Vishnu Institute of Technology

Department of EEE

Electrical Machines I

(Class Material)

UNIT 1: Introduction to DC Machines

Principles of electromechanical energy conversion – singly excited and multi excited system–Calculation of force and torque using the concept of co-energy.
Construction and principle of operation of DC generator – EMF equation for DC generator – Classification of DC machines based on excitation – OCC, Internal and External characteristics of DC Shunt generator.

1. Construction of DC Generator

A DC generator converts mechanical energy to electrical energy. The working of the DC generator is based on Faraday's Law of Electromagnetic Induction. As the name suggests, the output obtained is DC (Direct current), where the magnitude of current or voltage is constant with time. The construction of DC generator is same as that of the DC motor. It means, a DC machine can be used both as a generator and as a motor.

When the machine is driven by a prime mover, it converts mechanical energy into electrical energy and it is called DC generator. If an electrical supply is given as the input for DC machine, it gets converted into mechanical output and it is called as a DC motor. The DC machine consists of two parts: One part is rotating, called armature (rotor) and the other part is stationary, called field (stator).

The major components of a DC machine are

Magnetic frame or yoke
Pole core and pole shoe
Field coil or winding
Armature core and winding
Commutator
Brushes
Bearings and shaft

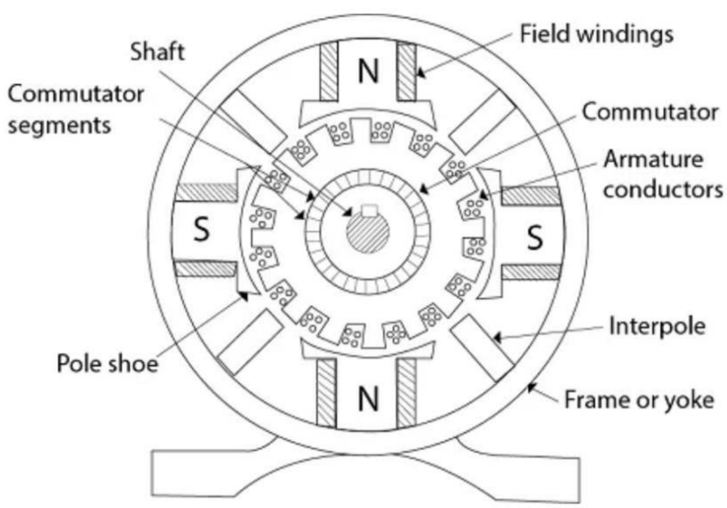


Fig. Construction of DC Generator

Magnetic Frame or Yoke

It is the stationary part of the machine in the shape of hollow cylinder. Poles are fixed at the inner periphery of the yoke. Yoke is usually made of cast iron for small machine, because of its cheapness. But for large machines, it is made of cast steel or rolled steel. It acts as the outer cover or frame for the entire machine and serves two main purposes

1. It is used to carry the magnetic flux produced by the poles.
2. It acts as mechanical support for the machine.

Pole Core and Pole Shoe

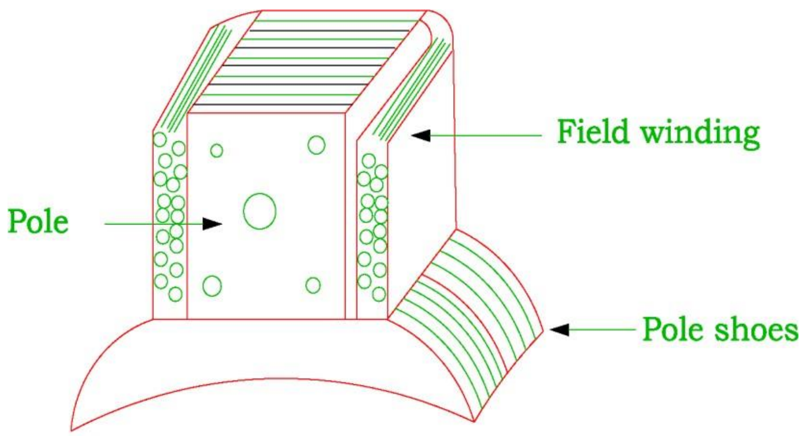


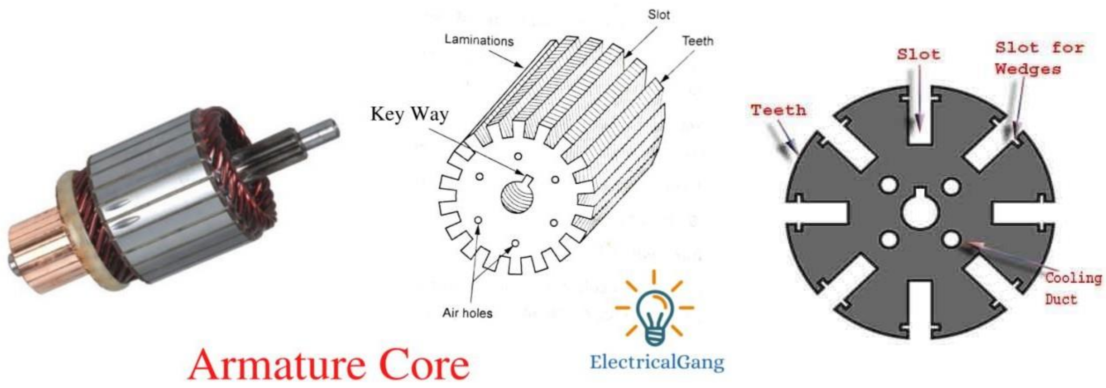
FIG C : POLE AND FIELD WINDING

The field pole consists of pole cores, pole shoes and field winding. The poles are made of thin laminated sheets, to avoid heating and eddy current loss. Pole cores are the projecting rectangular parts, which produce magnetic flux needed for the generator, when it is excited by the field winding. It is fitted to the yoke or frame by means of bolts and nuts or rivets. The pole shoes are located at the end of pole core. The purpose of providing pole shoe in the poles is to make the magnetic field uniform on the surface of the armature.

Field Coil or Winding

Field coil is made up of copper. They are mounted on the pole core and carry the DC current. The field coils are connected in such a way that adjacent poles have opposite polarity. When the coils carry DC current, the pole core become an electromagnet and produces the magnetic flux. The magnetic flux passes through the pole core, the air gap, the armature and the yoke. The number of poles in a DC Generator depends on the speed of the machine and the output for which the machine is designed.

Armature Core and Winding



In the construction of DC generator, armature core is designed as the rotating part and is built in cylindrical or drum shape with slots on its outer periphery. The purpose of armature is to house the winding and to rotate the conductors in the uniform magnetic field. It is mounted on the shaft. It is build up of steel lamination which is insulated by each other by thin paper or thin coating of varnish as insulation. The thickness of each lamination is about 0.5 mm. This lamination will reduce the eddy current loss. If silicon sheet is used for armature core, the hysteresis loss will also reduce. The armature winding or coil is placed on slots available on the armature's outer periphery. The ends of the coils are joined with commutator segments. Insulated higher conductivity copper wire is used for making the coils. There are two types of winding.

Lap Winding – Lap winding is used for high current, low voltage generators.

Wave Winding – Wave winding is used for high voltage, low current generators.

Commutator

The commutator provides the electrical connection between the rotating armature coil and the stationary external circuit. It is essentially a cylindrical structure and is built up of wedge shaped copper segments insulated from each other by mica sheets and mounted on the shaft of the machine. The commutator is a mechanical rectifier which converts the alternating EMF generator in the armature winding into direct voltage across the brushes. The ends of the armature coil or winding are connected to commutator segments.

Brushes

The function of brush is to collect the current from the commutator and supply it to the external load circuit. The brushes are manufactured in a variety of compositions to suit the commutation requirements.

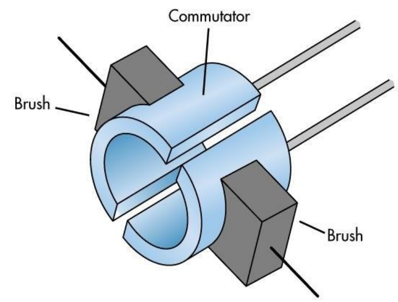


Fig. Commutator and brush assembly

It is made of carbon, graphite metal graphite or copper and is rectangular in shape. The brushes are placed in the brush holders which are mounted on rocker arm. The brushes are arranged in rocker arm in such a way that, it touches the commutator.

Bearings and Shaft

For construction of smaller DC generator, ball bearings are used at both the ends of the shaft but for larger machines, roller bearings are used at the driving end and ball bearings are used at the non driving end of the machine. The shaft is made up of mild steel having maximum breaking strength. It is used to transfer the mechanical power from or to the machine. All the rotating parts including the armature core, commutator, cooling parts are mounted and keyed to the shaft.

2. Principle Operation of DC Generator

DC generators produce electrical power based on the principle of Faraday’s law of electromagnetic induction. According to Faradays Law

1. When a conductor moves in a magnetic field it cuts magnetic lines of force, which induces an electromagnetic force (EMF) in the conductor.
2. The magnitude of this induced EMF depends upon the rate of change of flux (magnetic line force) linkage with the conductor. This EMF will cause a current to flow if the conductor circuit is closed.

Hence the most basic two essential parts of a generator are magnetic field and conductors which move inside that magnetic field.

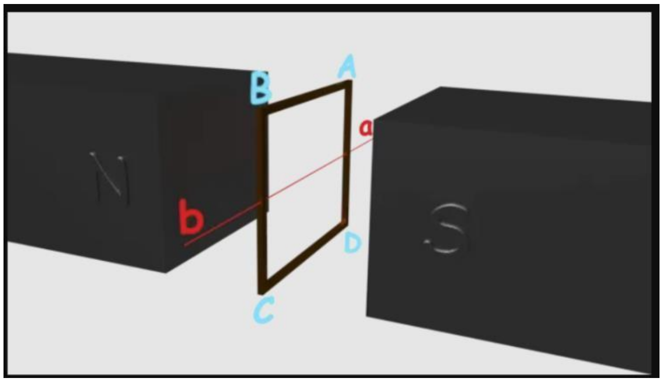
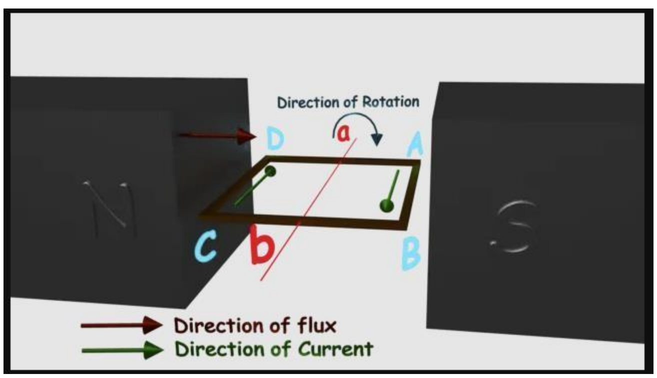
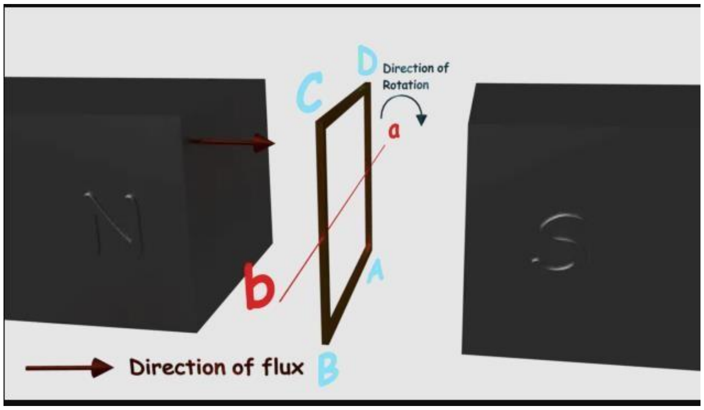


Fig. Single loop DC generator

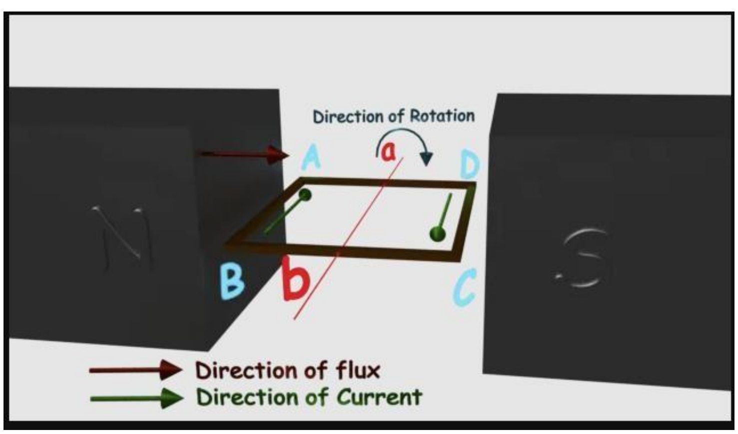
In the figure above, a single loop of conductor of rectangular shape is placed between two opposite poles of magnet. Let’s us consider, the rectangular loop of the conductor is ABCD which rotates inside the magnetic field about its axis **ab**. When the loop rotates from its vertical position to its horizontal position, it cuts the flux lines of the field. As during this movement two sides, i.e., AB and CD of the loop cut the flux lines there will be an EMF induced in these both of the sides (AB and BC) of the loop.



As the loop gets closed there will be a current circulating through the loop. The direction of the current can be determined by Fleming's right hand Rule. This rule says that if you stretch thumb, index finger and middle finger of your right-hand perpendicular to each other, then thumbs indicates the direction of motion of the conductor, index finger indicates the direction of magnetic field, i.e., N – pole to S – pole, and middle finger indicates the direction of flow of current through the conductor. Now if we apply this right-hand rule, we will see at this horizontal position of the loop, current will flow from point A to B and on the other side of the loop current will flow from point C to D.



Now if we allow the loop to move further, it will come again to its vertical position, but now the upper side of the loop will be CD, and lower side will be AB (just opposite of the previous vertical position). At this position, the tangential motion of the sides of the loop is parallel to the flux lines of the field. Hence there will be no question of flux cutting, and consequently, there will be no current in the loop. If the loop rotates further, it comes to again in a horizontal position. But now, said AB side of the loop comes in front of N pole, and CD comes in front of S pole, i.e., just opposite to the previous horizontal position as shown in the figure beside.



Here the tangential motion of the side of the loop is perpendicular to the flux lines; hence rate of flux cutting is maximum here, and according to Flemming's right-hand Rule, at this position current flows from B to A and on another side from D to C. Now if the loop is continued to rotate about its axis. Every time the side AB comes in front of S pole, the current flows from A to B. Again, when it comes in front of N pole, the current flows from B to A. Similarly, every time the side CD comes in front of S pole the current flows from C to D., when the side CD comes in front of N pole the current flows from D to C.

Now the loop is opened and connected it with a split ring as shown in the figure below. Split rings, made of a conducting cylinder, gets cut into two halves or segments insulated from each other. We connect the external load terminals with two carbon brushes which rest on these split slip ring segments.

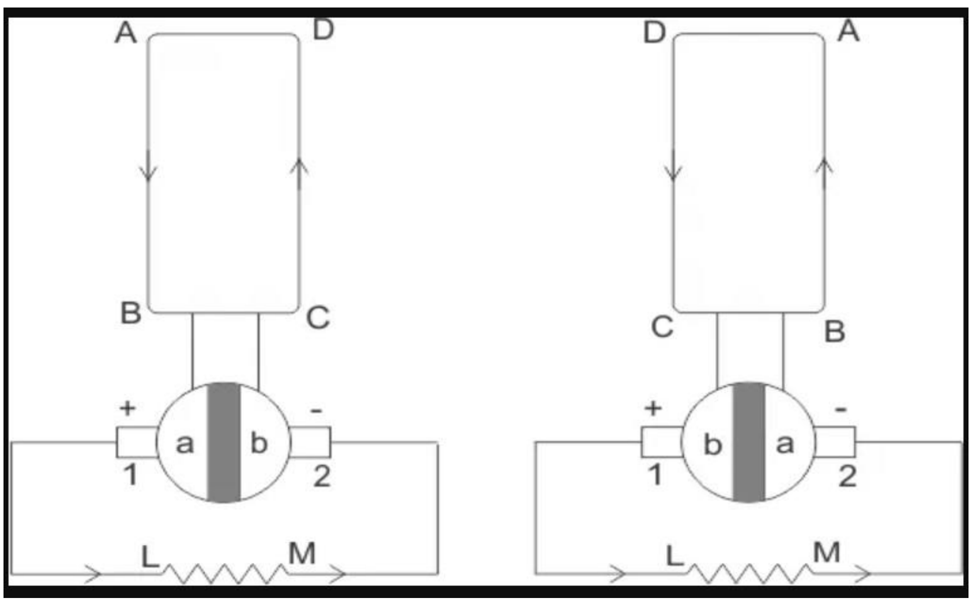


Fig. Principle operation of DC generator (simplified electric circuit)

We can see that in the first half of the revolution current always flows along ABLMCD, i.e., brush no 1 in contact with segment a. In the next half revolution, in the figure, the direction of the induced current in the coil is reversed. But at the same time the position of the segments a and b are also reversed which results that brush no 1 comes in touch with the segment b. Hence, the current in the load resistance again flows from L to M. The waveform of the current through the load circuit is as shown in the figure. This current is unidirectional.

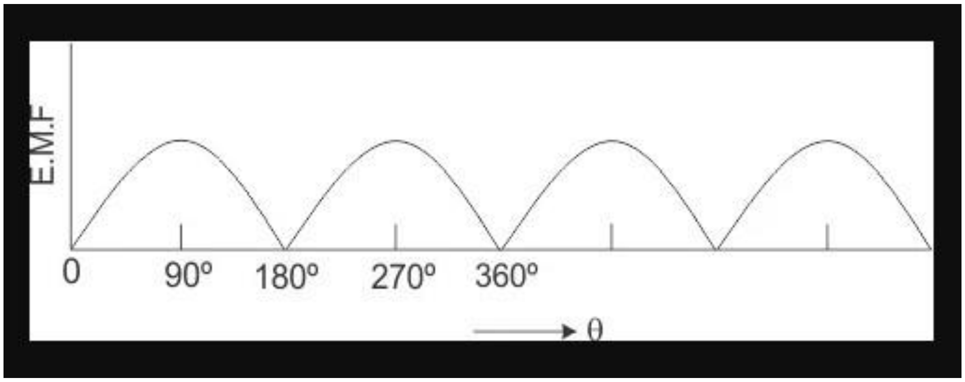
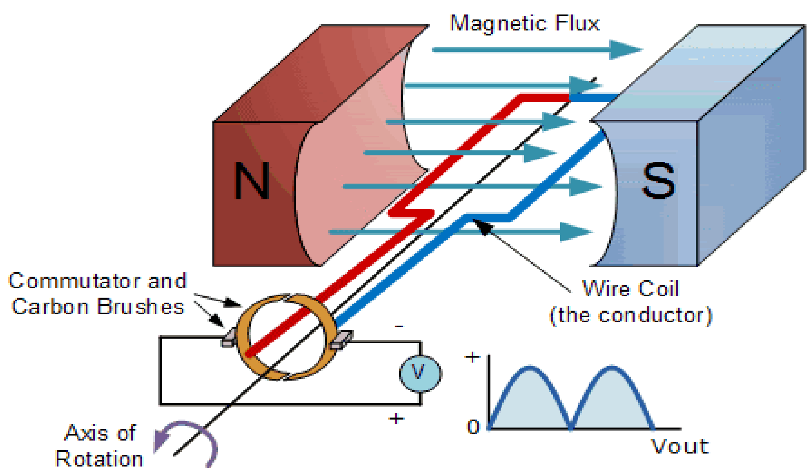
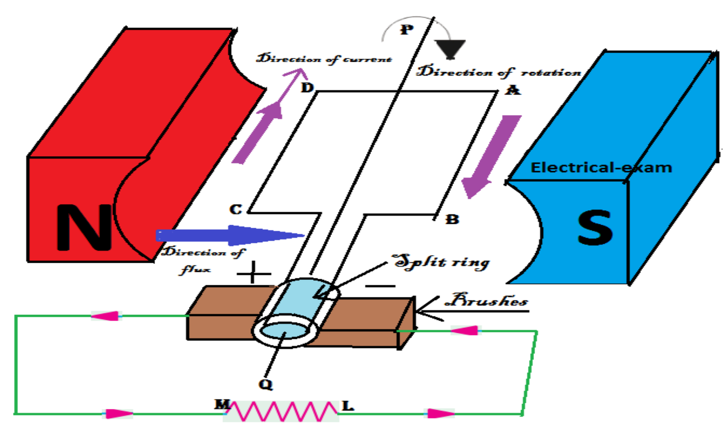


Fig. Final output voltage of DC generator

The above content is the basic working principle of DC generator, explained by single loop generator model. The positions of the brushes of DC generator are so that the change over of the segments a and b from one brush to other takes place when the plane of rotating coil is at a right angle to the plane of the lines of force. It is to become in that position, the induced EMF in the coil is zero.





3. EMF Equation of DC Generator

According to the working principle of dc generator, when conductors begin to cut the magnetic lines of force and therefore, the EMF induces in the conductors according to 'Faraday's Law of Electromagnetic Induction'. The value of induced EMF depends upon the lengths of the conductor, the magnetic field strength, and the speed at which the coil rotates. Let us see the equation for induced EMF

Let,

φ = Flux per pole in Weber.

Z = Total number of armature conductors.

N = Armature rotation in revolution per minute (RPM)

P = Number of poles.

A = Number of parallel paths in armature

E = EMF induced in any parallel path or generated EMF

According to Faraday's law of Electromagnetic induction, Average EMF generated per conductors,

$$= \frac{d\phi}{dt} = \frac{flux\ cut}{time\ taken}\ volt$$

Flux cut per Conductors in one revolution,

$$\mathbf{d\varphi = P\varphi \quad weber}$$

Number of revolutions per minute,

$$\mathbf{= N / 60}$$

Time taken for one revolution,

$$dt = 60 / N \text{ sec}$$

EMF generated per conductor is given by

$$= \frac{d\phi}{dt} = \frac{\phi P}{60 / N} \times \frac{\phi PN}{60} \; volt$$

Therefore, the total EMF (E) generated between the terminals if given as,

E = Average EMF generated per conductor * Number of conductor in each parallel path

$$= \frac{\phi PN}{60} \times \frac{Z}{A} \; volt$$

$$E = \frac{\phi PN}{60} \times \frac{Z}{A} \; volt$$

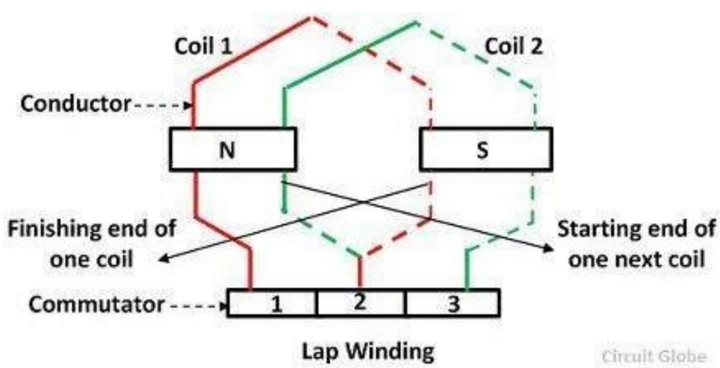
Where, A = P for lap winding & A = 2 for wave winding

Armature Windings

The armature winding is the most important part of the rotating machine. It is the place where energy conversion takes place, i.e., the mechanical energy is converted into electrical energy, and the electrical energy is converted into mechanical energy. The armature winding is mainly classified into types, i.e., **the lap winding and the wave winding.**

Lap Winding

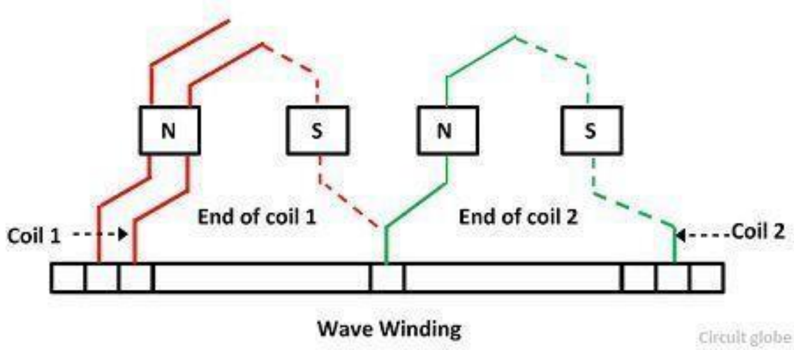
In lap winding, the conductors are joined in such a way that their parallel paths and poles are equal in number. The end of each armature coil is connected to the adjacent segment on the commutator. The number of brushes in the lap winding is equal to the number of parallel paths, and these brushes are equally divided into negative and positive polarity.



Wave Winding

In wave winding, only two parallel paths are provided between the positive and negative brushes.

The finishing end of the one armature coil is connected to the starting end of the other armature coil commutator segment at some distance apart. In this winding, the conductors are connected to two parallel paths irrespective of the number of poles of the machine. The number of brushes is equal to the number of parallel paths. The wave winding is mainly used in high voltage, low current machines.



Difference between lap and wave winding

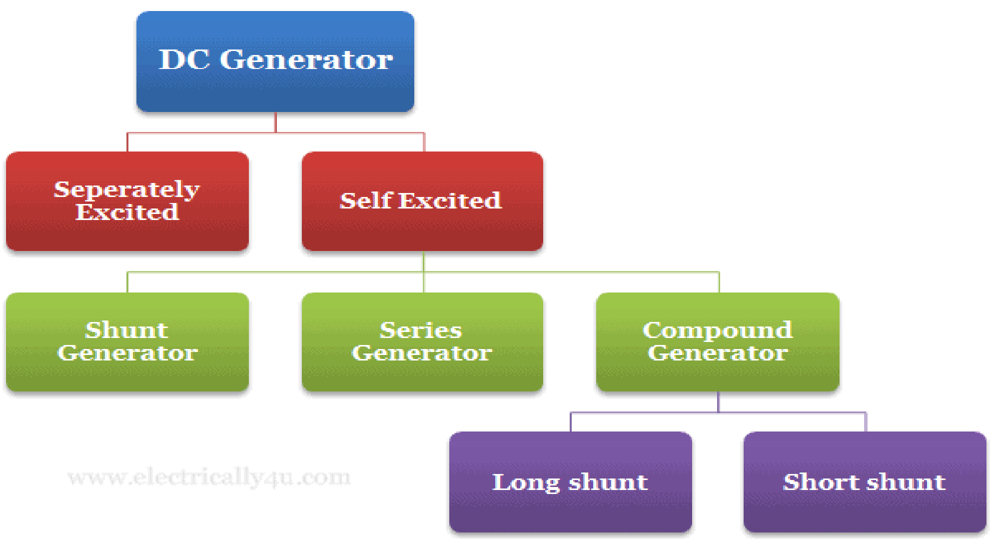
Lap Winding	Wave Winding
The lap winding can be defined as a coil which can be lap back toward the succeeding coil.	The wave winding can be defined as the loop of the winding can form the signal shape.
The numbers of the parallel path are equal to the total of number poles.	The number of parallel paths is equal to two.
Another name of lap winding is multiple winding otherwise Parallel Winding	Another name of wave winding is Series Winding otherwise Two-circuit
The EMF of lap winding is Less	The EMF of wave winding is More
The no. of brushes in lap winding is Equivalent to the no. of parallel paths.	The no. of brushes in wave winding is Equivalent to Two
The efficiency of the lap winding is Less	The efficiency of the wave winding is High
The winding cost of the lap winding is High	The winding cost of the wave winding is Low

The lap winding used for high current, low voltage machines.

The applications of wave winding include low current and high voltage machines.

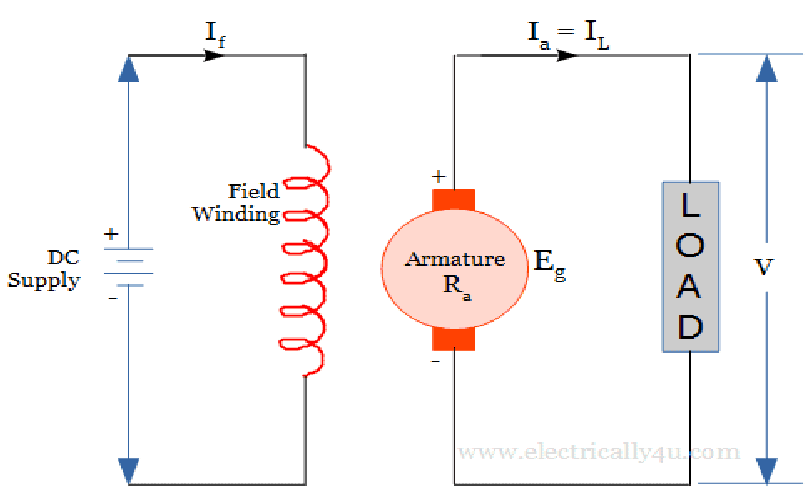
4. Classification of DC machines based on excitation :

In the case of a machine with field coils, a current must flow in the coils to generate the field, otherwise no power is transferred to or from the rotor. The process of generating a magnetic field by means of an electric current is called excitation. DC generators are classified based on the way in which the field windings are excited. The different types of DC generator are shown below.



Separately excited DC Generator

It is a type of DC generator, in which the field windings are excited from a separate source of supply. The following figure shows the circuit diagram of a separately excited DC generator.



I_f – Field current, **I_a** – Armature current, **I_L** – Load current,

R_a – Armature winding resistance, **V** – terminal voltage

Let **V_{br}** be the voltage drop at the brush contacts.

Armature current is given by, **$I_a = I_L$**

Applying Kirchhoff's Voltage Law to the armature circuit,

$$E_g - I_a R_a - V - V_{br} = 0$$

Thus, the generated EMF equation **$E_g = I_a R_a + V + V_{br}$**

Power developed in the DC generator = **$E_g I_a$**

Power delivered to the load = **$V I_a$**

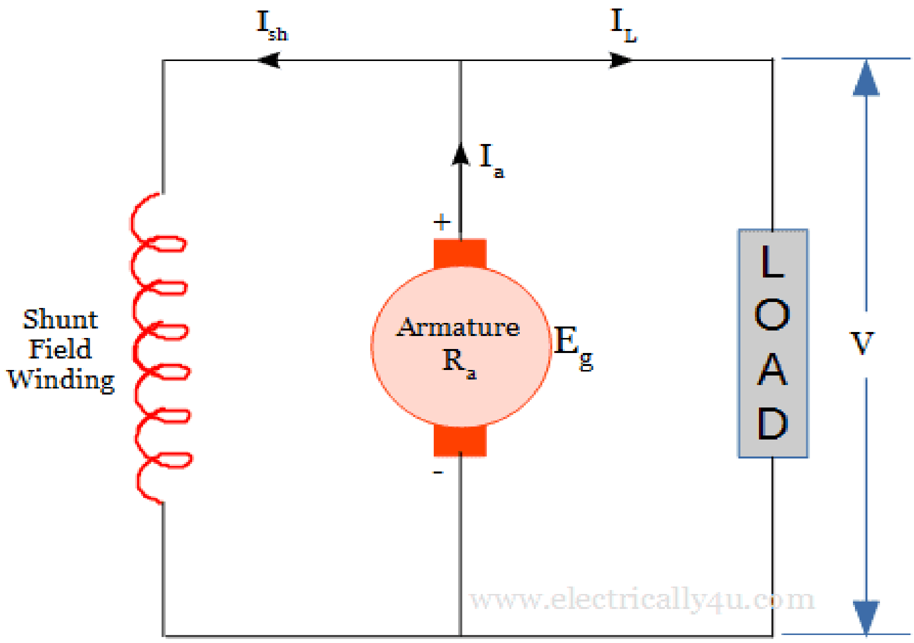
Self Excited DC Generator

The self-excited generator produces DC output, whose field windings are excited by the current produced by the generator itself. No separate source is used for filed excitation. In this type of generators, some flux is already present in the poles due to residual magnetism. When the armature is rotated with the residual flux, a small EMF and hence some current is induced in the armature conductors. This current will produce more flux, which in turn produces more current to flow through the field winding. It will continue until the field current reaches its rated value. Following are the types of self excited generators.

1. Shunt generator
2. Series generator
3. Compound generator

DC Shunt Generator

In DC shunt type generator, the field windings are connected across or in parallel with the armature conductors. The field winding has more number of turns and thin wire, having high resistance. The load is connected across the armature as shown in the diagram below. A small amount of current will flow through the field winding and more current flows through the armature.



$\mathbf{I_{sh}}$ – Shunt field current, $\mathbf{I_a}$ – Armature current, $\mathbf{I_L}$ – Load current,

$\mathbf{R_a}$ – Armature winding resistance, $\mathbf{V}$ – terminal voltage, $\mathbf{V_{br}}$ – Brush contact drop

Armature current is given by, $\mathbf{I_a = I_L + I_{sh}}$

Shunt field current $\mathbf{I_{sh} = V/R_{sh}}$, Where $\mathbf{R_{sh}}$ – shunt field resistance

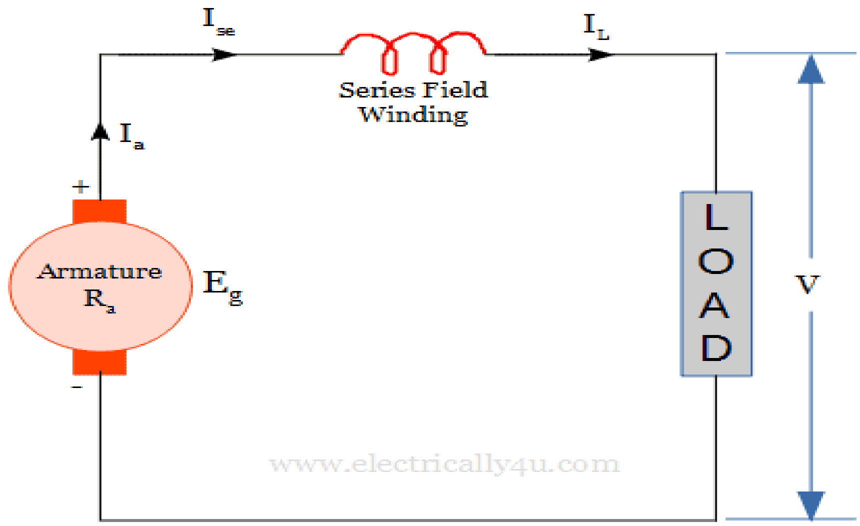
Terminal voltage equation is given by, $\mathbf{V = E_g - I_a R_a - V_{br}}$

Power developed in the DC generator = $\mathbf{E_g\ I_a}$

Power delivered to the load = $\mathbf{V\ I_L}$

DC Series Generator

As the name says, the field winding is connected in series with the armature conductors. Such generators are called a DC series Generator. They have less number of turns with a thick wire having low resistance. Here, the load is connected in series with the field winding and armature conductors. So all the current flows through field winding and load.



$\mathbf{I_{se}}$ – Shunt field current, $\mathbf{I_a}$ – Armature current, $\mathbf{I_L}$ – Load current,

$\mathbf{R_a}$ – Armature winding resistance, **V** – terminal voltage, $\mathbf{V_{br}}$ – Brush contact drop

Armature current is given by, $\mathbf{I_a = I_{se} = I_L}$

Terminal voltage equation is given by, $\mathbf{V = E_g - I_a R_a - I_a R_{se} - V_{br}}$

Power developed in the DC generator = $\mathbf{E_g I_a}$

Power delivered to the load = $\mathbf{V I_L}$

DC Compound Generator

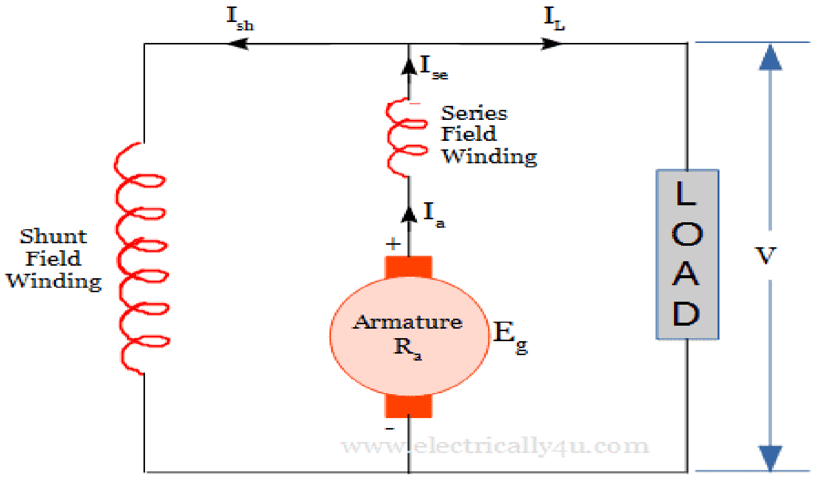
DC compound generator has both shunt field winding and series field winding. One field winding is connected in series with the armature and another field winding is connected in parallel with the armature.

DC Compound generator can be classified into two different types based on the way of connection

a) **Long shunt DC Compound generator**
b) **Short shunt DC Compound generator**

Long shunt DC Compound generator

The below figure shows the circuit diagram of long shunt DC compound generator. In this, shunt field winding is connected in parallel with a combination of series field winding and armature conductors.



I_{sh} – Shunt field current, **I_{se}** – Shunt field current, **I_a** – Armature current, **I_L** – Load current,

R_a – Armature winding resistance, **V** – terminal voltage, **V_{br}** – Brush contact drop

Armature current is given by, $\mathbf{I_a = I_{se} = I_L + I_{sh}}$

Shunt field current $\mathbf{I_{sh} = V/R_{sh}}$, Where **$R_{sh}$** – shunt field resistance

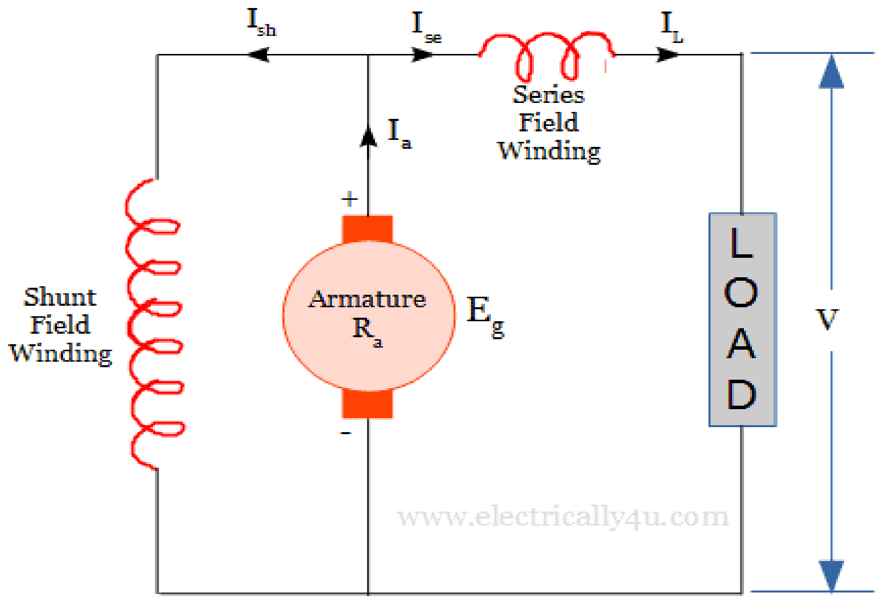
Terminal voltage equation is given by, $\mathbf{V = E_g - I_a R_a - I_a R_{se} - V_{br}}$

Power developed in the DC generator = $\mathbf{E_g I_a}$

Power delivered to the load = $\mathbf{V I_L}$

Short shunt DC Compound generator

In short shunt DC compound generator, the shunt field winding is connected across the armature conductors and this combination is connected in series with a series field winding. The following figure shows the circuit diagram of short shunt DC compound generator.



$\mathbf{I_{sh}}$ – Shunt field current, $\mathbf{I_{se}}$ – Shunt field current, $\mathbf{I_a}$ – Armature current, $\mathbf{I_L}$ – Load current, $\mathbf{R_a}$ – Armature winding resistance, $\mathbf{V}$ – terminal voltage, $\mathbf{V_{br}}$ **–** Brush contact drop

Armature current is given by, $\mathbf{I_a = I_L + I_{sh}}$ where $\mathbf{I_L = I_{se}}$

Terminal voltage equation is given by, $\mathbf{V = E_g - I_a R_a - I_{se} R_{se} - V_{br}}$

Generated EMF equation, $\mathbf{E_g = V + I_a R_a + I_{se} R_{se} + V_{br}}$

Voltage drop across shunt field winding = $\mathbf{I_{sh} R_{sh}}$

Shunt field current $\mathbf{I_{sh} = (E_g - I_a R_a - V_{br})/R_{sh}}$, Where $\mathbf{R_{sh}}$ – shunt field resistance

By substituting the value of E_g in the above equation, we get shunt field current $\mathbf{I_{sh} = (V + I_{se} R_{se})/R_{sh}}$

Power developed in the DC generator = $\mathbf{E_g I_a}$

Power delivered to the load = $\mathbf{V I_L}$

Differentially and cumulatively compound generator

The compound generator is constructed in such a way that, the shunt field winding is stronger than the series field winding. Based on this, the DC Compound generator can further be classified into two types:

a) **Differential compound generator**

b) **Cumulatively compound Generator**

If the flux produced by the series field winding opposes the shunt field flux, thereby weakening the total flux then it is called a differential compound generator.

If the series field flux aids the shunt field flux, thereby strengthening the field flux, then it is called a cumulatively compound Generator.

4. Characteristics of DC generator

When the load is connected to s DC generator, armature current starts flowing and supplies the necessary load current. While doing so, generator EMF has to overcome various voltage drops such as armature resistance drop, brush drop, drop due to armature reaction etc. As the load changes, the load current changes and due to this armature current also changes and hence all the drops also vary. Due to this, the terminal voltage changes and the variation in the terminal voltage or induced EMF due to this variation in the load current are called as load characteristics of a DC generator.

Characteristics of DC Shunt Generators

The DC generators have the following characteristics

a. Magnetization characteristics or Open circuit characteristics

b. Load characteristics (Internal and External characteristics)

a. Magnetization characteristics or Open circuit characteristics

Open circuit characteristic is also known as **magnetic characteristic** or **no-load saturation characteristic**. This characteristic shows the relation between generated EMF at no load **(E_0)** and the field current **(I_f)** at a given fixed speed. Field current is gradually increased and the corresponding terminal voltage is recorded.

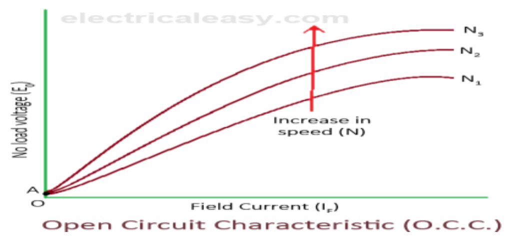


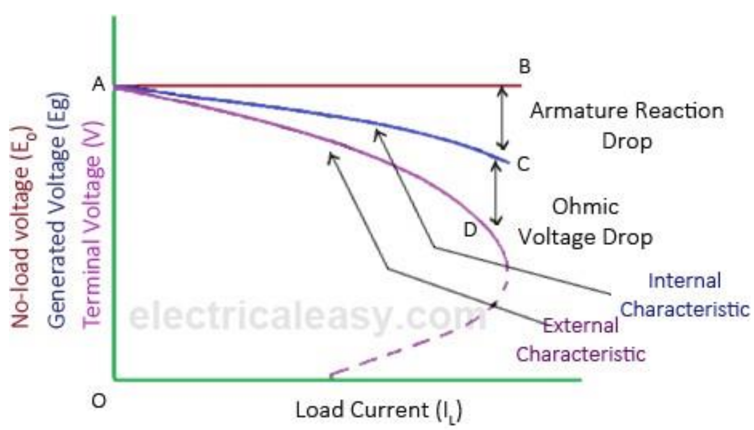
Fig. Open circuit characteristics curves

The O.C.C. curve is just the magnetization curve and it is practically similar for all type of generators.

b. Internal and External characteristics of DC Shunt Generator

To determine the internal and external load characteristics of a DC shunt generator the machine is allowed to build up its voltage before applying any external load. To build up voltage of a shunt generator, the generator is driven at the rated speed by a prime mover. Initial voltage is induced due to residual magnetism in the field poles. The generator builds up its voltage as explained by the O.C.C. curve. When the generator has built up the voltage, it is gradually loaded with resistive load and readings are taken at suitable intervals.

Unlike, separately excited DC generator, here, $\mathbf{I_L \neq I_a}$. For a shunt generator, $\mathbf{I_a = I_L + I_f}$.



Characteristics of DC shunt generator

During a normal running condition, when load resistance is decreased, the load current increases. But, as we go on decreasing the load resistance, terminal voltage also falls. So, load resistance can be decreased up to a certain limit, after which the terminal voltage drastically decreases due to excessive armature reaction at very high armature current and increased $\mathbf{I^2R}$ losses. Hence, beyond this limit any further decrease in load resistance results in decreasing load current. Consequently, the external characteristic curve turns back as shown by dotted line in the above figure.

Q.22 An 8 pole wave connected armature has 300 conductors and runs at 800 rpm. Determine the useful flux/pole if the electromotive force generated on open circuit is 500 V.

☞ **[JNTU : Part A, May-18, Marks 3]**

Ans. : P = 8, Z = 300, A = 2 as wave, E = 500 V, N = 800 rpm

$$E = \frac{\phi PNZ}{60A}$$

i.e. $$\phi = \frac{60\times 2\times 500}{8\times 800\times 300} = 31.25 \text{ mWb}$$

Q.23 An 8 pole d.c. generator has per pole flux of 40 mWb and winding is connected in lap with 960 conductors. Calculate the generated e.m.f. on open circuit when it runs at 400 r.p.m. If the armature is wave wound at what speed must the machine be driven to generate the same voltage.

☞ **[JNTU : Part B, Nov.-06,07, Feb.-07,08, Dec.-16]**

Ans. : P = 8, ϕ = 40 mWb, Lap hence A = P, Z = 960, N = 400 r.p.m.

$$E_g = \frac{\phi PNZ}{60\ A} = \frac{40\times 10^{-3}\times 8\times 400\times 960}{60\times 8} = 256 \text{ V}$$

Now it is wave connected hence A = 2, E_g = 256 V (same)

$\therefore$ $$256 = \frac{40\times 10^{-3}\times 8\times N_1\times 960}{60\times 2}$$

i.e. $$N_1 = 100 \text{ r.p.m.}$$

Q.25 A 6-pole, d.c. generator is running at 1500 r.p.m. It generates 230 V when the useful flux per pole is 0.02 Wb. If the armature has 60 slots, calculate the number of conductors per slot when the machine is i) Lap-wound and ii) Wave-wound. ☞ **[JNTU : Part B, Jan.-14, Marks 6]**

Ans. : P = 6, N = 1500 r.p.m, E_g = 230 V, ϕ = 0.02 Wb

i) Lab wound, A = P

$$\therefore \quad E_g = \frac{\phi PNZ}{60A} \quad \text{i.e.} \quad 230 = \frac{0.02 \times 6 \times 1500 \times Z}{60 \times 6}$$

$$\therefore \quad Z = 460 \quad \text{i.e.} \quad \text{conductors/slot} = \frac{460}{60}$$

$$= 7.66 \approx 8$$

ii) Wave wound, A = 2

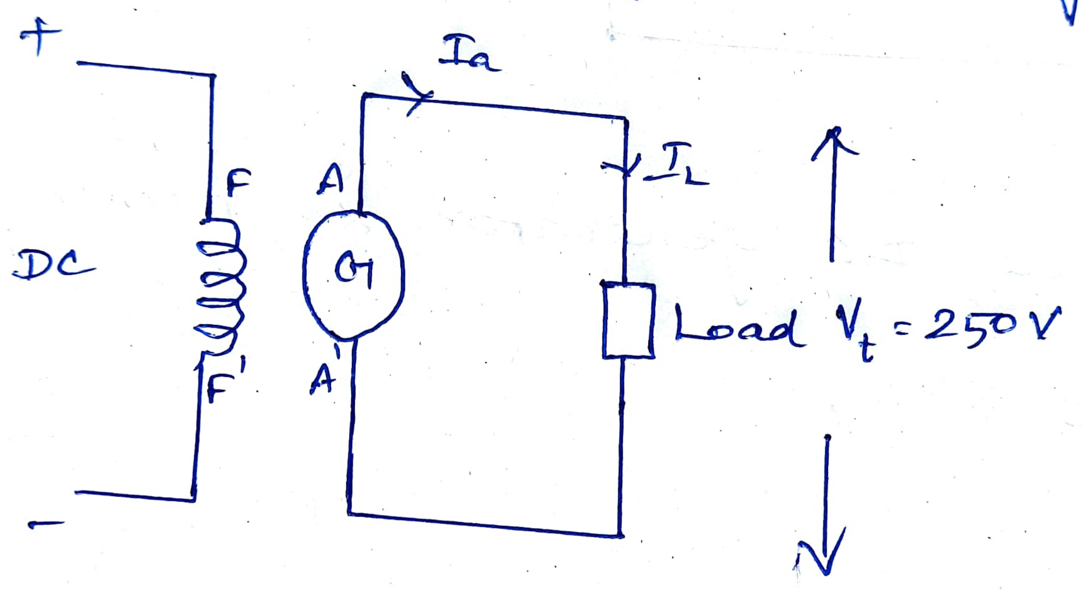
$$\therefore \quad 230 = \frac{0.02 \times 6 \times 1500 \times Z}{60 \times 2} \quad \text{i.e.} \quad Z = 153.33$$

$$\therefore \text{Conductors/slot} = \frac{153.33}{60} = 2.55 \approx 3$$

A 250V, 10 KW separately excited generator has an induced EMF of 255 V at full load. If the brush drop is 2 V per brush calculate the armature resistance of the generator.

given data:-

given separately excited generator



here $I_a = I_L$.

Note that 250V, 10 KW generator means the full load capacity of generator is to supply 10 KW at a terminal Voltage $V_t = 250$ V.

$\therefore V_t = 250\text{ V}$ and $P = 10\text{ KW}$

$\therefore P = V_t \times I_L$

$$\therefore I_L = \frac{P}{V_t} = \frac{10 \times 10^3}{250} = 40\text{ A}$$

$$\therefore \boxed{I_a = I_L = 40\text{ A}}$$

here is separately excited generator

$$E_g = V_t + I_a R_a + V_{br}$$

given brush drop is 2 V per brush

$\therefore$ Total 2 brushes $\therefore V_{br} = 2 \times 2$

$= 4\text{ V}$

$$\therefore E_g = 250 + 40 \times R_a + 4$$

given $E_g = 255$ on full load

$$\therefore 255 = 250 + 40 R_a + 4$$

$$\therefore \boxed{R_a = 0.025\,\Omega}$$

Example 4.1 : *A d.c. shunt generator has shunt field winding resistance of 100 Ω. It is supplying a load of 5 kW at a voltage of 250 V. If its armature resistance is 0.22 Ω, calculate the induced e.m.f. of generator.*

Solution : Consider shunt generator as shown in the Fig. 4.10.

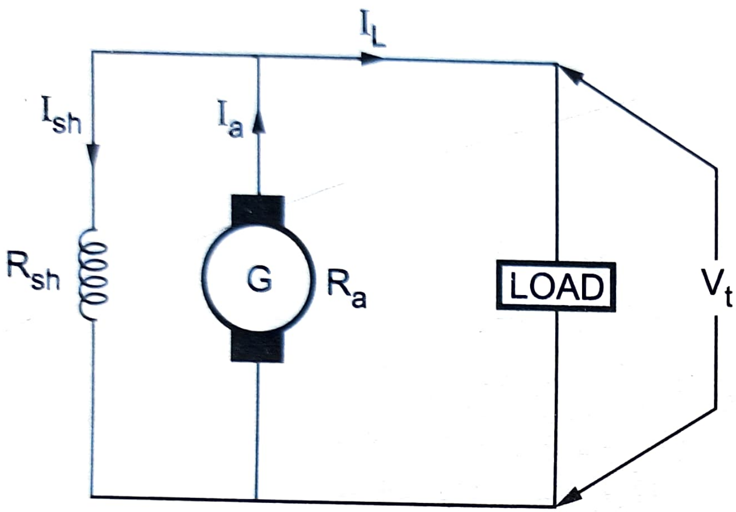


Fig. 4.10

$$I_a = I_L + I_{sh}$$

$$I_{sh} = \frac{V_t}{R_{sh}}$$

Now $\quad V_t = 250 \text{ V}$

and $\quad R_{sh} = 100\ \Omega$

$$\therefore \quad I_{sh} = \frac{250}{100} = 2.5 \text{ A}$$

Load power = 5 kW

$$\therefore \quad P = V_t \times I_L$$

$$\therefore \quad I_L = \frac{P}{V_t} = \frac{5 \times 10^3}{250} = 20 \text{ A}$$

$$\therefore \quad I_a = I_L + I_{sh} = 20 + 2.5 = 22.5 \text{ A}$$

Now $\quad E = V_t + I_a R_a$ (neglect V_{brush})

$$\therefore \quad E = 250 + 22.5 \times 0.22 = \mathbf{254.95 \text{ V}}$$

This is the induced e.m.f. to supply the given load.

Q.51 A series generator delivers 100 A at 250 V and the resistance of the series field and armature resistance are 0.055 Ω and 0.1 Ω respectively. Calculate the armature current and generated EMF. ☞ **[JNTU : Part B, May-13, Marks 6]**

Ans. : The armature current is same as load current.

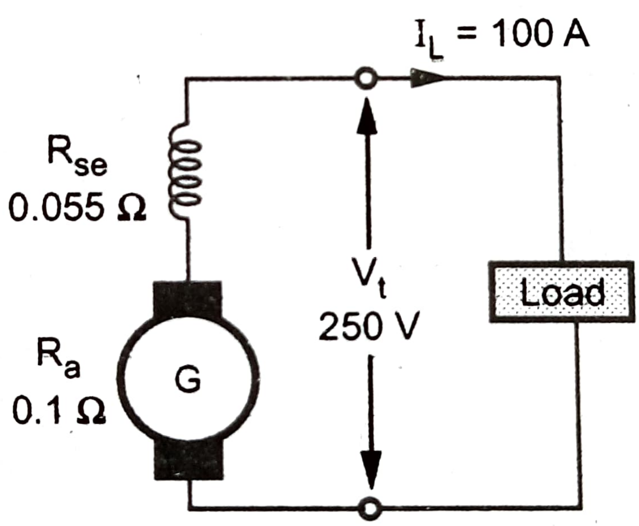


Fig. Q.51.1

$$\therefore \quad I_a = I_L = 100 \text{ A}$$

$$E_g = V_t + I_a R_a + I_a R_{se}$$

$$= 250 + 100\,(0.055 + 0.1) = \mathbf{265.5\ V}$$

Example 4.3 : *A d.c. series generator has armature resistance of 0.5 Ω and series field resistance of 0.03 Ω. It drives a load of 50 A. If it has 6 turns/coil and total 540 coils on the armature and is driven at 1500 r.p.m., calculate the terminal voltage at the load. Assume 4 poles, lap type winding, flux per pole as 2 mWb and total brush drop as 2 V.*

Solution : Consider the series generator as shown in Fig. 4.12.

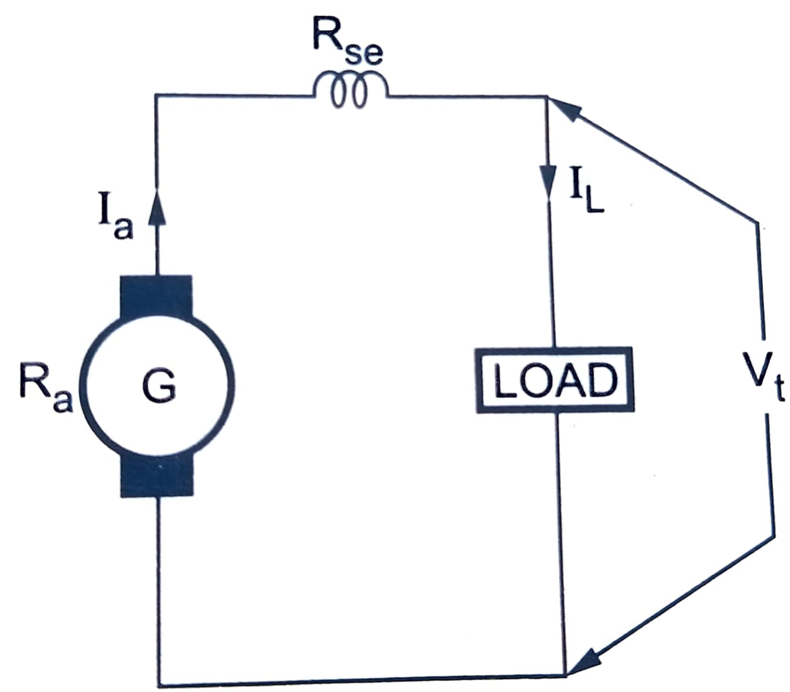


Fig. 4.12

$$R_a = 0.5\ \Omega,\quad R_{se} = 0.03\ \Omega$$

$$V_{brush} = 2\ V$$

$$N = 1500 \text{ r.p.m.}$$

Total coils are 540 with 6 turns/coil.

$$\therefore \quad \text{Total turns} = 540 \times 6 = 3240$$

$$\therefore \text{Total conductors } Z = 2 \times \text{turns}$$

$$= 2 \times 3240 = 6480$$

$$\therefore \quad E = \frac{\phi P N Z}{60\ A}$$

For lap type, $A = P$

and $\phi = 2 \text{ mWb} = 2 \times 10^{-3}$ Wb

$$\therefore \quad E = \frac{2 \times 10^{-3} \times 1500 \times 6480}{60}$$

$$= 324\ V$$

$$E = V_t + I_a (R_a + R_{se}) + V_{brush}$$

where $I_a = I_L = 50$ A

$$\therefore \quad 324 = V_t + 50\ (0.5 + 0.03) + 2$$

$$\therefore \quad V_t = \mathbf{295.5\ V}$$

Example 4.4 : *A short shunt compound d.c. generator supplies a current of 75 A at a voltage of 225 V. Calculate the generated voltage if the resistance of armature, shunt field and series field windings are 0.04 Ω , 90 Ω and 0.02 Ω respectively.*

Solution : Consider a short shunt generator as shown in the Fig. 4.13.

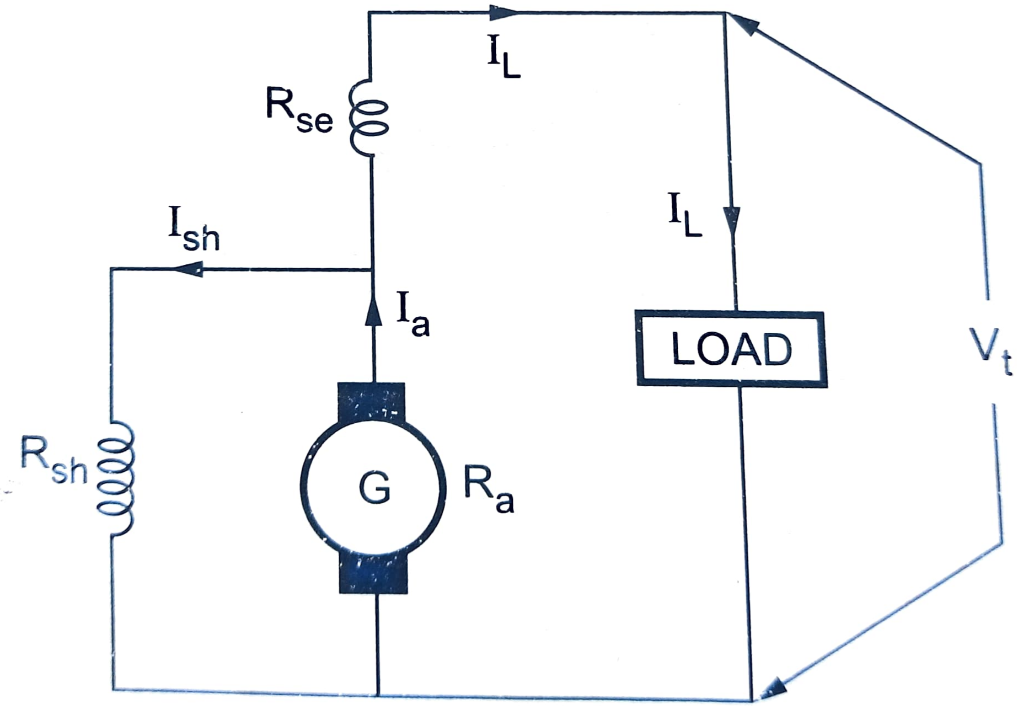


Fig. 4.13

$$R_a = 0.04\ \Omega, \quad R_{sh} = 90\ \Omega, \quad R_{se} = 0.02\ \Omega$$

$$V_t = 225\ \text{V}$$

$$I_L = 75\ \text{A}$$

$$I_a = I_L + I_{sh}$$

Now $$E = V_t + I_a R_a + I_L R_{se}$$

and drop across armature terminals is,

$$E - I_a R_a = V_t + I_L R_{se} = 225 + 75 \times 0.02 = 226.5\ \text{V}$$

$$\therefore \quad I_{sh} = \frac{E - I_a R_a}{R_{sh}} = \frac{V_t + I_L R_{se}}{R_{sh}}$$

$$= \frac{226.5}{90} = 2.5167\ \text{A}$$

$$\therefore \quad I_a = I_L + I_{sh} = 75 + 2.5167 = 77.5167\ \text{A}$$

$$\therefore \quad E = V_t + I_a R_a + I_L R_{se}$$

$$= 225 + 77.5167 \times 0.04 + 75 \times 0.02 = 229.6\ \text{V}$$

Q.52 In a 200 V d.c. compound generator the armature, shunt-field and series-field winding resistances are respectively 0.6 Ω, 150 Ω and 0.3 Ω. The machine is connected to a load of 15 kW, 200 V. Find the total emf generated and the armature current when the machine is connected in i) Long-shunt and ii) Short-shunt.

☞ **[JNTU : Part B, Nov.-12, Jan.-14, Marks 14]**

Ans. : $V_t = 200$ V, $R_a = 0.6\ \Omega$, $R_{sh} = 150\ \Omega$, $R_{se} = 0.3\ \Omega$

i) Long shunt : $V_t = 200$ V

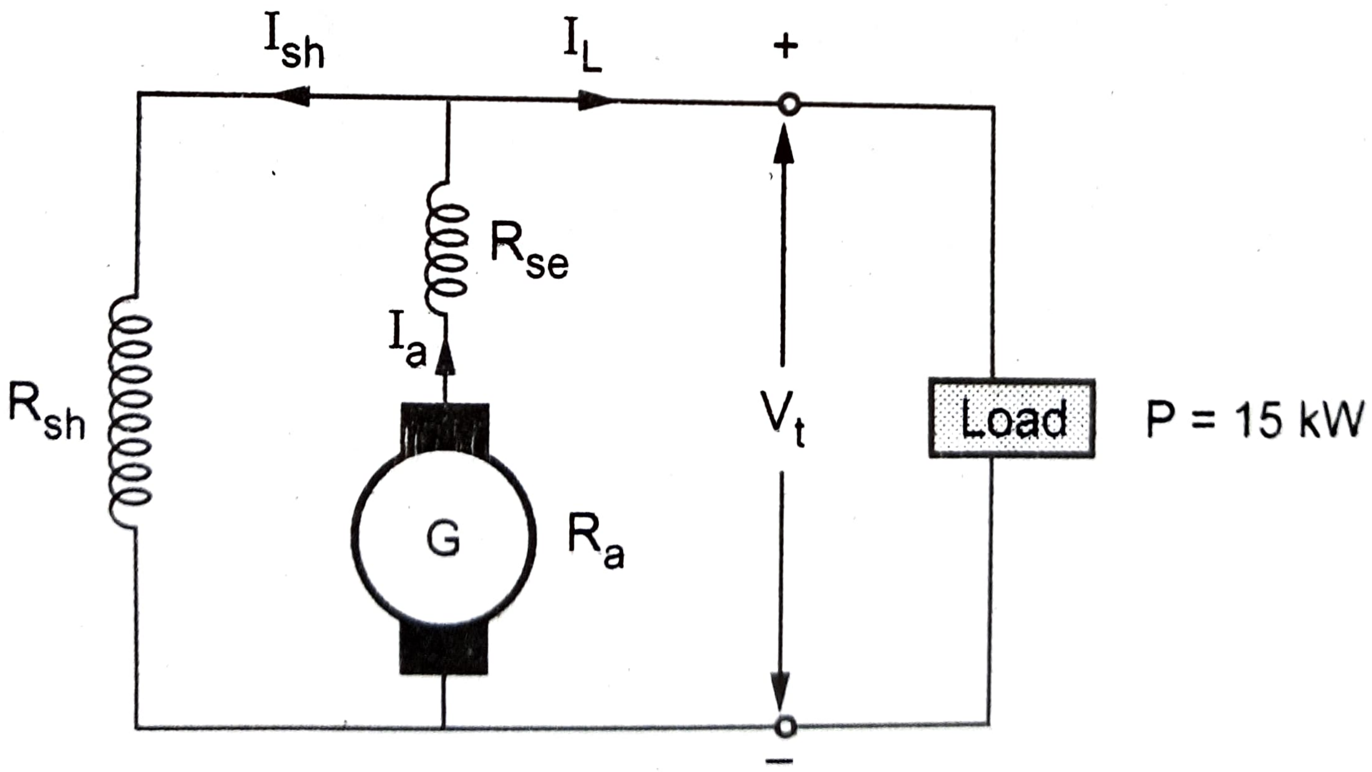


Fig. Q.52.1

$$I_L = \frac{P}{V_t} = \frac{15 \times 10^3}{200} = 75 \text{ A}$$

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{200}{150} = 1.333 \text{ A}$$

$$I_a = I_L + I_{sh} = \mathbf{76.333 \text{ A}}$$

$$E_g = V_t + I_a R_a + I_a R_{se} + V_{brush} \quad \ldots V_{brush} = 0$$

$$= 200 + 76.333\,(0.6 + 0.3) = \mathbf{268.7 \text{ V}}$$

ii) Short shunt : $V_t = 200$ V

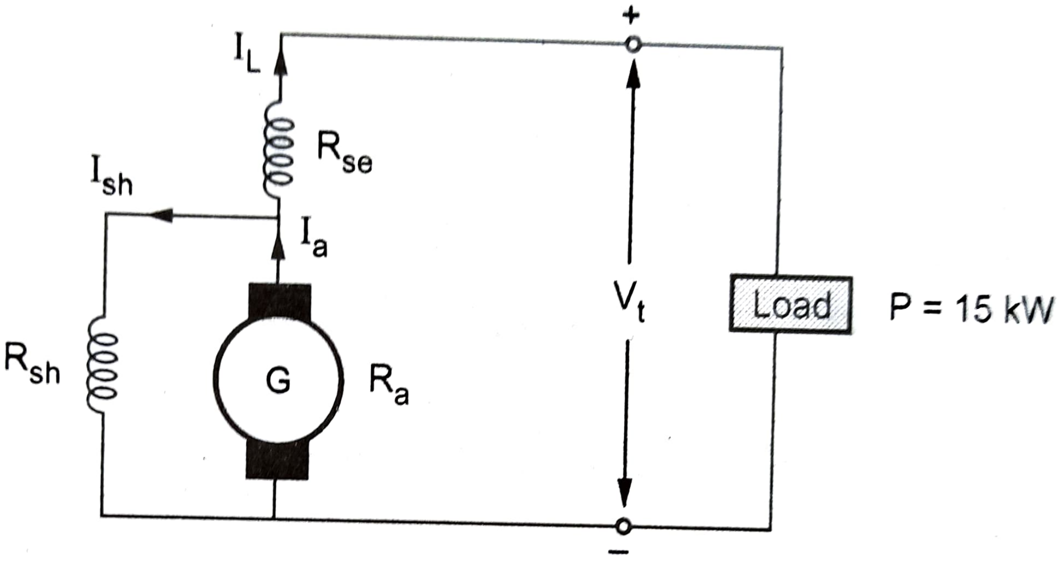


Fig. Q.52.2

$$\text{Drop across } R_{sh} = E_g - I_a R_a$$

$$= V_t + I_L R_{se}$$

where $$I_L = \frac{P}{V_t} = \frac{15 \times 10^3}{200} = 75 \text{ A}$$

$\therefore$ $$\text{Drop across } R_{sh} = 200 + 75 \times 0.3 = 222.5 \text{ V}$$

$\therefore$ $$I_{sh} = \frac{222.5}{R_{sh}} = \frac{222.5}{150} = 1.4833 \text{ A}$$

$\therefore$ $$I_a = I_L + I_{sh} = \mathbf{76.4833 \text{ A}}$$

$\therefore$ $$E_g = V_t + I_a R_a + I_L R_{se}$$

$$= 200 + 76.4833 \times 0.6 + 75 \times 0.3$$

$$= \mathbf{268.39 \text{ V}}$$

Unit - 1 - JNTU Problems

① A 4 pole wave connected armature ~~slots~~ has 100 slots. If the flux per pole is 0.04 Wb, calculate the number of conductors required per slot to generate 220 V. Take the speed of the generator as 300 rpm. Calculate the New value of flux due to change in the number of conductors per slot, if any.

given data:-

$P = 4$

$A = 2$ (wave connected)

Number of slots, $S = 100$

$\phi = 0.04$ Wb

$E_g = 220$ V

To find:-

Number of conductors/slot

first find, number of conductors & calculate conductors per slot

We Know

$$E_g = \frac{\phi Z N P}{60 A}$$

$$Z = \frac{E_g \cdot 60 \cdot A}{\phi N P} = \frac{220 \times 60 \times 2}{0.04 \times 300 \times 4}$$

$Z = 550$ conductors

$\therefore \frac{\text{Conductors}}{\text{Slot}} \Rightarrow \frac{Z}{S} = \frac{550}{100} = 5.5$

Conductors cannot be in fraction.

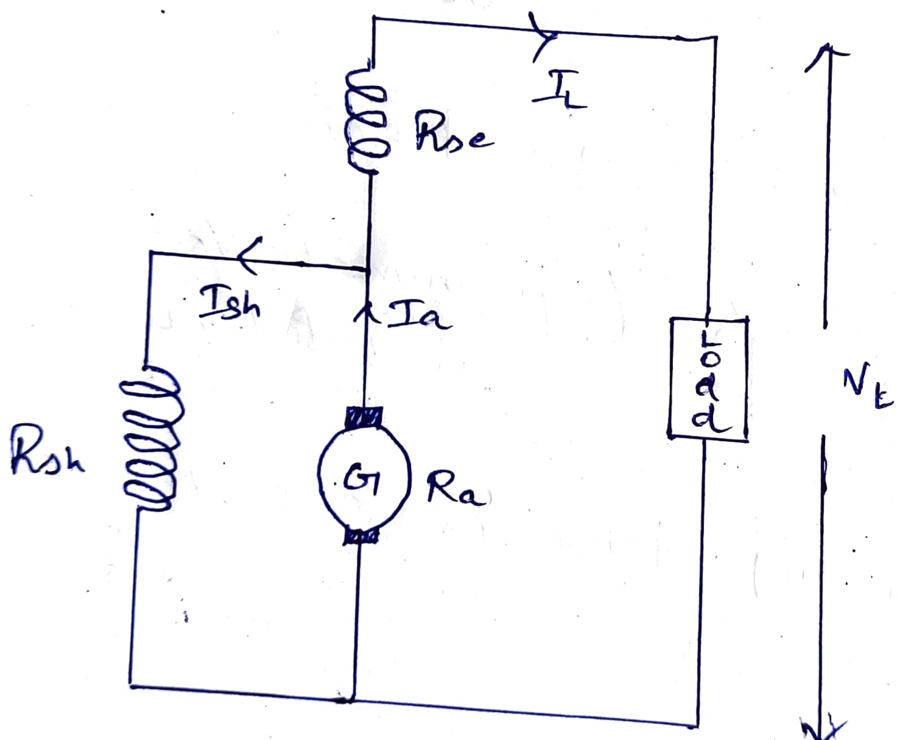
$\therefore 5.5 \simeq 6$ (approximate value in whole number)

$\therefore$ New value of flux is calculated as

$$\phi = \frac{E_g \times 60 \times A}{Z \times N \times P} \Rightarrow \frac{220 \times 60 \times 2}{6 \times 300 \times 4}$$

$$\boxed{\phi = 3.36 \text{ Wb}}$$

(2) A short shunt DC compound generator supplies 150 A at 100 V. The resistance of armature, series field and shunt field windings are 0.04, 0.03 and 60 Ω respectively. Find the EMF generated. Also find the EMF generated if the machine is connected as long shunt.



given $R_a = 0.04\ \Omega$

$R_{sh} = 60\ \Omega$

$R_{se} = 0.03\ \Omega$

$V_t = 100\ V$

$I_L = 150\ A$

here $I_a = I_L + I_{sh}$

$$E_g = V_t + I_a R_a + I_L R_{se}$$

$$E_g - I_a R_a = V_t + I_L R_{se}$$

$$\Rightarrow 100 + 150(0.03)$$

$$\Rightarrow 104.5$$

$$I_{sh} = \frac{E_g - I_a R_a}{R_{sh}} = \frac{V_t + I_L R_{se}}{R_{sh}}$$

$$I_{sh} \Rightarrow \frac{104.5}{60}$$

$$I_{sh} \Rightarrow 1.7416 \text{ A}$$

$\therefore$ $I_a = I_L + I_{sh}$

$$I_a \Rightarrow 150 + 1.7416$$

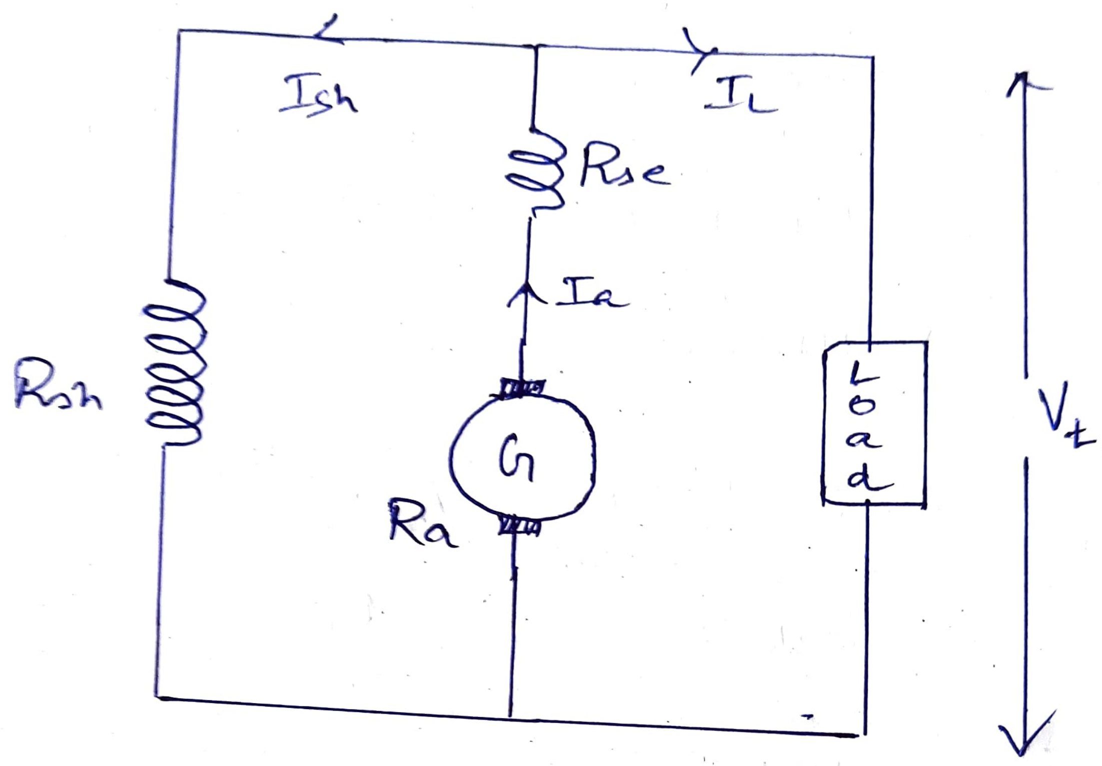
$$I_a = 151.74$$

$\therefore$ $E_g = V_t + I_a R_a + I_L R_{se}$

$$\Rightarrow 100 + 151.74 \times 0.04 + 150 \times 0.03$$

$$E_g \Rightarrow 110.56 \text{ V}$$

Now find the emf if the machine is connected as long shunt



$$E_g = V_t + I_a R_a + I_{se} R_{se}$$

here $I_a = I_{se}$

also, $I_a = I_L + I_{sh}$

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{100}{60} = 1.67\,A$$

$$I_a = 150 + 1.67$$

$$I_a = 151.67\,A$$

$\therefore E_g = 100 + 151.67(0.04) + 151.67(0.03)$

$E_g = 110.6167\ V$

(5) A 10 KW, 240 V, 6 pole, 1200 rpm lap connected DC generator has 500 armature conductors. At rated voltage and current, armature ohmic losses are 200 watts. Compute the useful flux per pole? Take 3V as the brush drop at full load.

given:-

$P_{out} = 10\ KW$

$V_t = 240\ V$

Poles, $P = 6$

$A = P$ (lap winding)

$I_a^2 R_a = 200\ W$

Brush drop $= 3V$

$Z = 500$

$$I_L = \frac{10 \times 10^3}{240} \Rightarrow 41.67\,A$$

I_{sh} Neglected (separately Excited)

$$\boxed{I_L = I_a}$$

$$I_a^2 R_a = 200$$

$$\therefore R_a = \frac{200}{(41.67)^2}$$

$$\therefore \boxed{R_a = 0.1152\,\Omega}$$

$$\therefore E_g = V_t + I_a R_a + V_{br}$$

$$\Rightarrow 240 + 0.1152 \times 41.67 + 3$$

$$E_g = 247.8\,V$$

$$E_g = \frac{\phi Z N P}{60 A}$$

$$\phi = \frac{E_g \times 60 \times A}{Z \times N \times P} = 24.78\ mWb.$$

(4) A 20 KW, 250V, 6 pole lap Connected DC generator runs at 1250 rpm. Armature has 550 Conductors. for full load armature - ohmic loss of 250 W, find the useful flux per pole. Take 2 V as the brush drop at full load.

Given:-

$P_{out} \Rightarrow 20$ KW

$V_t = 250$ V

poles $= 6$

$A = P = 6$ (lap Connected)

$Z = 550$

$I_a^2 R_a = 250$ W

brush drop, $V_{br} = 2$ V

$$I_L = \frac{20 \times 10^3}{250} = 80\ A$$

I_{sh} Neglected.

$\therefore\ I_L = I_a$

$I_a^2 R_a = 250$

$$R_a = \frac{250}{80^2}$$

$R_a = 0.039\ \Omega$

$\therefore\ E_g = V_t + I_a R_a + V_{br}$

$\Rightarrow 250 + 80 \times 0.039 + 2$

$E_g = 255.12\ V$

$$E_g = \frac{\phi Z N P}{60 A}$$

$$\phi = \frac{E_g \times 60 \times A}{Z \times N \times P} = \frac{255.12 \times 60 \times 6}{550 \times 1250 \times 6}$$

$\phi = 0.022$ Wb.

(5) A 4 pole lap wound DC shunt generator has a useful flux per pole of 0.07 Wb, the armature winding consist of 220 turns each of 0.004 Ω resistance. calculate the terminal Voltage when running at 900 rpm and the armature current is 50 A.

given data:-

$P = 4$ poles

$A = P = 4$ (lap)

$\phi = 0.07$ wb

220 turns / 0.004 Ω

$N = 900$ rpm

$I_a = 50$ A

To find :-

Terminal Voltage, $V_t = ?$

Number of armature Conductors

$Z = 2 \times$ turns $= 2 \times 220 = 440$

Since 2 conductors constitute 1 turn.

generated EMF is

$$E_g = \frac{\phi Z N P}{60 A} = \frac{0.07 \times 440 \times 900 \times 4}{60 \times 4}$$

$$E_g = 462 \text{ V}$$

Number of turns per path

$$\Rightarrow \frac{220}{4} \Rightarrow 55$$

Armature resistance per path

$\Rightarrow$ Number of turns per path $\times$ resistance of each turn

$$\Rightarrow 55 \times 0.004 \Rightarrow 0.22\ \Omega$$

Armature resistance,

$$R_a = \frac{\text{Armature resistance per path}}{\text{Number of parallel path}}$$

$$\therefore R_a = \frac{0.22}{4}$$

$$R_a = 0.055\ \Omega$$

Armature current, $I_a = 50\ A$

Terminal Voltage

$$V_t = E_g - I_a R_a \Rightarrow 462 - 50(0.055)$$

$$V_t \Rightarrow 459.25\ V$$

(b) A 4 pole DC shunt generator with wave wound armature has 40 slots each having 12 conductors. Armature resistance is $1\ \Omega$ and shunt field resistance is $200\ \Omega$. The flux per pole is 25 mWb. If a load of $50\ \Omega$ is connected across the armature terminal, calculate the voltage across the load when the generator is driven at 1000 rpm. What will be the load voltage if the generator is lap wound.

given data :-

$P = 4$ Pole

$A = 2$ (wave connected)

40 slots × 12 conductors per slot

$R_a = 1\,\Omega$

$R_{sh} = 200\,\Omega$ $\therefore Z = 40 \times 12$

$\phi = 25$ mWb

$R_L = 50\,\Omega$

$N = 1000$ rpm

To find :-

The terminal voltage for the above data

& terminal voltage for lap connected

$$E_g = \frac{\phi Z N P}{60 A} = \frac{25 \times 10^{-3} \times 40 \times 12 \times 1000}{60} \times \frac{4}{2}$$

$$E_g = 400\,V$$

Let the terminal voltage is V_t

Shunt field current $\Big\}$ $I_{sh} = \frac{V_t}{R_{sh}} = \frac{V_t}{200}$ A

Load current, $I_L = \dfrac{V_t}{R_L} = \dfrac{V_t}{50}$ A

Armature current I_a $\Big\}$ $= I_L + I_{sh}$

$$I_a = \frac{V_t}{200} + \frac{V_t}{50} = \frac{V_t}{40} \text{ A}$$

here $E_g = V_t + I_a R_a$ (neglecting V_{br})

$$\therefore\ 400 = V_t + \frac{V_t}{40} \times 1 = \frac{41}{40} V \quad \ldots\ (1)$$

$$\therefore\ V_t = \frac{400}{41/40} = 390.2\ V$$

(ii) if ~~wave~~ lap Connected, $V_t = ?$

$$E_g = \frac{25 \times 10^{-3} \times 40 \times 12 \times 1000 \times 4}{60 \times 4} = 200 V$$

$\therefore$ from Eq - (1)

$$V_t = \frac{E_g}{41/40} \Rightarrow \frac{200}{41/40} = 195.1\ V$$

Vishnu Institute of Technology

Department of EEE

Electrical Machines I

(Class Material)

UNIT 2: DC Motors

Principle of operation –Types– back emf and torque equation characteristics of separately-excited, shunt, series and compound motors – Necessity of starter – Starting by 3 point and 4 point starters – Speed control by armature voltage and field control methods - losses and efficiency – applications of dc motors.

Theoretically, the same DC machine can be used as a motor or generator. Therefore, construction of a DC motor is same as that of a DC generator.

Refer Unit 1 material for construction of DC motors

1. Principle Operation of DC Motor

An electric motor is an electrical machine which converts electrical energy into mechanical energy. The basic working principle of a DC motor is: "whenever a current carrying conductor is placed in a magnetic field, it experiences a mechanical force". The direction of this force is given by Fleming's left-hand rule and its magnitude is given by F = BIL. Where, B = magnetic flux density, I = current and L = length of the conductor within the magnetic field.

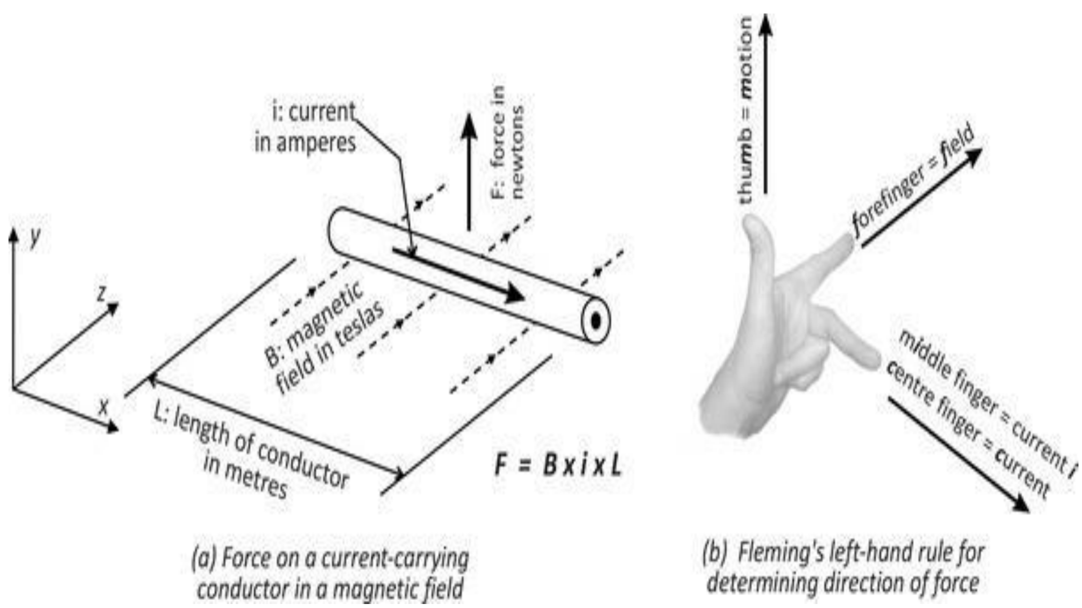


Fig. Fleming's left hand rule

Fleming's left hand rule: If we stretch the fore finger, middle finger and thumb of our left hand to be mutually perpendicular to each other, and the fore finger points out the direction of magnetic field, middle finger represents the direction of the current and then the thumb represents direction of the force (rotation) experienced by the current carrying conductor.

Consider a coil placed in a magnetic field with a flux density of B Tesla. When the coil is supplied with direct current by connecting it to a DC supply, a current I flows through the length of the coil. The electric current in the coil interacts with the magnetic field and the result is exertion of a force on the coil according to the Lorenz force equation. The force is proportional to the strength of the magnetic field and the current in the conductor.

Draw any 1 diagram

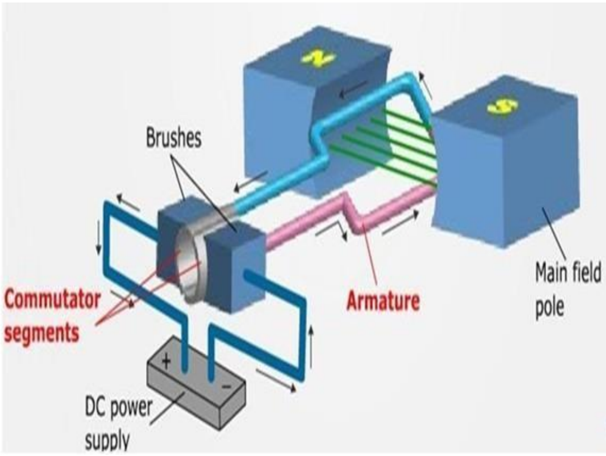


Fig. Working of DC motor

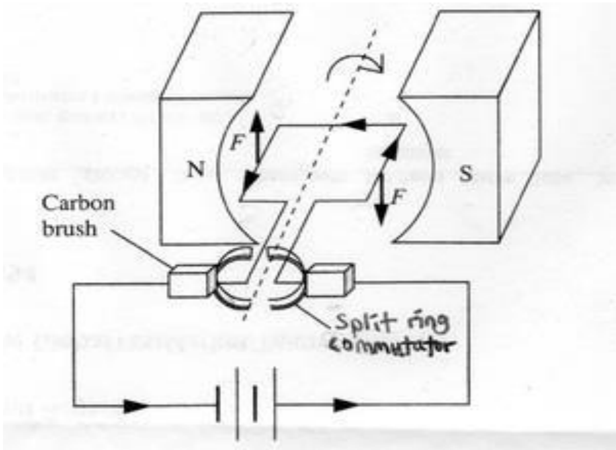


Fig. Working of DC motor

The same principle is used in DC motor and it consists of several coils that are wound on the armature and all the coils experience the same force. The result of this force is the rotation of the armature. The rotation of the conductor in the magnetic field will result in torque. The magnetic flux linking with the conductor is different at different positions of the coil in the magnetic field and these causes to induce an EMF in the coil according to the Faradays laws of electromagnetic induction. This EMF is referred to as back EMF. The direction of this EMF is opposite to the supply voltage which is responsible for current to flow in the conductor. Hence the total amount of current flowing in the armature is proportional to the difference between the supply voltage and the back EMF.

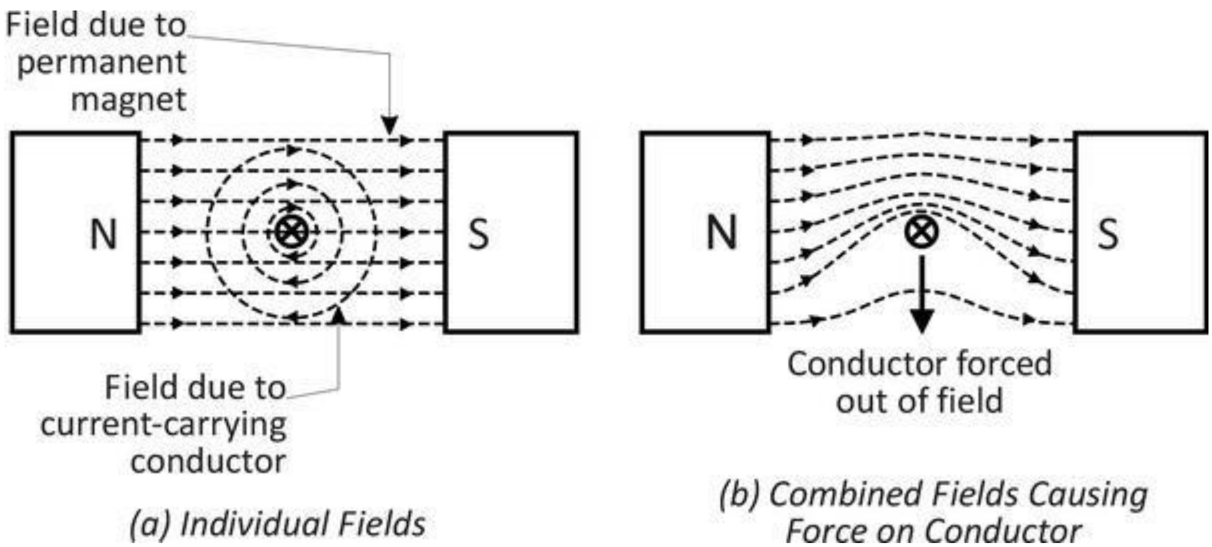


Fig. Force on Conductor in Magnetic Field

In Figure 1(a), a conductor is carrying current away from us, and is placed between the poles of a permanent magnet. The right-hand screw rule gives the direction of the flux lines around this conductor as clockwise, thus they reinforce the flux from the magnet above the conductor and oppose the flux from the magnet below the conductor. The result of the interaction between these fluxes is shown in Figure 1(b). The flux field above the conductor is strong and the lines are compressed. The flux field below the conductor is weak and flux lines are far apart. The tendency for the compressed lines to want to straighten and move further apart will exert a force on the conductor, attempting to push it downwards and out of the field. If the conductor current is reversed, or the magnet poles are reversed, this force will happen in the opposite direction and will push the conductor upwards.

Another Example to understand the operation of DC motor

In Fig. below a simple coil is placed inside two poles of a magnet. Now look at the current carrying conductor segment AB. The direction of the current is towards B, whereas in the conductor segment CD the direction is opposite. As the current is owing in opposite directions in the segments AB and CD, the direction of the motion of the segments would be in opposite directions according to Fleming’s le hand rule. When two ends of the coil experience force in opposite direction, they rotate.

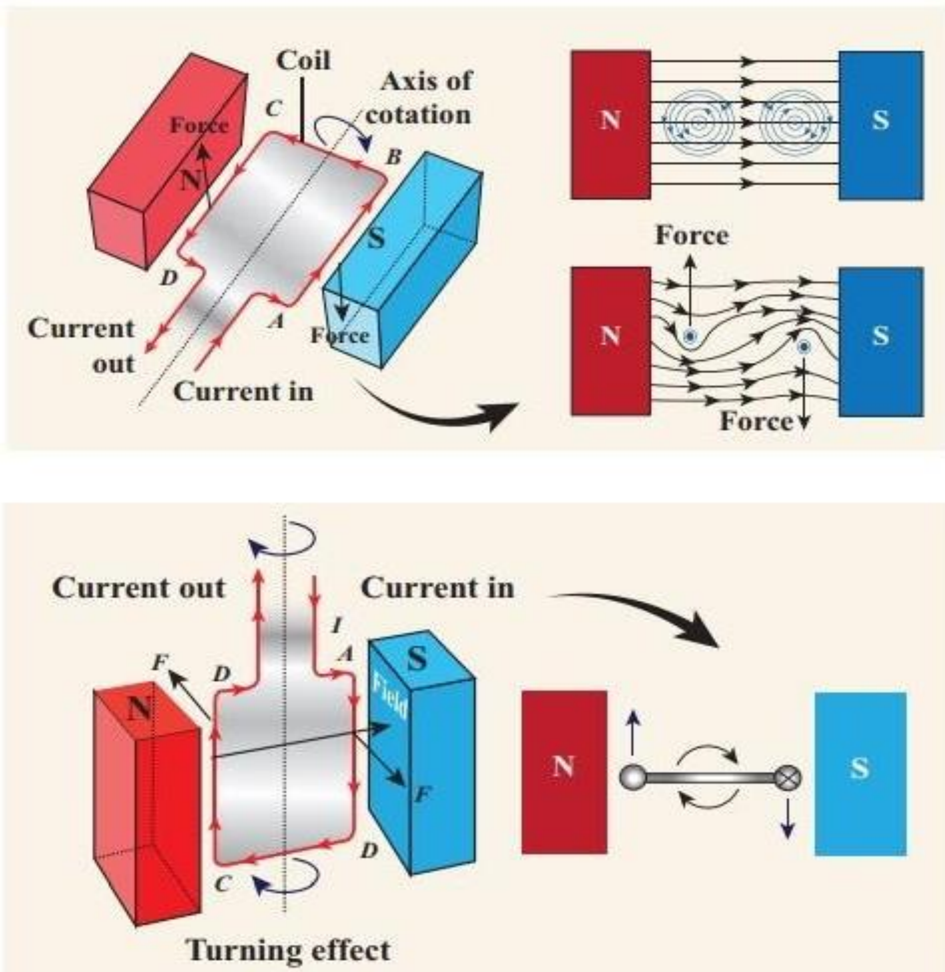


Figure 3.16 Turning effect in a coil

2. What is Back EMF?

When the armature of a DC motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field and hence EMF is induced in them as in a generator. The induced EMF acts in opposite direction to the applied voltage V (Lenz's law) and is known as **Back EMF or Counter EMF (E_b).**

The equation to find out back EMF in a DC motor is given below,

$$E_b = \frac{P\phi ZN}{60A}$$

The **back EMF** Eb (= PΦZN/60 A) is always less than the applied voltage V, although this difference is small when the motor is running under normal conditions.

3. Voltage Equation of a DC Motor

Input Voltage provided to the motor armature performs the following two tasks: Controls the induced Back E.M.F "Eb" of the Motor and Provides supply to the Ohmic IaRa drop.

$$\mathbf{V = E_b + I_aR_a} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \mathbf{(1)}$$

Where

$\mathbf{E_b}$= Back E.M.F

$\mathbf{I_aR_a}$ = Armature Current X Armature Resistance

The above relation is known as "**Voltage Equation of the DC Motor**".

4. Power Equation of a DC Motor

Multiplying both sides of Voltage Equation (1) by I_a, we get the power equation of a DC motor as follows.

$$VI_a = E_b I_a + I_a^2 R_a \quad \text{……………..} \quad (2)$$

Where,

VI_a = Input Power supply (Armature Input)

$E_b.I_a$ = Mechanical Power developed in Armature (Armature Output)

$I_a^2 R_a$ = Power loss in armature (Armature Copper (Cu) Loss

5. Condition for Maximum Power

Power of DC Motor

In a dc motor, the armature develops the mechanical power and the mathematical expression of the approximate mechanical power is

$$P_m = VI_a - I_a^2 R_a$$

Where V is the supply voltage, I_a and R_a are armature current and armature resistance respectively. We refer to this expression as approximate mechanical power since we have neglected here the shunt field current and series field resistance of the motor because both of them are quite small.

Differentiating both sides of the expression with respect to the armature current (I_a), we get,

$$\frac{dP_m}{dI_a} = V - 2I_a R_a = 0$$

$$\therefore I_a R_a = \frac{V}{2}$$

This is the condition of maximum power in the motor. Again, the voltage equation of the dc motor is,

$$E_b = V - I_a R_a$$

Therefore, at the maximum power condition,

$$E_b = V - \frac{V}{2}$$

$$\Rightarrow E_b = \frac{V}{2}$$

So, when the back emf of the motor becomes half of the supply voltage, the motor delivers the maximum mechanical power.

6. Torque Equation of a DC Motor

When armature conductors of a DC motor carry current in the presence of stator field flux, a mechanical torque is developed between the armature and the stator. Torque is given by the product of the force and the radius at which this force acts.

Torque $T = F \times r$ Newton Meter

Where, F = Force and r = Radius of the armature

Work done by this force in one revolution = **Force × distance moved in one revolution**

$= F \times 2\pi r$

Where, $2\pi r$ = circumference of the armature

Net power developed in the armature = **Word done / Time**

For N revolution it takes 60 s, therefore for 1 revolution 60/N s

Power developed in the armature = $(F \times 2\pi r) / 60/N$

Power developed in the armature = $(F \times 2\pi r \times N) / 60$

But, $F \times r = T$ **and** $2\pi N/60$ **= angular velocity** ω **in radians per second**. Putting these in the above power developed equation, we will get the expression

Net power developed in the armature = $P = T \times \omega$ **(Joules per second)**

Armature Torque (T_a)

The power developed in the armature can be given as, $P_a = T_a \times \omega = T_a \times 2\pi N/60$

The mechanical power developed in the armature is converted from the electrical power,

Therefore, mechanical power = electrical power

That means, $T_a \times 2\pi N/60 = E_b.I_a$

E_b. I_a is the electrical equivalent of mechanical power

We know, $E_b = P\Phi NZ / 60A$

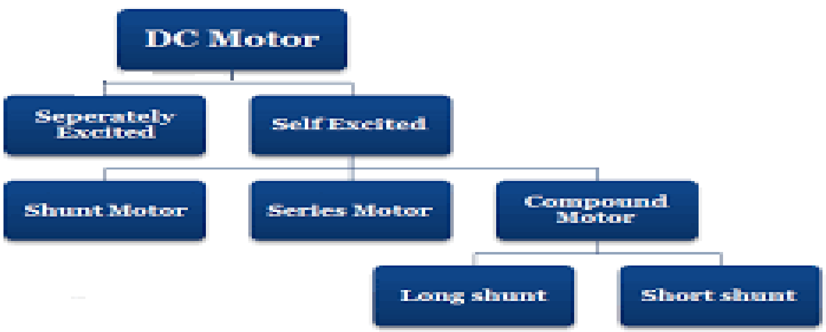
Therefore, $T_a \times 2\pi N/60 = (P\Phi NZ / 60A) \times I_a$

Rearranging the above equation,

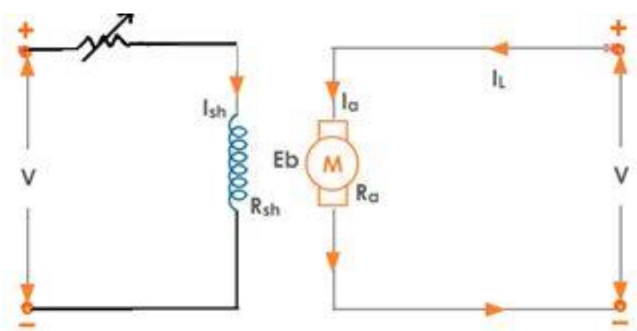
$$T_a = (PZ / 2\pi A) \times \Phi.I_a \text{ (N-m)}$$

The term $(PZ / 2\pi A)$ is practically constant for a DC machine. Thus, armature torque is directly proportional to the product of the flux and the armature current i.e. $T_a \propto \Phi.I_a$

7. Types of DC Motors:



1.Seperately excited DC Motors :



In a separately excited motor, armature and field windings are excited form two different dc supply voltages. In this motor,

Armature current Ia = Line current = IL = I

Back emf developed , Eb = V – I Ra

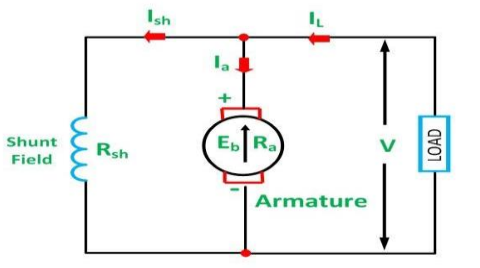
where V is the supply voltage and Ra is the armature resistance.

Power drawn from main supply , P = VI

Mechanical power developed ,

Pm = Power input to armature – power loss in armature

2.DC Shunt Motor:



When the DC voltage (V) is applied to the field and armature winding, the back EMF (Eb) is induced in the armature winding which opposes the applied voltage.

The armature current

$I_a = E_b/R_a$

Where, Ra- The armature resistance, E_b- Back EMF

The applied voltage (V) is equal to the Ia*Ra voltage drop plus back EMF(Eb).

$V = I_aR_a + E_b$

$I_a = I_{total} - I_{sh}$

$V = E_b + (I_{total} - I_{sh})R_a$

The field current is ;

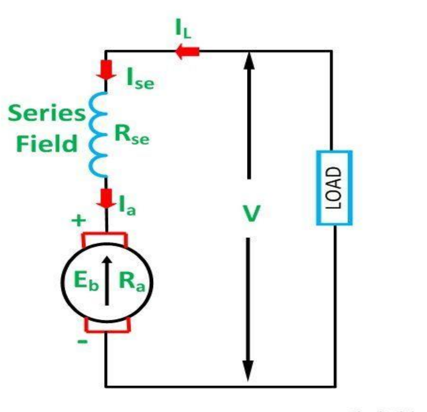
$I_{sh} = V/R_{sh}$

Where, R_{sh}- The Resistance of field winding

The field current remains constant for a fixed applied DC voltage(V).

DC Series Motor:

In this type of motor, the series field winding is connected in series with the armature and the supply, as shown in the Fig. 1.



Let R_{se} be the resistance of the series field winding. The value of R_{se} is very small and it is made of small number of turns having large cross-sectional area.

<u>1.1 Voltage and Current Relationship</u>

Let I_L be the total current drawn from the supply.

So $\quad I_L = I_{se} = I_a$

and $\quad V = E_b + I_a Ra + I_{se} R_{se} + V_{brush}$

$V = E_b + Ia\,(Ra + R_{se}) + V_{brush}$

Supply voltage has to overcome the drop across series field winding in addition to E_b and drop across armature winding.

Note : In series motor, entire armature current is passing through the series field winding. So flux produced is proportional to the armature current.

$$\boxed{\phi \propto I_{se} \propto I_a \quad \text{for series motor}}$$

Where

- E_a is the armature induced voltage
- I_a is the armature current
- R_a is the armature resistance
- R_{se} is the series field resistance

DC Compound Motor:

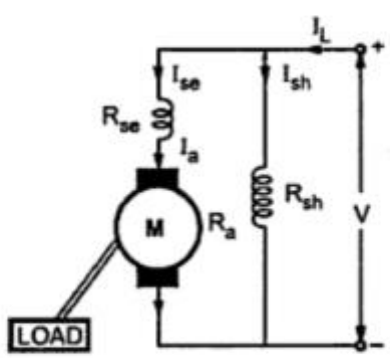
The compound motor consists of part of the field winding connected in series and part of the field winding connected in parallel with armature. It is further classified

a. Long Shunt Compound Motor

b. Short Shunt Compound Motor

a. Long Shunt Compound Motor:

In this type, the shunt field winding is connected across the combination of armature and the series field winding as shown in the Fig. 1.



Let Rse be the resistance of series field and R_{sh} be the resistance of shunt field winding. The total current drawn from supply is I_L.

So $\quad I_L = I_{se} + I_{sh}$

But $\quad I_{se} = I_a$

$\therefore \quad I_L = Ia + I_{sh}$

And $\quad I_{sh} = V/R_{sh}$

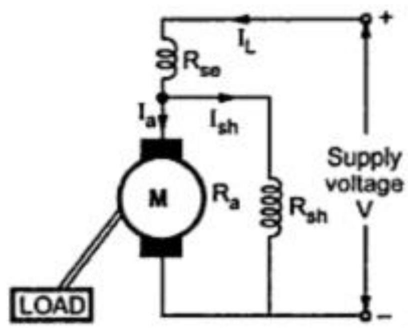
And $V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$

But as $I_{se} = I_a$,

$\therefore \quad V = E_b + I_a (R_a + R_{se}) + V_{brush}$

b. Short Shunt Compound Motor :

In this type, the shunt field is connected purely in parallel with armature and the series field is connected in series with this combination shown in the Fig. 2.



$I_L =$ Ise

The entire line current is passing through the series field winding.

and $I_L = I_a + I_{sh}$

Now the drop across the shunt field winding is to be calculated from the voltage equation.

So $V = E_b + I_{se} R_{se} + I_a R_a + V_{brush}$

but $I_{se} = I_L$

$\therefore \quad V = E_b + I_L R_{se} + I_a R_a + V_{brush}$

$\therefore$ Drop across shunt field winding is,

$= V - I_L R_{se} = E_b + I_a R_a +$ Vbrush

$$\therefore \quad \boxed{I_{sh} = \frac{V - I_L R_{se}}{R_{sh}} = \frac{E_b + I_a R_a + V_{brush}}{R_{sh}}}$$

Apart from these two, compound motor can be classified into two more types,

i) Cumulatively compound motors and ii) Differential compound motors.

Note : If the two field windings are wound in such a manner that the fluxes produced by the two always help each other, the motor is called cumulatively compound. If the fluxes produced by the two field windings are trying to cancel each other i.e. they are in opposite direction, the motor is called differential compound.

A long shunt compound motor can be of cumulative or differential type. Similarly short shunt compound motor can be cumulative or differential type.

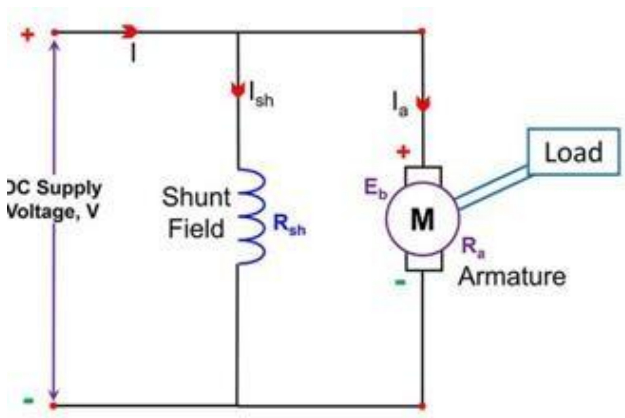
8. Characteristics of Shunt, Series and Compound Motors

Dc motors are widely used in various applications like variable-speed drives, constant-speed drives, and many more. For selecting a type dc motor for a particular application it is very important to study the characteristics of various dc motors. DC motor characteristics give the performance under various load conditions for recommending their field of applications.

Some of the important characteristics of dc motor are,

1. Speed-armature current characteristics (N vs I_a)
2. Torque-armature current characteristics (T vs I_a)
3. Speed-torque characteristics (N vs T)

Characteristics of DC Shunt Motor



DC Shunt Motor

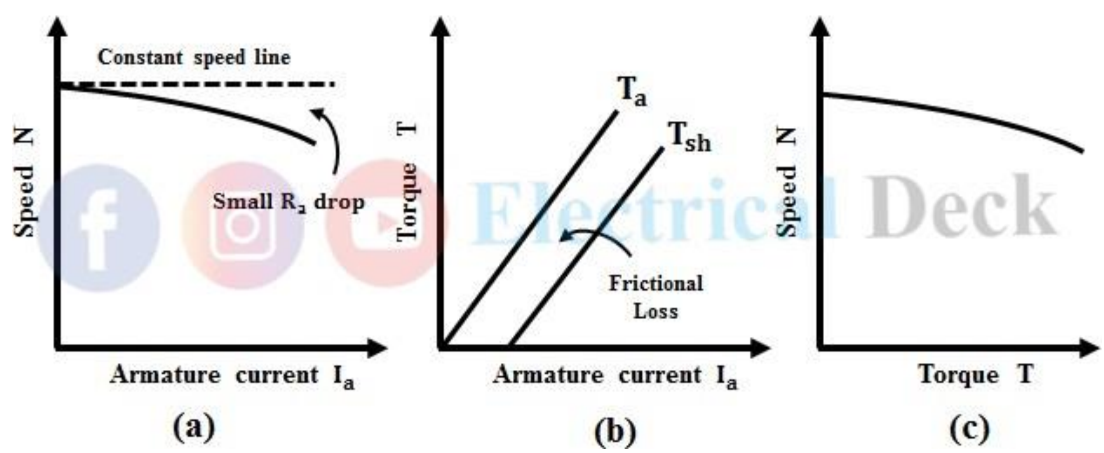
Speed-Armature Current Characteristics

The equation for back EMF induced in a dc motor is given as $E_b = P\phi NZ / 60A$. From this, the relation between speed and back EMF is expressed as $N \propto E_b/\varphi$. The expression for speed N of a dc motor is given as,

$$N = \frac{E_b}{\phi} = \frac{V - I_a R_a}{\phi}$$

$$N \propto I_a R_a \ (at\ constant\ \phi)$$

In a dc shunt motor, the field is connected across the armature terminals. Hence at constant supply voltage V, the field current remains constant producing constant φ. But in practice due to the armature reaction effect, the distribution of air-gap flux gets distorted. Thus reducing the resultant flux. Suppose if the load on the motor increases, the armature current I_a increases, thereby increasing the drop $I_a R_a$. This causes a small drop in speed because at constant φ there will be very little change in the difference ($V - I_a R_a$) since the armature resistance R_a of a dc motor is kept very small. The curve between speed and armature current with slightly dropping from no-load to full-load is shown in figure (a) below.



Torque-Armature Current Characteristics

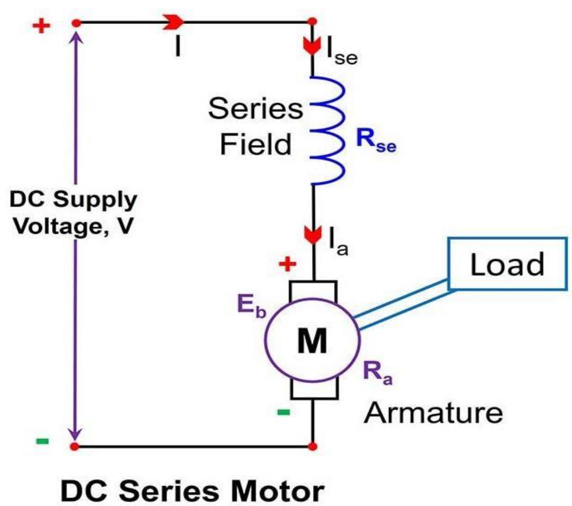
For a dc motor, the expression for torque is given as $T = K\ \varphi\ I_a$ i.e., torque is directly proportional to flux and armature current. At constant field current with constant φ, $T \propto I_a$. Now if the load increases, the load current increases (i.e., armature current increases), thereby increasing the armature torque. Therefore the curve will be a straight line from the origin as shown in figure (b).

But in practice, the output torque from the motor is less than armature torque due to frictional losses at the bearings. So that in order to generate high torque, the armature current should be very high which can damage the motor. Hence the dc shunt motor is well-suited for constant speed applications like conveyors, line shafts, etc.

Speed-Torque Characteristics

From T vs I_a and N vs I_a characteristics, the N vs T can be drawn as shown in figure (c). These characteristics are similar to speed and armature current characteristics, the speed remains almost constant for various load conditions below the full-load value.

Characteristics of DC Series Motor



Speed-Armature Current Characteristics

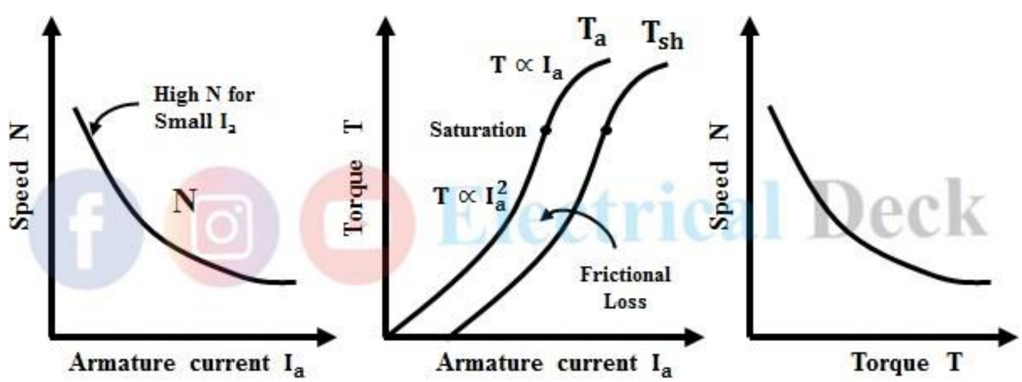
In a dc series motor, both field and armature windings are connected in series with the supply. Hence the supply voltage has to overcome the drop in the series winding also. Therefore, the expression for speed N of a dc series motor is given as,

$$N = \frac{V - I_a R_a - I_{se} R_{se}}{\phi}$$

$$N \propto \frac{E_b}{\phi} \propto \frac{V - I_a(R_a + R_{se})}{I_a}$$

$$(since\ \phi \propto I_a)$$

Due to the series field connection, the field current is proportional to the armature current and hence the flux, $\varphi \propto I_a$. Generally, R_a and R_{se} are kept small (with small I_a R_a and I_{se} R_{se} drops), thus for constant V i.e., $V \cong E_b$, if the load increases the speed decreases. The relation between N and I_a will be in the shape of a rectangular hyperbola as shown in figure (a). It is seen that, if the load is high, I_a will be high, and speed N will be low. But at no-load condition, I_a will be low and speed N will be very high. That is why a dc series motor never allowed to start without any load connected to it or with low-load conditions.



(a) (b) (c)

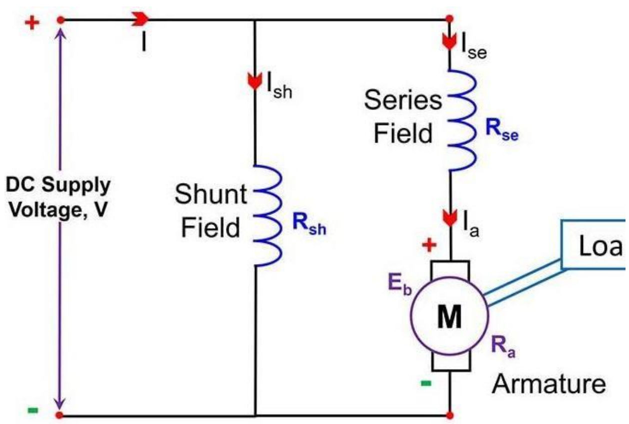
<u>Torque-Armature Current Characteristics</u>

For a dc motor, the expression for torque is given as $T = K\ \varphi\ I_a$. In a series motor, $\varphi \propto I_a$, thus $T \propto I_a^2$. Here torque is proportional to the square of the armature current, thus curve T vs I_a will be parabolic in nature as shown in figure (b). Also the curve, the parabolic will be up to a certain limit of I_a, then after since $I_a = I_{se}$, the field winding gets saturated and produces constant flux and hence constant T (curve will be a straight line). Therefore, from T vs I_a characteristics, a series motor when started with a load it develops high starting torque. Hence dc series motors are well suited for applications like electric trains, hoists, trolleys, etc.

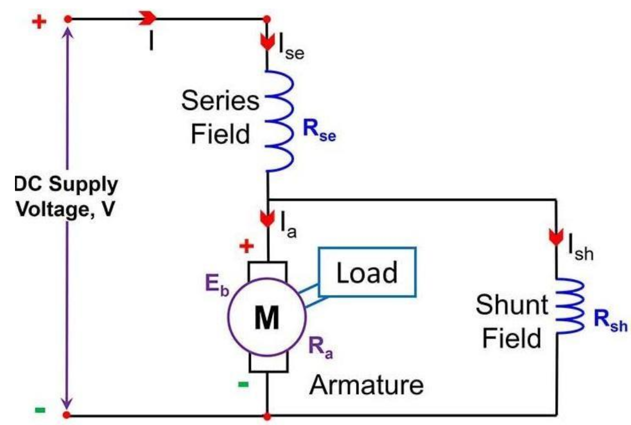
Speed-Torque Characteristics

Similar to the dc shunt motor, the N vs T relation of the dc series motor is similar to N vs I_a (a parabolic curve) as shown in figure (c). At no-load, I_a will be low, and hence torque, thus speed will be very high. As the load increases, torque increases, and speed decreases.

Characteristics of DC Compound Motor



Long Shunt DC Compound Motor



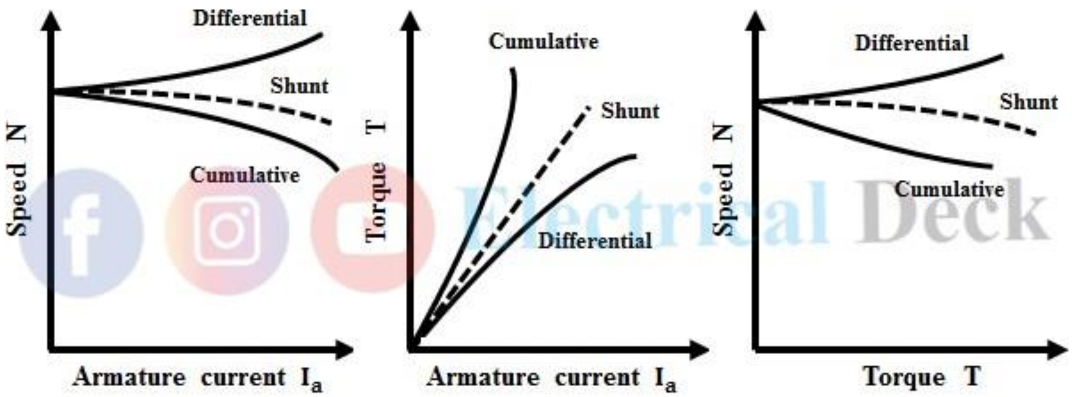
Short Shunt DC Compound Motor

We know that the compound motors are made with a combination of shunt and series field windings with the armature winding. Hence the characteristics of dc compound motors will be combined characteristics of shunt and series motors. But the characteristics depend upon how the two field windings are connected.

- In a cumulative compound motor, the flux produced by the series field winding assists the flux produced by the shunt field winding.
- In differential compound motor, the series field flux opposes the shunt field flux (reduction in resultant flux).

Speed-Armature Current Characteristics

The speed and armature current characteristics of dc cumulative and differential compound wound motors are shown in figure (a) below. Since resultant flux is more in cumulative compound motor with respect to load, the curve will be slightly more dropping compared to shunt motor.



(a) (b) (c)

Torque-Armature Current Characteristics

If the load on the cumulative compound motor increases the torque developed also increases. But the torque decreases with an increase in load in the case of a differential compound motor as shown in figure (b)

Speed-Torque Characteristics

When there is a sudden application of heavy loads, the speed decreases, the cumulative compound motors can develop large torque similar to series motors. At no-load, they run at definite speed due to shunt field flux, unlike series motor attain dangerous speeds. The speed remains almost constant in differential compound motors with increases in load. Hence shunt motors are more preferred compared to differential compound motors and are rarely used.

9. Losses and Efficiency

A DC generator converts mechanical power into electrical power and a dc motor converts electrical power into mechanical power. Thus, for a dc generator, input power is in the form of mechanical and the output power is in the form of electrical. On the other hand, for a dc motor, input power is in the form of electrical and output power is in the form of mechanical. In a practical machine, whole of the input power cannot be converted into output power as some power is lost in the conversion process. This causes the **efficiency of the machine** to be reduced. Efficiency is the ratio of output power to the input power. Thus, in order to design rotating dc machines (or any electrical machine) with higher efficiency, it is important to study the losses occurring in them. **Various losses in DC motor** can be characterized as follows

Losses in DC generator

Copper losses

- Armature Cu loss
- Field Cu loss
- Loss due to brush contact resistance

Iron Losses

- Hysteresis loss
- Eddy current loss

Mechanical losses

- Friction loss
- Windage loss

Copper Losses

These losses occur in armature and field copper windings. **Copper losses** consist of Armature copper loss, Field copper loss and loss due to brush contact resistance.

Armature copper loss = $\mathbf{I_a^2R_a}$ (where, I_a = Armature current and R_a= Armature resistance) This loss contributes about 30 to 40% to full load losses. The armature copper loss is variable and depends upon the amount of loading of the machine.

Field copper loss = **$I_f^2R_f$ or VR_f(**where, I_f = field current and R_f = field resistance) In the case of a shunt wounded field, field copper loss is practically constant. It contributes about 20 to 30% to full load losses.

Brush contact resistance also contributes to the copper losses. Generally, this loss is included into armature copper loss.

<u>Iron Losses (Core Losses)</u>

As the armature core is made of iron and it rotates in a magnetic field, a small current gets induced in the core itself too. Due to this current, **eddy current loss** and **hysteresis loss** occur in the armature iron core. Iron losses are also called as **Core losses or magnetic losses.**

Hysteresis loss is due to the reversal of magnetization of the armature core. When the core passes under one pair of poles, it undergoes one complete cycle of magnetic reversal. The frequency of magnetic reversal is given by, f=P.N/120 (where, P = no. of poles and N = Speed in rpm)

The loss depends upon the volume and grade of the iron, frequency of magnetic reversals and value of flux density. Hysteresis loss is given by, Steinmetz formula:

$W_h=\eta B_{max}^{1.6} fV$ (watts)

Where, η = Steinmetz hysteresis constant

V = volume of the core in m^3

Eddy current loss: When the armature core rotates in the magnetic field, an EMF is also induced in the core (just like it induces in armature conductors), according to the Faraday's law of electromagnetic induction. Though this induced EMF is small, it causes a large current to flow in the body due to the low resistance of the core. This current is known as eddy current. The power loss due to this current is known as eddy current loss.

<u>Mechanical Losses</u>

Mechanical losses consist of the losses due to friction in bearings and commutator. Air friction loss of rotating armature also contributes to these.

These losses are about 10 to 20% of full load losses.

Stray Losses

In addition to the losses stated above, there may be small losses present which are called as stray losses or miscellaneous losses. These losses are difficult to account. They are usually due to inaccuracies in the designing and modeling of the machine. Most of the times, stray losses are assumed to be 1% of the full load

The power flow from input to output is known as power stages or power flow diagram.

Power Stages

The various stages of energy transformation in a motor and also the various losses occurring in it are shown in the flow diagram of Fig. 29.26.

Overall or commercial efficiency $\eta_c = \frac{C}{A}$, Electrical efficiency $\eta_e = \frac{B}{A}$, Mechanical efficiency $\eta_m = \frac{C}{B}$.

The efficiency curve for a motor is similar in shape to that for a generator (Art. 24.35).

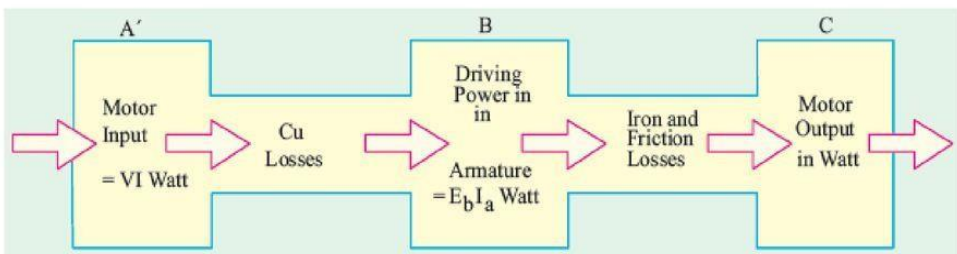
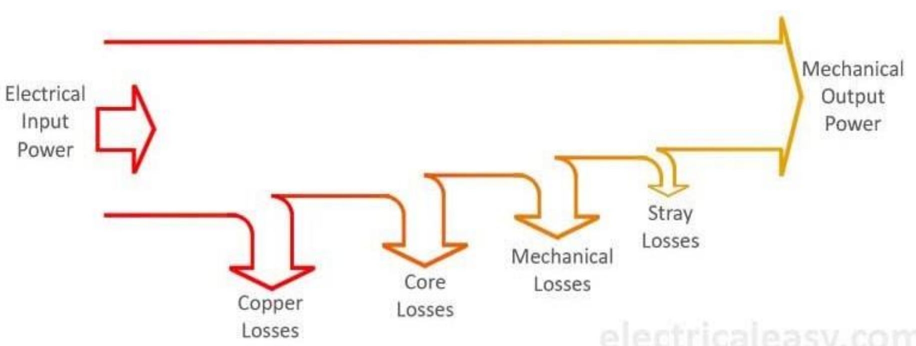


Fig. 29.26

It is seen that $A - B$ = copper losses and $B - C$ = iron and friction losses.

29 – 24



Power flow diagram of a DC motor

10. Starters

At starting when a rated voltage is applied across the stationary armature terminals of the dc motor, it draws more current comparatively greater than the rated current. When such heavy currents pass through the armature windings, it gets overheated and will damage the commutator and brushes. Hence to reduce these heavy inrush currents a resistance must be connected in series with the armature winding. Therefore a starter that consists of set resistances is connected to limit this starting current.

Starting of DC Motors

When the supply is connected to the armature terminals, motor draws huge currents more than its rated current. This is because the resistance of the armature circuit is relatively small. This can be understood from the expression of armature current I_a of the dc motor.

$$I_a = \frac{V - E_b}{R_a}$$

Where,

- V = Supply voltage
- R_a = Armature resistance
- E_b = Back EMF

From the above equation, we can see that armature current I_a is the ratio of voltage V to armature resistance R_a. Initially, when a dc motor is started the bach EMF E_b induced in the armature will be zero (since E = K φ N and N = 0). By substituting E_b = 0, the expression for armature current at starting is given as,

$$I_a(starting) = \frac{V - 0}{R_a}$$

$$= \frac{V}{R_a}$$

Generally, the armature resistance of a dc motor is kept very low (fraction of an ohm), and therefore armature draws current many times the full-load rated current (order of hundreds of amperes). These heavy currents when circulated through the armature winding can cause damage to the winding.

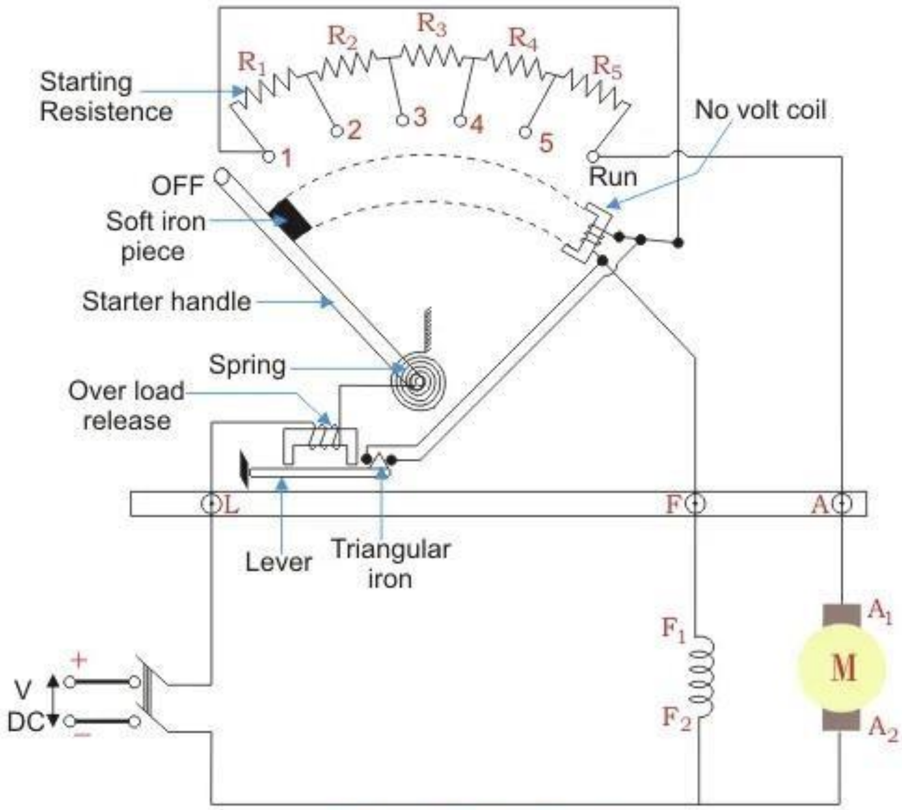
However once the motor starts rotating the back EMF in the motor builds, thus it gradually decreases the starting current (since the value V - E_b decreases) as the speed goes on increasing. At a moment the value of this current becomes less than the rated value. Hence the current should be limited at starting period only in order to prevent damage to the winding. To limit these starting currents flowing in armature winding, resistance is introduced in series with armature winding using Starters.

3 Point Starters

3 point starter is used for shunt and compound dc motor. it is called a **three-point starter** because it has three terminals named L, F, and A. the terminal named L must be connected to either side i.e the positive or negative side of the DC supply. Terminal A must be connected to the one of motor armature terminal and F terminal to the shunt field terminal of the motor (any of two field terminals).

And from there it gets the name 3 point starter. Now studying the **construction of 3 point starter** in further details reveals that the point 'L' is connected to an electromagnet called overload release (OLR) as shown in the figure. The other end of OLR is connected to the lower end of conducting lever of starter handle where spring is also attached with it, and the starter handle also contains a soft iron piece housed on it. This handle is free to move to the other side

RUN against the force of the spring. This spring brings back the handle to its original OFF position under the influence of its own force. Another parallel path is derived from the stud '1', given to another electromagnet called No Volt Coil (NVC) which is further connected to terminal 'F.' The starting resistance at starting is entirely in series with the armature. The OLR and NVC act as the two protecting devices of the starter.



Three Point Starter

Working of Three Point Starter

Having studied its construction, let us now go into the **working of the 3 point starter**. To start with the handle is in the OFF position when the supply to the DC motor is switched on. Then handle is slowly moved against the spring force to make contact with stud No. 1. At this point, field winding of the shunt or the compound motor gets supply through the parallel path provided to starting the resistance, through No Voltage Coil. While entire starting resistance comes in series with the armature. The high starting armature current thus gets limited as the current equation at this stage becomes:

As the handle is moved further, it goes on making contact with studs 2, 3, 4, etc., thus gradually cutting off the series resistance from the armature circuit as the motor gathers speed. Finally, when the starter handle is in ‘RUN’ position, the entire starting resistance is eliminated, and the motor runs with normal speed.

This is because back EMF is developed consequently with speed to counter the supply voltage and reduce the armature current.

So the external electrical resistance is not required anymore and is removed for optimum operation. The handle is moved manually from OFF to the RUN position with the development of speed. Now the obvious question is once the handle is taken to the RUN position how it is supposed to stay there, as long as the motor is running. To find the answer to this question let us look into the working of No Voltage Coil.

Working of No Voltage Coil of 3 Point Starter

The supply to the field winding is derived through no voltage coil. So when field current flows, the NVC is magnetized. Now when the handle is in the ‘RUN’ position, a soft iron piece is connected to the handle and gets attracted by the magnetic force produced by NVC, because of flow of current through it. The NVC is designed in such a way that it holds the handle in ‘RUN’ position against the force of the spring as long as supply is given to the motor. Thus NVC holds the handle in the ‘RUN’ position and hence also called **hold on coil**.

Now when there is any kind of supply failure, the current flow through NVC is affected and it immediately loses its magnetic property and is unable to keep the soft iron piece on the handle, attracted. At this point under the action of the spring force, the handle comes back to OFF position, opening the circuit and thus switching off the motor. So due to the combination of NVC and the spring, the starter handle always comes back to OFF position whenever there is any supply problem. Thus it also acts as a protective device safeguarding the motor from any kind of abnormality.

Drawbacks of a Three Point Starter

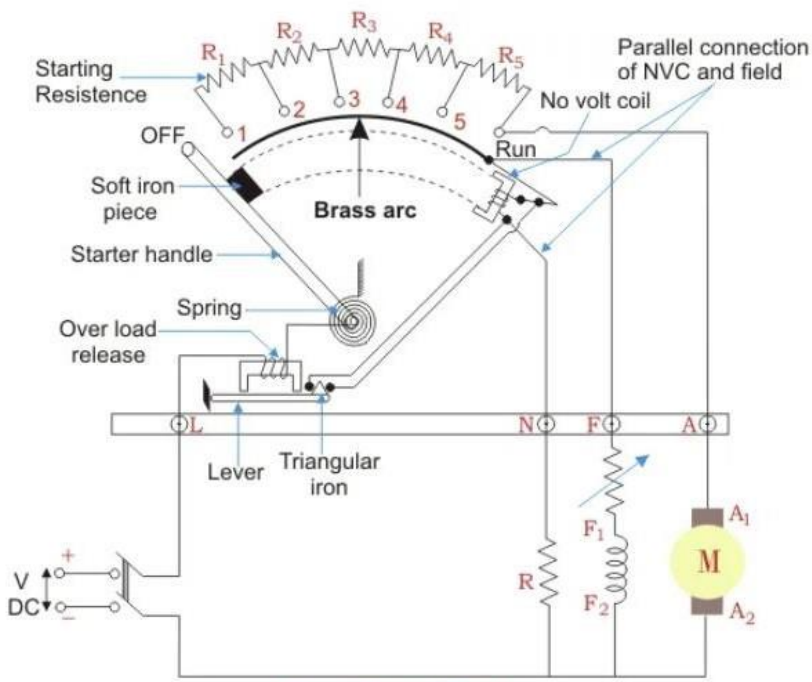
- The **3 point starter** suffers from a serious drawback for motors with a large variation of speed by adjustment of the field rheostat. To increase the speed of the motor field resistance can be increased. Therefore current through the shunt field is reduced.
- Field current becomes very low which results in holding electromagnet too weak to overcome the force exerted by the spring. The holding magnet may release the arm of the starter during the normal operation of the motor and thus disconnect the motor from the line. This is not desirable. A 4 point starter is thus used instead, which does not have this drawback.

4 Point Starters

A 4 point starter as the name suggests has 4 main operational points, namely

1. 'L' Line terminal (Connected to positive of supply.)
2. 'A' Armature terminal (Connected to the armature winding)
3. 'F' Field terminal. (Connected to the field winding.)
4. Like in the case of the 3 point starter, and in addition to it there is, A 4th point N (Connected to the No Voltage Coil NVC)

The remarkable difference in case of a 4 point starter is that the No Voltage Coil is connected independently across the supply through the fourth terminal called 'N' in addition to the 'L', 'F' and 'A'. As a direct consequence of that, any change in the field supply current does not bring about any difference in the performance of the NVC. Thus it must be ensured that no voltage coil always produce a force which is strong enough to hold the handle in its 'RUN' position, against the force of the spring, under all the operational conditions. Such a current is adjusted through No Voltage Coil with the help of fixed resistance R connected in series with the NVC using fourth point 'N' as shown in the figure below.



Four Point Starter

Apart from this above mentioned fact, the 4 point and 3 point starters are similar in all other ways like possessing is a variable resistance, integrated into number of sections as shown in the figure above. The contact points of these sections are called studs and are shown separately as OFF, 1, 2, 3, 4, 5, RUN, over which the handle is free to be maneuvered manually to regulate the starting current with gathering speed.

Now to understand its way of operating let's have a closer look at the diagram given above. Considering that supply is given and the handle is taken stud No.1, then the circuit is complete and the line current that starts flowing through the starter. In this situation we can see that the current will be divided into 3 parts, flowing through 3 different points.

1. 1 part flows through the starting resistance (R_1+ R_2 + R_3…..) and then to the armature.
2. A 2nd part flowing through the field winding F.
3. And a 3rd part flowing through the no voltage coil in series with the protective resistance R.

So the point to be noted here is that with this particular arrangement any change in the shunt field circuit does not bring about any change in the no voltage coil as the two circuits are independent of each other.

This essentially means that the electromagnet pull subjected upon the soft iron bar of the handle by the no voltage coil at all points of time should be high enough to keep the handle at its RUN position, or rather prevent the spring force from restoring the handle at its original OFF position, irrespective of how the field rheostat is adjusted. This marks the operational difference between a 4 point starter and a 3 point starter.

11. Applications of DC Motors

Type of Motor	Characteristics	Applications
Shunt	Speed is fairly constant and medium starting torque.	1. Blowers and fans 2. Centrifugal and reciprocating pumps 3. Lathe machines 4. Machine tools 5. Milling machines 6. Drilling machines
Series	High starting torque. No load condition is dangerous. Variable speed.	1. Cranes 2. Hoists, Elevators 3. Trolleys 4. Conveyors 5. Electric locomotives
Cumulative compound	High starting torque. No load condition is allowed.	1. Rolling mills 2. Punches 3. Shears 4. Heavy planers 5. Elevators
Differential compound	Speed increases as load increases.	Not suitable for any practical applications

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Vishnupur, BHIMAVARAM – 534 202

ELECTRICAL & ELECTRONICS ENGINEERING

III B. Tech. I Semester EEE R19

Electrical Machines – I

Material for Unit III-

PART-B

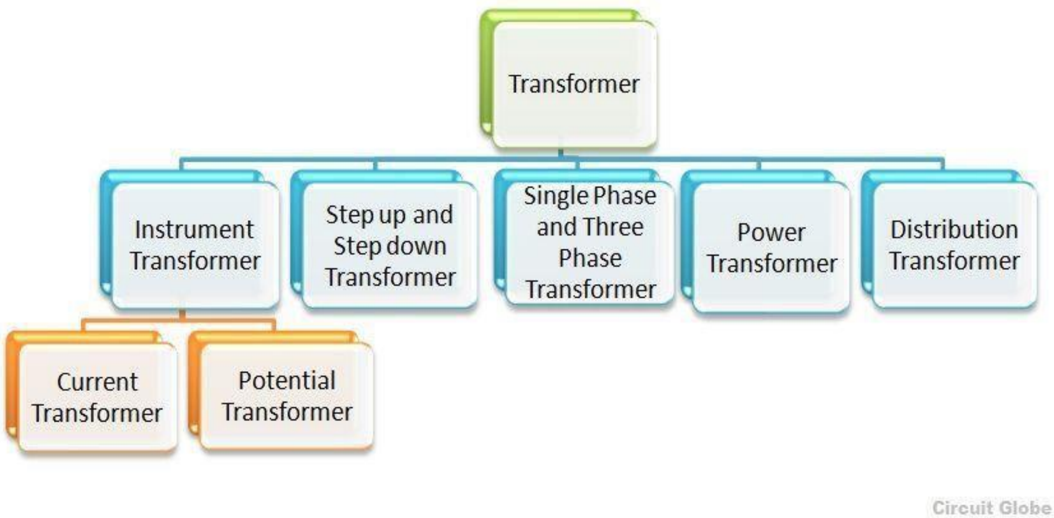
Single-phase Transformers:

Types and constructional details - principle of operation - EMF equation - operation on no load and on load – lagging, leading and unity power factors loads - phasor diagrams of transformers – equivalent circuit.

1. What is a transformer? Write about the types of transformer.

A single-phase transformer is an electrical device that accepts single-phase AC power and outputs single-phase AC. This is used in the distribution of power in non-urban areas as the overall demand and costs involved are lower than the 3-phase distribution transformer. They are used as a step-down transformer to decrease the home voltage to a suitable value without a change in frequency.

There are various types of transformer used in the electrical power system for different purposes, like generation, distribution and transmission and utilization of electrical power. The different types of transformer are Step up and Step down Transformer, Power Transformer, Distribution Transformer, Instrument transformer comprising current and Potential Transformer, Single phase and Three phase transformer, Auto transformer, etc.



Types of Transformer

Based on Construction

- Core type
- Shell type

Based on turns ratio

- Step up
- Step down

Based on services

- Power transformer
- Distribution transformer
- **Instrument transformer**
  - Current transformer
  - Potential transformer
  - Auto-transformer

Based on supply

- Single-phase
- Three-phase

Step up and Step down Transformer

This type of transformer is categorized on the basis of a number of turns in the primary and secondary windings and the induced emf.Step-up transformer transforms a low voltage, high current AC into a high voltage, low current AC system. In this type of transformer the number of turns in the secondary winding is greater than the number of turns in the primary winding. If ($V_2 > V_1$) the voltage is raised on the output side and is known as Step-up transformer. Step down transformer converts a high primary voltage associated with the low current into a low voltage, high current. With this type of transformer, the number of turns in the primary winding is greater than the number of turns in the secondary winding. If ($V_2 < V_1$) the voltage level is lowered on the output side and is known as Step down transformer

Power Transformer

The power transformers are used in the transmission networks of higher voltages. The ratings of the power transformer are as follows 400 KV, 200 KV, 110 KV, 66 KV and 33 KV. They are mainly rated above 200 MVA. Mainly installed at the generating stations and transmission substations. They are designed for maximum efficiency of 100%. They are larger in size as compared to the distribution transformer.

Distribution Transformer

This type of transformer has lower ratings like 11 KV, 6.6 KV, 3.3 KV, 440 V and 230 V. They are rated less than 200 MVA and used in the distribution network to provide voltage transformation in the power system by stepping down the voltage level where the electrical energy is distributed and utilized at the consumer end.

Instrument Transformer

They are generally known as an isolation transformer. Instrument transformer is an electrical device used to transform current as well as a voltage level. The most common use of instrument transformer is to safely isolate the secondary winding when the primary has high voltage and high current supply so that the measuring instrument, energy meters or relays which are connected to the secondary side of the transformer will not get damaged. The instrument transformer is further divided into two types

- Current Transformer (CT)
- Potential Transformer (PT)

Single Phase Transformer

A single-phase transformer is a static device, works on the principle of Faraday's law of mutual Induction. At a constant level of frequency and variation of voltage level, the transformer transfers AC power from one circuit to the other circuit. There are two types of windings in the transformer. The winding to which AC supply is given is termed as Primary winding and in the secondary winding, the load is connected.

Three Phase Transformer

If the three single-phase transformer is taken and connected together with their all the three primary winding connected to each other as one and all the three secondary windings to each other, forming as one secondary winding, the transformer is said to behave as a three-phase transformer, that means a bank of three single-phase transformer connected together which acts as a three-phase transformer.Three-phase supply is mainly used for electric power generation, transmission and distribution for industrial purpose.

2. Write about the construction of single phase transformers.

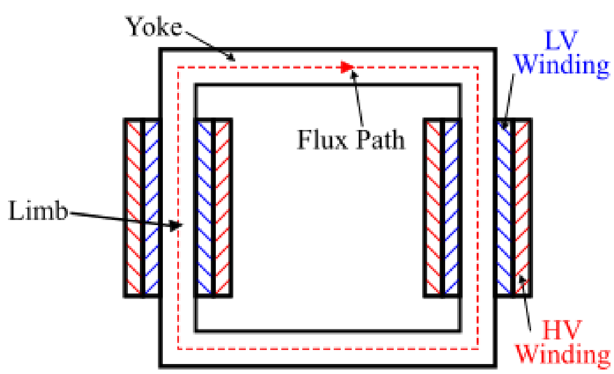
A single phase transformer consists of two windings viz. primary winding and secondary winding put on a magnetic core. The magnetic core is made from thin sheets (called laminations) of high graded silicon steel and provides a definite path to the magnetic flux. These laminations reduce the eddy-current losses while the silicon steel reduces the hysteresis losses.

The laminations are insulated from each other by enamel insulation coating. The thin laminations are stacked together to form the core of the transformer. The air-gap between the laminations should be minimum so that the excitation current being minimum.

For a single phase transformer, there are two types of transformer constructions viz. the core type and the shell type.

Core Type Transformer Construction

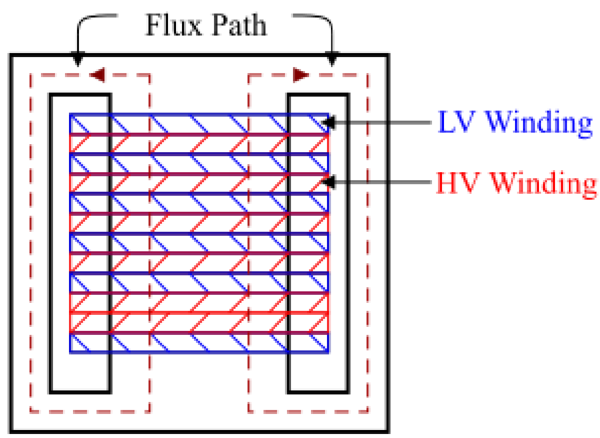
In *core type construction* of the transformer, the magnetic circuit consists of two vertical lags (called *limbs*) and two horizontal sections called *yokes*. To minimize the effect of leakage flux, half of each winding is placed on each limb (see the figure).The low-voltage winding is placed next to the core while the high-voltage winding over the low-voltage winding to reduce the insulation requirements. Therefore the two windings are arranged as *concentric coils* and known as *cylindrical winding*.



Core Type Construction

Shell Type Transformer Construction

In the *shell type construction* of transformer, the magnetic circuit consists of three limbs, both the primary and secondary windings are placed on the central limb and the two outer limbs complete the low reluctance flux path. The each winding is sub-divided into sections viz. the low voltage (LV) section and the high-voltage (HV) section, which are alternatively put one over the other in the form of sandwich (see the figure). Therefore, such windings are called *sandwich winding* or *disc winding*.



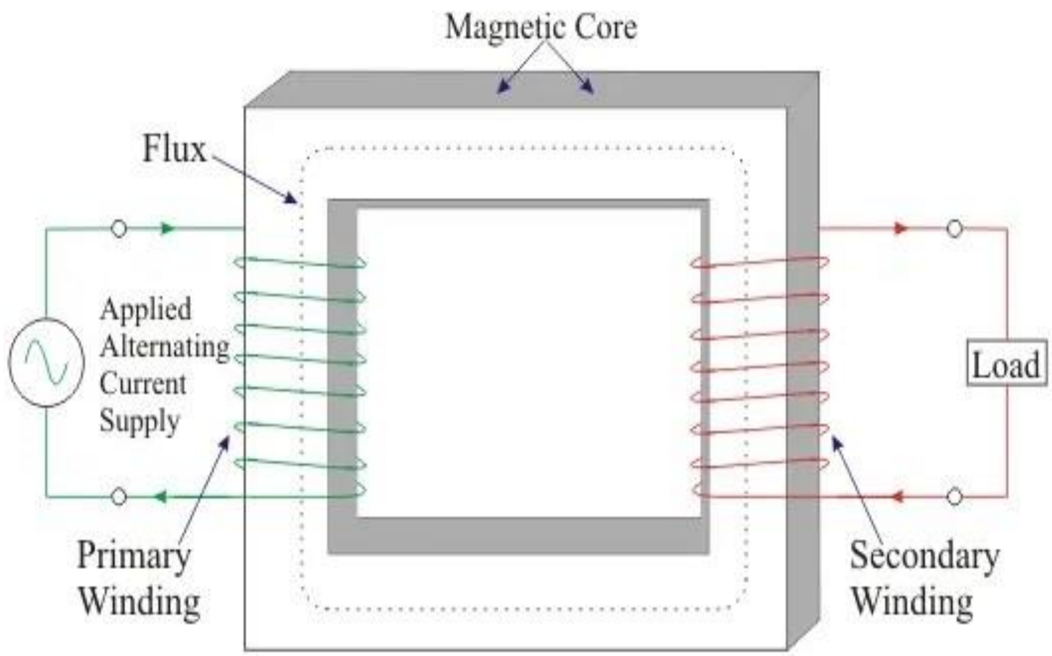
Shell Type Construction

3. Write about the principle operation of single phase transformers?

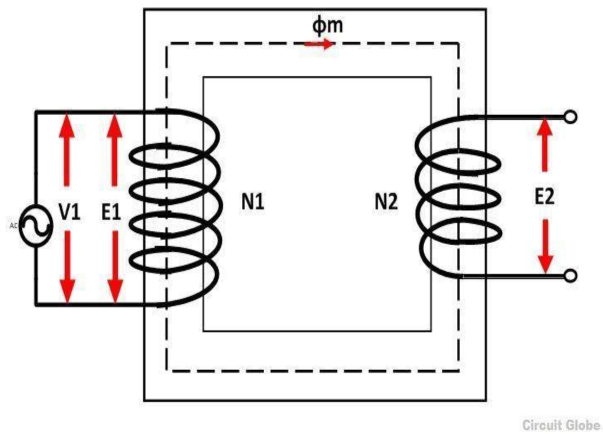
The single-phase transformer works on the principle of Faraday's Law of Electromagnetic Induction. Typically, mutual induction between primary and secondary windings is responsible for the transformer operation in an electrical transformer.

A transformer is a static device that transfers electric power in one circuit to another circuit of the same frequency. It consists of primary and secondary windings. This transformer operates on the principle of mutual inductance.

When the primary of a transformer is connected to an AC supply, the current flows in the coil and the magnetic field build-up. This condition is known as mutual inductance and the flow of current is as per the Faraday's Law of electromagnetic induction. As the current increases from zero to its maximum value, the magnetic field strengthens and is given by $d\phi/dt$.



Single Phase Transformer



This electromagnet forms the magnetic lines of force and expands outward from the coil forming a path of magnetic flux. The turns of both windings get linked by this magnetic flux. The strength of a magnetic field generated in the core depends on the number of turns in the winding and the amount of current. The magnetic flux and current are directly proportional to each other.

As the magnetic lines of flux flow around the core, it passes through the secondary winding, inducing voltage across it. The Faraday's Law is used to determine the voltage induced across the secondary coil and it is given by:

$$\mathbf{N.\ d\phi/dt}$$

Where,

'N' is the number of coil turns

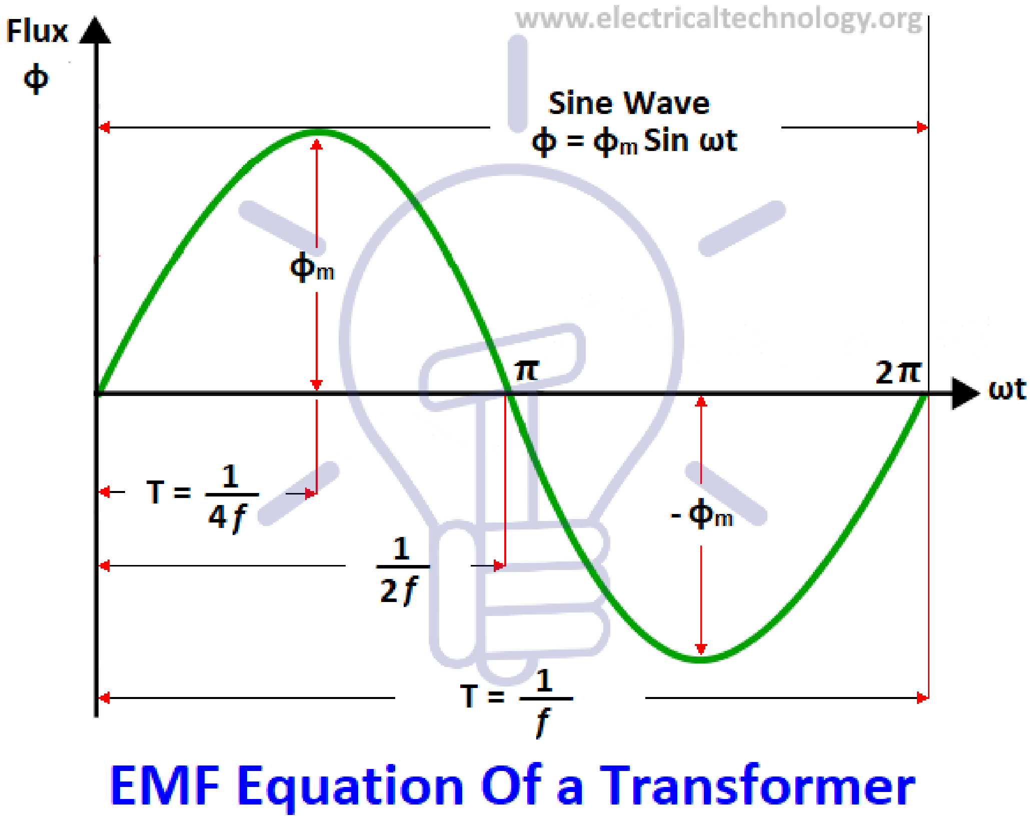
The frequency is the same in primary and secondary windings.

Thus, we can say that the voltage induced is the same in both the windings as the same magnetic flux links both the coils together. Also, the total voltage induced is directly proportional to the number of turns in the coil.

4. Derive the EMF equation of single phase transformers.

Let's,

- **N_1** = Number of turns in primary windings.
- **N_2** = Number of turns in second windings.
- **Φ_m** = Maximum flux in the core in Weber = (**Φ_m** = B_m.A)
- f = Frequency of A.C input in H_z.



As shown in fig above- flux increases from its zero value to maximum value **Φ_m** in one quarter of the cycle i.e. in ¼ **second**.

$$\therefore\ average\ rate\ of\ change\ of\ flux = \frac{\Phi m}{1/4f}$$

$$= \mathbf{4}\ f\ \Phi_m\ \textbf{Wb/s or volt}$$

Now rate of change of flux per turn means induced EMF in volts

$$\textbf{Average EMF / per turn} = 4f\,\Phi_m \text{ volt.}$$

If flux Φ_m varies sinusoidally, then RMS value of induced EMF is obtained by multiplying the average value with form factor.

$$\textbf{Form Factor} = \textit{RMS value / Average value} = 1.11$$

$$\text{RMS value of EMF / turn} = 1.11.\ 4f\,\Phi_m = 4.44f\,\Phi_m \text{ volt}$$

Now RMS value of the induced EMF in the whole primary winding= (Induced**EMF / turn**) x **number of primary turns**

$$E_1 = 4.44 \times f \times N_1\ \Phi_m \ldots\ldots\ldots\ldots\ldots (i)$$

$$E_1 = 4.44 \times f\ N_1\ B_m A \ \ldots\ [as\ (\Phi_m = B_m A)]$$

Similarly, RMS value of the EMF induced in secondary is,

$$E_2 = 4.44 \times f\ N_2\ \Phi_m \ldots\ldots\ldots\ldots\ldots (ii)$$

$$E_2 = 4.44 \times f\ N_2\ B_m A. \ \ldots\ [as\ (\Phi_m = B_m A)]$$

It's seen from *(i)* and *(ii)* that: **EMF Equation of the Transformer =**

$$E_1 / N_1 = E_2 / N_2 = 4.44 \times f\ \Phi_m \ldots\ldots\ldots (iii)$$

It means that **EMF / turn is the same in both the primary and secondary windings in the transformer** i.e. flux in Primary and Secondary Winding of the Transformer is same.Moreover, we already know that from the power equation of the transformer, i.e, in ideal Transformer (there are no losses in transformer) on no-load,

$$V_1 = E_1$$

and

$$\mathbf{E_2 = V_2}$$

Where,

- $\mathbf{V_1 =}$ supply voltage of primary winding
- $\mathbf{E_2}$ = terminal voltage induced in the secondary winding of the transformer.

Voltage Transformation Ratio (K)

As we have derived from the above EMF equation of the transformer

$$\frac{E_2}{E_1} = \frac{4.44fN_2\varphi_m}{4.44fN_1\varphi_m}$$

Or

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

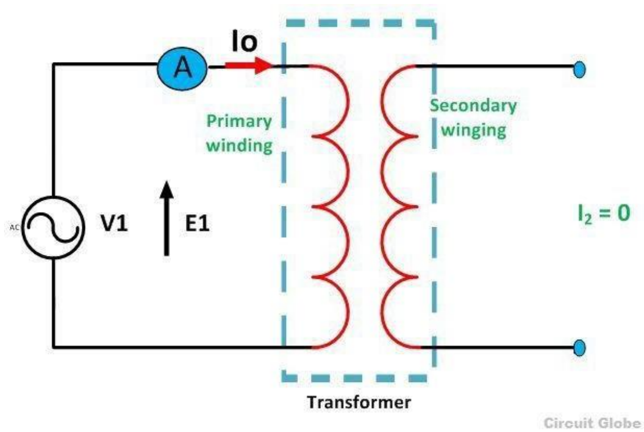
The above equation is called the **turn ratio** where **K** is known as the transformation ratio.

i) If ***N2*** > ***N1*** i.e K > 1, then transformer is called step-up transformer.

ii) If ***N2*** < ***N1*** i.e K < 1, then transformer is called step-down transformer.

5. Single phase transformers – No load operation &Phasor diagram

When the transformer is operating at no load, the secondary winding is open-circuited, which means there is no load on the secondary side of the transformer and, therefore, current in the secondary will be zero. While primary winding carries a small current I_0 called no-load current which is **2 to 10% of the rated current**.This current is responsible for supplying the iron losses (hysteresis and eddy current losses) in the core and a very small amount of copper losses in the primary winding. The angle of lag depends upon the losses in the transformer. The power factor is very low and varies from **0.1 to 0.15**.



The no-load current consists of two components:

- **Reactive or magnetizing component (I_m)**

  (It is in quadrature with the applied voltage V_1. It produces flux in the core and does not consume any power).

- **Active or power component (I_w or I_c)**, also known as a working component

  (It is in phase with the applied voltage **V_1.** It supplies the iron losses and a small amount of primary copper loss).

The following steps are given below to draw the phasor diagram:

1. The function of the magnetizing component is to produce the magnetizing flux, and thus, it will be in phase with the flux.

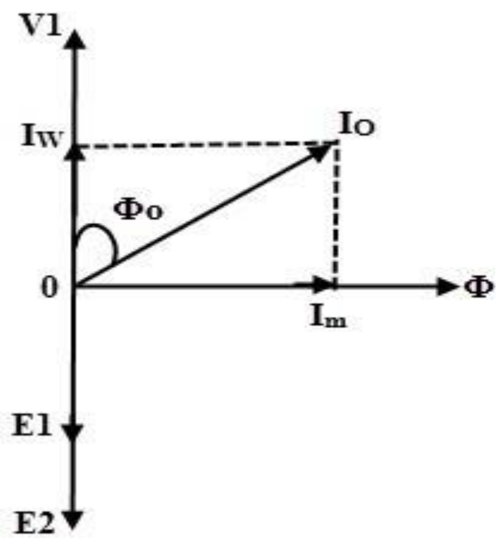
2. Induced emf in the primary and the secondary winding lags the flux ϕ by 90 degrees.

3. The primary copper loss is neglected, and secondary current losses are zero as $I_2 = 0$. Therefore, the current I_0 lags behind the voltage vector V_1 by an angle ϕ_0 called the no-load power factor angle and is shown in the phasor diagram above.

4. The applied voltage V_1 is drawn equal and opposite to the induced emf E_1 because the difference between the two, at no load, is negligible.

5. Active component I_w or I_c is drawn in phase with the applied voltage V_1.

6. The phasor sum of magnetizing current I_m and the working current I_w or I_c gives the no-load current I_0.



From the phasor diagram drawn above, the following conclusions are made

The magnetizing component of no load current, $I_m = I_o \sin \Phi o$

The core loss component of no load current, I_w or $I_c = I_o \cos \Phi o$

The no load current magnitude, $I_o = \sqrt{(I^2_m + I^2_c)}$

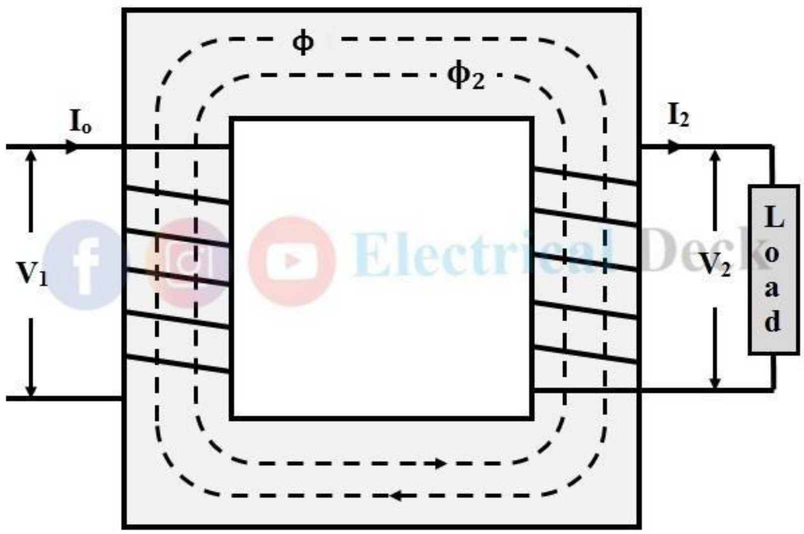
Where Φo no load angle between primary voltage and current.

Total power input on no load, $W_o = V_1 \times I_o \times \cos \Phi o$

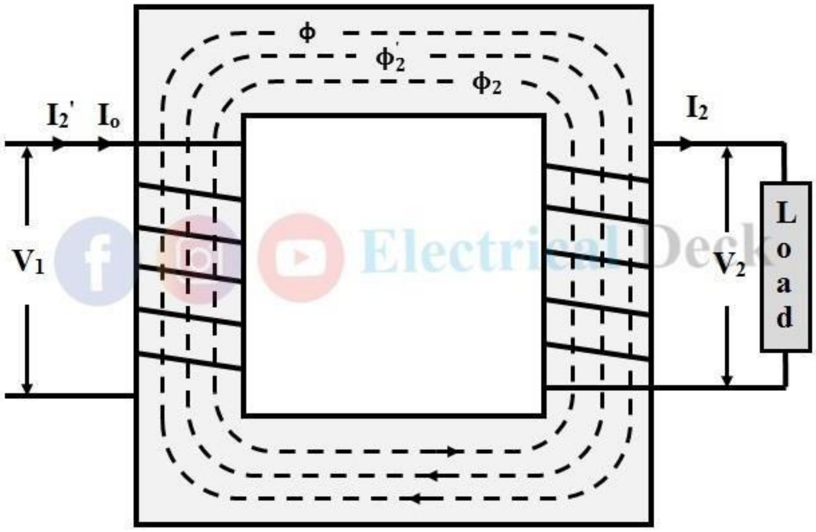
6. Single phase transformers – Load operation & Phasor diagram

When the transformer is loaded i.e., its secondary is connected to the load terminals. The connected load can be resistive, inductive, and capacitive. At this condition, the secondary current I_2 starts flowing in the secondary winding. The phase angle of this secondary current I_2 with respect to the secondary voltage V_2 will depend upon the nature of the load. The current I_2 will be in-phase with the secondary voltage V_2 if the load is purely resistive, it lags behind the voltage if the load is inductive, and it leads the voltage if it is capacitive.

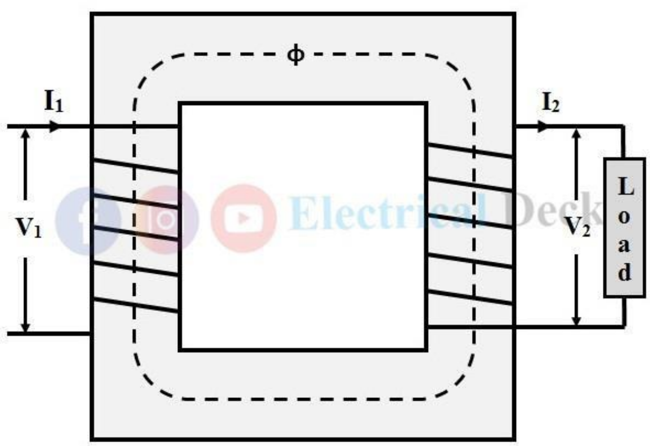
- Now, this current I_2 circulates and produces its own flux Φ_2, and it is in opposition to the primary winding flux Φ which is due to I_o. This opposition causes to weakens the Φ and hence the resultant flux in the transformer reduces which in turn reduces the primary induced EMF E_1 as shown in the below figure.



- At a moment when E_1 goes on decreasing, the voltage V_1 gains the upper hand over E_1. This causes the primary winding to draw additional current I_2' in order to restore the primary flux Φ so that $E_1 = V_1$. This additional current I_2' produces additional flux Φ_2' in the primary as shown below.



- The nature of the Φ_2' is in such a way that it cancels out the flux Φ_2 produced by the secondary current I_2, but it is in the same direction as the main flux Φ. This completely causes to neutralize the magnetic effect of the secondary current. This whole process starts at the instant when the transformer is load. Therefore, the net flux in the transformer when it is load is only due to primary (i.e., φ) which is the same as that of the no-load condition shown in the below figure.



Therefore, we can conclude that whatever will be the condition of the transformer (either no-load or on-load). The operation will be the same if the voltage drop across in the windings is not assumed and the core losses will remain constant under all conditions.

$$since\ \phi_2 = \phi_2'$$

$$N_2 I_2 = N_1 I_2'$$

$$I_2' = \frac{N_2}{N_1} \times I_2$$

$$\therefore\ I_2' = K\, I_2$$

Hence, when the transformer is loaded, the primary has two components currents in it i.e., no-load I_o and load component of primary current I_2'. The total primary current is the vector sum of I_o and I_2' as given by,

$$I_1 = \sqrt{(I_2' + I_o)^2}$$

$$Where$$

$$I_o = \sqrt{\left(I_\mu + I_W\right)^2}$$

Phasor Diagram of Transformer on Load Conditions:

The phasor or vector diagrams for a transformer on resistive, inductive, and capacitive loads are drawn by taking flux Φ as the reference. Let,

- V_1 = Primary supply voltage.
- E_1 & E_2 = Primary and secondary induced emf's.
- I_o = No-load primary input current.
- I_2 = Primary current.
- I_2 = Secondary current.
- I_2' = Balancing current or load component of the primary current.

For Resistive (non-inductive) Load:

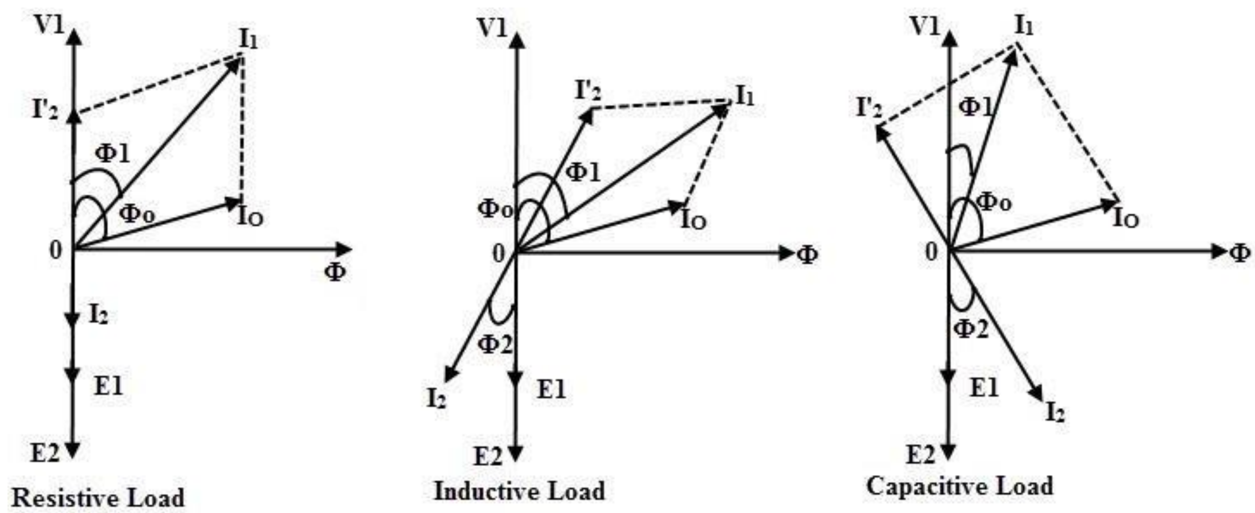
When the transformer secondary is connected to a resistive load, the current will be in phase with the voltage.

For Inductive Load:

When the transformer secondary is connected to an inductive load, the current flowing will lags with respect to the voltage as shown below.

For Capacitive Load:

Similarly, when the transformer secondary is connected to a capacitive load, the current flowing will leads the respective voltage as shown below.



In a transformer when it is loaded as compared to current I_2', I_o is very small, and if we neglect it. Then primary current I_1 is equal to I_2' ($I_1 = I_2'$). We get,

$$N_1\, I_2' = N_2\, I_2$$

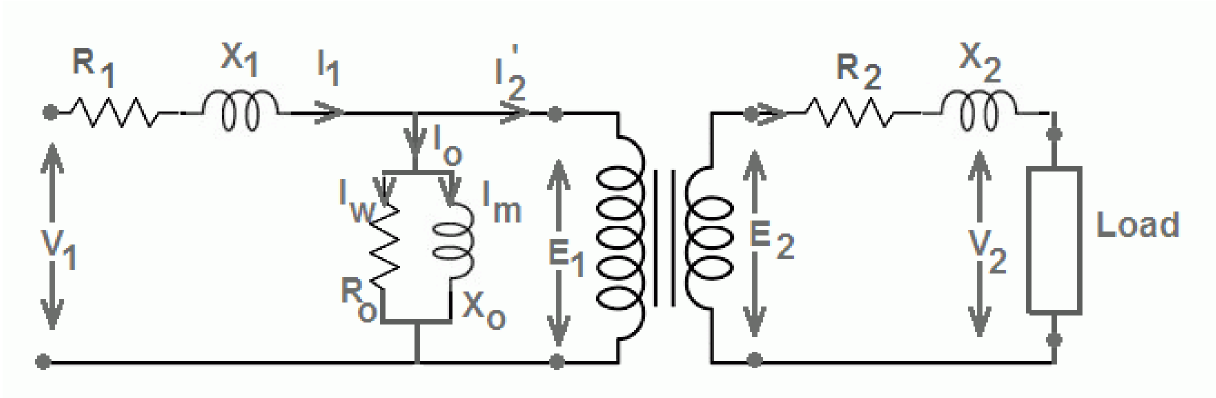
$$or$$

$$N_1\, I_1 = N_2\, I_2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{N_2}{N_1} = K$$

7. Equivalent circuit of Single phase transformers

The **equivalent circuit of transformer** is shown in the figure.



Equivalent circuit of transformer

No load Components

The no-load primary current $\mathbf{I}_o$ has two components, namely $\mathbf{I}_m$ and $\mathbf{I}_w$.

Where $\mathbf{I}_m$ = magnetizing component = $\mathbf{I}_o \sin \varphi_o$ and $\mathbf{I}_w$ or $\mathbf{I}_c$ = core-loss component = $\mathbf{I}_o \cos\varphi_o$.

- $\mathbf{I}_w$ or $\mathbf{I}_c$ supplies for the no-load losses and is assumed to flow through the no-load resistance which is also known as core-loss resistance (R_o).
- The magnetizing component, $\mathbf{I}_m$ is assumed to be flowing through a reactance which is known as magnetizing reactance, X_o.
- The parallel combination of R_o and X_o is also known as the **exciting circuit**.From the **equivalent circuit of transformer**, $R_o = V_1/\mathbf{I}_w$ and $X_o = V_1/\mathbf{I}_m$.
- The core-loss resistance (R_o) and the magnetizing reactance (X_o) of a transformer are determined by the open circuit test of transformer.

Primary Components

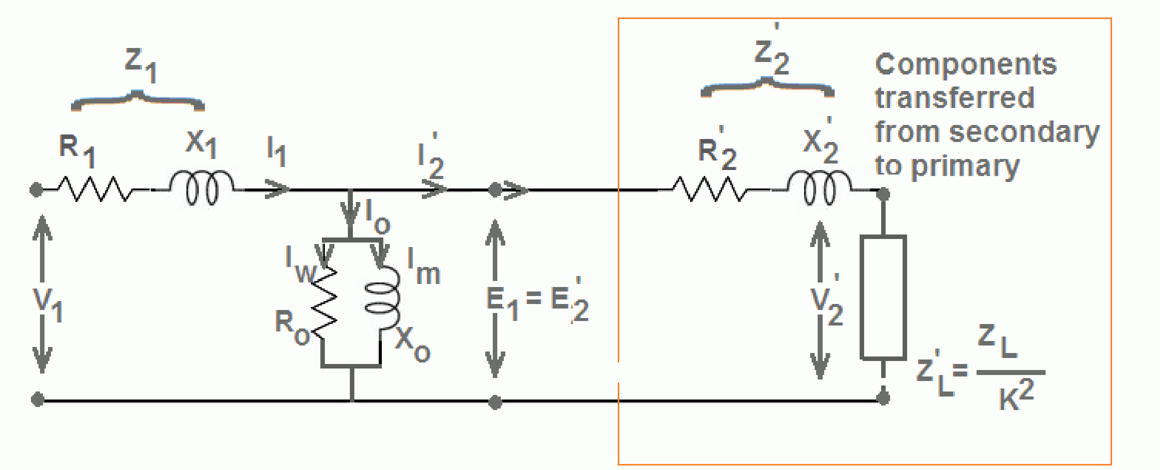
- The resistance R_1 and reactance X_1 correspond to the winding resistance (DC resistance) and leakage reactance of the primary winding.

- The total current $\mathbf{I}_1$ on the primary side is equal to the phasor sum of $\mathbf{I}_o$ and $\mathbf{I}_2$'.
- $\mathbf{I}_2' = K\mathbf{I}^2$ is the additional primary current which flows due to the load connected on the secondary side of the transformer.

Secondary Components

- The resistance R_2 and reactance X_2 correspond to the winding resistance and leakage reactance of the secondary winding.
- Load impedance Z_L can be resistive, inductive or capacitive.
- The equivalent circuit of single phase transformer is further simplified by transferring all the quantities to either primary or secondary side.
- This is done in order to make the calculations easy.

Equivalent Circuit of Transformer Referred to Primary



Equivalent circuit of the transformer referred to the primary

All the components on the secondary side of the transformer are transferred to the primary side as shown in the figure.

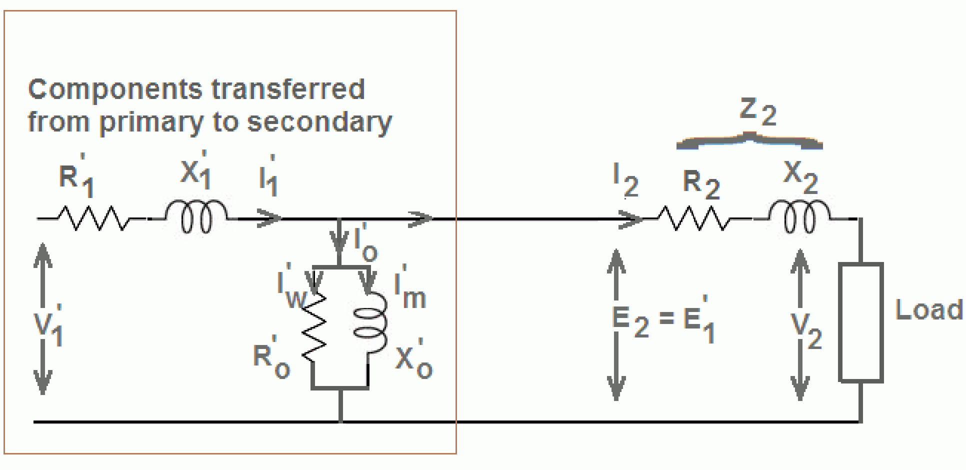
- R_2', X_2' and Z_L' are the values of R_2, X_2 and Z_L referred to primary respectively.
- The values of these components are obtained as follows:

  $R_2' = R_2/K^2$, $X_2' = X_2/K^2$ and $Z_L' = Z_L/K^2$ where $K = N_2/N_1$ (transformation ratio).

- The current $\mathbf{I}_2$ and voltage E_2 are also transferred to the primary side as $\mathbf{I}_2$' and E_2' respectively. The expressions for $\mathbf{I}_2$' and E_2' are as follows:

  $E_2' = E_2/K$ and $\mathbf{I}_2' = K\mathbf{I}_2$

Equivalent Circuit of Transformer Referred to Secondary



Equivalent circuit of transformer referred to the secondary

The equivalent circuit of transformer referred to the secondary side is shown in the figure.

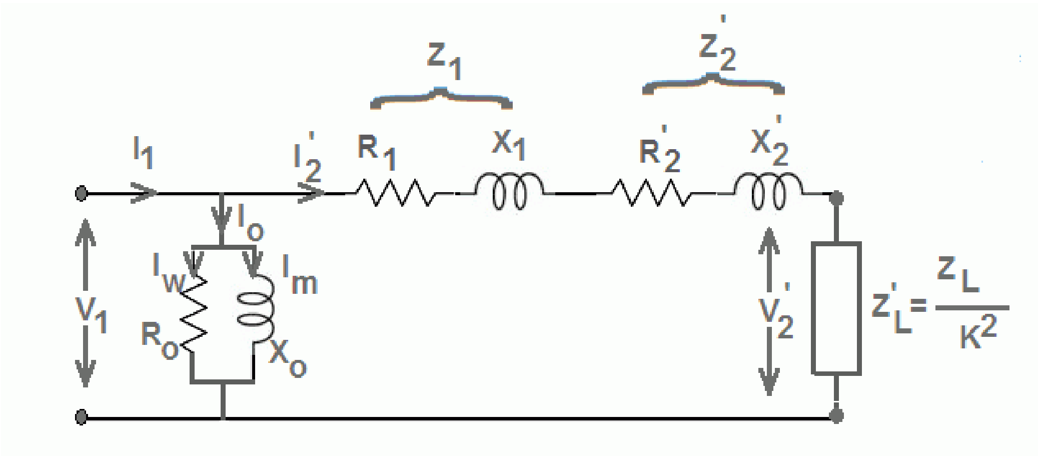
- Components R_1', X_1', R_o' and X_o' are the primary components referred to secondary. The expressions for these components are as follows:

  $R_1' = K^2R_1$, $X_1' = K^2X_1$, $R_o' = K^2R_o$, $X_o' = K^2X_o$

- The primary voltages and currents also get transferred to the secondary side as $\mathbf{I}_1$', V_1', $\mathbf{I}_o$', E_1' respectively and are given by:

  $\mathbf{I}_1' = \mathbf{I}_1/K$, $E_1' = KE_1$, $\mathbf{I}_o' = \mathbf{I}_o/K$ where $K = N_2/N_1$ (transformation ratio).

Approximate Equivalent Circuit of Transformer



Approximate equivalent circuit referred to the primary

An approximate equivalent circuit is one which is obtained by shifting the exciting circuit to the left of R_1 and X_1 as shown in the figure. Although this shifting creates an error in the voltage drop across R_1 and X_1 yet it greatly simplifies the calculation work and gives much simplified equivalent circuit.

- Now it is possible to combine the resistances R_1 with R_2' and X_1 with X_2'. So R_1 and R_2' are combined to obtain the equivalent resistance of transformer referred to the primary R_{01}.

  Therefore, $R_{1e} = R_1 + R_2' = R_1 + R_2/K^2$

- Similarly X_1 and X_2' can be combined to obtain the equivalent reactance of transformer referred to primary X_{01}.

  Therefore, $X_{1e} = R_1 + R_2' = R_1 + R_2/K^2$

Now the impedance of the transformer referred to the primary is given by, $Z_{1e} = R_{1e} + jX_{1e}$

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Vishnupur, BHIMAVARAM – 534 202

ELECTRICAL & ELECTRONICS ENGINEERING

III B. Tech. I Semester EEE R20

Electrical Machines – I

Material for Unit IV

Unit 2: Testing of Single-phase Transformers: Open Circuit & Short Circuit test – regulation,– losses and efficiency– Sumpner's test -separation of losses – parallel operation - All day efficiency – Auto Transformer (Qualitative Treatment Only)

1. O.C. and S.C. Tests on Single Phase Transformer

The efficiency and regulation of a transformer on any load condition and at any power factor condition can be predetermined by indirect loading method. In this method, the actual load is not used on transformer. But the equivalent circuit parameters of a transformer are determined by conducting two tests on a transformer which are,

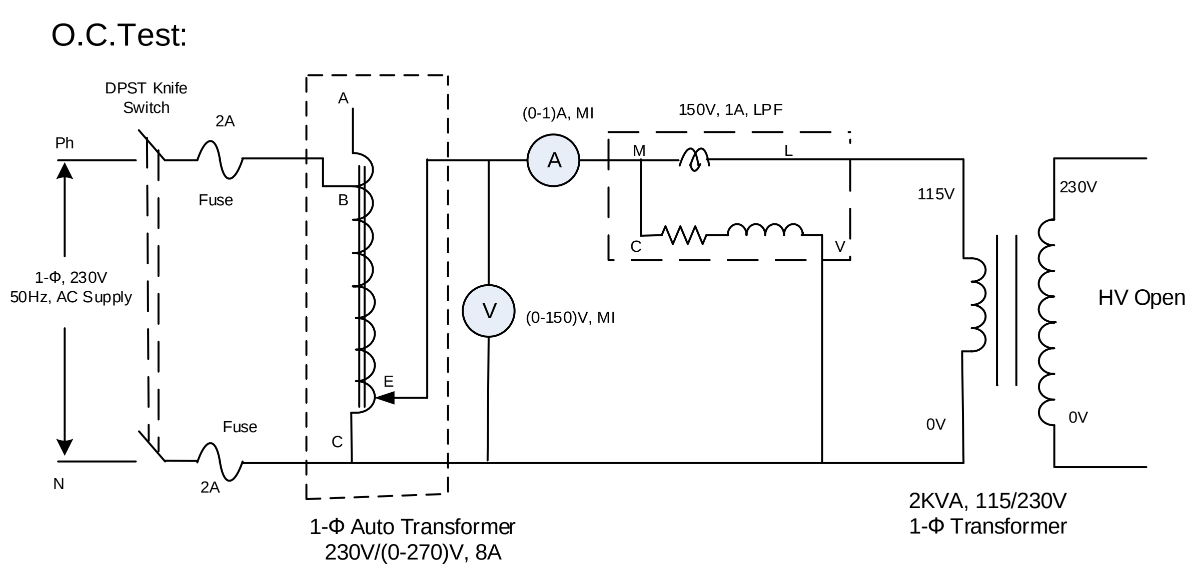
1. Open circuit test (O.C Test)

2. Short circuit test (S.C Test)

The parameters calculated from these test results are effective in determining the regulation and efficiency of a transformer at any load and power factor condition, without actually loading the transformer. The advantage of this method is that without much power loss the tests can be performed and results can be obtained. Let us discuss in detail how to perform these tests and how to use the results to calculate equivalent circuit parameters

Open Circuit Test (O.C. Test)

The experimental circuit to conduct O.C test is shown in the Figure.



The transformer primary is connected to AC supply through ammeter, wattmeter and variac. The secondary of transformer is kept open. Usually low voltage side is used as primary and high voltage side as secondary to conduct O.C test. The primary is excited by rated voltage, which is adjusted precisely with the help of a variac. The wattmeter measures input power. The ammeter measures input current. The voltmeter gives the value of rated primary voltage applied at rated frequency.

Sometimes a voltmeter may be connected across secondary to measure secondary voltage which is $V_2 = E_2$ when primary is supplied with rated voltage. As voltmeter resistance is very high, though voltmeter is connected, secondary is treated to be open circuit as voltmeter current is always negligibly small.

When the primary voltage is adjusted to its rated value with the help of variac, readings of ammeter and wattmeter are to be recorded. The observation table is as follows

V_o volts	I_o amperes	W_o watts
Rated		

V_o = Rated voltage

W_o = Input power

I_o = Input current = no load current

As transformer secondary is open, it is on no load. So current drawn by the primary is no load current I_o. The two components of this no load current are,

$$I_m = I_o \sin \Phi_o$$

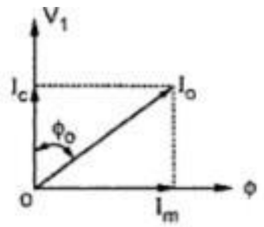
$$I_c = I_o \cos \Phi_o$$

Where $\cos \Phi_o$ = No load power factor

And hence power input can be written as,

$W_o = V_o I_o \cos \Phi_o$

The phasor diagram is shown in the Fig. 2.



As secondary is open, $I_2 = 0$. Thus its reflected current on primary is also zero. So we have primary current $I_1 = I_o$. The transformer no load current is always very small, hardly 2 to 4 % of its full load value. As $I_2 = 0$, secondary copper losses are zero. And $I_1 = I_o$ is very low hence copper losses on primary are also very very low. Thus the total copper losses in O.C. test are negligibly small. As against this the input voltage is rated at rated frequency hence flux density in the core is at its maximum value. Hence iron losses are at rated voltage. As output power is zero and copper losses are very low, the total input power is used to supply iron losses. This power is measured by the wattmeter i.e. W_o. Hence the wattmeter in O.C. test gives iron losses which remain constant for all the loads.

$W_o = P_i =$ Iron losses

Calculations : We know that,

$W_o = V_o I_o \cos \Phi$

$\cos \Phi_o = W_o / (V_o I_o)$ = no load power factor

Once $\cos \Phi_o$ is known we can obtain,

$I_c = I_o \cos \Phi_o$

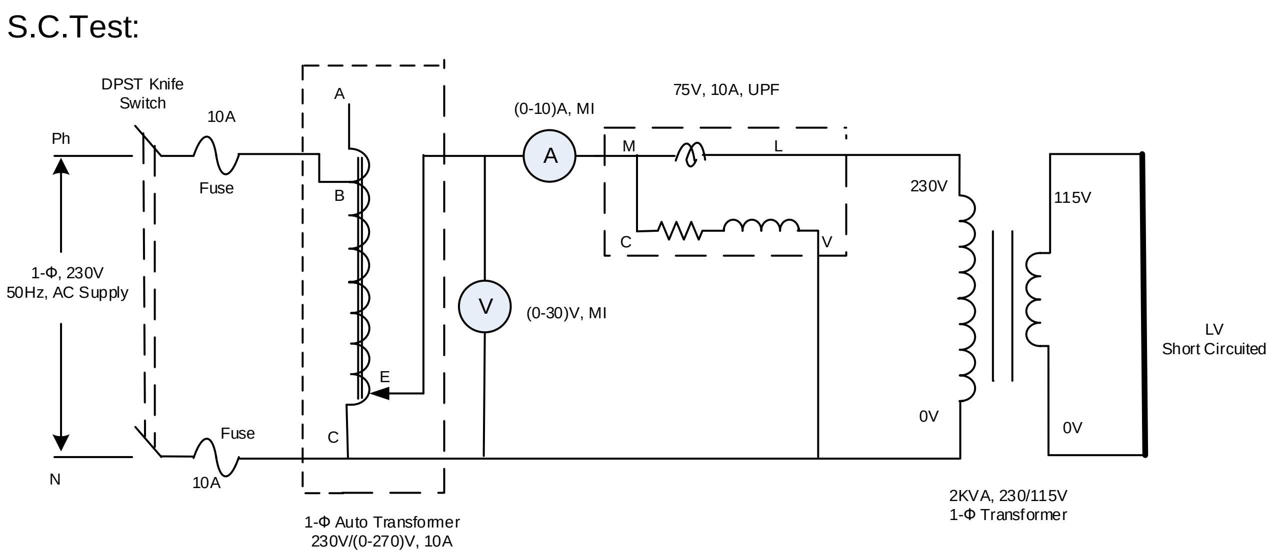
and $I_m = I_o \sin \Phi_o$

Once I_c and I_m are known we can determine exciting circuit parameters as,

$$Ro = V_o / I_c \, \Omega \text{ and } X_o = V_o / I_m \, \Omega$$

Short Circuit Test (S.C. Test)

In this test, primary is connected to a.c. supply through variac, ammeter and voltmeter as shown in the Fig. 3.



The secondary is short circuited with the help of thick copper wire or solid link. As high voltage side is always low current side, it is convenient to connect high voltage side to supply and shorting the low voltage side. As secondary is shorted, its resistance is very very small and on rated voltage it may draw very large current. Such large current can cause overheating and burning of the transformer. To limit this short circuit current, primary is supplied with low voltage which is just enough to cause rated current to flow through primary which can be observed on an ammeter. The low voltage can be adjusted with the help of variac. Hence this test is also called low voltage test or reduced voltage test. The wattmeter reading as well as voltmeter, ammeter readings are recorded. The observation table is as follows

V_{sc} volts	I_{sc} amperes	W_{sc} watts
	Rated	

Now the current flowing through the windings are rated current hence the total copper loss is full load copper loss. Now the voltage supplied is low which is a small fraction of the rated voltage. The iron losses are function of applied voltage. So the iron losses in reduced voltage test are very small. Hence the wattmeter reading is the power loss which is equal to full load copper losses as

$W_{sc} = (P_{cu})$ F.L. = Full load copper loss

Calculations : From S.C. test readings we can write,

$W_{sc} = V_{sc} I_{sc} \cos \Phi_{sc}$.

$\cos \Phi_{sc} = V_{sc} I_{sc} / W_{sc}$ = short circuit power factor

$W_{sc} = I_{sc}^2 R_{1e}$ = copper loss

$R_{1e} = W_{sc} / I_{sc}^2$

While $Z_{1e} = V_{sc} / I_{sc} = \sqrt{(R1e^2 + X1e^2)}$

. .. $X_{1e} = \sqrt{(Z_{1e}^2 - R_{1e}^2)}$

Thus we get the equivalent circuit parameters R1e, X1e and Z1e. Knowing the transformation ratio K, the equivalent circuit parameters referred to secondary also can be obtained.

Calculation of Efficiency from O.C. and S.C. Tests

We know that,

From O.C. test, $W_o = P_i$

From S.C. test, $W_{sc} = (P_{cu})$ F.L.

$$\therefore \quad \% \eta \text{ on full load } = \frac{V_2 (I_2) \text{F.L.} \cos \phi}{V_2 (I_2) \text{F.L.} \cos + W_o + W_{sc}} \times 100$$

Thus for any PF $\cos\Phi_2$ the efficiency can be predetermined. Similarly at any load which is fraction of full load then also efficiency can be predetermined as,

$$\boxed{\% \eta \text{ at any load} = \frac{n \times (\text{VA rating}) \times \cos\phi}{n \times (\text{VA rating}) \times \cos\phi + W_o + n^2 W_{sc}} \times 100}$$

Where n = fraction of full load

$$\text{or} \qquad \% \eta = \frac{n V_2 I_2 \cos\phi}{n V_2 I_2 \cos\phi + W_o + n^2 W_{sc}} \times 100$$

Where $I_2 = n\,(I_2)$ F.L.

Calculation of Regulation

From S.C. test we get the equivalent circuit parameters referred to primary or secondary.

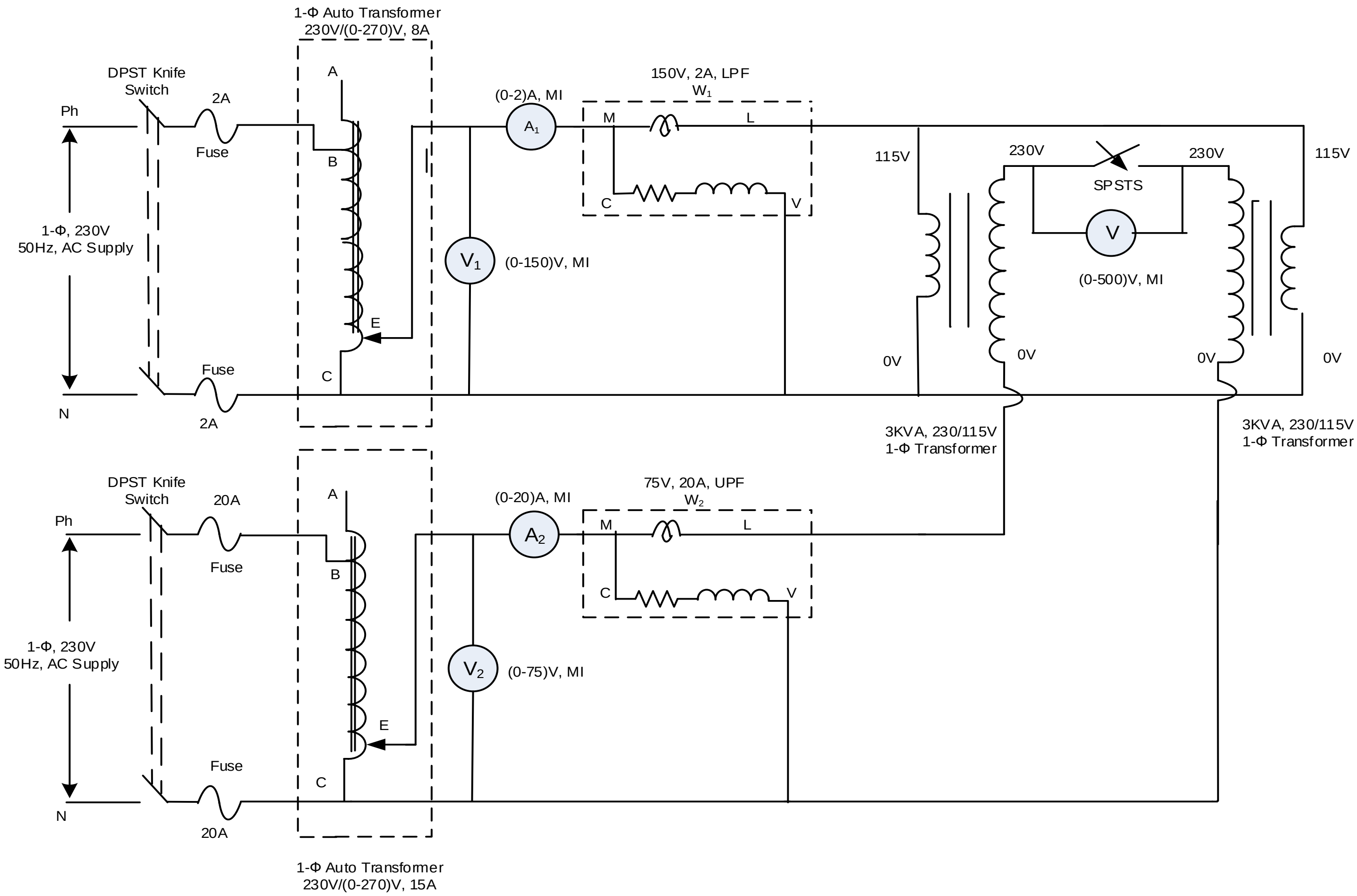
The rated voltages V_1, V_2 and rated currents (I_1) F.L. and (I_2) F.L. are known for the given transformer. Hence the regulation can be determined as,

$$\boxed{\begin{aligned} \% R &= \frac{I_2 R_{2e} \cos\phi \pm I_2 X_{2e} \sin\phi}{V_2} \times 100 \\ &= \frac{I_1 R_{1e} \cos\phi \pm I_1 X_{1e} \sin\phi}{V_1} \times 100 \end{aligned}}$$

Where I_1, I_2 are rated currents for full load regulation. For any other load the currents I_1, I_2 must be changed by fraction n.

I_1, I_2 at any other load = $n\,(I_1)$ F.L., $n\,(I_2)$ F.L

2. Sumpners Test on Two Single Phase Transformer



The Sumpner's test requires two identical transformers. Both the transformers are connected to the supply such that one transformer is loaded on the other. Thus power taken from the supply is that much necessary for supplying the losses of both the transformers and there is very small loss in the control circuit.

While conducting this test, the primaries of the two identical transformers are connected in parallel across the supply V_1. While the secondaries are connected in series opposition so that induced e.m.f.s in the two secondaries oppose each other. The secondaries are supplied from another low voltage supply are connected in each circuit to get the readings. The connection diagram is shown in the Fig. 1.

T_1 and T_2 are two identical transformers. The secondaries of T_1 and T_2 are connected in series opposition. Therefore the induced EMF in two secondaries is equal and acts in opposite direction to each other and cancels each other. So net voltage ion the local circuit of secondaries is zero, when primaries are excited by supply 1 of rated voltage and frequency. So there is no current flowing in the loop formed by two secondaries. The series opposition can be checked by another voltmeter connected in the secondary circuit as per polarity test. If it reads zero, the secondaries are in series opposition and if it reads double the induced e.m.f. in each secondary, it is necessary to reverse the connections of one of the secondaries.

As per superposition theorem, if V_2 is assumed zero then due to phase opposition to current flows through secondary and both the transformers T_1, T_2 are as good as on no load. So O.C. test gets simulated. The current drawn from source V_1 in such case is 2 I_o where I_o is no load current of each transformer. The input power as measured by wattmeter W_1 thus reads the iron losses of both the transformers.

∴ P_i per transformer $= W_1/2$ as T_1, T_2 are identical

Then a small voltage V_2 is injected into the secondary with the help of low voltage transformer, by closing the switch S. With regulation mechanism, the voltage V_2 is adjusted so that the rated secondary current I_2 flows through the secondaries as shown. I_2 flows from E to F and then from H to G. The flow of I_1 is restricted to the loop B A I J C D L K B and it does not pass through W_1. Hence W_1 continues to read core losses. Both primaries and secondaries carry rated current so S.C. test condition gets simulated. Thus the wattmeter W_2 reads the total full load copper losses of both the transformers.

∴ (P_{cu}) F.L. per transformer $= W_2/2$

Thus in the sumpner's test without supplying the load, full iron loss occurs in the core while full copper loss occurs in the windings simultaneously. Hence heat run test can be conducted on the two transformers. In O.C. and S.C. test, both the losses do not occur simultaneously hence heat run test cannot be conducted. This is the advantage of Sumpner's test.

From the test results the full load efficiency of each transformer can be calculated as,

$$\boxed{\% \, \eta_{FL} \text{ of each transformer} = \frac{\text{Output}}{\text{Output} + \frac{W_1}{2} + \frac{W_2}{2}} \times 100}$$

Where output = VA rating x cos Φ_2

5 14 Voltage Regulation of Transformer

- Because of the voltage drop across the primary and secondary impedances it is observed that the secondary terminal voltage drops from its no load x alue (E_2) to load value (V_2) as load and load current increases.
- This decrease in the secondary terminal voltage expressed as a fraction of the no load secondary terminal voltage is called regulation of a transformer.

Let E_2 – Secondary terminal voltage on no load

V_2 = Secondar y termin al voltage on given load

- Then mathematically voltage regulation given load can be expressed as,

$$\% \text{ voltage regulation} = \frac{E_2 - V_2}{V_2} \times 100$$

- The ratio $(E_2 - V_2 / V_2)$ is called **per unit**

- The secondary terminal voltage does not depend only on the magnitude of the load current but also on the nature of the power factor of the load. If is determined for full load and sp ecified power toctcv cooc4ition the regulation is called full load i”e¿q•ulation.

- .As load current I_L increases, the voltage drops tend tea increase and V_2 drops more and more. In case of lagging power factor $V_2 < E_2$ and we get positive oltage regulation, while for leading power factor $E_2 < V_2$ and we get negative voltage regulation. This is shown in the Fig. 5.2.1.

Key Point : The voltage drop should be as small as possible hence less the regulation better is the perfrarmance of a transformer.

5.14.1 Expression for Voltage Regulation

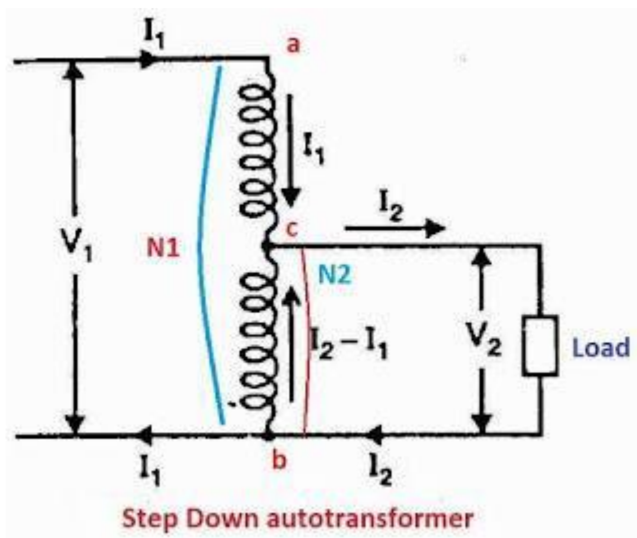
- Mathematically percentage voltage regulation is defined as.

$$\% R = \frac{E_2 - V_2}{V_2} \times 100 = \frac{\text{Total voltage drop}}{} \times 100$$

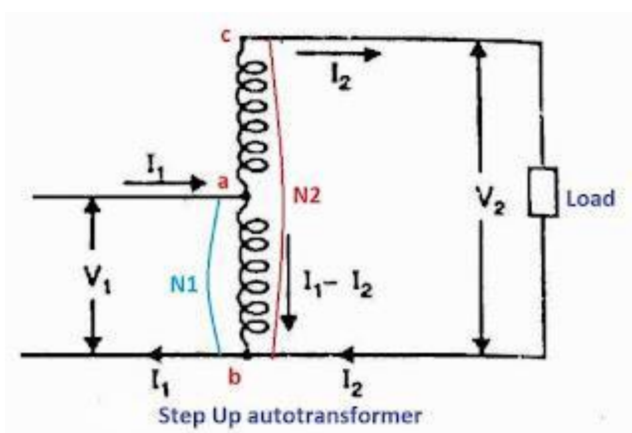
- The total voltage drop depends on the nature of the ower fac

6. Autotransformer

Autotransformer is a transformer in which part of the winding is common to both the primary winding circuit and a secondary winding. In normal two winding transformers, primary and secondary winding are electrically isolated but in the case of an autotransformer, the two winding are connected electrically as well as magnetically. Autotransformer also called as variac or variable autotransformer. Autotransformer works as a voltage regulator. With the help of autotransformer, we can get variable voltage at the output.



The above diagram (a) shows the connection diagram of step down auto-transformer and figure (b) shows the connection diagram of step-up autotransformer.



In both cases step up and step down, the winding "ab" is having N_1 turns is primary winding circuit and winding "bc" having N_2 turns is secondary winding of the auto transformer. Hence Power from the primary is transferred to the secondary winding conductively as well as Transformer action by mutual induction. The above diagram depicts the connections of the loaded step-down and step-up autotransformer. In both cases, I_1 is the input current and I_2 is output current, Regardless of Step Up/Step down autotransformer, the current in the section of winding that is common to both the primary and secondary is the difference between these two currents I_1 and I_2.

The direction of the current through the common part of the winding depends upon the connection of the autotransformer. Because the type of connection decides whether input current I_1 or output current I_2 is larger. For a step-down type $I_2 > I_1$ so $I_2 - I_1$ current flows through the common part of the winding. For step Up autotransformer $I_2 < I_1$ hence $I_1 - I_2$ current flows in the common part of the winding.

Power Transfer in Autotransformer

The primary and secondary windings of autotransformer are connected magnetically as well as electrically; the power transfer from the primary circuit to the secondary is in the form of induction as well as conduction.

Output Apparent power = $V_2.I_2$

Apparent power transfer by induction= $V_2 (I_2 - I_1) = V_2 (I_2 - KI_2) = V_2 I_2 (1- K) = V_1 I_1 (1- K)$

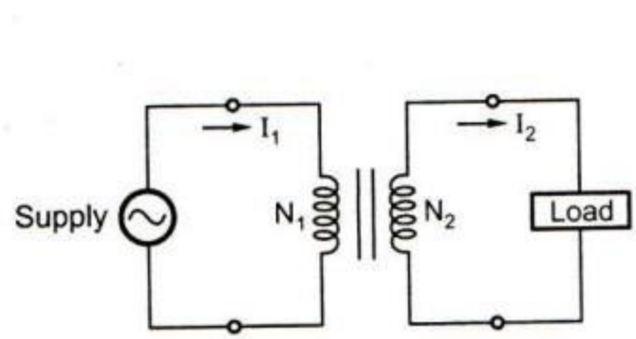
Power transfer inductively, P_t = Input X (1- K)

Therefore, Power transfer conductively, P_c = Input X (1- K)

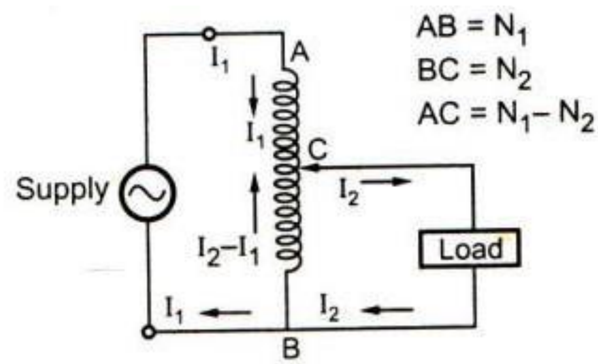
Copper saving in autotransformer

For same output and voltage transformation, an autotransformer required less copper transformer required less copper than ordinary transformer in first diagram ordinate transformer is depicted and the second diagram shows at a transformer having the same output and voltage transformation ratio k

The length of copper required in autotransformer winding is directly proportional to the turns, and the area of cross-section of the winding wire is proportional to the current rating of auto transformer, therefore, copper required for winding is proportional to the current X turn.



(f) Two winding transformer



(g) Step down transformer

Let WTw = total weight of copper in two winding transformer

WAT = weight of copper in autotransformer

In two winding transformer,

Weight of copper of primary a N1 11

Weight of copper of secondary a N2 Ii

WTw a N1!1 + N2 b ....total weight of Cu

In case of step-down autotransformer.

Weight of copper of section AC a (N1 - N2) 11

Weight of copper of section BC a N2 (b - li)

WAT a (N1 - N2) 11 + N2 (b - 11) .. ... total weight of Cu

Taking ratio of the two Weights,

$$\frac{W_{TW}}{W_{AT}} = \frac{N_1I_1+N_2I_2}{(N_1-N_2)I_1+N_2(I_2-I_1)}$$

$$= \frac{N_1I_1+N_2I_2}{N111 \; N2I1+N2I2-N2I1}$$

$$\frac{N1I1+N2I2}{N1Ji+N2'2-2N2I1}$$

But K = N2/Ni =!.!, 12

$$\frac{WTw}{WAT} = \frac{N.J.+KN.(\;)}{N1I1+KN1(\;)-2KN1I1}$$

$$= \frac{2N_1 1_1}{2N1'1-2kN1I1} \quad \frac{1}{1-K}$$

WAT = (1-K) WTw

Saving of copper = Ww - WAT = WTw - (1-K) Wrw

Saving of copper = K WTw................. For step down autotransformer

Thus saving of copper is **K** times the total weight of copper in two winding transforPler.

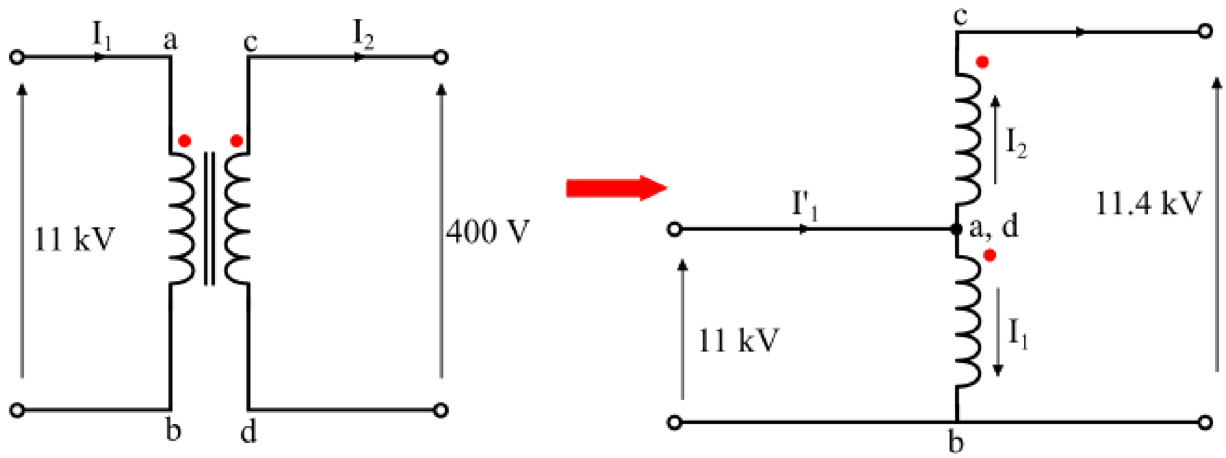
And Savmg of copper $= \frac{1}{K}$ Wrw .. ... For step up autotransformer

Conversion of Two Winding Transformer to an Autotransformer

A conventional two-winding transformer can be converted into an autotransformer as shown in the figures given below. It can be converted into a step-up autotransformer by connecting the two windings electrically in series with additive polarities. If the windings are connected electrically in series with subtractive polarities, then a step-down autotransformer is obtained.

To understand the conversion of a two winding transformer into an autotransformer, consider a conventional two-winding transformer of 20 kVA, 11000/400 V. This transformer has to be connected in autotransformer

Case 1 – With Additive Polarity



The figure shows the series connections of the winding with additive polarity. As the polarity of the connection is additive, therefore, the output voltage on the secondary side is given by,

$$v_2=11000+400=11400V=11.4kV$$

The primary side voltage is given by,

$$v_1=11000V=11kV$$

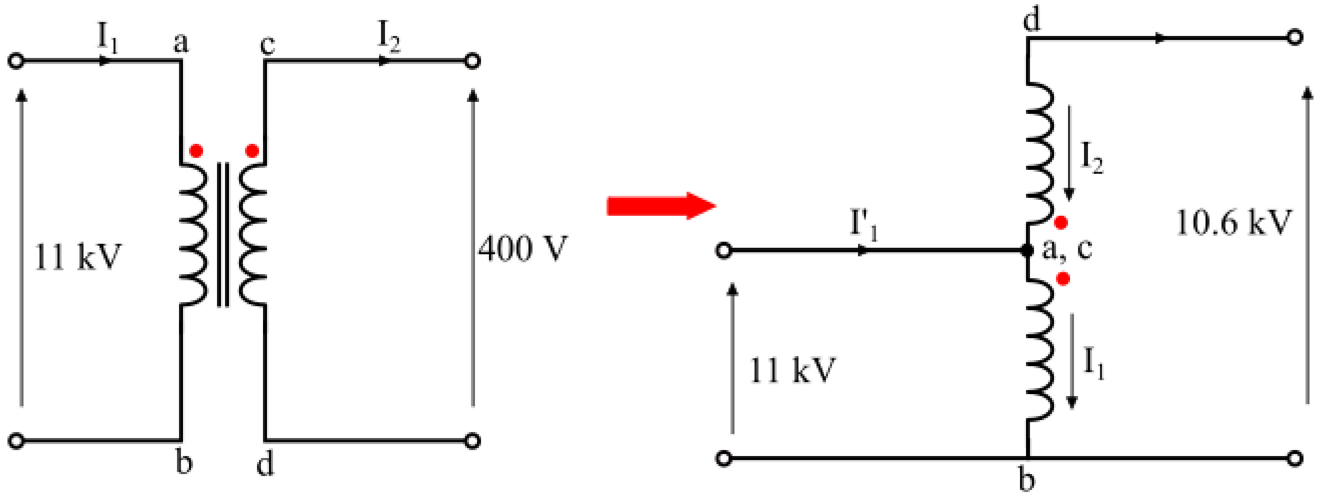
Also, the current drawn by the autotransformer is given by,

$$I'_1=I_1+I_2$$

Hence, in this case, the transformer acts as a step-up autotransformer.

Case 2 – With Subtractive Polarity

The connection diagram of this arrangement is shown in the figure. Here, the two windings are connected in series opposing so that it produces a reduced voltage across the secondary winding terminals. Hence, the transformer with subtractive polarity acts as a step-down autotransformer.



For this case, the primary side and secondary side voltages are given as follows −

$$\mathbf{V_1=11000V=11kV}$$

$$\mathbf{V_2=11000-400=1060}$$

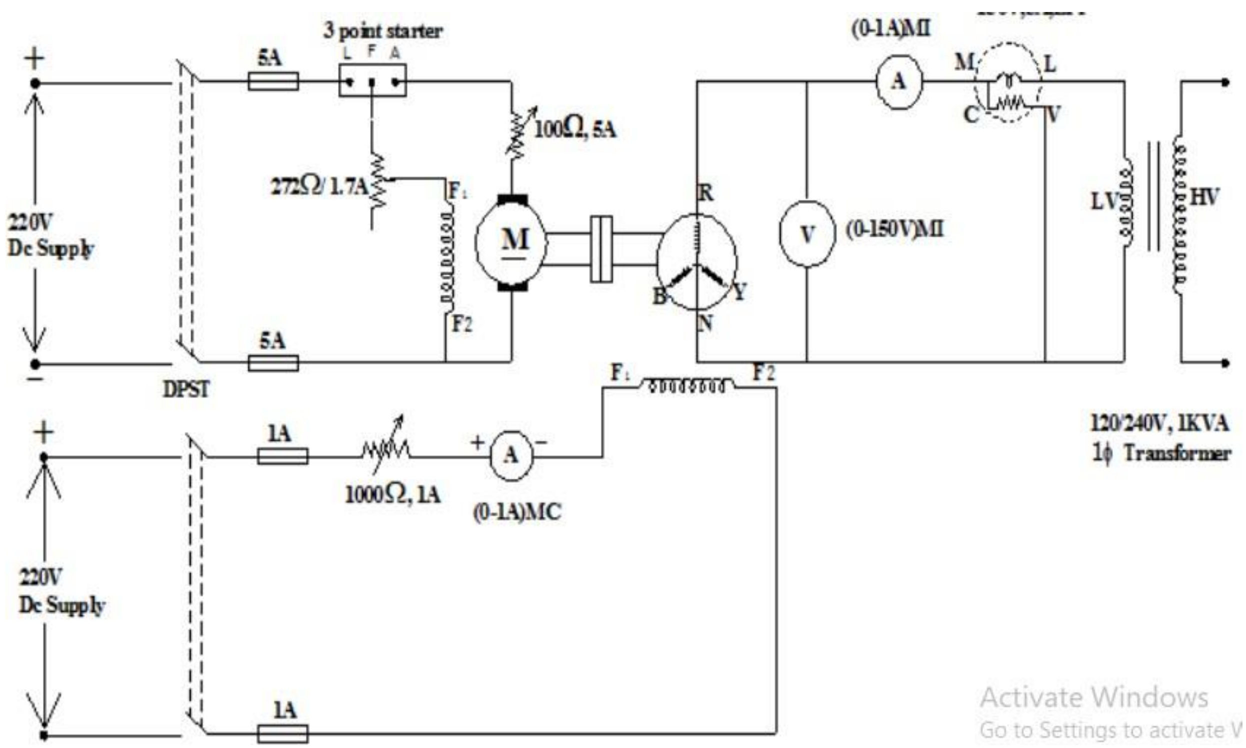
$$\mathbf{0V=10.6kV}$$

Also, the current drawn by the autotransformer is given by,

$$\mathbf{I'_1=I_1+I_2}$$

6. Separation of Losses in single phase transformers

Hysteresis loss and eddy current loss are the components of the iron losses. For the applied flux density B_{max} to the core. Hysteresis loss and eddy current loss both depend upon magnetic properties of the materials used to construct the core of transformer and its design. So these losses in transformer are fixed and do not depend upon the load current. So core losses in transformer which is alternatively known as iron loss in transformer can be considered as constant for all range of load.



Case:1:

Hysteresis loss in transformer is denoted as,

$$\text{Hysteresis loss} = K_h B^{1.6} f V$$

Where B is the flux density, V is volume of core and f is the frequency

$$\text{Hysteresis loss} = Af$$

Where **A** is constant = $K_h B^{1.6} V$

Eddy current loss in transformer is denoted as,

$$\text{Eddy current loss} = K_e\, B^2 f^2\, t^2$$

Where B is the flux density, t is the thickness of the core and f is the frequency

$$\text{Eddy current loss} = Bf^2$$

$$\text{Where } \mathbf{B} \text{ is constant} = K_e\, B^2 t^2$$

$$\text{Therefore, Core loss} = \text{Hysteresis Loss} + \text{Eddy Current Loss} = Af + Bf^2$$

Case-2:

V/F ≠ Constant

Bm ≠ Constant

$$W_h = A\,(V/F)^{1.6} F$$

$$= A\, V^{1.6}\, F^{-0.6}$$

$$W_e = B\,(V/F)^2 f^2$$

$$= B\, V^2$$

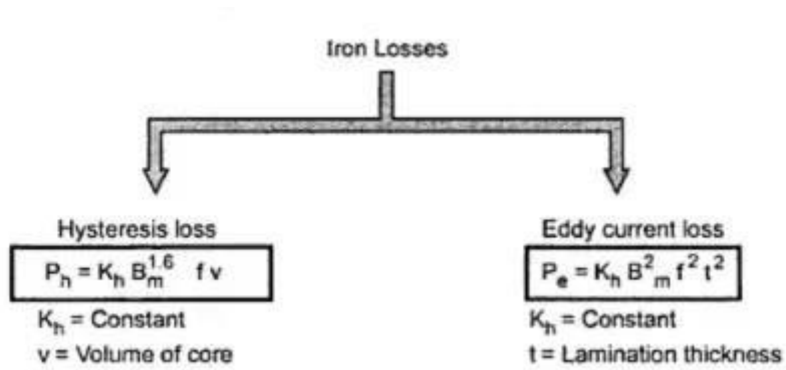
$$\text{Core loss} = Wi + We$$

Practically, conduct two tests on transformers at two different frequencies f_1 and f_2, keeping maximum flux density B_m in the core same. The results are to be used in the to obtain the constants A and B. Thus the core losses i.e iron losses are separated into hysteresis and eddy current losses.

7. Effect of frequency and supply voltage on iron losses

The iron loss of a transformer includes two types of losses - **1. Hysteresis loss & 2. Eddy current loss.**

For a given volume and thickness of laminations, these losses depend on the operating frequency, maximum flux density and the voltage.



It is known that for a transformer,

$$\mathbf{V = 4.44\, \boldsymbol{f}\, \Phi_m\, N = .44\, \boldsymbol{f}\, B_m A N}$$

[*as* (Φ_m= B_mA)]

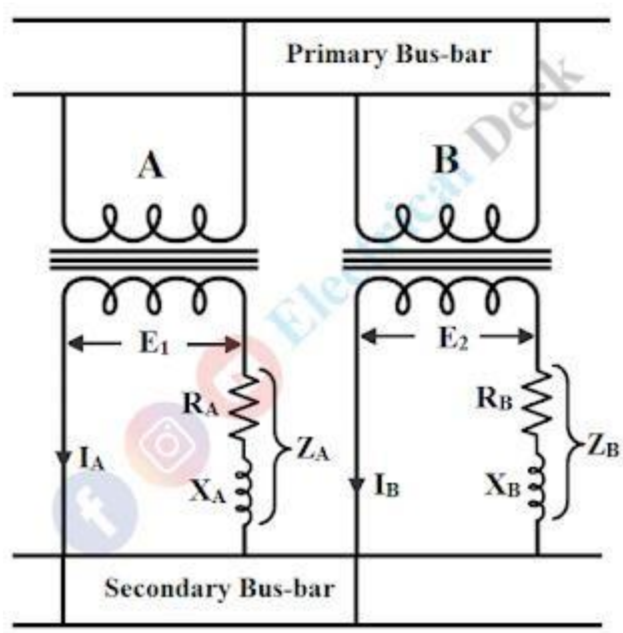
A is the core area.

$$\mathbf{B_m\ \alpha\ V/\boldsymbol{f}}$$

Thus as voltage changes, maximum flux density changes and both eddy & hysteresis losses also changes. As the voltage increases, the maximum flux density in the core increases and total iron loss increases. As frequency increases, the flux density in the core decreases but as the iron loss is directly proportional to the frequency hence effect of increased frequency is to increase the iron losses.

8. Parallel operation of two single phase transformers

Parallel operation of two or more Transformers means that all the Transformers Primary is connected with the common supply and their Secondary are feeding to a common bus through which load is connected. Parallel operation of Transformers requires that their Primaries as well as Secondaries are connected in parallel.



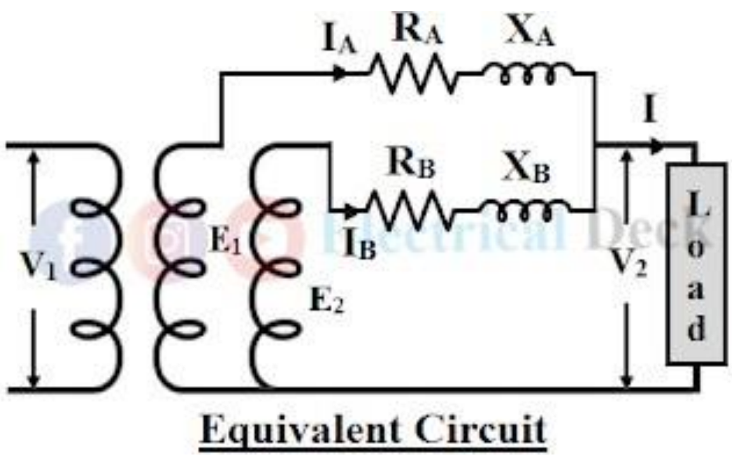
Parallel operation of two or more Transformers has many advantages when compared with a single large Transformer. Though using single large Transformer instead of two or more Transformers connected in parallel are cheap but still due to the following advantages, parallel operation of Transformers are preferred where required:

- With two or Transformers, the Power System becomes more reliable. Let one Transformer develops a fault, then the faulty Transformer can be removed from the circuit while maintaining the power supply at a reduced level through healthy Transformers. Thus in this way, Power System becomes more reliable.
- Depending upon the load, Transformers can be switched ON / OFF. In this way, Transformer losses are reduced and the system becomes more efficient and economical.
- If the power demand increases with time then extra spare Transformer can be taken into service to meet the power demand.

Conditions for satisfactory parallel operation of single-phase transformers

- The transformers should be suitable for the supply system voltage and frequency.
- The polarity of the transformer should be ensured that properly connected.
- Transformation or turn ratios of the transformers should be same.
- The percentage (per-unit) impedance of the transformers should be same.
- The transformer's ratios of resistance to reactance should be same.

For satisfactory parallel operation, the transformers must meet the above conditions. Otherwise, under the no-load condition, the localized circulating current will be produced due to unequal induced EMFs in the secondaries to equalize the voltage in each transformer. Thus when the two transformers have loaded this results in an unequal share of the load with respect to their full kVA output and cause another transformer to overload. The equivalent circuit of the two transformers with unequal induced EMFs is shown below.



However, it is difficult in practice to build such identical transformers. In order to reduce the circulating current, the difference between the voltage ratios of the transformers should least as possible. This is possible when the difference between the magnetizing current of the transformers is very small.

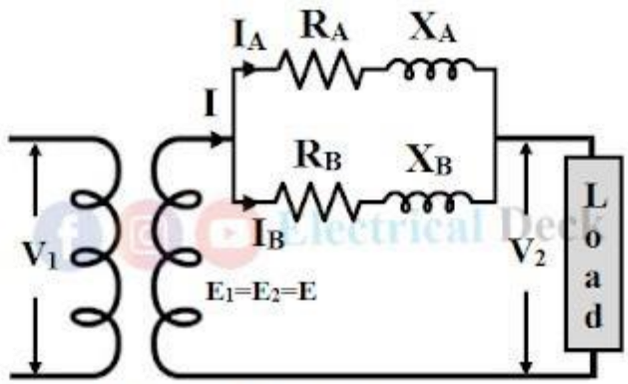
The circulating current will cause undesirable effects like,

- The full kVA output of the transformers cannot be utilized to a full extent.
- It overloads one transformer from the parallel connected group.

- The copper losses in the transformers windings increases.

With Equal Voltage Ratio:

When two transformers operating in parallel having equal voltage ratios (i.e., $E_A = E_B = E$), the total load current will be divided between them inversely as their equivalent impedance. Since the voltage ratios are the same if we neglect magnetizing currents of two transformers the equivalent circuit referred to as secondary is represented as shown below.



Equivalent Circuit

Let,

- Z_A & Z_B = Impedances of the transformers A and B respectively.
- I_A & I_B = Currents supplied to the load from transformers A and B respectively.
- V_2 = Common terminal load voltage.
- I = Combined load current
- V_1 = Common supply input voltage.

From the above circuit, the voltage equation may be written as,

$$E_2 = V_2 + I_A Z_A = V_2 + I_B Z_B$$

$$or$$

$$I_A Z_A = I_B Z_B = I Z_{AB} \ldots (1)$$

Where Z_{AB} = combined impedance of Z_A and Z_B. Since both are in parallel the equivalent impedance is given by,

$$\frac{1}{Z_{AB}} = \frac{1}{Z_A} = \frac{1}{Z_B}$$

$$Z_{AB} = \frac{Z_A Z_B}{Z_A + Z_B} \ \dots (2)$$

From equation (1), I_A is given by,

$$I_A = \frac{I Z_{AB}}{Z_A} \ \dots (3)$$

Substituting the value of Z_{AB} from equation (2) in equation (3),

$$I_A = \frac{I Z_B}{(Z_A + Z_B)}$$

Similarly, I_B is given by,

$$I_B = \frac{I Z_{AB}}{Z_B} = \frac{I Z_A}{(Z_A + Z_B)}$$

Multiplying on both sides by $V_2 \times 10^{-3}$ we have,

$$V_2 I_A \times 10^{-3} = V_2 I \times 10^{-3} \frac{Z_B}{(Z_A + Z_B)}$$

$$and$$

$$V_2 I_B \times 10^{-3} = V_2 I \times 10^{-3} \frac{Z_A}{(Z_A + Z_B)}$$

Let,

- $Q = V_2 I \times 10^{-3}$ = The combined load kVA
- $Q = V_2 I_A \times 10^{-3}$ = kVA carried by transformer A
- $Q = V_2 I_B \times 10^{-3}$ = kVA carried by transformer B

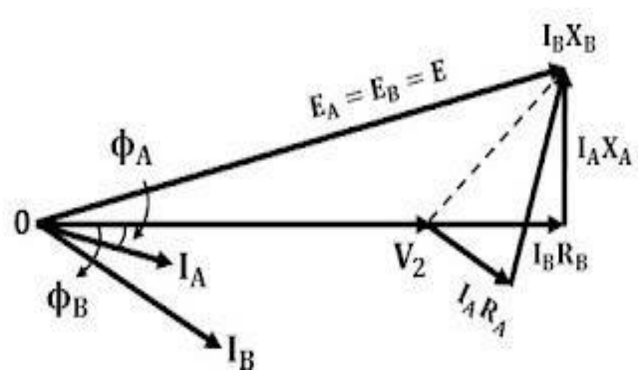
$$Q_A = Q \frac{Z_B}{(Z_A + Z_B)}$$

$$Q_B = Q \frac{Z_A}{(Z_A + Z_B)}$$

If the ratios of resistance to reactance are not equal the power factors of the transformers A and B will be different and the expression for kVA supplied will remain the same. Hence, there must be an inverse relationship between the kVA carried and the impedance of the transformer to share the load to its full kVA output without overloading.

$$\frac{Q_A}{Q_B} = \frac{Z_B}{Z_A}$$

The phasor diagram of the transformers operating in parallel with their quantities is shown below.



4. Losses in Single Phase Transformers

In a transformer there exist two types of losses

1. Core gets subjected to an alternating flux, causing core losses or iron losses
2. Winding carry currents when transformer is loaded, causing copper losses

1. Iron Losses or Core Losses

Iron losses are caused by the alternating flux in the core of the transformer as this loss occurs in the core it is also known as **Core loss**. Iron loss is further divided into **hysteresis and eddy current loss.**

a. Hysteresis Loss

The core of the transformer is subjected to an alternating magnetizing force, and for each cycle of EMF, a hysteresis loop is traced out. Power is dissipated in the form of heat known as hysteresis loss and given by the equation shown below:

$$P_h = K\eta B_{max}^{1.6} f V \quad \text{watts}$$

Where

- $K\eta$ is a proportionality constant which depends upon the volume and quality of the material of the core used in the transformer,
- f is the supply frequency,
- B_{max} is the maximum or peak value of the flux density.

The iron or core losses can be minimized by using silicon steel material for the construction of the core of the transformer.

b. Eddy Current Loss

When the flux links with a closed circuit, an EMF is induced in the circuit and the current flows, the value of the current depends upon the amount of EMF around the circuit and the resistance of the circuit. Since the core is made of conducting material, these EMFs circulate currents within the body of the material. These circulating currents are called **Eddy Currents**. They will occur when the conductor experiences a changing magnetic field. As these currents are not responsible for doing any useful work, and it produces a loss (I^2R loss) in the magnetic material known as an **Eddy Current Loss**. The eddy current loss is minimized by making the core with thin laminations.

The equation of the eddy current loss is given as:

$$P_e = K_e B_m^2 t^2 f^2 V \qquad \text{watts}$$

Where,

- K_e – coefficient of eddy current. Its value depends upon the nature of magnetic material like volume and resistivity of core material, the thickness of laminations
- B_m – maximum value of flux density in wb/m^2
- T – thickness of lamination in meters
- F – frequency of reversal of the magnetic field in Hz
- V – the volume of magnetic material in m^3

Therefore, Iron loss = Eddy current loss + Hysteresis loss

2. **<u>Copper Loss or Ohmic Loss</u>**

These losses occur due to ohmic resistance of the transformer windings. If I_1 and I_2 are the primary and the secondary current. R_1 and R_2 are the resistance of primary and secondary winding then the copper losses occurring in the primary and secondary winding will be $I_1^2R_1$ and $I_2^2R_2$ respectively.

Therefore, the total copper losses will be

$$P_c = I_1^2R_1 + I_2^2R_2$$

OR

$$\mathbf{P_{cu} = I^2{}_1 R_{1e} = I^2{}_2 R_{2e}}$$

These losses varied according to the load and known hence it is also known as variable losses. Copper losses vary as the square of the load current.

Therefore, for a transformer

Total losses = Iron loss + Copper loss

2.5 Efficiency of a Transformer

Due to the losses in a transformer, the output power of a transformer is less than the input power supplied.

$$\therefore \quad \text{Power output} = \text{Power input} - \text{Total losses}$$

$$\therefore \quad \text{Power input} = \text{Power output} + \text{Total losses}$$

$$= \text{Power output} + P_i + P_{cu}$$

The efficiency of any device is defined as the ratio of the power output to power input. So for a transformer the efficiency can be expressed as,

$$\eta = \frac{\text{Power output}}{\text{Power input}}$$

$$\therefore \quad \eta = \frac{\text{Power output}}{\text{Power output} + P_i + P_{cu}}$$

$$\text{Now Power output} = V_2 I_2 \cos\phi$$

$$\text{where} \quad \cos\phi = \text{Load power factor}$$

The transformer supplies full load of current I_2 and with terminal voltage V_2.

$$P_{cu} = \text{Copper losses on full load} = I_2^2 R_{2e}$$

$$\therefore \quad \eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}$$

But

$$V_2 I_2 = \text{VA rating of a transformer}$$

$$\therefore \quad \eta = \frac{(\text{VA rating}) \times \cos\phi}{(\text{VA rating}) \times \cos\phi + P_i + I_2^2 R_{2e}}$$

$$\therefore \quad \boxed{\% \eta = \frac{(\text{VA rating}) \times \cos\phi}{(\text{VA rating}) \times \cos\phi + P_i + I_2^2 R_{2e}} \times 100}$$

This is full load percentage efficiency with,

$$I_2 = \text{Full load secondary current}$$

But if the transformer is subjected to fractional load then using the appropriate values of various quantities, the efficiency can be obtained.

Let

$$\boxed{n = \text{Fraction by which load is less than full load} = \frac{\text{Actual load}}{\text{Full load}}}$$

For example, if transformer is subjected to half load then,

$$n = \frac{\text{Half load}}{\text{Full load}} = \frac{(1/2)}{1} = 0.5$$

When load changes, the load current changes by same proportion.

$$\therefore \quad \boxed{\text{new } I_2 = n\,(I_2)\text{ F.L.}}$$

Similarly the output $V_2\, I_2 \cos\phi_2$ also reduces by the same fraction. Thus fraction of VA rating is available at the output.

Similarly as copper losses are proportional to square of current then,

$$\boxed{\text{new } P_{cu} = n^2\,(P_{cu})\text{ F.L.}}$$

Key Point: *So copper losses get reduced by n^2.*

In general for fractional load the efficiency is given by,

$$\boxed{\%\,\eta = \frac{n\,(\text{VA rating})\cos\phi}{n\,(\text{VA rating})\cos\phi + P_i + n^2\,(P_{cu})\text{ F.L.}} \times 100}$$

where n = **Fraction by which load is less than full load.**

Key Point : *For all types of load power factor also ...*

2.6 Condition for Maximum Efficiency

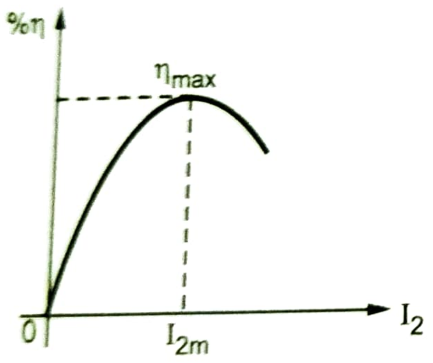


Fig. 2.11

When a transformer works on a constant input voltage and frequency then efficiency varies with the load. As load increases, the efficiency increases. At a certain load current, it achieves a maximum value. If the transformer is loaded further the efficiency starts decreasing. The graph of efficiency against load current I_2 is shown in the Fig. 2.11.

The load current at which the efficiency attains maximum value is denoted as I_{2m} and maximum efficiency is denoted as η_{max}.

Let us determine,

1. Condition for maximum efficiency.
2. Load current at which η_{max} occurs.
3. kVA supplied at maximum efficiency.

The efficiency is a function of load i.e. load current I_2 assuming $\cos \phi_2$ constant. The secondary terminal voltage V_2 is also assumed constant. So for maximum efficiency,

$$\frac{d\eta}{dI_2} = 0$$

Now
$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}$$

$$\therefore \quad \frac{d\eta}{dI_2} = \frac{d}{dI_2}\left[\frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}\right] = 0$$

$$\therefore (V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}) \frac{d}{dI_2}(V_2 I_2 \cos\phi_2)$$

$$-(V_2 I_2 \cos\phi_2) \cdot \frac{d}{dI_2}(V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}) = 0$$

$$\therefore (V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e})(V_2 \cos\phi_2) - (V_2 I_2 \cos\phi_2)(V_2 \cos\phi_2 + 2I_2 R_{2e}) = 0$$

Cancelling $(V_2 \cos\phi_2)$ from both the terms we get,

$$V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e} - V_2 I_2 \cos\phi_2 - 2 I_2^2 R_{2e} = 0$$

$$\therefore \quad P_i - I_2^2 R_{2e} = 0$$

$$\therefore \quad P_i = I_2^2 R_{2e} = P_{cu}$$

So condition to achieve maximum efficiency is that,

$$\boxed{\text{Copper losses} = \text{Iron losses}}$$

2.6.1 Load Current I_{2m} at Maximum Efficiency

For η_{max}, $\quad I_2^2 R_{2e} = P_i$ but $I_2 = I_{2m}$

$$\therefore \quad I_{2m}^2 R_{2e} = P_i$$

$$\therefore \quad I_{2m} = \sqrt{\frac{P_i}{R_{2e}}}$$

This is the load current at η_{max}.

Let $\quad (I_2)F.L. = $ Full load current

$$\therefore \quad \frac{I_{2m}}{(I_2)F.L.} = \frac{1}{(I_2)F.L.}\sqrt{\frac{P_i}{R_{2e}}}$$

$$\therefore \quad \frac{I_{2m}}{(I_2)F.L.} = \sqrt{\frac{P_i}{[(I_2)F.L.]^2 R_{2e}}} = \sqrt{\frac{P_i}{(P_{cu})F.L.}}$$

$$\therefore \quad \boxed{I_{2m} = (I_2)F.L. \sqrt{\frac{P_i}{(P_{cu})F.L.}}}$$

This is the load current at η_{max} interms of full load current.

2.6.2 kVA Supplied at Maximum Efficiency

For constant V_2 the kVA supplied is the function of load current.

$$\therefore \quad \text{kVA at } \eta_{max} = I_{2m} V_2 = V_2 (I_2)\text{F.L.} \times \sqrt{\frac{P_i}{(P_{cu})\,\text{F.L.}}}$$

$$\therefore \quad \boxed{\text{kVA at } \eta_{max} = (\text{kVA rating}) \times \sqrt{\frac{P_i}{(P_{cu})\,\text{F.L.}}}}$$

Substituting condition for η_{max} in the expression of efficiency, we can write expression for η_{max} as,

$$\% \eta_{max} = \frac{V_2 I_{2m} \cos\phi}{V_2 I_{2m} \cos\phi + 2P_i} \times 100 \qquad \text{as } P_{cu} = P_i$$

i.e.

$$\% \eta_{max} = \frac{\text{kVA for } \eta_{max} \cos\phi}{\text{kVA for } \eta_{max} \cos\phi + 2P_i}$$

6. All day Efficiency of Transformers

For a transformer, the efficiency is defined as the ratio of output power to input power.

Transformer Efficiency= Output Power /Input Power

The above equation is efficiency of any transformer. But for some special types of transformers such as distribution transformers power efficiency is not the true measure of the performance. **For that purpose distribution transformer we calculate all day efficiency**. Distribution transformer serves residential and commercial loads. The load on distribution transformers vary considerably during the period of the day. For most period of the day these transformers are working at 30 to 40 % of full load only or even less than that. But the primary of such transformers is energized at its rated voltage for 24 hours, to provide continuous supply to the consumer.

The core loss which depends on voltage, takes place continuously for all the loads. But copper loss depends on the load condition. For no load, copper loss is negligibly small while on full load it is at its rated value. Hence power efficiency cannot give the measure of **true efficiency of distribution transformers**. In such transformers, the energy output is calculated in kilo watt hour (kWh). Then ratio of total energy output to total energy input (output + losses) is calculated. Such **ratio is called energy efficiency or All Day Efficiency of a transformer.**

Based on this efficiency, the performance of various distribution transformers is compared. All day efficiency is defined as,

$$\boxed{\begin{aligned}\% \text{ All day } \eta &= \frac{\text{Output energy in kWh during a day}}{\text{Input energy in kWh during a day}} \times 100 \\ &= \frac{\text{Output energy in kWh during a day}}{\text{Output energy + Energy spent for total losses}} \times 100\end{aligned}}$$

While calculating energies, all energies can be expressed in watt hour (Wh) instead of kilo watt hour (kWh). Such distribution transformers are designed to have very low core losses. This is achieved by **limiting the core flux density to lower value by using a relative higher core**

cross-section i.e. larger iron to copper weight ratio. The **maximum efficiency in distribution transformers occurs at about 60-70 % of the full load**. So by proper designing, high energy efficiencies can be achieved for distribution transformers.

Problems

(Must Refer the Problems Solved in Class Hours)

$kVA \text{ for } \eta_{max} \cos\phi + 2P_i$

Example 2.4 : *A 250 kVA single phase transformer has iron loss of 1.8 kW. The full load copper loss is 2000 watts. Calculate*

i) Efficiency at full load, 0.8 lagging p.f.

ii) kVA supplied at maximum efficiency.

iii) Maximum efficiency at 0.8 lagging p.f.

Solution : The given values are,

$$P_i = 1800 \text{ W}, \quad (P_{cu})\text{F.L.} = 2000 \text{ W}$$

i)

$$\% \eta = \frac{(\text{VA rating}) \cos\phi}{(\text{VA rating}) \cos\phi + P_i + (P_{cu})\,\text{F.L.}} \times 100$$

$$= \frac{250 \times 10^3 \times 0.8}{250 \times 10^3 \times 0.8 + 1800 + 2000} \times 100$$

$$= \mathbf{98.135\ \%}$$

ii)

$$\text{kVA at } \eta_{max} = \text{kVA rating} \times \sqrt{\frac{P_i}{(P_{cu})\,\text{F.L.}}} = 250 \times \sqrt{\frac{1800}{2000}}$$

$$= \mathbf{237.1708\ kVA}$$

iii)

$$\% \eta_{max} = \frac{\text{kVA at } \eta_{max} \times \cos\phi}{\text{kVA at } \eta_{max} \times \cos\phi + P_i + P_i} \times 100$$

where

$$P_{cu} = P_i = 1800 \text{ W}$$

$\therefore$

$$\% \eta_{max} = \frac{237.1708 \times 10^3 \times 0.8}{237.1708 \times 10^3 \times 0.8 + 2 \times 1800} \times 100$$

$$= \mathbf{98.137\ \%}$$

Example 2.5 : *A 400 kVA, distribution transformer has full load iron loss of 2.5 kW and copper loss of 3.5 kW. During a day, its load cycle for 24 hours is,*

6 hours 300 kW at 0.8 p.f.

10 hours 200 kW at 0.7 p.f.

4 hours 100 kW at 0.9 p.f.

4 hours No load

Determine its all day efficiency.

Solution : Given values are,

$$P_i = 2.5 \text{ kW}, \qquad (P_{cu})F.L. = 3.5 \text{ kW}, \qquad 400 \text{ kVA}$$

Iron losses are constant for 24 hours. So energy spent due to iron losses for 24 hours is,

$$P_i = 2.5 \times 24 \text{ hours} = 60 \text{ kWh}$$

Total energy output in a day from given load cycle is,

$$\text{Energy output} = 300 \times 6 \text{ hours} + 200 \times 10 \text{ hours} + 100 \times 4 \text{ hours}$$

$$= 4200 \text{ kWh}$$

To calculate energy spent due to copper loss,

i) Load 1 of 300 kW at $\cos\phi = 0.8$

$$\therefore \quad \text{kVA supplied} = \frac{\text{kW}}{\cos\phi} = \frac{300}{0.8} = 375 \text{ kVA}$$

$$\therefore \quad n = \frac{\text{Load kVA}}{\text{kVA rating}} = \frac{375}{400} = 0.9375$$

Copper losses are proportional to square of kVA ratio i.e. n^2.

$$\therefore \quad \text{Load 1 } P_{cu} = n^2 \times (P_{cu}) \text{ F.L.} = (0.9375)^2 \times 3.5 = 3.076 \text{ kW}$$

$$\therefore \quad \text{Energy spent} = 3.076 \times 6 \text{ hours} = 18.457 \text{ kWh}$$

ii) Load 2 of 200 kW at $\cos\phi = 0.7$

$$\therefore \quad \text{kVA supplied} = \frac{\text{kW}}{\cos\phi} = \frac{200}{0.7} = 285.7142 \text{ kVA}$$

$$\therefore \quad n = \frac{\text{Load kVA}}{\text{kVA rating}} = \frac{285.7142}{400} = 0.7142$$

$$\therefore \quad \text{Load 2 } P_{cu} = n^2 \times (P_{cu}) \text{ F.L.} = (0.7142)^2 \times 3.5 = 1.7857 \text{ kW}$$

$$\therefore \quad \text{Energy spent} = 1.7857 \times 10 = 17.857 \text{ kWh}$$

iii) Load 3 of 100 kW at $\cos\phi = 0.9$

$$\therefore \quad \text{kVA supplied} = \frac{\text{kW}}{\cos\phi} = \frac{100}{0.9} = 111.111 \text{ kVA}$$

$$\therefore \quad n = \frac{111.111}{400} = 0.2778$$

$$\therefore \quad \text{Load 3 } P_{cu} = n^2 \times (P_{cu}) \text{ F.L.} = (0.2778)^2 \times 3.5 = 0.2701 \text{ kW}$$

$$\therefore \quad \text{Energy spent} = 0.2701 \times 4 = 1.0804 \text{ kWh}$$

iv) No load hence negligible copper losses.

$$\therefore \quad \text{Total energy spent} = \text{Energy spent due to [Iron loss + Total copper loss]}$$

$$= 60 + 18.457 + 17.857 + 1.0804 = 97.3944 \text{ kWh}$$

and $\quad \text{Total output} = 4200 \text{ kWh}$

$$\therefore \quad \text{All day } \eta = \frac{\text{Total output for 24 hours}}{\text{Total output for 24 hours} + \text{Total energy spent for 24 hours}} \times 100$$

$$= \frac{4200}{4200 + 97.3944} \times 100 = \mathbf{97.73\ \%}$$

Example 2.6 : *A 500 kVA transformer has an iron loss of 500 W and full load copper loss of 600 W. Calculate the efficiency at $\frac{3}{4}^{th}$ full load and 0.8 power factor. Also calculate the maximum efficiency at that power factor.*

Solution : $P_i = 500$ W, $P_{cu}(F.L.) = 600$ W, $n = 3/4 = 0.75$, $\cos\phi = 0.8$

$$\% \eta = \frac{n \text{ kVA} \cos\phi}{n \text{ kVA} \cos\phi + P_i + n^2 P_{cu}(F.L.)} \times 100$$

$$= \frac{0.75 \times 500 \times 10^3 \times 0.8}{0.75 \times 500 \times 10^3 \times 0.8 + 500 + (0.75)^2 \times 600} \times 100$$

$$= \mathbf{99.7216\ \%}$$

$$\text{kVA for } \eta_{max} = \text{kVA} \times \sqrt{\frac{P_i}{P_{cu}(F.L.)}} = 500 \times \sqrt{\frac{500}{600}}$$

$$= 456.435 \text{ kVA}$$

and $$P_{cu} = P_i = 500 \text{ W for } \eta_{max}$$

$\therefore$ $$\% \eta_{max} = \frac{\text{kVA for } \eta_{max} \cos\phi}{\text{kVA for } \eta_{max} \cos\phi + P_i + P_i} \times 100$$

$$= \frac{456.435 \times 10^3 \times 0.8}{456.435 \times 10^3 \times 0.8 + 500 + 500} \times 100$$

$$= \mathbf{99.7268\ \%}$$

Example 2.19 : *Find the All day efficiency of single phase transformer having maximum efficiency fo 98 % at 15 kVA at UPF and loaded as follows.*

12 hours -	*2 kW at 0.5 power factor lagging.*
6 hours -	*12 kW at 0.8 power factor lagging.*
6 hours -	*No load*

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Solution : $\eta_{max} = 98\%$ at 15 kVA, $\cos\phi = 1$

$$\therefore \quad \% \eta_{max} = \frac{[\text{VA for } \eta_{max}] \times \cos\phi}{[\text{VA for } \eta_{max}] \times \cos\phi + 2P_i} \times 100$$

$$\therefore \quad 0.98 = \frac{15 \times 10^3 \times 1}{15 \times 10^3 \times 1 + 2P_i}$$

$P_i = P_{cu}$

$$\therefore \quad P_i = 153.061 \text{ W} \qquad \text{... Iron loss}$$

P_{cu} at 15 kVA = P_i = 153.061 W ... For η_{max} $P_{cu} = P_i$

Energy output = $[2 \text{ kW} \times 12 \text{ hours}] + [12 \text{ kW} \times 6 \text{ hours}]$ = 96 kWh

Energy spent due to iron loss = $P_i \times 24$ hours = 3.6734 kWh

To calculate energy spent due to copper loss :

i) Load 1, 2 kW, $\cos\phi = 0.5$

$n = \frac{\text{Load KVA}}{\text{KVA rating}} = \frac{4}{15}$

$$\therefore \quad \text{kVA supplied} = \frac{\text{kW}}{\cos\phi} = \frac{2}{0.5} = 4 \text{ kVA}$$

Load 1 $P_{cu} = n^2 \times P_{cu}$ (FL)

$$\therefore \quad \text{Load 1 } P_{cu} = P_{cu} \text{ at 15 kVA } \times \left(\frac{4}{15}\right)^2 = 153.061 \times 0.07111 = 10.8843 \text{ W}$$

$$\therefore \quad \text{Energy spent} = \text{Load 1 } P_{cu} \times \text{hours} = 10.8843 \times 12 = 130.6116 \text{ Wh} = 0.1306 \text{ kWh}$$

ii) Load 2, 12 kW at $\cos\phi$ = 0.8 for 6 hours

$$\therefore \quad \text{kVA supplied} = \frac{\text{kW}}{\cos\phi} = \frac{12}{0.8} = 15 \text{ kVA}$$

$n = \frac{15}{15} = 1$

$$\therefore \quad \text{Load 2 } P_{cu} = P_{cu} \text{ at 15 kVA} = 153.061 \text{ W}$$

$$\therefore \quad \text{Energy spent} = 153.061 \times 6 \text{ hours} = 918.366 \text{ Wh} = 0.9183 \text{ kWh}$$

iii) No copper loss for no load

$$\therefore \text{Total energy spent} = \underbrace{3.6734}_{P_i} + \underbrace{0.1306 + 0.91836}_{P_{cu}} = 4.72236 \text{ kWh}$$

$$\therefore \quad \text{All day } \eta = \frac{\text{Total energy output in 24 hours}}{\text{Total energy output in 24 hours} + \text{Energy spent in 24 hours}} \times 100$$

$$= \frac{96}{96 + 4.72236} \times 100 = \mathbf{95.311\ \%}$$

Example 3.1 : *A 5 kVA, 500 / 250 V, 50 Hz, single phase transformer gave the following readings,*

HV wdg 250V/500V

O.C. Test : 500 V, 1 A, 50 W (L.V. side open)

S.C. Test : 25 V, 10 A, 60 W (L.V. side shorted)

Determine : i) The efficiency on full load, 0.8 lagging p.f.

ii) The voltage regulaion on full load, 0.8 leading p.f.

iii) The efficiency on 60 % of full load, 0.8 leading p.f.

iv) Draw the equivalent circuit referred to primary and insert all the values in it.

Solution : In both the tests, meters are on H.V. side which is primary of the transformer. Hence the parameters obtained from test results will be referred to primary.

From O.C. test, $V_0 = 500\text{ V},\ I_0 = 1\text{ A},\ W_0 = 50\text{ W}$

$$\therefore \quad \cos\phi_0 = \frac{W_0}{V_0 I_0} = \frac{50}{500 \times 1} = 0.1$$

$$\therefore \quad I_c = I_0 \cos\phi_0 = 1 \times 0.1 = 0.1\text{ A}$$

and $$I_m = I_0 \sin\phi_0 = 1 \times 0.9949 = 0.9949\text{ A}$$

$$\therefore \quad R_0 \text{ (HV)} = \frac{V_0}{I_c} = \frac{500}{0.1} = 5000\ \Omega$$

and $$X_0 \text{ (HV)} = \frac{V_0}{I_m} = \frac{500}{0.9949} = 502.52\ \Omega$$

and $$W_0 = P_i = \text{Iron losses} = 50\text{ W}$$

From S.C. test, $V_{sc} = 25\text{ V},\ I_{sc} = 10\text{ A},\ W_{sc} = 60\text{ W}$

$$\therefore \quad R_{1e} = \frac{W_{sc}}{I_{sc}^2} = \frac{60}{(10)^2} = 0.6\ \Omega$$

$$Z_{1e} = \frac{V_{sc}}{I_{sc}} = \frac{25}{10} = 2.5\ \Omega$$

$\therefore \quad X_{1e} = \sqrt{(2.5)^2 - (0.6)^2} = 2.4269\ \Omega$

$$(I_1)\text{ F.L.} = \frac{\text{VA rating}}{V_1} = \frac{5\times 10^3}{500} = 10\text{ A}$$

$I_{2l} = \frac{\text{VA rating}}{V_2} = \frac{5\times10^3}{250} = 20$

and $\quad I_{sc} = (I_1)\text{ F.L.}$

$\therefore \quad W_{sc} = (P_{cu})\text{ F.L.} = 60\text{ W}$

i) η on full load, $\quad \cos\phi_2 = 0.8$ lagging

$$\%\eta = \frac{(\text{VA rating})\cos\phi_2}{(\text{VA rating})\cos\phi_2 + P_i + (P_{cu})\text{ F.L.}}\times 100$$

$$= \frac{5\times10^3\times0.8}{5\times10^3\times0.8+50+60}\times100 = \mathbf{97.32\ \%}$$

$\frac{I_1 R_{1e}\cos\phi - I_1 X_{1e}\sin\phi}{V_1}$

ii) Regulation on full load, $\cos\phi = 0.8$ leading

$$\%\ R = \frac{(I_1)\text{ F.L. } R_{1e}\cos\phi - (I_1)\text{ F.L. } X_{1e}\sin\phi}{V_1}\times100$$

$$= \frac{10\times0.6\times0.8-10\times2.4269\times0.6}{500}\times100 = \mathbf{-\ 1.95\ \%}$$

iii) For 60 % of full load, n = 0.6 and $\cos\phi_2 = 0.8$ leading

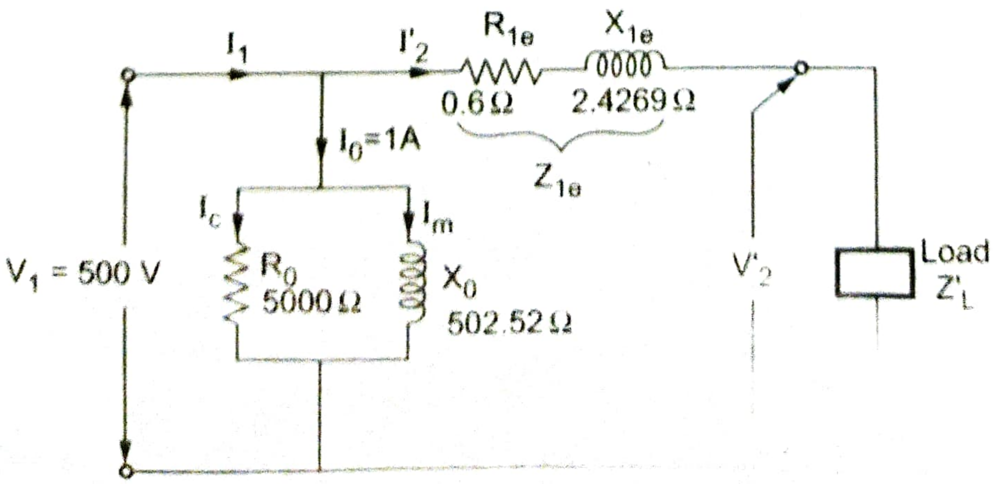
$\therefore \quad P_{cu} = \text{Copper loss on new load} = n^2\times(P_{cu})\text{ F.L.}$

$$= (0.6)^2\times60 = 21.6\text{ W}$$

$$\therefore \quad \%\eta = \frac{n(\text{VA rating})\cos\phi_2}{n(\text{VA rating})\cos\phi_2 + P_i + n^2(P_{cu})\text{ F.L.}}\times100$$

$$= \frac{0.6\times5\times10^3\times0.8}{0.6\times5\times10^3\times0.8+50+21.6}\times100 = \mathbf{97.103\ \%}$$

iv) The equivalent circuit referred to primary is shown in the Fig. 3.4.



Example 3.2 : *The open circuit and short circuit tests on a 10 kVA, 125 / 250 V, 50 Hz, single phase transformer gave the following results :*

O.C. test : 125 V, 0.6 A, 50 W (on L.V. side)

S.C. test : 15 V, 30 A, 100 W (on H.V. side)

Calculate : i) copper loss on full load

ii) full load efficiency at 0.8 leading p.f.

iii) half load efficiency at 0.8 leading p.f.

iv) regulation at full load, 0.9 leading p.f.

Solution : From O.C. test we can write,

$$W_0 = P_i = 50 \text{ W} = \text{Iron loss}$$

From S.C. test we can find the parameters of equivalent circuit. Now S.C. test is conducted on H.V. side i.e. meters are on H.V. side which is transformer secondary. Hence parameters from S.C. test results will be referred to secondary.

$$V_{sc} = 15 \text{ V}, I_{sc} = 30 \text{ A}, W_{sc} = 100 \text{ W}$$

$$\therefore \quad R_{2e} = \frac{W_{sc}}{(I_{sc})^2} = \frac{10}{(30)^2} = 0.111 \ \Omega$$

$$Z_{2e} = \frac{V_{sc}}{I_{sc}} = \frac{15}{30} = 0.5 \ \Omega$$

$$\therefore \quad X_{2e} = \sqrt{Z_{2e}^2 - R_{2e}^2} = 0.4875 \ \Omega$$

i) Copper loss on full load

$$(I_2) \text{ F.L.} = \frac{\text{VA rating}}{V_2} = \frac{10 \times 10^3}{250} = 40 \text{ A}$$

In short circuit test, $I_{sc} = 30$ A and not equal to full load value 40 A.

Hence W_{sc} does not give copper loss on full load

$$\therefore \quad W_{sc} = P_{cu} \text{ at } 30 \text{ A} = 100 \text{ W}$$

$$\text{Now} \quad P_{cu} \propto I^2$$

$$\therefore \quad \frac{P_{cu} \text{ at } 30 \text{ A}}{P_{cu} \text{ at } 40 \text{ A}} = \left(\frac{30}{40}\right)^2$$

$$\therefore \quad \frac{100}{P_{cu} \text{ at } 40 \text{ A}} = \frac{900}{1600}$$

$$\therefore \quad P_{cu} \text{ at } 40 \text{ A} = 177.78 \text{ W}$$

$\therefore \quad (P_{cu})$ F.L. $=$ **177.78 W**

ii) Full load η , $\cos \phi_2 = 0.8$

$$\% \eta \text{ on full load} = \frac{V_2 (I_2) \text{ F.L. } \cos \phi_2}{V_2 (I_2) \text{ F.L.} \cos \phi_2 + P_i + (P_{cu}) \text{ F.L.}} \times 100$$

$$= \frac{250 \times 40 \times 0.8}{250 \times 40 \times 0.8 + 50 + 177.78} \times 100 = \mathbf{97.23 \%}$$

iii) Half load η , $\cos \phi_2 = 0.8$

$$n = 0.5 \text{ as half load,} \quad (I_2) \text{ H.L.} = \frac{1}{2} \times 40 = 20 \text{ A}$$

$$\therefore \% \eta \text{ on half load} = \frac{V_2 (I_2) \text{ H.L. } \cos \phi_2}{V_2 (I_2) \text{ H.L.} \cos \phi_2 + P_i + n^2 (P_{cu}) \text{ F.L.}} \times 100$$

$$= \frac{n \text{ (VA rating) } \cos \phi_2}{n \text{ (VA rating) } \cos \phi_2 + P_i + n^2 (P_{cu}) \text{ F.L.}} \times 100$$

$$= \frac{0.5 \times 10 \times 10^3 \times 0.8}{0.5 \times 10 \times 10^3 \times 0.8 + 50 + (0.5)^2 \times 177.78} \times 100 = \mathbf{97.69 \%}$$

iv) Regulation at full load, $\cos \phi = 0.9$ leading

$$\% R = \frac{(I_2) \text{ F.L.} R_{2e} \cos \phi - (I_2) \text{ F.L. } X_{2e} \sin \phi}{V_2} \times 100$$

$$= \frac{40 \times 0.111 \times 0.9 - 40 \times 0.4875 \times 0.4358}{250} \times 100 = \mathbf{-1.8015 \%}$$

economical.

Example 3.3 : *Two similar 200 kVA, single phase transformers gave the following results when tested by Sumpner's test :*

Mains wattmeter $W_1 = 4$ *kW*

Series wattmeter $W_2 = 6$ *kW at full load current*

Find out individual transformer efficiencies at,

i) Full load at unity p.f.

ii) Half load at 0.8 p.f. lead

Solution : The given values are,

$$\text{Rating} = 200 \text{ kVA}, \quad W_1 = 4 \text{ kW}, \quad W_2 = 6 \text{ kW}$$

$$W_1 = \text{Iron loss of both the transformers}$$

$$\therefore \quad P_i = \frac{W_1}{2} = \frac{4}{2} = 2 \text{ kW for individual transformer}$$

$$W_2 = \text{Full load copper loss for both the transformers}$$

$$\therefore \quad (P_{cu})\text{F.L.} = \frac{W_2}{2} = \frac{6}{2} = 3 \text{ kW for individual transformer}$$

i) At full load, $$\% \eta = \frac{\text{VA rating} \cos\phi_2}{\text{VA rating} \cos\phi_2 + P_i + (P_{cu})\text{ F.L.}} \times 100 \qquad \text{with } \cos\phi_2 = 1$$

$$= \frac{200 \times 10^3 \times 1}{200 \times 10^3 \times 1 + 2 \times 10^3 + 3 \times 10^3} \times 100 = \mathbf{97.56\ \%}$$

ii) At half load, $\cos\phi_2 = 0.8$ and $n = \dfrac{1}{2} = 0.5$

$$\therefore \quad \% \eta = \frac{n \times (\text{VA rating}) \times \cos\phi_2}{n \times (\text{VA rating}) \times \cos\phi_2 + P_i + n^2 \times (P_{cu})\,\text{F.L.}} \times 100$$

$$(P_{cu})\text{H.L.} = n^2 \times (P_{cu})\,\text{F.L. where } n = \text{Fraction of full load}$$

$$\therefore \quad \% \eta = \frac{0.5 \times 200 \times 10^3 \times 0.8}{0.5 \times 200 \times 10^3 \times 0.8 + 2 \times 10^3 + (0.5)^2 \times 3 \times 10^3} \times 100 = \mathbf{96.67\ \%}$$

Example 3.7 : *Two 250 kVA transformers suppling a network are connected in parallel on both primary and secondary sides. Their voltage ratios are the same. The resistance drops are 1.5 % and 0.9 % and the reactance drops are 3.33 % and 4 % respectively. Calculate the kVA loading on each transformer and its power factor when the total load on the transformers is 500 kVA and at 0.707 lagging p.f.*

Solution : Given values are,

kVA of transformer 1 = 250

kVA of transformer 2 = 250

$$\% R_1 = 1.5, \quad \% R_2 = 0.9, \quad \% X_1 = 3.33, \quad \% X_2 = 4$$

Load combined, $Q = 500$ kVA, p.f. of Load = 0.707 lag

Since the voltage ratios of the two transformers are same,

Load shared by transformer 1 $= Q\left(\dfrac{Z_2}{Z_1+Z_2}\right)$

We have

$$\frac{\% Z_2}{\% Z_1 + \% Z_2} = \frac{Z_2}{Z_1+Z_2} = \frac{0.9+j\,4}{(1.5+3.33)+(0.9+j\,4)}$$

$$= \frac{0.9+j\,4}{2.5+j\,7.33} = \frac{4.1\angle 77.31^\circ}{7.71\angle 71.87^\circ}$$

$$= 0.5316\angle 5.44^\circ$$

Similarly $$\frac{\% Z_1}{\% Z_1 + \% Z_2} = \frac{Z_1}{Z_1 + Z_2} = \frac{1.5 + j\,3.33}{(1.5 + 3.33) + (0.9 + j\,4)}$$

$$= \frac{1.5 + j\,3.33}{2.4 + j\,7.33} = \frac{3.6522 \angle 65.75°}{7.71 \angle 71.87°}$$

$$= 0.4736 \angle -6.12°$$

Load shared by transformer $$1 = Q\left(\frac{Z_2}{Z_1 + Z_2}\right)$$

$$= (500 \angle -45°)\ (0.5316 \angle\ 5.44°)$$

$$= \mathbf{265.8\ kVA \angle -39.55°}$$

$$\text{p.f.} = \cos 39.55 = \mathbf{0.7709\ (lag)}$$

Load shared by transformer $$2 = Q\left(\frac{Z_1}{Z_1 + Z_2}\right)$$

$$= (500 \angle -45°)\ \ (0.4736 \angle -6.12°)$$

$$= \mathbf{236.8 \angle -51.12°\ kVA}$$

$$\text{p.f.} = \mathbf{\cos 51.12 = 0.6276\ (lag)}$$

Example 3.11 : *A 250/500 V, 50 Hz, single phase transformer gave the following test results :*

Short circuit test : 20 V, 12 A, 100 W with low voltage winding shorted

Open circuit test : 250 V, 1 A, 80 W with high voltage winding open

Draw the equivalent circuit with all the constants and calculate the applied voltage and efficiency when the output is 10 A at 500 V with 0.8 p.f. lagging.

Solution : At S.C. test L.V. winding is shorted i.e. readings are on secondary. So the parameters to be obtained from S.C. test are referred to secondary.

$$V_{sc} = 20\text{V},\ I_{sc} = 12\ \text{A},\ W_{sc} = 100\ \text{W}$$

$$\therefore \quad R_{2e} = \frac{W_{sc}}{(I_{sc})^2} = \frac{100}{(12)^2} = 0.6944\ \Omega$$

and $$Z_{2e} = \frac{V_{sc}}{I_{sc}} = \frac{20}{12} = 1.667\ \Omega$$

$$\therefore \quad X_{2e} = \sqrt{Z_{2e}^2 - R_{2e}^2} = 1.5154\ \Omega$$

$$K = \frac{V_2}{V_1} = \frac{500}{250} = 2$$

$$\therefore \quad R_{1e} = \frac{R_{2e}}{K^2} = \frac{0.6944}{4} = 0.1736\ \Omega$$

$$\therefore \quad X_{1e} = \frac{X_{2e}}{K^2} = \frac{1.5154}{4} = 0.37885\ \Omega$$

$$\therefore \quad Z_{1e} = \frac{Z_{2e}}{K^2} = \frac{1.667}{4} = 0.41675\ \Omega$$

The L.V. side is high current and low impedance side.

In O.C. test, readings are on primary side.

$$V_0 = 250\ \text{V},\ I_0 = 1\ \text{A},\ W_0 = 80\ \text{W}$$

$$\therefore \quad \cos\phi_0 = \frac{W_0}{V_0 I_0} = \frac{80}{250} = 0.32$$

$$\therefore \quad I_c = I_0 \cos\phi_0 = 0.32\ \text{A}$$

and $$I_m = I_0 \sin\phi_0 = 0.9474\ \text{A}$$

$$\therefore \quad R_0 = \frac{V_0}{I_c} = \frac{250}{0.32} = 781.25\ \Omega$$

and $$X_0 = \frac{V_0}{I_m} = \frac{250}{0.9474} = 263.88\ \Omega$$

The equivalent circuit referred to primary is shown in the Fig. 3.18.

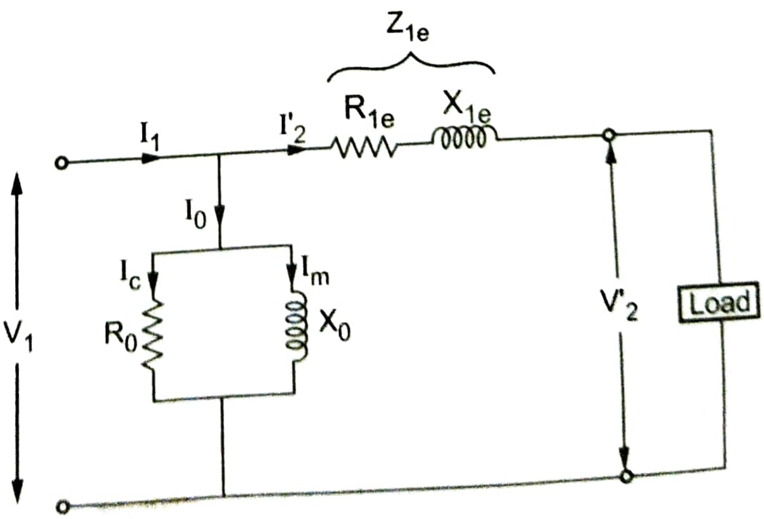


Fig. 3.18

The iron loss,

$$P_i = W_0 = 80\ \text{W}$$

Now $$P_{cu}(\text{F.L.}) = W_{sc} = 100\ \text{W}$$

$$\therefore \quad \% \eta = \frac{n\,(kVA)\cos\phi}{n\,(kVA)\cos\phi + P_i + n^2 P_{cu}(F.L.)} \times 100$$

Now on S.C. test, I_{sc} = 12 A. **Assuming it to be full load current on secondary**. We can say that for I_2 = 10 A the fraction of load applied is **n** = $\frac{10}{12}$ = **0.833** i.e. 83.33 %

$$kVA = 500 \times 12 = 6000 \text{ VA} = 6 \text{ kVA}$$

$$\therefore \quad \% \eta = \frac{0.8333 \times (6 \times 10^3) \times 0.8}{0.8333 \times 6 \times 10^3 \times 0.8 + 80 + (0.833)^2 \times 100} \times 100 = \mathbf{96.4\ \%}$$

The regulation at 10 A and 0.8 p.f. lagging is,

$$\% R = \frac{I_2[R_{2e}\cos\phi + X_{2e}\sin\phi]}{V_2} \times 100 = 2.92\ \%$$

Let V_1 is changed to V_1' to get V_2 = 500 V with same regulation.

$$\therefore \quad \% R = \frac{V_1' - V_1}{V_1} \times 100 \qquad \text{where} \quad V_1 = 250 \text{ V}$$

$$2.92 = \frac{V_1' - 250}{250} \times 100$$

$$\therefore \quad V_1' = \mathbf{257.324\ V}$$

This should be the applied voltage to get 500 V at 10 A, 0.8 p.f. to the load.

Example 3.15 : *Two transformers each of 800 kVA are connected in parallel. One has a resistance and reactance of 1 % and 4 % respectively and the other has resistance and reactance of 1.5 % and 6 % respectively. Calculate the load shared by each transformer and corresponding power factor when the total load shared is 100 kVA at 0.8 power factor lagging.*

Solution :

$$\text{kVA of transformer 1} = 800$$

$$\text{kVA of transformer 2} = 800$$

$$\% R_1 = 1, \quad \% X_1 = 4$$

$$\% R_2 = 1.5, \quad \% X_2 = 6$$

$$\text{Combined load} \quad Q = 100 \text{ kVA},$$

$$\text{p.f. of Load} = 0.8 \text{ lag}$$

Assuming the voltage ratios of the two transformers are same

$$\text{Load shared by transformer } 1 = Q\left(\frac{Z_2}{Z_1 + Z_2}\right)$$

We have

$$\frac{\% Z_2}{\% Z_1 + \% Z_2} = \frac{Z_2}{Z_1 + Z_2} = \frac{1.5 + j\,6}{(1 + j\,4) + (1.5 + j\,6)}$$

$$= \frac{1.5 + j\,6}{2.5 + j\,10} = \frac{6.1846 \angle 75.96^\circ}{10.3077 \angle 75.96^\circ}$$

$$= 0.5999 \angle 0^\circ \approx 0.6 \angle 0^\circ$$

Also $$\frac{\% Z_1}{\% Z_1 + \% Z_2} = \frac{Z_1}{Z_1 + Z_2} = \frac{1+j4}{(1+j4)+(1.5+j6)}$$

$$= \frac{1+j4}{2.5+j10} = \frac{4.1231 \angle 75.96^\circ}{10.3077 \angle 75.96^\circ}$$

$$= 0.4 \angle 0^\circ$$

Load shared by transformer $$1 = Q\left(\frac{Z_1}{Z_1 + Z_2}\right)$$

$$= (100)\ (0.6) \angle -36.86^\circ$$

$$= \mathbf{60\ kVA} \angle -\mathbf{36.86^\circ}$$

p.f. = cos 36.86° = **0.8 (lag)** of transformer 1

Load shared by transformer $$2 = Q\left(\frac{Z_1}{Z_1 + Z_2}\right)$$

$$= (100)\ (0.4) \angle -36.86^\circ$$

$$= \mathbf{40\ kVA} \angle -\mathbf{36.86^\circ}$$

p.f. = cos 36.86° = **0.8 (lag)** of transformer 2

Example 3.16 : *Two single phase transformers A and B are connected in parallel. They have same kVA ratings but their resistances are respectively 0.05 and 0.1 per unit and their leakage reactance 0.05 and 0.04 per unit. If A is operated on full load at a p.f. of 0.8 lagging, what will be the load and p.f. of B.*

Solution : Given

Two transformers A and B are working in parallel

$$R_A = 0.05\ \Omega\ \text{p.u.},\ X_A = 0.05\ \Omega\ \text{p.u.}$$

$$R_B = 0.1\ \Omega\ \text{p.u.},\ \ X_B = 0.04\ \Omega\ \text{p.u.}$$

Load shared by transformer A, $$Q_A = Q\frac{Z_B}{Z_A + Z_B} \quad \dots (I)$$

Load shared by transformer B, $$Q_B = Q\frac{Z_A}{Z_A + Z_B} \quad \dots (II)$$

Dividing equation (II) by equation (I),

$$\frac{Q_B}{Q_A} = \frac{Z_A}{Z_B}$$

$$Z_A = (R_A + j\,X_A) = (0.05 + j\,0.05)\ \Omega\ \text{p.u.},$$

$$\therefore \quad \% Z_A = (5 + j\,5)\ \Omega$$

$$Z_B = (R_B + j\,X_B) = (0.1 + j\,0.04)\ \Omega \text{ p.u.},$$

$$\therefore \qquad \%\,Z_B = (10 + j\,4)\ \Omega$$

Transformer A is operating at $\cos^{-1}0.8 = 36.86$ lagging p.f.

$$Q_B = Q_A \cdot \frac{Z_A}{Z_B} = [Q_A \angle -36.86^\circ]\left[\frac{5+j5}{10+j4}\right]$$

$$= (Q_A \angle -36.86^\circ)\left[\frac{7.071\angle 45^\circ}{10.77\angle 21.80^\circ}\right]$$

$$= 0.6565\,Q_A \angle -13.66^\circ$$

Load shared by transformer B, $\quad Q_B = \mathbf{0.6565\,Q_A}$

p.f. of transformer $\quad B = \cos\phi_B = \cos(13.66^\circ)$

$$= \mathbf{0.9717\ (lag)}$$

Example 3.27 : *Two single phase transformers with equal turns have impedances of* (0.5 + j3) *ohm and* (0.6 + j10) *ohm with respect to the secondary. If they operate in parallel, determine how they will share a total load of* 100 kW *at p.f.* 0.8 *lagging ?*

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Solution :

$$Z_1 = 0.5 + j\,3\ \Omega$$

$$Z_2 = 0.6 + j\,10\ \Omega$$

$$\text{Total load} = 100\ \text{kW}$$

$$\text{p.f. of load} = 0..8\ \text{lag},\quad \cos\phi = 0.8,\quad \therefore\ \phi = \cos^{-1} 0.8 = 36.86^\circ$$

$$\text{kVA of load} = \frac{100}{0.8} = 125$$

$$\therefore \quad Q = 125 \angle -36.86^\circ\ \text{kVA}$$

$$\text{Load shared by transformer 1} = Q\left(\frac{Z_2}{Z_1 + Z_2}\right)$$

$$= [125 \angle -36.86^\circ]\left[\frac{0.6 + j\,10}{(0.5 + j\,3) + (0.6 + j\,10)}\right]$$

$$= \frac{(125 \angle -36.86^\circ)\ (10.017 \angle 86.56^\circ)}{1.1 + j\,13}$$

$$= \frac{1252.125 \angle 49.7^\circ}{13.046 \angle 85.16^\circ} = \mathbf{95.97 \angle -35.46^\circ\ kVA}$$

$$\textbf{p.f.} = \mathbf{\cos 35.46^\circ = 0.8145\ lag}$$

$$\text{Load shared by transformer 2} = Q\left(\frac{Z_1}{Z_1 + Z_2}\right)$$

$$= [125 \angle -36.86^\circ]\left[\frac{0.5 + j\,3}{(0.5 + j\,3) + (0.6 + j10)}\right]$$

$$= \frac{125 \angle -36.86^\circ)\ (3.041 \angle 80.53^\circ)}{13.046 \angle 85.16^\circ}$$

$$= \frac{380.125 \angle 43.67^\circ}{13.046 \angle 85.16^\circ} = \mathbf{29.13 \angle -41.49^\circ\ kVA}$$

$$\textbf{p.f.} = \mathbf{\cos 41.49^\circ = 0.7490\ lag}$$

A × p.f = kW

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ELECTRICAL & ELECTRONICS ENGINEERING

III B. Tech. I Semester EEE R20

Electrical Machines – I

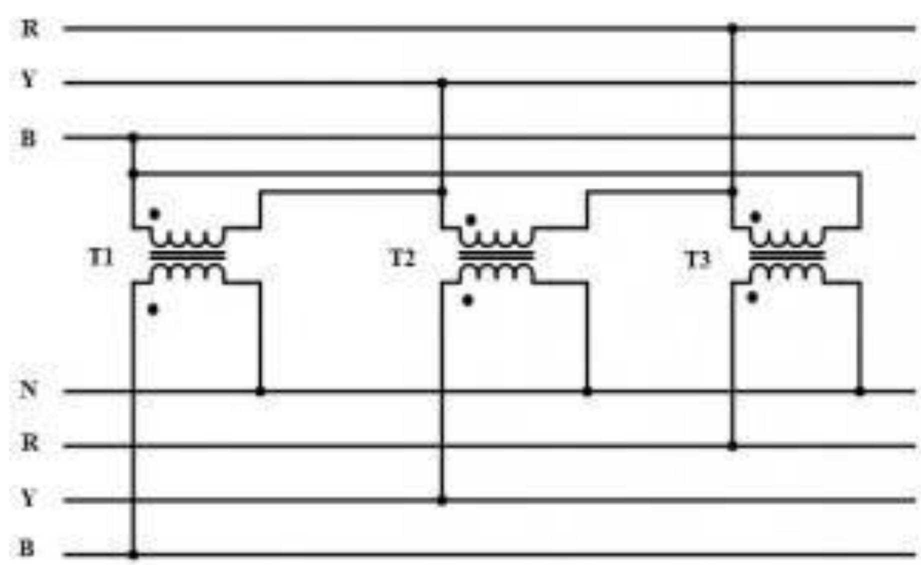
Material for Unit V

UNIT–III: 3-Phase Transformers:
Poly phase connections – Y/Y, Y/Δ, Δ/Y, Δ/Δ and open Δ – Third harmonics in phase voltages – Three winding transformers: determination of Z_p, Z_s and Z_t – off load and on load tap changers – Scott connection.

1. What is a three transformer?

Three phase transformers are used to step-up or step-down the high voltages in various stages of power transmission system. The power generated at various generating stations is in three phase nature and the voltages are in the range of 13.2KV or 22KV. In order to reduce the power loss to the distribution end, the power is transmitted at somewhat higher voltages like 132 or 400KV. Hence, for transmission of the power at higher voltages, three phase step-up transformer is used to increase the voltage. Also at the end of the transmission or distribution, these high voltages are step-down to levels of 6600, 400, 230 volts, etc. For this, a three phase step down transformer is used.

A three phase transformer can be built in two ways; a bank of three single phase transformers or single unit of three phase transformer. The former one is built by suitably connecting three single phase transformers having same ratings and operating characteristics. In this case if the fault occurs in any one of the transformers, the system still retained at reduced capacity by other two transformers with open delta connection. Hence, continuity of the supply is maintained by this type of connection. These are used in mines because easier to transport individual single phase transformers.



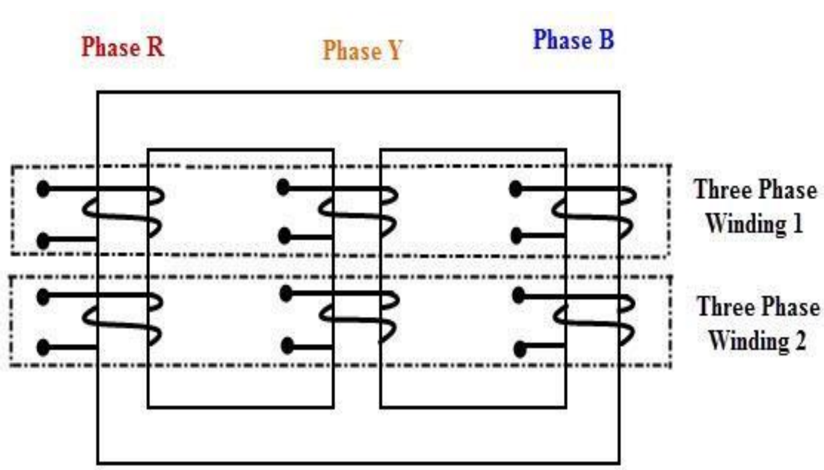
Instead of using three single phase transformers, a three phase bank can be constructed with a single three phase transformer consisting of six windings on a common multi-legged core. Due to this single unit, weight as well as the cost is reduced as compared to three units of the same rating and also windings, the amount of iron in the core and insulation materials are saved. Space required to install a single unit is less compared with three unit bank. But the only disadvantage with single unit three phase transformers is if the fault occurs in any one of the phase, then entire unit must be removed from the service.

Construction of Three Phase Transformers

A three phase transformer can be constructed by using common magnetic core for both primary and secondary windings. As we discussed in the case of single phase transformers, construction can be core type or shell type. So for a bank of three phase core type transformer, three core type single phase transformers are combined. Similarly, a bank of three phase shell type transformer is get by properly combining three shell type single phase transformers. In a shell type transformer, EI laminated core surrounds the coils whereas in core type coil surrounds the core.

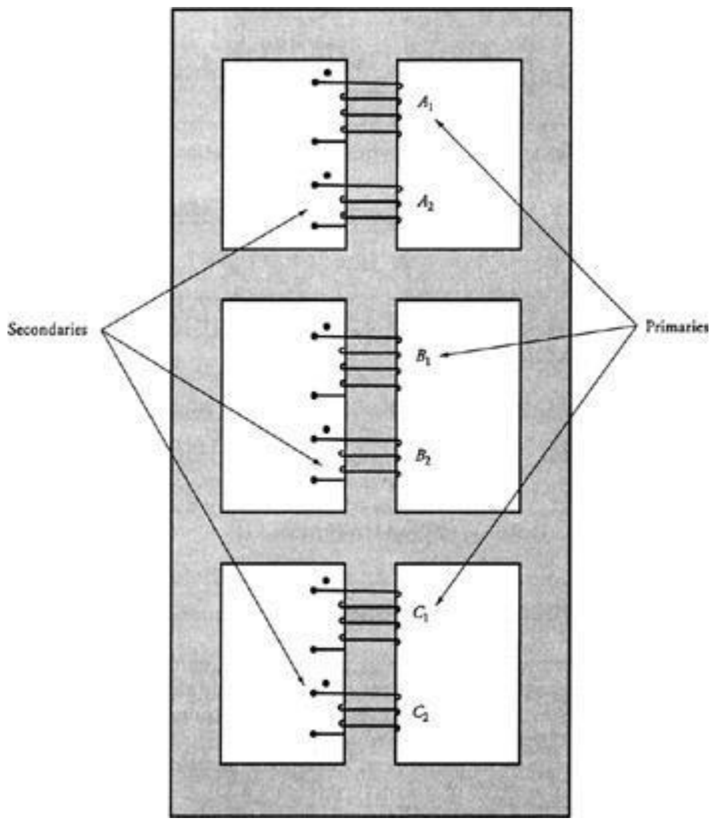
Core Type Construction

In core type three phase transformer, core is made up of three limbs or legs and two yokes. The magnetic path is formed between these yokes and limbs. On each limb both primary and secondary windings are wounded concentrically. Circular cylindrical coils are used as the windings for this type of transformer. The primary and secondary windings of one phase are wounded on one leg. Under balanced condition, the magnetic flux in each phase of the leg adds up to zero. Therefore, under normal conditions, no return leg is needed. But in case of unbalanced loads, high circulating current flows and hence it may be best to use three single phase transformers.



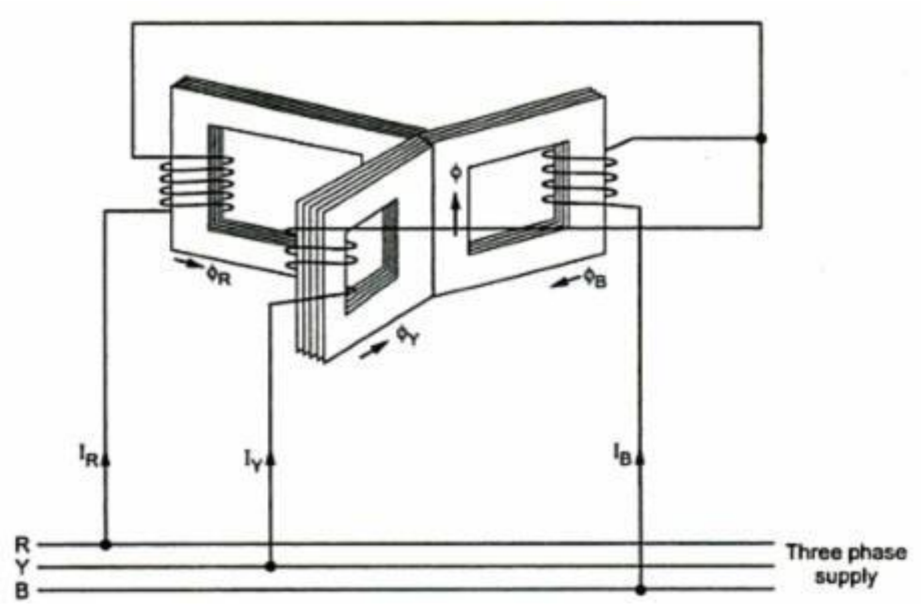
Shell Type Construction

In shell type, three phases are more independent because each phase has independent magnetic circuit compared with core type transformer. The construction is similar to the single phase shell type transformer built on top of another. The magnetic circuits of this type of transformer are in parallel. Due to this, the saturation effects in common magnetic paths are neglected. However, shell type constructed transformers are rarely used in practice.



Working of Three Phase Transformers

Consider the below figure in which the primary of the transformer is connected in star fashion on the cores. For simplicity, only primary winding is shown in the figure which is connected across the three phase AC supply. The three cores are arranged at an angle of 120 degrees to each other. The empty leg of each core is combined in such that they form center leg as shown in figure.



When the primary is excited with the three phase supply source, the currents IR, IY and IB are starts flowing through individual phase windings. These currents produce the magnetic fluxes ΦR, ΦY and ΦB in the respective cores. Since the center leg is common for all the cores, the sum of all three fluxes are carried by it. In three phase system, at any instant the vector sum of all the currents is zero. In turn, at the instant the sum of all the fluxes is same. Hence, the center leg doesn't carry any flux at any instant. So even if the center leg is removed it makes no difference in other conditions of the transformer. Likewise, in three phase system where any two conductors acts as return for the current in third conductor, any two legs acts as a return path of the flux for the third leg if the center leg is removed in case of three phase transformer. Therefore, while designing the three phase transformer, this principle is used. These fluxes induce the secondary EMFs in respective phase such that they maintain their phase angle between them. These EMFs drives the currents in the secondary and hence to the load. Depends on the type of connection used and number of turns on each phase, the voltage induced will be varied for obtaining step-up or step-down of voltages.

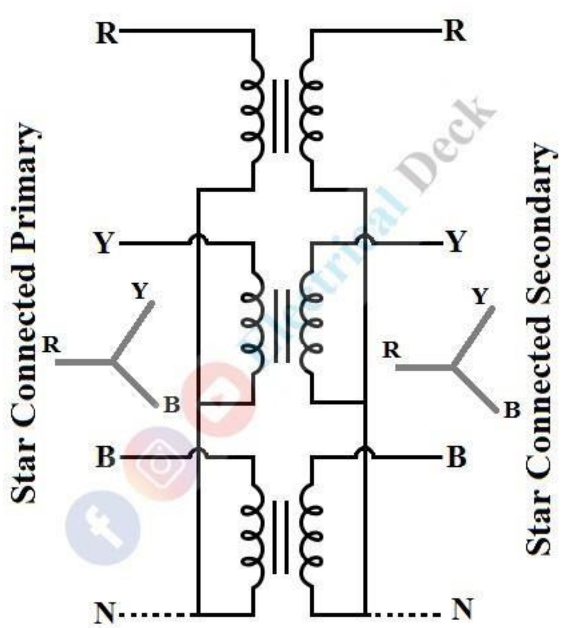
Three Phase Transformer Winding Connections

In a single-phase as well as three-phase transformer the windings are placed on the core structure which is made up of a laminated steel core. This core structure which supports the two windings of the transformer is placed inside the transformer tank (immersed in the oil for cooling). The terminals of the primary and secondary windings are brought out through the transformer tank. But before bringing out, the three windings (each winding for each phase) of primary and secondary are connected inside the transformer. Let us see the possible way of connecting the primary and secondary of a three-phase transformer.

The different connections used for three-phase transformer windings are :

- Star-Star (Y - Y),
- Delta-Delta (Δ - Δ),
- Star-Delta (Y - Δ), and
- Delta-Star (Δ - Y)

Star-Star (Y - Y) Connection:



In a star-star type of connection, the terminals of a three-phase winding on both primary and secondary sides are connected in the shape of the letter 'Y'. Since, both primary and secondary windings are connected in the same manner (i.e., star). It is known as the Star-to-Star connection. This type of connection is most economical for transformers that work for small currents and high voltages.

Advantages of Star-Star Connection:

- In star connection phase voltage $V_{ph} = 1/\sqrt{3} \times$ line voltage. We can see that line voltage is $\sqrt{3}$ times the phase voltage these intern decreases amount of coil turns required per phase.
- The insulation required for windings is also decreased for coil turns which are most economical.
- The phase displacement between the primary and secondary voltages is almost zero. Hence both the primary and secondary voltages are in phase.
- Transformer windings with star connections are good enough to use when the load is balanced.

Disadvantages of Star-Star Connection:

- If there is no earthing connected to the neutral star point of the load side winding. It is difficult to control the phase voltage on the load side when the load is unbalanced. This can be overcome by connecting the neutral point (star point) of the primary winding to the neutral point of the generating station.
- The waveform of the flux in the transformer core gets distorted if there is no proper neutral point connected to the primary winding. Because the transformer cannot draw its third harmonics part. Hence there can be a change in the magnitude of the magnetizing current and produces harmonics in supply voltage.

Star - Star Connection

$Y - Y$

1) $V_{ph2} \rightarrow ?$

$$K = \frac{V_{ph2}}{V_{ph1}}$$

$$V_{ph2} = K \cdot V_{ph.1}$$

$$V_{ph2} = K \cdot \frac{V_{L1}}{\sqrt{3}}$$

$$\boxed{V_{ph2} = K \cdot \frac{V_{L1}}{\sqrt{3}}}$$

2) $V_{L2} \rightarrow ?$

$$V_{ph2} = K \cdot \frac{V_{L1}}{\sqrt{3}}$$

$$\frac{V_{L2}}{\sqrt{3}} = K \cdot \frac{V_{L1}}{\sqrt{3}}$$

$$\boxed{\therefore V_{L2} = K \cdot V_{L1}}$$

3) $I_{ph2} \rightarrow ?$

$$K = \frac{I_{ph1}}{I_{ph2}}$$

$$I_{ph2} = \frac{I_{ph1}}{K} = \frac{I_{L1}}{K}$$

$$\therefore \boxed{I_{ph2} = \frac{I_{L1}}{K}}$$

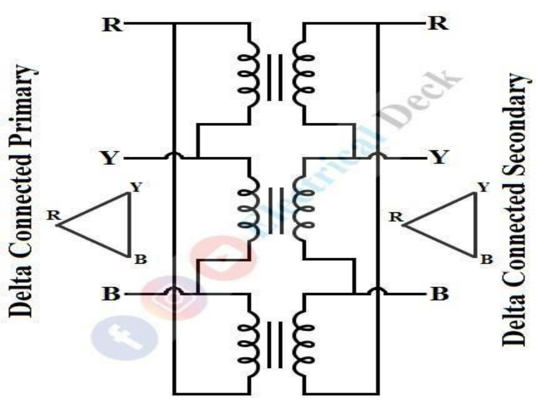
4) $I_{L2} \rightarrow ?$

$$I_{ph2} = \frac{I_{L1}}{K}$$

$$\boxed{I_{L2} = \frac{I_{L1}}{K}}$$

Delta-Delta (Δ - Δ) Connection:

In this type of connection, the end of one coil of a three-phase winding is connected to the end of the other coil. In this way, the three coils are connected such that it forms in a shape delta. The delta connected windings can generate large currents with low values of voltages. The below figure shows the Δ - Δ connections and voltage triangles.



Advantages of Delta-Delta Connection:

- There is no phase displacement between the primary and secondary voltages.
- In delta connected winding third harmonics can flow without any neutral point connected. Therefore, the chances of distortion in the waveform of the flux will be less.
- The phase current is reduced by √3 times. Hence the current carrying capacity of the conductor decreases thereby reducing the overall cost of the conductor.
- This connection can work satisfactorily when the load is unbalanced compared to star connection.
- Continuity is more i.e. if one of the phases is removed (fault) in Δ - Δ connection, operation continues on open-delta (or in V - V) with reduced (58%) capacity.

Disadvantages Delta-Delta Connection:

- In Δ - connected winding line voltage is equal to the phase voltage. This increases the insulation cost required when compared to Y - Y connection.
- This type of connection is not suitable for distribution purposes where there is a need for a return path (neutral point).

Delta - Delta Connection

$\Delta - \Delta$

1) $V_{ph2} \rightarrow ?$

$$K = \frac{V_{ph2}}{V_{ph1}}$$

$$V_{ph2} = K \cdot V_{ph1}$$

$$\boxed{V_{ph2} = K \cdot V_{L1}}$$

2) $V_{L2} \rightarrow ?$

$$V_{ph2} = K \cdot V_{L1}$$

$$\boxed{V_{L2} = K \cdot V_{L1}}$$

3) $I_{ph2} \rightarrow ?$

$$K = \frac{I_{ph1}}{I_{ph2}}$$

$$I_{ph2} = \frac{I_{ph1}}{K} \Rightarrow \frac{I_{L1}}{\sqrt{3} \cdot K}$$

$$\boxed{I_{ph2} = \frac{I_{L1}}{\sqrt{3} K}}$$

4) $I_{L2} \rightarrow ?$

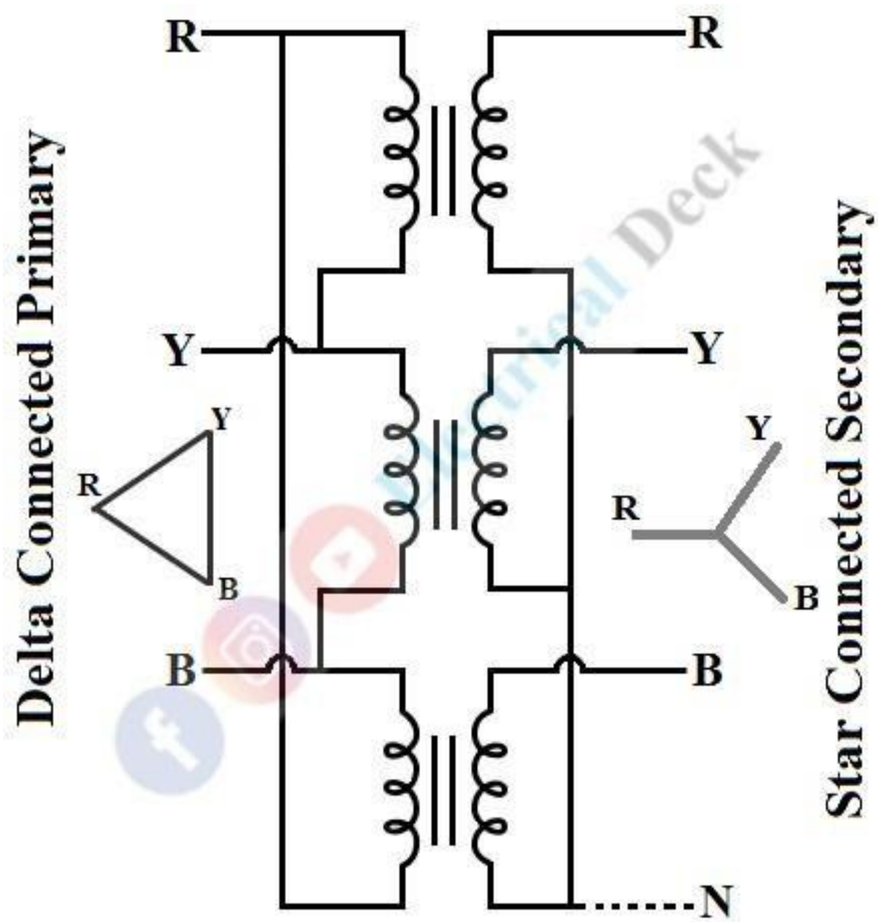
$$I_{ph2} = \frac{I_{L1}}{\sqrt{3} K}$$

$$\frac{I_{L2}}{\sqrt{3}} = \frac{I_{L1}}{\sqrt{3} K}$$

$$\boxed{I_{L2} = \frac{I_{L1}}{K}}$$

Delta-Star (Δ - Y) Connection:

It is a combination of delta connected winding on the primary side and star connection on the secondary side as shown below. The delta-star connection can be used where there is a need in increasing the voltage levels or stepping up.



This type of connection is well suited for distribution purposes due to the availability of a 3-phase, 4-wire system at the secondary side. But limited to its applications due to the existence of a phase shift between primary and secondary winding.

Delta - Star Connection

$\Delta - Y$

1) $V_{ph2} \rightarrow ?$

$$K = \frac{V_{ph2}}{V_{ph1}}$$

$$V_{ph2} = K \cdot V_{ph1}$$

$$\boxed{V_{ph2} = K \cdot V_{L1}}$$

2) $V_{L2} \rightarrow ?$

$$V_{ph2} = K \cdot V_{L1}$$

$$\frac{V_{L2}}{\sqrt{3}} = K \cdot V_{L1}$$

$$\boxed{V_{L2} = \sqrt{3} \cdot K \cdot V_{L1}}$$

3) $I_{ph2} \rightarrow ?$

$$K = \frac{I_{ph1}}{I_{ph2}}$$

$$I_{ph2} = \frac{I_{ph1}}{K}$$

$$\boxed{I_{ph2} = \frac{I_{L1}}{\sqrt{3} \cdot K}}$$

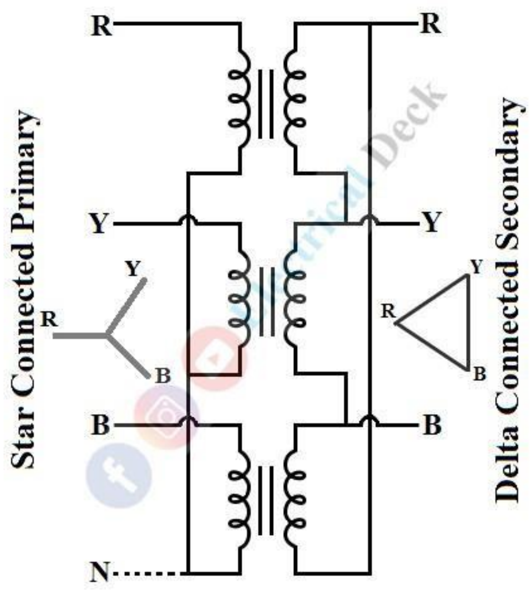
4) $I_{L2} = ?$

$$I_{ph2} = \frac{I_{L1}}{\sqrt{3} \cdot K}$$

$$\boxed{I_{L2} = \frac{I_{L1}}{\sqrt{3} \cdot K}}$$

Star-Delta (Y - Δ) Connection:

This type of winding connection is used where we need to decrease the voltages levels, for example, at the end of a transmission line. Here, the neutral of the transformer's primary winding is earthed.



In this system transformation ratio, K is equal to the ratio of secondary phase voltage to the primary phase voltage. Also third harmonic current flow in the Δ - connected winding can control the system even when it is unbalanced with minimized distortion in the waveform of the flux. On the primary side of the transformer, the winding is connected in a star. Here in the star connection line voltage is √3 times of phase voltage which decreases the effective use of per phase insulation required. But there will be a phase displacement between the primary and secondary voltages.

$Y - \Delta$

1)

$\longrightarrow$ $\longrightarrow$

$K = \frac{V_{ph2}}{}$ K V,_/

, 2 =

$$\boxed{V_{L2} = \frac{K \cdot V_{L1}}{\sqrt{3}}}$$

$V_{ph} \longrightarrow$

$V_{ph2} \quad \frac{}{\sqrt{3}}$

$$\boxed{V_{ph2} = \frac{K \cdot V_{L1}}{\sqrt{3}}}$$

$\longrightarrow$

3) I_{ph}

I_{ph2} $\frac{I_{L2}}{\sqrt{3}}$ $\frac{I_{L1}}{}$

$= \frac{}{K}$

$\frac{\sqrt{3} \cdot I_{L1}}{}$

$$\boxed{I_{ph2} = \frac{I_{L1}}{K}}$$

OFF LOAD & ON LOAD TAP CHANGER:

What is Tap-changing Transformer ? Off-load & On-load

In transmission and distribution systems there can be voltage fluctuations (i.e., increase or decrease in voltage levels) when the load on the system varies. These fluctuations can also be caused due to a voltage drop in the distribution system. Sometimes these variations in voltage levels can result in quite unsatisfactory performance.

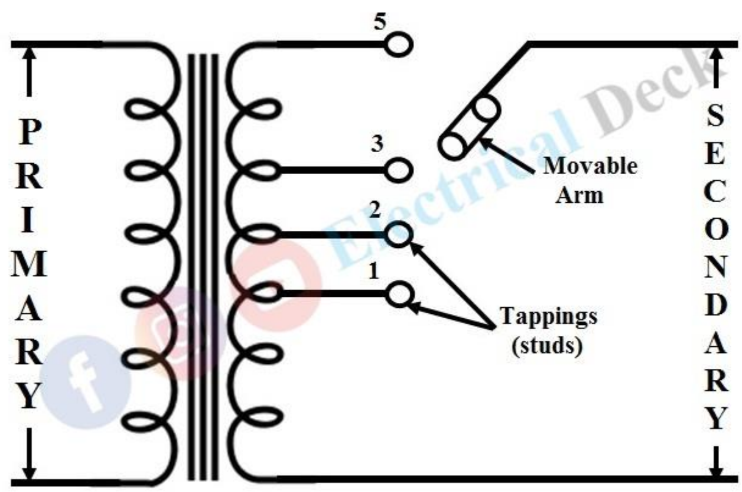
In order to maintain a constant voltage or to maintain within the prescribed limits transformer tap-changing is used. In tap-changing, the tappings on the coils of the transformer are placed so that by varying the turn-ratio voltage induced can be varied.

This arrangement is done externally to the transformer by taking coil terminals out of the transformer tank. Usually, the maximum allowable variation of the turn-ratio can be up to ±2% to 5%. There are two types of tap-changing transformers,

- Off-Load Tap-Changing Transformer.
- On-Load Tap-Changing Transformer.

Off-Load Tap-Changing Transformer :

The below figure shows the off-load tap-changing transformer provided with tappings (1 to 5) on the secondary winding. The position of the movable arm on the first stud will give minimum secondary voltage and on the fifth stud will give maximum voltage across secondary.

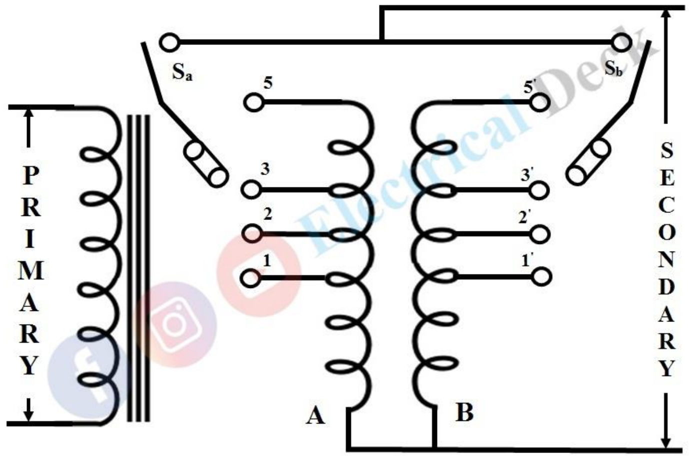


During the light load period, the movable arm is placed on the first stud and with an increase in load, the movable arm is taken to a stud (2, 3, 4, or 5) giving higher turns-ratio so that voltage drop in the line is compensated and the output secondary voltage is maintained.

The disadvantage of this scheme is whenever the tapping is to change load must be disconnected first from the transformer thus it is referred to as off-load tap-changing. This type of tap-changing cannot be used where continuity of the supply to the load is the main priority and it is limited where there will be a need for only slight changing in the turn-ratio.

On-Load Tap-Changing Transformer :

The drawback of off-load tap-changing can be overcome by using a special arrangement of coil connections to the transformer known as on-load tap-changing of the transformer. The transformer connection for on-load tap-changing is shown below.



Here, the coils of the winding in which tappings are to be done are divided into two parallel sections with equal tapping on both sections of the coil. This forms the two winding A & B as shown above. Under normal operating conditions both the switches (S_a & S_b) are in the closed (short-circuit) condition with identical tappings (i.e., 1 & 1'). As the winding is divided into parallel sections the total current will be the sum of the currents in winding A and B.

When the tappings are to be changed, to maintain the continuity of the supply the tap-changing process is made in such a way that,

- At first, any one of the winding (either A or B) from the parallel section is to be disconnected by open the respective switch.
- Now, the tap-changing is to be done to the disconnected winding.
- At this instant, the full-load current will pass through the connected winding (i.e., double of its rated current).
- After changing the tapping to the disconnected winding is reconnected by closing the switch.
- At this moment there will be an unequal share of the load on both windings due to their different turn ratios.
- Now the other winding is disconnected and tapping is to be changed (which is equal to the tapping of previously disconnected winding).
- So that there will be an equal amount of load share on both the windings (A & B).

In this way, the continuity of the supply is maintained and more turn-ratio of tap-changing can possibly compare to off-load tap-changing of the transformer. Care must be taken to prevent the short-circuit with the windings while the tap-changing process.

Example 5.1 : *A three phase transformer has delta connected primary and a star connected secondary working on 50 Hz three phase supply. The line voltages of primary and secondary are 3300 V and 400 V respectively. The line current on the primary side is 12 A and secondary has a balanced load at 0.8 lagging p.f. Determine the secondary phase voltage line current and the output.*

Solution : Primary side connected with delta

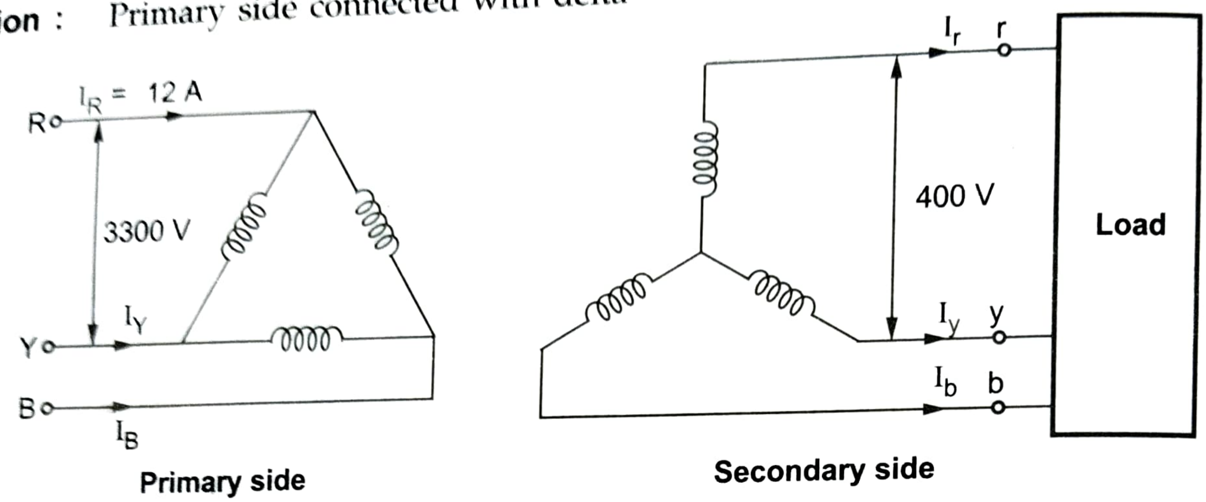


Fig. 5.13

$$V_{L_1} = 3300 \text{ V}, \quad I_{L1} = 12 \text{ A}$$

Secondary side star connected with p.f. 0.8

Primary side : $\quad V_{ph1} = V_{L1} = 3300 \text{ V}$

Secondary side : $\quad V_{ph2} = \dfrac{V_{L2}}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 230.94 \text{ V}$

Transformation Ratio, $\quad K = \dfrac{V_{ph2}}{V_{ph1}} = \dfrac{230.94}{3300} = 0.0699$

Primary side, $\quad I_{L1} = 12 \text{ A}$

$$I_{ph1} = \frac{I_{L1}}{\sqrt{3}} = \frac{12}{\sqrt{3}} = 6.928 \text{ A}$$

Secondary side, $\quad K = \dfrac{I_{ph1}}{I_{ph2}}$

$\therefore$

$$I_{ph_2} = \frac{I_{ph1}}{K} = \frac{6.928}{0.0699} = 99.11 \text{ A}$$

Since secondary is connected in star

$$I_{ph2} = I_{L2} = 99.11 \text{ A}$$

$$\text{Output} = \sqrt{3}\, V_{L2}\, I_{L2} \cos\phi$$

$$= \sqrt{3} \times 400 \times 99.11 \times 0.8$$

$$= \mathbf{54935.62\ W = 54.94\ kW}$$

Example 5.2 : *A 3 phase step down transformer is connected to 6600 volts mains and it takes 10 A. Calculate the secondary line voltage, line current, and output for the following connections*

a) Delta - Delta b) Star - Star c) Star - Delta d) Delta - Star

Turns ratio/phase is 12. Draw connection diagrams **JNTU : April-2004, Set-3**

Solution :

$$\text{Primary line voltage } V_{L_1} = 6600 \text{ volts}$$

$$\text{Primary line current } I_{L_1} = 10 \text{ A}$$

$$\text{Transformation ratio or turns ratio, } K = \frac{1}{12}$$

i) Delta-Delta connection

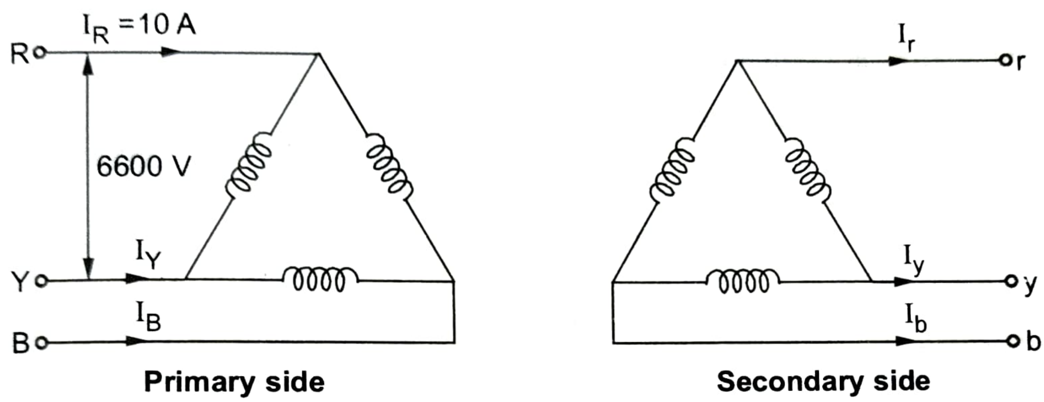


Fig. 5.14

Since primary is connected in delta

$$V_{L_1} = V_{ph_1} = 6600 \text{ V}$$

$$I_{ph_1} = \frac{I_{L_1}}{\sqrt{3}} = \frac{10}{\sqrt{3}} = 5.773 \text{ A}$$

$$K = \frac{V_{ph_2}}{V_{ph_1}}$$

$$V_{ph_2} = K \cdot V_{ph_1} = \frac{1}{12} \cdot 6600 = 550 \text{ V}$$

Since secondary is also connected in delta

$$V_{L_2} = V_{ph_2} = 550 \text{ V}$$

$$K = \frac{I_{ph_1}}{I_{ph_2}} \quad ; \quad I_{ph_2} = \frac{I_{ph_1}}{K} = \frac{5.773}{\left(\frac{1}{12}\right)} = 69.28 \text{ A}$$

$$I_{L_2} = \sqrt{3} I_{ph_2} = \sqrt{3} \times 69.28 = 120 \text{ A}$$

$$\text{Secondary Output} = \sqrt{3}\, V_{L_2} I_{L_2} = \sqrt{3} \times 550 \times 120$$

$$= 114.315 \text{ kVA}$$

ii) Star-Star Connection

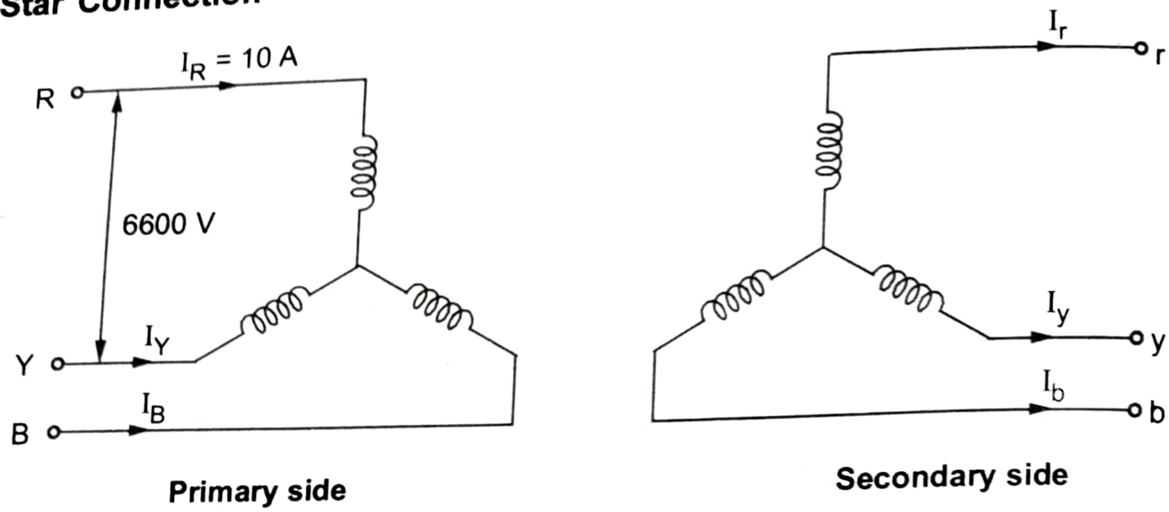


Fig. 5.15

Since primary side is connected in star

$$V_{L_1} = 6600 \text{ V}$$

$$V_{ph_1} = \frac{V_{L_1}}{\sqrt{3}} = \frac{6600}{\sqrt{3}} = 3810.5118 \text{ V}$$

$$I_{L_1} = I_{ph_1} = 10 \text{ A}$$

$$K = \frac{I_{ph_1}}{I_{ph_2}}$$

$$\therefore \quad I_{ph_2} = \frac{I_{ph_1}}{K} = \frac{10}{\left(\frac{1}{12}\right)} = 120 \text{ A}$$

Since secondary side is also connected in star

$$I_{L_2} = I_{ph_2} = 120 \text{ A}$$

Now, $$\frac{V_{ph_2}}{V_{ph_1}} = K$$

$$\therefore \quad V_{ph_2} = K \cdot V_{ph_1} = \left(\frac{1}{12}\right)(3810.5118) = 317.543 \text{ V}$$

$$\therefore \quad V_{L_2} = \sqrt{3}\, V_{ph_2} = \sqrt{3} \times 317.54 = 550 \text{ V}$$

Secondary output $= \sqrt{3}\, V_{L_2} \cdot I_{L_2}$

$= \sqrt{3} \times 550 \times 120$

$= 114.315 \text{ kVA}$

iii) Star-Delta Connection

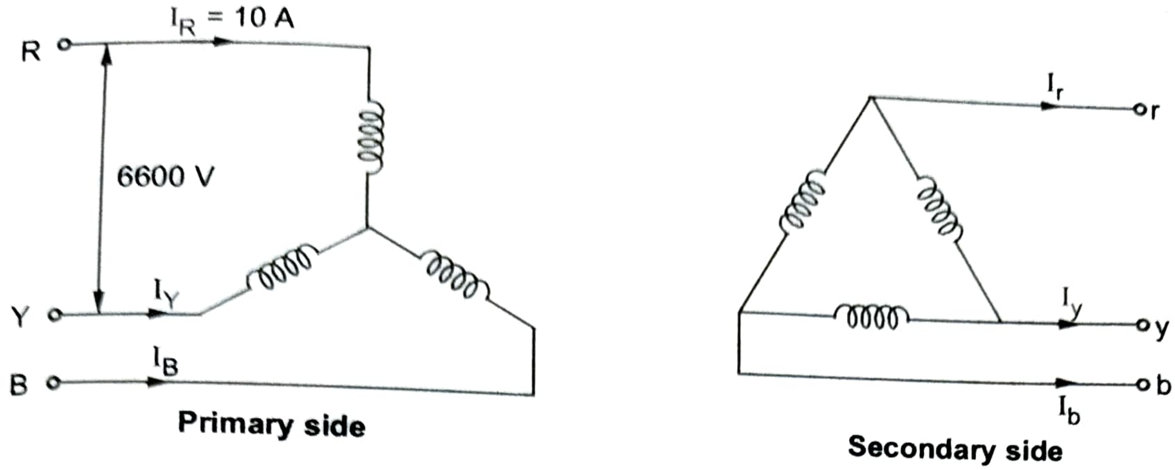


Fig. 5.16

Since primary side is connected in star

$$V_{L_1} = 6600 \text{ V}$$

$$V_{ph_1} = \frac{V_{L_1}}{\sqrt{3}} = \frac{6600}{\sqrt{3}} = 3810.51 \text{ V}$$

$$K = \frac{V_{ph_2}}{V_{ph_1}}$$

$$\therefore \quad V_{ph_2} = K\, V_{ph_1} = \frac{1}{12} \times 3810.51 = 317.543 \text{ V}$$

Since secondary side is connected in delta

$$V_{L_2} = V_{ph_2} = 317.543 \text{ V}$$

$$I_{L_1} = 10 \text{ A} = I_{ph_1}$$

$$K = \frac{I_{ph_1}}{I_{ph_2}}$$

$$I_{ph_2} = \frac{I_{ph_1}}{K} = \frac{10}{\left(\frac{1}{12}\right)} = 120 \text{ A}$$

$$I_{L_2} = \sqrt{3}\, I_{ph_2} = \sqrt{3} \times 120 = 207.846 \text{ A}$$

$$\text{Secondary output} = \sqrt{3}\, V_{L_2} I_{L_2} = \sqrt{3} \times 317.543 \times 207.846$$

$$= \mathbf{114.315 \text{ kVA}}$$

$I_{L2} = \frac{I_L\sqrt{3}}{K}$.

=

iv) Delta - Star connection

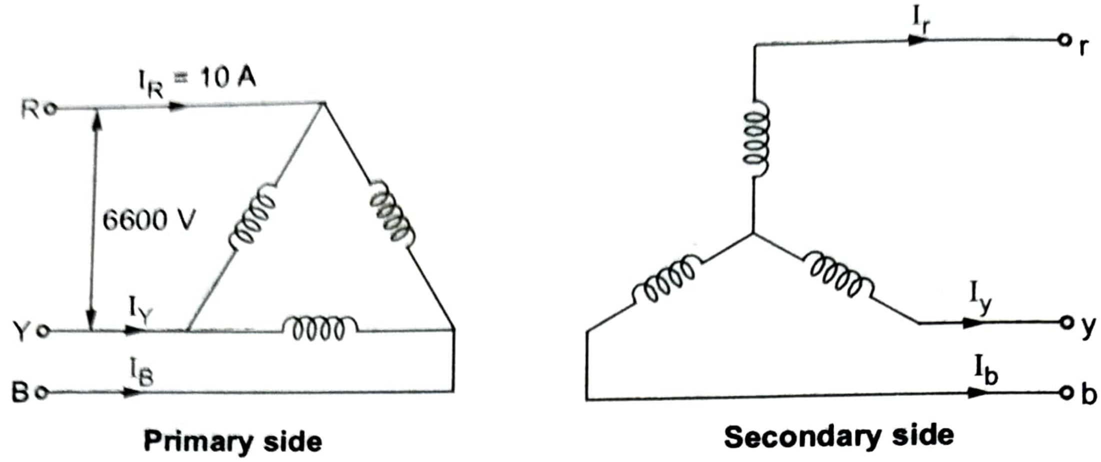


Fig. 5.17

Since primary is connected in delta

$$V_{L_1} = V_{ph_1} = 6600 \text{ V}$$

$$I_{L_1} = \sqrt{3}\, I_{ph_1}$$

$$I_{ph_1} = \frac{I_{L_1}}{\sqrt{3}} = \frac{10}{\sqrt{3}} = 5.7735 \text{ A}$$

$$K = \frac{I_{ph_1}}{I_{ph_2}}$$

$$I_{ph_2} = \frac{I_{ph_1}}{K} = \frac{5.7735}{\left(\frac{1}{12}\right)} = 69.282 \text{ A}$$

Since secondary is connected in star

$$I_{L_2} = I_{ph_2} = 69.282 \text{ A}$$

$$K = \frac{V_{ph_2}}{V_{ph_1}}$$

$$V_{ph_2} = K\, V_{ph_1} = \left(\frac{1}{12}\right)(6600) = 550 \text{ V}$$

$$V_{L_2} = \sqrt{3} \times 550 = 952.62 \text{ V}$$

$$\text{Secondary output} = \sqrt{3}\, V_{L_2} I_{L_2} = \sqrt{3} \times 952.62 \times 69.282$$

$$= \mathbf{114.315 \text{ kVA}}$$

Open Delta Connection (V-V Connection)

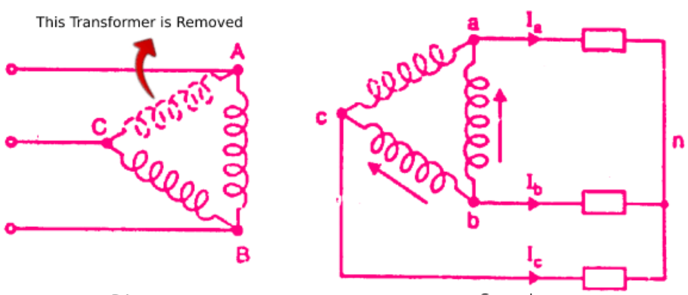
If one transformer breaks down in a star-star connected system of 3 single-phase transformers in a substation, three-phase power cannot be supplied until the defective transformer has been replaced or repaired.

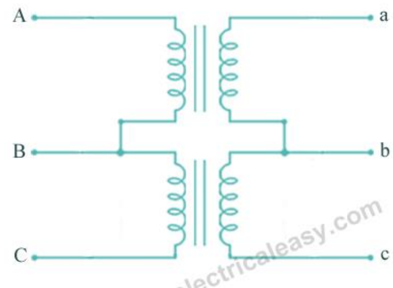
To eliminate this undesirable condition, single-phase transformers are generally **connected in Δ-Δ**.

In this case, if one transformer breaks down, it is possible to continue supplying three-phase power with the other two transformers because this arrangement maintains correct voltage and phase relations on the secondary.

If one transformer of delta-delta connected transformer bank is damaged or opened, then rest of the system will continue to supply the 3-phase power. If this damaged transformer is isolated, the remaining two transformers will function as a 3-phase bank with rating reduced to about 58 % that of the original delta-delta bank. This type of arrangement is known as *open-delta* or *V-V connection*.

Therefore, in the case of open-delta or V-V connection, two instead of three 1-phase transformers are used for three-phase operation.





Open Delta (V-V) Connection

However, **with two transformers, the capacity of the bank is reduced to 57.7%** of what it was with all three transformers in service (i.e., complete Δ-Δ circuit).

Open-Delta Connection Calculations and Formulae

If V_{2} and I_{2} are the rated secondary voltage and rated secondary current respectively of the delta-delta connected transformer. Then, the line current to the load of a delta connected system is √3I_{2}. Therefore, the normal delta load VA is,

$$S_{\triangle-\triangle} = \sqrt{3} \times \text{line voltage} \times \text{line current}$$

$$\Rightarrow S_{\triangle-\triangle} = \sqrt{3} \times V_2 \times (\sqrt{3} I_2) = 3V_2 I_2 \ldots (3)$$

Now, one transformer is removed, the delta-delta connection becomes open-delta connection and the lines are in series with the windings of the transformer. Thus, the secondary line currents is equal to the rated secondary current. Therefore, the load VA carried by the opendelta connection without exceeding the ratings of the transformer is,

$$S_{V-V} = \sqrt{3} V_2 I_2 \ldots (4)$$

Therefore,

$$\frac{S_{V-V}}{S_{\triangle-\triangle}} = \frac{\sqrt{3} V_2 I_2}{3 V_2 I_2} = \frac{1}{\sqrt{3}} = 0.577$$

$$\Rightarrow S_{V-V} = 57.7\% \text{ of } S_{\triangle-\triangle} \quad \ldots (5)$$

Hence, the load that can be carried by an open-delta transformer without exceeding the ratings of it is 57.7 % of the original load that is carried by the normal delta-delta transformer

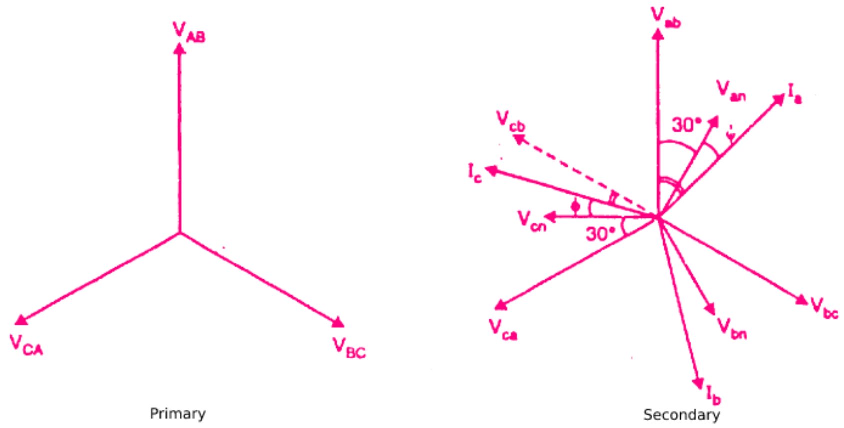
Also,

$$\frac{\text{VA per transformer}}{\text{Total 3} - \text{phase VA}} = \frac{V_2 I_2}{\sqrt{3} V_2 I_2} = \frac{1}{\sqrt{3}} = 0.577 \ldots (6)$$

From the eq. (6), it is clear that the VA supplied by each transformer in an open-delta system is also 57.7 % of the total 3-phase VA.

Now, if three 1-phase transformers are connected in delta-delta fashion and supplying rated load. As soon as it becomes an open-delta transformer, the current in each winding is increased by √3 times, i.e., full line current flows in each of the remaining two phase windings of the transformer. Hence, each transformer in the open-delta system is overloaded by 73.2%. Therefore, it is an important precaution that the load should be reduced by √3 times in the case of V-V connection of the transformers. Otherwise, the remaining two transformers may breakdown due to overheating.

Power Supplied by Open-Delta (V-V) Connected Transformer



When an open-delta bank of two transformers supplies a balanced three phase load of power factor cos φ, then the phase angle between the line voltage and the line current in one transformer is (30°+φ) whereas the phase angle between the line voltage and the line current in the other transformer is (30°-φ). Hence, one transformer operates at a power factor of cos(30°+φ) and the other at cos(30°-φ). Therefore, the power supplied by the transformers is given by,

$$P_1 = V_L I_L \cos(30^\circ + \varphi)$$

$$P_2 = V_L I_L \cos(30^\circ - \varphi)$$

The total power supplied by the transformers is

$$P = P_1 + P_2 = V_L I_L \cos(30^\circ + \varphi) + V_L I_L \cos(30^\circ - \varphi)$$

$$\Rightarrow P = V_L I_L (\cos 30^\circ \cos\varphi - \sin 30^\circ \sin\varphi + \cos 30^\circ \cos\varphi + \sin 30^\circ \sin\varphi)$$

$$\Rightarrow P = 2V_L I_L \cos 30^\circ \cos\varphi$$

$$\Rightarrow P = \sqrt{3} V_L I_L \cos\varphi \ldots (7)$$

At the load of unity power factor i.e.

$$\cos\varphi = 1 \Rightarrow \varphi = 0^\circ$$

Hence, the power supplied by each transformer is

$$P_1 = P_2 = V_L I_L \cos 30^\circ = \frac{\sqrt{3}}{2} V_L I_L \ldots (8)$$

Applications of the Open Delta or V-V Connection

Following are the applications in which open-delta system is used −

- As a temporary measure, when one transformer of a delta-delta bank is damaged and removed for maintenance.
- The V-V connected transformers are used to supply a combination of large 1-phase and smaller 3-phase loads.
- The V-V connected transformers are used to provide service in a new development area where the full growth of load requires several years. In such cases, an open-delta system is installed in the initial stage and whenever the need arises at a future date to accommodate the growth in the power demand, a third transformer is added for deltadelta operation. This third transformer increases the capacity of the bank by 73.2%.

Third Harmonics in Phase voltages:

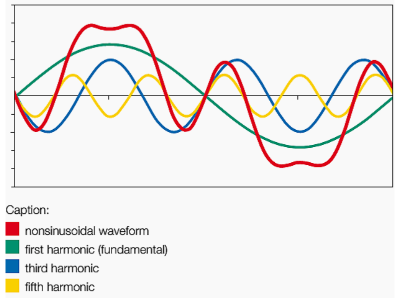
The third harmonic present in the phase magnetising current of three phase transformer is not present in the line current. ... The third harmonic components are **flows rounds the closed loop of the delta**. The delta connection only allows a sinusoidal flux and voltage with no third harmonic current in the transmission line.

It is a fraction of the energy exhibiting behaviour at a frequency 3 times the fundamental, but at a lower amplitude.

Any time you chop the top off a sine wave, it introduces a long series of odd harmonics, of diminishing amplitude. The first, and largest of these is the third. The transformers usually act as a low pass filter, so the others get progressively attenuated.

Transformers tend to distort the tops of sine waves, because the BH magnetisiation curve is not a straight line, but a sort of S curve, and for reasons of economy (we want to use as little iron as possible) the tops of the sine waves tend to push into the most non-linear region of the magnetising curve

So it is perfectly normal for transformers to introduce harmonics, principally the 3rd. There are various configurations and design tricks that are used to minimise this.



Third-harmonic voltages can be large. If a three-phase set of voltages is applied to a Y - Y transformer, the voltages in any phase will be 1200 apart from the voltages in any other phase. However, the third-harmonic components of each of the three phases will be in phase with each other, since there are three cycles in the third harmonic for each cycle of the fundamental frequency. There are always some third-harmonic components in a transformer because of the nonlinearity of the core, and these components add up. The result is a very large third-harmonic component of voltage on top of the 50 or 6O-Hz fundamental voltage. This third-harmonic voltage can be larger than the fundamental voltage itself.

Both the unbalance problem and the third-harmonic problem can be solved using one of the two following techniques:

1.Solidly ground the neutrals of the transformers, especially the primary winding's neutral. This connection permits the additive third-harmonic components to cause a current flow in the neutral instead of building up large voltages. The neutral also provides a return path for any current imbalances in the load.

2. Add a third (tertiary) winding connected in Δ to the transformer bank. If a third Δ connected winding is added to the transformer, then the third-harmonic components of voltage in the Δwill add up, causing a circulating current flow within the winding.

This suppresses the third-harmonic components of voltage in the same manner as grounding the transformer neutrals. The Δ connected tertiary windings need not even be brought out of the transformer case, but they often are used to supply lights and auxiliary power within the substation where it is located. The tertiary windings must be large enough to handle the circulating currents, so they are usually made about one-third the power rating of the two main windings. One or the other of these correction techniques must be used any time a Y-Y transformer is installed. In practice, very few Y-Y transformers are used, since the same jobs can be done by one of the other types of three-phase transformers.

Determination of impedance of 3-winding transformers:

Generally, large power transformers have three windings. The third winding is known as a tertiary winding which may be used for the following purposes

1. To supply a load at a voltage different from the secondary voltage.

2. To provide a low impedance for the flow of certain abnormal currents, such a third harmonic currents.

3. To provide for the excitation of a regulating transformer.

Note: When one winding is left open, the three winding transformer behaves as two winding transformer and standard short circuit tests can be used to evaluate per unit leakage impedances which are defined as follows

Z_{ps} = per unit leakage impedance measured from primary with secondary shorted and tertiary open.

Z_{pt} = per unit leakage impedance measured from primary with tertiary shorted and secondary open.

Z_{st} = per unit leakage impedance measured from secondary with tertiary shorted and primary pen.

$Z_{ps} = Z_p + Z_s$ ……………(a)

$Z_{pt} = Z_p + Z_t$ ………………(b)

$Z_{st} = Z_s + Z_t$ ………………(c)

Where Z_p, Z_s , and Z_t : the impedances of primary, secondary and tertiary.

Solving these equations we find

$Z_p = \frac{1}{2} (Z_{ps} + Z_{pt} - Z_{st})$ ……..(d)

$Z_s = \frac{1}{2} (Z_{ps} + Z_{st} - Z_{pt})$ ……….. (e)

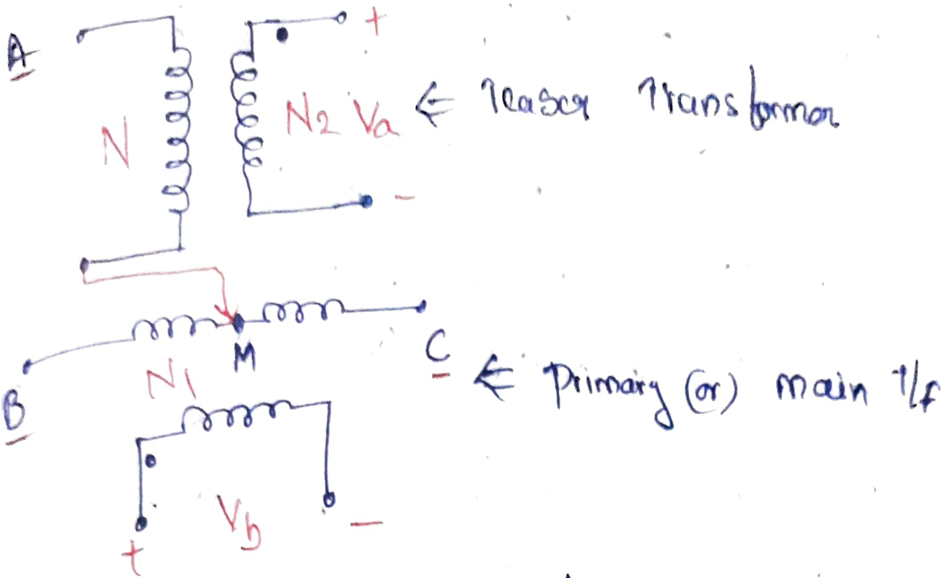
$Z_t = \frac{1}{2} (Z_{pt} + Z_{st} - Z_{ps})$ ………..(f)

These equations can be used to evaluate the per unit series impedances Z_p, Z_s , and Z_t of three winding transformer equivalent circuit from the per unit impedances Z_{ps}, Z_{pt} and Z_{st} which in turn are determined from short circuit tests. Note. The impedances Z_p, Z_s , and Z_t of the three windings are connected in star .

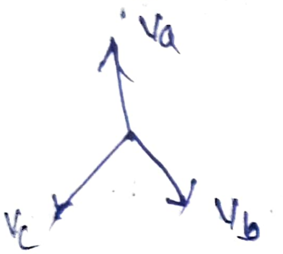
SCOTT (T-T) CONNECTION

Definition: The Scott-T Connection is the method of connecting two single phase transformer to perform the 3-phase to 2-phase conversion and vice-versa. The two transformers are connected electrically but not magnetically. One of the transformers is called the main transformer, and the other is called the auxiliary or teaser transformer.

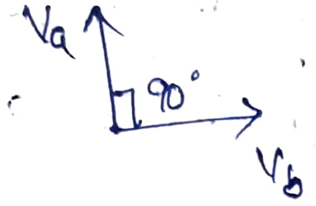
① We have to design - Scott Connection



3φ System



2φ System



We have to convert 3φ to 2φ System

Requirements:

① Voltage phasor must be 90° to Each other

② $|V_a| = |V_b|$

Here we have taken two transformers - 1φ

three phase supply is given through A, B & C terminals.

Primary winding of main transformer is divided in to two parts i.e. Center tapped with M.

Assume No of turns of teaser transformer - N

Primary → No of turns of main transformer - N_1

Secondary winding of both primary T/F

and Teaser transformer - N_2

using dot potential the voltage across teaser

transformer can be represended as shown

with potentials given.

Similarly the secondary of main T/F can

be represented as shown in figure.

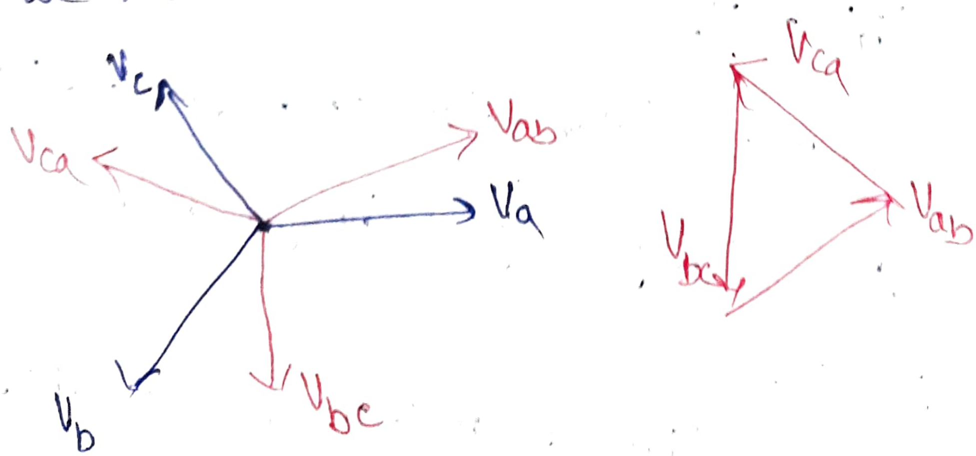
and also Primary second terminal is connected

to M point of main T/F as shown

Three phase supply is given to A, B, & C

now we can draw phasor diagram,

we know that



I am taking V_{BC} as reference

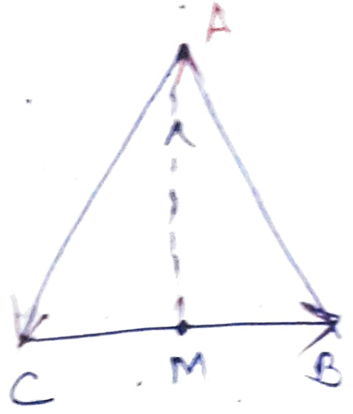


fig ②

point M lie in between V_{BC} phase, hence

V_{AM} is lear to V_{BC}

Now the Transformation Ratio is ⇒

as we know $\frac{V_2}{V_1} \Rightarrow \frac{N_2}{N_1}$, hence

for Teaser T/F $= \frac{V_a}{V_{AM}} = \frac{N_2}{N_0} \Rightarrow V_a = \frac{N_2}{N} V_{AM}$ — ①

for main T/F $= \frac{V_b}{V_{BC}} = \frac{N_2}{N_1} \Rightarrow V_b = \frac{N_2}{N_1} V_{BC}$ — ②

we know that, the phase voltage must be parallel to each other in 1φ transformer hence

from fig ②, we can represent

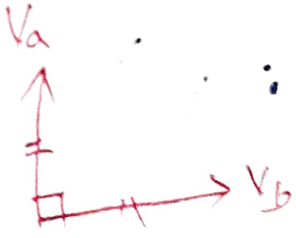


fig ③

∴ V_a is ~~inphase~~ parallel to V_{AM} &

V_b is in parallel to V_{BC}, hence our first requirement ie, two phase output is Exactly Equal to 90° to Each other.

We can to fulfill second requirement ie V_a & V_b

Case ②

Magnitude should be Equal. So Equate Equ ① & Equ ②

hence $V_a = \frac{N_2}{N} V_{Am}$ & $V_b = \frac{N_2}{N_1} V_{BC}$

hence $\frac{\cancel{N_2}}{N} V_{AM} = \frac{\cancel{N_2}}{N_1} V_{BC}$

$$\boxed{\frac{N}{N_1} = \frac{V_{AM}}{V_{BC}}}$$

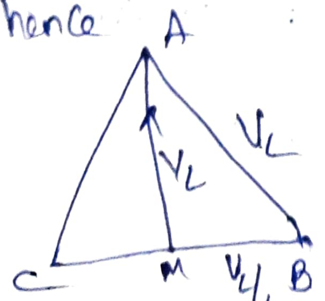
$\Rightarrow$ $N = N_1 \left(\frac{V_{AM}}{V_{BC}}\right)$ ——— ③

ie No of turns of tease T/F $= N_2$ $N_1 \left(\frac{N_{AM}}{V_{BC}}\right)$

Calculate V_{AM} $\Rightarrow$

WKT $V_{BC}, V_{AB}, V_{CA} = V_L$

hence



hence

$V_{AM} = \sqrt{V_{AB}^2 - V_{MB}^2}$

$V_{AM} = \sqrt{V_L^2 - \frac{V_L^2}{4}}$

$V_{AM} = \frac{\sqrt{3}\, V_L}{2}$

WKT $V_{BC} = V_L$

Subs V_{AM} & V_{BC} in Eq (8)

$$N = N_1 \frac{\frac{\sqrt{3}}{2}\cancel{V_L}}{\cancel{V_L}}$$

$$N = \frac{\sqrt{3}}{2} N_1$$

$$\therefore \boxed{N = 0.866\, N_1}$$

we get magnitude ie $|V_a| = |V_b|$ when $N = 0.866 N_1$

$N \Rightarrow \underline{86.7\% \, N_1}$

Hence the circuit can be modified.

we can conclude that

(1) $V_a = \frac{N_2}{N_1} V_{AM}$ $\qquad V_{AM} = \frac{\sqrt{3}}{2} V_L$

$$= \frac{N_2}{\frac{\sqrt{3}}{2} N_1} \times \frac{\sqrt{3}}{2} V_L$$

$$\boxed{V_a = \frac{N_2}{N_1} V_L} \leftarrow (4)$$

$$V_b = \frac{N_2}{N_1} V_{BC}$$

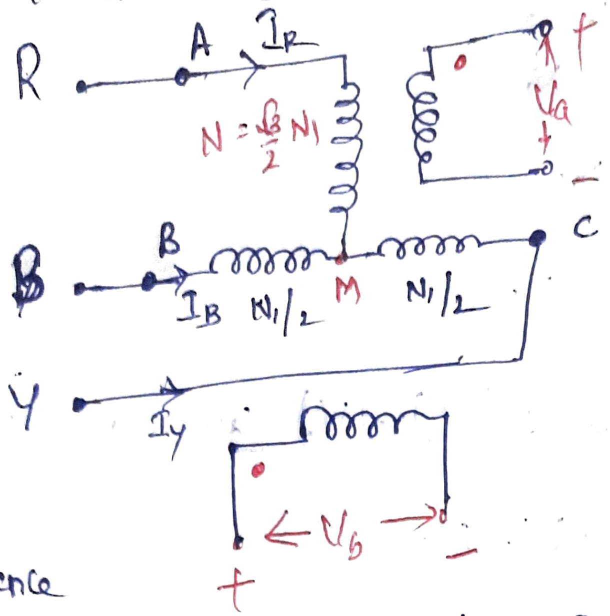
$$\boxed{V_b = \frac{N_2}{N_1} V_L} - (5)$$

By observing (4) & (5)

we can prove that $|V_a| = |V_b|$

& $\angle V_a$ & $\angle V_b = \underline{90^\circ}$

Hence the final circuit is



Hence

- → Main Transformer has Centertapped primary at M
- → Main transformer has N_1 no of turns
- → main transformer primary is connected between Y and B.
- → Teaser transformer primary connected between R phase and M
- → Teaser Primary has 0.866 of N_1 turns
- → Secondary of both have Equal turns
- → $|V_a| = |V_b|$ and and V_b leads V_a by 90°