Exercise 1

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Q1.

$$p(y \mid \mu, \kappa) = \frac{1}{2\pi I_0(\kappa)} \exp(\kappa \cos(y - \mu)),$$

Log-likelihood. Taking the logarithm and using $\log(ab) = \log a + \log b$,

$$\log p(y \mid \mu, \kappa) = \kappa \cos(y - \mu) - \log(2\pi) - \log I_0(\kappa).$$

$$\mathrm{NLL}(\mu, \kappa; y) = -\log p(y \mid \mu, \kappa) = -\kappa \cos(y - \mu) + \log(2\pi) + \log I_0(\kappa).$$

Q2.

$$\mathrm{NLL}(\mu, s; y) = -e^s \cos(y - \mu) + \log(2\pi) + \log I_0(e^s).$$

Hence,

$$\mathrm{NLL}(\mu, s; y) = \log I_0(e^s) - e^s \cos(y - \mu).$$

Q3. Predicting $s = \log \kappa$ instead of κ directly is advantageous because it automatically enforces the constraint $\kappa \geq 0$ without requiring additional constraints or clipping during optimization.