真空出现在不同区域的推导

主要利用下面这篇文章中cut link的套路

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Symmetries, topological phases, and bound states in the one-dimensional quantum walk

J. K. Asbóth
Institute for Solid State Physics and Optics, Wigner Research Centre, Hungarian Academy of Sciences, H-1525 Budapest P.O. Box 49, Hungary

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Discrete-time quantum walks have been shown to simulate all known topological phases in one and two dimensions. Being periodically driven quantum systems, their topological description, however, is more complex than that of closed Hamiltonian systems. We map out the topological phases of the particle-hole symmetric one-dimensional discrete-time quantum walk. We find that there is no chiral symmetry in this system: its topology arises from the particle-hole symmetry alone. We calculate the $\mathbb{Z}_2 \times \mathbb{Z}_2$ topological invariant in a simple way that is consistent with a general definition for one-dimensional periodically driven quantum systems. These results allow for a transparent interpretation of the edge states on a finite lattice via the the bulk-boundary correspondence. We find that the bulk Floquet operator does not contain all the information needed for the topological invariant. As an illustration to this statement, we show that in the split-step quantum walk, the edges between two bulks with the same Floquet operator can host topologically protected edge states.

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that is cut, it has to end up in a state which is unaccessible to it from any other state. The only states that are "not taken" are those to either sides of a cut link. Therefore the only option to implement a totally cut link, is to introduce a spin flip instead of a jump. It is still possible to include a phase shift along with the spin flip. To retain PHS, this phase shift can only be chosen to be ± 1. In much the same way as with the reflective coin above, without loss of generality, we can fix a phase of -1 upon reflection from one of the sides. Cutting the link between sites y and $y+1$ is implemented by altering the shift operator S:

$$S_{(y)} = \sum_{x \neq y} S_{x,x+1} \pm C_{y,y+1}. \tag{26}$$

Here, the shift operators for the "link" and "cut link" between sites x and $x+1$ are defined as

$$S_{x,x+1} = |x,\downarrow\rangle\langle x+1,\downarrow| + |x+1,\uparrow\rangle\langle x,\uparrow|, \tag{27}$$

$$C_{x,x+1} = |x+1,\uparrow\rangle\langle x+1,\downarrow| - |x,\downarrow\rangle\langle x,\uparrow|. \tag{28}$$

C. Partially cut links in the bulk

In order to use bulk-boundary correspondence, we need to connect the "cut link" to the "uncut link" by way of a continuous parameter in the Floquet operator. The first idea here, the introduction of an additional "link rotation angle" ϕ, works,

$$S_{x,x+1}(\phi) = \cos(\phi)\, S_{x,x+1} + \sin(\phi)\, C_{x,x+1}. \tag{29}$$

In the bulk, this is equivalent to the "split-step" walk of Kitagawa *et al.*,[12] where the spin-z dependent displacement is broken down to two successive steps:

$$S_\downarrow = \sum_{x=1}^{N} (|x-1\rangle\langle x| \otimes |\downarrow\rangle\langle\downarrow| + |x\rangle\langle x| \otimes |\uparrow\rangle\langle\uparrow|), \tag{30}$$

$$S_\uparrow = \sum_{x=1}^{N} (|x\rangle\langle x| \otimes |\downarrow\rangle\langle\downarrow| + |x+1\rangle\langle x| \otimes |\uparrow\rangle\langle\uparrow|), \tag{31}$$

$$U_2(\theta,\phi) = S(\phi)R(\theta) = S_\uparrow e^{-i\phi\sigma_y} S_\downarrow e^{-i\theta\sigma_y}. \tag{32}$$

As shown in Ref. 12, $S(\phi = 0) = S_\uparrow S_\downarrow = S$.

说的这样一个事情，对分步的quantum walk进行分解，也就是另一种理解方式。分解为cut link和uncut link。

$S_{x,x+1}(\phi)=\cos(\phi)S_{x,x+1}+\sin(\phi)C_{x,x+1}$，S是uncut link，C是cut link，这里的ϕ就是第二个翻转的角度（我们这里其实是$\phi=-\theta_1/2$）。我们的系统演化与他们有些不同，所以可以重新写一下S和C（其实就是差了些正负号，决定了真空的结果）。

$$S_{\downarrow}=\sum_{x=1}^{N}(|x-1\rangle\langle x|\otimes|\downarrow\rangle\langle\downarrow|-|x\rangle\langle x|\otimes|\uparrow\rangle\langle\uparrow|)$$

$$S_{\uparrow}=\sum_{x=1}^{N}(-|x\rangle\langle x|\otimes|\downarrow\rangle\langle\downarrow|+|x+1\rangle\langle x|\otimes|\uparrow\rangle\langle\uparrow|)$$

$U_2(\theta,\phi)=S(\phi)R(\theta)=S_{\uparrow}e^{-i\phi\sigma_y}S_{\downarrow}e^{-i\theta\sigma_y}=S_{\uparrow}e^{i\theta_1/2\sigma_y}S_{\downarrow}e^{-i\theta_2/2\sigma_y}$（我们激光设计的结果），则有：

$S(\phi=0)=S_{\uparrow}S_{\downarrow}=S_{x,x+1}$

$S(\phi=\pi/2,\theta_1=-\pi)=C_{x,x+1}=-|x+1,\uparrow\rangle\langle x+1,\downarrow|+|x,\downarrow\rangle\langle x,\uparrow|$

然后我们用$C_{-1,0}=-|0,\uparrow\rangle\langle 0\downarrow|+|-1,\downarrow\rangle\langle -1,\uparrow|$来理解真空。原文这样处理是因为他做的是周期性边界，所以从左从右都会入射到壁上，在-1和0处都存在一个壁。而我们的壁恰好是

$-|0,\uparrow\rangle\langle 0\downarrow|R(\theta_2)$时，其实对应$\theta_1=-\pi$所在的区域为真空

$|0,\uparrow\rangle\langle 0\downarrow|R(\theta_2)$时，其实对应$\theta_1=\pi$所在的区域为真空，

这也与昨天发给您的结果相同。