"Question #1"

(a) $\lim\limits_{(x,y)\to(2,1)} \dfrac{x^2-2xy}{x^2-4y^2}$

$\lim\limits_{(x,y)\to(2,1)} \dfrac{(2)^2-2(2)(1)}{(2)^2-4(1)^2}$

$= \cancel{4-4}$

$x^2 = 4y^2 \Rightarrow x = 4y.$

$y = x/4.$

$$\frac{x^2/16 - 2(x)(x/4)}{x^2 - 4\left(\frac{x^2}{16}\right)}$$

$$\frac{\frac{x^2}{16} - \frac{x^2}{2}}{x^2 - \frac{x^2}{4}} = \frac{(x^2-8x^2)\,4}{4\,16\,(4x^2-x^2)}$$

$$= \frac{-7x^2}{4(3x^2)} \Rightarrow -7/12$$

(b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x-4y}{6y+7x}$

$6y = 7x$

$y = 7x/6$

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$$\frac{x - 4(7x/6)}{7x + 7x}$$

$$\frac{x - 28x/6}{14x}$$

$$\frac{6x - 28x}{6(14x)} \Rightarrow \frac{-\cancel{22}^{11}x}{3\cancel{6}(14)\cancel{x}} \Rightarrow \frac{-11}{52}.$$

(c) $\lim_{(x,y)\to(0,0)} \frac{x^2 - y^6}{xy^3}$

let $y = mx$

$$\frac{x^2 - m^6 x^6}{x\, m^3 x^3}$$

$$\frac{\cancel{x}^2 (1 - m^6 x^3)}{\cancel{x}^{4\,2} m^3}$$

$$\frac{1 - m^6 x^3}{m^3 x^2}$$

(d) $\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3 - ze^2 y}{6x + 2y - 3z}$

$$\frac{(-1)^3 - 4(0)e^2}{6(-1) + 2(0) - 3(4)}$$

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$$\frac{-1-0}{-6+2-12} \Rightarrow \frac{+1}{+16} \Rightarrow 1/16.$$

Question # 2

(a) $f(x,y) = \cos(x/y)$ in $v = (3,-4)$.

$$f\nabla = \frac{\partial}{\partial x}(\cos(x/y))i + \frac{\partial}{\partial y}(\cos(x/y))j$$

$$= \frac{-1}{y}\sin(x/y)\, i + x(-\sin x/y)\left(-\frac{1}{y^2}\right)j.$$

$$= -\frac{1}{y}\sin\left(\frac{x}{y}\right)i + \frac{x}{y^2}\sin(x/y)j.$$

unitvector :

$$\frac{3i-4j}{\sqrt{9+16}} \Rightarrow \frac{3}{\sqrt{25}}i - \frac{4}{\sqrt{25}}j$$

$$D\vec{u}f = -\frac{3}{5y}\sin\left(\frac{x}{y}\right) + \frac{4x}{5y^2}\sin(x/y).$$

$$D\vec{u}f = \frac{1}{5y}\sin(x/y)\left(\frac{4x}{y} - 3\right).$$

(b) $f(x,y,z) = x^2y^3 - 4xz$; $\vec{v} = (-1,2,0)$

$$\nabla f = (2y^3xi + (-4z)i) + 3y^2x^2j - 4xk.$$

$$\hat{v} = \frac{-i+2j+0k}{\sqrt{1+4}} \Rightarrow \frac{-1}{\sqrt{5}}i + \frac{2}{\sqrt{5}}j + 0k$$

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$$D_u f = -\frac{1}{\sqrt{5}}(2y^3x - 4z) + \frac{2}{\sqrt{5}}(3y^2x^2) + 0.$$

Question #3

$$f(x,y,z) = 4x - y^2 e^{3xz}$$

$$\nabla f = 4 - y^2 e^{3xz}(3z)\,i - 2ye^{3xz_0}\,j + y^2 e^{3xz} \cdot 3x\,k$$

$$\nabla f\big|_{(3,-1,0)} = 4 - (-1)^2 e^{3(3)(0)}\,(2)(3)\,i - 2(-1)e^{3(3)(0)}\,j - (-1)^2 e^{3(3)0} \cdot 3(3)$$

$$= (4-0)\,i - 2j - 9k.$$

$$v = (-1, 4, 2)$$

$$\hat{v} = \frac{-1i + 4j + 2k}{\sqrt{1+16+4}} \Rightarrow \frac{-1}{\sqrt{21}}i + \frac{4}{\sqrt{21}}j + \frac{2}{\sqrt{21}}k$$

$$\frac{-1}{\sqrt{21}}(4) - 2\frac{(4)}{\sqrt{21}} - 9\frac{(2)}{\sqrt{21}}$$

$$\frac{-4}{21} - \frac{8}{\sqrt{21}} - \frac{18}{\sqrt{21}} \Rightarrow \frac{-4-8-18}{\sqrt{21}} \Rightarrow \frac{-32}{\sqrt{21}}$$

Question # 4

(a) $f(x,y) = \sqrt{x^2+y^3}$ at $(-2,3)$

$$\nabla f = \frac{1}{2}(x^2+y^3)^{-1/2}(2x)i + \frac{1}{2}(x^2+y^3)^{-1/2}(3y^2)j$$

$$\nabla f(-2,3) = (4+9)^{-1/2}(-2)i + \frac{1}{2}(4+9)^{-1/2}(3(9))j$$

$$= \frac{-2}{\sqrt{13}}i + \frac{27}{2\sqrt{13}}j$$

(b) $f(x,y,z) = e^{2x}\cos(y-2z)$ at $(4,-2,0)$

$$\nabla f = (e^{2x}.2\cos(y-2z))i + e^{2x}(-\sin(y-2z)(1))j$$
$$+ e^{2x}(-\sin(y-2z)(-2)k$$

$$\nabla f(4,-2,0) = e^{4x}.2\cos(-2-0)i + e^{4x}(-\sin(-2))j$$
$$+ e^{4x}(-\sin(-2))(-2)$$

$$\nabla f(4,-2,0) = e^{4x}(-2\cos 2\, i - \sin(-2)j + 2\sin(-2)k).$$

Question # 5

(a) $F = x^2 y \hat{i} - 2(z^3 - 3x)\hat{j} + 4y^2 \hat{k}$

$\nabla = 2xyi - 0j + 0.$

$\text{Div } f = \nabla f \cdot f.$

$= (2xy\, i) \cdot (x^2 y\, i - (z^3 - 3x) j + 4y^2 k).$

$\text{Div } f = 2x^3 y^2 + 0j$

$\text{curve} = \nabla f \times f.$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & -(z^3 - x3) & 4y^2 \end{vmatrix}$$

$= i(8y + 3z^2) - j(0-0) + k(+3x^2 - x^2).$

$= (8y + 3z^2) i - 0j + (2x^2) k.$

(b) $F = (2x + 2z^2) i + \frac{x^3 y^2}{z} j - (z - 7x) k.$

$\text{div} = \nabla f \cdot f.$

$= \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right) \cdot \left((2x + 2z^2) i + \frac{x^3 y^2}{z} j - (z -\right.$

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$$= 2 + \frac{2x^3}{z}y - (1)$$

$$= 2 + \frac{2x^3}{z}y - 1.$$

curve $= \nabla f \times f$

$$\begin{vmatrix} i & j & k. \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x + 2x^2 & \frac{x^3y^2}{z} & -(z - 7x) \end{vmatrix}$$

$$i\left(0 + \frac{x^3y^2}{z^2}\right) - j(7 - 4x) + k\left(\frac{3x^2}{z}y^2 + 0\right)$$

$$\frac{x^3y^2}{z^2}i - (7 - 4x)j + \left(\frac{3x^2}{z}y^2\right)k.$$

Q8:

$F = x^2 y i -$

$F = \left(4x^2 + \frac{3x^2y}{z^2}\right)i + \left(8xy + \frac{x^3}{z^2}\right)\hat{j} + \left(11 - \frac{2x^3y}{z^3}\right)$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}, \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

$$F = \left(4y^2 + \frac{3x^2y}{z^2}\right)i + \left(8xy + \frac{x^3}{z^3}\right)j + \left(11 - \frac{2x^3y}{z^3}\right)\hat{k} = P$$

$$\frac{\partial M}{\partial y} = 8y + \frac{3x^2}{z}, \quad \frac{\partial N}{\partial x} = 8y - \frac{3x^2}{z^2}$$

$$\frac{\partial N}{\partial z} = x^3 z^{-2} = x^3(-2)z^{-3} = -\frac{2x^3}{z^3}$$

$$\frac{\partial P}{\partial y} = \frac{-2x^3}{z^3}$$

$$\frac{\partial M}{\partial z} = 4x^2 + \frac{3x^2y}{z^2}$$

$$= 3x^2y(-2)z^{-3}$$

$$= \frac{-6x^2y}{z^3}$$

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x}\left(11 - \frac{2x^2y}{z^3}\right) = \frac{-6x^2y}{z^3}$$

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) $\vec{F} = 6x\hat{i} + (2x - y^2)\hat{j} + (6z - x^3)\hat{k}$

$\frac{\partial M}{\partial y} = \frac{\partial (6x)}{\partial y} = 0$,

$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(2x - y^2) = 2$

$\frac{\partial N}{\partial z} = \frac{\partial}{\partial z}(2x - y^2) = 0$,

$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}(6z - x^3) = 0$

$\frac{\partial M}{\partial z} = \frac{\partial}{\partial z} 6x = 0$

$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x}(6z - x^3) = -3x^2$

so;

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$, $\frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}$; $\frac{\partial M}{\partial z} \neq \frac{\partial P}{\partial x}$

It is not conservative

$4x^{\dots} y \quad dx$

Q7:

a) $z = \frac{x^2 - w}{y^4}$, $x = t^3 + 7$

$y = \cos(2t)$, $w = 4t$

$\frac{dz}{dt} = \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{y} \cdot \frac{dy}{dt} + \frac{dz}{dw} \frac{dw}{dt}$

$\frac{dz}{dt} = 3t^2$, $\frac{dz}{dy} = (x^2 - w) \cdot 4 y^{-5}$

$\frac{\partial z}{\partial y} = \frac{-4(x^2 - w)}{y^5}$

$\frac{dz}{dt} = \frac{2x}{y^4}$, $\frac{\partial z}{\partial y} = (x^2 - w) y^{-4}$

$\frac{dy}{dt} = \frac{d}{dt} \cos 2t = -\sin 2t \cdot 2$

$= -2 \sin 2t$

$\frac{\partial z}{\partial w} = \frac{-1}{y^4}$ $\frac{dw}{dt} = 4$

$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{dy} \cdot \frac{dy}{dt} + \frac{\partial z}{\partial w} \cdot \frac{dw}{dt}$

$= \frac{2x^2}{y^4} \cdot 3t^2 + \left(\frac{-4(x^2 - w)}{y^5}\right) \cdot -2\sin 2t$

$+\left(\frac{-1}{y^4}\right)4$

$= \frac{(2x+1)^2}{y^4} + \frac{8(x^2 \cdots)\sin 2t}{y^5} - \frac{y\omega}{y^4}$

b) $z = x^2y^4 - 2y \quad , y = \sin(x^2)$

$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$

$\frac{dz}{dy} = \frac{\partial}{\partial y}(x^2y^4 - 2y)$

$= (4y^3 x^2 - 2)$

$= (4x^2y^3 - 2)$

$\frac{dy}{dx} = \frac{\partial}{\partial x}\sin(x^2)$

$= \cos x^2 \cdot 2x$

$= 2x\cos x^2$

$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial x}$

$= (4x^2y^3 - 2)(2x\cos^2 x)$

$= 8x^3y^3\cos x^2 - 4x\cos x^2$

) $x^2y^4 - 3 = \sin(xy)$

$\frac{\partial}{\partial x}(x^2y^4 - 3) = \frac{\partial}{\partial x}(\sin(xy))$

$2xy^4 + x^2 4y^3 \frac{dy}{dx} = y\cos(xy)$

$2xy^4 + 4x^2y^3 \frac{dy}{dx} = y\cos(xy)$

$4x^2y^3 \frac{dy}{dx} = y\cos(xy) - 2xy^4$

$\frac{dy}{dx} = y\left(\frac{\cos(xy) - 2xy^3}{4x^2y^2}\right)$