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Adaptive hierarchical formation control for uncertain Euler–Lagrange systems using distributed inverse dynamics[☆]

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ABSTRACT

This paper establishes a novel adaptive hierarchical formation control method for uncertain heterogeneous nonlinear agents described by Euler–Lagrange (EL) dynamics. Formation control is framed as a synchronization problem where a distributed model reference adaptive control is used to synchronize the EL systems. The idea behind the proposed adaptive formation algorithm is that each agent must converge to the model defined by its hierarchically superior neighbors. Using a distributed inverse dynamics structure, we prove that distributed nonlinear matching conditions between connected agents hold, so that matching gains exist to make the entire formation converge to same homogeneous dynamics: to compensate for the presence of uncertainties, estimation laws are devised for such matching gains, leading to adaptive synchronization. An appropriately designed distributed Lyapunov function is used to derive asymptotic convergence of the synchronization error. The effectiveness of the proposed methodology is supported by simulations of a formation of Unmanned Aerial Vehicles (UAVs).

1. Introduction

In recent years, the synchronization of multi-agent systems has been an emerging research direction drawing the attention of the control community. Synchronization represents a potential solution for coordination of large-scale networked systems [48,49], encompassing spacecraft attitude control [50], sensor networks [27], smart buildings and smart grids [30,47], unmanned aerial, ground and underwater vehicles [14,21,38]. To coordinate multi-agent systems, either a centralized approach or a distributed approach can be adopted. The centralized approach introduces a central node that utilizes the information stemming from all agents to control them. On the contrary, the distributed approach uses, for each agent, a controller that utilizes local information, i.e. neighbors' information. The distributed approach gives more advantages due to its applicability in the presence of communication constraints [13,16,51]. The research direction in distributed control can be grouped into several directions that may be overlapping, such as synchronization (sometimes referred to as consensus or rendezvous when the synchronizing behavior is constant), distributed formation control (sometimes referred to as flocking in the presence of collision avoidance capabilities), distributed optimization and estimation [10]. In this work, distributed synchronization is studied as a way to control formations of uncertain agents with Euler–Lagrange (EL) dynamics.

Several fixed-gain approaches to synchronization of EL systems have been proposed in the past, e.g. under sampled-data communication [32], under communication delays and intermittent information exchange [1,2], under time-varying or probabilistic sampled-data control [33,52], or by distributed average tracking [11] or finite-time tracking [39]. Most of these fixed-gain approaches guarantee some robustness to uncertain parameters, as studied in [35,53], or can be extended to include some estimation mechanisms [3,36,40]. Obviously, parametric uncertainty is a big concern in practice, which might require adaptive gains. In the approach of [28] some agents have exactly known parameters and others are with parametric uncertainties. A robust consensus tracking problem is addressed in [12] for multiple unknown EL systems, where the robust distributed control law is based on

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the sliding mode methodology (i.e. with discontinuous protocols). Other sliding mode approaches appear in [23,29,31,34,41] for synchronization, and in [15,45] for flocking. While being robust to uncertainty, sliding mode approaches require continuous communication due to chattering: therefore, implementation in networked environments is not straightforward. In addition, in the presence of a leader moving with time-varying velocity, sliding modes appear both in the controller and in the estimator of the leader velocity. In some cases, discontinuities in the controller or in the estimator of the leader velocity can be avoided, like in [3,36], where either a constant possibly unknown trajectory or a globally known trajectory are considered: however, their extension in the presence of non-constant trajectories or local information is an open problem. By substituting the signum function with saturation-like functions, one can obtain continuous protocols in place of discontinuous ones: however, one has to renounce to asymptotic synchronization in favor of bounded synchronization. Similarly, the neural network approaches adopted in [26,41] lead to bounded synchronization, in view of the approximation error of the neural networks. Assuming that the leader is a linear system, Cai and Huang [9] avoids any sliding mode via a distributed observer to generate the estimation of the leaders signal for multiple uncertain EL systems: in this approach like in most of the aforementioned approaches, agents must exchange auxiliary variables among neighbors in addition to state/outputs. Auxiliary variables are typically distributed observer variables needed for reconstruction of the leader's signal.

Recently, a framework for linear heterogeneous uncertain agents has been proposed which can lead to asymptotic synchronization without any sliding mode [5,6]. The framework is based on a distributed model reference adaptive control idea where not only state/outputs, but also inputs can be exchanged among neighbors. Such a framework turns out to have as a special case some platooning protocols proposed in literature [17,18]. Two things shown in these works must be remarked: first, exchanging the inputs makes any exchange of auxiliary variables unnecessary, resulting overall in less communication effort (because the input to be exchanged has typically a smaller dimension than the observer variables); second, arbitrarily time-varying leader trajectories can be handled without requiring any global knowledge of such trajectory neither any sliding mode. On the other hand, the approach has been proven only on networks of linear agents. Therefore, it is very relevant to study if such framework can transfer with similar properties to the nonlinear domain. The main motivation of this work is to give a positive answer, in a nonlinear domain and for hierarchical (i.e. directed acyclic) networks, to the following questions: can we get rid of exchanging auxiliary variables? Can we handle time-varying leader trajectories without requiring any sliding mode mechanisms? We focus in particular on the class of (heterogeneous uncertain) EL dynamics, which are extremely relevant in the robotic field. We obtain, for this setting, a continuous protocol with the capability of handling large parametric uncertainties and arbitrary leader trajectories: on the one hand, considering hierarchical networks is a consequence of exchanging control inputs among neighbors, because the mutual dependence of control inputs will bring well-posedness problems if the inputs are generated without a prescribed priority [46]; on the other hand, it was shown in [5] that the distributed model reference adaptive framework can work, with appropriate modifications, also in the presence of cyclic networks.

The article is organized as follows: Section 2 introduces preliminary results to support the proposed methodology. Section 3 presents the proposed methodology for leader-reference model synchronization. Section 4 extends the concepts to follower–leader synchronization. Section 5 presents a test case based on formation control of fixed-wing UAVs, with simulations in Section 6. Conclusions are in Section 7.

Notation: The notation in this article is standard. The notation $P = P' > 0$ indicates a symmetric positive definite matrix. The identity matrix of compatible dimensions is denoted by $\mathbb{1}$, and diag{...} represents a block-diagonal matrix. The set $\mathbb{R}$ represents the set of real numbers. A vector signal $x \in \mathbb{R}^n$ is said to belong to the $\mathscr{L}_2$ class if $\int_0^t ||x(\tau)||^2 d\tau < \infty$, $\forall t > 0$. A vector signal $x \in \mathbb{R}^n$ is said to belong to $\mathscr{L}_\infty$ class if $\max ||x(t)|| < \infty$, $\forall t > 0$.

2. Preliminaries results

2.1. EL systems

The dynamics of a network of EL agents can be described by

$$D_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i, \quad i = \{1, \ldots, N\} \tag{1}$$

where the term $D_i(q_i)\ddot{q}$ is proportional to the second derivatives of the generalized coordinates, the term $C_i(q_i, \dot{q}_i)\dot{q}$ is the vector of centrifugal/Coriolis forces, proportional to the first derivatives of the generalized coordinates, and the term $g_i(q_i)$ is the vector of potential forces. Finally, the term τ_i represents the external force applied to the system. Throughout this work the following assumptions, standard in literature [37], will be adopted:

Assumption 1. The inertia matrix $D_i(q_i)$ is symmetric positive definite, and both $D_i(q_i)$ and $D_i(q_i)^{-1}$ are uniformly bounded.

Assumption 2. There is an independent control input for each degree of freedom of the system.

Assumption 3. All the parameters of interest such as link masses, moments of inertia, etc. appears in the linear-in-the parameter form, i.e. as coefficient of known function of the generalized coordinates.

Remark 1. For most EL agents of practical interests, like robotic manipulators and mobile robots, Assumptions 1 and 3 hold [37]. Assumption 2 implies that the system is fully actuated, which is not always the case in practice. For under-actuated EL systems, a control allocator should be put in place to transform the input τ_i into the actual inputs to the system: this will introduce unmodeled dynamics which can be handled modifying the proposed methodology in a robust adaptive sense [42]. While this is a relevant practical aspect, in this work we focus for compactness on fully-actuated EL dynamics, as done in most synchronization literature we are aware of [1–3,9,11,12,15,23,26,28,29,31–36,39–41,45,52,53].

2.2. Inverse dynamic based control

Inverse dynamic based control is a typical control method for EL systems [37], which is here recalled for completeness. Given the dynamics (1), the objective of inverse dynamic based control is to cancel all the non-linearities in the system and introduce simple PD control so that the closed-loop system is linear. In the known parameter case, the cancellation of non-linearities can be achieved via the controller

$$\tau_i = D_i(q_i)a_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) \tag{2}$$

where the term a_i is defined as

$$a_i = \ddot{q}^d - K_v\dot{e}_i - K_p e_i \tag{3}$$

with $e_i = q_i - q^d$ and K_p, K_v being the proportional and derivative gains of the (multivariable) PD controller. Note that q^d, $\dot{q}^d$, and $\ddot{q}^d$ are desired trajectories, velocities, and accelerations to be specified by the user. Substituting (2) into (1) gives us

$$D_i(q_i)(\ddot{q}_i - \ddot{q}^d + K_v\dot{e}_i + K_p e_i) = 0$$

$$\ddot{e}_i + K_v \dot{e}_i + K_p e_i = 0. \tag{4}$$

The resulting second-order error equation can be written as

$$\begin{bmatrix} \dot{e}_i \\ \ddot{e}_i \end{bmatrix} = \begin{bmatrix} 0 & \mathbb{1} \\ -K_p & -K_v \end{bmatrix} \begin{bmatrix} e_i \\ \dot{e}_i \end{bmatrix} \tag{5}$$

or equivalently,

$$\begin{bmatrix} \dot{q}_i \\ \ddot{q}_i \end{bmatrix} = \begin{bmatrix} 0 & \mathbb{1} \\ -K_p & -K_v \end{bmatrix} \begin{bmatrix} q_i \\ \dot{q}_i \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbb{1} \end{bmatrix} (\ddot{q}^d + K_v \dot{q}^d + K_p q^d). \tag{6}$$

The closed-loop systems (6) is a second-order state-space system whose state matrix must be Hurwitz by construction (by appropriately selecting K_p and K_v). Two things must be noted about (2): the first is that the control law (2) requires the dynamics to be perfectly known, so as to operate a perfect inversion; the second is that the control law (2) requires agent i to know q^d, $\dot{q}^d$, and $\ddot{q}^d$. It clear that in practice the dynamics are not known due to parametric uncertainty, leading to imperfect inversion. In addition, in a multi-agent setting, the desired positions, velocities and accelerations may not be available to all systems. In fact, due to communication constraints, it might be impossible to communicate q^d, $\dot{q}^d$, and $\ddot{q}^d$ to the entire formation, except for a few agents, e.g. the leading agents. Therefore, one cannot implement (3) in a distributed way and in the presence of uncertainty. In Section 3, we will design an adaptive distributed version of the inverse dynamic based control, which can be implemented in the presence of uncertainty and using local measurements from neighboring agents.

2.3. Communication graph

We consider networks of EL agent which are linked to each other via a *communication graph* that describes the allowed information flow. In other words, we say that system i has a *directed* connection to system j if the second can receive information from the first. In our case, information includes both state measurements $(q_i, \dot{q}_i)$ and input measurements τ_i from neighbors. In a communication graph, a special role is played by the *pinning* node, which is an agent (typically indicated as system 0) that does not receive information from any other agents in the network. Note that the pinner can be a virtual or a real agent, depending on the particular application. The communication graph describing the allowed information flow between all the systems, pinner excluded, is completely defined by the pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \ldots, N\}$ is a finite nonempty set of nodes, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of pairs of nodes, called edges. To include the presence of the pinner in the network we define $\bar{\mathcal{G}} = \{\mathcal{V}, \mathcal{E}, \mathcal{T}\}$, where $\mathcal{T} \subseteq \mathcal{V}$ is the set of those nodes, called *target nodes*, which receive information from the pinner. Fig. 1 provides a simple communication graph where $\mathcal{V} = \{1, 2, 3, 4\}$, $\mathcal{E} = \{(1, 2), (3, 4)\}$, and $\mathcal{T} = \{1, 3\}$. Note that the target nodes will be referred to as leaders in this work, because they can access the information of the pinner: with this directed connection, follower 2 can observe the measurement from leader 1, and follower 4 can observe the measurement from leader 3, but not vice versa. Let us introduce the *adjacency matrix* $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ of a directed communication graph, which is defined as $a_{ii} = 0$ and $a_{ij} = 1$ if $(i, j) \in \mathcal{E}$, where $i \neq j$. The adjacency matrix corresponding to the example in Fig. 1 is

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

In addition, we define a vector, the *target vector* $\mathcal{M} = [a_{j0}] \in \mathbb{R}^N$, to describe the directed communication of the pinner with the target nodes. Specifically, the target matrix is defined as $a_{j0} = 1$ if $j \in \mathcal{T}$ and $a_{j0} = 0$ otherwise. In the example of Fig. 1, we have $\mathcal{M} = [1, 0, 1, 0]'$. A directed graph $\bar{\mathcal{G}}$ is said to be *hierarchical* if the vertices of the graph can be sorted in such a way that the adjacency matrix has upper triangular form with only zeros in the diagonal, which is the case of the graph in Fig. 1: this is equivalent to saying that the directed communication graph is acyclic. Also, the graph is said to contain a *directed spanning tree* with the pinner as the root node, if for every agent j there is a directed path that leads from j to 0. In this work we will consider hierarchical networks containing a directed spanning tree with the pinner as the root node. Considering hierarchical networks is a necessary consequence of exchanging control inputs among neighbors, because the mutual dependence of control inputs will bring well-posedness problems if the inputs are generated without a prescribed priority [46]; however, it was shown in [5] that the distributed model reference adaptive framework can work, with appropriate modifications, also in the presence of cyclic networks. We are now ready to give the problem formulation.

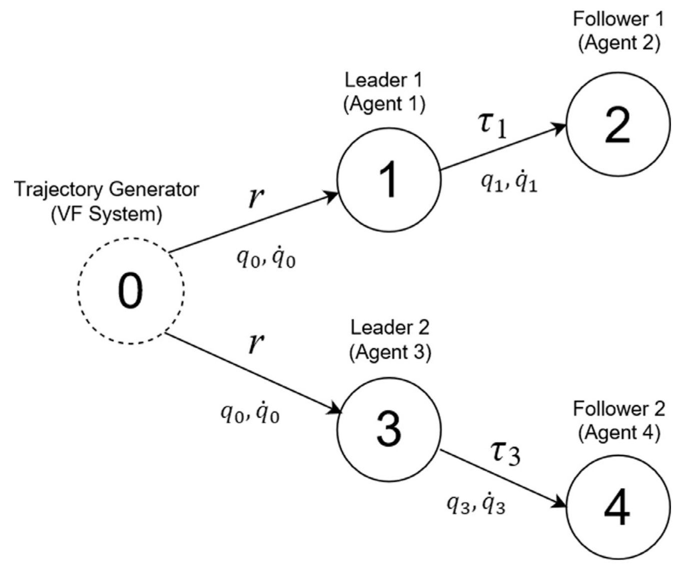

Fig. 1. Example of multi-agent system communication graph.

Problem 1. Given a hierarchical network $\bar{\mathcal{G}}$ of EL heterogeneous uncertain agents (1), a pinner with state $(q_0, \dot{q}_0)$, find a distributed strategy for the inputs τ_i that respects the communication graph, that does not require knowledge of the EL matrices, and that leads to synchronization of the network, i.e. $[q_i, \dot{q}_i] \to [q_0, \dot{q}_0]$ as $t \to \infty$, for every agent i.

3. Adaptive synchronization of the leader to the reference model

This section will focus on the control for a leading agent. Because the leading agent has access to q^d, $\dot{q}^d$, and $\ddot{q}^d$, the main problem for this agent is to cope with uncertainty. Without loss of generality we denote a leading agent with the index 1: in addition, we focus on a single leading agent because the presence of multiple leaders is a trivial extension of the proposed control law. Let us start by formulating some reference model dynamics

$$\begin{bmatrix} \dot{q}_0 \\ \ddot{q}_0 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \mathbb{1} \\ -K_p & -K_v \end{bmatrix}}_{A_m} \underbrace{\begin{bmatrix} q_0 \\ \dot{q}_0 \end{bmatrix}}_{x_m} + \underbrace{\begin{bmatrix} 0 \\ \mathbb{1} \end{bmatrix}}_{B_m} r \tag{7}$$

where q_0, $\dot{q}_0 \in \mathbb{R}^n$ is the state of the reference model and $r = \ddot{q}^d + K_v \dot{q}^d + K_p q^d$ is a user-specified reference input. The reference dynamics (7) basically represent some homogeneous dynamics all agents should synchronize to. Any leader dynamics in the form (2) can be written in the state-space form

$$\begin{bmatrix}\dot q_1\\ \ddot q_1\end{bmatrix} = \underbrace{\begin{bmatrix}0 & \mathbb{1}\\ 0 & -D_1^{-1}C_1\end{bmatrix}}_{A_1}\underbrace{\begin{bmatrix}q_1\\ \dot q_1\end{bmatrix}}_{x_1} + \begin{bmatrix}0\\ -D_1^{-1}g_1\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ D_1^{-1}\end{bmatrix}}_{B_1}\tau_1 \tag{8}$$

where the dependence of the matrices on $q_1, \dot q_1$ will be omitted whenever obvious. The main idea is to formulate a nonlinear version of the model reference adaptive control method [19,42], by designing a controller to match the leader dynamics (8) to the reference model dynamics (7). To this purpose, we propose a controller in the form

$$\tau_1^* = \underbrace{\begin{bmatrix}\bar K_1^{*\prime} & \bar{\bar K}_1^{*\prime}\end{bmatrix}}_{K_1^{*\prime}}\begin{bmatrix}q_1\\ \dot q_1\end{bmatrix} + G_1^{*\prime} + L_1^{*\prime} r \tag{9}$$

where the superscript * indicates an ideal controller whose gains possibly require the knowledge of the system dynamics. The following lemma tells how to find such matching gains.

Lemma 1. *There exists an ideal control law in the form of (9) that matches the leader dynamics (8) to the reference model dynamics (7), and whose control gains $\bar K_1^*, \bar{\bar K}_1^*, L_1^*$, and G_1^* are*

$$\begin{aligned}\bar K_1^{*\prime} &= -D_1K_p \qquad L_1^{*\prime} = D_1\\ \bar{\bar K}_1^{*\prime} &= -D_1K_v + C_1 \quad G_i^{*\prime} = g_1.\end{aligned} \tag{10}$$

Proof. By direct substitution of (9) into (8), we have the leader closed-loop dynamics

$$\begin{aligned}\begin{bmatrix}\dot q_1\\ \ddot q_1\end{bmatrix} &= \begin{bmatrix}0 & \mathbb{1}\\ D_1^{-1}\bar K_1^{*\prime} & D_1^{-1}(\bar{\bar K}_1^{*\prime} - C_1)\end{bmatrix}\begin{bmatrix}q_1\\ \dot q_1\end{bmatrix}\\ &\quad + \begin{bmatrix}0\\ -D_1^{-1}(g_1 - G_1^{*\prime})\end{bmatrix} + \begin{bmatrix}0\\ D_1^{-1}L_1^{*\prime}\end{bmatrix} r.\end{aligned} \tag{11}$$

We see that Lemma 1 is verified for the ideal control law

$$\tau_1^* = -D_1K_pq_1 - D_1K_v\dot q_1 + C_1\dot q_1 + g_1 + D_1 r \tag{12}$$

from which we derive the control gains in (10). This concludes the proof. □

Being the system matrices in (8) unknown, the controller (12) cannot be implemented, and the synchronization task has to be achieved adaptively. Then, inspired by the ideal controller (12), we propose the controller

$$\tau_1 = \underbrace{\Theta_{D_1}'\xi_{D_1}}_{\hat D_1}(-K_pq_1 - K_v\dot q_1 + r) + \underbrace{\Theta_{C_1}'\xi_{C_1}}_{\hat C_1}\dot q_1 + \underbrace{\Theta_{g_1}'\xi_{g_1}}_{\hat g_1} \tag{13}$$

where the estimates $\hat D_1, \hat C_1, \hat g_1$ of the ideal matrices have been split in a linear-in-the-parameter form. In fact, in view of Assumption 3, an appropriate linear-in-the-parameter form $D_1 = \Theta_{D_1}^{*\prime}\xi_{D_1}$, $C_1 = \Theta_{C_1}^{*\prime}\xi_{C_1}$ and $g_1 = \Theta_{g_1}^{*\prime}\xi_{g_1}$ can always be found. A specific form of regressand Θ and regressor ξ will be derived later in Section 5 for the example of Unmanned Aerial Vehicles. Let us define the error $e_1 = x_1 - x_m$, whose dynamics are

$$\begin{aligned}\dot e_1 &= A_me_1 + B_1(\tilde{\bar K}_1'q_1 + \tilde{\bar{\bar K}}_1'\dot q_1 + \tilde G_1' + \tilde L_1' r)\\ &= A_me_1 + B_1(\tilde\Theta_{D_1}'\xi_{D_1}(-K_pq_1 - K_v\dot q_1 + r) + \tilde\Theta_{C_1}'\xi_{C_1}\dot q_1 + \tilde\Theta_{g_1}'\xi_{g_1})\end{aligned} \tag{14}$$

where $\tilde{\bar K}_1 = \bar K_1 - \bar K_1^*$, $\tilde{\bar{\bar K}}_1 = \bar{\bar K}_1 - \bar{\bar K}_1^*$, $\tilde L_1 = L_1 - L_1^*$, $\tilde\Theta_{D_1} = \Theta_{D_1} - \Theta_{D_1}^*$, $\tilde\Theta_{C_1} = \Theta_{C_1} - \Theta_{C_1}^*$ and $\tilde\Theta_{g_1} = \Theta_{g_1} - \Theta_{g_1}^*$. The following theorem provides the synchronization result between the leader and the reference model.[1]

Theorem 1. *Consider the reference model (7), the unknown leader dynamics (8), and controller (13). Under the assumption that a matrix S_1 exists such that*

$$L_1^*S_1 = S_1'L_1^{*\prime} > 0 \tag{15}$$

then, the adaptive laws

$$\begin{aligned}\dot\Theta_{D_1}' &= -S_1B_m'Pe_1(-K_pq_1 - K_v\dot q_1 + r)'\xi_{D_1}'\\ \dot\Theta_{C_1}' &= -S_1B_m'Pe_1\dot q_1'\xi_{C_1}'\\ \dot\Theta_{g_1}' &= -S_1B_m'Pe_1\xi_{g_1}'\end{aligned} \tag{16}$$

where $P = P' > 0$ is such that

$$PA_m + A_m'P = -Q, \quad Q > 0 \tag{17}$$

guarantee synchronization of the leader dynamics (8) to the reference model (7), i.e. $e_1 \to 0$.

Proof. To analytically show the asymptotic convergence of the synchronization error between the leader and the reference model, let us introduce the following Lyapunov function

$$\begin{aligned}V_1(e_1, \tilde\Theta_{D_1}, \tilde\Theta_{C_1}, \tilde\Theta_{g_1}) &= e_1'Pe_1 + tr(\tilde\Theta_{D_1}S_1^{-1}L_1^{*-1}\tilde\Theta_{D_1}')\\ &\quad + tr(\tilde\Theta_{C_1}S_1^{-1}L_1^{*-1}\tilde\Theta_{C_1}') + tr(\tilde\Theta_{g_1}S_1^{-1}L_1^{*-1}\tilde\Theta_{g_1}').\end{aligned} \tag{18}$$

Then it is possible to verify

$$\begin{aligned}&\dot V_1(e_1, \tilde\Theta_{D_1}, \tilde\Theta_{C_1}, \tilde\Theta_{g_1})\\ &= e_1'(PA_m + A_m'P)e_1\\ &\quad + 2e_1'PB_1(\tilde\Theta_{D_1}'\xi_{D_1}(-K_pq_1 - K_v\dot q_1 + r) + \tilde\Theta_{C_1}'\xi_{C_1}\dot q_1 + \tilde\Theta_{g_1}'\xi_{g_1})\\ &\quad + 2tr(\tilde\Theta_{D_1}S_1^{-1}L_1^{*-1}\dot{\tilde\Theta}_{D_1}') + 2tr(\tilde\Theta_{C_1}S_1^{-1}L_1^{*-1}\dot{\tilde\Theta}_{C_1}')\\ &\quad + 2tr(\tilde\Theta_{g_1}S_1^{-1}L_1^{*-1}\tilde\Theta_{g_1}')\\ &= -e_1'Qe_1 + 2tr\\ &\quad \times(\tilde\Theta_{D_1}L_1^{*-1}(B_m'Pe_1(-K_pq_1 - K_v\dot q_1 + r)'\xi_{D_1}' + S_1^{-1}\dot{\tilde\Theta}_{D_1}'))\\ &\quad + 2tr(\tilde\Theta_{C_1}L_1^{*-1}(B_m'Pe_1\dot q_1'\xi_{C_1}' + S_1^{-1}\dot{\tilde\Theta}_{C_1}'))\\ &\quad + 2tr(\tilde\Theta_{g_1}L_1^{*-1}(B_m'Pe_1\xi_{g_1}' + S_1^{-1}\dot{\tilde\Theta}_{g_1}'))\\ &= -e_1'Qe_1.\end{aligned} \tag{19}$$

Here we used the property $a'b = tr(ab')$. From (19), we deduce that V_1 has a finite limit, so $e_1, \tilde\Theta_{D_1}, \tilde\Theta_{C_1}, \tilde\Theta_{g_1} \in \mathcal{L}_\infty$. Because $e_1 = x_1 - x_m \in \mathcal{L}_\infty$ and $x_m \in \mathcal{L}_\infty$, we have $x_1 \in \mathcal{L}_\infty$. This implies $x_1, \tilde\Theta_{D_1}, \tilde\Theta_{C_1}, \tilde\Theta_{g_1} \in \mathcal{L}_\infty$. Consequently, we can deduce $\tau_1 \in \mathcal{L}_\infty$. Therefore, all signals in the closed-loop systems are bounded. From (19), we can establish that V_1 has a bounded integral, so that we have $e_1 \in \mathcal{L}_2$. Then by using $\tilde\Theta_{D_1}, \tilde\Theta_{C_1}, \tilde\Theta_{g_1}, e_1 \in \mathcal{L}_\infty$, we have $\dot e_1 \in \mathcal{L}_\infty$. This concludes the proof of the boundedness of all closed-loop signal and convergence $e_1 \to 0$ as $t \to \infty$. □

Remark 2. Condition (15) is mutuated from the well-known condition of multivariable MRAC [43]: even though such condition might sound restrictive because it involves a possibly unknown matrix L_1^*, it can be easily satisfied in most EL systems of practical interests. In fact, in most EL systems like robotic manipulators and mobile robots, the matrix D_i is symmetric in view of some symmetrical geometry of the robot: this implies that L_i^*, even if unknown, is

[1] From here till the end of the work we assume for simplicity the inertia matrix to be constant. This is done to make the presentation consistent with the well-known methods of multivariable adaptive control [43]. Note that most single-body EL systems have a coordinate-independent inertia matrix (cf. the UAV example in Section 5). For multi-body EL systems with coordinate-dependent inertia matrices, a similar Lyapunov-based design applies provided that the inverse of the estimated inertia matrix is included in the regressors, cf. [37, Section 3.2].

symmetric. Therefore, (15) is satisfied by simply selecting $S_i = \gamma I$, for any positive scalar γ. For coordinate-dependent inertia matrices, as indicated in the footnote before Theorem 1, one should follow the approach in [37, Section 3.2]: in such a case, the term L_1^{*-1} should not be included in the Lyapunov function, and the adaptive gain simply results in a positive definite matrix. However, please note that, by including the inverse of the estimated inertia matrix in the regressor, one should guarantee that the estimated inertia matrix is invertible, as explained in [37, Section 3.2]. This can be obtained by appropriate parameter projection.

Remark 3. It is now clear that in the presence of multiple leaders it suffices for each one to implement a control law in the form (12) to achieve synchronization to the reference model dynamics (7).

4. Adaptive synchronization of a follower to a neighbor

In this section we explain how a follower agent that has no access to the desired trajectories q^d, $\dot{q}^d$, and $\ddot{q}^d$ can still synchronize to the reference model dynamics (7) by exploiting the signals of a neighboring agents for adaptation. By looking at Fig. 1 and without loss of generality, the follower dynamics are denoted with subscript 2, while the dynamics of the neighboring (hierarchically superior) agent are denoted with subscript 1. The dynamics of any follower in the form (2) can be written in the state-space form

$$\begin{bmatrix} \dot{q}_2 \\ \ddot{q}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \mathbb{1} \\ 0 & -D_2^{-1}C_2 \end{bmatrix}}_{A_2} \underbrace{\begin{bmatrix} q_2 \\ \dot{q}_2 \end{bmatrix}}_{x_2} + \begin{bmatrix} 0 \\ -D_2^{-1}g_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ D_2^{-1} \end{bmatrix}}_{B_2} \tau_2. \tag{20}$$

Analogously to the previous section, we aim to find a matching controller for agent 2: however, since the reference model signals are not available to this agent, we assume the dynamics of the neighboring agent 1 to act as a reference model. Then, let us propose the following controller to match the follower dynamics (20) to the leader dynamics (8)

$$\tau_2^* = \underbrace{\begin{bmatrix} \bar{K}_{21}^{*\prime} & \bar{\bar{K}}_{21}^{*\prime} \end{bmatrix}}_{K_{21}^{*\prime}} \begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix} + \underbrace{\begin{bmatrix} \bar{K}_2^{*\prime} & \bar{\bar{K}}_2^{*\prime} \end{bmatrix}}_{K_2^{*\prime}} \underbrace{\begin{bmatrix} q_2 - q_1 \\ \dot{q}_2 - \dot{q}_1 \end{bmatrix}}_{e_{21}} + G_2^{*\prime} + L_{21}^{*\prime}\tau_1. \tag{21}$$

The following lemma explains how to find the matching control gains in (21).

Lemma 2. *There exists an ideal control law in the form (21) that matches the follower dynamics (20) to the leader dynamics (8), and whose gains $\bar{K}_2^*, \bar{\bar{K}}_2^*, \bar{K}_{21}^*, \bar{\bar{K}}_{21}^*, L_{21}^*$, and G_2^* are*

$$\begin{aligned} \bar{K}_2^{*\prime} &= -D_2K_p & \bar{K}_{21}^{*\prime} &= 0 \quad G_2^{*\prime} = g_2 \\ \bar{\bar{K}}_2^{*\prime} &= -D_2K_v + C_2 & \bar{\bar{K}}_{21}^{*\prime} &= C_2 - D_2D_1^{-1}C_1 \quad L_{21}^{*\prime} = D_2D_1^{-1}. \end{aligned} \tag{22}$$

Proof. By direct substitution of (21) into (20), we have the leader closed-loop dynamics

$$\begin{aligned} \begin{bmatrix} \dot{q}_2 \\ \ddot{q}_2 \end{bmatrix} &= \begin{bmatrix} 0 & \mathbb{1} \\ D_2^{-1}\bar{K}_2^{*\prime} & D_2^{-1}(\bar{\bar{K}}_2^{*\prime} - C_2) \end{bmatrix} \begin{bmatrix} q_2 \\ \dot{q}_2 \end{bmatrix} \\ &+ \begin{bmatrix} 0 & 0 \\ D_2^{-1}(\bar{K}_{21}^{*\prime} - \bar{K}_2^{*\prime}) & D_2^{-1}(\bar{\bar{K}}_{21}^{*\prime} - \bar{\bar{K}}_2^{*\prime}) \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ -D_2^{-1}(-g_2 + G_2^{*\prime}) \end{bmatrix} + \begin{bmatrix} 0 \\ D_2^{-1}L_{21}^{*\prime} \end{bmatrix} \tau_1 \end{aligned} \tag{23}$$

from which we see that matching is achieved for the ideal control law

$$\begin{aligned} \tau_2^* &= C_2\dot{q}_1 - D_2D_1^{-1}C_1\dot{q}_1 - D_2K_p\bar{e}_{21} \\ &\quad - D_2K_v\bar{\bar{e}}_{21} + C_2\bar{\bar{e}}_{21} + g_2 + D_2D_1^{-1}\tau_1 \\ &= C_2\dot{q}_2 + D_2D_1^{-1}\tau_1 - D_2D_1^{-1}C_1\dot{q}_1 - D_2(K_p\bar{e}_{21} \\ &\quad + K_v\bar{\bar{e}}_{21}) + g_2 \end{aligned} \tag{24}$$

where we have defined $\bar{e}_{21} = q_2 - q_1$, $\bar{\bar{e}}_{21} = \dot{q}_2 - \dot{q}_1$. From (24) we find the control gains (22). This concludes the proof. □

Remark 4. Differently from Lemma 1, which gives us matching conditions between an agent and the reference model dynamics, Lemma 2 gives us matching conditions among neighboring agent. In fact, it is easy to show how (24) implies the existence of coupling gains $\bar{K}_{21}^{*\prime}, \bar{\bar{K}}_{21}^{*\prime}, L_{21}^{*\prime}$ satisfying

$$\begin{aligned} \bar{K}_{21}^{*\prime} &= \bar{K}_2^{*\prime} - L_{21}^{*\prime}\bar{K}_1^{*\prime} \\ \bar{\bar{K}}_{21}^{*\prime} &= \bar{\bar{K}}_2^{*\prime} - L_{21}^{\prime}\bar{\bar{K}}_1^{*\prime} \\ L_{21}^{*\prime} &= L_2^{*\prime}(L_1^{*\prime})^{-1} \end{aligned} \tag{25}$$

where $L_2^* = D_2$. Therefore, Lemma 2 can be interpreted as a distributed matching condition among neighboring agents.

Being the system matrices in (20) unknown, the control (24) cannot be implemented, and the synchronization task has to be achieved adaptively. Then, inspired by the ideal controller (24), we propose the controller

$$\begin{aligned} \tau_2 = &- \underbrace{\Theta_{D_2}^{\prime}\xi_{D_2}}_{\hat{D}_2}(K_p\bar{e}_{21} + K_v\bar{\bar{e}}_{21}) + \underbrace{\Theta_{C_2}^{\prime}\xi_{C_2}}_{\hat{C}_2}\dot{q}_2 \\ &+ \underbrace{\Theta_{D_2D_1}^{\prime}\xi_{D_2D_1}}_{\widehat{D_2D_1}}\tau_1 - \underbrace{\Theta_{D_2D_1C_1}^{\prime}\xi_{D_2D_1C_1}}_{\widehat{D_2D_1C_1}}\dot{q}_1 + \underbrace{\Theta_{g_2}^{\prime}\xi_{g_2}}_{\hat{g}_2} \end{aligned} \tag{26}$$

where the estimates $\hat{D}_2$, $\hat{C}_2$, $\widehat{D_2D_1}$, $\widehat{D_2D_1C_1}$, $\hat{g}_2$ of the ideal matrices have been split in a linear-in-the-parameter form, in view of Assumption 3. In fact, Assumption 3 guarantees $D_2 = \Theta_{D_2}^{*\prime}\xi_{D_2}$, $C_2 = \Theta_{C_2}^{*\prime}\xi_{C_2}$, $g_2 = \Theta_{g_2}^{*\prime}\xi_{g_2}$, $D_2D_1 = \Theta_{D_2D_1}^{*\prime}\xi_{D_2D_1}$ and $D_2D_1C_1 = \Theta_{D_2D_1C_1}^{*\prime}\xi_{D_2D_1C_1}$: again, a specific form of regressand Θ and regressor ξ will be revealed in Section 5 for the example of Unmanned Aerial Vehicles. Let us define the error $e_{21} = x_2 - x_1$, whose dynamics are

$$\begin{aligned} \dot{e}_{21} &= A_me_{21} + B_2(\tilde{K}_2{}^{\prime}e_{21} + \tilde{K}_{21}{}^{\prime}x_1 + \tilde{L}_{21}^{\prime}\tau_1 + \tilde{G}_2^{\prime}) \\ &= A_me_{21} + B_2(\tilde{\bar{K}}_2^{\prime}\bar{e}_{21} + \tilde{\bar{\bar{K}}}_2^{\prime}\bar{\bar{e}}_{21} + \tilde{\bar{K}}_{21}^{\prime}q_1 + \tilde{\bar{\bar{K}}}_{21}^{\prime}\dot{q}_1 + \tilde{L}_{21}^{\prime}\tau_1 + \tilde{G}_2^{\prime}) \\ &= A_me_{21} + B_2(\tilde{\Theta}_{C_2}^{\prime}\xi_{C_2}\dot{q}_2 + \tilde{\Theta}_{D_2D_1}^{\prime}\xi_{D_2D_1}\tau_1 - \tilde{\Theta}_{D_2D_1C_1}^{\prime}\xi_{D_2D_1C_1}\dot{q}_1 \\ &\quad - \tilde{\Theta}_{D_2}^{\prime}\xi_{D_2}(K_p\bar{e}_{21} + K_v\bar{\bar{e}}_{21}) + \tilde{\Theta}_{g_2}^{\prime}\xi_{g_2}) \end{aligned} \tag{27}$$

where $\tilde{K}_2 = K_2 - K_2^*$, $\tilde{K}_{21} = K_{21} - K_{21}^*$, $\tilde{L}_{21} = L_{21} - L_{21}^*$, $\tilde{\Theta}_{D_2} = \Theta_{D_2} - \Theta_{D_2}^*$, $\tilde{\Theta}_{C_2} = \Theta_{C_2} - \Theta_{C_2}^*$, $\tilde{\Theta}_{g_2} = \Theta_{g_2} - \Theta_{g_2}^*$, $\tilde{\Theta}_{D_2D_1} = \Theta_{D_2D_1} - \Theta_{D_2D_1}^*$ and $\tilde{\Theta}_{D_2D_1C_1} = \Theta_{D_2D_1C_1} - \Theta_{D_2D_1C_1}^*$. The following theorem provides the follower–leader synchronization.

Theorem 2. *Consider the reference model (7), the unknown leader dynamics (8), the unknown follower dynamics (20), and controller (26). Provided that there exists a matrix S_2 such that*

$$L_2^*S_2 = S_2^{\prime}L_2^{*\prime} > 0 \tag{28}$$

then, the adaptive laws

$$\begin{aligned} \dot{\Theta}_{C_2}^{\prime} &= -S_2B_m^{\prime}Pe_{21}\dot{q}_2^{\prime}\xi_{C_2}^{\prime} \quad \dot{\Theta}_{D_2}^{\prime} = S_2B_m^{\prime}Pe_{21}(K_p\bar{e}_{21} + K_v\bar{\bar{e}}_{21})^{\prime}\xi_{D_2}^{\prime} \\ \dot{\Theta}_{D_2D_1}^{\prime} &= -S_2B_m^{\prime}Pe_{21}\tau_1^{\prime}\xi_{D_2D_1}^{\prime} \quad \dot{\Theta}_{g_2}^{\prime} = -S_2B_m^{\prime}Pe_{21}\xi_{g_2}^{\prime} \\ \dot{\Theta}_{D_2D_1C_1}^{\prime} &= S_2B_m^{\prime}Pe_{21}\dot{q}_1^{\prime}\xi_{D_2D_1C_1}^{\prime} \end{aligned} \tag{29}$$

where $P = P^{\prime} > 0$ is such that (17) holds, guarantee synchronization of the follower dynamics (20) to the leader dynamics (8), i.e. $e_{21} \to 0$.

Proof. To analytically show the asymptotic convergence of the synchronization error between the follower and the leader, let us introduce the following Lyapunov function

$V_2(e_{21}, \tilde{\Theta}_{C_2}, \tilde{\Theta}_{D_2D_1}, \tilde{\Theta}_{D_2D_1C_1}, \tilde{\Theta}_{D_2}, \tilde{\Theta}_{g_2})$

$$
\begin{aligned}
&= e_{21}' P e_{21} + tr(\tilde{\Theta}_{C_2} S_2^{-1} L_2^{*-1} \tilde{\Theta}_{C_2}') + tr(\tilde{\Theta}_{D_2D_1} S_2^{-1} L_2^{*-1} \tilde{\Theta}_{D_2D_1}') \\
&\quad + tr(\tilde{\Theta}_{D_2D_1C_1} S_2^{-1} L_2^{*-1} \tilde{\Theta}_{D_2D_1C_1}') + tr(\tilde{\Theta}_{D_2} S_2^{-1} L_2^{*-1} \tilde{\Theta}_{D_2}') \\
&\quad + tr(\tilde{\Theta}_{g_2} S_2^{-1} L_2^{*-1} \tilde{\Theta}_{g_2}').
\end{aligned} \tag{30}
$$

Then it is possible to verify

$$
\begin{aligned}
\dot{V}_2 &= -e_{21}' Q e_{21} + 2e_{21}' P B_2 (\tilde{\Theta}_{C_2}' \xi_{C_2} \dot{q}_2 + \tilde{\Theta}_{D_2D_1}' \xi_{D_2D_1} \tau_1 \\
&\quad - \tilde{\Theta}_{D_2D_1C_1}' \xi_{D_2D_1C_1} \dot{q}_1 - \tilde{\Theta}_{D_2}' \xi_{D_2} (K_p \bar{e}_{21} + K_v \bar{\bar{e}}_{21}) + \tilde{\Theta}_{g_2}' \xi_{g_2}) \\
&\quad + 2tr(\tilde{\Theta}_{C_2} S_2^{-1} L_2^{*-1} \dot{\tilde{\Theta}}_{C_2}') + 2tr(\tilde{\Theta}_{D_2D_1} S_2^{-1} L_2^{*-1} \dot{\tilde{\Theta}}_{D_2D_1}') \\
&\quad + 2tr(\tilde{\Theta}_{D_2D_1C_1} S_2^{-1} L_2^{*-1} \dot{\tilde{\Theta}}_{D_2D_1C_1}') \\
&\quad + 2tr(\tilde{\Theta}_{D_2} S_2^{-1} L_2^{*-1} \dot{\tilde{\Theta}}_{D_2}') + 2tr(\tilde{\Theta}_{g_2} S_2^{-1} L_2^{*-1} \dot{\tilde{\Theta}}_{g_2}') \\
&= -e_{21}' Q e_{21} + 2tr(\tilde{\Theta}_{C_2} L_2^{*-1} (B_m' P e_{21} \dot{q}_2' \xi_{C_2}' + S_2^{-1} \dot{\tilde{\Theta}}_{C_2}')) \\
&\quad + 2tr(\tilde{\Theta}_{D_2D_1} L_2^{*-1} (B_m' P e_{21} \tau_1' \xi_{D_2D_1}' + S_2^{-1} \dot{\tilde{\Theta}}_{D_2D_1}')) \\
&\quad - 2tr(\tilde{\Theta}_{D_2D_1C_1} L_2^{*-1} (B_m' P e_{21} \dot{q}_1' \xi_{D_2D_1C_1}' - S_2^{-1} \dot{\tilde{\Theta}}_{D_2D_1C_1}')) \\
&\quad - 2tr(\tilde{\Theta}_{D_2} L_2^{*-1} (B_m' P e_{21} (K_p \bar{e}_{21} + K_v \bar{\bar{e}}_{21})' \xi_{D_2}' - S_2^{-1} \dot{\tilde{\Theta}}_{D_2}')) \\
&\quad + 2tr(\tilde{\Theta}_{g_2} L_2^{*-1} (B_m' P e_{21} \xi_{g_2}' + S_2^{-1} \dot{\tilde{\Theta}}_{g_2}')) \\
&= -e_{21}' Q e_{21}.
\end{aligned} \tag{31}
$$

Following similar steps as in the proof of Theorem 1, from (31) we deduce that V_2 has a finite limit, so e_2, $\tilde{\Theta}_{C_2}$, $\tilde{\Theta}_{D_2D_1}$, $\tilde{\Theta}_{D_2D_1C_1}$, $\tilde{\Theta}_{D_2}$, $\tilde{\Theta}_{g_2} \in \mathscr{L}_\infty$. Because $e_{21} = x_2 - x_1 \in \mathscr{L}_\infty$ and $x_1 \in \mathscr{L}_\infty$, we have $x_2 \in \mathscr{L}_\infty$. This implies x_2, $\tilde{\Theta}_{C_2}$, $\tilde{\Theta}_{D_2D_1}$, $\tilde{\Theta}_{D_2D_1C_1}$, $\tilde{\Theta}_{D_2}$, $\tilde{\Theta}_{g_2} \in \mathscr{L}_\infty$. Consequently, we can deduce $\tau_2 \in \mathscr{L}_\infty$. Therefore, all signals in the closed-loop systems are bounded. From (31), we can establish that V_2 has a bounded integral, so that we have $e_{21} \in \mathscr{L}_2$. Then by using $\tilde{\Theta}_{C_2}$, $\tilde{\Theta}_{D_2D_1}$, $\tilde{\Theta}_{D_2D_1C_1}$, $\tilde{\Theta}_{D_2}$, $\tilde{\Theta}_{g_2}, e_{21} \in \mathscr{L}_\infty$, we have $\dot{e}_{21} \in \mathscr{L}_\infty$. This concludes the proof of the boundedness of all closed-loop signal and convergence $e_{21} \to 0$ as $t \to \infty$. □

Remark 5. In line with Assumptions 3, the distributed gains in (23) should be written in the linear in the parameter form: this in general requires some reparameterization or overparameterization (e.g. collecting two or more parameters in a new parameter to be estimated), as shown in the UAV case of Section 5.

Remark 6. It is worth remarking that the controller (26) and the adaptive law (29) do not require the knowledge of the desired trajectories q^d, $\dot{q}^d$, and $\ddot{q}^d$. On the other hand, local information from the neighboring agent 1 is needed to agent 2, namely the state q_1, $\dot{q}_1$ and the input τ_1. Now, comparing controller (26) with [9,15] or similar approaches, we see that the proposed protocol is essentially simpler, because it does not require an extra dynamical system to exchange in a distributed manner the observer variables and reconstruct the leader's signal: therefore, the distributed observer mechanism has been replaces with local exchange of input information among neighbors. In addition, the proposed approach is continuous even when the velocity of the leader is not constant, whereas in state-of-the-art works a time-varying leader velocity requires sliding mode [15,41]: this is because the proposed approach makes use of the proportional-derivative leader dynamics defined by K_p and K_d.

4.1. Extension to hierarchical graphs

At this point we have shown that the proposed theory is consistent, modulo the nonlinear setting, with the theory in [6]. Therefore, it is not difficult to extend the results of Theorems 1 and 2 to hierarchical communication graphs. Only the main points are given for lack of space. It has to be remarked that hierarchical communication graphs are sometimes also referred to as Directed Acyclic Graphs (DAGs), and they find application in scheduling, data processing networks, citation graphs, data compression, just to name a few.

Except for the leaders, which use controller (13) and adaptive laws (16), the following controller is proposed for the other agents

$$
\begin{aligned}
\tau_j &= -\frac{\sum_{i=1}^N a_{ij} \hat{D}_j (K_p(q_j - q_i) + K_v(\dot{q}_j - \dot{q}_i))}{\sum_{i=1}^N a_{ij}} + \hat{C}_j \dot{q}_j \\
&\quad + \frac{\sum_{i=1}^N a_{ij} \widehat{D_j D_i}_j \tau_i}{\sum_{i=1}^N a_{ij}} - \frac{\sum_{i=1}^N a_{ij} \widehat{D_j D_i C}_{ij} \dot{q}_i}{\sum_{i=1}^N a_{ij}} + \hat{g}_j
\end{aligned} \tag{32}
$$

where the terms a_{ij} indicate the entries of the adjacency matrix $\mathcal{A}$ and of the target vector $\mathcal{M}$, as explained in Section 2.3 (one can verify that, with the appropriate adjacency matrices, the adaptive controller (32) reduces to the special case (26)). The following result holds.

Theorem 3. *Consider the unknown EL agents (1) connected in a hierarchical network $\bar{\mathcal{G}}$, the reference model (7), controllers (13), (32), and adaptive laws (16) and*

$$
\begin{aligned}
\dot{\Theta}_{D_j}' &= S_j B_m' P \left[\sum_{i=1}^N a_{ij}(x_j - x_i) \right] \\
&\quad \times \left[\sum_{i=1}^N a_{ij}(K_p(q_j - q_i) + K_v(\dot{q}_j - \dot{q}_i)' \xi_{D_jD_i}' \right] \\
\dot{\Theta}_{D_jD_i}' &= -S_j B_m' P \left[\sum_{i=1}^N a_{ij}(x_j - x_i) \right] \left[\sum_{i=1}^N a_{ij} \tau_i' \xi_{D_jD_i}' \right] \\
\dot{\Theta}_{C_j}' &= -S_j B_m' P \left[\sum_{i=1}^N a_{ij}(x_j - x_i) \right] \dot{q}_j' \xi_{C_j}' \\
\dot{\Theta}_{D_jD_iC_i}' &= S_j B_m' P \left[\sum_{i=1}^N a_{ij}(x_j - x_i) \right] \left[\sum_{i=1}^N a_{ij} \dot{q}_i' \xi_{D_jD_iC_i}' \right] \\
\dot{\Theta}_{g_j}' &= -S_j B_m' P \left[\sum_{i=1}^N a_{ij}(x_j - x_i) \right] \xi_{g_j}'
\end{aligned} \tag{33}
$$

where Θ_{C_j}, $\Theta_{D_jD_i}$, $\Theta_{D_jD_iC_i}$, Θ_{D_j}, Θ_{g_j} are the estimate of the regressand of C_j, D_jD_i, $D_jD_iC_i$, D_j, g_j respectively. Then, all closed-loop signals are bounded and, for any (i, j) such that $a_{ij} \neq 0$, we have $e_{ji} = x_j - x_i \to 0$ as $t \to \infty$. In addition, for every agent j we have $e_j = x_j - x_m \to 0$ as $t \to \infty$.

Proof. Let us adopt similar tools as Theorems 1 and 2 by considering the distributed Lyapunov function

$$
\begin{aligned}
V_j &= \sum_{j=1}^N \left[\sum_{i=0}^N a_{ij} e_{ji} \right]' P \left[\sum_{i=0}^N a_{ij} e_{ji} \right] + \sum_{j=1}^N tr\left[\tilde{\Theta}_{C_j} S_j^{-1} L_j^{*-1} \tilde{\Theta}_{C_j}' \right] \\
&\quad + \sum_{j=1}^N \sum_{i=1}^N a_{ij} tr\left[\tilde{\Theta}_{D_jD_i} S_j^{-1} L_j^{*-1} \tilde{\Theta}_{D_jD_i}' \right] \\
&\quad + \sum_{j=1}^N \sum_{i=1}^N a_{ij} tr\left[\tilde{\Theta}_{D_jD_iC_i} S_j^{-1} L_j^{*-1} \tilde{\Theta}_{D_jD_iC_i}' \right] \\
&\quad + \sum_{j=1}^N tr\left[\tilde{\Theta}_{D_j} S_j^{-1} L_j^{*-1} \tilde{\Theta}_{D_j}' \right] \\
&\quad + \sum_{j=1}^N tr\left[\tilde{\Theta}_{g_j} S_j^{-1} L_j^{*-1} \tilde{\Theta}_{g_j}' \right]
\end{aligned} \tag{34}
$$

where the index $i = 0$ is used for the reference model, i.e. $e_{j0} = e_j = x_j - x_m$. Let us define $\bar{e}_{ji} = q_j - q_i$ and $\bar{\bar{e}}_{ji} = \dot{q}_j - \dot{q}_i$. Then, from

(1) and (32), we obtain the following error dynamics

$$\begin{aligned}\dot e_{ji} &= A_m e_{ji} + B_j(\tilde{K}_j{}' e_{ji} + \tilde{K}_{ji}{}' x_i + \tilde{L}'_{ji}\tau_i + \tilde{G}'_j)\\ &= A_m e_{ji} + B_j(\tilde\Theta'_{C_j}\xi_{C_j}\dot q_j + \tilde\Theta'_{D_jD_i}\xi_{D_jD_i}\tau_i\\ &\quad - \tilde\Theta'_{D_jD_iC_i}\xi_{D_jD_iC_i}\dot q_i - \tilde\Theta'_{D_j}\xi_{D_j}(K_p\bar e_{ji} + K_v\bar{\bar e}_{ji}) + \tilde\Theta'_{g_j}\xi_{g_j}). \end{aligned} \tag{35}$$

The error dynamics have a similar structure as in the previously developed two-agent case: in addition, such dynamics are valid over any hierarchical graphs even when a node has more than one parent (cf. [6] for details). Then, using (35) it is possible to verify that

$$\begin{aligned}\dot V_j = &-\sum_{j=1}^{N}\left[\sum_{i=0}^{N} a_{ij}e_{ji}\right]' Q\left[\sum_{i=0}^{N} a_{ij}e_{ji}\right]\\ &+2\left[\sum_{i=0}^{N} a_{ij}e_{ji}\right]' Pb_j\left[\tilde\Theta'_{C_j}\xi_{C_j}\sum_{j=1}^{N} a_{ij}\dot q_j + \sum_{j=1}^{N} a_{ij}\tilde\Theta'_{D_jD_i}\xi_{D_jD_i}\tau_i\right.\\ &-\sum_{j=1}^{N} a_{ij}\tilde\Theta'_{D_jD_iC_i}\xi_{D_jD_iC_i}\dot q_i - \sum_{j=1}^{N} a_{ij}\tilde\Theta'_{D_j}\xi_{D_j}(K_p\bar e_{ji} + K_v\bar{\bar e}_{ji})\\ &\left.+\tilde\Theta'_{g_j}\xi_{g_j}\sum_{j=1}^{N} a_{ij}\right] + 2\sum_{j=1}^{N} tr\left[\tilde\Theta_{C_j}S_j^{-1}L_j^{*-1}\dot{\tilde\Theta}'_{C_j}\right]\\ &+2\sum_{j=1}^{N} tr\left[\tilde\Theta_{g_j}S_j^{-1}L_j^{*-1}\dot{\tilde\Theta}'_{g_j}\right]\\ &+2\sum_{j=1}^{N}\sum_{i=1}^{N} a_{ij}tr\left[\tilde\Theta_{D_jD_iC_i}S_j^{-1}L_j^{*-1}\dot{\tilde\Theta}'_{D_jD_iC_i}\right]\\ &+2\sum_{j=1}^{N} tr\left[\tilde\Theta_{D_j}S_j^{-1}L_j^{*-1}\dot{\tilde\Theta}'_{D_j}\right]\\ &+2\sum_{j=1}^{N}\sum_{i=1}^{N} a_{ij}tr\left[\tilde\Theta_{D_jD_i}S_j^{-1}L_j^{*-1}\dot{\tilde\Theta}'_{D_jD_i}\right]\\ = &-\sum_{j=1}^{N}\left[\sum_{i=0}^{N} a_{ij}e_{ji}\right]' Q\left[\sum_{i=0}^{N} a_{ij}e_{ji}\right]. \end{aligned}\tag{36}$$

This can be used to derive boundedness of all closed-loop signals and convergence of e_{ji} to zero, which can be proved by using the Barbalat's lemma procedure already adopted in Theorems 1 and 2. Convergence of $x_j - x_m$ to zero follows according to [5] by using the hierarchical structure of the graph. This concludes the proof. □

Remark 7. Because the proposed approach is a distributed-input approach (i.e. the input of an agent turns out to depend on the input of the neighbors), handling non-hierarchical networks is not trivial, due to the creation of algebraic loops that can make the input not well posed. For example, in [46] this problem has been solved by assigning priorities to remove the cycles. Some of the authors have studied this problem as well: in [7] it was shown that a parameterization similar to (35) turns out to be independent on the network connectivity (thus it holds also for non-hierarchical networks). However, because the input of each agent may be not well defined for all time instants on general graphs, Lesser et al. [4] have studied how to tackle this problem by embedding appropriate parameter projection in the adaptive law.

Remark 8. Other aspects in the proposed results worth remarking are the convergence analysis and its robustness. We have adopted a standard convergence analysis (via Barbalat's lemma) as it can be found, for example, in [20,44]. Alternatively and almost equivalently, one can use the analysis via the LaSalle-Yoshizawa theorem (cf. [24]) which embeds Barbalat's lemma and furthermore provides a uniform global stability characterization of the origin. With respect to robustness, it is well known that the LaSalle-Yoshizawa's (or Barbalat's) analysis do not provide robustness of the adaptive loop. Uniform persistency of excitation for the reference input (leader's input) would be required to ensure uniform global asymptotic stability, and thus inherent robustness. On the other hand, as it is quite established in adaptive control literature that persistency of excitation is not desirable and sometimes not possible, one would require techniques like parameter projection or leakage so as to achieve robust adaptive control results (cf. the excellent textbook [20]). For distributed-input approaches, the robustness issue has been studied for linear uncertain systems [6], and would translate to the EL setting quite straightforwardly in view of the linear-in-the-parameter form (Assumption 3).

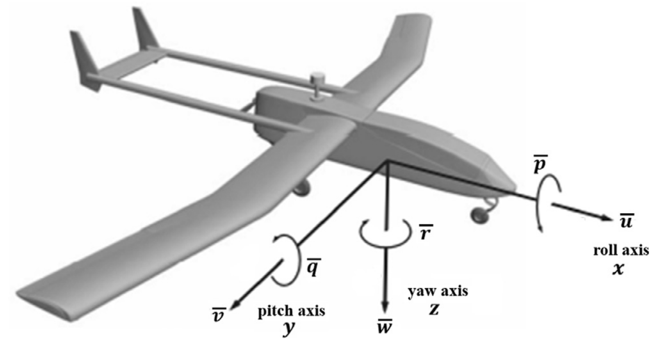

Fig. 2. Fixed-wing UAV body frame.

Remark 9. All the synchronization results have been given for errors in the form $e_{ji} = x_j - x_i$. This implies that upon synchronization all the agents will converge to a common trajectory depending on the desired trajectories q^d, $\dot q^d$, and $\ddot q^d$, and on the reference model dynamics K_p and K_v. It is not difficult to show that the proposed synchronization protocol can be extended to include formation gaps, provided that the error

$$e_{ji} = x_j - x_i + d_{ji} = \begin{bmatrix} q_j\\ \dot q_j\end{bmatrix} - \begin{bmatrix} q_i\\ \dot q_i\end{bmatrix} + \begin{bmatrix} \bar d_{ji}\\ 0\end{bmatrix} \tag{37}$$

is considered, where d_{ji} contains the desired formation displacement $\bar d_{ji}$ among agents j and i. Crucial to this extension is the fact that

$$\begin{bmatrix} 0 & \mathbb{1}\\ 0 & -D_i^{-1}C_i\end{bmatrix}\begin{bmatrix} \bar d_{ji}\\ 0\end{bmatrix} = \begin{bmatrix} 0\\ 0\end{bmatrix} \tag{38}$$

implying that the added displacement does not contribute to the error dynamics. Defining the formation as a desired set of displacements is the standard way most formations are defined, e.g. vehicle formations or other robotic formations [17,18].

5. The unmanned aerial vehicle case

In this section, we consider a formation of fixed-wing UAVs as a test case for the proposed adaptive formation control algorithm. Let us start by introducing the EL dynamics of a fixed-wing UAV. The translational motion and rotational motion will be considered. Let us introduce ϕ, θ, ψ to be the Euler Angles, $\bar u, \bar v, \bar w \in \mathbb{R}$ to be the velocities and $\bar p, \bar q, \bar r \in \mathbb{R}$ to be the angular velocities the UAV along axis ***x, y, z*** in the body frame: the axes ***x, y, z*** are shown in Fig. 2. The equation of motion are first derived in the inertial (earth) frame, and later the dynamics of inertial velocity will be expressed in body frame. The EL equation for translational motion in the earth frame are

$$m\begin{bmatrix}\dot{\bar{u}}\\ \dot{\bar{v}}\\ \dot{\bar{w}}\end{bmatrix}^e-\begin{bmatrix}0\\ 0\\ mg\end{bmatrix}^e=\begin{bmatrix}\tau_1\\ \tau_2\\ \tau_3\end{bmatrix}^e. \tag{39}$$

where the superscript e indicates the earth frame, m the mass of the UAV, g the gravitational constant and τ_1, τ_2, τ_3 (with the superscript ^{e}) are the forces acting in ***x, y, z*** coordinate, respectively. The corresponding dynamics of inertial velocity expressed in body frame [22] is

$$\begin{bmatrix}m&0&0\\ 0&m&0\\ 0&0&m\end{bmatrix}\begin{bmatrix}\dot{\bar{u}}\\ \dot{\bar{v}}\\ \dot{\bar{w}}\end{bmatrix}+\begin{bmatrix}0&-m\bar{r}&m\bar{q}\\ m\bar{r}&0&-m\bar{p}\\ -m\bar{q}&m\bar{p}&0\end{bmatrix}\begin{bmatrix}\bar{u}\\ \bar{v}\\ \bar{w}\end{bmatrix}$$
$$+\begin{bmatrix}\sin\theta mg\\ -\sin\phi\cos\theta mg\\ -\cos\phi\cos\theta mg\end{bmatrix}=\begin{bmatrix}\tau_1\\ \tau_2\\ \tau_3\end{bmatrix} \tag{40}$$

where we have used no superscript to express quantities in the body frame. Next, we derive the rotational motion, first in the inertial frame

$$\underbrace{\begin{bmatrix}I_x&0&-I_{xz}\\ 0&I_y&0\\ -I_{xz}&0&I_z\end{bmatrix}}_{I}\begin{bmatrix}\dot{\bar{p}}\\ \dot{\bar{q}}\\ \dot{\bar{r}}\end{bmatrix}^e=\begin{bmatrix}\tau_4\\ \tau_5\\ \tau_6\end{bmatrix}^e \tag{41}$$

where I is inertia tensor, and τ_4, τ_5, τ_6 (with the superscript ^{e}) are the moments acting in ***x, y, z*** coordinate, respectively. Note that the form of I takes into account that the fixed-wing UAV is symmetric with respect to axes ***x*** and ***z***, as well as the fact that the inertia on the planes ***xy*** and ***yz*** is negligible. The dynamics of the inertial rotational velocity expressed in body frame [22] is

$$\begin{bmatrix}I_x&0&-I_{xz}\\ 0&I_y&0\\ -I_{xz}&0&I_z\end{bmatrix}\begin{bmatrix}\dot{\bar{p}}\\ \dot{\bar{q}}\\ \dot{\bar{r}}\end{bmatrix}$$
$$+\begin{bmatrix}0&I_z\bar{r}-I_{xz}\bar{p}&-I_y\bar{q}\\ -I_z\bar{r}+I_{xz}\bar{p}&0&I_x\bar{p}-I_{xz}\bar{r}\\ I_y\bar{q}&-I_x\bar{p}+I_{xz}\bar{r}&0\end{bmatrix}\begin{bmatrix}\bar{p}\\ \bar{q}\\ \bar{r}\end{bmatrix}=\begin{bmatrix}\tau_4\\ \tau_5\\ \tau_6\end{bmatrix}. \tag{42}$$

The EL translational and rotational dynamics can be merged in the compact form

$$\underbrace{\begin{bmatrix}m&0&0&0&0&0\\ 0&m&0&0&0&0\\ 0&0&m&0&0&0\\ 0&0&0&I_x&0&-I_{xz}\\ 0&0&0&0&I_y&0\\ 0&0&0&-I_{xz}&0&I_z\end{bmatrix}}_{D}\underbrace{\begin{bmatrix}\dot{\bar{u}}\\ \dot{\bar{v}}\\ \dot{\bar{w}}\\ \dot{\bar{p}}\\ \dot{\bar{q}}\\ \dot{\bar{r}}\end{bmatrix}}_{\ddot{q}}$$
$$+\underbrace{\begin{bmatrix}0&-m\bar{r}&m\bar{q}&0&0&0\\ m\bar{r}&0&-m\bar{p}&0&0&0\\ -m\bar{q}&m\bar{p}&0&0&0&0\\ 0&0&0&0&I_z\bar{r}-I_{xz}\bar{p}&-I_y\bar{q}\\ 0&0&0&-I_z\bar{r}+I_{xz}\bar{p}&0&I_x\bar{p}-I_{xz}\bar{r}\\ 0&0&0&I_y\bar{q}&-I_x\bar{p}+I_{xz}\bar{r}&0\end{bmatrix}}_{C(\dot{q})}$$
$$\times\underbrace{\begin{bmatrix}\bar{u}\\ \bar{v}\\ \bar{w}\\ \bar{p}\\ \bar{q}\\ \bar{r}\end{bmatrix}}_{\dot{q}}+\underbrace{\begin{bmatrix}\sin\theta mg\\ -\sin\phi\cos\theta mg\\ -\cos\phi\cos\theta mg\\ 0\\ 0\\ 0\end{bmatrix}}_{g}=\begin{bmatrix}\tau_1\\ \tau_2\\ \tau_3\\ \tau_4\\ \tau_5\\ \tau_6\end{bmatrix} \tag{43}$$

where all generalized coordinates are to be intended as expressed in body frame. From (43) it is possible to see that Assumptions 1 and 3 are verified. With respect to Assumption 2, we focus on an 'ideal' UAV situation where full actuation is present. In practice, as mentioned in Remark 1, one should consider a control allocator that allocates all forces and moments to the aileron, rudder, elevator and thrust. Such control allocator will introduce some unmodeled dynamics because not all forces and moments can be perfectly allocated at all time instants. We have verified by simulations that, upon relaxing asymptotic synchronization to bounded synchronization, these unmodeled dynamics can be handled by robust adaptive laws, in the spirit of [19]. In order to be consistent with the theory, in the rest of the section we focus on the ideal fully-actuated case, and we would like to study the under-actuated case more extensively as future work.

Next, let us derive the control law in the form (13) for a UAV indicated by subscript i, and with dynamics as in (43): it is possible to show that the linear-in-the-parameter forms for D_i, C_i, g_i are

$$\Theta^{*\prime}_{D_i}=\begin{bmatrix}m_i&0&0&0&0&0\\ 0&m_i&0&0&0&0\\ 0&0&m_i&0&0&0\\ 0&0&0&I_{x_i}&0&-I_{xz_i}\\ 0&0&0&0&I_{y_i}&0\\ 0&0&0&-I_{xz_i}&0&I_{z_i}\end{bmatrix}$$

$$\xi_{D_i}=\begin{bmatrix}1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1\end{bmatrix}$$

$$\Theta^{*\prime}_{g_i}=\begin{bmatrix}m_i&0&0&0&0&0\\ 0&m_i&0&0&0&0\\ 0&0&m_i&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\end{bmatrix}\xi_{g_i}=\begin{bmatrix}\sin\theta g\\ -\sin\phi\cos\theta g\\ -\cos\phi\cos\theta g\\ 0\\ 0\\ 0\end{bmatrix}$$

$$\Theta^{*\prime}_{C_i}=\begin{bmatrix}m_i&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&m_i&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&m_i&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&I_{x_i}&0&0&I_{y_i}&0&0&I_{z_i}&0&0&I_{xz_i}&0&0\\ 0&0&0&0&I_{x_i}&0&0&I_{y_i}&0&0&I_{z_i}&0&0&I_{xz_i}&0\\ 0&0&0&0&0&I_{x_i}&0&0&I_{y_i}&0&0&I_{z_i}&0&0&I_{xz_i}\end{bmatrix} \tag{44}$$

$$\xi'_{C_i}=\begin{bmatrix}0&\bar{r}_i&-\bar{q}_i&0&0&0&0&0&0&0&0&0&0&0&0\\ -\bar{r}_i&0&\bar{p}_i&0&0&0&0&0&0&0&0&0&0&0&0\\ \bar{q}_i&-\bar{p}_i&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&\bar{q}_i&0&-\bar{r}_i&0&0&\bar{p}_i&0\\ 0&0&0&0&0&-\bar{p}_i&0&0&0&\bar{r}_i&0&0&-\bar{p}_i&0&\bar{r}_i\\ 0&0&0&0&\bar{p}_i&0&-\bar{q}_i&0&0&0&0&0&0&-\bar{r}_i&0\end{bmatrix}.$$

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Next, we derive the control law in the form (26), for two neighboring UAVs, indicated by subscripts i and j: it is not difficult to show that the linear-in-the-parameter forms of D_jD_i and $D_jD_iC_i$ are

$$\Theta^{*\prime}_{D_jD_i} = \begin{bmatrix} \frac{m_j}{m_i} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{m_j}{m_i} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{m_j}{m_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{I_{z_i}I_{x_j}I_{xz_j}}{I_{x_i}I_{z_i}-I_{xz_i}I_{xz_i}} & 0 & \frac{I_{xz_i}I_{x_j}I_{xz_j}}{I_{x_i}I_{z_i}-I_{xz_i}I_{xz_i}} \\ 0 & 0 & 0 & 0 & \frac{I_{y_j}}{I_{y_i}} & 0 \\ 0 & 0 & 0 & \frac{I_{xz_i}I_{xz_j}I_{z_j}}{I_{x_i}I_{z_i}-I_{xz_i}I_{xz_i}} & 0 & -\frac{I_{x_i}I_{xz_j}I_{z_j}}{I_{x_i}I_{z_i}-II_{xz_i}I_{xz_i}} \end{bmatrix}$$

$$\xi_{D_jD_i} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{45}$$

$$\Theta^{*}_{D_jD_iC_i} = \begin{bmatrix} m_j & 0 & 0 & 0 & 0 & 0 \\ 0 & m_j & 0 & 0 & 0 & 0 \\ 0 & 0 & m_j & 0 & 0 & 0 \\ 0 & 0 & 0 & \Gamma_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_1 \\ 0 & 0 & 0 & \Gamma_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_2 \\ 0 & 0 & 0 & \Gamma_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_3 \\ 0 & 0 & 0 & \Gamma_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_4 \\ 0 & 0 & 0 & \Gamma_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_5 \\ 0 & 0 & 0 & \Gamma_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_6 \\ 0 & 0 & 0 & \Gamma_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_7 \\ 0 & 0 & 0 & \Gamma_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_8 \\ 0 & 0 & 0 & \Gamma_9 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_9 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_9 \\ 0 & 0 & 0 & \Gamma_{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_{10} \\ 0 & 0 & 0 & \Gamma_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_{11} \\ 0 & 0 & 0 & \Gamma_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_{12} \end{bmatrix}$$

$$\xi'_{D_jD_iC_i} = \begin{bmatrix} 0 & -\bar{r}_i & \bar{q}_i & 0 & 0 & 0 \\ \bar{r}_i & 0 & -\bar{p}_i & 0 & 0 & 0 \\ -\bar{q}_i & \bar{p}_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\bar{q}_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\bar{r}_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{p}_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{q}_i & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{r}_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\bar{p}_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{r}_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\bar{p}_i & 0 \\ 0 & 0 & 0 & 0 & 0 & -\bar{q}_i \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\bar{r}_i \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{p}_i \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{q}_i \end{bmatrix} \tag{46}$$

where

$$D_jD_i^{-1} = \begin{bmatrix} \frac{m_j}{m_i} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{m_j}{m_i} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{m_j}{m_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{I_{z_i}I_{x_j}I_{xz_j}}{I_{x_i}I_{z_i}-I_{xz_i}I_{xz_i}} & 0 & \frac{I_{xz_i}I_{x_j}I_{xz_j}}{I_{x_i}I_{z_i}-I_{xz_i}I_{xz_i}} \\ 0 & 0 & 0 & 0 & \frac{I_{y_j}}{I_{y_i}} & 0 \\ 0 & 0 & 0 & \frac{I_{xz_i}I_{xz_j}I_{z_j}}{I_{x_i}I_{z_i}-I_{xz_i}I_{xz_i}} & 0 & -\frac{I_{x_i}I_{xz_j}I_{z_j}}{I_{x_i}I_{z_i}-I_{xz_i}I_{xz_i}} \end{bmatrix} \tag{47}$$

$$D_jD_i^{-1}C_i = \begin{bmatrix} 0 & -m_j\bar{r}_i & m_j\bar{q}_i & 0 & 0 & 0 \\ m_j\bar{r}_i & 0 & -m_j\bar{p}_i & 0 & 0 & 0 \\ -m_j\bar{q}_i & m_j\bar{p}_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{q}_i\Gamma_1 & \bar{r}_i\Gamma_5 - \bar{p}_i\Gamma_6 & -\bar{q}_i\Gamma_9 \\ 0 & 0 & 0 & -\bar{r}_i\Gamma_2 + \bar{p}_i\Gamma_3 & 0 & -\bar{r}_i\Gamma_{10} + \bar{p}_i\Gamma_{11} \\ 0 & 0 & 0 & \bar{q}_i\Gamma_4 & \bar{r}_i\Gamma_7 - \bar{p}_i\Gamma_8 & \bar{q}_i\Gamma_{12} \end{bmatrix} \tag{48}$$

$$\Gamma_1 = \frac{I_{y_i}(I_{xz_j}I_{x_i} - I_{xz_i}I_{x_j})}{I_{xz_i}^2 - I_{x_i}I_{z_i}}$$

$$\Gamma_7 = \frac{(I_{xz_j}I_{xz_i} - I_{x_i}I_{z_j})I_{xz_i} - (I_{z_j}I_{xz_i} - I_{z_i}I_{xz_j})I_{z_i}}{I_{xz_i}^2 - I_{x_i}I_{z_i}}$$

$$\Gamma_2 = \frac{I_{y_j}I_{z_i}}{I_{y_i}} \quad \Gamma_8 = \frac{(I_{xz_j}I_{z_i} - I_{x_i}I_{z_j})I_{x_i} - (I_{z_j}I_{xz_i} - I_{z_i}I_{xz_j})I_{xz_i}}{I_{xz_i}^2 - I_{x_i}I_{z_i}}$$

Table 1
Fixed-wing UAVs parameters and initial conditions.

	Mass (kg)	Initial cond. $[x, y, z, \phi, \theta, \psi]'(0)$	Initial cond. $[\bar{u}, \bar{v}, \bar{w}, \bar{p}, \bar{q}, \bar{r}]'(0)$	Moment of Inertia (kg m^2)
Agent 0 (Trajectory generator)	10	[0,490,−50,0,0,0]'	[15,0,0,0,0,0]'	$\begin{bmatrix} 0.02 & 0 & -0.01 \\ 0 & 0.026 & 0 \\ -0.01 & 0 & 0.053 \end{bmatrix}$
Agent 1 (Leader 1)	20	[−5, 525, −52.5,0.5,0.05,0.5]'	[25,0,0,0,0,0]'	$\begin{bmatrix} 0.1 & 0 & -0.01 \\ 0 & 0.05 & 0 \\ -0.01 & 0 & 0.1 \end{bmatrix}$
Agent 2 (Follower 1)	30	[−10, 550, −55,1,0.1,1]'	[5,0,0,0,0,0]'	$\begin{bmatrix} 0.2 & 0 & -0.02 \\ 0 & 0.1 & 0 \\ -0.02 & 0 & 0.2 \end{bmatrix}$
Agent 3 (Leader 2)	40	[−5, 475, −57.5, −0.5, −0.05, −0.5]'	[20,0,0,0,0,0]'	$\begin{bmatrix} 0.4 & 0 & -0.04 \\ 0 & 0.2 & 0 \\ -0.04 & 0 & 0.4 \end{bmatrix}$
Agent 4 (Follower 2)	50	[−10, 450, −60, −1, −0.1, −1]'	[10,0,0,0,0,0]'	$\begin{bmatrix} 0.8 & 0 & -0.08 \\ 0 & 0.4 & 0 \\ -0.08 & 0 & 0.8 \end{bmatrix}$

$$\Gamma_3 = \frac{I_{y_j} I_{xz_i}}{I_{y_i}} \quad \Gamma_9 = \frac{I_{y_i}(I_{xz_j} I_{xz_i} - I_{z_i} I_{x_j})}{I_{xz_i}^2 - I_{x_i} I_{z_i}}$$
$$\Gamma_4 = \frac{I_{y_i}(I_{xz_j} I_{xz_i} - I_{x_i} I_{z_j})}{I_{xz_i}^2 - I_{x_i} I_{z_i}} \quad \Gamma_{10} = \frac{I_{y_j} I_{xz_i}}{I_{y_i}}$$
$$\Gamma_5 = \frac{(I_{xz_j} I_{x_i} - I_{xz_i} I_{x_j}) I_{xz_i} + (I_{xz_j} I_{xz_i} - I_{z_i} I_{x_j}) I_{z_i}}{I_{xz_i}^2 - I_{x_i} I_{z_i}} \quad \Gamma_{11} = \frac{I_{y_j} I_{x_i}}{I_{y_i}}$$
$$\Gamma_6 = \frac{(I_{xz_j} I_{x_i} - I_{xz_i} I_{x_j}) I_{x_i} + (I_{xz_j} I_{xz_i} - I_{z_i} I_{x_j}) I_{xz_i}}{I_{xz_i}^2 - I_{x_i} I_{z_i}}$$
$$\Gamma_{12} = \frac{I_{y_i}(I_{z_j} I_{xz_i} - I_{z_i} I_{xz_j})}{I_{xz_i}^2 - I_{x_i} I_{z_i}}. \tag{49}$$

Remark 10. Note that the regressand $\Theta^*_{D_i}$, $\Theta^*_{C_i}$, $\Theta^*_{g_i}$, $\Theta^*_{D_j D_i}$ and $\Theta^*_{D_j D_i C_i}$ are generally sparse matrices whose structure is a priori known, as it can be seen from the UAV test case: one can use this a priori knowledge to create estimates Θ_{D_i}, Θ_{C_i}, Θ_{g_i}, $\Theta_{D_j D_i}$ and $\Theta_{D_j D_i C_i}$ with the same sparse structure and to project to zero the corresponding components in the estimation law (cf. parameter estimation mechanisms in adaptive laws in [19,42]). This would sensibly reduce the total number of parameters to be estimated.

Let us now find some matrices S_i satisfying conditions (15) or (28)

$$L_i = D_i = \begin{bmatrix} m_i & 0 & 0 & 0 & 0 & 0 \\ 0 & m_i & 0 & 0 & 0 & 0 \\ 0 & 0 & m_i & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{x_i} & 0 & -I_{xz_i} \\ 0 & 0 & 0 & 0 & I_{y_i} & 0 \\ 0 & 0 & 0 & -I_{xz_i} & 0 & I_{z_i} \end{bmatrix}$$
$$S_i = \begin{bmatrix} S_{i1} & 0 & 0 & 0 & 0 & 0 \\ 0 & S_{i2} & 0 & 0 & 0 & 0 \\ 0 & 0 & S_{i3} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{i4} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{i5} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{i4} \end{bmatrix} \tag{50}$$

where S_{i4} appears twice on the diagonal. Therefore, S_i can be the identity matrix or any positive multiple. Now that we have verified that all the theoretical conditions hold, we are ready to present the simulations.

6. Numerical simulations

Simulations are performed using the following reference model dynamics and parameters:

$$A_m = \begin{bmatrix} 0 & \mathbb{1} \\ -k_p \mathbb{1} & -k_v \mathbb{1} \end{bmatrix} \quad B_m = \begin{bmatrix} 0 \\ \mathbb{1} \end{bmatrix} \quad r = \begin{bmatrix} k_p x^d + k_v \dot{x}^d + \ddot{x}^d \\ k_p y^d + k_v \dot{y}^d + \ddot{y}^d \\ k_p z^d + k_v \dot{z}^d + \ddot{z}^d \\ k_p \phi^d + k_v \dot{\phi}^d + \ddot{\phi}^d \\ k_p \theta^d + k_v \dot{\theta}^d + \ddot{\theta}^d \\ k_p \psi^d + k_v \dot{\psi}^d + \ddot{\psi}^d \end{bmatrix}$$
$$Q = 100\mathbb{1} \qquad k_p = 50 \qquad k_v = 50 \qquad S_1 = \cdots = S_4 = 100\mathbb{1} \tag{51}$$

where, for each agent j, the state is $x_j = \begin{bmatrix} q'_j & \dot{q}'_j \end{bmatrix}'$, being q_j the generalized UAV coordinates expressed in the body frame. In addition, q^d, $\dot{q}^d$, and $\ddot{q}^d$ are obtained from an UAV flying in a windy environment and implementing a vector field approach as in [54]: this UAV represents the pinner UAV. We consider constant airspeed $V_a = 15$ m/s, constant altitude $h_m = 50$ m, and slowly-varying wind with amplitude $A(t) = 3\sin(0.01t)$ and slowly-varying wind angle $\psi_w(t) = \pi \sin(0.01t)$. The control parameters of the vector field approach are $\kappa = \frac{\pi}{2}$, $k = 0.1$, $\epsilon = 1$, $\Gamma = 0.1$, and $\sigma = 0$, whose exact meaning can be retrieved from [54].

In line with most UAV path generation approaches, the path is composed of straight lines and orbits. For these simulations we take a path consisting of a straight line followed an orbit with radius $R = 50$ m and orbit center $c = [500\text{ m}, 250\text{ m}]$. The simulations of the multi-UAV formation are carried out for 4 UAVs and a pinner UAV under the same communication graph shown in Fig. 1. Table 1 shows the parameters of the fixed-wing UAVs, which are used only for the sake of simulations and are unknown for the purpose of control design.

Figs. 3 and 4 show the state synchronization for all UAVs, where it is immediate to see that all states converge to the reference model. The states are reported in the inertial frame for an easier interpretation of the results. Fig. 5 shows how the UAVs behave in the inertial ***xy*** plane. It can be seen that by synchronizing to the reference model, all the agents perform the path consisting of the straight line and the orbit.

Clearly, in Fig. 5, the UAVs converge to the same point (rendezvous) due to the fact that we did not implement any desired formation (no formation gaps). In order to do so, we provide the desired gaps among agents, in such a way to describe a V formation. For convenience, we introduce first the gaps $\bar{d}^e_{ji}$ in the inertial frame: $\bar{d}^e_{10}$, $\bar{d}^e_{21} = [50, -100, 0, 0, 0, 0]$, and $\bar{d}^e_{30}$, $\bar{d}^e_{43} = [50, 100, 0, 0, 0, 0]$. These gaps describe the V formation: obviously,

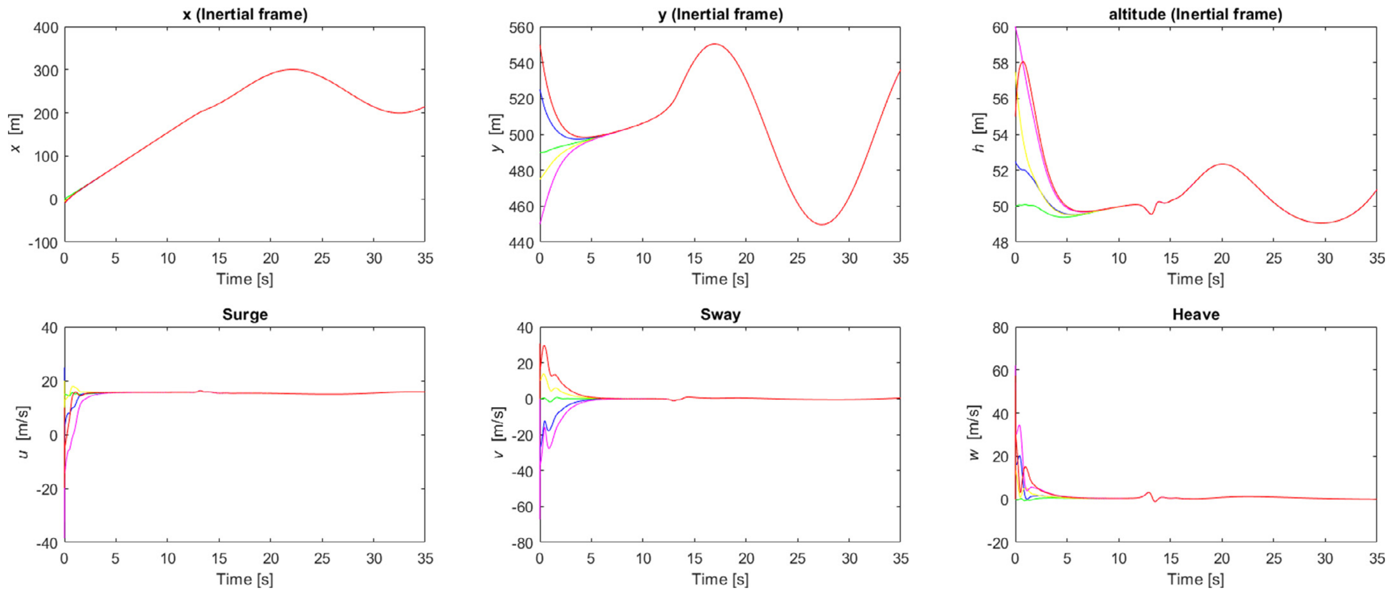

Fig. 3. Adaptive state synchronization for states $(x, y, z, \bar{u}, \bar{v}, \bar{w})$.

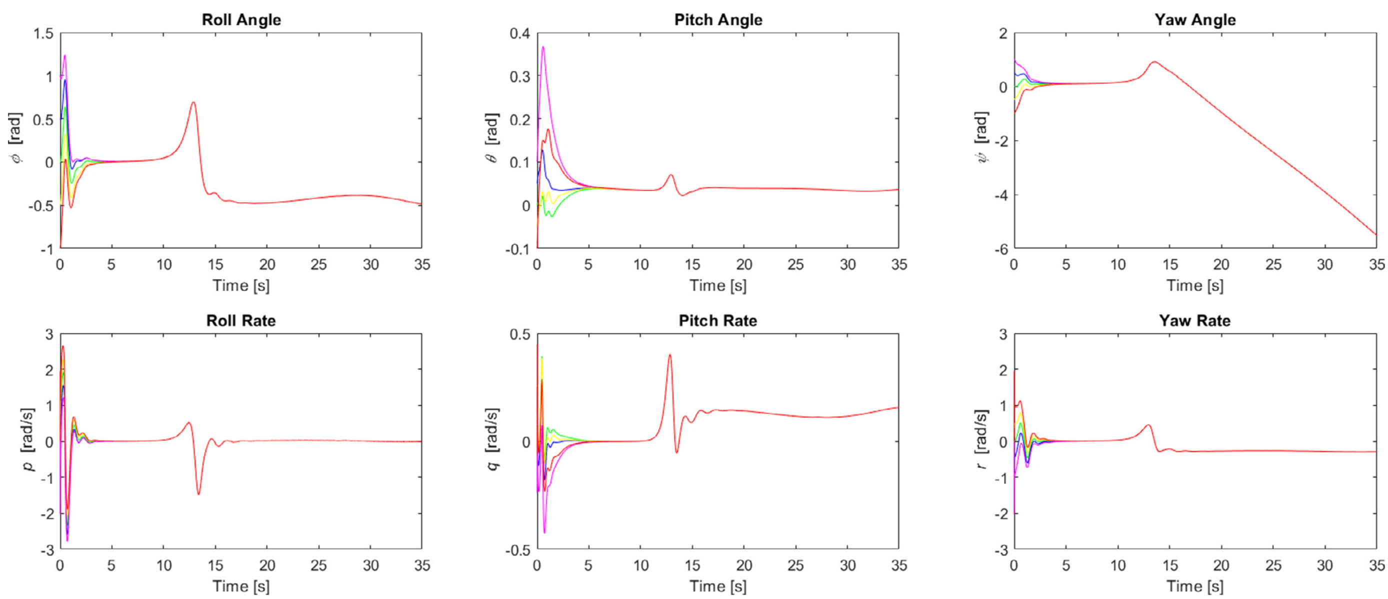

Fig. 4. Adaptive state synchronization for states $(\phi, \theta, \psi, \bar{p}, \bar{q}, \bar{r})$.

it is possible to introduce more complex gaps in the ***x, y, z*** space to perform more complicated flight formations. The error e^e_{ji} in inertial frame is

$$e^e_{ji} = x^e_j - x^e_i + \bar{d}^e_{ji}. \tag{52}$$

Then, we express the error to the body frame by introducing the rotation matrix $\mathcal{R}$

$$e_{ji} = \mathcal{R} e^e_{ji} \tag{53}$$

where

$$\mathcal{R}(\phi, \theta, \psi) = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi & -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi & \cos\phi\cos\theta \end{bmatrix} \tag{54}$$

so as to express control and adaptive laws in the body frame. Fig. 6 shows, in the inertial ***xy*** plane, that the V-formation of fixed wing UAVs can be achieved, which demonstrates the effectiveness of the proposed adaptive formation algorithm.

One final point should be remarked: the Barbalat's (or the LaSalle-Yoshizawa's) analysis of adaptive loops does not provide any guarantee of robustness. One reason is that only uniform global stability characterization of the origin can be derived, in place of the more desirable uniform global asymptotic stability. Lack of asymptotic stability implies that convergence of the estimates to the actual parameters cannot be guaranteed: it is well known in adaptive literature that uniform persistency of excitation conditions would ensure uniform global asymptotic stability, and thus convergence to the true parameters, and thus inherent robustness of the adaptive loop. However, persistency of excitation cannot be guaranteed a priori: for example, the leader trajectory in the proposed simulations does not guarantee convergence of the estimates to the actual parameters. This can bee seen, for system 1,

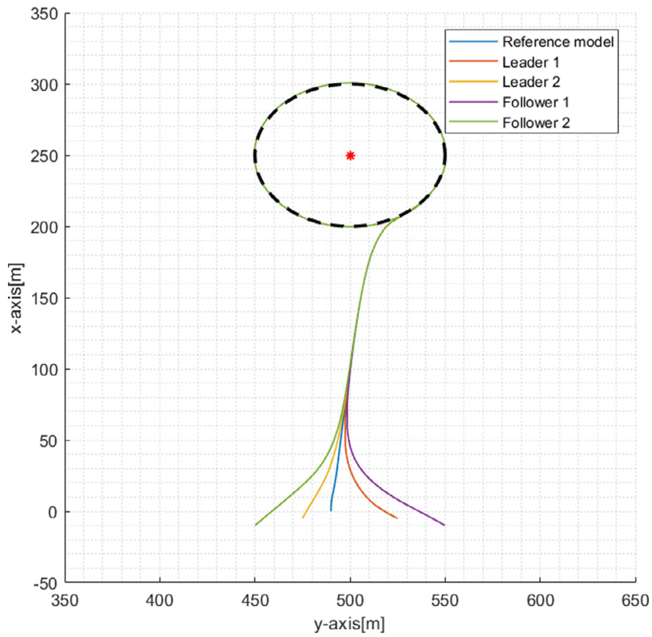

Fig. 5. Adaptive state synchronization in the inertial ***xy*** plane without formation gaps.

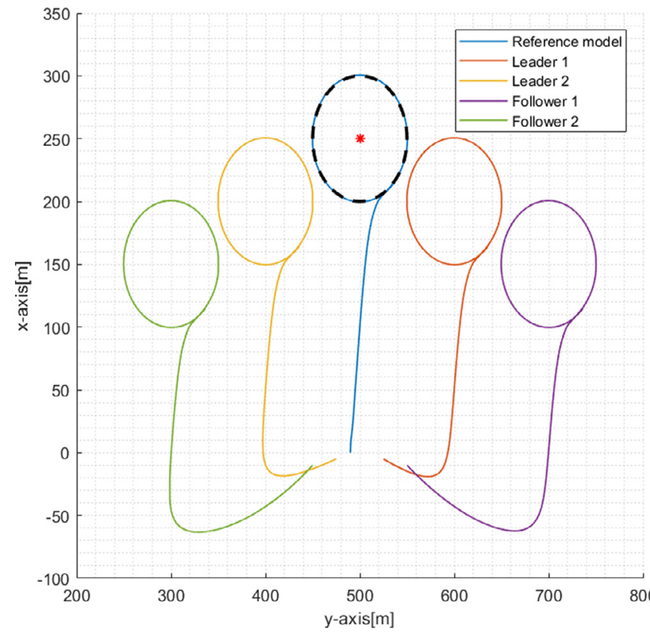

Fig. 6. Adaptive formation control in the inertial ***xy*** plane for V formation flight.

in Figs. 7–9 (only a few representative parameters are shown for better readability). Therefore, in the absence of persistence of excitation, techniques like parameter projection or leakage should be include to provide some level of robustness, as discussed in Remark 8.

7. Conclusion

This work has shown the possibility to synchronize uncertain heterogeneous agents with Euler–Lagrange dynamics. The formation control problem was defined as a synchronization problem where a distributed model reference adaptive control is used to synchronize the Euler–Lagrange systems. The adaptive formation control algorithm is distributed among the agent and utilizes local state and input information, without any extra auxiliary variables nor sliding modes. The idea behind the proposed adaptive algorithm was to make each agent converge to the model defined by its neighbors. The presence of uncertainty is handled first by showing that distributed nonlinear matching gains exist between neighboring agents, and then by developing adaptive laws to estimate these gains. Such distributed nonlinear matching conditions and adaptive laws allow all agents to converge to same homogeneous reference model dynamics, and thus synchronize. The stability of the proposed controlled was derived analytically by introducing an appropriately defined distributed Lyapunov function. The effectiveness of the proposed methodology is verified via numerical simulations of a formation of Unmanned Aerial Vehicles.

Future work will consider the presence of under-actuated Euler–Lagrange dynamics, and possibly non-hierarchical networks.

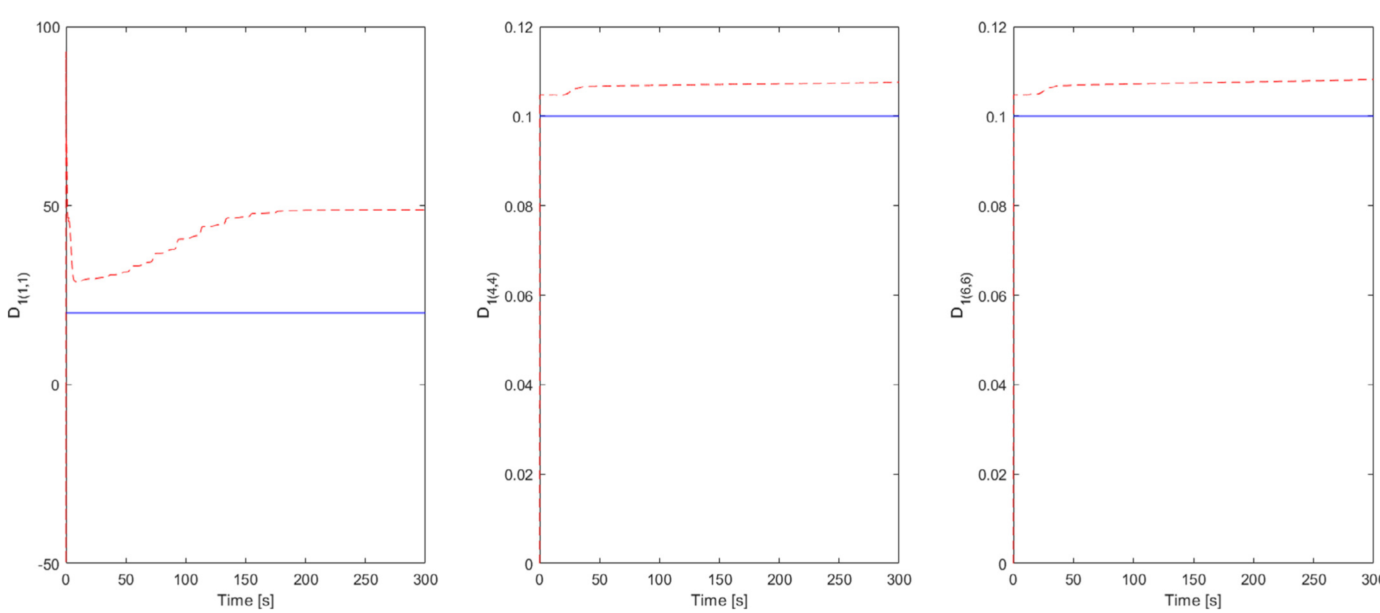

Fig. 7. Estimation of Θ_{D_1}: actual (blue-solid line) and estimated (red-dashed line) parameters. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

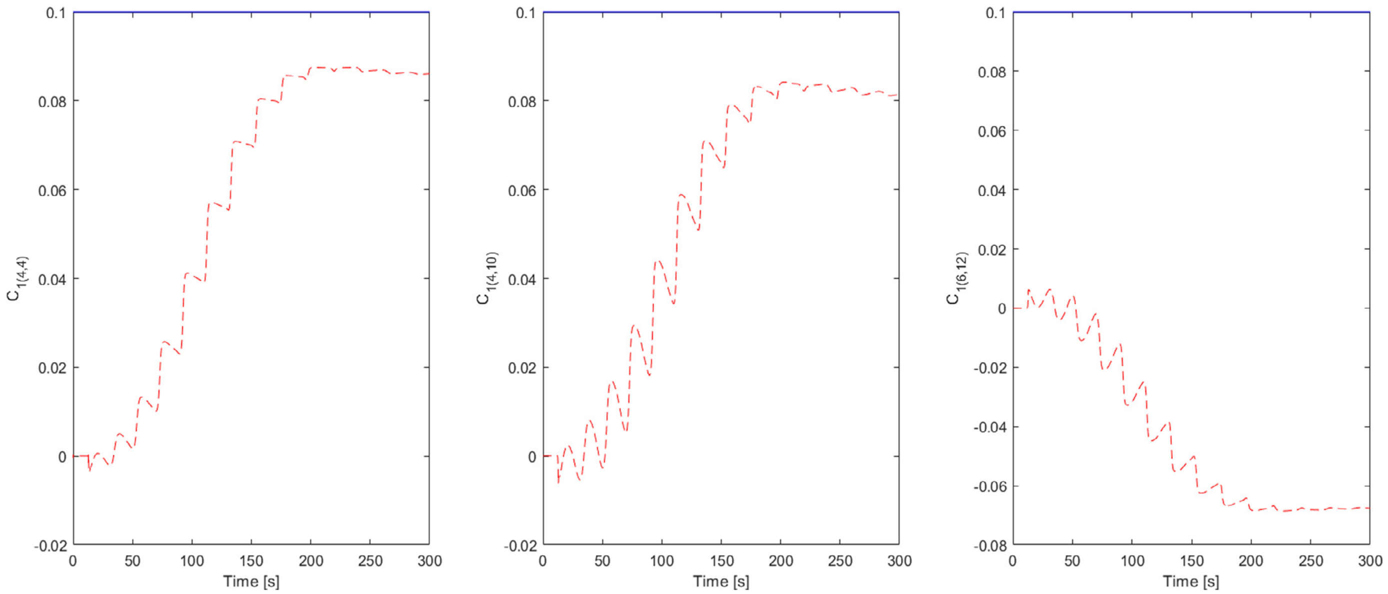

Fig. 8. Estimation of Θ_{C_1}: actual (blue-solid line) and estimated (red-dashed line) parameters. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

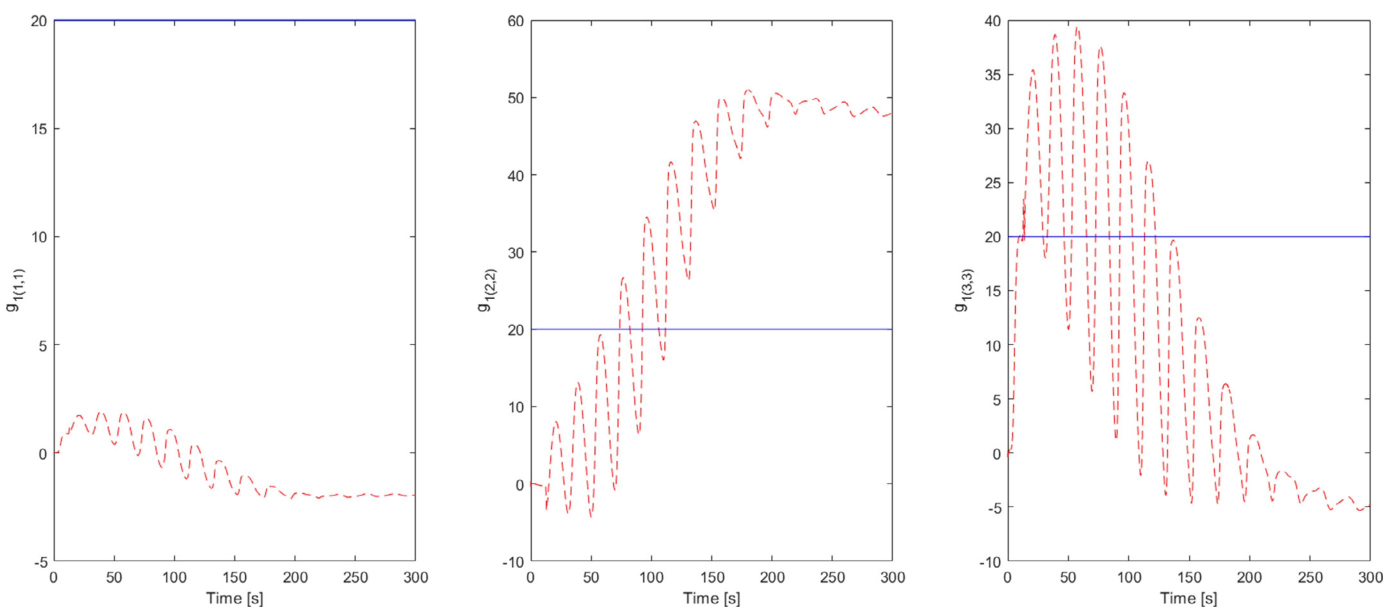

Fig. 9. Estimation of Θ_{g_1}: actual (blue-solid line) and estimated (red-dashed line) parameters. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Other relevant directions are, in line with [8], to address persistency of excitation requirements for estimation, whereas, in line with [25], one can consider practical constraints such as actuator saturation.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejcon.2018.11.001.

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