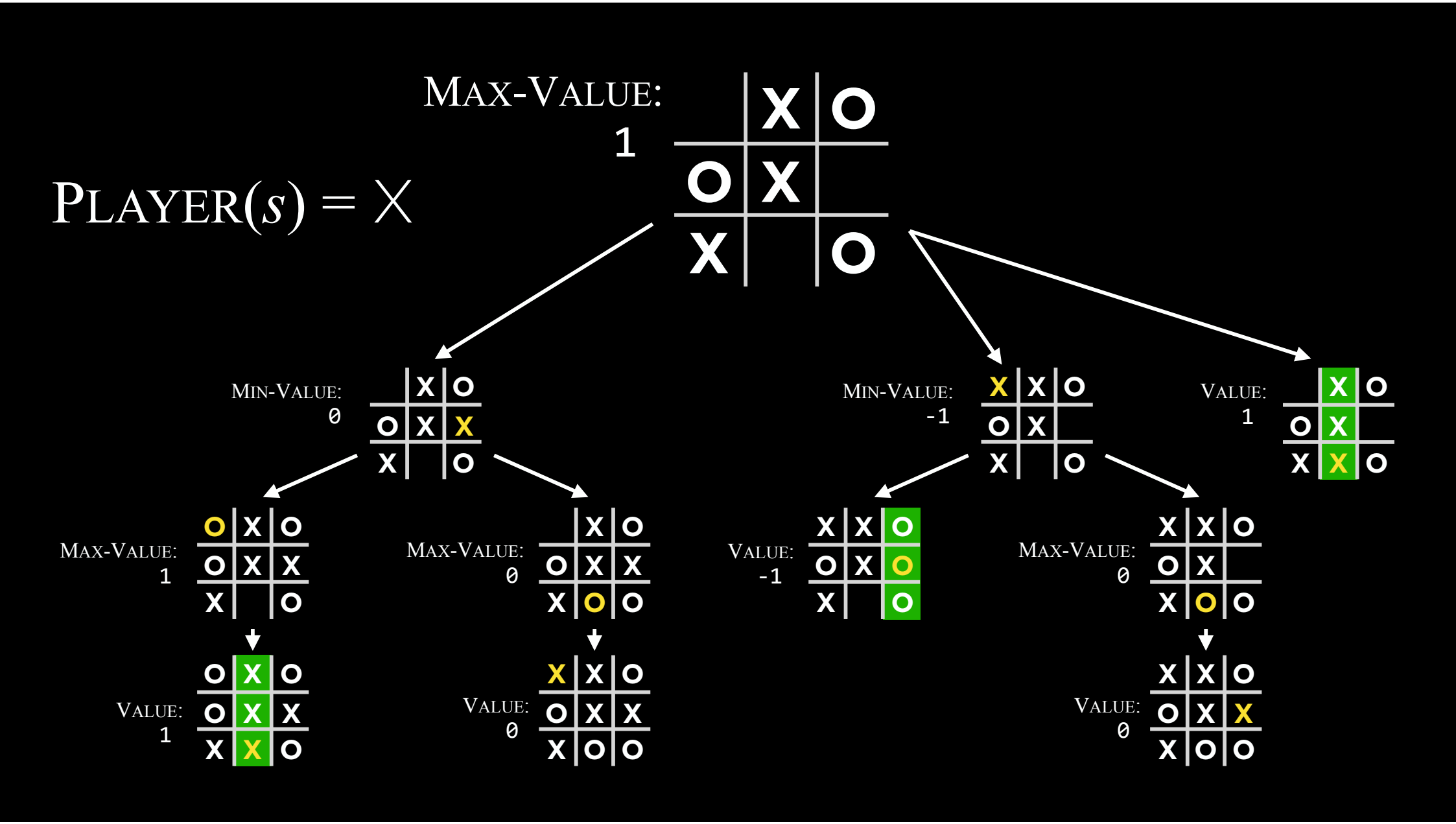
UTILITY( ) = -1

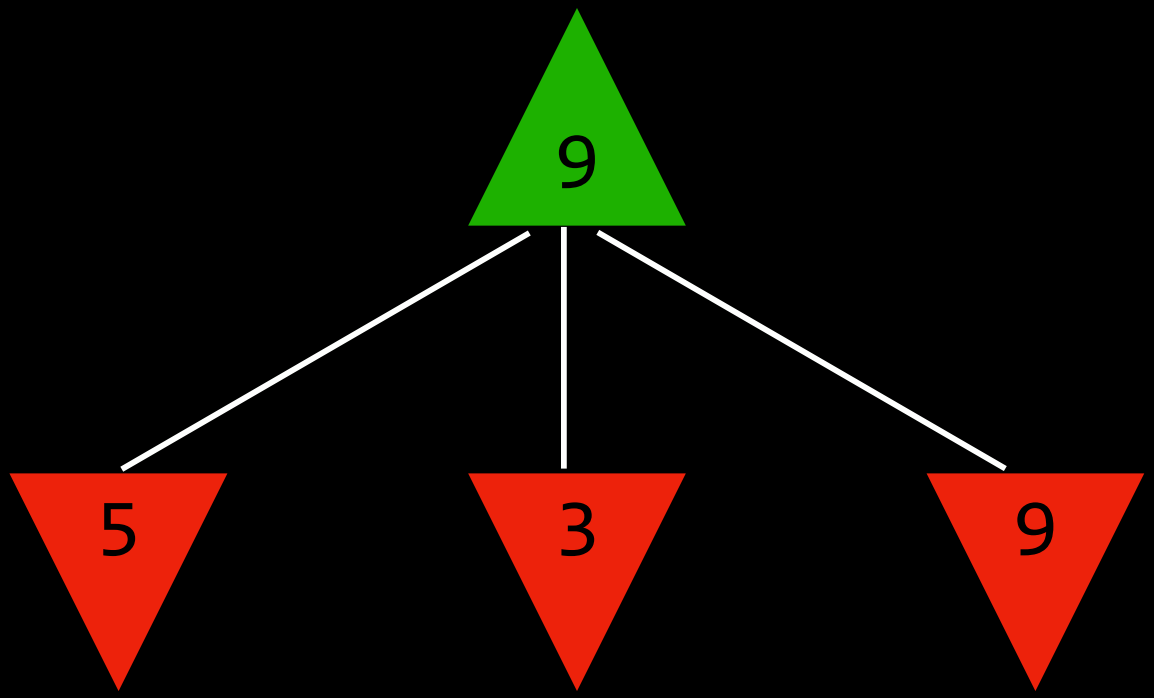


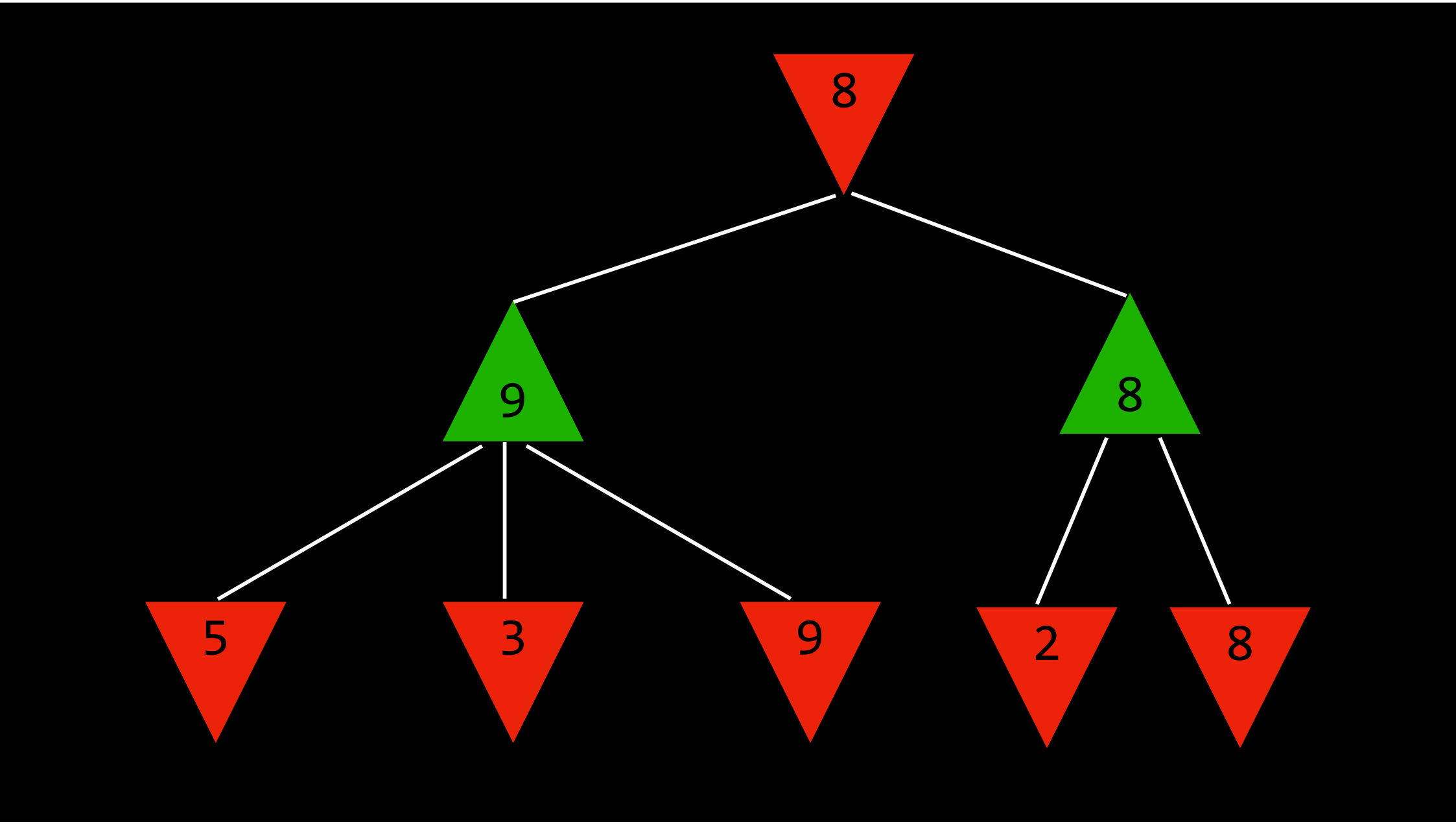
VALUE: 1


PLAYER(s) = O
MIN-VALUE:
0
MAX-VALUE:
1
MAX-VALUE:
0
VALUE:
1
VALUE:
0


PLAYER(s) = O
MIN-VALUE: 0
MAX-VALUE: 1
MAX-VALUE: 0
VALUE: 1
VALUE: 0


PLAYER(s) = X
MAX-VALUE:
1
MIN-VALUE:
0
MIN-VALUE:
-1
VALUE:
1
MAX-VALUE:
1
MAX-VALUE:
0
VALUE:
-1
MAX-VALUE:
0
VALUE:
1
VALUE:
0
VALUE:
0


9
5
3
9


8
9
8
5
3
9
2
8

Minimax

- Given a state s:
  - MAX picks action a in ACTIONS(s) that produces highest value of MIN-VALUE(RESULT(s, a))
  - MIN picks action a in ACTIONS(s) that produces smallest value of MAX-VALUE(RESULT(s, a))

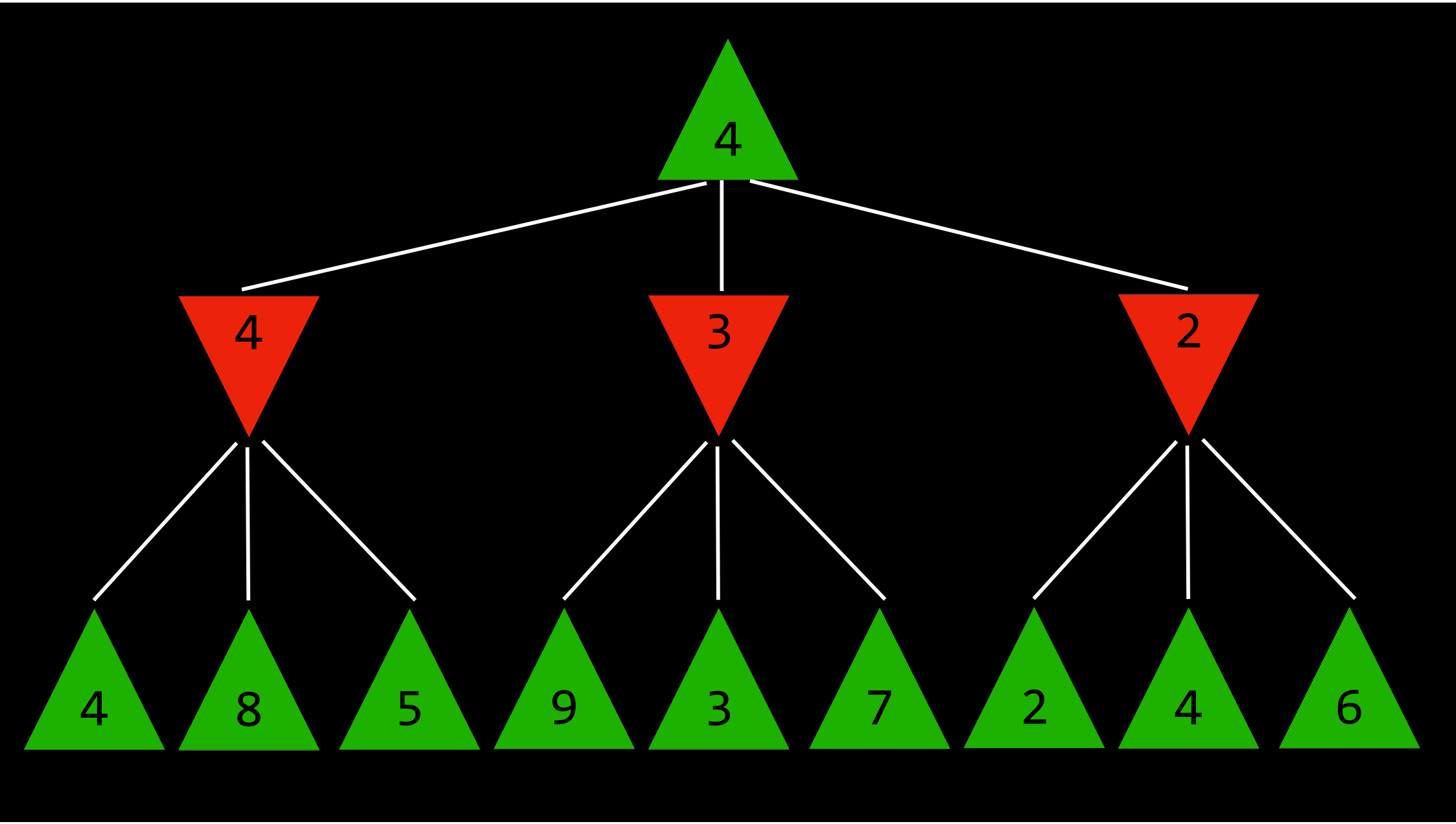
Minimax

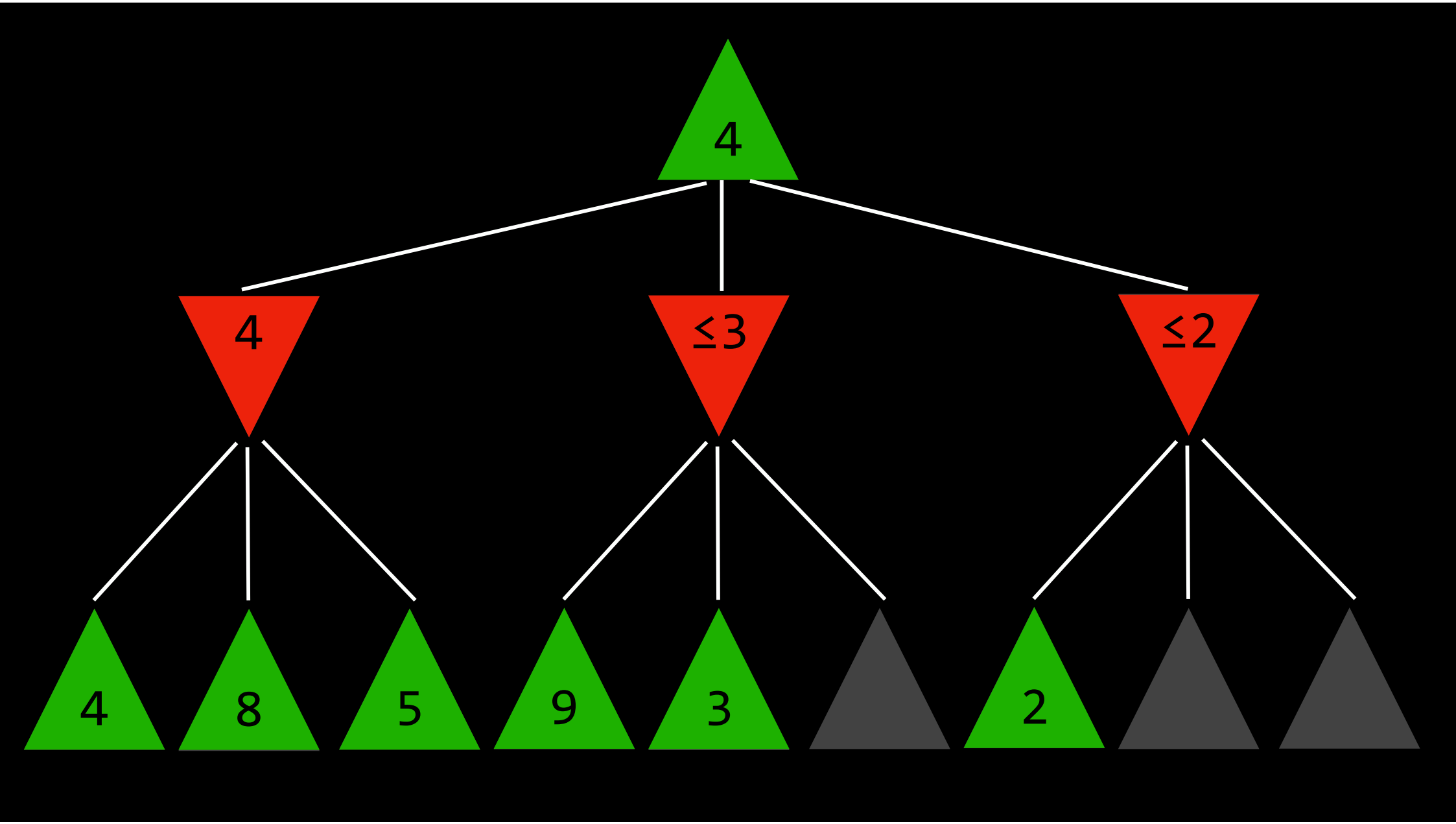
```
function MAX-VALUE(state):
    if TERMINAL(state):
        return UTILITY(state)
    v = -∞
    for action in ACTIONS(state):
        v = MAX(v, MIN-VALUE(RESULT(state, action)))
    return v
```

Minimax

```
function MIN-VALUE(state):
    if TERMINAL(state):
        return UTILITY(state)
    v = ∞
    for action in ACTIONS(state):
        v = MIN(v, MAX-VALUE(RESULT(state, action)))
    return v
```

Optimizations


4
4
3
2
4
8
5
9
3
7
2
4
6


4
4
≤3
≤2
4
8
5
9
3
2

Alpha-Beta Pruning

255,168

total possible Tic-Tac-Toe games

288,000,000,000

total possible chess games
after four moves each

10^{29000}

total possible chess games
(lower bound)