Scaling and Programming the quantum bus of trapped ions by tailoring motional modes with optical tweezers

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Trapped ion system is an ideal platform for quantum computation, where the spins of the trapped atoms supply the quantum registers, and the motional modes of trapped atoms interacted by Coulomb force account for the quantum bus.

the limit of a quantum computer based on trapped ions is not the size of the registers, the number of the spin can be trapped to hundreds in current apparatus. The

We proposed a programmable quantum bus to tailor scheme to tailor and control the motional modes in a ion trap, by addressing optical tweezers on required ions.

the scheme has ability to extend ion number to hundreds in a chain, while two ion entanglement gate keeping a high fidelity. Combing with fast gates operation

realistic way to quantum supremacy.

I. INTRODUCTION

Recent years, the size of the controllable quantum system is increased to tens of qubits, such as Ryder-berg atoms , superconductive circuit. The trapped ions system is one of most potentials for quantum information research, benefiting from combining the advantages of the high fidelity, good stability and long-range interactions.

Transverse modes of trapped ions provide a flexible way to addressing and entangle ions, which have potential to scale the [1], however the transverse modes will be dense and close two each other with the ion

Fast gate [2]

Scaling the trapped ion system with microtraps [3]

the Coulomb forces between the ions can support long range interactions, the motions of trapped ions can be coupled to different motional modes, which act as a quantum bus to entangle the trapped ions. The motional modes. To programmable control

Optical tweezers are widely used in AMO research in recent years. Tightly focused light can provide a strong optical force by the AC Stark shift effect, which can trap the atom with a high motional frequency. In this work, we proposed a scheme to scaling quantum computation on a large ions chain, which based on tailoring and controlling the motional modes by addressing optical tweezers on required ions. As optical traps impact on the ion chain in the ion trap can be programmed, individual ion coupling to the quantum bus can be controlled. Our method is compatible with fast gates schemes, which will tailor

Monroe hundred ions chain ,

The text is organized as following: In the first part, the basic scheme is described, which is mainly based on fast operation Raman lasers and an additional dipole laser, and $^{171}\mathrm{Yb}^{+}$ ion system is taken as an example the next studies. In the second part, tailoring effects of ions motional modes under optical dipoles traps are studied. In the third part, we discussed the two and multi qubit gate under the tailored modes. In order to study the performance of the quantum gate, two ion entanglement gates are numerically simulated, which explore high fidelity on a long chain with hundred $^{171}\mathrm{Yb}^{+}$ ions, even in case that two operated ions are separated by tens of ion distance. Finally, a discussion is present, such as multi ion entanglement gates, further extension of the scheme.

The motional frequency of a atom trapped in an optical tweezer can reach MHz order of magnitude in current experiment , which is same as the secular motion frequency of a trapped ion in Paul trap. The tailor effect of by a optical standing wave has been observed in experiment. motional

Multi gates can be also,

II. SCHEME

a chain of ions is trapped in a linear trap along with y direction, a pair of Raman lasers beams are used to operate spin states, and a strong dipole laser are used to generate optical dipole traps. Both the Raman lasers and dipole laser are modulated by AODs, so the laser beams can be focused to multi spots on ions. The Raman beams oppositely propagate along with x direction, creating a walking standing wave, and are focused to address ions by the side objective lenses (L_1, L_2) , which is used to operate and entangle the spin state of addressed ions through the transverse ion motional modes. The dipole laser beam can be focused to a series of dipoles trapped by the top objective lens (L_0), to tailor the motional modes of ion, that is to program the quantum bus. (b) An example energy level with $^{171}\mathrm{Yb}^{+}$ ion and the corresponding Raman laser configuration.

The Raman beams have a detuning of μ, which will cre-

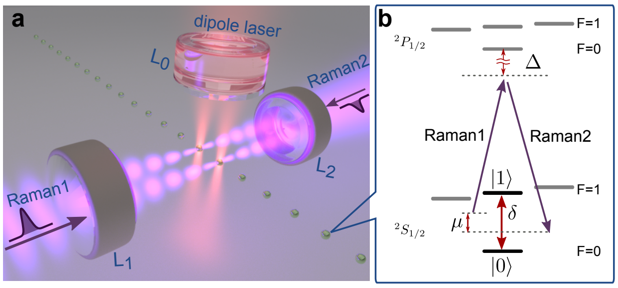

FIG. 1. The scheme to scaling a trapped ion processor by programming the quantum bus with optical tweezers. (a) a chain of ions is trapped in a linear trap along with y direction, a pair of Raman lasers beams are used to operate spin states, and a strong dipole laser are used to generate optical dipole traps. Both the Raman lasers and dipole laser are modulated by AODs, so the laser beams can be focused to multi spots on ions. The Raman beams oppositely propagate along with x direction, creating a walking standing wave, and are focused to address ions by the side objective lenses (L_1, L_2) , which is used to operate and entangle the spin state of addressed ions through the transverse ion motional modes. The dipole laser beam can be focused to a series of dipoles trapped by the top objective lens (L_0), to tailor the motional modes of ion, that is to program the quantum bus. (b) An example energy level with $^{171}\mathrm{Yb}^+$ ion and the corresponding Raman laser configuration, the detuning parameters satisfy $\mu \ll \delta \ll \Delta$.

ate a walking standing wave to operate the ion motion. An example energy level with $^{171}\mathrm{Yb}^+$ ion and the corresponding Raman laser configuration is shown in Fig. 1b, the detuning of ground nuclear spins $\delta = 12.643$ GHz, the detuning between Raman beams μ is in order MHz, and Δ is the laser detuning to the excitation levels, which is in range of hundreds of GHz. So the detuning parameters are in condition of $\mu \ll \delta \ll \Delta$.

III. MOTINAL MODES TAILORED BY OPTICAL DIPOLES

IV. TWO ION ENTANGLEMENT GATES

1. nebeigh ions gate
2. none neibeigh ion gate
3. multi ion gate
4. long-distance ion gate

A. Advantages

tailor modes and program the interactions

improving gate speed. combining the fast operation schemes [2].

can be extend to 2D trapped ion systems.

support fast operations, high fidelity and large Q number of operation.

B. Hamiltonian of system

1. *Focused Gaussian laser beam trap and Standing wave trap*

Light induced shift of the atoms energy level, so call ac-Stack shift, can bring trapping effect. For example, an atom in the ground state acts as a potential U_{dip} in which the atom moves. In the case of frequency detuning to red, the dipole force acts in the direction of increasing I: atoms in a tight-focused laser beam are attracted towards the region of high intensity, both in racial direction and along the axis of the beam. The dipole force confine the atom and create a dipole trap. The optical dipole force is strong in a standing wave condition because the intensity changes a lot in the scaling of wavelength.

The potential energy of the dipole trap for different ground hyperfine states is[4]

$$U_{dip,i}(r) = \frac{3\pi c^2 \Gamma}{2\omega_0^3} \sum_j \frac{c_{ij}^2}{\Delta_{ij}} I(r), \tag{1}$$

where ω_0 is the angular frequency of the laser beam, Δ_{ij} is the detuning from resonance between level i and j, $c_{i,j}$ is the CG coefficient between i and j level, and Γ is the spontaneous rate. The light intensity has the form of Gauss beam: $I(r,z) = I_0 \left(\frac{\sigma_0}{\sigma(z)}\right)^2 \exp\left(\frac{-2r^2}{\sigma(z)^2}\right)$, and $\sigma(z) = \sigma_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}$. In our scheme, the qubit is defined by the ground hyperfine states of $^{171}\mathrm{Yb}^+$ ions: $|0\rangle \equiv {}^2S_{1/2}\,|F=0, m_F=0\rangle$ and $|1\rangle \equiv {}^2S_{1/2}\,|F=1, m_F=0\rangle$, with an energy splitting $\delta = 2\pi \times 12.643\,\mathrm{GHz}$, so only the energy shift of these two energy levels marked as $U_{dip,0}$, and $U_{dip,1}$ are studied. The dipole lasers have a red detuning Δ (negative value), if the dipole trap laser is set in near resonance condition with $\Delta_{i,j} \ll \omega_0$, $S_{1/2}$ to $P_{1/2}$transition mainly contribute to the dipole potential, $c_{i,j}^2$ will be 1/3. The potentials are

$$U_{dip,0}(r) = \frac{\pi c^2 \Gamma}{\omega_0^3(\Delta - \delta/2)} I(r), \tag{2}$$

$$U_{dip,1}(r) = \frac{\pi c^2 \Gamma}{\omega_0^3(\Delta + \delta/2)} I(r), \tag{3}$$

which can be decomposed to an common potential $U_{dip,C}$ (spin independent potential) and differential potential $U_{dip,D}$ (spin dependent potential)

$$\begin{cases} U_{dip,C} = \frac{1}{2}(U_{dip,0} + U_{dip,1}) \\ U_{dip,D} = \frac{1}{2}(U_{dip,0} - U_{dip,1}) \end{cases}. \tag{4}$$

The light for the optical tweezers is a Gauss beam vertical to the ion chain and propagate along with z axis, which is focused on the ions at $z = 0$. So the restraint of the atom in the radial x direction, which is the direction of motional modes can be operated by the Raman beams, has the form of

$$U_T(x) = U_T(0)e^{-2\left(\frac{x}{\sigma_D}\right)^2} \simeq U_T(0)\left[1 - 2\left(\frac{x}{\sigma_T}\right)^2\right], \quad (5)$$

where σ_T and $U_T(0) = U_{Tweezer}(\vec{r} = \mathbf{0})$ are the beam waist and the potential depth of the focused dipole beam, respectively. As the potential is close to harmonica, when the motion amplitude $x_{max} \ll \sigma_T$, it will lead to a trapping frequency at the focus

$$\nu_T = \sqrt{\frac{4\,|U_T(0)|}{m\sigma_T^2}}. \quad (6)$$

The Raman beam propagate in the opposite direction with the wave vector k and $-k$ and have a frequency difference μ, they will create a walking standing wave field which generate optical forces on the ions, the standing wave leads to a potential form:

$$\begin{aligned} U_R(x,t) &\simeq U_R(0)\cos^2(kx + (\mu t + \phi_0)/2)\left[1 - 2\left(\frac{x}{\sigma_R}\right)^2\right] \\ &\simeq \frac{1}{2}U_R(0)\left[1 + \cos(2kx + \phi(t))\right] \end{aligned} \quad (7)$$

where σ_R and $U_R(0) = U_{Raman}(\vec{r} = \mathbf{0})$ are the beam waist and the potential depth of the focused Raman beams, respectively, and $\phi(t) = \mu t + \phi_0$ is the phase difference between the beams at $x = 0$. In our scheme for ion entanglement gates, we set the Raman beams with a constant detuning μ, and a programmable phase $\phi(t)$.

2. The Hamiltonian

Suppose a single ion with mass of m is trapped, one of its motion direction is along x axis, which is oscillating with frequency of ν_0 in the bare ion trap, after applying the optical dipole beam and the Raman beams, the system Hamiltonian is

$$\begin{aligned} \hat{H} &= \left(\frac{\hat{p}^2}{2m} + \frac{1}{2}m\nu_0^2\hat{x}^2\right)\otimes\mathbb{I} + \left(U_{R,0}(\hat{x},t) + U_{T,0}(\hat{x})\right)\otimes|0\rangle\langle 0| + \left(U_{R,1}(\hat{x},t) + U_{T,1}(\hat{x})\right)\otimes|1\rangle\langle 1| + \frac{1}{2}\hbar\delta\mathbb{I}\otimes\sigma_z \\ &= \left(\frac{\hat{p}^2}{2m} + \frac{1}{2}m\nu_0^2\hat{x}^2 + U_{T,C}(\hat{x})\right)\otimes\mathbb{I} + U_{R,C}(\hat{x},t)\otimes\mathbb{I} + U_{R,D}(\hat{x},t)\otimes\sigma_z + U_{T,D}(\hat{x})\otimes\sigma_z + \frac{1}{2}\hbar\delta\mathbb{I}\otimes\sigma_z \\ &\simeq \underbrace{\left(\frac{\hat{p}^2}{2m} + \frac{1}{2}m(\nu_0^2 + v_T^2)\hat{x}^2\right)\otimes\mathbb{I}} + \underbrace{\frac{1}{2}\cos(2k\hat{x} + \Phi(t))\left[U_{R,C}(0)\otimes\mathbb{I} + U_{R,D}(0)\otimes\sigma_z\right]} + \\ &\quad \underbrace{\frac{1}{2}(U_{R,D}(0) + \delta)\mathbb{I}\otimes\sigma_z} - \underbrace{2U_{T,D}(0)\left(\frac{\hat{x}^2}{\sigma_T}\right)\otimes\sigma_z} + \underbrace{U_{const}\mathbb{I}\otimes\mathbb{I}} \end{aligned}, \quad (8)$$

where σ_z is the Pauli matrix, $U_{R(T),C}$, $U_{R(T),D}$ are the Raman (dipole) laser induced spin independent potential and spin dependent potential, respectively, which can be expressed in Eq. . The fist term shows the tailored motion by the dipole laser, the final motion frequency is $\nu = \sqrt{\nu_0^2 + v_T^2}$; the second and third term the Hamiltonian of the Raman laser to control qubit state, which will further studied; the forth term is spin dependent force induced by the dipole laser, as the dipole laser is far detuning from the transition, $\Delta \gg \delta$, so $U_{T,D}(0) \to 0$ and this term can be neglected; the last term is a constant energy, which can be ignored.

The Hamiltonian of the Raman laser can be further expanded in Lamb-Dicke approximation $|2kx| \ll 1$, from the approximation

$$\cos(2k\hat{x} + \phi) \simeq \cos\phi - 2k\hat{x}\sin\phi,$$

and the definition $\hat{x} = \sqrt{\frac{\hbar}{2m\nu}}(a + a^+) = x_0(a + a^+)$, the system Hamiltonian is derived as $H = H_0 + H_1$, where

$$\begin{aligned} H_0 &= \hbar\left[\nu a^+ a + \frac{1}{2}\left(\Omega^D\cos\phi + \Omega^D + \delta\right)\sigma_z\right], \\ H_1 &= -\hbar\eta\left(\Omega^C + \Omega^D\sigma_z\right)\sin\phi\otimes(a^+ + a), \end{aligned}$$

here $\Omega^C = U_{R,C}(0)/\hbar$, $\Omega^D = U_{R,D}(0)/\hbar$, and $\eta = kx_0$ is the Lamb-Dicke parameter.

The Hamiltonian can be extended to multi ions by taking motion of each ion into the mode. The position of of lth ion $\hat{x}_l$on a chain with N ions, can be decomposed into the motional modes operators,

$$k\hat{x}_l = \sum_m^N b_l^m k\sqrt{\frac{\hbar}{2m\nu_m}}(a_m + a_m^+) = \sum_m^N b_l^m\eta_m(a_m + a_m^+), \quad (9)$$

η_m is the Lamb-Dicke parameter of the mth motional modes, a_m, a_m^+ is the annihilation and creation operator of the mth montional modes. On this ion chain, if two ions are addressed and operated, which are at ith and jth ions, then the Hamiltonian is

$$H_0 =\hbar \left[\sum_m^N \nu_m a_m^+ a_m + \frac{1}{2} \sum_l^{i,j} \left(\Omega_l^D \cos\phi_l + \Omega_l^D + \delta \right) \sigma_z^l \right], \tag{10}$$

$$H_1 = -\hbar \sum_m^N \sum_l^{i,j} \eta_m b_l^m \sin\phi_l (a_m^+ + a_m) \otimes \left(\Omega_l^C + \Omega_l^D \sigma_z^l \right), \tag{11}$$

where ν_m is the motional frequency of mth mode, $\phi_l(t)$ is the phase difference of the Raman lasers on the lth ion. By rotating the motion to an inaction frame, the interaction Hamiltonian is

$$H_{I,1} = -\hbar \sum_m^N \sum_l^{i,j} f_l^m(t) a_m^+ + f_l^{m*}(t) a_m,$$

here $f_l^m(t) = \eta_m b_l^m \sin\phi_l \left(\Omega_l^C + \Omega_l^D \sigma_z^l \right) e^{i\nu_m t}$, the operation after the some evolution time t is

$$U = \exp\left(i \sum_m \Phi_m \right) \Pi_m D(\alpha_m)$$

where $\alpha_m = \sum_l^{i,j} \int^t f_l^m(t) dt$ is the displacement value of mth mode in the phase space, $\Phi_m = Im\left(\int \alpha_m^* d\alpha_m \right)$ is the geometric phase. The geometric phase can be calculated as four part: $\Phi_m = \Phi_{0,m} + \Phi_{1,m}\sigma_z^i + \Phi_{1,m}\sigma_z^j + \Phi_{2,m}\sigma_z^i \otimes \sigma_z^i$, where $\Phi_{2,m}$ is the part contributing to the two qubit phase gate, which should be $\pi/4$.

There are two modulation schemes on the Raman lasers to obtain the desired two qubit phase gate: amplitude modulation or phase modulation. We note that $\Omega_l^C(t)$ and $\Omega_l^D(t)$ have the similar form as they both relate to the amplitude modulation function, so they can be expressed as $\Omega_{l,0}^{C(D)} g_l(t)$, where $g_l(t)$ is the amplitude function. Both schemes require that the displacement is zero after the gate operation, which is

$$\int g_l(t) \sin(\phi_l(t)) e^{i\nu_m t} dt = 0.$$

For the phase modulation, we need optimize the phase function$\phi_l(t)$ while keeping $g_l(t)$ constant. Suppose $g_l(t) = 1$ and $\phi_l(t) = \phi(t) + \phi_0$ is same for ith and jth ion, where ϕ_0 is the initial phase difference between two Raman beam, we can get

$$\begin{aligned}\Phi_{2,m} &= 2M_m \iint \sin\left[\phi(t_1) + \phi_0\right] \left[\sin\phi(t_2) + \phi_0\right] \sin(\nu_m(t_2 - t_1)) dt_1 dt_2 \\ &= M_m \iint \left[\cos(\phi(t_1) - \phi(t_2)) - \cos(\phi(t_1) + \phi(t_2) + 2\phi_0)\right] \sin(\nu_m(t_2 - t_1)) dt_1 dt_2,\end{aligned}$$

where $M_m = \eta_m^2 b_i^m b_j^m \Omega_{i,0}^D \Omega_{j,0}^D$. The entanglement gate requires $\Phi_{2,m} = \pi/4$. In order to let phase gate robust, it should be insensitive to the initial phase ϕ_0, so the final condition for $\phi(t)$ is

$$\begin{aligned}\sum_m M_m \iint \cos(\phi(t_1) - \phi(t_2)) \sin(\nu_m(t_2 - t_1)) dt_1 dt_2 &= \pi/4 \\ \iint \cos(\phi(t_1) + \phi(t_2)) \sin(\nu_m(t_2 - t_1)) dt_1 dt_2 &= 0 \\ \iint \sin(\phi(t_1) + \phi(t_2)) \sin(\nu_m(t_2 - t_1)) dt_1 dt_2 &= 0.\end{aligned}$$

For the amplitude modulation, we first fix the phase function as $\phi(t) = \mu t + \phi_0$, the condition for $g(t)$is

$$\begin{aligned}\sum_m M_m \iint g(t_2) g(t_1) \cos(\mu(t_2 - t_1)) \sin(\nu_m(t_2 - t_1)) dt_1 dt_2 &= \pi/4 \\ \iint g(t_2) g(t_1) \cos(\mu(t_2 + t_1)) \sin(\nu_m(t_2 - t_1)) dt_1 dt_2 &= 0 \\ \iint g(t_2) g(t_1) \sin(\mu(t_2 + t_1)) \sin(\nu_m(t_2 - t_1)) dt_1 dt_2 &= 0.\end{aligned}$$

To obtain a two-bits entanglement gate, the geometric phase can be calculated as below:

$$\begin{aligned}\hat{U} =& \exp\left(-i\int H_{I,1}(t)dt/\hbar\right)\\ =&\Pi_m D(\alpha_{i,C}^m + \alpha_{i,D}^m\sigma_i^z)D(\alpha_{j,C}^m + \alpha_{j,D}^m\sigma_j^z)\times\\ & e^{i\theta}e^{i\theta_i\sigma_i^z}e^{i\theta_i\sigma_j^z}e^{i\theta_{i,j}\sigma_i^z\sigma_j^z}\end{aligned} \tag{12}$$

here $a_{i(j),C(D)}^m = \int g_{i(j)}^m \Omega_l^{C(D)}(t)\sin\Phi_l(t)dt$ is the displacement of the ith (jth) ion with the mth motional modes in the phase space, define $\mathit{\Phi}_{i,j}^m = \nu_m(t_2 - t_1) + \Phi_i(t_2) - \Phi_j(t_1)$, $\theta = \sum_{m,l}\iint (g_l^m)^2(\Omega_l^D(t_1)\Omega_l^D(t_2) + \Omega_l^C(t_1)\Omega_l^C(t_2))\sin\mathit{\Phi}_{l,l}^m dt_1 dt_2$ is global phase which is irrelevant with the spin, $\theta_i = \sum_m \iint g_i^m g_j^m \Omega_i^D(t_1)\Omega_j^C(t_2)\sin\mathit{\Phi}_{j,i}^m dt_1 dt_2$, $\theta_j = \sum_m \iint g_i^m g_j^m \Omega_i^C(t_1)\Omega_j^D(t_2)\sin\mathit{\Phi}_{j,i}^m dt_1 dt_2$ are the . $\theta_{i,j} = 2\sum_m \iint g_i^m g_j^m \Omega_i^D(t_1)\Omega_j^D(t_2)\sin\mathit{\Phi}_{j,i}^m dt_1 dt_2$ is the spin dependent geometry phase of two ions, to entangle the two ions, which is expected to be $\pi/4$. We need to optimize the laser parameter to satisfy the following constrain (τ is the gate time):

$$\begin{aligned} a_{i,C}^k(\tau) &= 0\\ a_{i,D}^k(\tau) &= 0\\ \theta_{i,j}(\tau) &= \frac{\pi}{4}\end{aligned} \tag{13}$$

3. *Further detail*

Given a intuitive understand of the process: the focus beam woks as the frequency tailoring, which will rearrange the modes. The transverse mode is so close and may need more complicated waveform and longer gate time. Using the mode after rearrangement, two mode will be selected and have a big gap with other modes. Then the multiple ions optimal problem will be simplified to two ions optimal problem because the coupling between two ions and other ions is quite small.

4. *Optimal result and Fidelity*

A lot of has talked about the optimal methods in laser sequence. However, as the increasing of the number of ions the optimal process has became harder and more controllable parameter is needed. 1D N ions chain will involved in N motional modes, and the gap between transverse modes will become tighter which makes it hard to address single mode. The gate time will increase and according to the former part: when operate on M ions, $M\times N + \frac{N(N-1)}{2}$ constrains needed to be considered which means more complex waveform or more segments are required. Just adding auxiliary frequency tailoring, the motional modes will be rearranged. In this paper, we will give a simple vision of optimize and show that this methods really bring the convience in constructing fast and high-fidelity entanglement gate. We give an example of optimize the gate between the 5th and 6th ions in a ten ions chain. The waveform is chosen to be simple: $\Omega_{m,n} = \Omega\sin(\mu t + \phi(t))$, amplitude of the Raman beams Ω is setting to be constant, μ is the detuning between the beam and $\phi(t)$ is the additional initial phase. We choose $\mu$to be constant and divide $\phi(t)$ into 4 parts. The gate time is fixed to be $\tau = 5\,\mu$s and $\phi(t)$ is antisymmetry with $\tau/2$ (every segment is setting to be constant)

$$\phi(t) = \begin{cases} \phi_1 & [0, \tau_1]\\ \phi_2 & [\tau_1, \frac{\tau}{2}]\\ -\phi_2 & [\frac{\tau}{2}, \tau - \tau_1]\\ -\phi_1 & [\tau - \tau_1, \tau]\end{cases}, \tag{14}$$

so only 3 optimal parameter (τ_1, ϕ_1, ϕ_2) is needed. Ω_{Stand} is setting to satisfy $\theta_{m,n}(\tau) = \frac{\pi}{4}$ and to be same for two ions(if considering more ions gate the different amplitude is needed). The simple waveform means the optimize become simpler. All the integral can be calculate analytically. The $\phi(t)$ have four segments:

$$\Theta_{m,n}^{j,j'} = \begin{cases} \int_{t_{j-1}}^{t_j} dt_1 \int_{t_{j'-1}}^{t_{j'}} dt_2 \sin(\mu t_1 + \phi_j)\sin(\mu t_2 + \phi_{j'})\sin(\omega_k(t_1 - t_2)) & j > j'\\ \int_{t_{j-1}}^{t_j} dt_1 \int_{t_{j-1}}^{t_1} dt_2 \sin(\mu t_1 + \phi_j)\sin(\mu t_2 + \phi_j)\sin(\omega_k(t_1 - t_2)) & j = j'\\ 0 & j < j'\end{cases}, \tag{15}$$

$$\theta_{m,n} = 2\sum_k\sum_{j,j'} g_m^k g_n^k \Theta_{m,n}^{j,j'}, \tag{16}$$

$$a_{m,n}^k = \int dt \sin(\mu t + \phi(t))e^{i\nu_k t}, \tag{17}$$

The real and image part can be calculate. Because ν_T and Ω_R part can be control independently, ν_T can also to be adjusted to change distribution. But we don't do this for simplification. There is always a big gap between two modes and the other modes, so only the constrains on enclose in the phase space became simple because you only to optimize COM mode and stench mode(other modes are so small for their coupling b_j^m). The gate fidelity can be written as:

$$F = |\langle\psi_{ideal}|\, U(\tau)\, |\psi_0\rangle|^2 \tag{18}$$
$$= \frac{1}{8}\left[2 + 2\left(\Gamma_i + \Gamma_j\right) + \Gamma_+ + \Gamma_-\right], \tag{19}$$

where $\Gamma_{i(j)} = \exp\left[-\sum_k \left|\alpha_{i(j)}^k(\tau)\right|^2 \overline{\beta}_k/2\right]$ and $\Gamma_\pm = \exp\left[-\sum_k \left|\alpha_i^k(\tau) \pm \alpha_j^k(\tau)\right|^2 \overline{\beta}_k/2\right]$. As the $\theta_{m,n} = \pi/4$ can be always satisfied the optimal function can be written as:

$$\sum_{k=c,s} \sum_{j=m,n} |a_j^k|^2 \tag{20}$$

In the optimal process, sometimes over optimization may appear: $|a_{m,n}^{c,s}|^2$ can decline to 10^{-9} and the other displacement remains 10^{-6}. So regularization term($\sum_{allmodes} \sum_{j=m,n} |a_j^k|^2$) can be considered into and the resullt can be more stabilizing. The optimal function can be futher rewritten as:

$$\sum_{k=c,s} \sum_{j=m,n} |a_j^k|^2 + \beta \sum_k \sum_{j=m,n} |a_j^k|^2 \tag{21}$$

To make phase space displacement become more average. In the following, we give an example of optimize the phase gate in the 10 ions chain. The result of the gate between the neighbor 5th and 6th ion is showed in the following scheme. In Fig. 2, we take the operated 5th ion for example. The extra frequency by the dipole is set to be $2\pi\times 2$MHz. The gate time is set to be $5\mu s$ and the optimization parameter is the phase ϕ_1, ϕ_2, τ. Fig. 2(a,b) shows the stimulated trajectory of the 5th ion coupling to the COM mode and the strench mode which the circle return to the origin after the gate time. We give the opitimization result without specially optimizing for the other mode but it's easy to observe that the other mode leave their origin at scale of 10^{-3}. It's because the frenquency tailoring which rearrange the motional mode, selecting out only two modes and reducing the coupling between ions and the other modes. Fig. 3 shows the robustness of this scheme: the gate error is plotted as the function of the frequency drifts. We find that the gate can tolerate frequency errors up to 2 KHz under the typical error threshold for high-fidelity gates 0.01%. To show the method bring convenience in optimization procedure, we plot the result with different optimal functions for all modes and two modes in Fig. 3. The result has slight differences as expected.

To show the scalability of the scheme, we give an example in Fig. 4 of two ion gate in 101 ions chain. The first operated one is the 51th ion (center) and the ion distance shows the distance between the first ion and the second ion. When ps

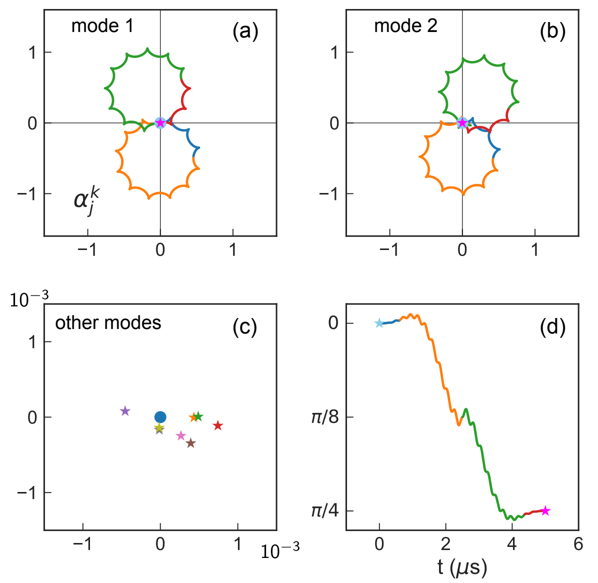

FIG. 2. Simulated phase space trajectory. The horizontal and vertical axes represent $x_k^j \sim b_k^j(a_k + a_k^+)$ and $p_k^j \sim ib_k^j(a_k^+ - a_k)$. The following take motional trajectory a_k^j of the operated 5th ion for example. (a) the trajectories of 5th ion coupling to the COM mode. (b) the trajectories of 5th ion coupling to the stretch mode. (c) the end points of the other mode in the phase space without specially optimization for them. (d) Accumulation of the state-dependent phase over the evolution time. All the coupling strengths optimized to obtained the desired $\pi/4$.

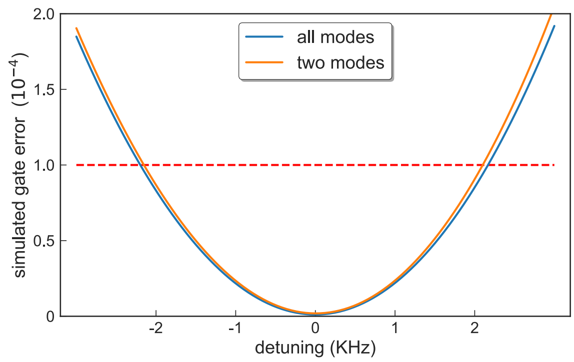

FIG. 3. Simulated gate error: the error of detuning between the Raman beam. The red dashed line represent the typical error threshold for high-fidelity gates is 0.01%, and the gate we optimized can tolerate frequency error up to ±2 kHz. The blue line represents optimization for all modes and the orange line represents optimization for two modes. It can be seen there are only slight differences between them.

C. Some formula on entangle gate

This part give the relative formula about this work.

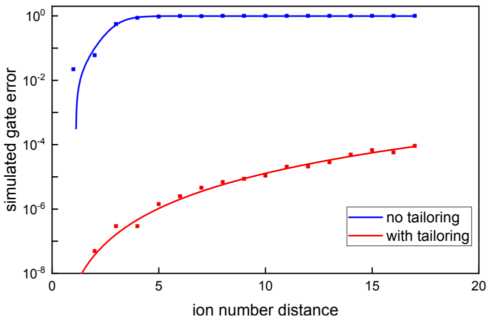

FIG. 4. Example of the Scalability in this scheme: Stimulated gate error of two ion gate in the 101 ions chain. The first operated one is the 51th ion (center) and the ion distance shows the distance between the first ion and the second ion. The square points shows the optimized result and the line represents the fitted curve. The red one is the result with frequency tailoring and the blue one is the result without frequency tailoring. It can be found that without tailoring, the gate becomes hard to optimize with the distance between two operated ions increasing to five. While using frequency tailoring the gate fidelity remains high even the distance increasing to 20.

1. AC Stark shift

As we know, when we irradiate on the atoms with light, the energy levels of the atoms will have a shift. The optically induced shift is knowned as light shift or the ac stack shift. Different energy levels will have different shift. Our qubit is defined by the ground hyperfine states of $^{171}\mathrm{Yb}^{+}$ ions: $2S_{1/2}$—F=0¿ and $2S_{1/2}$—F=1¿ with an energy splitting $\delta = 2\pi \times 12.6428\,\mathrm{GHz}$. The strongly forced Raman beams(wavelength $\lambda = 369$ nm) are added perpendicular to the ions chain to address one ion with a mean detuning $\Delta = -200GHz$ and the frequency difference μ ($\Delta \gg \delta \gg \mu$). The energy shift of the two hyperfine level:

$$\Delta_{\uparrow} = \frac{3\pi c^2}{2\omega^3}\left(\frac{\Gamma}{\Delta - \delta/2} + \frac{\Gamma}{\Delta - \delta/2 - \mu}\right)I \tag{22}$$

$$\Delta_{\downarrow} = \frac{3\pi c^2}{2\omega^3}\left(\frac{\Gamma}{\Delta + \delta/2} + \frac{\Gamma}{\Delta + \delta/2 + \mu}\right)I \tag{23}$$

$$U_{dip} = \frac{\Delta_{\uparrow} + \Delta_{\downarrow}}{2} = \frac{3\pi c^2 \Gamma I}{\omega^2 \Delta} \tag{24}$$

$$\frac{\Delta_{\uparrow} - \Delta_{\downarrow}}{2} = \frac{3\pi c^2 \Gamma I}{\omega^2 \Delta^2}\delta \tag{25}$$

$$\Omega = \frac{3\pi c^2 \Gamma I}{\omega^2 \Delta^2 \hbar}\delta \tag{26}$$

$$\Omega_{SDF}\hbar = F x_0 \tag{27}$$

$$\Omega_{SDF} = \frac{U_0 x_0 4k\eta\delta}{\triangle\hbar} = \frac{2U_0\delta\eta^2}{\triangle\hbar} \tag{28}$$

2. Frequency Tailoring

The common part of the shift contribute as the locality enhanced harmonic strength. Our proposal is to design a scalable transverse with faster speed. We consider a system of N ions confined in a linear trap with the external harmonic trapping potential characterizing by the trap frequency $\nu_x \gg \nu_z$, and the montional Hamiltonian: $H_0 = \sum_{k=1}^{N} \hbar w_{x,k}(a_k^{+} a_k + 1/2)$. To solve the transverse motional mode, we can give the matrix and solve the eigenfrequencies and eigenstate. However, the confination in the transverse mode is so strong that the frequency spliting between the N eigenfrequencies is so close so is hard to address the phonon mode. So as discussed above when we add the additional strongly focus beam, the common part can work as a local enhanced potential. So we introduce the $\nu_{dip} = 2k\sqrt{\frac{2U_{dip}}{m}}$ to rearrange the modes. Now the equilibrium position:

$$u_j - \sum_{n=1}^{j-1} 1/(u_j - u_n)^2 + \sum_{n=j+1}^{N} 1/(u_j - u_n)^2 = 0 \tag{29}$$

$$u_n \equiv z_n^0/\ell (n = 1, 2, \ldots, N)$$

where $\ell \equiv \sqrt[3]{e^2/4\pi\epsilon_0 M\omega_z^2}$. The above solves the axis equilibrium position, and it will not change under the rearrangement.

The matrix:

$$A_{nj}^{x} = \begin{cases} \nu_{dip,j}^2 + (\frac{\nu_x}{\nu_z})^2 - \sum\limits_{p=1, p\neq j}^{N} 1/|u_j - u_p|^3 & (n = j) \\ 1/|u_j - u_n|^3 & (n \neq j) \end{cases} \tag{30}$$

Diagonalize the matrix and we will obtain the eigenfrequencies $\nu_k = \sqrt{\lambda_k}\nu_x$ and the eigen-vectors $\mathbf{b}_j^k$, with $\sum_n A_{nj}\mathbf{b}_n^k = \lambda_k \mathbf{b}_j^k$ (ignore the index x). In this case, we add two additional Raman beams to seperately address two ions, which means only $\nu_{dip,j_1}, \nu_{dip,j_2} \neq 0$. As the ν_{dip} climbing, two of the mode will be selected out, which it's easy to imagine is the common and strench mode of the two selected ions. When the gap between the two modes and the other mode increasing, the coupling between them becaming weaker and the frequencies of the two mode will increase so gate time will be shorter.

3. *Formalism for quantam entangling gate*

As expressed in the first part, we state that in the condition the raman beams are far red detuning from the upper $2P^{1/2}$, two hyperfine state will both experience a energy level drop. The common part works as two optical dipole trap; the different part works as the state-dependant ac-Stark shift. Using two laser beams of equal intensity, wave vector difference $2k$ and frequence difference μ,

4. *Further details and some problems*

The polarization of raman beams:we can choose the beam with different circular polarization($\sigma^{\pm}$), then both ground state with different m_F are shift by the same amount because of the simple symmetry reason.

The ac-Stack shift vs Raman process: both provide state-dependent force. The former Hamiltonian has the form of σ_z with the wavelength of 369nm and the other(spin flop) has the form of σ_x with the wavelength of 355nm.

The traveling wave field vs the standing wave filed

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