

# PINGALA ASSIGNMENTS

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### 1 JEE 2019

Let  $\alpha$  and  $\beta$  ( $\alpha > \beta$ ) be the roots of the equation  $z^2 - z - 1 = 0$ . Define,

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \quad n \geq 1 \quad (1.1)$$

$$b_n = a_{n-1} - a_{n+1}, \quad n \geq 2, \quad b_1 = 1 \quad (1.2)$$

Verify the following using a python code.

1.1

$$\sum_{k=1}^n a_k = a_{n+2} - 1, \quad n \geq 1 \quad (1.3)$$

#### Solution:

Download the Python code using

```
$ wget https://github.com/HARI-donk
-EY/sig_pros/tree/main/pingala/codes/1
_1.py
```

and run it using,

```
$ python3 1_1.py
```

From Fig. 1.1, both the graphs are similar for *LHS* and *RHS*.

Hence 1.1 is true.

1.2

$$\sum_{k=1}^{\infty} \frac{a_k}{10^k} = \frac{10}{89} \quad (1.4)$$

1.3

**Solution:** Download the Python code using

```
$ wget https://github.com/HARI-donk
-EY/sig_pros/tree/main/pingala/codes/1
_2.py
```

and run it using,

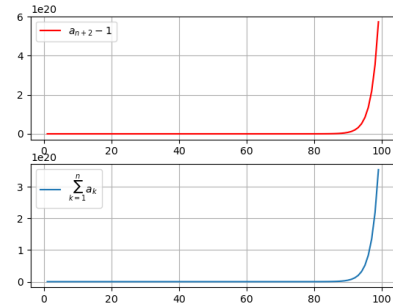


Fig. 1.1

```
$ python3 1_2.py
```

The Fig. 1.2 shows that the difference between *LHS* and *RHS* tends to zero as the value of  $k$  increases.

It shows that for a large value of  $k$ , the

$$LHS \rightarrow RHS$$

Hence 1.2 is true.

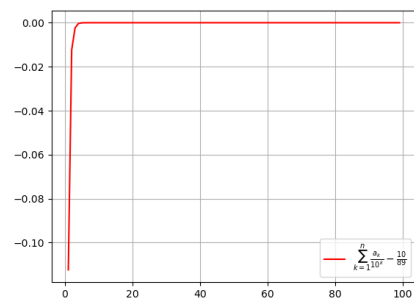


Fig. 1.2

$$b_n = \alpha^n + \beta^n, \quad n \geq 1 \quad (1.5)$$

**Solution:** Download the Python code using

```
$ wget https://github.com/HARI-donk
-EY/sig_pros/tree/main/pingala/codes/1
_3.py
```

and run it using,

```
$ python3 1_3.py
```

From Fig. 1.3, both the graphs are similar for *LHS* and *RHS*.

Hence 1.3 is true.

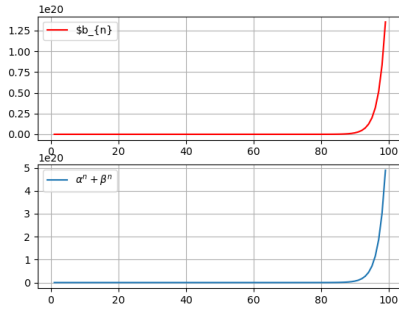


Fig. 1.3

1.4

$$\sum_{k=1}^{\infty} \frac{b_k}{10^k} = \frac{8}{89} \quad (1.6)$$

**Solution:**

Download the Python code using

```
$ wget https://https://github.com/HARI-donk-EY/sig_pros/tree/main/pingala/codes/1_4.py
```

and run it using,

```
$ python3 1_4.py
```

The Fig. 1.4 shows that the difference between *LHS* and *RHS* tends to  $\frac{12}{89}$  as the value of  $k$  increases.

It shows that for a large value of  $k$ , the

$$LHS \rightarrow RHS$$

Hence 1.4 is false.

## 2 PINGALA SERIES

2.1 The *one sided* Z-transform of  $x(n)$  is defined as

$$X^+(z) = \sum_{n=0}^{\infty} x(n)z^{-n}, \quad z \in \mathbb{C} \quad (2.1)$$

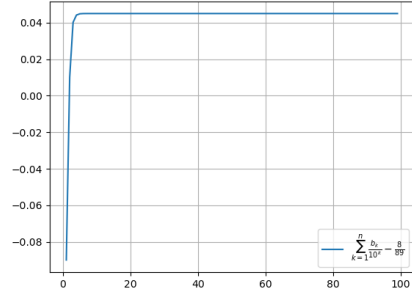


Fig. 1.4

2.2 The *Pingala* series is generated using the difference equation

$$x(n+2) = x(n+1) + x(n) \quad (2.2)$$

$$x(0) = x(1) = 1, \quad n \geq 0 \quad (2.3)$$

Generate a stem plot for  $x(n)$ .

**Solution:**

Obtain the python code to generate the plot using

```
$ wget https://github.com/HARI-donk-EY/sig_pros/tree/main/pingala/codes/2_2.py
```

Run the code using

```
$ python3 2_2.py
```

The following Fig. 2.2 is obtained

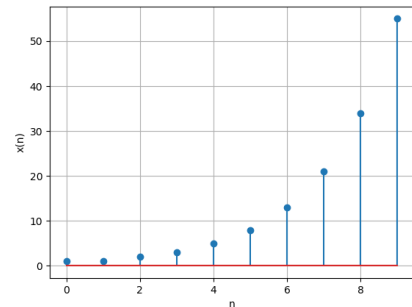


Fig. 2.2

2.3 Find  $X^+(z)$ .

**Solution:**

$$x(n+2) = x(n+1) + x(n) \quad (2.4)$$

Applying positive Z-transform on both sides as we know that Z-transform is a linear operator.

$$\sum_{k=0}^{\infty} x(k+2)z^{-k} = \sum_{k=0}^{\infty} x(k+1) + \sum_{k=0}^{\infty} x(k) \quad (2.5)$$

$$z^2 (X^+(z) - x(0) - x(1)) = X^+(z) + z(X^+(z) - x(0)) \quad (2.6)$$

$$X^+(z) = \frac{z^2}{z^2 - z - 1} \quad (2.7)$$

$$X^+(z) = \frac{1}{1 - z^{-1} - z^{-2}} \quad (2.8)$$

2.4 Find  $x(n)$ .

**Solution:**

$$X^+(z) = \frac{1}{(1 - \alpha z)(1 - \beta z)} \quad (2.9)$$

where  $\alpha, \beta$  are the roots of the equation

$$z^2 - z - 1 = 0 \quad (2.10)$$

Co-efficient of  $z^{-k}$  in the above expression is  $x(k)$ , so by comparing co-efficients.

$$X^+(z) = \frac{1}{\alpha - \beta} \left( \frac{\alpha}{1 - \alpha z^{-1}} - \frac{\beta}{1 - \beta z^{-1}} \right) \quad (2.11)$$

Using binomial theorem, we get

$$x(k) = \frac{\alpha^{k+1} - \beta^{k+1}}{\alpha - \beta} \quad (2.12)$$

2.5 Sketch

$$y(n) = x(n-1) + x(n+1), \quad n \geq 0 \quad (2.13)$$

**Solution:**

Obtain the python code to generate the plot using

```
$ wget https://github.com/HARI-donk-EY/sig_pros/tree/main/pingala/codes/2_5.py
```

Run the code using

```
$ python3 2_5.py
```

The following Fig. 2.5 is obtained

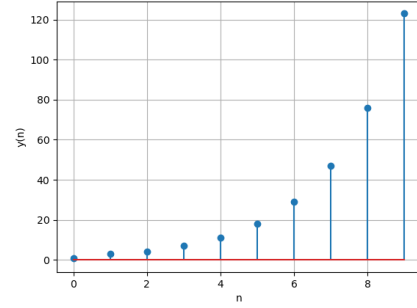


Fig. 2.5

Take +ve Z-transform on both sides of (2.13).

$$\sum_{k=0}^{\infty} y(k)z^{-k} = \sum_{k=0}^{\infty} x(k+1)z^{-k} + \sum_{k=0}^{\infty} x(k)z^{-k} \quad (2.14)$$

$$Y^+(z) = z(X^+(z) - x(0)) + X^+(z) \quad (2.15)$$

$$\therefore x(-1) = 0$$

$$Y^+(z) = \frac{z + z^{-1}}{1 - z^{-1} - z^{-2}} - z \quad (2.16)$$

$$\therefore Y^+(z) = \frac{1 + 2z^{-1}}{1 - z^{-1} - z^{-2}} \quad (2.17)$$

2.7 Find  $y(n)$ .

**Solution:**

Co-efficient of  $z^{-n}$  in  $Y^+(z)$  will be  $y(n)$ .

$$Y^+(z) = \frac{1}{1 - z^{-1} - z^{-2}} + \frac{2z^{-2}}{1 - z^{-1} - z^{-2}} \quad (2.18)$$

$$y(k) = \frac{\alpha^{k+1} - \beta^{k+1}}{\alpha - \beta} + 2 \frac{\alpha^k - \beta^k}{\alpha - \beta} \quad (2.19)$$

$$y(k) = \frac{\alpha^{k+2} + \alpha^k - \beta^k - \beta^{k+2}}{\alpha - \beta} \quad (2.20)$$

$$y(k) = \frac{\alpha^{k+2} - \beta\alpha^{k+1} + \alpha\beta^{k+1} - \beta^{k+2}}{\alpha - \beta} \quad (2.21)$$

$$[\because \alpha\beta = -1]$$

$$\therefore y(k) = \alpha^{k+1} + \beta^{k+1} \quad (2.22)$$

2.6 Find  $Y^+(z)$ .

**Solution:**