

## Q1

(1)

Radioactive decay follows the first-order differential equation:

$$\frac{dN}{dt} = -\frac{1}{\tau}N$$

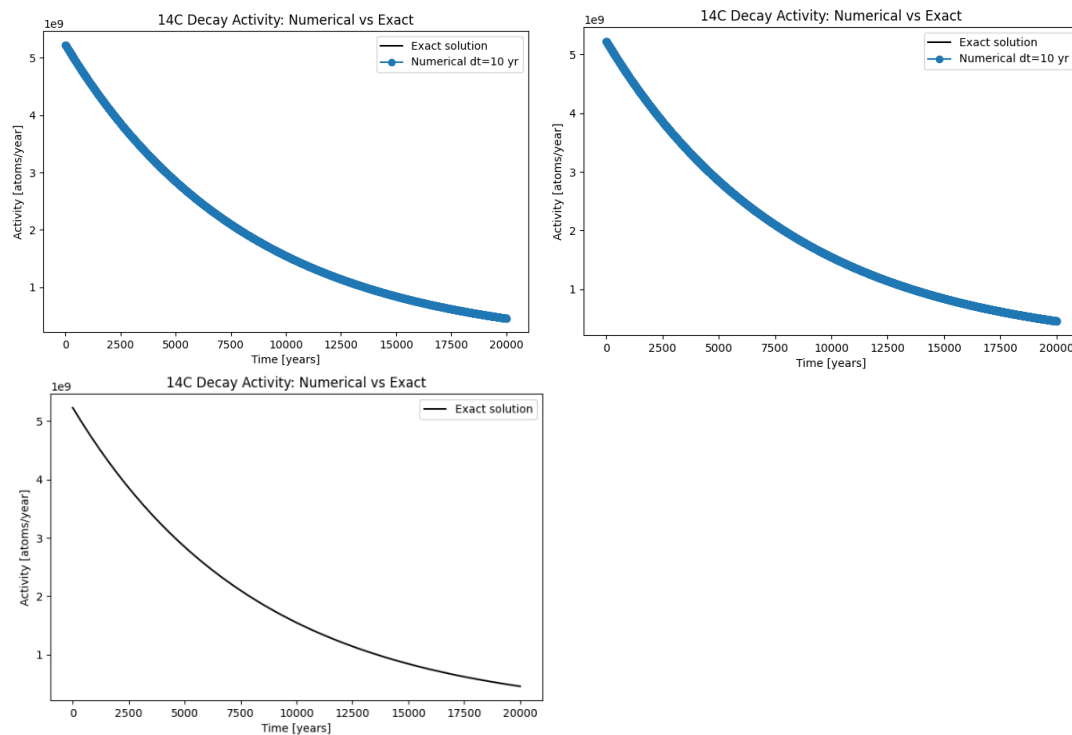
$$\Rightarrow N(t) = N_0 e^{-t/\tau}$$

Plug in:  $N(T_{1/2}) = \frac{N_0}{2}$

$$\frac{N_0}{2} = N_0 e^{-\frac{T_{1/2}}{\tau}}$$

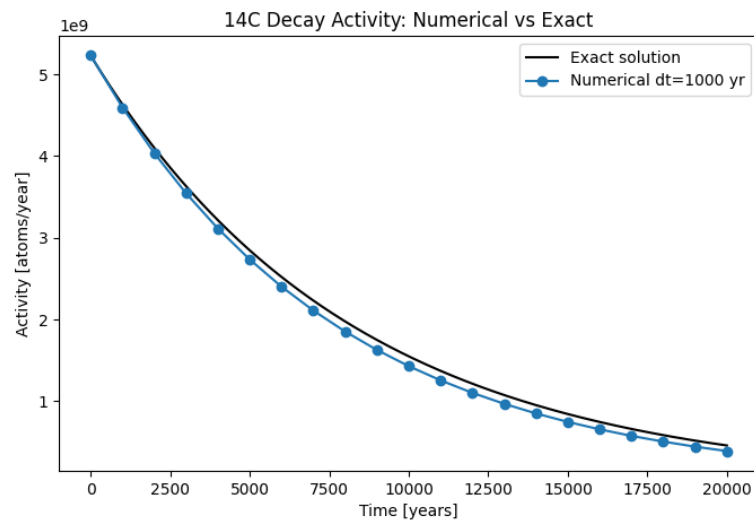
$$\Rightarrow \tau = \frac{T_{1/2}}{\ln 2}$$

(2)



The numerical data with time-step widths of 10 and 100 years are match with the exact solution.

(3)



$$N_{exact} = N_0 \cdot 2^{-2} = \frac{N_0}{4} = 1.075 \times 10^{13}$$

Numerical  $N_{num}$  at 2 half-lives:

$$N_{num} = 9.827 \times 10^{12}$$
$$error = \frac{|N_{num} - N_{exact}|}{N_{exact}} \times 100\% \approx 8.59\%$$

The numerical solution with time-step widths of 1000 years has some errors but they are still acceptable.

Q2

