

# Digital Signal Processing

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## 1 SOFTWARE INSTALLATION

Run the following commands

```
sudo apt-get update
sudo apt-get install libffi-dev libsndfile1 python3
    -scipy python3-numpy python3-matplotlib
sudo pip install cffi pysoundfile
```

## 2 DIGITAL FILTER

### 2.1 Download the sound file from

```
wget https://github.com/HARI-donk-EY/
    sig_pros/blob/main/codes/2/sound_files/
    Sound_Noise.wav
```

### 2.2 You will find a spectrogram at <https://academo.org/demos/spectrum-analyzer>.

Upload the sound file that you downloaded in Problem 2.1 in the spectrogram and play. Observe the spectrogram. What do you find?

**Solution:** There are a lot of yellow lines between 440 Hz to 5.1 KHz. These represent the synthesizer key tones. Also, the key strokes are audible along with background noise.

### 2.3 Write the python code for removal of out of band noise and execute the code.

**Solution:**

```
import soundfile as sf
from scipy import signal

#read .wav file

input_signal,fs = sf.read('sound_files/
    Sound_Noise.wav')

#sampling frequency of Input signal

sampl_freq=fs

#order of the filter

order=4

#cutoff frequency 4kHz

cutoff_freq=4000.0

#digital frequency

Wn=2*cutoff_freq/sampl_freq

# b and a are numerator and
    denominatorpolynomials respectively

b, a = signal.butter(order,Wn, 'low')

#filter the input signal with butterworth filter

output_signal = signal.filtfilt(b, a,
    input_signal)

#output signal = signal.lfilter(b, a, input
    signal)

#write the output signal into .wav file

sf.write('sound_files/
    Sound_With_ReducedNoise.wav',
    output_signal, fs)
```

## 4 Z-TRANSFORM

2.4 The output of the python script in Problem 2.3 is the audio file Sound\_With\_ReducedNoise.wav. Play the file in the spectrogram in Problem 2.2. What do you observe?

**Solution:** The key strokes as well as background noise is subdued in the audio. Also, the signal is blank for frequencies above 5.1 kHz.

## 3 DIFFERENCE EQUATION

3.1 Let

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \quad (3.1)$$

Sketch  $x(n)$ .

3.2 Let

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2),$$

$$y(n) = 0, n < 0 \quad (3.2)$$

Sketch  $y(n)$ .

**Solution:** The following code yields Fig. 3.2.

```
wget https://github.com/HARI-donk-EY/
sig_pros/blob/main/codes/3/xnyn.py
```

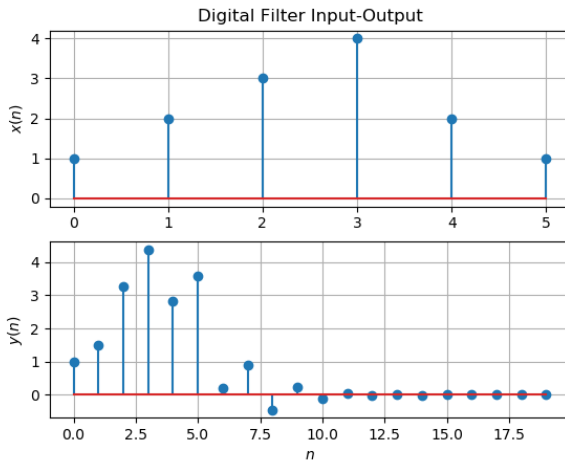


Fig. 3.2

4.1 The Z-transform of  $x(n)$  is defined as

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \quad (4.1)$$

Show that

$$\mathcal{Z}\{x(n-1)\} = z^{-1}X(z) \quad (4.2)$$

and find

$$\mathcal{Z}\{x(n-k)\} \quad (4.3)$$

**Solution:** From (4.1),

$$\mathcal{Z}\{x(n-1)\} = \sum_{n=-\infty}^{\infty} x(n-1)z^{-n} \quad (4.4)$$

$$= \sum_{n=-\infty}^{\infty} x(n)z^{-n-1} = z^{-1} \sum_{n=-\infty}^{\infty} x(n)z^{-n} \quad (4.5)$$

resulting in (4.2). Similarly, it can be shown that

$$\mathcal{Z}\{x(n-k)\} = z^{-k}X(z) \quad (4.6)$$

4.2 Find

$$H(z) = \frac{Y(z)}{X(z)} \quad (4.7)$$

from (3.2) assuming that the Z-transform is a linear operation.

**Solution:** Applying (4.6) in (3.2),

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \quad (4.8)$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} \quad (4.9)$$

4.3 Find the Z transform of

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} \quad (4.10)$$

and show that the Z-transform of

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (4.11)$$

is

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \quad (4.12)$$

**Solution:** It is easy to show that

$$\delta(n) \stackrel{Z}{=} 1 \quad (4.13)$$

and from (4.11),

$$U(z) = \sum_{n=0}^{\infty} z^{-n} \quad (4.14)$$

$$= \frac{1}{1 - z^{-1}}, \quad |z| > 1 \quad (4.15)$$

using the formula for the sum of an infinite geometric progression.

4.4 Show that

$$a^n u(n) \stackrel{Z}{=} \frac{1}{1 - az^{-1}} \quad |z| > |a| \quad (4.16)$$

**Solution:** From (4.11), we get,

$$a^n u(n) = \begin{cases} a^n & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (4.17)$$

from above,

$$\begin{aligned} U_a(z) &= \sum_{n=0}^{\infty} a^n z^{-n} \quad (4.18) \\ &= \sum_{n=0}^{\infty} (az^{-1})^n = \frac{1}{1 - az^{-1}}, \quad |z| > 1 \end{aligned} \quad (4.19)$$

using the formula for the sum of an infinite geometric progression.

4.5 Let

$$H(e^{j\omega}) = H(z = e^{j\omega}). \quad (4.20)$$

Plot  $|H(e^{j\omega})|$ . Comment.  $H(e^{j\omega})$  is known as the *Discret Time Fourier Transform* (DTFT) of  $x(n)$ .

**Solution:** The following code plots Fig. 4.5.

```
wget https://github.com/HARI-donk-EY/
sig_pros/blob/main/codes/4/dtft.py
```

We can note that the graph in Fig. 4.5 is an even periodic function.

## 5 IMPULSE RESPONSE

5.1 Find an expression for  $h(n)$  using  $H(z)$ , given that

$$h(n) \stackrel{Z}{=} H(z) \quad (5.1)$$

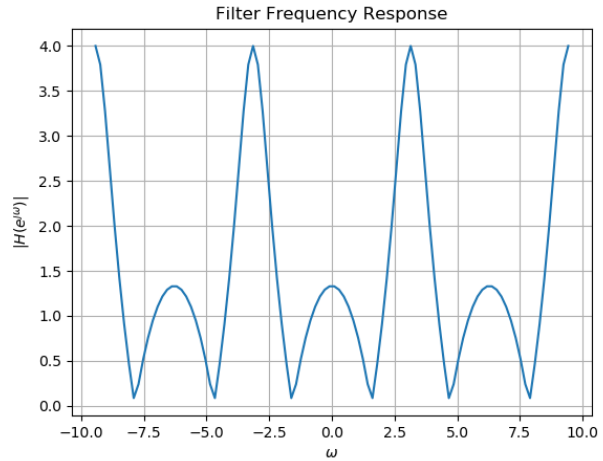


Fig. 4.5:  $|H(e^{j\omega})|$

and there is a one to one relationship between  $h(n)$  and  $H(z)$ .  $h(n)$  is known as the *impulse response* of the system defined by (3.2).

Take  $H(z)$  from (4.7)

**Solution:** From (4.9),

$$H(z) = \frac{1}{1 + \frac{1}{2}z^{-1}} + \frac{z^{-2}}{1 + \frac{1}{2}z^{-1}} \quad (5.2)$$

$$\Rightarrow h(n) = \left(-\frac{1}{2}\right)^n u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2) \quad (5.3)$$

using (4.16) and (4.6).

5.2 Sketch  $h(n)$ . Is it bounded? Convergent?

**Solution:** The following code plots Fig. 5.2.

```
wget https://github.com/HARI-donk-EY/
sig_pros/blob/main/codes/5/hn.py
```

5.3 Compute and sketch  $h(n)$  using

$$h(n) + \frac{1}{2}h(n-1) = \delta(n) + \delta(n-2), \quad (5.4)$$

This is the definition of  $h(n)$ .

**Solution:** The following code plots Fig. 5.3. Note that this is the same as Fig. 5.2.

```
wget https://raw.githubusercontent.com/
gadepall/EE1310/master/filter/codes/hndef
.py
```

5.4 Compute

$$y(n) = x(n) * h(n) = \sum_{n=-\infty}^{\infty} x(k)h(n-k) \quad (5.5)$$

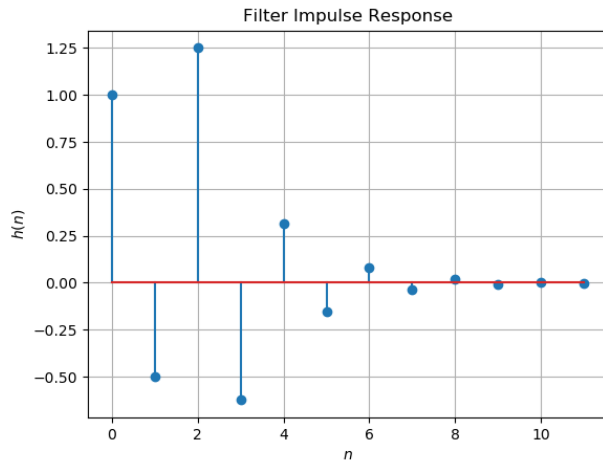


Fig. 5.2:  $h(n)$  as the inverse of  $H(z)$

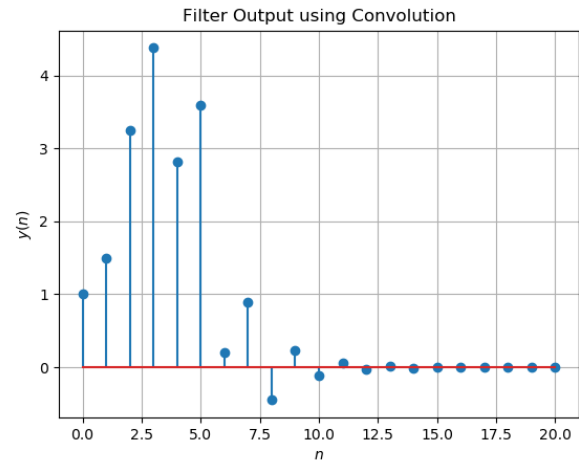


Fig. 5.4:  $y(n)$  from the definition of convolution

to get

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k) \quad (5.7)$$

$$= \sum_{n-k=-\infty}^{\infty} x(n-k) h(k) \quad (5.8)$$

$$= \sum_{k=-\infty}^{\infty} x(n-k) h(k) \quad (5.9)$$

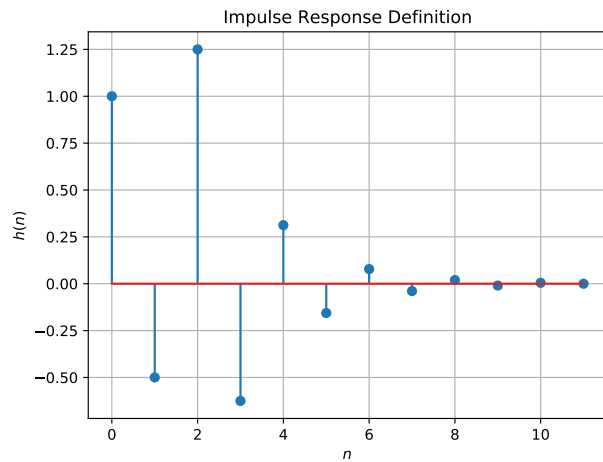


Fig. 5.3:  $h(n)$  from the definition

Comment. The operation in (5.5) is known as *convolution*.

**Solution:** The following code plots Fig. 5.4. Note that this is the same as  $y(n)$  in Fig. ??.

```
wget https://raw.githubusercontent.com/
gadepall/EE1310/master/filter/codes/
ynconv.py
```

5.5 Show that

$$y(n) = \sum_{n=-\infty}^{\infty} x(n-k) h(k) \quad (5.6)$$

**Solution:** From (5.5), we substitute  $k := n - k$