Q1

(1)

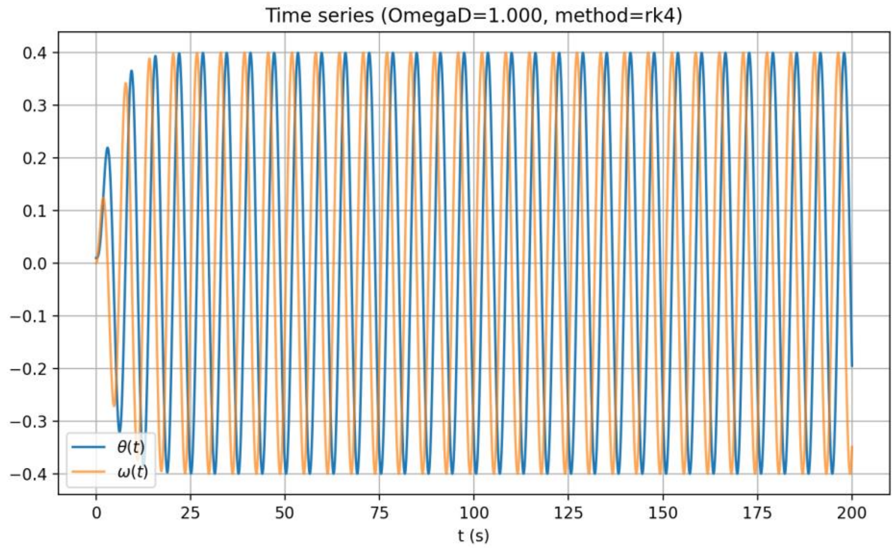
Small-angle natural frequency $\omega_0 = \sqrt{g/l} = 1.0\ s^{-1}$

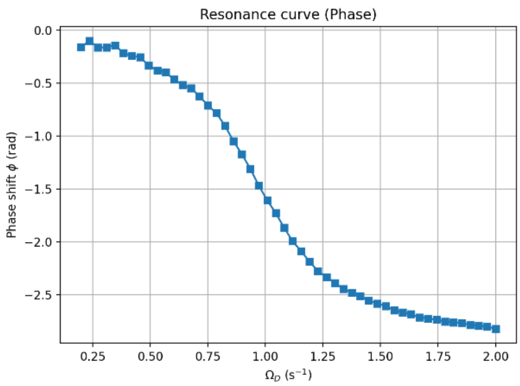
Resonant response will be near ω_0; for weak damping peak is close to ω_0 but slightly shifted.

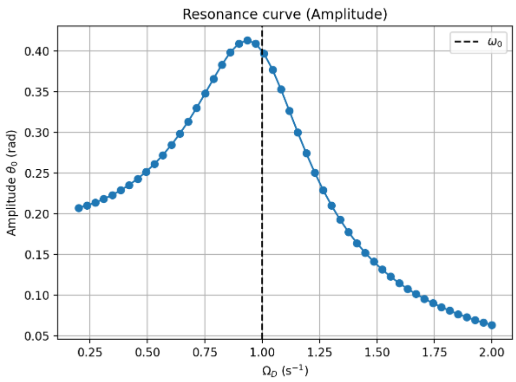
Damping $Y = 0.250\ s^{-1}$; quality factor $Q \approx \frac{\omega_0}{2Y} = 2.0$

Estimated resonance near $Omega_D \approx 1.0\ s^{-1}$

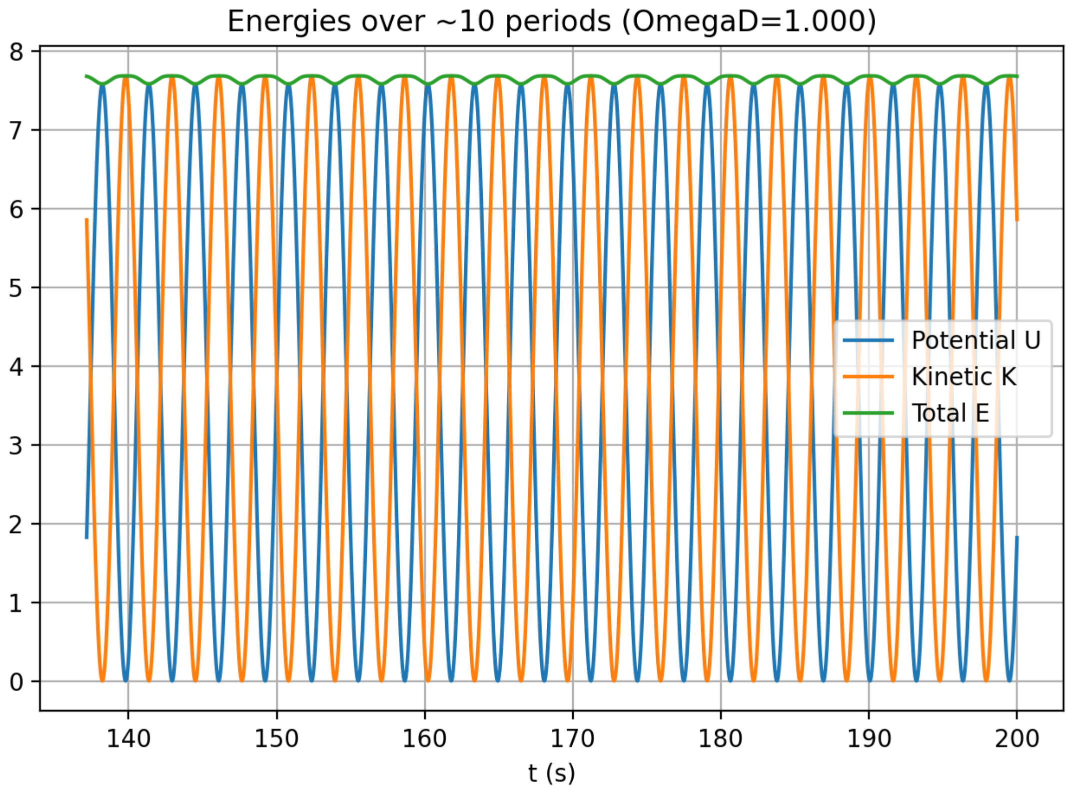
(2)



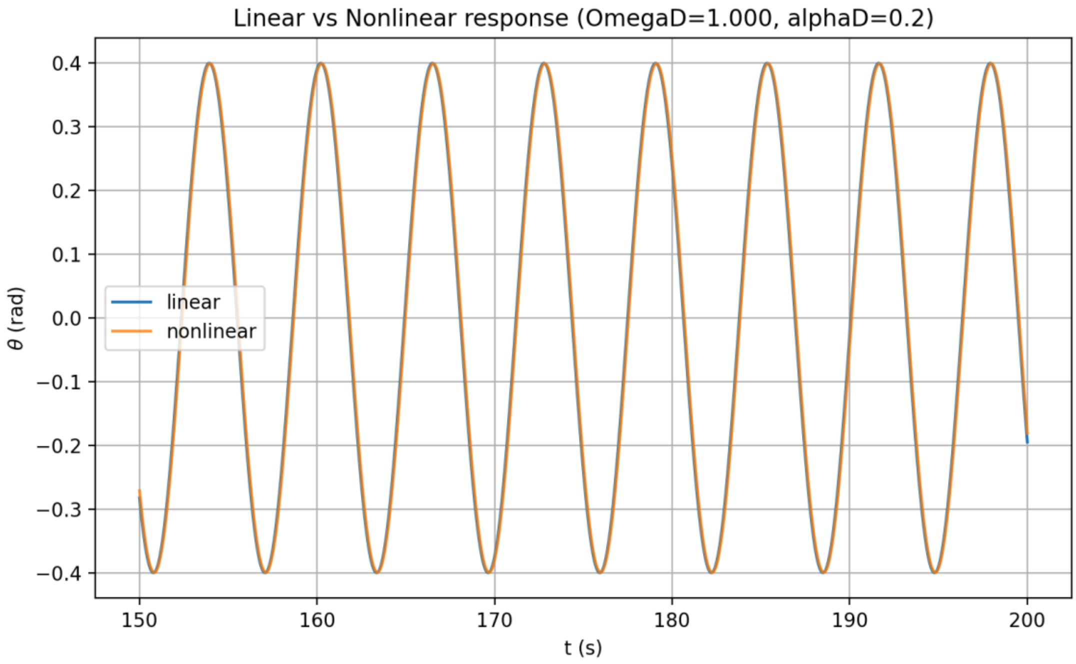




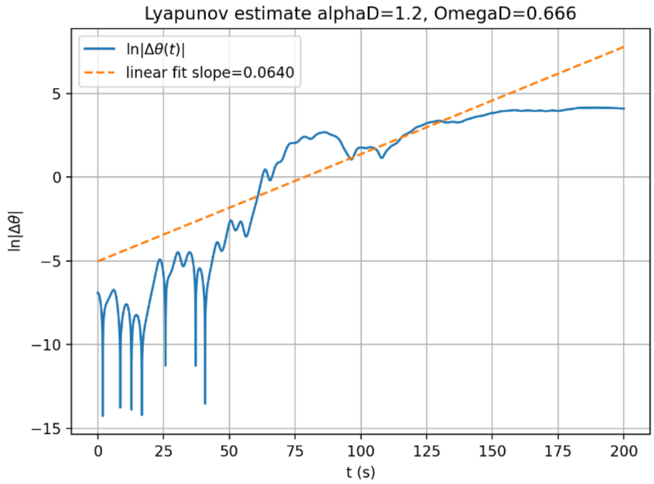
(3)


Energies over ~10 periods (OmegaD=1.000)
8
7
6
5
4
3
2
1
0
140
150
160
170
180
190
200
t (s)
Potential U
Kinetic K
Total E

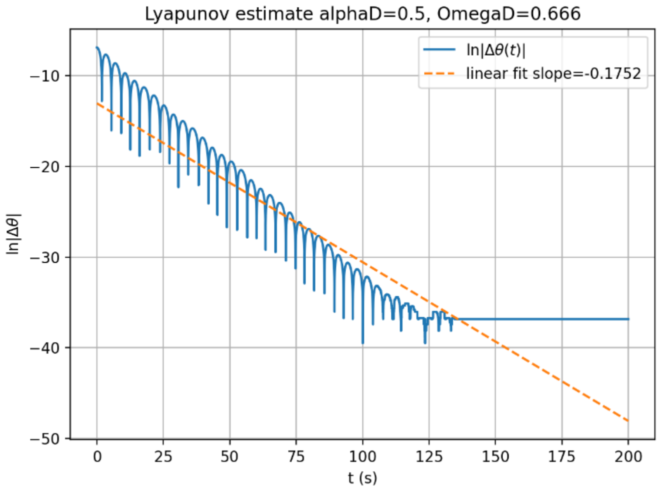
(4)


Linear vs Nonlinear response (OmegaD=1.000, alphaD=0.2)
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
θ (rad)
150
160
170
180
190
200
t (s)
linear
nonlinear

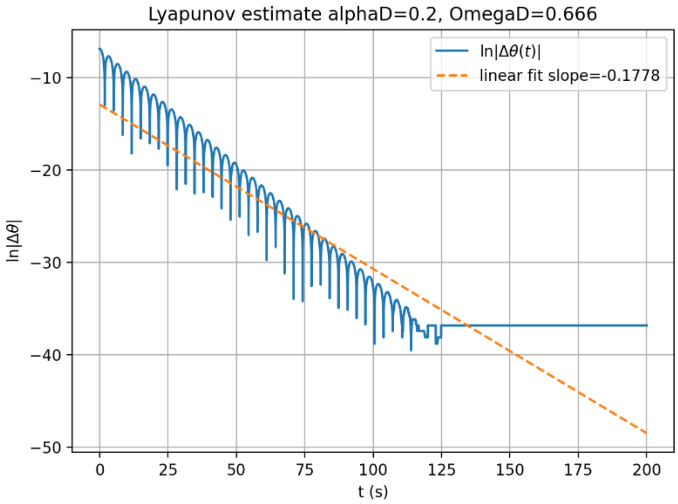
(5)



Lyapunov estimate alphaD=1.2, OmegaD=0.666
ln|Δθ(t)|
linear fit slope=0.0640
5
0
−5
−10
−15
0
25
50
75
100
125
150
175
200
t (s)
ln|Δθ|



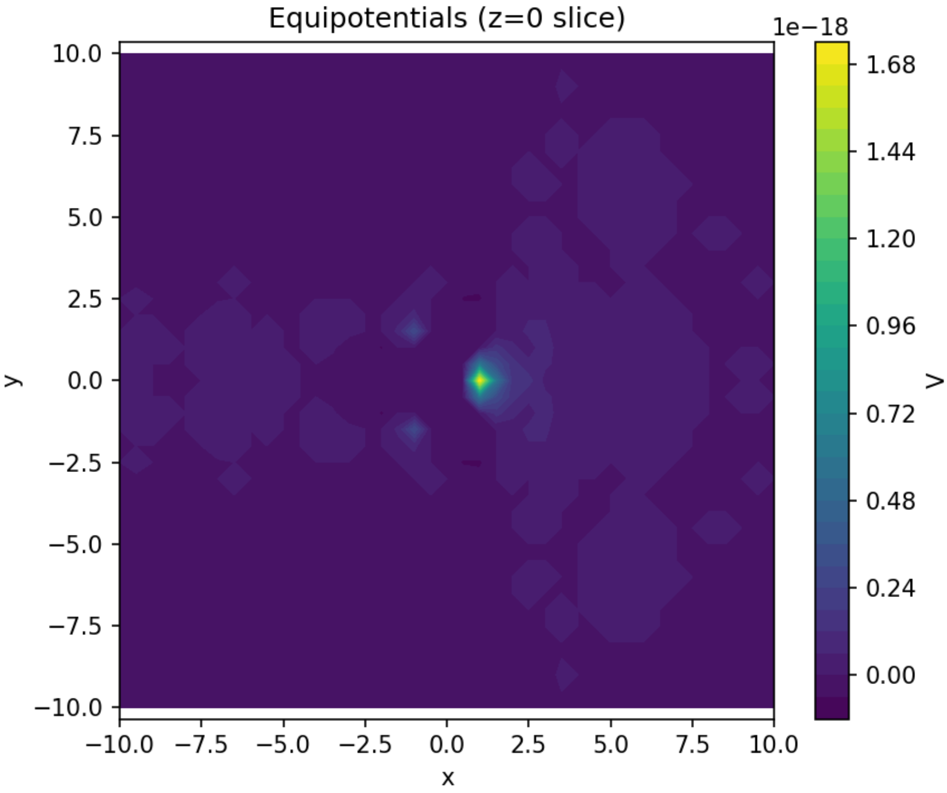
Lyapunov estimate alphaD=0.5, OmegaD=0.666
ln|Δθ(t)|
linear fit slope=-0.1752
−10
−20
−30
−40
−50
0
25
50
75
100
125
150
175
200
t (s)
ln|Δθ|



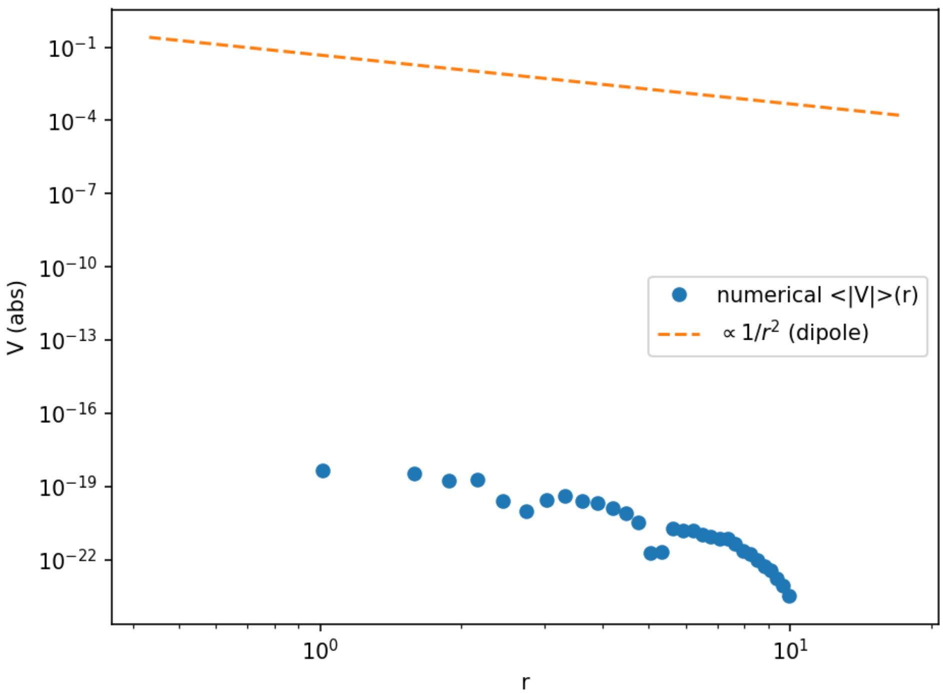
Lyapunov estimate alphaD=0.2, OmegaD=0.666
ln|Δθ(t)|
linear fit slope=-0.1778
−10
−20
−30
−40
−50
0
25
50
75
100
125
150
175
200
t (s)
ln|Δθ|

Q2

(1)

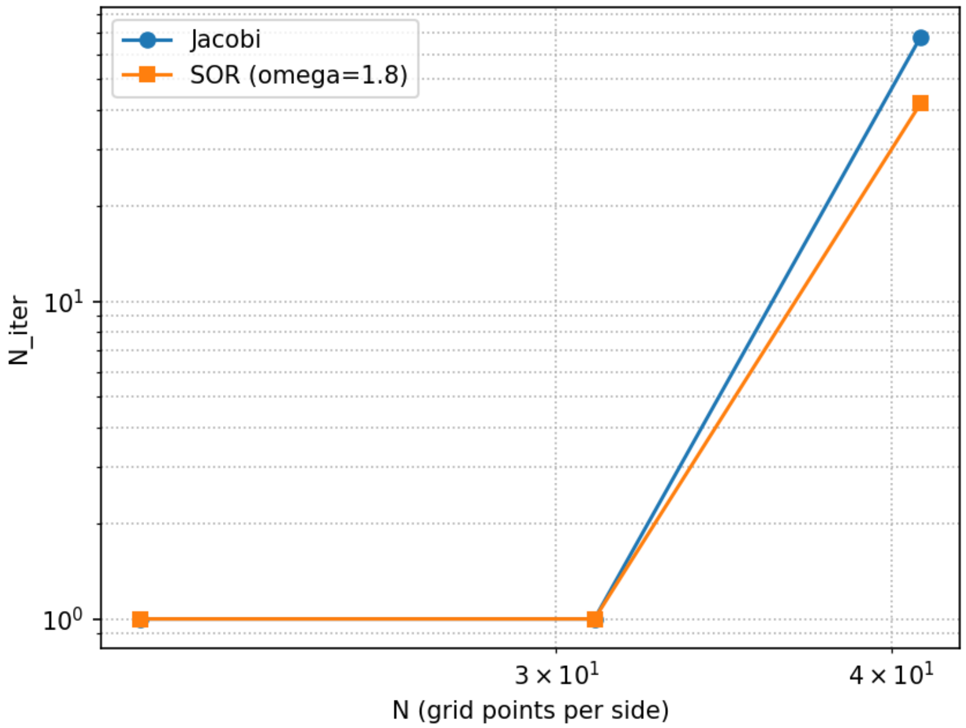


Equipotentials (z=0 slice)
1e−18
1.68
1.44
1.20
0.96
0.72
0.48
0.24
0.00
V
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
y
−10.0 −7.5 −5.0 −2.5 0.0 2.5 5.0 7.5 10.0
x

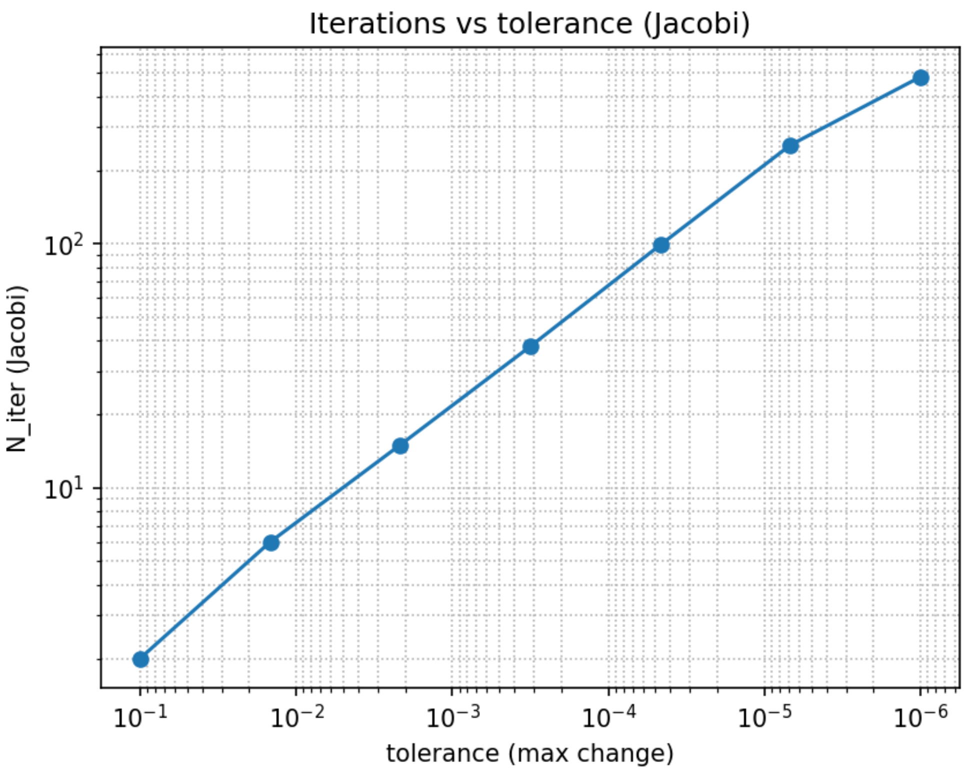


10^{-1}
10^{-4}
10^{-7}
10^{-10}
10^{-13}
10^{-16}
10^{-19}
10^{-22}
V (abs)
numerical <|V|>(r)
$\propto 1/r^2$ (dipole)
10^0
10^1
r

(2)



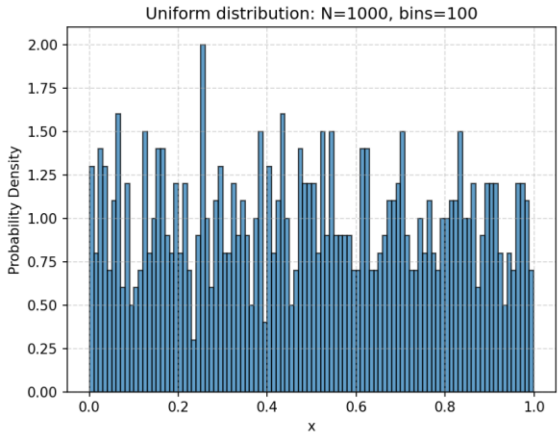
Jacobi
SOR (omega=1.8)
N_iter
10^1
10^0
3×10^1
4×10^1
N (grid points per side)

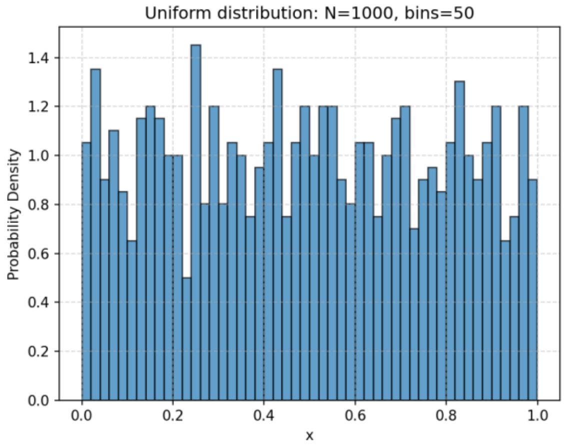


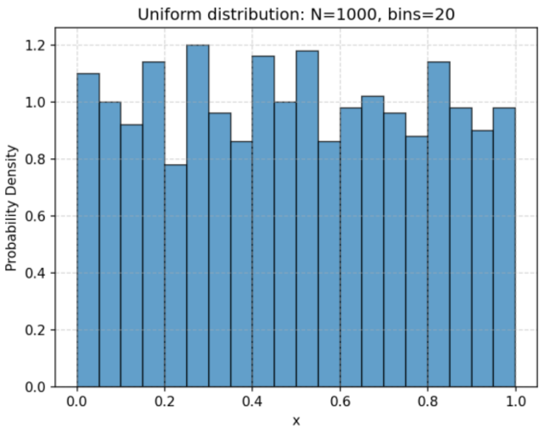
Iterations vs tolerance (Jacobi)
N_iter (Jacobi)
10^2
10^1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
tolerance (max change)

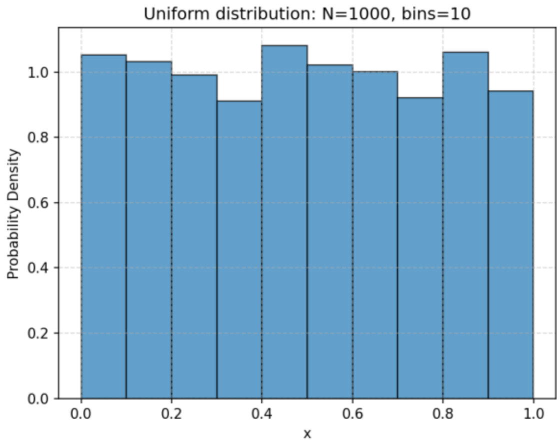
Q3

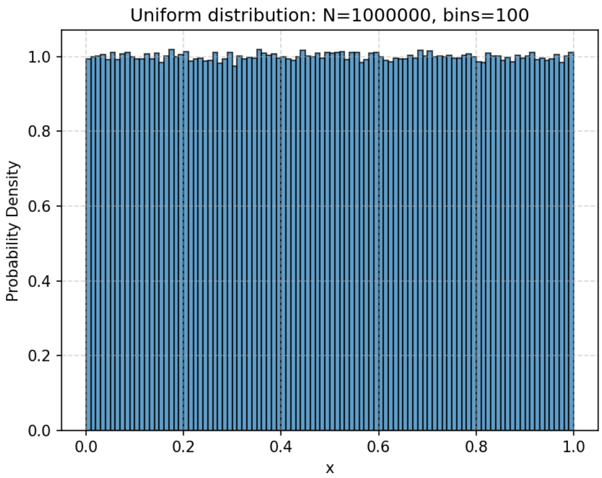
(1)

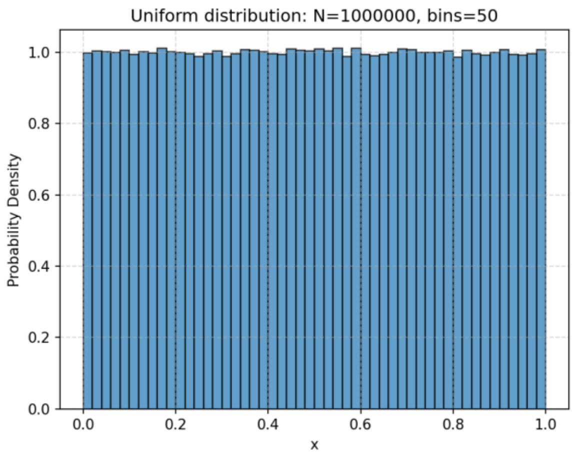

Uniform distribution: N=1000, bins=100
Probability Density
x

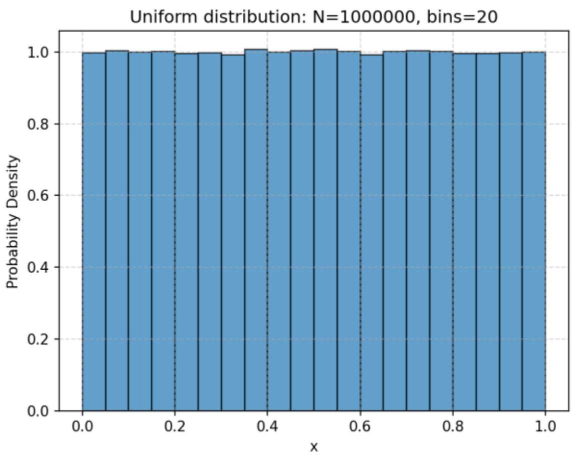

Uniform distribution: N=1000, bins=50
Probability Density
x

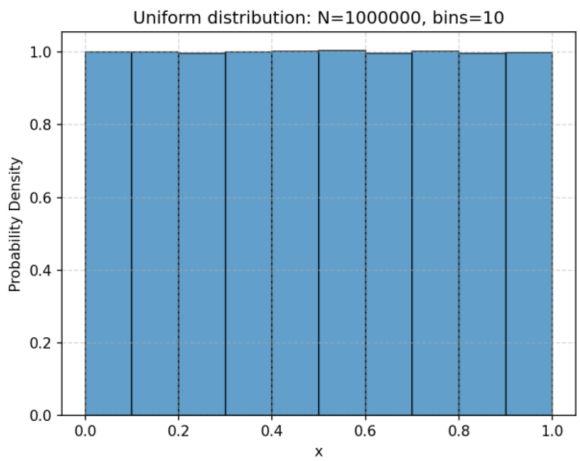

Uniform distribution: N=1000, bins=20
Probability Density
x


Uniform distribution: N=1000, bins=10
Probability Density
x

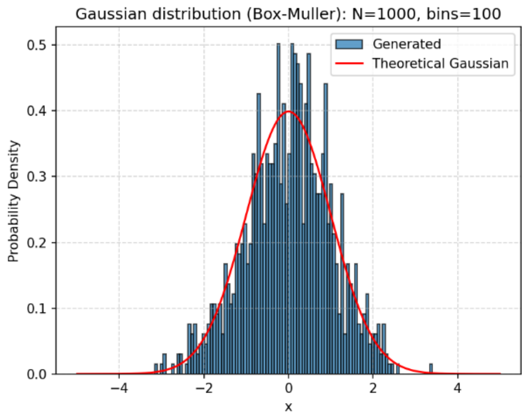

Uniform distribution: N=1000000, bins=100
Probability Density
x

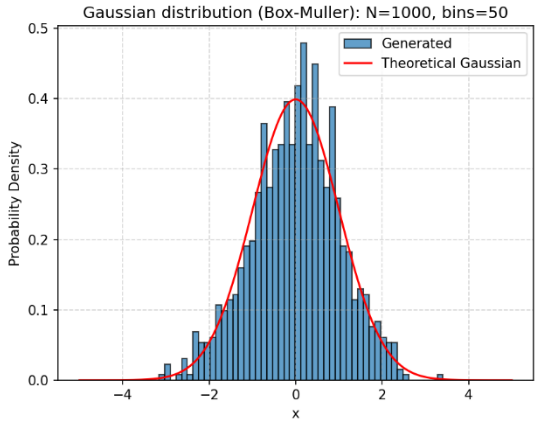

Uniform distribution: N=1000000, bins=50
Probability Density
x

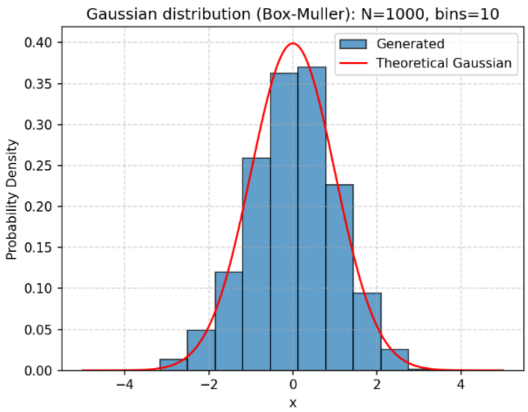

Uniform distribution: N=1000000, bins=20
Probability Density
x

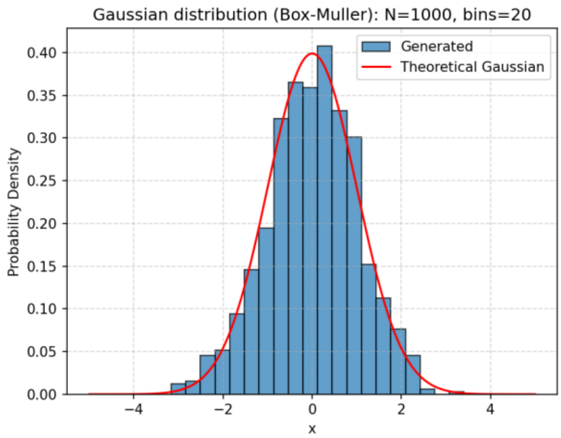

Uniform distribution: N=1000000, bins=10
Probability Density
x

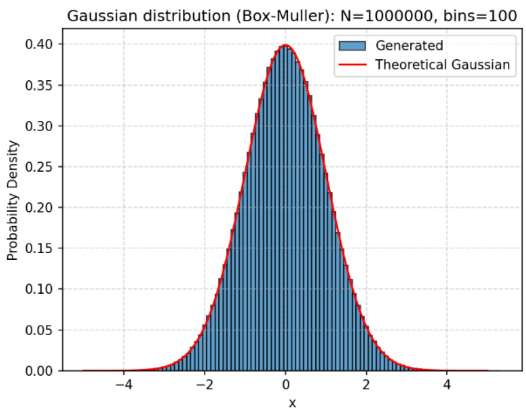
(2)

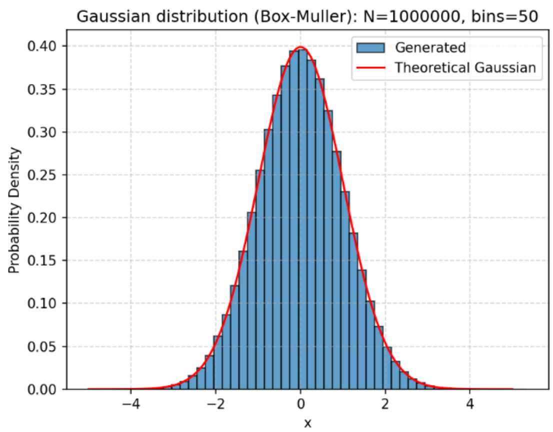

Gaussian distribution (Box-Muller): N=1000, bins=100
Generated
Theoretical Gaussian
Probability Density
x

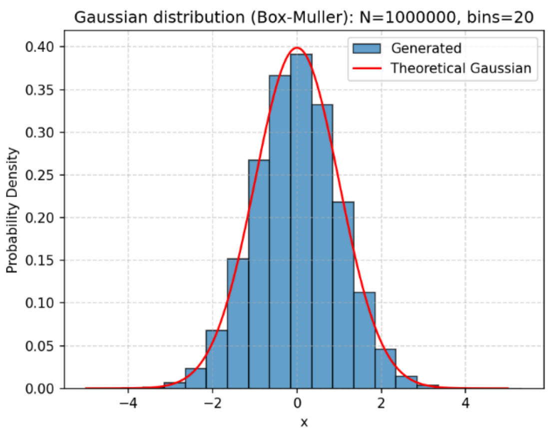

Gaussian distribution (Box-Muller): N=1000, bins=50
Generated
Theoretical Gaussian
Probability Density
x

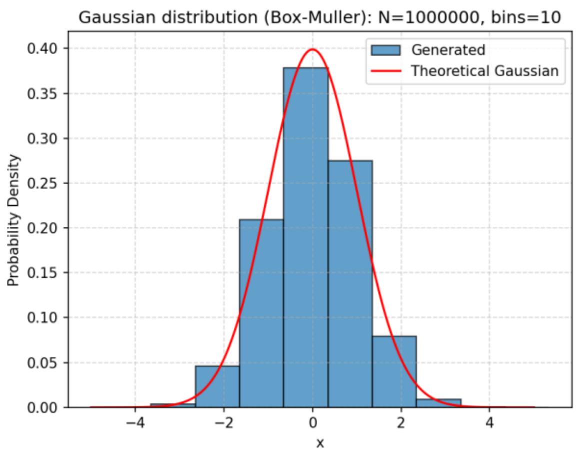

Gaussian distribution (Box-Muller): N=1000, bins=10
Generated
Theoretical Gaussian
Probability Density
x


Gaussian distribution (Box-Muller): N=1000, bins=20
Generated
Theoretical Gaussian
Probability Density
x


Gaussian distribution (Box-Muller): N=1000000, bins=100
Generated
Theoretical Gaussian
Probability Density
x

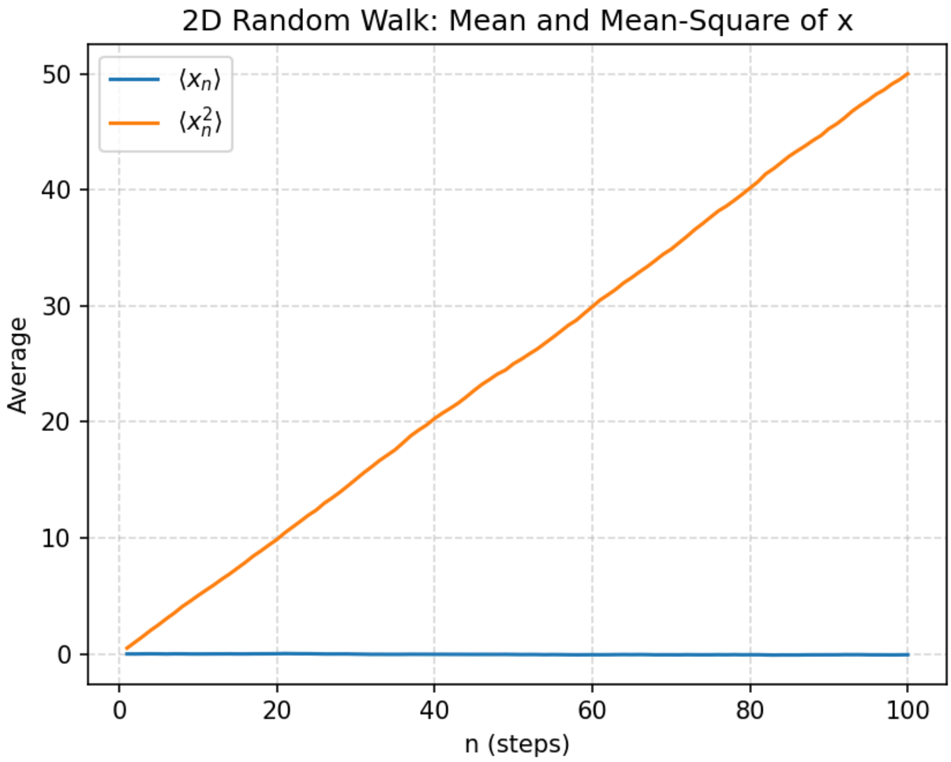

Gaussian distribution (Box-Muller): N=1000000, bins=50
Generated
Theoretical Gaussian
Probability Density
x


Gaussian distribution (Box-Muller): N=1000000, bins=20
Generated
Theoretical Gaussian
Probability Density
x


Gaussian distribution (Box-Muller): N=1000000, bins=10
Generated
Theoretical Gaussian
Probability Density
x

Q4

(1)


2D Random Walk: Mean and Mean-Square of x
$\langle x_n \rangle$
$\langle x_n^2 \rangle$
Average
n (steps)
0
10
20
30
40
50
0
20
40
60
80
100

(2)


2D Random Walk: Diffusive Behavior
$\langle r^2 \rangle$ (data)
Linear fit (D ≈ 0.250)
$\langle r^2 \rangle$
Steps (t)
0
20
40
60
80
100
0
20
40
60
80
100

Q5

(1)

For $p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{x^2}{2\sigma^2}\right)$

$$\langle x^2 \rangle = \int_{-\infty}^{+\infty} x^2 p(x) dx$$

$$= \int_{-\infty}^{+\infty} \frac{x^2}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx$$

Using the Gaussian integral:

$$\int x^2 \exp(-ax^2) dx = \frac{\sqrt{\pi}}{2a^{1.5}}$$

So:

$$\langle x^2 \rangle = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \frac{\sqrt{\pi}}{2\left(\frac{\sigma}{2}\right)^{1.5}}$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \sqrt{\pi} \cdot \sqrt{2}\sigma^3$$

$$= \sigma^2$$

(2)