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To: SRT Group

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Subject: Interferometer geometry calculations

For the “VLBI” mode we start with the latitude, longitude and height of each end of the “baseline” and convert to geocentric right handed  $x, y, z$  coordinates. This coordinate conversion is done by function

$$x = (n + hgt) \cos(lat) \cos(lon)$$

$$y = (n + hgt) \cos(lat) \sin(lon)$$

$$z = (n + (1 - e) + hgt) \sin(lat)$$

where  $n = a / (1 - e \sin^2(lat))^{\frac{1}{2}}$

$$a = 6378137 \text{ m} \quad \text{WGS84}$$

$$e = 2f \cdot f^2$$

$$f = 1/298.257223563 \quad \text{WGS84}$$

The vector baseline is defined as the vector from site1 (the “reference” site) to site2 (the remote site)

$$b_x = x_2 - x_1$$

$$b_y = y_2 - y_1$$

$$b_z = z_2 - z_1$$

The delay  $\tau$  of a signal’s arrival at the remote site is  $\tau = -\vec{b} \cdot \hat{s} / c = -(b_x s_x + b_y s_y + b_z s_z) / c$

Where  $c$  = velocity of propagation

$\hat{s}$  = unit vector in the direction of the source

$$s_x = \cos(dec) \cos(gha)$$

$$s_y = -\cos(dec) \sin(gha)$$

$$s_z = \sin(dec)$$

where  $gha = gst - ra$  = Greenwich hour angle

$gst$  = Greenwich sidereal time

$ra$  = apparent right ascension  
 $dec$  = apparent declination

or from the derivatives of the phase with respect to  $ra$  and  $dec$

$$\phi = (2\pi/\lambda) (\cos(dec)\cos(gha)b_x - \cos(dec)\sin(gha)b_y + \sin(dec)b_z)$$

In units of fringes per arc second

$$u = (b_x \sin(gha) + b_y \cos(gha)) (\pi/648,000\lambda)$$

$$v = (b_z \cos(dec) - b_x \cos(gha)\sin(dec) + b_y \sin(gha)\sin(dec)) (\pi/648,000\lambda)$$

The interferometer phase (normally defined as being positive (NRAO's convention) when the signal arrives earlier at the 2<sup>nd</sup> site is

$$\phi = +2\pi \vec{b} \cdot \hat{s} / \lambda \quad (radians)$$

$$\text{or } \phi = -2\pi \tau f \quad (radians)$$

where  $\lambda$  = wavelength (m)

$f$  = frequency (Hz)

The components of the baseline projected in the direction of the source in the directions of increasing RA and increasing declination are known as  $u$  and  $v$  and are often expressed in units of fringes per arc second. These can be derived from the baseline projections

$$u = b_x \sin(gha) + b_y \cos(gha)$$

$$v = b_z \cos(dec) - b_x \cos(gha)\sin(dec) + b_y \sin(gha)\sin(dec)$$