Unit-1: Measuring Instruments

Characteristics of an instruments

The performance characteristics of an instrument are mainly divided into two categories: 1) Static characteristics 2) Dynamic characteristics

1) Static characteristics: of a measurement system are in general those that must be considered when the system or instrument is used to a condition not varying with time.

i) Precision: refers to the degree of consistency and reproducibility of the results obtained from a measuring instrument or process. It indicates how close repeated measurements of the same quantity are to each other.

ii) Accuracy: refers to how close the measured values are to the true or target value. It is a measure of how well the measurements represent the actual quantity being measured.

iii) Sensitivity: The ratio of the change in output (response) of the instrument to a change of input or measured variable

iv) Resolution: The smallest change in a measured variable to which an instrument will respond

v) Tolerance: refers to the acceptable range of variation or deviation from a specified value or dimension.

2) Dynamic characteristics : we must examine the dynamic relations which exist between the output and input, this is normally done with the help of differential equations performance criteria based up on dynamic relations constitute the dynamic characteristics.

i) Speed of response: It is defined as the rapidity with which a measurement system responds to changes in the measured quantity.

ii) Measuring lag: It is the retardation or delay in the response of a measurement system to changes in the measured quantity. The measuring lags are of two types:

- **a) Retardation type:** In this case the response of the measurement system begins immediately afterthe change in measured quantity has occurred.
- **b) Time delay lag:** In this case the response of the measurement system begins after a dead time after the application of the input.

iii) Fidelity: It is defined as the degree to which a measurement system indicates changes in the measure and quantity without dynamic error

iv) Dynamic error: It is the difference between the true value of the quantity changing with time & the value indicated by the measurement system if no static error is assumed. It is also called measurement error.

<u>ERROR:</u> is defined as the difference between the measured value & the true value of the quantity (expected value). Then

$$\text{Error } (\delta A) = A_m - A_t, \quad \text{Relative Error} = €_r = (\delta A/A_t) = (A_m - A_t / A_t)$$

Relative Accuracy (A) = $1 - \left| A_m - A_t/A_t \right|$

%Accuracy (a) = A*100% = 100% - %Error

TYPES OF ERRORS

The types of errors are follows

a) Gross Errors:

This error is mainly due to human mistakes in reading or in using instruments or errors in recording observations. Errors may also occur due to incorrect adjustments of instruments and computational mistakes. These errors cannot be treated mathematically. The complete elimination of gross errors is not possible, but one can minimize them. Some errors are easily detected while others may be elusive. One of the basic gross errors that occur frequently is the improper use of an Instrument. The error can be minimized by taking proper care in reading and recording the measurement parameter

b) Systematic errors

There are basically three types of systematic errors

(i) **Instrumental Errors**: Instrumental errors are inherent in measuring instruments, because of their mechanical structure. For example, in the D'Arsonval movement friction in the bearings of various moving components, irregular spring tensions, stretching of the spring or reduction in tension due to improper handling or over loading of the instrument. Instrumental errors can be avoided by (a) Selecting a suitable instrument for the particular measurement applications. (b) Applying correction factors after determining the amount of instrumental error (c) Calibrating the instrument against a standard instrument.

(ii) **Environmental Errors:** Environmental errors are due to conditions external to the measuring device, including conditions in the area surrounding the instrument, such as the effects of change in temperature, humidity, barometric pressure or of magnetic or electrostatic fields. These errors can also be avoided by (i) air conditioning, (ii) hermetically sealing certain components in the instruments and (iii) using magnetic shields.

(iii) **Observational Errors**: Observational errors are errors introduced by the observer. The most common error is the **parallax error** introduced in reading a meter scale, and the error of estimation when obtaining a reading from a meter scale. These errors are caused by the habits of individual observers. For example, an observer may always introduce an error by consistently holding his head too far to the left while reading a needle and scale reading. In general, systematic errors can also be subdivided into static and dynamic errors. Static errors are caused by limitations of the measuring device or the physical laws governing its behavior. Dynamic errors are caused by the instrument not responding fast enough to follow the changes in a measured variable

c) **Random errors:** Some errors still present, after the systematic and instrumental errors are reduced or accounted. We have no control over these errors. The causes of such errors are unknown and hence the errors are called random errors

Unit-I

MEASURING INSTRUMENTS

Measurements are the basic means of acquiring knowledge about the parameters and variables involved in the operation of a physical system. Measurement, generally involves using an instrument as a physical means of determining a quantity or variable. An instrument or a measuring instrument is, therefore, defined as a device for determining the value or magnitude of a quantity or variable. The electrical measuring instrument, as its name implies, is based on electrical principles for its measurement function.

These days a number of measuring instruments, both analog as well as digital ones, are available for the measurement of electrical quantities like voltage, current, power energy, frequency, power factor, etc. First Analog devices are worth concerning.

Analog instruments may be divided into three groups:

(a) Electromechanical instruments;

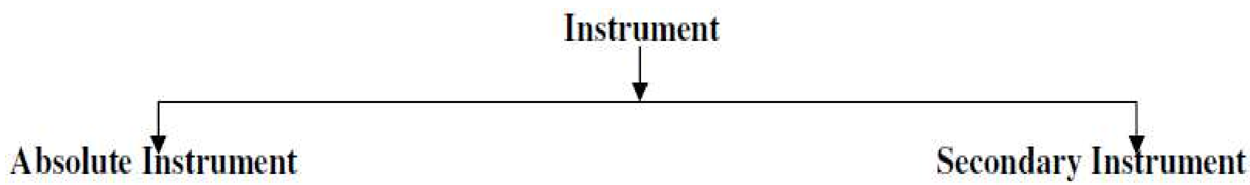
(b) Electronic instruments which are often constructed by the addition of electronic circuits to electromechanical indicators thus increasing their sensitivity and input impedances; and

(c) Graphical instruments which are electromechanical and electronic instruments having a modified display arrangement so that a graphical trace, that is, a display of instantaneous values against time is obtained.

Direct Measurement:- In direct method the unknown quantity is measured directly such as measurement of current by an ammeter, voltage by voltmeter, resistance by ohm meter, power by wattmeter etc.

Indirect Measurement:- In the indirect method of measurement the unknown quantity is determined by measuring other functionally related quantities and calculating the desired quantity rather than measuring it directly with an instrument. For example resistance of a conductor may be determined by measuring voltage across the conductor V, and current flowing through the conductor I, and then calculating it by R=V/I

Classification of instruments



Absolute instruments or Primary Instruments:- These instruments gives the magnitude of quantity under measurement in terms of physical constants of the instrument e.g. Tangent Galvanometer. These instruments do not require comparison with any other standard instrument.

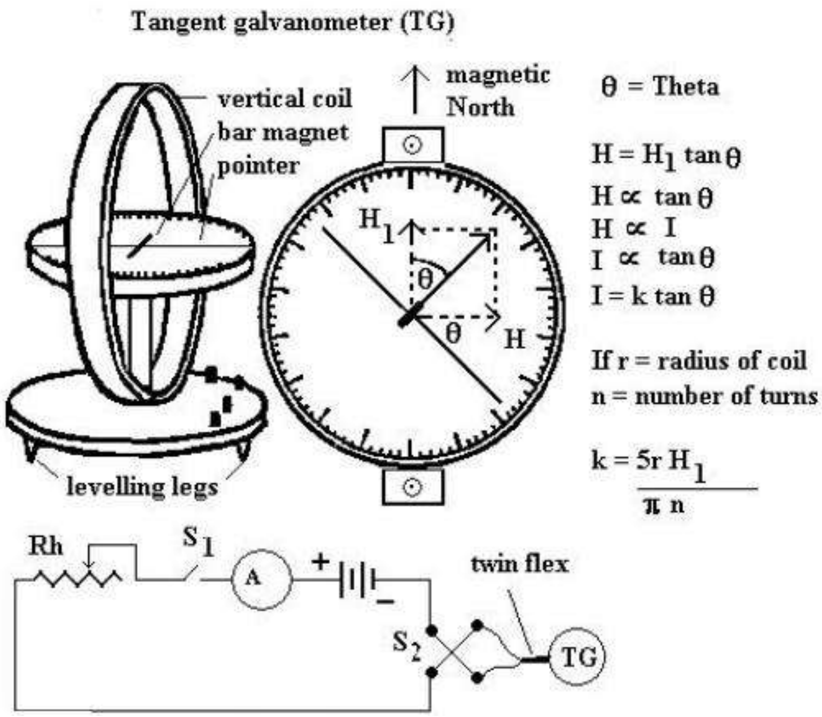
These instruments give the value of the electrical quantity in terms of absolute quantities (or some constants) of the instruments and their deflections. In this type of instruments no calibration or comparison with other instruments is necessary. They are generally not used in laboratories and are seldom used in practice by electricians and engineers. They are mostly used as means of standard measurements and are maintained lay national laboratories and similar institutions.

Some of the examples of absolute instruments are:

Tangent galvanometer

Raleigh current balance

Absolute electrometer



Secondary instruments:-

These instruments are so constructed that the quantity being measured can only be determined by the output indicated by the instrument. These instruments are calibrated by comparison with an absolute instrument or another secondary instrument, which has already been calibrated against an absolute instrument.

Working with absolute instruments for routine work is time consuming since every time a measurement is made, it takes a lot of time to compute the magnitude of quantity under measurement. Therefore secondary instruments are most commonly used.

• They are direct reading instruments. The quantity to be measured by these instruments can be determined from the deflection of the instruments.

• They are often calibrated by comparing them with either some absolute instruments or with those which have already been calibrated.

• The deflections obtained with secondary instruments will be meaningless until it is not calibrated.

•These instruments are used in general for all laboratory purposes.

Some of the very widely used secondary instruments are: ammeters, voltmeter, wattmeter, energy meter (watt-hour meter), ampere-hour meters etc.

Classification of Secondary Instruments:

(a) Classification based on the various effects of electric current (or voltage) upon which their operation depend.

They are:

• Magnetic effect: Used in ammeters, voltmeters, watt-meters, integrating meters etc.

• Heating effect: Used in ammeters and voltmeters.

• Chemical effect: Used in dc ampere hour meters.

• Electrostatic effect: Used in voltmeters.

• Electromagnetic induction effect: Used in ac ammeters, voltmeters, watt meters and integrating meters.

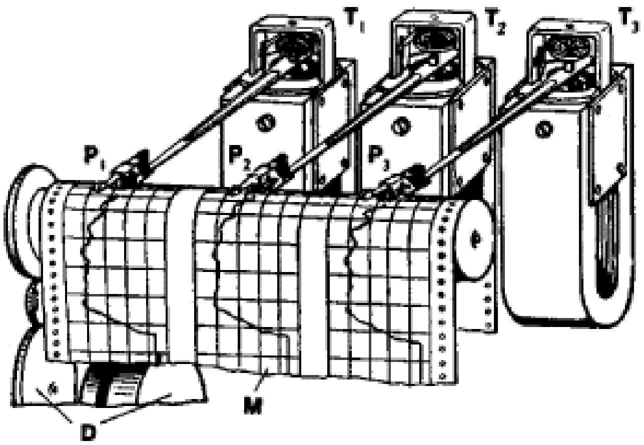
Generally the magnetic effect and the electromagnetic induction effect are utilized for the construction of the commercial instruments. Some of the instruments are also named based on the above effect such as electrostatic voltmeter, induction instruments, etc.

(b) Classification based on the Nature of their Operations:

We have the following instruments.

• **Indicating instruments**: Indicating instruments indicate, generally the quantity to be measured by means of a pointer which moves on a scale. Examples are ammeter, voltmeter, wattmeter etc.

• Recording instruments: These instruments record continuously the variation of any electrical quantity with respect to time. In principle, these are indicating instruments but so arranged that a permanent continuous record of the indication is made on a chart or dial. The recording is generally made by a pen on a graph paper which is rotated on a dice or drum at a uniform speed. The amount of the quantity at any time (instant) may be read from the traced chart. Any variation in the quantity with time is recorded by these instruments. Any electrical quantity like current, voltage, power etc., (which may be measured lay the indicating instruments) may be arranged to be recorded by a suitable recording mechanism.



• **Integrating instruments:** These instruments record the consumption of the total quantity of electricity, energy etc., during a particular period of time. That is, these instruments totalize events over a specified period of time. No indication of the rate or variation or the amount at a particular instant are available from them. Some widely used integrating instruments are: Ampere-hour meter: kilowatt-hour (kWh) meter, kilovolt-ampere-hour (kVARh) meter.

Fig. Energy meter

(c) Classification based on the Kind of Current that can be Measurand:

Under this heading, we have:

• Direct current (dc) instruments

• Alternating current (ac) instruments

• Both direct current and alternating current instruments (dc/ac instruments).

(d) Classification based on the method used.

Under this category, we have:

• **Direct measuring instruments:** These instruments converts the energy of the measured quantity directly into energy that actuates the instrument and the value of the unknown quantity is measured or displayed or recorded directly. These instruments are most widely used in engineering practice because they are simple and inexpensive. Also, time involved in the measurement is shortest. Examples are Ammeter, Voltmeter, Watt meter etc.

• **Comparison instruments:** These instruments measure the unknown quantity by comparison with a standard. Examples are dc and ac bridges and potentiometers. They are used when a higher accuracy of measurements is desired.

1.3.4 Electromechanical indicating instrument:

For satisfactory operation electromechanical indicating instrument, three forces are necessary.
They are

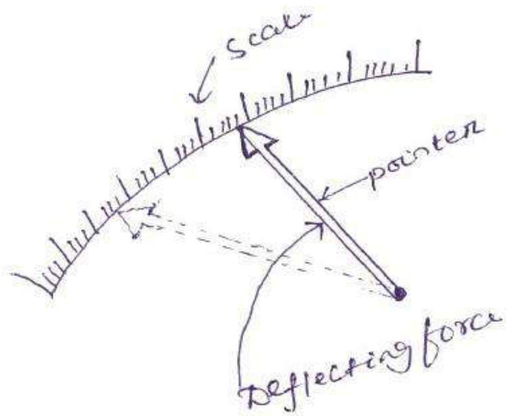
(a) Deflecting force

(b) Controlling force

(c)Damping force

1.4 Deflecting force:

When there is no input signal to the instrument, the pointer will be at its zero position. To deflect the pointer from its zero position, a force is necessary which is known as deflecting force. A system which produces the deflecting force is known as a deflecting system. Generally a deflecting system converts an electrical signal to a mechanical force.



1.5 Controlling force:

To make the measurement indicated by the pointer definite (constant) a force is necessary which will be acting in the opposite direction to the deflecting force. This force is known as controlling force. A system which produces this force is known as a controlled system. When the external signal to be measured by the instrument is removed, the pointer should return back to the zero position. This is possibly due to the controlling force and the pointer will be indicating a steady value when the deflecting torque is equal to controlling torque.

$$T_d = T_c$$

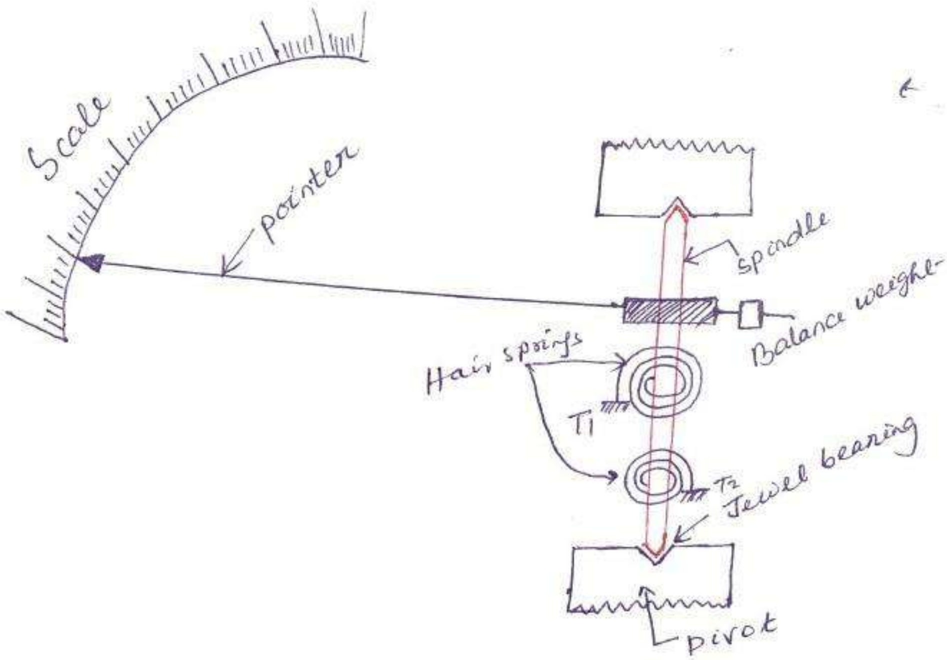
1.5.1 Spring control:

Two springs are attached on either end of spindle (Fig. 1.5).The spindle is placed in jeweled bearing, so that the frictional force between the pivot and spindle will be minimum. Two springs are provided in opposite direction to compensate the temperature error. The spring is made of phosphorous bronze.

$$T_C \propto \theta$$

The deflecting torque produced Td proportional to ‘I’. When T_C = Td the pointer will come to a steady position. Therefore

$$\theta \propto I$$



Since, θ and I are directly proportional to the scale of such instrument which uses spring controlled is uniform

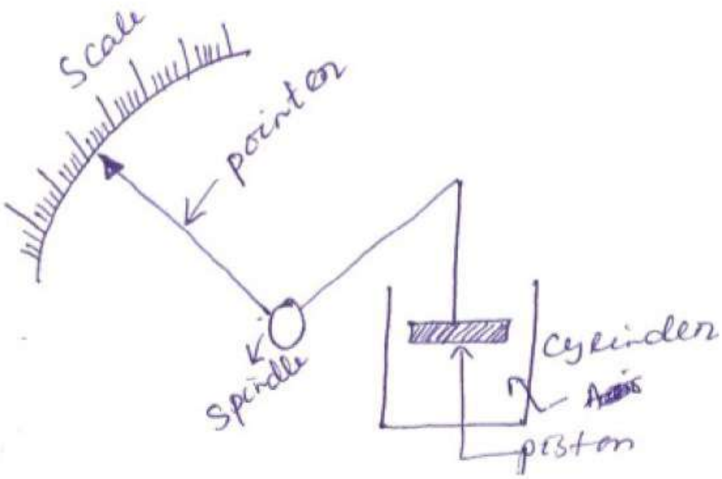
1.6 Damping force:

The deflection torque and controlling torque produced by systems are electro mechanical. Due to inertia produced by this system, the pointer oscillates about it final steady position before coming to rest. The time required to take the measurement is more. To damp out the oscillations quickly, a damping force is necessary. This force is produced by different systems.

(a) Air friction damping
(b) Fluid friction damping
(c) Eddy current damping

1.6.1 Air friction damping:

The piston is mechanically connected to a spindle through the connecting rod (Fig. 1.6). The pointer is fixed to the spindle moves over a calibrated dial. When the pointer oscillates in clockwise direction, the piston goes inside and the cylinder gets compressed. The air pushes the piston upwards and the pointer tends to move in anticlockwise direction



If the pointer oscillates in anticlockwise direction the piston moves away and the pressure of the air inside cylinder gets reduced. The external pressure is more than that of the internal pressure. Therefore the piston moves down wards. The pointer tends to move in clock wise direction.

1.6.2 Eddy current damping:

An aluminum circular disc is fixed to the spindle (Fig. 1.6). This disc is made to move in the magnetic field produced by a permanent magnet.

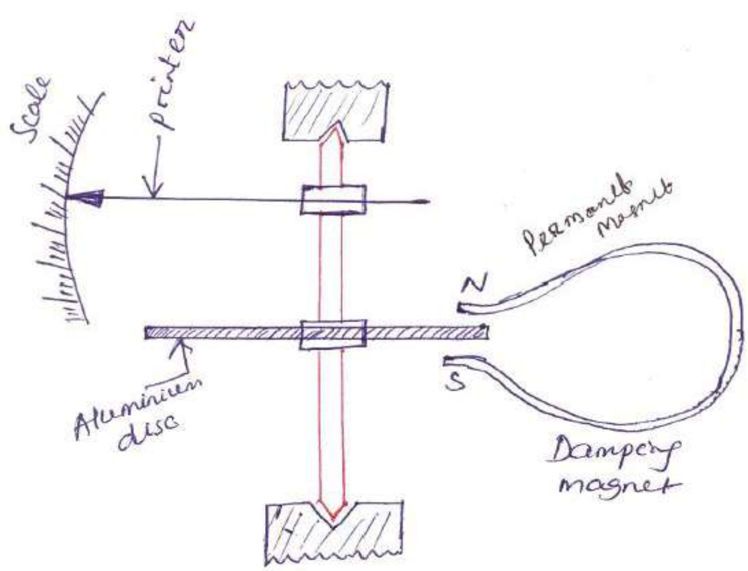


Fig. 1.6 Disc type

When the disc oscillates it cuts the magnetic flux produced by damping magnet. An emf is induced in the circular disc by faradays law. Eddy currents are established in the disc since it has several closed paths. By Lenz's law, the current carrying disc produced a force in a direction opposite to oscillating force. The damping force can be varied by varying the projection of the magnet over the circular disc.

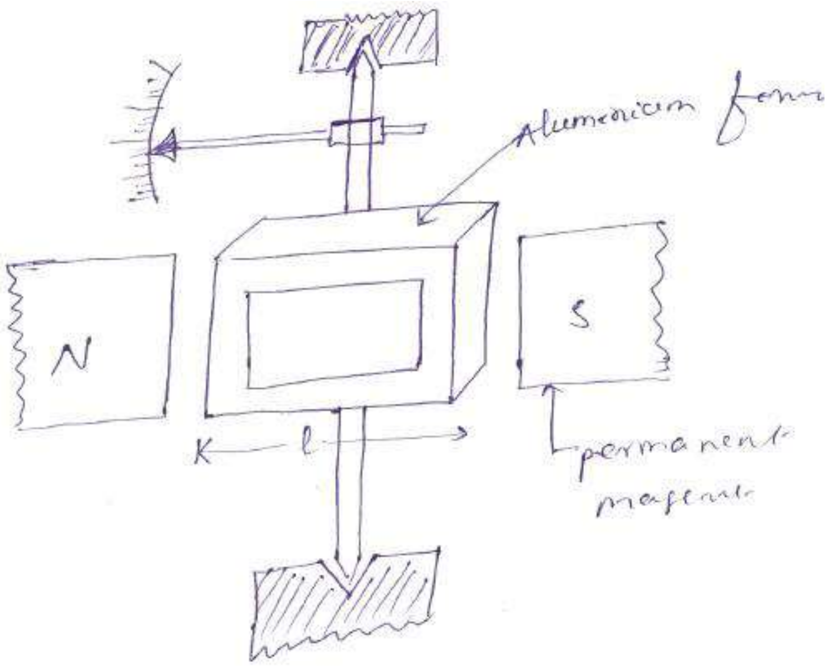


Fig. 1.6 Rectangular type

1.7 Permanent Magnet Moving Coil (PMMC) instrument:

One of the most accurate type of instrument used for D.C. measurements is PMMC instrument.

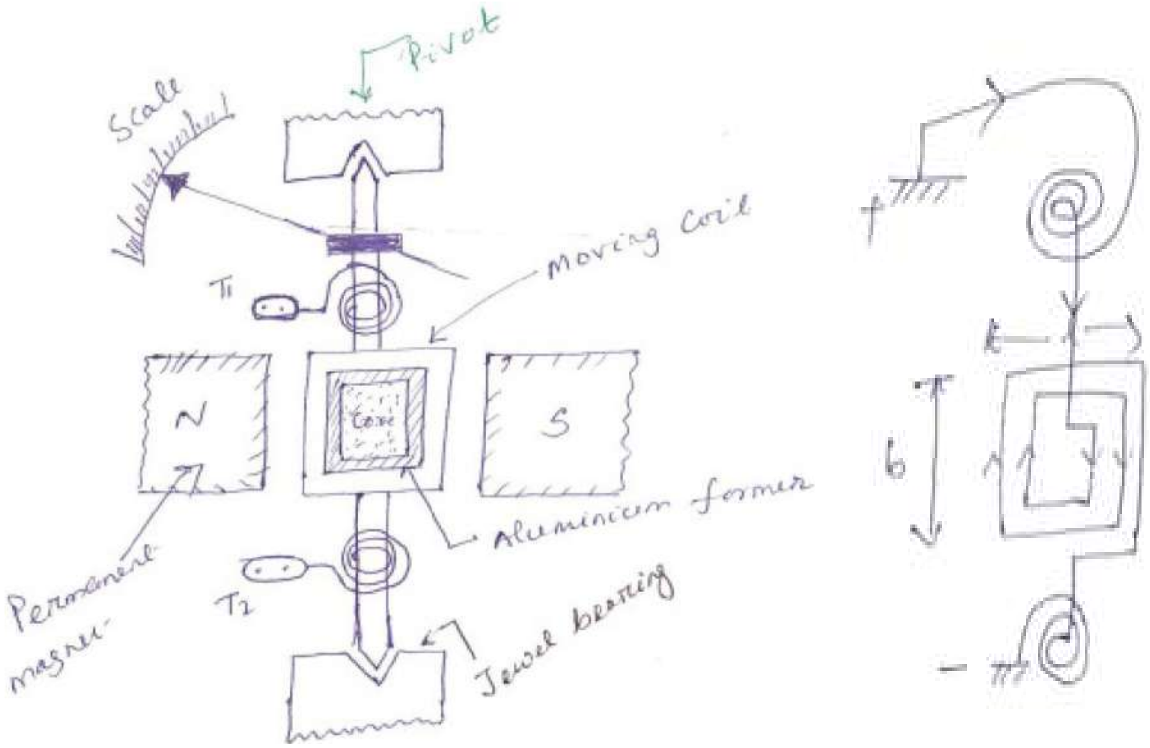
Construction: A permanent magnet is used in this type instrument. Aluminum former is provided in the cylindrical in between two poles of the permanent magnet (Fig. 1.7). Coils are wound on the aluminum former which is connected with the spindle. This spindle is supported with jeweled bearing. Two springs are attached on either end of the spindle. The terminals of the moving coils are connected to the spring. Therefore the current flows through spring 1, moving coil and spring 2.

Damping: Eddy current damping is used. This is produced by aluminum former.

Control: Spring control is used.

Principle of operation:

When D.C. supply is given to the moving coil, D.C. current flows through it. When the current carrying coil is kept in the magnetic field, it experiences a force. This force produces a torque and the former rotates. The pointer is attached with the spindle. When the former rotates, the pointer moves over the calibrated scale. When the polarity is reversed a torque is produced in the opposite direction. The mechanical stopper does not allow the deflection in the opposite direction. Therefore the polarity should be maintained with PMMC instrument.



If A.C. is supplied, a reversing torque is produced. This cannot produce a continuous deflection. Therefore this instrument cannot be used in A.C.

Torque developed by PMMC:

Let Td =deflecting torque

TC = controlling torque

θ = angle of deflection

K=spring constant

b=width of the coil

l=height of the coil or length of coil

N=No. of turns

I=current

B=Flux density

A=area of the coil

The force produced in the coil is given by F = BIL sinθ

When $\theta = 90^{\circ}$

For N turns, F = NBIL

Torque produced Td = F* perpendicular distance

T_d =NBIL* b

= BINA

T_d =BANI

$T_d \alpha I$

Advantages:

- Torque/weight is high
- Power consumption is less
- Scale is uniform
- Damping is very effective
- Since operating field is very strong, the effect of stray field is negligible
- Range of instrument can be extended

Disadvantages:

- Use only for D.C.
- Cost is high
- Error is produced due to ageing effect of PMMC
- Friction and temperature error are present

ERRORS IN PMMC INSTRUMENT:

The basic sources of errors in PMMC instruments are friction, temperature and aging of various parts. To reduce the frictional errors ratio of torque to weight is made very high.

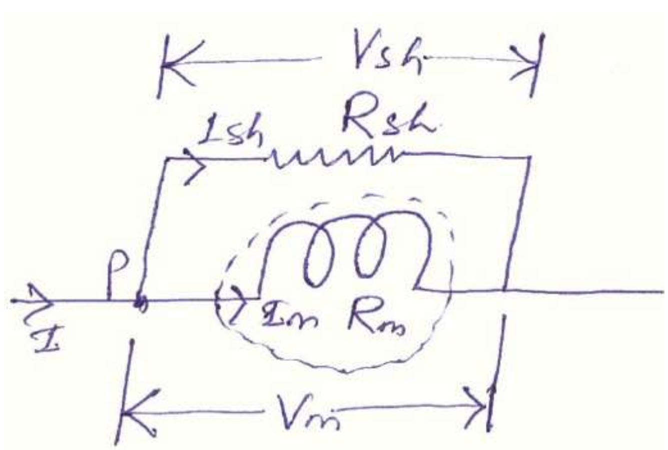
The most serious errors are produced by the heat generated or by changes in the temperature. This changes the resistance of the working coil, causing large errors. In case of voltmeters, a large series resistance of very low temperature coefficient is used. This reduces the temperature errors.

The aging of permanent magnet and control springs also cause errors. The weakening of magnet and springs cause opposite errors. The weakening of magnet cause less deflection while weakening of the control springs cause large deflection, for a particular value of current. The proper use of material and preageing during manufacturing can reduce the errors due to weakening of the control springs.

. **Extension range of PMMC instrument:**

Case-I: Shunt:

A low shunt resistance connected in parallel with the ammeter to extent the range of current. Large current can be measured using low current rated ammeter by using a shunt.



Let R_m =Resistance of meter

R_{sh} =Resistance of shunt

I_m = Current through meter

I_{sh} =current through shunt

I = current to be measure

$$\therefore V_m = V_{sh}$$

$$I_m R_m = I_{sh} R_{sh}$$

$$\frac{I_m}{I_{sh}} = \frac{R_{sh}}{R_m}$$

Apply KCL at 'P' $I = I_m + I_{sh}$

Eq^n (1.12) ÷ by I_m

$$\frac{I}{I_m} = 1 + \frac{I_{sh}}{I_m}$$

$$\frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}}$$

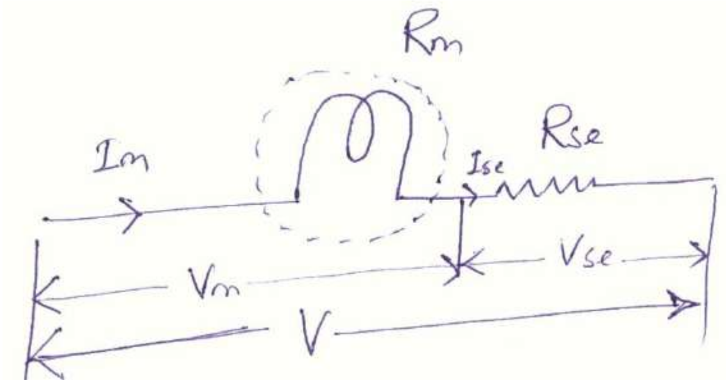
$$\therefore I = I_m\left(1 + \frac{R_m}{R_{sh}}\right)$$

$\left(1 + \frac{R_m}{R_{sh}}\right)$ is called multiplication factor

Shunt resistance is made of manganin. This has least thermo electric e.m.f. The change is resistance, due to change in temperature is negligibl

Case (II): Multiplier:

A large resistance is connected in series with voltmeter is called multiplier (Fig. 1.9). A large voltage can be measured using a voltmeter o f small rating with a multiplier.



(Fig. 1.9)

Let R = resistance of meter

R_{se} = resistance of multiplier

V_m = Voltage across meter

V_{se} = Voltage across series resistance

V = voltage to be measured

$$I_m = I_{se}$$

$$\frac{V_m}{R_m} = \frac{V_{se}}{R_{se}}$$

$$\therefore \frac{V_{se}}{V_m} = \frac{R_{se}}{R_m}$$

Apply KVL, $V = V_m + V_{se}$

Eq^n (1.19) $\div V_m$

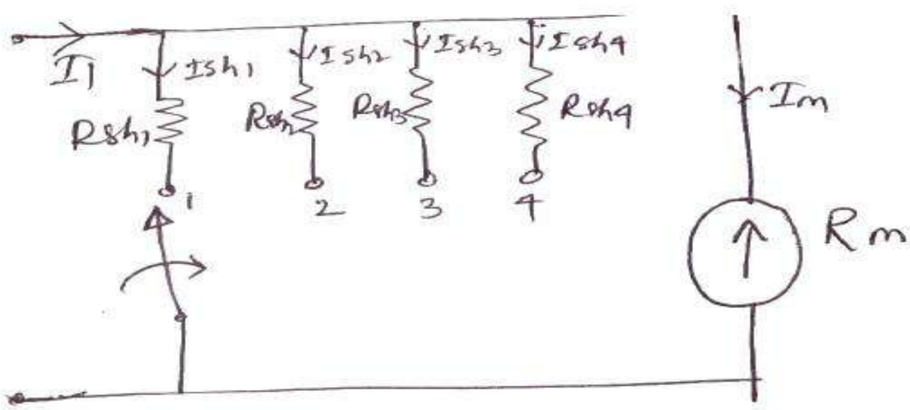
$$\frac{V}{V_m} = 1 + \frac{V_{se}}{V_m} = \left(1 + \frac{R_{se}}{R_m}\right)$$

$$\therefore V = V_m\left(1 + \frac{R_{se}}{R_m}\right)$$

$$\left(1 + \frac{R_{se}}{R_m}\right) \rightarrow \text{Multiplication factor}$$

MULTI RANGE AMMETER

When the switch is connected to position (1), the supplied current I_1



$$I_{sh1} R_{sh1} = I_m R_m$$

$$R_{sh1} = \frac{I_m R_m}{I_{sh1}} = \frac{I_m R_m}{I_1 - I_m}$$

$$R_{sh1} = \frac{R_m}{\frac{I_1}{I_m} - 1}, R_{sh1} = \frac{R_m}{m_1 - 1}, m_1 = \frac{I_1}{I_m} = \text{ Multiplying power of shunt}$$

$$R_{sh2} = \frac{R_m}{m_2 - 1}, m_2 = \frac{I_2}{I_m}$$

$$R_{sh3} = \frac{R_m}{m_3 - 1}, m_3 = \frac{I_3}{I_m}$$

$$R_{sh4} = \frac{R_m}{m_4 - 1}, m_4 = \frac{I_4}{I_m}$$

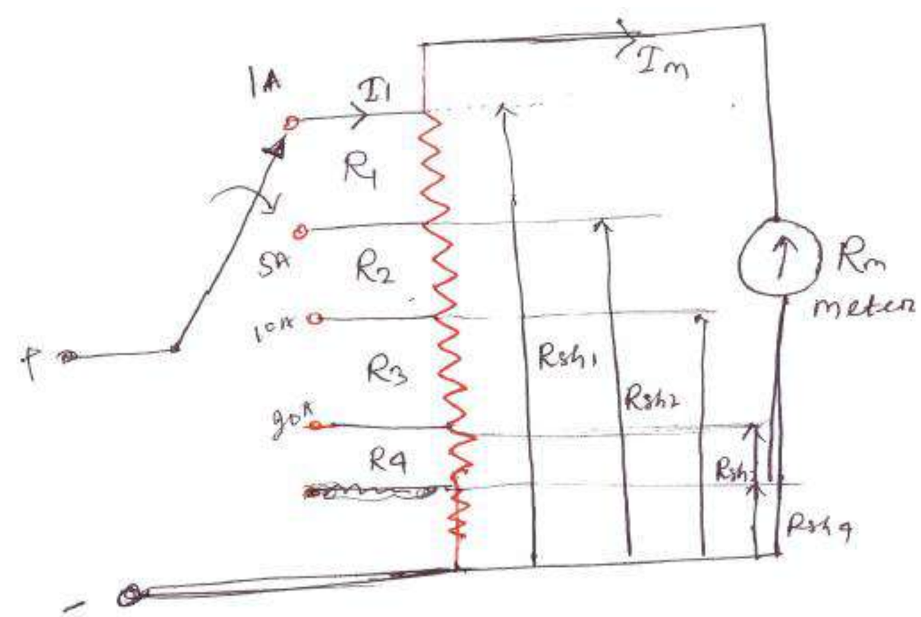
THE Ayton (or) Universal shunts

$$R_1 = R_{sh1} - R_{sh2}$$

$$R_2 = R_{sh2} - R_{sh3}$$

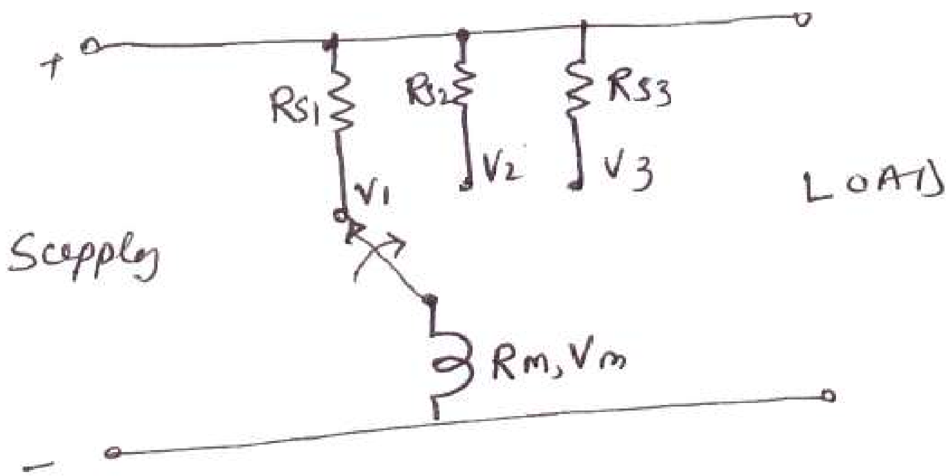
$$R_3 = R_{sh3} - R_{sh4}$$

$$R_4 = R_{sh4}$$



Ayton shunt is also called universal shunt. Ayton shunt has more sections of resistance. Taps are brought out from various points of the resistor. The variable points in the o/p can reconnected to any position. Various meters require different types of shunts. The Ayton shunt is used in the lab, so that any value of resistance between minimum and maximum specified can be used. It eliminates the possibility of having the meter in the circuit without a shunt

Multirange range dc voltmeter



$$R_{s1} = R_m(m_1 - 1)$$
$$R_{s2} = R_m(m_2 - 1)$$
$$R_{s3} = R_m(m_3 - 1)$$

$$m_1 = \frac{V_1}{V_m}, m_2 = \frac{V_2}{V_m}, m_3 = \frac{V_3}{V_m}$$

We can obtain different Voltage ranges by connecting different value of multiplier resistor in series with the meter. The number of these resistors is equal to the number of ranges required**.**

Moving Iron (MI) instruments

One of the most accurate instruments used for both AC and DC measurement is moving iron instrument. There are two types of moving iron instrument.

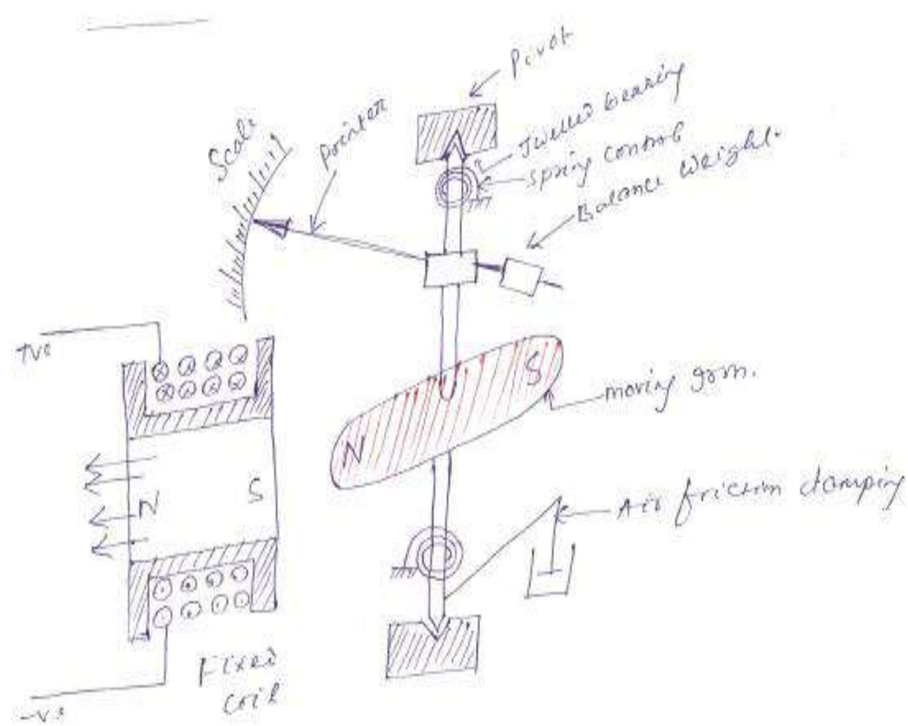
1. Attraction type
2. Repulsion type

Attraction type M.I. instrument

Construction: The moving iron fixed to the spindle is kept near the hollow fixed coil (Fig. 1.10).The pointer and balance weight are attached to the spindle, which is supported with jeweled bearing. Here air friction damping is used.

Principle of operation

The current to be measured is passed through the fixed coil. As the current is flow through the fixed coil, a magnetic field is produced. By magnetic induction the moving iron gets magnetized. The north pole of moving coil is attracted by the south pole of fixed coil. Thus the deflecting force is produced due to force of attraction. Since the moving iron is attached with the spindle, the spindle rotates and the pointer moves over the calibrated scale. But the force of attraction depends on the current flowing through the coil.



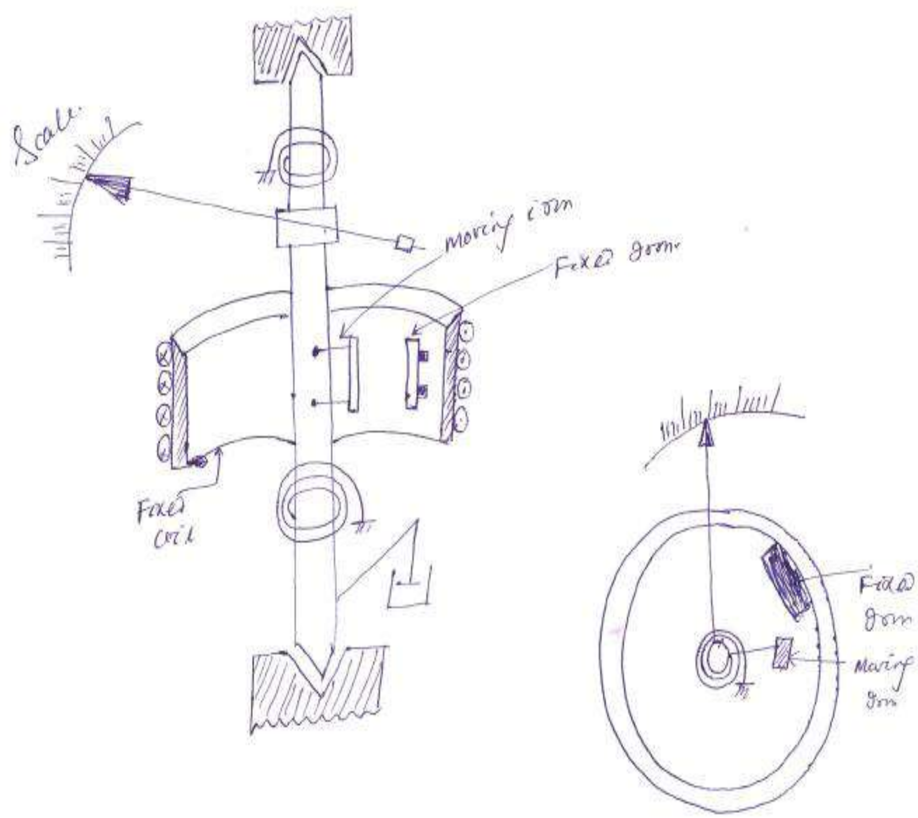
Repulsion type moving iron instrument

Construction: The repulsion type instrument has a hollow fixed iron attached to it (Fig. 1.12).The moving iron is connected to the spindle. The pointer is also attached to the spindle in supported with jeweled bearing.

Principle of operation: When the current flows through the coil, a magnetic field is produced by it. So both fixed iron and moving iron are magnetized with the same polarity, since they are kept in the same magnetic field. Similar poles of fixed and moving iron get repelled. Thus the deflecting torque is produced due to magnetic repulsion. Since moving iron is attached to spindle, the spindle will move. So that pointer moves over the calibrated scale.

Damping: Air friction damping is used to reduce the oscillation.

Control: Spring control is used.



Torque developed by M.I(moving iron instrument)

Consider a small increment in current supplied to the coil of the instrument. due to this current let $d\theta$ be the deflection under the deflecting torque T_d. Due to such deflection, some mechanical work will be done.

$\therefore$ Mechanical Work = $T_d \; d\theta$

There will be a change in the energy stored i the magnetic field due to the change in inductance. This is because the vane tries to occupy the position of minimum reluctance. The inductance is inversely proportional to the reluctance of the magnetic circuit of coil.

Let I = initial current

L=instrument inductance

θ = deflection

dI = increase in current

$d\theta$ = change in deflection

dL = change in inductance

In order to effect an increment dL in the current, there must be an increase in the applied voltage given by,

$$e = \frac{d(LI)}{dt}$$

$$= I\frac{dL}{dt} + L\frac{dI}{dt} \qquad \text{as both I and L are changing.}$$

The electrical energy supplied is given by,

$$eIdt = \left(I\frac{dL}{dt} + L\frac{dI}{dt}\right)Idt$$

$$= I^2\, dL + IL\, dI$$

The stored energy increases from $\frac{1}{2}LI^2$ to $\frac{1}{2}(L+dL)(I+dI)^2$

Hence the change in stored energy is given by,

$$= \frac{1}{2}(L+dL)(I+dI)^2 - \frac{1}{2}LI^2$$

Neglecting higher order terms, this becomes, $IL\,dI + \frac{1}{2} I^2\,dL$

The energy supplied in nothing but increase in stored energy plus the energy required for mechanical work done

$$\therefore \quad I^2 dL + IL\,dI = IL\,dI + \frac{1}{2} I^2\,dL + T_d \cdot d\theta$$

$$\therefore \quad T_d \cdot d\theta = \frac{1}{2} I^2\,dL$$

$$\therefore \quad \boxed{T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}}$$

While the controlling torque is given by,

$$T_c = K\theta$$

where K = spring constant

$$\therefore \quad K\theta = \frac{1}{2} I^2 \frac{dL}{d\theta} \qquad \text{under equilibrium}$$

$$\therefore \quad \theta = \frac{1}{2}\frac{I^2}{K}\frac{dL}{d\theta}$$

Thus the deflection is proportional to the square of the current through the coil. And the instrument gives square law response

Advantages

1. MI can be used in AC and DC
2. It is cheap
3. Supply is given to a fixed coil, not in moving coil.
4. Simple construction
5. Less friction error.

Disadvantages

1. It suffers from eddy current and hysteresis error
2. Scale is not uniform
3. It consumed more power
4. Calibration is different for AC and DC operation

Errors in moving iron instruments

The various errors in the moving iron instruments are,

1) **Hysteresis error:** Due to hysteresis effect, the flux density for the same current while ascending and descending values is different. While descending, the flux density is higher and while ascending it is lesser. So meter reads higher for descending values of current or voltage. So remedy for this is to use smaller iron parts which can demagnetise quickly or to work with lower flux densities.
2) **2) Temperature error** : The temperature error arises due to the effect of temperature on the temperature coefficient of the spring. This error is of the order of 0.02 % per ^{o}C change in temperature. Errors can cause due to self heating of the coil and due to which change in resistance of the coil. So coil and series resistance must have low temperature coefficient. Hence mangnin is generally used for the series resistance.
3) **3) Stray magnetic Field Error**: The operating magnetic field in case of moving iron instruments is very low. Hence effect of external i.e. stray magnetic field can cause error. This effect depends on the direction of the stray magnetic field with respect to the operating field of the instrument.
4) **4) Frequency Error** : These are related to a.c. operation of the instrument. The change in frequency affects the reactance of the working coil and also affects the magnitude of the eddy currents. This cause error in the instrument.

5) **Eddy Current Error**: When instrument is used for a.c. measurements the eddy currents are produced in the iron parts of the instrument. The eddy current affects the instrument current causing the change in the deflection torque. This produce the error in the meter reading. As eddy current are frequency dependent, frequency changes cause eddy current errors.

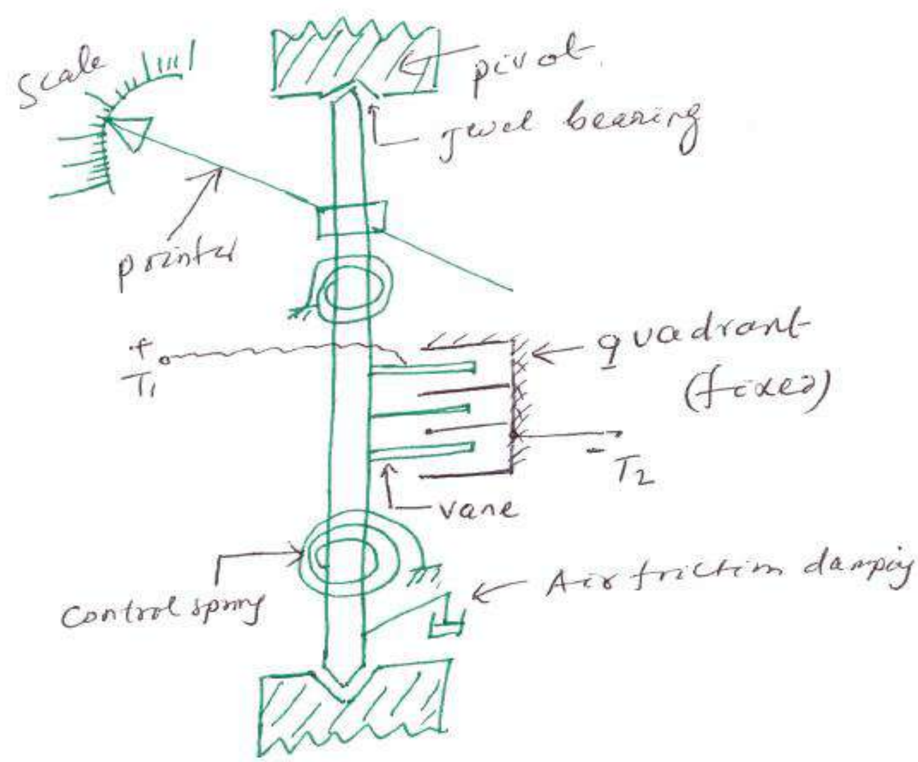
Electro static meter:

Basically electrostatic instruments are all voltmeters .the main advantages of such instruments is the measurement of high voltage in both AC and DC circuits without any errors due to eddy currents loss and hysteresis.

In multi cellular construction several vans and quadrants are provided. The voltage is to be Measured is applied between the vanes and quadrant. The force of attraction between the vanes And quadrant produces a deflecting torque. Controlling torque is produced by spring control.

Air Friction damping is used

The instrument is generally used for measuring medium and high voltage. The voltage is reduced to low value by using capacitor potential divider. The force of attraction is proportional to the Square of the voltage.



Torque developed by electro static voltmeter

V=Voltage applied between vane and quadrant

C=capacitance between vane and quadrant

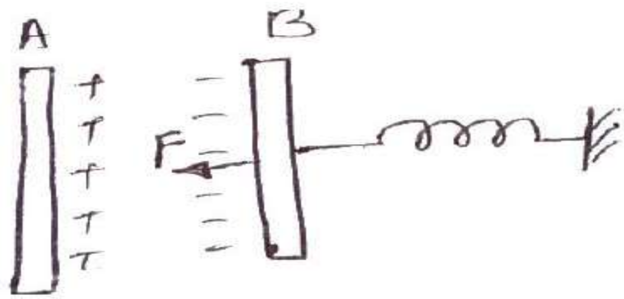
Energy stored=$\frac{1}{2}CV^2$

Let 'θ' be the deflection corresponding to a voltage V.

Let the voltage increases by dv, the corresponding deflection is'$\theta + d\theta$'

When the voltage is being increased, a capacitive current flows

$$i = \frac{dq}{dt} = \frac{d(CV)}{dt} = \frac{dC}{dt}V + C\frac{dV}{dt}$$



$V \cdot dt$ multiply on both side of equation

$$Vidt = \frac{dC}{dt}V^2 dt + CV\frac{dV}{dt}dt$$

$$Vidt = V^2 dC + CVdV$$

Change in stored energy=$\frac{1}{2}(C+dC)(V+dV)^2 - \frac{1}{2}CV^2$

$$= \frac{1}{2}\left[(C+dC)V^2 + dV^2 + 2VdV\right] - \frac{1}{2}CV^2$$

$$= \frac{1}{2}\left[CV^2 + CdV^2 + 2CVdV + V^2dC + dCdV^2 + 2VdVdC\right] - \frac{1}{2}CV^2$$

$$= \frac{1}{2}V^2dC + CVdV$$

$$V^2dC + CVdV = \frac{1}{2}V^2dC + CVdV + F \times rd\theta$$

$$T_d \times d\theta = \frac{1}{2}V^2dC$$

At steady state condition, $T_d = T_C$

$$K\theta = \frac{1}{2}V^2\left(\frac{dC}{d\theta}\right)$$

$$\theta = \frac{1}{2K}V^2\left(\frac{dC}{d\theta}\right)$$

Advantages

1. It is used in both AC and DC.

2. There is no frequency error.

3. There is no hysteresis error.

4. There is no stray magnetic field error. Because the instrument works on electrostatic Principle.

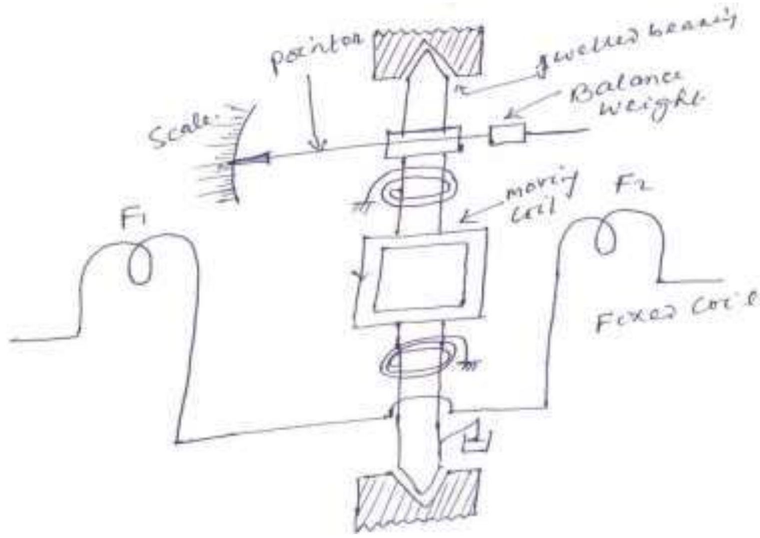
5. It is used for high voltage

6. Power consumption is negligible

Disadvantages

1. Scale is not uniform
2. Large in size
3. These are expensive instruments.

Dynamo meter (or) Electromagnetic moving coil instrument (EMMC)



This instrument can be used for the measurement of voltage, current and power. The difference between the PMMC and dynamometer type instrument is that the permanent magnet is replaced by an electromagnet.

<u>Construction</u>: A fixed coil is divided in to two equal half. The moving coil is placed betweenthe two half of the fixed coil. Both the fixed and moving coils are air cored. So that the hysteresis effect will be zero. The pointer is attached with the spindle. In a non metallic former the moving coil is wounded.

Control: Spring control is used

Damping: Air friction damping is used.

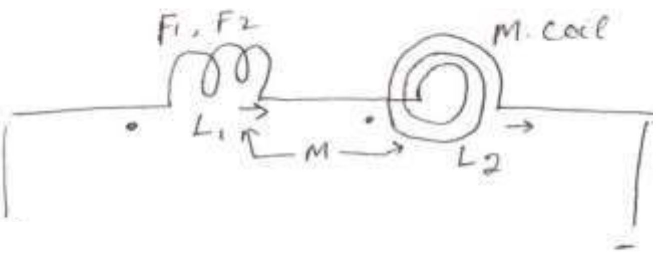
<u>Principleofoperation</u>:

When the current flows through the fixed coil, it produced a magnetic field, whose flux density is proportional to the current through the fixed coil. The moving coil is kept in between the fixed coil. When the current passes through the moving coil, amagnetic field is produced by this coil.

The magnetic poles are produced in such a way that the torque produced on the moving coil deflects the pointer over the calibrated scale. This instrument works on AC and DC. When AC voltage is applied, alternating current flows through the fixed coil and moving coil.

When the current in the fixed coil reverses, the current in the moving coil also reverses. Torque remains in the same direction. Since the current i_1 and i_2 reverse simultaneously. This is because the fixed and moving coils are either connected in series or parallel.

Torque developed by EMMC



Let

L_1=Self inductance of fixed coil

L_2= Self inductance of moving coil

M=mutual inductance between fixed coil and moving coil

i_1=current through fixed coil

i_2=current through moving coil

Total inductance of system,

$$L_{total} = L_1 + L_2 + 2M$$

But we know that in case of M.I

$$T_d = \frac{1}{2} i^2 \frac{d(L)}{d\theta}$$

$$T_d = \frac{1}{2} i^2 \frac{d}{d\theta}(L_1 + L_2 + 2M)$$

The value of L_1 and L_2 are independent of 'θ' but 'M' varies with θ

$$T_d = \frac{1}{2} i^2 \times 2 \frac{dM}{d\theta}$$

$$T_d = i^2 \frac{dM}{d\theta}$$

If the coils are not connected in series $i_1 \neq i_2$

$$\therefore T_d = i_1 i_2 \frac{dM}{d\theta}$$

$$T_C = T_d$$

$$\therefore \theta = \frac{i_1 i_2}{K} \frac{dM}{d\theta}$$

Hence the deflection of pointer is proportional to the current passing through fixed coil and moving coil

We can obtain different Voltage ranges by connecting different value of multiplier resistor inserieswiththemeter.Thenumberoftheseresistorsisequaltothenumberofrangesrequired**.**

1) A PMMC ammeter has the following specification

Coil dimension are 1cm×1cm. Spring constant is $0.15\times10^{-6}N-m/rad$, Flux density is $1.5\times10^{-3}wb/m^2$.Determine the no. of turns required to produce a deflection of 90^0 when a current 2mA flows through the coil.

At steady state **condition** $T_d = T_C$

$$BANI = K\theta$$

$$\Rightarrow N = \frac{K\theta}{BAI}$$

A=$1\times10^{-4}m^2$

K=$0.15\times10^{-6}\frac{N-m}{rad}$

B=$1.5\times10^{-3}wb/m^2$

I=$2\times10^{-3}A$

$$\theta = 90^{\circ} = \frac{\Pi}{2}rad$$

N=785 ans.

2) The pointer of a moving coil instrument gives full scale deflection of 20mA. The potential difference across the meter when carrying 20mA is 400mV.The instrument to be used is 200A for full scale deflection. Find the shunt resistance required to achieve this, if the instrument to be used as a voltmeter for full scale reading with 1000V. Find the series resistance to be connected it?

Case-1

V_m=400mV

$I_m = 20mA$

I=200A

$$R_m = \frac{V_m}{I_m} = \frac{400}{20} = 20\Omega$$

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)$$

$$200 = 20\times10^{-3}\left[1+\frac{20}{R_{sh}}\right]$$

$$R_{sh} = 2\times10^{-3}\Omega$$

Case-II

V=1000V

$$V = V_m\left(1+\frac{R_{se}}{R_m}\right)$$

$$4000 = 400\times10^{-3}\left(1+\frac{R_{se}}{20}\right)$$

$$R_{se} = 49.98k\Omega$$

3) A 150 v moving iron voltmeter is intended for 50HZ, has a resistance of 3kΩ. Find the series resistance required to extent the range of instrument to 300v. If the 300V instrument is used to measure a d.c. voltage of 200V. Find the voltage across the meter?

Solution:

$R_m = 3k\Omega, V_m = 150V, V = 300V$

$$V = V_m\left(1+\frac{R_{se}}{R_m}\right)$$

$$300 = 150\left(1+\frac{R_{se}}{3}\right) \Rightarrow R_{se} = 3k\Omega$$

Case-II $\quad V = V_m\left(1+\frac{R_{se}}{R_m}\right)$

$$200 = V_m\left(1+\frac{3}{3}\right)$$

$\therefore V_m = 100V$ Ans

4) What is the value of series resistance to be used to extent '0'to 200V range of 20,000Ω/volt voltmeter to 0 to 2000 volt?

Solution:

$$V_{se} = V - V = 1800$$

$$I_{FSD} = \frac{1}{20000} = \frac{1}{Sensitivity}$$

$$V_{se} = R_{se} \times i_{FSD} \Rightarrow R_{se} = 36M\Omega$$ ans.

5) For a certain dynamometer ammeter the mutual inductance 'M' varies with deflection θ as $M = -6\cos(\theta + 30^{\circ})mH$ .Find the deflecting torque produced by a direct current of 50mA corresponding to a deflection of 60^0.

Solution:

$$T_d = I_1 I_2 \frac{dM}{d\theta} = I^2 \frac{dM}{d\theta}$$

$$M = -6\cos(\theta + 30^{\circ})$$

$$\frac{dM}{d\theta} = 6\sin(\theta + 30)mH$$

$$\frac{dM}{d\theta}\Big|_{\theta=60} = 6\sin 90 = 6mH/\deg$$

$$T_d = I^2 \frac{dM}{d\theta} = (50\times10^{-3})^2 \times 6\times10^{-3} = 15\times10^{-6} N-m$$

6) The inductance of a moving iron ammeter with a full scale deflection of 90^0 at 1.5A, is given by the expression $L = 200 + 40\theta - 4\theta^2 - \theta^3 \mu H$, where θ is deflection in radian from the zero position. Estimate the angular deflection of the pointer for a current of 1.0A.

$$L = 200 + 40\theta - 4\theta^2 - \theta^3 \mu H$$

$$\frac{dL}{d\theta}\Big|_{\theta=90^{\circ}} = 40 - 8\theta - 3\theta^2 \mu H/rad$$

$$\frac{dL}{d\theta}\Big|_{\theta=90^{\circ}} = 40 - 8\times\frac{\Pi}{2} - 3(\frac{\Pi}{2})^2 \mu H/rad = 20\mu H/rad$$

$$\therefore \theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta}\right)$$

$$\frac{\Pi}{2} = \frac{1}{2}\frac{(1.5)^2}{K} \times 20 \times 10^{-6}$$

K=Spring constant=$14.32 \times 10^{-6} N-m/rad$

For I=1A, $\therefore \theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta} \right)$

$$\therefore \theta = \frac{1}{2} \times \frac{(1)^2}{14.32 \times 10^{-6}} \left(40 - 8\theta - 3\theta^2\right)$$

$$3\theta + 36.64\theta^2 - 40 = 0$$

$$\theta = 1.008 rad, 57.8^\circ$$

7) The inductance of a moving iron instrument is given by $L = 10 + 5\theta - \theta^2 - \theta^3 \mu H$, where θ is the deflection in radian from zero position. The spring constant is $12 \times 10^{-6} N-m/rad$. Estimate the deflection for a current of 5A.

$$\frac{dL}{d\theta} = (5 - 2\theta)\frac{\mu H}{rad}$$

$$\therefore \theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta} \right)$$

$$\therefore \theta = \frac{1}{2} \times \frac{(5)^2}{12 \times 10^{-6}} (5 - 2\theta) \times 10^{-6}$$

$$\therefore \theta = 1.69 rad, 96.8^\circ$$

8) Design a multi range d.c. mille ammeter using a basic movement with an internal resistance $R_m = 50\Omega$ and a full scale deflection current $I_m = 1mA$. The ranges required are 0-10mA; 0-50mA; 0-100mA and 0-500mA.

Solution:

Case-I 0-10mA

Multiplying power $m = \frac{I}{I_m} = \frac{10}{1} = 10$

$\therefore$ Shunt resistance $R_{sh1} = \frac{R_m}{m-1} = \frac{50}{10-1} = 5.55\Omega$

Case-II 0-50mA

$$m = \frac{50}{1} = 50$$

$$R_{sh2} = \frac{R_m}{m-1} = \frac{50}{50-1} = 1.03\Omega$$

Case-III 0-100mA, $m = \frac{100}{1} = 100\Omega$

$$R_{sh3} = \frac{R_m}{m-1} = \frac{50}{100-1} = 0.506\Omega$$

Case-IV 0-500mA, $m = \frac{500}{1} = 500\Omega$

$$R_{sh4} = \frac{R_m}{m-1} = \frac{50}{500-1} = 0.1\Omega$$

9) A moving coil voltmeter with a resistance of 20Ω gives a full scale deflection of 120^0, when a potential difference of 100mV is applied across it. The moving coil has dimension of 30mm*25mm and is wounded with 100 turns. The control spring constant is $0.375\times10^{-6}\,N-m/\deg ree$. Find the flux density, in the air gap. Find also the diameter of copper wire of coil winding if 30% of instrument resistance is due to coil winding. The specific resistance for copper=$1.7\times10^{-8}\,\Omega m$.

Solution:

Data given

$$V_m = 100mV$$

$$R_m = 20\Omega$$

$$\theta = 120^\circ$$

N=100

$K = 0.375\times10^{-6}\, N-m/\deg ree$

$R_C = 30\% of R_m$

$\rho = 1.7\times10^{-8}\,\Omega m$

$I_m = \frac{V_m}{R_m} = 5\times10^{-3} A$

$T_d = BANI, T_C = K\theta = 0.375\times10^{-6}\times120 = 45\times10^{-6} N-m$

$$B = \frac{T_d}{ANI} = \frac{45\times10^{-6}}{30\times25\times10^{-6}\times100\times5\times10^{-3}} = 0.12 wb/m^2$$

$R_C = 0.3\times20 = 6\Omega$

Length of mean turn path =2(a+b) =2(55)=110mm

$$R_C = N\left(\frac{\rho l}{A}\right)$$

$$A = \frac{N\times\rho\times(l_t)}{R_C} = \frac{100\times1.7\times10^{-8}\times110\times10^{-3}}{6}$$

$$= 3.116\times10^{-8} m^2$$

$$= 31.16\times10^{-3} mm^2$$

$$A = \frac{\Pi}{4}d^2 \Rightarrow d = 0.2mm$$

10) A moving coil instrument gives a full scale deflection of 10mA, when the potential difference across its terminal is 100mV. Calculate

(1) The shunt resistance for a full scale deflection corresponding to 100A

(2) The resistance for full scale reading with 1000V.

Calculate the power dissipation in each case?

Solution:

Data given

$I_m = 10mA$

$V_m = 100mV$

$I = 100A$

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)$$

$$100 = 10\times10^{-3}\left(1+\frac{10}{R_{sh}}\right)$$

$R_{sh} = 1.001\times10^{-3}\Omega$

$R_{se} = ??, V = 1000V$

$$R_m = \frac{V_m}{I_m} = \frac{100}{10} = 10\Omega$$

$$V = V_m\left(1+\frac{R_{se}}{R_m}\right)$$

$$1000 = 100\times10^{-3}\left(1+\frac{R_{se}}{10}\right)$$

$\therefore R_{se} = 99.99K\Omega$

11. Design an Aryton shunt to provide an ammeter with current ranges of 1A,5A,10A and 20A. A basic meter with an internal resistance of 50w and a full scale deflection current of 1mA is to be used.

$$I_m = 1\times10^{-3}A \quad R_m = 50\Omega \quad \begin{vmatrix} I_1 = 1A \\ I_2 = 5A \\ I_3 = 10A \\ I_4 = 20A \end{vmatrix} \quad \begin{vmatrix} m_1 = \frac{I_1}{I_m} = 1000A \\ m_2 = \frac{I_2}{I_m} = 5000A \\ m_3 = \frac{I_3}{I_m} = 10000A \\ m_4 = \frac{I_4}{I_m} = 20000A \end{vmatrix}$$

$$R_{sh1} = \frac{R_m}{m_1 - 1} = \frac{50}{1000-1} = 0.05\Omega$$
$$R_{sh2} = \frac{R_m}{m_2 - 1} = \frac{50}{5000-1} = 0.01\Omega$$
$$R_{sh3} = \frac{R_m}{m_3 - 1} = \frac{50}{10000-1} = 0.005\Omega$$
$$R_{sh4} = \frac{R_m}{m_4 - 1} = \frac{50}{20000-1} = 0.0025\Omega$$

$\therefore$ The resistances of the various section of the universal shunt are

$$R_1 = R_{sh1} - R_{sh2} = 0.05 - 0.01 = 0.04\Omega$$
$$R_2 = R_{sh2} - R_{sh3} = 0.01 - 0.005 = 0.005\Omega$$
$$R_3 = R_{sh3} - R_{sh4} = 0.005 - 0.025 = 0.0025\Omega$$
$$R_4 = R_{sh4} = 0.0025\Omega$$

12) A basic d' Arsonval meter movement with an internal resistance $R_m = 100\Omega$ and a full scale current of $I_m = 1mA$ is to be converted in to a multi range d.c. voltmeter with ranges of 0-10V, 0-50V, 0-250V, 0-500V. Find the values of various resistances using the potential divider arrangement.

Solution:

Data given

$$R_m = 100\Omega$$
$$I_m = 1mA$$
$$V_m = I_m \times R_m$$
$$V_m = 100 \times 1 \times 10^{-3}$$
$$V_m = 100mV$$

$$m_1 = \frac{V_1}{V_m} = \frac{10}{100 \times 10^{-3}} = 100$$
$$m_2 = \frac{V_2}{V_m} = \frac{50}{100 \times 10^{-3}} = 500$$
$$m_3 = \frac{V_3}{V_m} = \frac{250}{100 \times 10^{-3}} = 2500$$
$$m_4 = \frac{V_4}{V_m} = \frac{500}{100 \times 10^{-3}} = 5000$$

$$R_1 = (m_1 - 1)R_m = (100 - 1) \times 100 = 9900\Omega$$
$$R_2 = (m_2 - m_1)R_m = (500 - 100) \times 100 = 40K\Omega$$
$$R_3 = (m_3 - m_2)R_m = (2500 - 500) \times 100 = 200K\Omega$$
$$R_4 = (m_4 - m_3)R_m = (5000 - 2500) \times 100 = 250K\Omega$$

13) A moving coil instrument whose resistance is 25Ω gives a full scale deflection with a current of 1mA. This instrument is to be used with a manganin shunt, to extent its range to 100mA. Calculate the error caused by a 10^0C rise in temperature when:

(a) Copper moving coil is connected directly across the manganin shunt.

(b) A 75 ohm manganin resistance is used in series with the instrument moving coil.

The temperature co-efficient of copper is 0.004/^{0}C and that of manganin is 0.00015^0/C.

Solution:

Case-1

$I_m = 1mA$

$R_m = 25\Omega$

I=100mA

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)$$

$$100 = 1\left(1+\frac{25}{R_{sh}}\right) \Rightarrow \frac{25}{R_{sh}} = 99$$

$$\Rightarrow R_{sh} = \frac{25}{99} = 0.2525\Omega$$

Instrument resistance for 10^0C rise in temperature, $R_{mt} = 25(1+0.004\times10)$

$$R_t = R_o(1+\rho_t \times t)$$

$$R_{m/t=10^\circ} = 26\Omega$$

Shunt resistance for 10^0C, rise in temperature

$$R_{sh/t=10^\circ} = 0.2525(1+0.00015\times10) = 0.2529\Omega$$

Current through the meter for 100mA in the main circuit for 10^0C rise in temperature

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)\bigg|_{t=10^\circ C}$$

$$100 = I_{mt}\left(1+\frac{26}{0.2529}\right)$$

$$I_m\big|_{t=10} = 0.963mA$$

But normal meter current=1mA

Error due to rise in temperature=(0.963-1)*100=-3.7%

Case-b As voltmeter

Total resistance in the meter circuit= $R_m + R_{sh} = 25+75 = 100\Omega$

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)$$

$$100 = 1\left(1+\frac{100}{R_{sh}}\right)$$

$$R_{sh} = \frac{100}{100-1} = 1.01\Omega$$

Resistance of the instrument circuit for 10^0C rise in temperature

$$R_m\big|_{t=10} = 25(1+0.004\times10)+75(1+0.00015\times10) = 101.11\Omega$$

Shunt resistance for 10^0C rise in temperature

$$R_{sh}\big|_{t=10} = 1.01(1+0.00015\times10) = 1.0115\Omega$$

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)$$

$$100 = I_m\left(1+\frac{101.11}{1.0115}\right)$$

$$I_m\big|t = 10^\circ = 0.9905mA$$

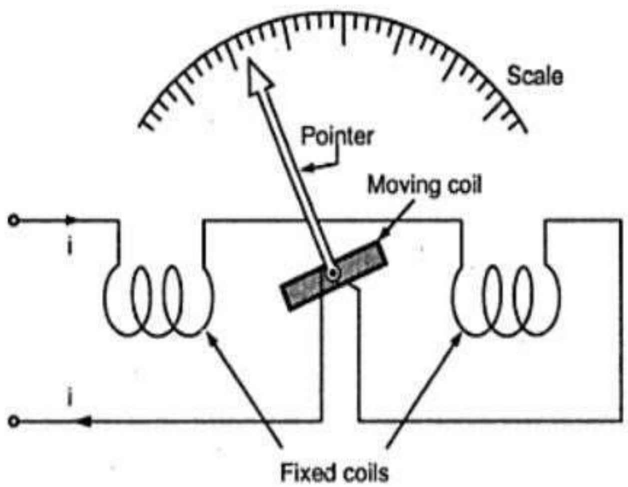
Error =(0.9905-1)*100=-0.95%

EMI: UNIT-II

MEASUREMENT OF POWER

Single phase dynamometer instrument

The Fig. 1 shows the construction of electrodynamometer type instrument.



Construction

The various type of the electrodynamometer type instrument are:

Fixed Coils: The necessary field required for the operation of the instrument is produced by the fixed coils. A uniform field is obtained near the center of coil due to division of coil in two sections. These coils are air cored. Fixed coils are wound with fine wire for using as voltmeter, while for ammeters and wattmeters it is wound with heavy wire. The coils are usually varnished. They are clamped in place against the coil supports. This makes the construction rigid.

Ceramic is usually used for mounting supports. If metal parts would have been used then it would weaken the field of the fixed coil.

Moving Coil: The moving coil is wound either as a self sustaining coil or else on a non-metallic former. If metallic former is used, then it would induce eddy currents in it. The construction of moving coil is made light as well as rigid. It is air cored.

Controlling: The controlling torque is provided by springs. These springs act as leads to the moving coil.

Moving System: The moving coil is mounted on an aluminium spindle. It consists of counter weights and pointer. Sometimes a suspension may be used, in case a high accuracy is desired.

Damping: The damping torque is provided by air friction, by a pair of aluminium vanes which are attached to the spindle at the bottom. They move in sector shaped chambers. As operating field would be distorted by eddy current damping, it is not employed.

Shielding: The field produced by these instruments is very weak. Even earth's magnetic field considerably affects the reading. So shielding is done to protect it from stray magnetic fields. It is done by enclosing in a casing high permeability alloy.

Cases and Scales: Laboratory standard instruments are usually contained in polished wooden or metal cases which are rigid. The case is supported by adjustable levelling screws.

A spirit level may be provided to ensure proper levelling.

For using electrodynamometer instrument as ammeter, fixed and moving coils are connected in series and carry the same current. A suitable shunt is connected to these coils to limit current through them upto desired limit.

The electrodynamometer instruments can be used as a voltmeter by connecting the fixed and moving coils in series with a high non-inductive resistance. It is most accurate type of voltmeter.

For using electrodynamometer instrument as a wattmeter to measure the power, the fixed coils acts as a current coil and must be connected in series with the load. The moving coils acts as a voltage coil or pressure oil and must be connected across the supply terminals. The wattmeter indicates the supply power. When current passes through the fixed and moving coils, both coils produce the magnetic fields. The field produced by fixed coil is proportional to the load current while the field produced by the moving coil is proportional to the voltage. As the deflecting torque is produced due to the interaction of these two fields, the deflection is proportional to the power supplied to the load.

Torque Equation

Let i_1 = Instantaneous value of current in fixed coil

i_2 = Instantaneous value of current in moving coil

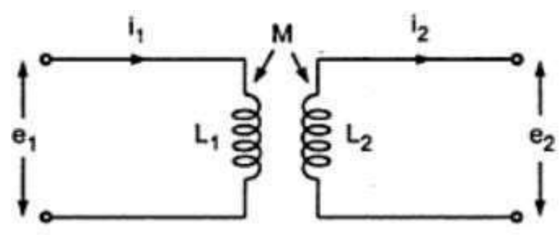
L_1 = Self inductance of fixed coil

L_2 = self inductance of moving coil

M = Mutual inductance between fixed and moving coils

The electrodynamometer instrument can be represented by an equivalent circuit as shown in the

Fig.2.



The flux linkages of coil 1 are,

$$\phi_1 = L_1 i_1 + M i_2$$

The flux linkages of coil 2 are,

$$\phi_2 = L_2 i_2 + M i_1$$

Now $e_1 = \frac{d\phi_1}{dt}$

and $e_2 = \frac{d\phi_2}{dt}$

$$\begin{aligned}\text{Electrical input energy} &= e_1 i_1 dt + e_2 i_2\, dt \\ &= i_1\, d\,\phi_1 + i_2\, d\,\phi_2 \\ &= i_1\, d\,(L_1 i_1 + M\, i_2) + i_2\, d\,(L_2 i_2 + M\, i_1) \\ &= i_1 L_1\, d\, i_1 + i_1^2\, d\, L_1 + i_1\, i_2\, d\, M + i_1\, M\, d\, i_2 + \\ &\quad i_2 L_2\, d\, i_2 + i_2^2\, d\, L_2 + i_1\, i_2\, d\, M + i_2\, M\, d\, i_1 \qquad \ldots(1)\end{aligned}$$

The energy stored in the magnetic field due to L_1, L_2 and M is given by,

$$\text{Energy stored} = \frac{1}{2} L_1\, i_1^2 + \frac{1}{2} L_2\, i_2^2 + i_1\, i_2\, M$$

$$\text{Change in stored energy} = d\left[\frac{1}{2} L_1\, i_1^2 + \frac{1}{2} L_2\, i_2^2 + i_1\, i_2\, M\right]$$

$$\begin{aligned}&= i_1\, L_1\, d\, i_1 + \frac{1}{2} i_1^2\, d\, L_1 + i_2\, L_2\, d\, i_2 + \frac{1}{2} i_2^2\, d\, L_2 + \\ &\quad i_1\, M\, d\, i_2 + i_2\, M\, d\, i_1 + i_1\, i_2\, d\, M \qquad \ldots(2)\end{aligned}$$

From the principle of conversation of energy, Energy input = Energy stored + Mechanical energy

∴ Mechanical energy = Energy input - Energy stored

Substraction (2) from equation (1),

$$\text{Mechanical energy} = \frac{1}{2} i_1^2\, d\, L_1 + \frac{1}{2} i_2^2\, d\, L_2 + i_1\, i_2\, d\, M$$

The self inductance L_1 and L_2 are constant and hence dL_1 and dL_2 are zero. Mechanical energy = i1 i2 dM

If Ti is the instantaneous deflecting torque and dθ is the change in the deflection then Mechanical energy = Mechanical work done

$i_1\ i_2\, d\, M = T_i\ d\theta$

$$\boxed{T_i = i_1 i_2 \frac{dM}{d\theta}}$$

This is the expression for the instantaneous deflection torque. Let us see its operation on a.c. and

$$T_d = I_1 I_2 \frac{dM}{d\theta}$$

D.C. operation : For d.c current of I_1 and I_2,

The controlling torque is provided by springs hence

$$T_c = K\theta$$

In steady state,

$$T_d = T_c$$

$$\therefore \quad I_1 I_2 \frac{dM}{d\theta} = K\theta$$

$$\therefore \quad \boxed{\theta = \frac{I_1 I_2}{K} \cdot \frac{dM}{d\theta}}$$

Thus the deflection is proportional to the product of the two currents and the rate of change of mutual inductance.

A.c. operation : In a.c. operation, the total deflecting torque over a cycle must be obtained by integrating T_i over one period.

Average deflecting torque over one cycle is,

$$T_d = \frac{1}{T}\int_0^T T_i \, dt$$

T = Time period of one cycle

$$T_d = \frac{dM}{d\theta} \cdot \frac{1}{T}\int_0^T i_1 i_2 \, dt$$

Now if two currents are sinusoidal and displaced by a phase angle then

$$i_1 = I_{m1} \sin \omega t$$

and $$i_2 = I_{m2} \sin(\omega t - \phi)$$

$$\therefore \quad T_d = \frac{dM}{d\theta} \cdot \frac{1}{T} \int_0^T I_{m1} \sin \omega t \cdot I_{m2} \sin(\omega t - \phi) \, d(\omega t)$$

$$= \left(\frac{I_{m1} \, I_{m2}}{2} \right) \cos \phi \frac{dM}{d\theta}$$

$$= I_1 \, I_2 \cos \phi \frac{dM}{d\theta}$$

where i_1, i_2 are the r.m.s. values of the two currents as,

$$I_1 = \frac{I_{m1}}{\sqrt{2}} \quad \text{and} \quad I_2 = \frac{I_{m2}}{\sqrt{2}}$$

As $$T_c = K \theta$$

Hence in steady state, $$T_c = T_d$$

$$\therefore \quad I_1 \, I_2 \cos \phi \frac{dM}{d\theta} = K \theta$$

$$\therefore \quad \boxed{\theta = \frac{I_1 \, I_2}{K} \cos \phi \frac{dM}{d\theta}}$$

Thus the deflection is decided by the product of r.m.s. values of two currents, cosine of the phase angle (power factor) and rate of change of mutual inductance.

For d.c. use, the deflection is proportional to square of current and the scale is nonuniform and crowded at the ends. For a.c. use the instantaneous torque is proportional to the square of the instantaneous current. The i^2 is positive and as current varies, the deflecting torque also varies.

But moving system, due to inertia cannot follow rapid variations and thus finally meter shows the average torque.

Thus the deflection is the function of the mean of the squared current. The scale is thus calibrated in terms of the square root of the average current squared i.e. r.m.s value of the a.c. quantity to be measured.

If an electrodynamometer instrument is calibrated with a d.v. current if 1 A and pointer indicates 1 A

d.c. on scale then on a.c., the pointer will deflect upto the same mark but 1A in this case will indicate r.m.s value.

Thus as it is a transfer instrument, there is direct connection between a.c. and d.c. Hence the instrument is often used as a calibration instrument.

The instrument can be used as an ammeter to measure currents upto 20 A while using as a voltmeter it can have low sensitivity of about 10 to 30 Ω/v

The Fig. 3(a), (b) and (c) shows the connections of the electrodynamometer instrument as ammeter, voltmeter and the wattmeter

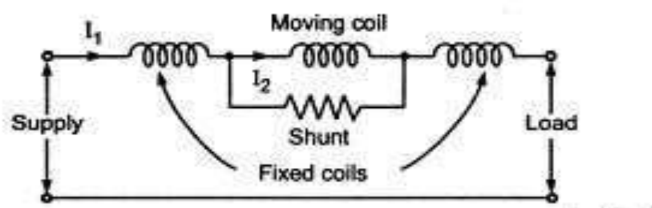


Fig. 3 (a) Electrodynamometer ammeter upto 100 mA

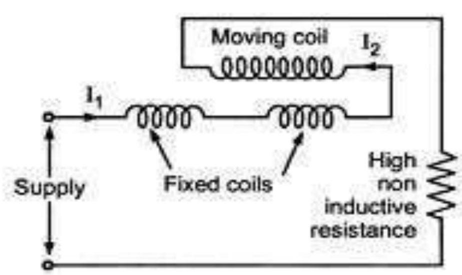


Fig. 3 (b) Electrodynamometer voltmeter

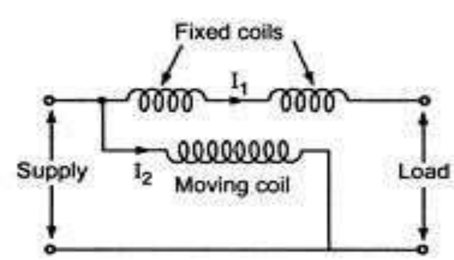


Fig. 3 (c) Electrodynamometer wattmeter

Single phase dynamometer wattmeter:

An electro dynamo meter type wattmeter is used to measure power. It has two coils, fixed coil which is current coil and moving coil which is pressure coil or voltage coil. The current coil carries the current of the circuit while pressure coil carries current proportional to the voltage in the circuit. This is achieved by connecting a series resistance in voltage circuit. The connections of an electrodynamometer wattmeter in the circuit are shown in fig

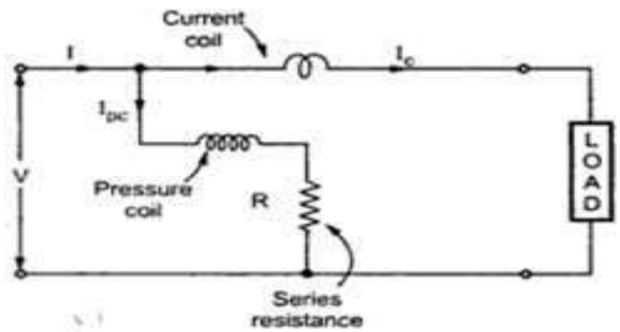


Fig: electro dynamometer wattmeter

I_C = current through current coil

I_{PC} = current through pressure coil

R = series resistance.

V= R.M.S. value of supply voltage

I= R.M.S. value of current.

Torque equation:

According to theory of electrodynamics instruments

$T_i = i_1 i_2 \frac{dM}{d\theta}$

Let v = instantaneous voltage = $V_m \sin\omega t = \sqrt{2}V \sin\omega t$

Due to the high resistance, pressure coil is treated to be purely resistive.

I_{PC} = instantaneous value = $\frac{V}{R_P}$

Where $R_P = r_{Pc} + R$

$I_{PC} = \frac{\sqrt{2}\,V}{R_P} \sin \omega t$

If current coil lags the voltage by angle $\emptyset$ then its instantaneous value is

$$i_c = \sqrt{2}\, I_C \sin(\omega t - \emptyset)$$

Now $i_1 = i_c$ and $i_2 = i_{pc}$ hence

$$T_i = [\sqrt{2} I_{PC} \sin \omega t][\sqrt{2}\, I_C \sin(\omega t - \emptyset)] \frac{dM}{d\theta}$$

$$= 2 I_{PC} I_C \sin(\omega t)\sin(\omega t - \emptyset)\frac{dM}{d\theta}$$

$$T_i = I_{PC} I_C [\cos\emptyset - \cos(2\omega t - \emptyset)]\frac{dM}{d\theta}$$

$$T_d = \text{average deflecting torque} = \frac{1}{T}\int_0^T T_i \, d(\omega t)$$

$$= \frac{1}{T}\int_0 I_{PC} I_C [\cos\emptyset - \cos(2\omega t - \emptyset)] \frac{dM}{d\theta} d(\omega t)$$

$$T_d = I_{PC} I_C \cos\emptyset \frac{dM}{d\theta}$$

Where $I_{PC} = \frac{V}{R_P}$

For a spring controlled wattmeter

$T_c = k\,\theta$ But $T_d = T_c$

$$I_{PC} I_C \cos\emptyset \frac{dM}{d\theta} = k\,\theta$$

$$\theta = \frac{1}{k} I_{PC} I_C \cos\emptyset \frac{dM}{d\theta} = K_1 I_{PC} I_C \cos\emptyset$$

Where $K_1 = \frac{1}{K}\frac{dM}{d\theta}$

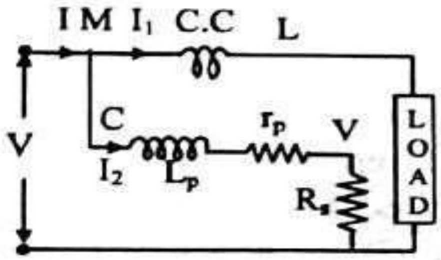
$$\theta = K_1 I_C \frac{V}{R_P}\cos\emptyset = K_2 P$$

$K_2 = \frac{K1}{R_P}$ and P = $V I_C \cos\emptyset$ = power

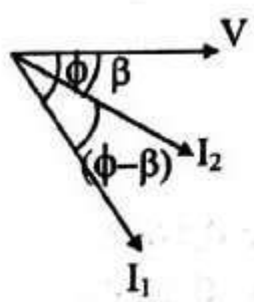
$\theta \propto P$

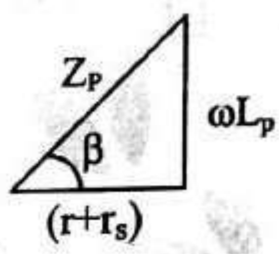
Errors in electro dynamometer wattmeter

1) **Error due to pressure coil inductance:**



L_p = inductance of PC
r_p = resistance of PC ($r_p << R_s$)
R_s = series multiplier resistance (KΩ)
Z_p = impedance of PC
$= ((r_p + r_s) + j\omega L_p)$
β = PC phase angle





$$\cos\beta = \left(\frac{R_S}{Z_p}\right)$$

$$\beta = \tan^{-1}\left(\frac{\omega L_p}{r_p + r_s}\right)$$

$$T_d \propto I_1 I_2 \cos(\phi-\beta)\frac{dM}{d\theta}$$

W_M (measured power)$\propto\theta$

$$= \frac{V}{Z_p}\frac{I}{K_c}\cos(\phi-\beta)\frac{dM}{d\theta}$$

$$W_M = \frac{VI\cos\beta\cos(\phi-\beta)}{R_s k_c}\ \frac{dM}{d\theta}$$

W_M(True power) without 'L_p'

$$T_d \propto I_1 I_2 \cos\phi \frac{dM}{d\theta}$$

$$T_d \propto \frac{VI\cos\phi}{R_s}\cdot\frac{dM}{d\theta}$$

$$\boxed{W_T = \frac{VI}{R_s}\cos\phi\frac{dM}{d\theta}}$$

$$\frac{W_T}{W_m} = \frac{\cos\phi}{\cos\beta\cos(\phi-\beta)}$$

$$\text{Correction factor (c.f)} = \frac{\cos\phi}{\cos\beta.\cos(\phi\mp\beta)}$$

'–' for lag
'+' for lead

$$W_T = \frac{\cos\phi}{\cos\beta(\cos\phi.\cos\beta+\sin\phi.\sin\beta)}.W_m$$

$$\frac{W_T}{W_m} = \frac{\cos\phi}{\cos\beta\cos\phi.\cos\beta(1+\tan\phi.\tan\beta)}$$

$$W_T = \frac{1+\tan^2\beta}{1+\tan\phi\tan\beta}.W_m$$

$$\boxed{W_T = \frac{1}{1+\tan\phi\tan\beta}.W_m}$$

$$W_m - W_T = W_T(\tan\phi\tan\beta)$$

$$W_m - W_T = VI.\cos\phi.\frac{\sin\phi}{\cos\phi}.\sin\beta$$

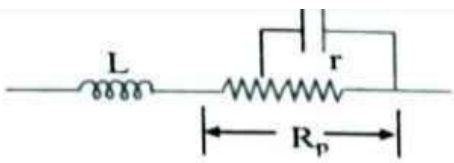
$$\boxed{\text{Error} = W_m - W_T = VI.\sin\phi.\tan\beta}$$

$$\text{error} = \frac{W_m - W_T}{W_T} \times 100$$

$$= \frac{VI.\sin\phi.\tan\beta}{VI\cos\phi} \times 100$$

$$\%\ \text{error} = \pm(\tan\phi\tan\beta \times 100)$$

Compensation of error due to pressure coil inductance



$$Z = (R_p - r) + j\omega L + \left(r \,||\, \frac{1}{j\omega C}\right)$$

$$Z = (R_p - r) + j\omega L + \frac{\left(r \times \frac{1}{j\omega C}\right)}{\left(r + \frac{1}{j\omega C}\right)}$$

$$= (R_p - r) + j\omega L + \frac{r}{1 + j\omega rC}$$

$$= (R_p - r) + j\omega L + \frac{r \times (1 - j\omega rC)}{(1 + j\omega rC)(1 - j\omega rC)}$$

$$= (R_p - r) + j\omega L + \frac{r - j\omega Cr^2}{(1 + \omega^2 r^2 C^2)}$$

Neglect $\omega^2 C^2 r^2$ term

$Z = (R_p - r) + j\omega L + r - j\omega Cr^2$

$Z \simeq R_p + j\omega(L - Cr^2)$

To eliminate the frequency term,

$(L - Cr^2)$ terms becomes zero

$L - Cr^2 = 0$

$C = \frac{L}{r^2} \rightarrow$ Approximate value

If $\omega^2 C^2 r^2$ term is not neglected then exact value of $C = 0.41\frac{L}{r^2}$

2) Error due to method of connection:

There are two ways of connecting wattmeter in a given circuit. These are respectively shown in the Fig. 3.12 (a) and (b).

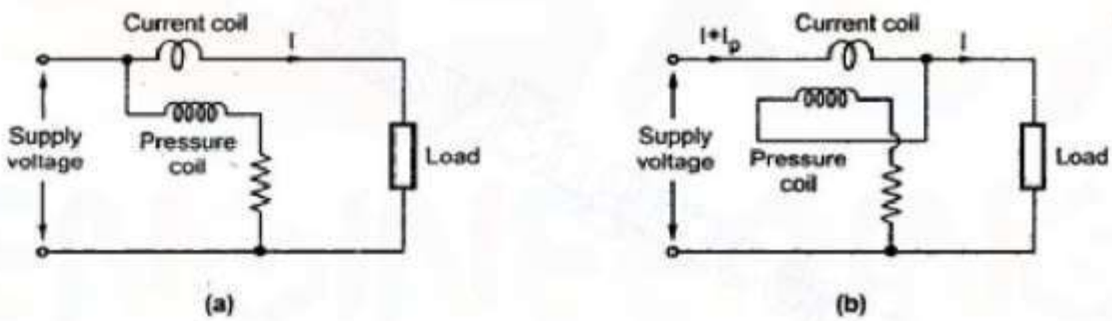


Fig. 3.12

Because of the power loss in the current and pressure coils, error is introduced in the measurement of power.

In connection shown in the Fig. 3.12 (a), pressure coil is connected on the supply side and therefore the voltage applied to the pressure coil is the voltage across the load plus the voltage drop across current coil. Thus wattmeter measures power loss in its current coil in addition to power consumed by load.

Power indicated by wattmeter = Power consumed by load + Power loss in current coil

$\therefore$ Power indicated by wattmeter = Power consumed by load + $I^2 R_c$

If wattmeter connections are as shown in the Fig. 3.12 (b) the current coil is on supply side and hence it carries pressure coil current plus the load current. Thus wattmeter reads in addition to power consumed in load, the power loss in pressure coil.

Power indicated by wattmeter = Power consumed by load + Power loss in pressure coil circuit

= Power consumed by load + V^2 / R_p

With small load current, the voltage drop in current coil is small so connections in Fig. 3.12 (a) introduces small error. Alternatively if load current is large, the pressure coil current is very small as compared with load current. Hence power loss in pressure coil circuit is small as compared with power consumed by load. Thus connection shown in Fig. 3.12 (a) is preferable for small currents while for large currents the connections shown in Fig. 3.12 (b) are preferable.

But if load current is high and the power factor is small, connection shown in Fig. 3.12 (b) results in large error as the total power measured is small. In this case a compensating coil may be used for compensation of error which is explained further in low power factor wattmeters.

3) Error due to pressure coil capacitance:

The pressure coil circuit may have capacitance in addition with inductance. This capacitance mainly due to the inter turn capacitance of the series résistance. The effect of capacitance is opposite to that due to inductance. Therefore the wattmeter will read high when the load power factor is leading

The inductance in pressure coil circuit will always more than inductance, hence the error caused by capacitance will be nullified by that due to inductance

4) Eddy current errors: Eddy currents are induced in the solid metal parts and within the thick conductors by the alternating magnetic field produced by the current coil.

This eddy current produce their own magnetic field and it will alter that produced by the main current in the current coil and thus error occurred.

This error can be minimized by avoiding solid metal parts as much as Possible and by using stranded conductors for high current applications

5) Error due to mutual inductance:

Errors may occur due to the mutual inductance between the current and pressure coils of the watt meter.These errors are quite low at power frequencies.But they increased with increase in frequencies.

The effect of mutual inductance can be avoid by arranging the coil system in such a way that they have no mutual inductance.So we can eliminate the errors due to mutual inductance.The Drysdale Torsion head wattmeter is an example for such type.

6) **Stray Magnetic field Errors:**

The electrodynamometer type watt meter has a weak operating field and therefore it Effected by stray magnetic fields it will resulting in serious errors. Hence these instruments Should be shielded against stray magnetic field.

7) Errors caused by vibration of moving system:

The torque on the moving system varies with frequency which is twice that of voltage. If the part of the moving system has a natural frequency which is resonance with the frequency of torque pulsation, the moving system would vibrate with considerable amplitude. These vibrations will cause errors. This error can be reduced by design.

8) Temperature Error:

The change in room temperature may affect the indication of wattmeter. This is because of change in temperature will change in resistance of pressure coil and stiffness of springs which provide controlling torque. This effect are opposite in nature and cancel each other.The use of material of having negligible temperature coefficient of resistance will reduce change in resistance the pressure coil with change in temperature.

<u>Extension of range of wattmeter using instrument transformer:</u>

For very high voltage circuits, the high rating wattmeters are not available to measure the power. The range of wattmeter can be extended using instrument transformers, in such high voltage circuits. The connections are as shown in fig.

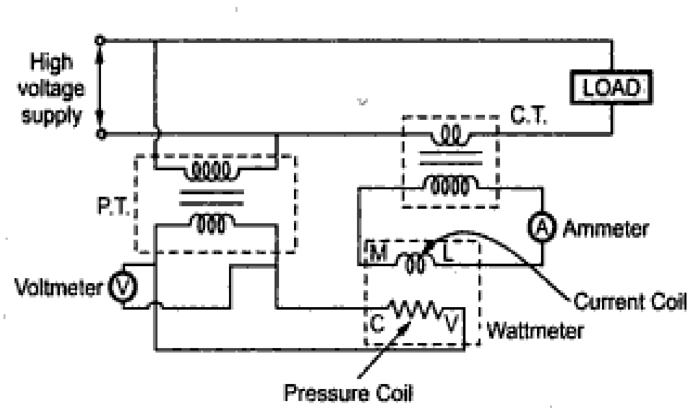


Fig: power measurement using C.T and P.T

The primary winding of C.T is connected in series with the load and secondary is connected in series with an ammeter and the current coil of a wattmeter.

The primary winding of P.T is connected across the supply and secondary is connected across voltmeter and the pressure coil of the wattmeter. One secondary terminal of each transformer and the casings are grounded.

Now both C.T and P.T have errors like ratio and phase angle error. For precise measurements, these errors must be considered. If not considered, these errors may cause inaccurate measurements. The correction must be applied to such errors to get the accurate results.

Low Power Factor Electro-Dynamamometer Type Wattmeter

If any circuit is operating at low power factor then power in that circuit is difficult to measure with ordinary electrodynamometer wattmeters. The reading of the wattmeter is inaccurate on account of following reasons,

1. The deflecting torque on the moving system is small as the power factor is low even though the current and pressure coils are fully excited.
2. The inductance of pressure coil introduces considerable error at low power factors.

In order to get accurate reading from the wattmeter when it is measuring low power, extra adjustments are required to be made so that there will be compensation of the errors.

When power to be measured is low then the current in the circuit is high as the power factor is low. Thus in this case pressure coil can not be connected to supply side as otherwise large error will be produced because of large current flowing in current coil and corresponding power loss in current coil circuit is measured by wattmeter.

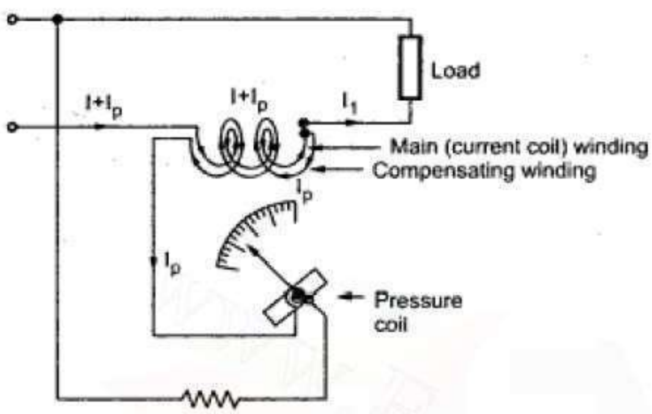


Fig. 3.13

If pressure coil is connected to load side, power consumed by pressure coil is measured by wattmeter which is appreciable in comparison with power to be measured which is small. Hence it is necessary to compensate for pressure coil current in low power factor wattmeter. The compensated wattmeter is shown in Fig. 3.13.

As shown in the Fig. 3.13 the compensating coil is connected in series with the potential coil and is made as identical and coincident with current coil as possible. The current coil carries current $I + I_p$ and produces its own field proportional to this current. The compensating coil carries current I_p and produces field proportional to this current. This field acts in opposite direction to the field produced by current coil.

Thus the resultant field is due to current I only. Hence error due to pressure coil current is neutralized.

Thus at no load condition, the wattmeter should not deflect as the resultant current coil field is zero.

In case of low power factor wattmeter, the pressure coil circuit is designed for low resistance to increase the current flowing through it so as to have increased torque. In low power factor wattmeter the value of pressure coil current is 10 times the current in case of high power factor wattmeters.

We have already seen in previous section that the pressure coil inductance introduces error whose magnitude is given by $VI \sin\phi \tan\beta$. If power factor is low then ϕ is large and hence $\sin\phi$ is large. Thus the error introduced in the measurement is appreciable which must be compensated. It is compensated by connecting a capacitor across a part of series resistance in the pressure coil circuit which is shown in the Fig. 3.14.

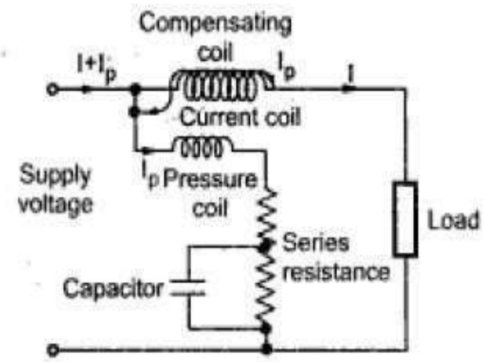


Fig. 3.14 Low power factor wattmeter

Blondel"s theorem

The Blondel's theorem is about number of wattmeters required in a polyphase system for the measurement of total power. It states that,

If a network is supplied using n conductors, the total power is the sum of the readings of n wattmeters so arranged that a current coil of each wattmeter is in each line and the corresponding pressure coil is connected between that line and a common point. If the common point is located on one of the lines then the power may be measured by n - 1 wattmeters.

Thus in a three phase system if common point for connecting pressure coils is located on one of the lines then only 2 wattmeters are sufficient for measuring the power. This is called two wattmeter method.

Two wattmeter method

Method of Connection :

The current coils of the two wattmeters are connected in any two lines while the voltage coil of each wattmeter is connected between its own current coil terminal and the line without a current coil. For example, the current coils are inserted in the lines R and Y then the pressure coils are connected between R-B for one wattmeter and Y-B for other wattmeter, as shown in the Fig. 3.24.

The connections are same for star or delta connected load. It can be shown that when two wattmeters are connected in this way, the algebraic sum of the two wattmeter readings gives the total power dissipated in the three phase circuit.

If W_1 and W_2 are the two wattmeter readings then total power is,

$$W = W_1 + W_2 = \text{Three phase power}$$

Proof of two wattmeter method

Consider star connected load and two wattmeters connected as shown in the Fig.3.24.

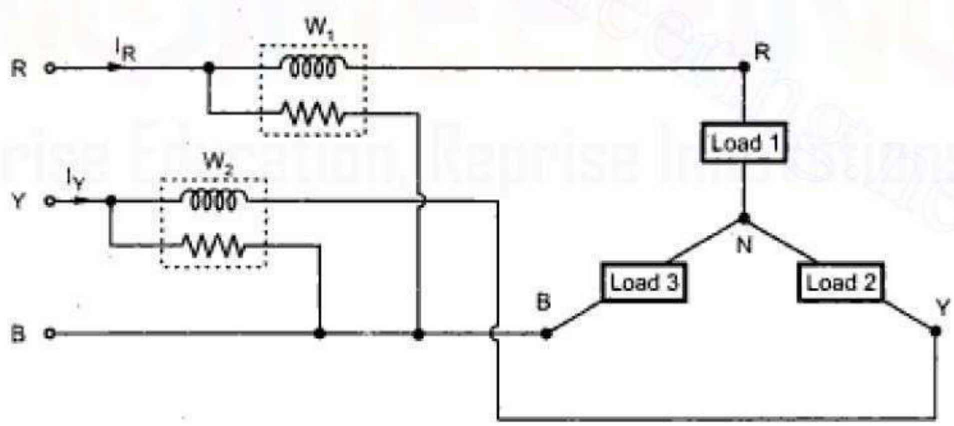


Fig. 3.24 Two wattmeter method for star connected load

a) For unbalanced load

Let us consider the instantaneous values of different-current and voltages.

Note that the r.m.s. values are indicated in capital letters like I_R, V_{RY} etc. while instantaneous values are indicated in small letters like i_r, v_{ry} etc.

It is important to note that wattmeter gives average value which is $W = I_c \times V_{pc} \times \cos\left(I_c \hat{} V_{pc}\right)$ if I_c and V_{pc} are r.m.s. values of current though current coil and voltage across voltage coil, respectively. And instantaneously wattmeter reads just the product of instantaneous values of current through current coil and voltage across pressure coil.

$$W_{instantaneous} = i_c \times v_{pc}$$

According to the connections shown in the Fig. 3.24 instantaneously W_1 and W_2 will show following readings.

$$W_1 = i_r \times v_{rb} \quad \text{and} \quad W_2 = i_y \times v_{yb}$$

Now $$v_{rb} = v_r - v_b \quad \text{and} \quad v_{yb} = v_y - v_b$$

where v_r, v_y and v_b are instantaneous values of phase voltages V_R, V_Y and V_B respectively.

$\therefore$ Substituting in W_1 and W_2,

$$W_1 = i_r \times (v_r - v_b) \quad \text{and} \quad W_2 = i_y \times \left(v_y - v_b\right)$$

$$W_1 = i_r \times (v_r - v_b) \text{ and } W_2 = i_y \times (v_y - v_b)$$

$$\therefore \quad W_1 + W_2 = i_r(v_r - v_b) + i_y(v_y - v_b)$$

$$= i_r v_r - i_r v_b + i_y v_y - i_y v_b$$

$$= i_r v_r + i_y v_y - (i_r + i_y) v_b$$

Now according to Kirchhoff's current law to neutral point.

$$i_r + i_y + i_b = 0$$

$$\therefore \quad i_r + i_y = -i_b$$

Substituting above, $W_1 + W_2 = i_r v_r + i_y v_y + i_b v_b$

As v_r, v_y and v_b are instantaneous values of phase voltages and i_r, i_y, i_b are instantaneous values of line currents which are same as phase currents as load is star connected,

$$\boxed{W_1 + W_2 = P_R + P_y + P_B}$$

where P_R, P_y and P_B are instantaneous values of powers consumed by each phase of the load at the instant considered regardless of power factor. Hence at any instant, addition of two wattmeter readings always gives instantaneous total power consumed by a three phase load.

The result can be easily proved for delta connected load. Hence by two wattmeter method we can measure total power though load is star or delta connected, balanced or unbalanced.

Note that instantaneously power is always fluctuating and wattmeter pointer also tries to show this fluctuating power. But due to inertia of the moving system, pointer can not respond to these fluctuations. And hence wattmeter reads average value of the power and hence $W_1 + W_2$ gives the average value of the total power consumed by 3 phase load.

b) For balanced load : Let us consider the r.m.s. values of the currents and voltages to prove that sum of two wattmeter gives total power consumed by three phase load.

$$W_1 = I_R \times V_{RB} \times \cos\left(I_R \,\hat{}\, V_{RB}\right)$$

$$W_2 = I_Y \times V_{YB} \times \cos\left(I_Y \,\hat{}\, V_{YB}\right)$$

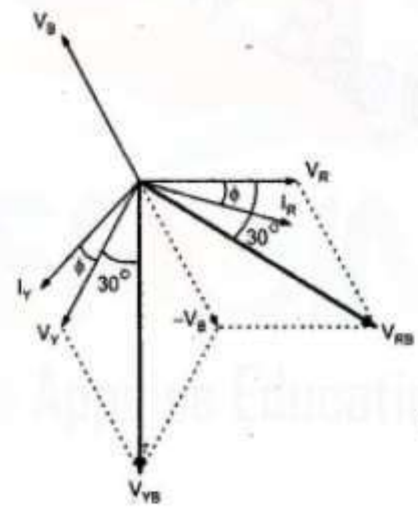


Fig. 3.25

To find angle between (I_R and V_{RB}) and (I_Y and V_{YB}) let us draw phasor diagram. (Assuming load p.f. be $\cos\phi$ lagging)

$$\overline{V}_{RB} = \overline{V}_R - \overline{V}_B$$

and $$\overline{V}_{YB} = \overline{V}_Y - \overline{V}_B$$

$$V_R \wedge I_R = \phi \text{ and } V_Y \wedge I_Y = \phi$$

$$V_R = V_Y = V_B = V_{ph}$$

and $$V_{RB} = V_{YB} = V_L$$

$$I_R = I_Y = I_L = I_{ph}$$

From Fig. 3.25, $I_R \wedge V_{RB} = 30 - \phi$

and $I_Y \wedge V_{YB} = 30 + \phi$

$\therefore$ $$W_1 = I_R V_{RB} \cos(30 - \phi)$$

$\therefore$ $$\boxed{W_1 = V_L I_L \cos(30 - \phi)}$$

and $$W_2 = I_Y V_{YB} \cos(30 + \phi)$$

$\therefore$ $$\boxed{W_2 = V_L I_L \cos(30 + \phi)}$$

$\therefore$ $$W_1 + W_2 = V_L I_L [\cos(30 - \phi) + \cos(30 + \phi)]$$

$$= V_L I_L [\cos 30 \cos\phi + \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi]$$

$$= 2\ V_L I_L \cos 30 \cos\phi = 2 V_L I_L \frac{\sqrt{3}}{2} \cos\phi$$

$$\boxed{\therefore \quad W_1 + W_2 = \sqrt{3}\, V_L\, I_L \cos\phi = \text{Total 3 phase power}}$$

Power factor calculation by two wattmeter method

In case of balanced load, the p.f. can be calculated from W_1 and W_2 readings.

For balanced, lagging p.f. load, $W_1 = V_L I_L \cos(30-\phi)$

$$W_2 = V_L I_L \cos(30+\phi)$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi \quad \text{...(1)}$$

$$\begin{aligned} W_1 - W_2 &= V_L I_L [\cos(30-\phi) - \cos(30+\phi)] \\ &= V_L I_L [\cos 30 \cos\phi + \sin 30 \sin\phi - \cos 30 \cos\phi + \sin 30 \sin\phi] \\ &= V_L I_L [2 \sin 30 \sin\phi] = V_L I_L \left[2 \times \frac{1}{2} \times \sin\phi\right] \end{aligned}$$

$$\therefore \quad W_1 - W_2 = V_L I_L \sin(\phi) \quad \text{...(2)}$$

Taking ratio of (1) and (2),

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \frac{\tan\phi}{\sqrt{3}}$$

$$\therefore \quad \tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

$$\therefore \quad \phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]$$

$$\therefore \quad \boxed{\text{p.f. } \cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}}$$

For **leading p.f.** we get **tan ϕ negative.** But cosine of negative angle is positive.

Key Point : *So cos ϕ is always positive but its nature must be determined by observing sign of t*

3.13 Effect of P.F. on Wattmeter Readings

For a lagging p.f.

$$W_1 = V_L I_L \cos(30 - \phi)$$

$$W_2 = V_L I_L \cos(30 + \phi)$$

Consider different cases,

Case i) $\cos\phi = 0 \quad \phi = 90°$

$$\therefore \quad W_1 = V_L I_L \cos(30 - 90) = +\frac{1}{2} V_L I_L$$

$$W_2 = V_L I_L \cos(30 + 90) = -\frac{1}{2} V_L I_L$$

i.e. $\quad W_1 + W_2 = 0$

$$|W_1| = |W_2| \qquad \text{but} \quad W_2 = -W_1$$

In the case discussed above W_1 will show positive reading with normal connections while W_2 will try to deflect in negative direction and hence W_2 reading must be obtained by reversing connections of either of the two coils and must be taken as negative.

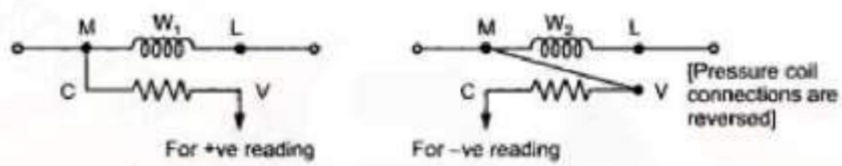


Fig. 3.29 Negative reading on wattmeter

So on wattmeter $W_1 = W_2$ but W_2 must be taken as negative as this reading will be obtained by reversing connections of any one coil.

Case ii) $\cos\phi = 0.5, \ \phi = 60°$

$$\therefore \quad W_1 = V_L I_L \cos(30 - 60) = V_L I_L \cos 30$$

$$= \text{Positive}$$

$$= \text{Positive}$$

$$W_2 = V_L I_L \cos(30+60) = 0$$

$$\therefore \quad W_1 + W_2 = W_1 = \text{Total power.}$$

One wattmeter shows zero reading for $\cos\phi = 0.5$. For all power factors between 0 to 0.5 W_2 shows negative and W_1 shows positive, **for lagging p.f.**

Case iii) $\cos\phi = 1, \ \phi = 0°$

$$W_1 = V_L I_L \cos(30+0) = V_L I_L \cos 30 = +\text{ve}$$

$$W_2 = V_L I_L \cos(30-0) = V_L I_L \cos 30 = +\text{ve}$$

$\therefore$ Both W_1 and W_2 are equal and positive. For all power factors between 0.5 to 1 both wattmeter gives +ve reading.

In short, the result can be summarised as,

Range of p.f.	Range of ' ϕ '	W_1 sign	W_2 sign	Remark
$\cos\phi = 0$	$\phi = 90°$	positive	negative	$\lvert W_1\rvert = \lvert W_2\rvert$
$0 \angle \cos\phi \angle 0.5$	$90° \angle \phi \angle 60°$	positive	negative	
$\cos\phi = 0.5$	$\phi = 60°$	positive	0	
$0.5 \angle \cos\phi \angle 1$	$60° \angle \phi \angle 0°$	positive	positive	
$\cos\phi = 1$	$\phi = 0°$	positive	positive	$W_1 = W_2$

Table 3.2

Key Point : *The Table 3.2 is applicable for lagging power factors but same table is applicable for leading power factors by interchanging columns of W_1 and W_2.*

The curve of p.f. against K is shown in the Fig. 3.30 where,

$$K = \frac{\text{Smaller reading}}{\text{Larger reading}}$$

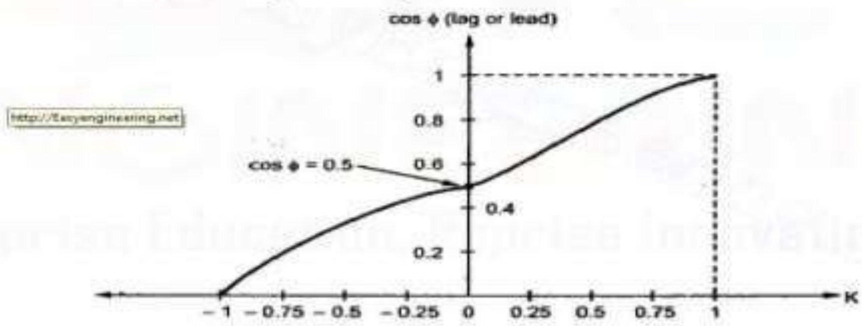


Fig. 3.30 Effect of p.f. on wattmeter readings

Reactive volt-ampere by two watt meter method

We have seen that,

$$W_1 - W_2 = V_L I_L \sin\phi$$

The total reactive volt-amperes for a 3 phase circuit is given by,

$$Q = \sqrt{3}\, V_L I_L \sin\phi = \sqrt{3}\,(W_1 - W_2) \quad \text{VAR}$$

Thus reactive volt-amperes of a 3 phase circuit can be obtained by multiplying the difference of two wattmeter readings by $\sqrt{3}$.

So,

S = Apparent power $\sqrt{3}\, V_L I_L$ VA or kVA
P = Active power = $\sqrt{3}\, V_L I_L \cos\phi = W_1 + W_2$ W or kW
Q = Reactive power = $\sqrt{3}\, V_L I_L \sin\phi = \sqrt{3}\,(W_1 - W_2)$ VAR or kVAR

Example 3.4 : *Two wattmeters connected to measure the input to a balanced 3 ϕ circuit indicate 2000 W and 500 W respectively. Find the power factor of the circuit :-*

i) When both readings are positive.

ii) When the latter is obtained after reversing the connection to the current coil of one instrument.

Solution : Case i) Both readings positive

$$W_1 = 2000\text{ W} \quad \text{and} \quad W_2 = 500\text{ W}$$

$$\therefore \quad \cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\} = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(2000-500)}{(2000+500)}\right]\right\}$$

$$= \cos\{\tan^{-1}(1.0392)\} = \cos(46.102^\circ)$$

$$= \mathbf{0.6933 \quad lagging}$$

Case ii) W_2 obtained by reversing the connections hence negative

$$\therefore \quad W_1 = 2000\text{ W} \quad \text{and} \quad W_2 = -500\text{ W}$$

$$\therefore \quad \cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

$$= \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(2000-(-500))}{(2000-500)}\right]\right\}$$

$$= \cos\{\tan^{-1}(2.8867)\} = \cos(70.89^\circ)$$

$$= \mathbf{0.3273 \quad lagging}$$

Advantages of two watt-meter method

The various advantages of two wattmeter method are,

1. The method is applicable for balanced as well as unbalanced loads.
2. Neutral point for star connected load is not necessary to connect the wattmeters.
3. The delta connected load, need not be opened for connecting the wattmeters.
4. Only two wattmeters are sufficient to measure total 3 phase power.
5. If the load is balanced not only the power but power factor also can be determined.
6. Total reactive volt amperes can be obtained using two wattmeter readings for balanced loads.

dis- advantages of two wattmeter method

The few disadvantages of this method are,

1. Not applicable for three phase, 4 wire system
2. The signs of W_1 and W_2 must be identified and noted down correctly otherwise it may lead to the wrong results.

One wattmeter method

This method can be used only for balanced loads. In this method, only one wattmeter is used. Its current coil is introduced in any one line while its voltage coil is connected to other two lines, one after the other in sequence, with the help of two way switch.

The sum of the two readings gives us total active power in three phase circuit.

Its connection can be shown as in the Fig. 3.31.

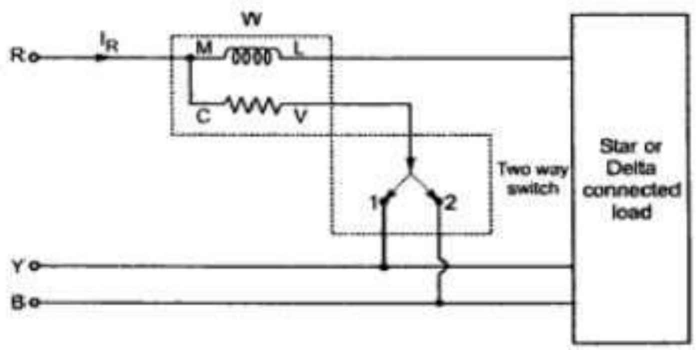


Fig. 3.31 Modified 2 wattmater method as 1 wattmeter method

In the Fig. 3.31 current coil is introduced in line R and hence it carries current I_R.

While one end of pressure coil is connected to 'R' only and second end gets connected to 'Y' when switch is in position 1 and gets connected to 'B' when switch is in position 2.

Assume Star Connected Load : Let us find the two readings on that wattmeter. Assume load having p.f. cos ϕ lagging.

In switch position 1 :
$$I_c = I_R$$
$$V_{pc} = V_{RY}$$
$$\therefore \quad W_1 = I_R V_{RY} \cos(I_R \wedge V_{RY})$$

In switch position 2 :
$$I_c = I_R$$
$$V_{pc} = V_{RB}$$
$$\therefore \quad W_2 = I_R V_{RB} \cos(I_R \wedge V_{RB})$$

Let us find $I_R \wedge V_{RY}$ and $I_R \wedge V_{RB}$ from phasor diagram as shown in the Fig. 3.32.

Let us find $I_R \wedge V_{RY}$ and $I_R \wedge V_{RB}$ from phasor diagram as shown in the Fig. 3.32.

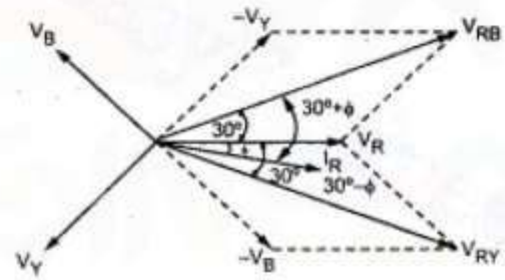


Fig. 3.32

$$\bar{V}_{RB} = \bar{V}_R - \bar{V}_B$$

$$\bar{V}_{RY} = \bar{V}_R - \bar{V}_Y$$

I_R lags V_R by angle 'ϕ'.

$$\therefore \quad I_R \wedge V_{RY} = (30 - \phi)$$

$$I_R \wedge V_{RB} = (30 + \phi)$$

$\therefore$ In position 1, $W_1 = I_R V_{RY} \cos(30-\phi) = I_L V_L \cos(30-\phi)$

and In position 2, $W_2 = I_R V_{RB} \cos(30+\phi) = I_L V_L \cos(30+\phi)$

$$\therefore \quad \boxed{W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\phi = \text{Total power.}}$$

One wattmeter method for reactive volt-ampere measurement

This can be used for balanced load. In this method, current coil of wattmeter is connected in any one line and pressure coil is connected across remaining two lines. The connection is as shown in the Fig. 3.33.

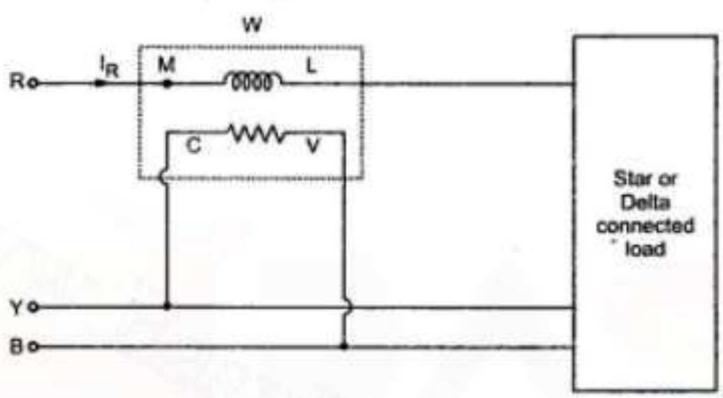
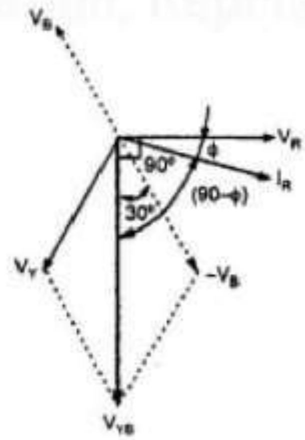


Fig. 3.33 Reactive voltamperes measurement

$$W = I_c V_{pc} \cos(I_{C} \wedge V_{PC})$$
$$= I_R V_{YB} \cos(I_R \wedge V_{YB})$$

To find $I_R \wedge V_{YB}$, assume load to be star connected having $\cos\phi$ lagging p.f.

$\therefore$ Phasor diagram is as in the Fig. 3.34.



$$\bar{V}_{YB} = \bar{V}_Y - \bar{V}_B$$
$$I_R \wedge V_{YB} = 90 - \phi$$
$$\therefore \quad W = I_R V_{YB} \cos(90-\phi)$$
$$= I_R V_{YB} \sin\phi = I_L V_L \sin\phi$$

Thus in this method the wattmeter reading is,

$$\boxed{W = V_L I_L \sin\phi}$$

But total reactive volt amperes are $\sqrt{3}\, V_L I_L \sin\phi$.

Key Point : *This reading must be multiplied by $\sqrt{3}$ to get total reactive volt amperes.*

$$\therefore \quad \boxed{\sqrt{3}\, W = \text{Total reactive volt amperes}}$$

Example 3.5 : *In a particular test the two wattmeter readings are 4 kW and 1 kW. Calculate the power and power factor if* *i) Both meters read direct ii) One meter connections reversed.*

Solution : i) Both meters read direct

$$\therefore \quad W_1 = +4 \text{ kW} \qquad W_2 = +1 \text{ kW}$$

$$\therefore \quad P = W_1 + W_2 = 5 \text{ kW}$$

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\} = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\times 3}{5}\right]\right\}$$

$$\therefore \quad \cos\phi = \cos(46.102^\circ) = 0.6933 \text{ lagging}$$

ii) When one meter reversed

$$W_1 = +4 \text{ kW} \qquad W_2 = -1 \text{ kW}$$

$$\therefore \quad P = W_1 + W_2 = 4 - 1 = 3 \text{ kW}$$

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}[4-(-1)]}{[4-1]}\right]\right\} = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\times 5}{3}\right]\right\}$$

$$= \cos(70.8933^\circ) = 0.3273 \text{ lagging}$$

Three phase wattmeter

Similar to single phase dynamometer wattmeter, a three phase dynamometer wattmeter is available. It consists of two sets of fixed and moving coils, mounted together in one case. The moving coils are placed on the same spindle. The Fig.3.40. shows the construction of a three phase wattmeter.

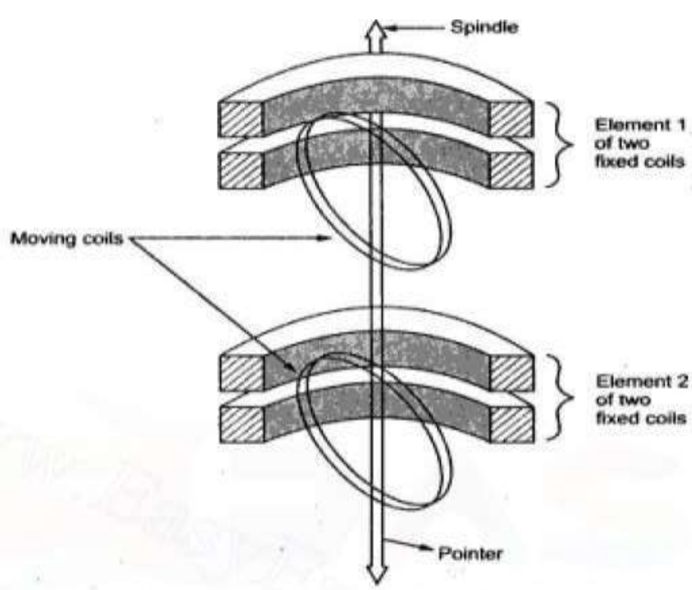


Fig. 3.40 Three phase two element wattmeter

Due to two sets of current and pressure coils, its connections are similar to the connections of two single phase wattmeters to measure three phase power. Each element experiences a torque which is proportional to the power measured by that element. The net torque on the moving system is the sum of the deflecting torques produced on each of the two elements.

$\therefore \qquad T_{d1} \propto W_1 \quad \text{and} \quad T_{d2} \propto W_2$

$\therefore \qquad T_d \propto (T_{d1} + T_{d2}) \propto (W_1 + W_2) \propto W$

where $\qquad T_d$ = Total deflecting torque

W_1 = Power measured by element 1

W_2 = Power measured by element 2

W = Total power

Thus the total deflecting torque is proportional to the total power.

As the coils are mounted very near each other, errors due to mutual interference are possible. To eliminate such errors, the laminated iron shield is placed in between the two elements.

The compensation for mutual interference can be obtained by using the resistances as shown in the Fig. 3.41.

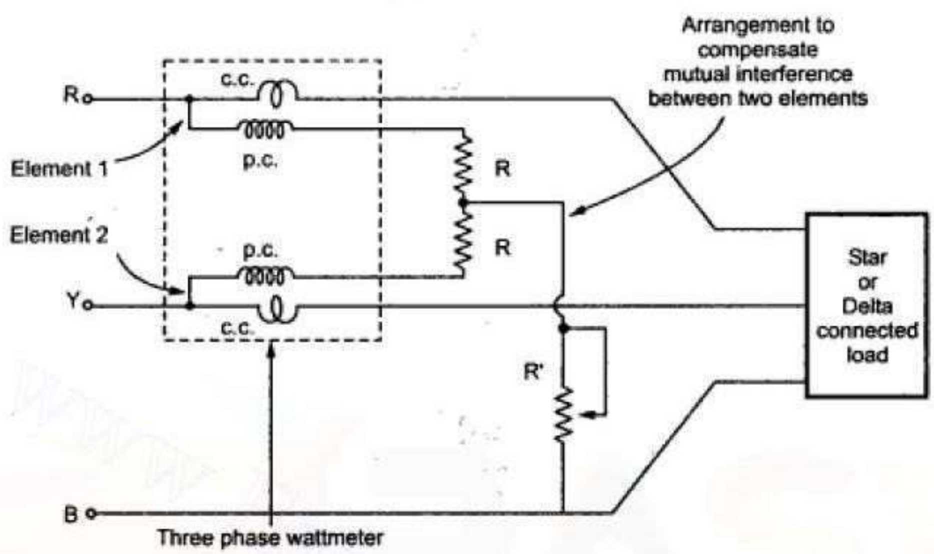


Fig. 3.41 Connections of three phase wattmeter

The value of R can be adjusted using R' to compensate the errors due to mutual effects between the two elements.

1) A 3 Phase, 10 KVA load has power factor of 0.342. The power is measured by two wattmeter method. Find reading of each wattmeter when i) Power Factor is leading ii) Power Factor is lagging

i)

$V_L.I_L = 5.77$ kVA

$\cos\varphi = 0.342 = \varphi = 70°$

Reading of each wattmeter when the power factor is leading

$W_1 = V_L.I_L.\text{Cos}(30 + \varphi)$

$W_1 = 5.77 \times 10^3 .\text{Cos}(30 + 70°) = -1\text{kW}$

$W_2 = V_L.I_L.\text{Cos}(30 - \varphi)$

$W_2 = V_L.I_L.\text{Cos}(30 - 70) = 4.42$ kW

ii)

Given power factor = 0.342

$\Rightarrow \cos\phi = 0.342$

$\Rightarrow \phi = 70°$

S = 10 kVA

$\Rightarrow \sqrt{3}\, V_L I_L = 10$

$\Rightarrow V_L I_L = 5.77$ kVA

$W_2 = V_L I_L \cos(30 + \phi) = 5.77 \cos(30 + 70) = -1$ kW

$W_1 = V_L I_L \cos(30 - \phi) = 5.77 \cos(30 - 70) = 4.4$ kW

Example 3.4 : *Two wattmeters connected to measure the input to a balanced 3 ϕ circuit indicate 2000 W and 500 W respectively. Find the power factor of the circuit :-*

i) When both readings are positive.

ii) When the latter is obtained after reversing the connection to the current coil of one instrument.

Solution : Case i) Both readings positive

$$W_1 = 2000 \text{ W} \quad \text{and} \quad W_2 = 500 \text{ W}$$

$$\therefore \quad \cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\} = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(2000-500)}{(2000+500)}\right]\right\}$$

$$= \cos\{\tan^{-1}(1.0392)\} = \cos(46.102°)$$

$$= 0.6933 \quad \textbf{lagging}$$

Case ii) W_2 obtained by reversing the connections hence negative

$$\therefore \quad W_1 = 2000 \text{ W} \quad \text{and} \quad W_2 = -500 \text{ W}$$

$$\therefore \quad \cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

$$= \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(2000-(-500))}{(2000-500)}\right]\right\}$$

$$= \cos\{\tan^{-1}(2.8867)\} = \cos(70.89°)$$

$$= \mathbf{0.3273 \quad lagging}$$

➠ **Example 3.5 :** *In a particular test the two wattmeter readings are 4 kW and 1 kW. Calculate the power and power factor if*

i) Both meters read direct ii) One meter connections reversed.

Solution : i) Both meters read direct

$$\therefore \quad W_1 = +4 \text{ kW} \qquad W_2 = +1 \text{ kW}$$

$$\therefore \quad P = W_1 + W_2 = \mathbf{5 \text{ kW}}$$

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\} = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\times 3}{5}\right]\right\}$$

$$\therefore \quad \cos\phi = \cos(46.102°) = \mathbf{0.6933 \text{ lagging}}$$

ii) When one meter reversed

$$W_1 = +4 \text{ kW} \qquad W_2 = -1 \text{ kW}$$

$$\therefore \quad P = W_1 + W_2 = 4 - 1 = \mathbf{3 \text{ kW}}$$

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}[4-(-1)]}{[4-1]}\right]\right\} = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\times 5}{3}\right]\right\}$$

$$= \cos(70.8933°) = \mathbf{0.3273 \text{ lagging}}$$

Example 3.6 : *A three phase, 400 V load has power factor of 0.6 lagging. The two wattmeters read a total input power of 20 kW. Find the reading of each wattmeter.*

Solution : $W_1 + W_2 = 20\text{ kW} = P,\ V_L = 400\text{ V},\ \cos\phi = 0.6$

$$\text{Now} \qquad P = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$\therefore \qquad 20\times10^3 = \sqrt{3}\times400\times I_L\times0.6$$

$$\therefore \qquad I_L = 48.1125\ \text{A}$$

$$\phi = \cos^{-1} 0.6 = 53.13°$$

$$\therefore \qquad W_1 = V_L\, I_L \cos(30-\phi) = 400\times48.1125\times\cos(30°-53.13°)$$

$$= 17698\ \text{W}$$

$$= \mathbf{17.698\ kW}$$

$$\text{And} \qquad W_2 = V_L\, I_L \cos(30+\phi) = 400\times48.1125\times\cos(30°+53.13°)$$

$$= 2302\ \text{W}$$

$$= \mathbf{2.302\ kW}$$

Unit-III: MEASUREMENT OF ENERGY

1)a) Explain the construction and working of Induction type single phase energy meter with a neat diagram.
b) An energy meter is designed to make 100 revolutions of the disc for one unit of energy. Calculate the number of revolutions made by it, when connected to a load carrying 40 A at 230V and 0.4 p.f. for 1 hour. If it actually makes 360 revolutions, find the percentage error.

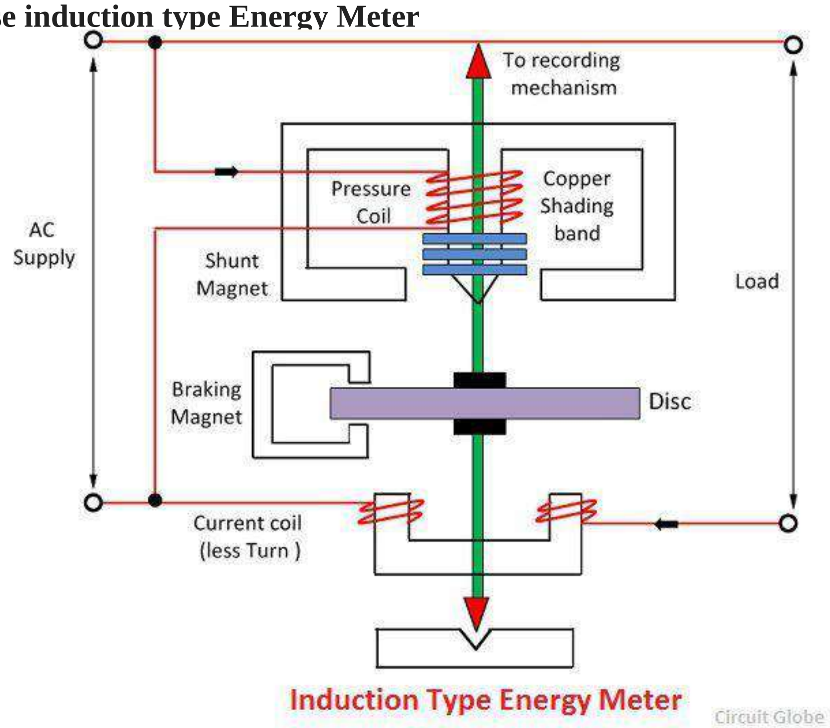
2)a) Derive the expression for torque and no of revolutions in a Induction type single phase energy meter?
b) A 50 A, 230V meter on full load test makes 61 revolutions in 37 seconds. If the normal disc speed is 520 revolutions per KWH, find the percentage error.

3)a) Explain the different sources of errors in Induction type energy meter and how they can be adjusted/compensated.
b) The disc of an energy meter makes 600 revolutions per unit of energy. When a 1000 watt load is connected, the disc rotates at 10.2 r.p.m if the load is on for 12 hours, how many units are recorded as error ?

<u>Construction and working of Induction type single phase Energy meter</u>

The meter which is used for measuring the energy utilises by the electric load is known as the energy meter. The energy is the total power consumed and utilised by the load at a particular interval of time. It is used in domestic and industrial AC circuit for measuring the power consumption. The meter is less expensive and accurate.

Single phase induction type Energy Meter



The energy meter has four main parts. They are the

1. Driving System
2. Moving System
3. Braking System
4. Registering System

1. **Driving System** – The electromagnet is the main component of the driving system. It is the temporary magnet which is excited by the current flow through their coil. The core of the electromagnet is made up of silicon steel lamination. The driving system has two electromagnets. The upper one is called the shunt electromagnet, and the lower one is called series electromagnet. The series electromagnet is excited by the load current flow through the current coil. The coil of the shunt electromagnet is directly connected with the supply and hence carries the current proportional to the shunt voltage. This coil is called the pressure coil. The centre limb of the magnet has the copper band. These bands are adjustable. The main function of the copper band is to align the flux produced by the shunt magnet in such a way that it is exactly perpendicular to the supplied voltage.
2. **Moving System** – The moving system is the aluminium disc mounted on the shaft of the alloy. The disc is placed in the air gap of the two electromagnets. The eddy current is induced in the disc because of the change of the magnetic field. This eddy current is cut by the magnetic flux. The interaction of the flux and the disc induces the deflecting torque. When the devices consume power, the aluminium disc starts rotating, and after some number of rotations, the disc displays the unit used by the load. The number of rotations of the disc is counted at particular interval of time. The disc measured the power consumption in kilowatt hours.
3. **Braking system** – The permanent magnet is used for reducing the rotation of the aluminium disc. The aluminium disc induces the eddy current because of their rotation. The eddy current cut the magnetic flux of the permanent magnet and hence produces the braking torque. This braking torque opposes the movement of the disc, thus reduces their speed. The permanent magnet is adjustable due to which the braking torque is also adjusted by shifting the magnet to the other radial position.
4. **Registration (Counting Mechanism)** – The main function of the registration or counting mechanism is to record the number of rotations of the aluminium disc. Their rotation is directly proportional to the energy consumed by the loads in the kilowatt hour. The rotation of the disc is transmitted to the pointers of the different dial for recording the different readings. The reading in kWh is obtained by multiply the number of rotations of the disc with the meter constant.

Working of the Energy Meter

The energy meter has the aluminium disc whose rotation determines the power consumption of the load. The disc is placed between the air gap of the series and shunt electromagnets. The shunt magnet has the pressure coil, and the series magnet has the current coil. The pressure coil creates the magnetic field because of the supply voltage, and the current coil produces flux because of the current. The field induces by the voltage coil is lagging by 90º on the magnetic field of the current coil because of which eddy current induced in the disc. The interaction of the eddy current and the magnetic field causes torque, which exerts a force on the disc. Thus, the disc starts rotating. The force on the disc is proportional to the current and voltage of the coil. The permanent magnet controls their rotation. The permanent magnet opposes the movement of the disc and equalises it on the power consumption. The cyclometer counts the rotation of the disc.

The pressure coil has the number of turns which makes it more inductive. The current I_p flows through the pressure coil because of the supply voltage, and it lags by 90º. The flux Φ_p is proportional to the applied voltage, and it is in phase with I_p. This flux is alternating and hence induces an eddy induced emf E_{ep} and as a result current I_{ep} flows in the disc.

The load current passes through the current coil induces the flux Φ_s. This flux induces eddy emf E_{es} and causes the eddy current I_{es} to flow on the disc. The eddy current I_{es} interacts with the flux Φ_p, and the eddy current I_{ep} interacts with Φ_s to produce the another torque. These torques are opposite in direction, and the net torque is the difference between these two.

I_p and I_s are currents passing through current coil and pressure coil

$V \rightarrow I_p \rightarrow \phi_p \rightarrow E_{ep} \rightarrow I_{ep}$

$I \rightarrow I_s \rightarrow \phi_s \rightarrow E_{es} \rightarrow I_{es}$

τ_d is the driving torque and $\tau_d \propto T_{d_1} - T_{d_2}$

τ_{d_1} is driving torque generated due to interaction of ϕ_p & I_{es}

$\tau_{d_1} \propto \phi_p I_{es} \cos(\phi_p \,\&\, I_{es})$

$\propto VI \cos(\phi_p \,\&\, I_{es})$

$\tau_{d_2} \propto \phi_s I_{ep} \cos(\phi_s \,\&\, I_{ep})$

$\propto VI \cos(\phi_s \,\&\, I_{ep})$

$\tau_d \propto \tau_{d_1} - \tau_{d_2}$

$\propto VI \left(\cos((90+\alpha) - (\Delta-\phi)) - \cos((90+\alpha)+(\Delta-\phi))\right)$

$a = 90+\alpha$, $b = \Delta - \phi$

$\propto VI(\cos(a-b) - \cos(a+b))$

$\propto VI(\cancel{\cos a \cos b} + \sin a \sin b - \cancel{\cos a \cos b} + \sin a \sin b)$

$\propto 2VI \sin a \sin b$

$\propto 2VI \sin(90+\alpha) \sin(\Delta - \phi)$

Here if disc is pure resistive $\alpha = 0$

$\propto 2VI \times 1 \times \sin(\Delta - \phi)$

If pressure coil is pure inductive $\Delta = 90°$

$\tau_d \propto VI \cos\phi$

$\tau_d \propto P$

$\tau_d = k_1 P$

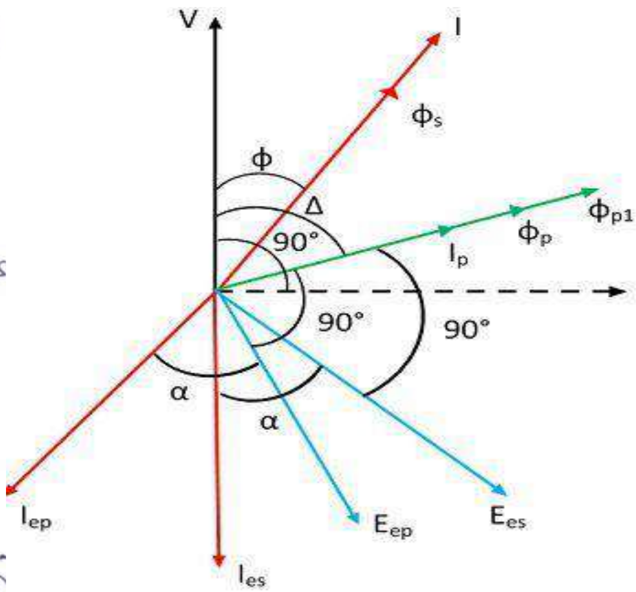
If N is steady speed, braking torque $T_B = k_2 N$

At steady state driving torque = Braking torque

$k_2 N = k_1 P$

$\therefore$ Speed of rotation is proportional to power

$\boxed{\text{Total no. of revolutions} = \int N dt = k \int P dt = k \times \text{Energy}}$



Phasor Diagram of Energy Meter

$\angle \phi_p \,\&\, I_{es} = \alpha + 90 - (\Delta - \phi)$

$= 90 + \alpha - (\Delta - \phi)$

$\angle \phi_s \,\&\, I_{ep} = \alpha + 90 + (\Delta - \phi)$

$= (90+\alpha) + (\Delta - \phi)$

Errors in energy meters

1. Creeping error in Energy Meter: Creeping in energy meter is the phenomenon in which the aluminium disc rotates continuously when only the voltage is supplied to the pressure coil, and no current flows through the current coil. In other words, the creeping is the kind of error in which the energy meter consumes a very small amount of energy even when no load is attached to the meter.

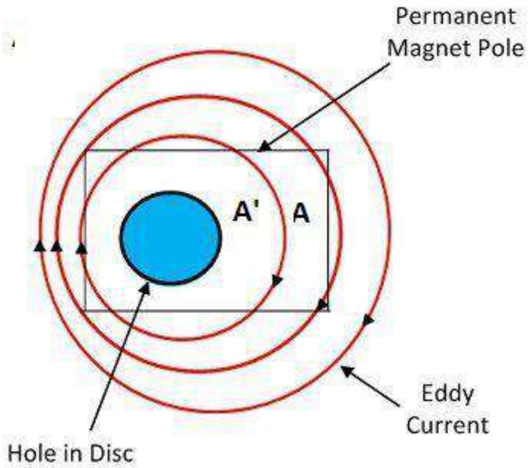
The creeping increases the speed of the disc even under the light load condition which increases the meter reading. The vibration, stray magnetic field and the extra voltage across the potential coil are also responsible for the creeping.

The creeping error occurs because of the excessive compensation for friction. The main driving torque is absent at no load. Hence the disc rotates because of the additional torque provided by the compensating vane.

Compensation to creeping error

The creeping is avoided by drilling the hole in the disc. The holes are diametrically opposite to each other. The aluminium disc stops rotating even when the small edge of the disc comes under the pole of the magnet. The holes will limit the revolution of the disc.

This action can easily be understood by considering the figure shown below.



Prevention of creep by drilling holes in disc

The circular eddy current path of the disc will disturb when the hole comes under the pole of the magnet. The A' is the centre point of the magnetic pole which is produced by the current. The force acting on the disc will move the centre point A' away from the poles axis A.

The disc will rotate at no load until the holes reach under the edges of the magnet. Their movement is opposed by the torque produced in this condition.

In some cases, the small piece of iron is attached to the edge of the disc. The force of attraction occurs between the pole magnets and the iron piece, which prevent the creeping of the disc.

2. **Speed error**: Due to the incorrect position of the brake magnet, the braking torque is not correctly developed. This can be tested when meter runs at its full load current alternatively

on loads of unity power factor and a low lagging power factor. The speed can be adjusted to the correct value by varying the position of the braking magnet towards the centre of the disc or away from the centre and the shielding loop. If the meter runs fast on inductive load and correctly on non- inductive load, the shielding loop must be moved towards the disc. On the other hand, if the meter runs slow on non-inductive load, the brake magnet must be moved towards the centre of the disc.

3. Meter phase error: An error due to incorrect adjustment of the position of shading band results an incorrect phase displacement between the magnetic flux and the supply voltage (not in quadrature). This is tested with 0.5 p.f. load at the rated load condition. By adjusting the position of the copper shading band in the central limb of the shunt magnet this error can be eliminated.

4.Friction error: An additional amount of driving torque is required to compensate this error. The two shading bands on the limbs are adjusted to create this extra torque. This adjustment is done at low load (at about 1/4th of full load at unity p.f.).

5.Temperature effect: Energy meters are almost inherently free from errors due to temperature variations. Temperature affects both driving and braking torques equally (with the increase in temperature the resistance of the induced current path in the disc is also increases) and so produces negligible error. A flux level in the brake magnet decreases with increase in temperature and introduces a small error in the meter readings. This error is frequently taken as negligible, but in modern energy meters compensation is adopted in the form of flux divider on the break magnet.

Energy meter constant K is defined as K = No. of revolutions/kw

In commercial meters the speed of the disc is of the order of 1800 revolutions per hour at full load.

Advantages of Induction meter

- ✓ Construction is simple and strong
- ✓ It is cheap in cost
- ✓ High torque is to weight ratio.
- ✓ More accurate and less maintenance
- ✓ Less affected by stray magnetic field.
- ✓ Range can be extended using Instrumental transformers

Disadvantages of Induction meter

- ✓ Without proper compensation measures, a considerable amount of errors are caused in the measurement due to temperature, waveform and frequency changes.
- ✓ Induction meters can use only for AC measurements.
- ✓ The creeping can cause errors
- ✓

Problems

1. An energy meter is designed to make 100 revolutions of the disc for one unit of energy. Calculate the number of revolutions made by it, when connected to a load carrying 40 A at 230V and 0.4 p.f. for 1 hour. If it actually makes 360 revolutions, find the percentage error.

Solution:

Energy consumed in one hour = Kwh= V*I*cosΦ*t= 230 × 40 × 0.4 × 1/1000

= 3.68 kWh (units)

No. of revolutions the meter should make if it is correct = No. Of units * revolutions per unit

$=3.68 \times 100$

$= 368$

No. of revolutions actually made = 360

∴ Percentage error = (368 − 360) × 100/368 = **2.17 %**

2) A 50 A, 230V meter on full load test makes 61 revolutions in 37 seconds. If the normal disc speed is 520 revolutions per KWH, find the percentage error.

Solution

Unity power-factor is assumed.

Energy consumed, in kWh, in 37 seconds = (50 x 230)/1000 x 37/3600

= 0.1182kWh

Number of revolutions corresponding to this energy = 520 × 0.1182

= 61.464

The meter makes 61 revolutions

∴ % Error = (61.464-61)/61.464 x 100%

= 0.755%

3)A single phase KWh meter makes 500 revolutions per KWh. It is found, on testing, as making 40 revolutions in 58 seconds at 5 KW full load. Find out the percentage error

Solution

Energy consumed, in kWh, in 58 seconds = 5kW x 58/3600

= 0.0805 kWh

Number of revolutions corresponding to this energy = 500 × 0.0805

= 40.278

The meter makes 40 revolutions

∴ % Error = (40.278-60)/40.278x 100%

= 0.7%

4)The disc of an energy meter makes 600 revolutions per unit of energy. When a 1000 watt load is connected, the disc rotates at 10.2 r.p.m if the load is on for 12 hours, how many units are recorded as error ?

Solution

Energy actually consumed is = (1000/1000) × 12 hours = 12 kWh

Total number of revolutions made by the disc during the period of 12 hours

= 10.2 × 60 × 12 = 7,344 revolutions

Energy recorded by the meter is = 7,344/600 = 12.24 kWh

Hence, 0.24 unit is recorded extra by the energy meter.

Three phase energy meter

In a three phase, four wire system, the measurement of energy is to be carried out by a three phase energy meter. For three phase, three wire system, the energy

measurement can be carried out by two element energy meter, the connections of which are similar to the connections of two wattmeters for power measurement in a three phase, three wire system. So these meters are classified as i) three element energymeter and ii) two element energymeter.

Three element energy meter

This meter consists of three elements. The construction of an individual element is similar to that of a single phase energymeter. The pressure coils are denoted as P_1, P_2 and P_3. The current coils are denoted as C_1, C_2 and C_3. All the elements are mounted in a vertical line in common case and have a common spindle, gearing and registering mechanism. The coils are connected in such a manner that the net torque produced is sum of the torques due to all the three elements. These are employed for three phase, four wire system where fourth wire is a neutral wire.

The current coils are connected in series with the lines while pressure coils are connected across a line and a neutral. Fig. 4.9 shows a three phase energymeter.

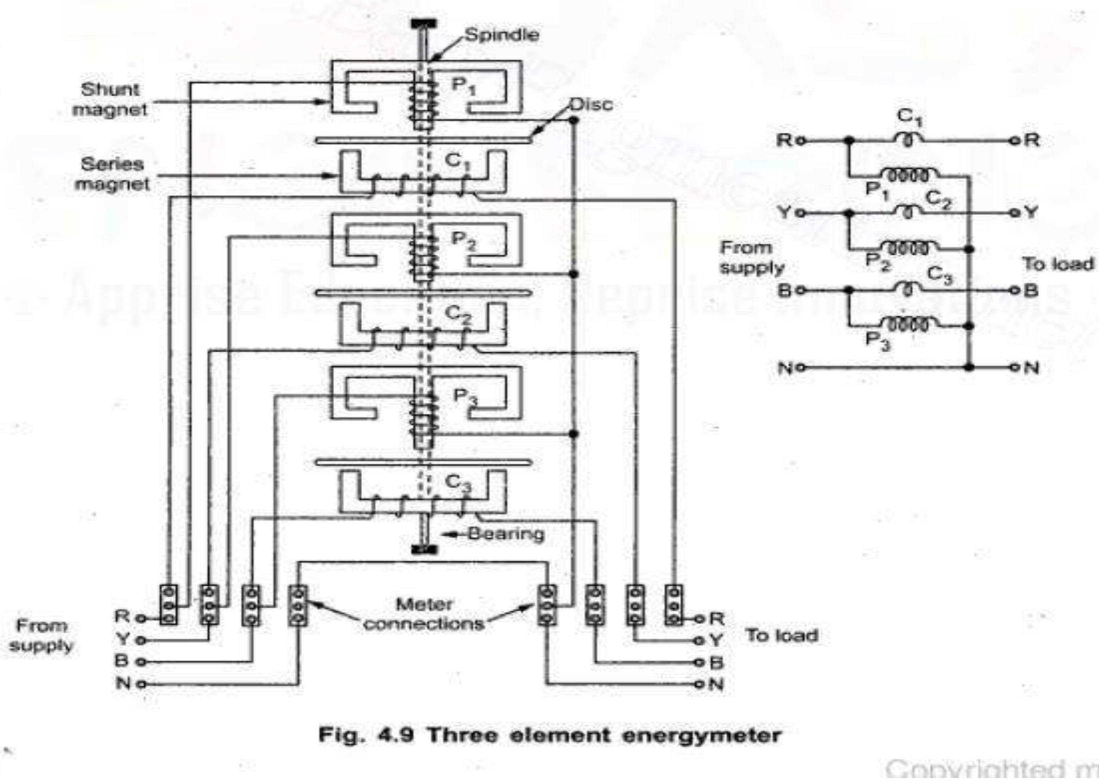


Fig. 4.9 Three element energymeter

One unit of three element, three phase element is always cheaper than three units of single phase energymeter. But due to interaction between eddy currents produced by one element with the flux produced by another element, there may be errors in the measurement by three phase energymeter. Such errors may be reduced by suitable adjustments.

Two element Energy meter

The Fig. 4.10 shows a two element energymeter and a simplified connection diagram.

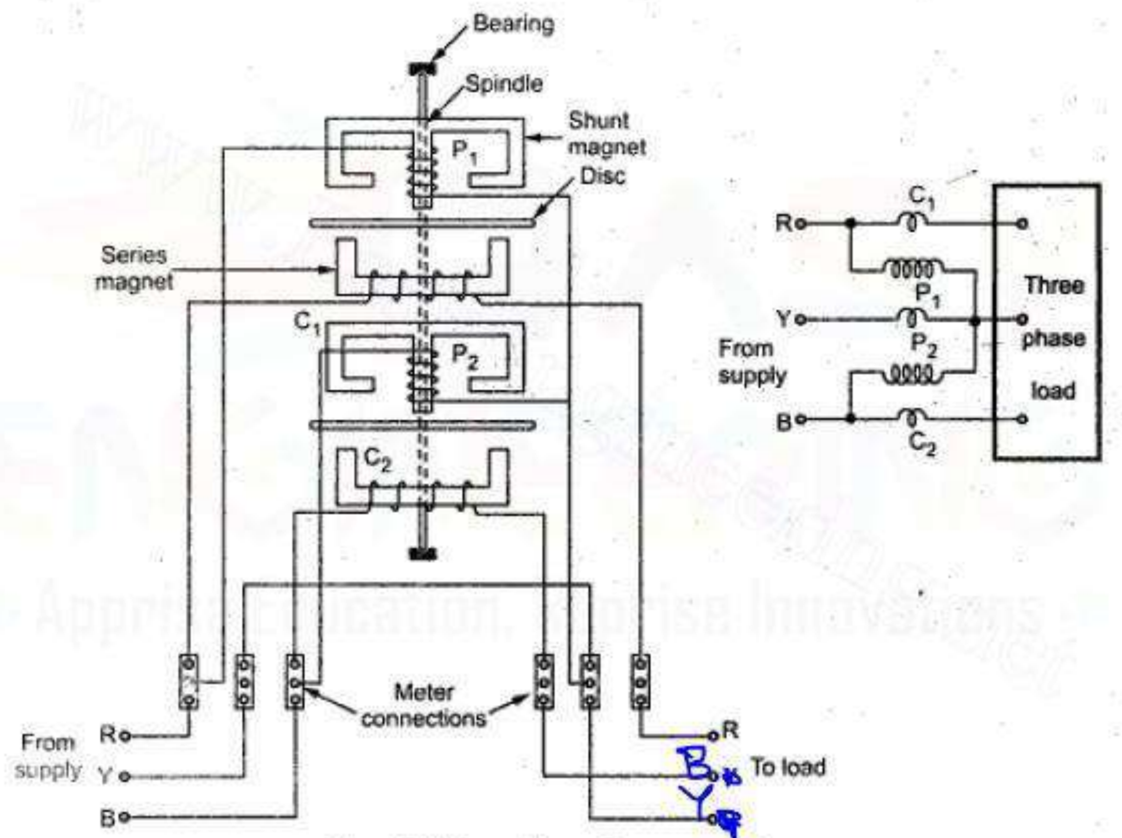


Fig. 4.10 Two element energymeter

This energymeter is used for three phase, three wire systems. The meter is provided with two discs each for an element. The shunt magnet is carrying pressure coil while a series magnet carries a current coil. The pressure coils are connected in parallel and the current coils in series. The connections are similar to the connections of two wattmeters for power measurement in three phase, three wire system. Torque is produced in same manner as in a single phase energymeter, in each element. The total torque on the registering mechanism connected to moving system, is sum of the torques of the individual elements.

Circuit for testing of energy meter with phantom loading

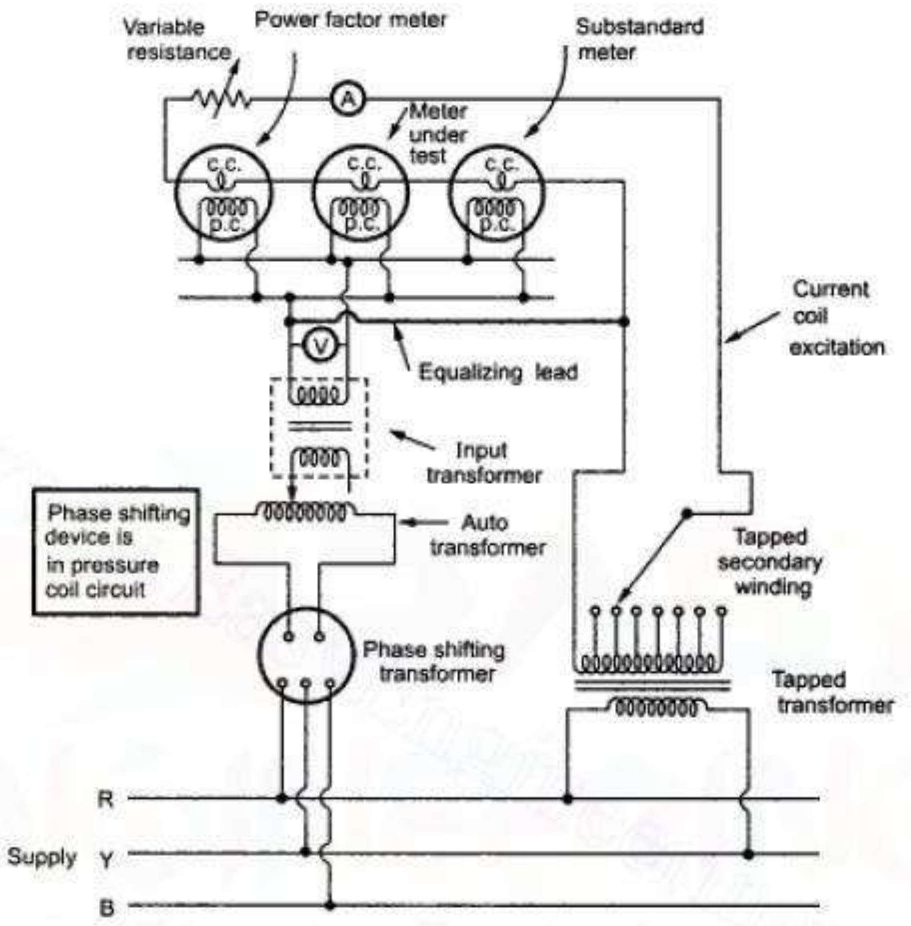


Fig. 4.19 Phantom loading of a.c. meter using rotating substandard meter

[redacted]: In this method, the phase shifting device is in the pressure coil circuit. The pressure coil is supplied through an input transformer having output voltage of 250 V. The autotransformer on primary side helps to adjust the voltage accurately to 250 V. The current coil is supplied seperately through a transformer, the primary of which is connected to same supply as that of pressure coil circuit. To adjust the necessary current, the tappings are provided on the transform secondary. Fine adjustment is possible using variable resistor.

The phase shifting transformer is employed in the circuit supplying the pressure coil circuit. Thus the phase of the pressure coil can be changed by changing the rotor of the phase shifting transformer. Hence any power factor condition, required for testing can be obtained. The arrangement is shown in the Fig. 4.19.

This test is conducted for short period of time hence called **short period test.** A rotating substandard meter is used alongwith the meter under test. The current coils of two meters are connected in series while pressure coils in parallel. The two meters are started and stopped simultaneously for short period of time.

When the predetermined load is adjusted, then the meter under test is allowed to make certain number of revolutions. At the same time, the number of revolutions made by rotating substandard meter, in the same time are observed.

If the constants of meters are same then error can be directly obtained. But if meter constants are different then error is required to be calculated.

Let K_x = Meter constant in number of revolutions per kWh for meter under test

K_s = Meter constant in number of revolutions per kWh for substandard meter

N_x = Number of revolutions made by meter under test

N_s = Number of revolutions made by substandard meter

E_r = Energy recorded by meter under test = $\frac{N_x}{K_x}$

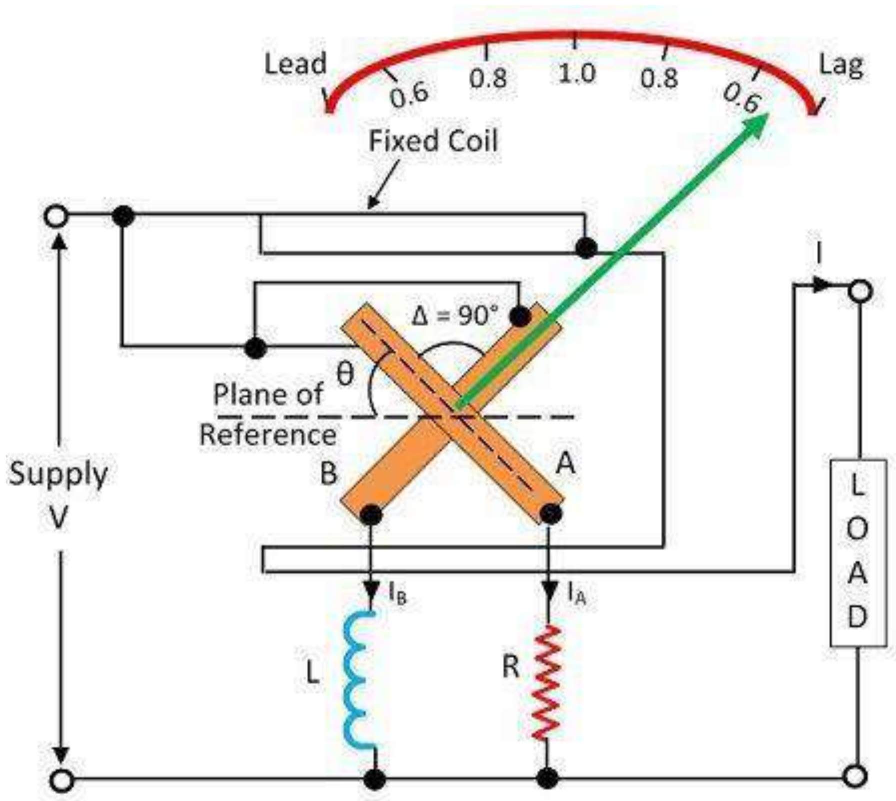
E_t = True energy recorded by substandard meter = $\frac{N_s}{K_s}$

$$\therefore \quad \boxed{\% \text{ error } = \frac{\frac{N_x}{K_x} - \frac{N_s}{K_s}}{\frac{N_s}{K_s}} \times 100}$$

Before conducting the test, the meters are allowed to run for 15 to 30 minutes with full load, to attain steady temperature.

Single phase electro dynamometer type power factor meter

The construction of the single phase electrodynamometer is shown in the figure below. The meter has fixed coil which acts as a current coil. This coil is split into two parts and carries the current under test. The magnetic field of the coil is directly proportional to the current flow through the coil. The meter has two identical pressure coils A and B. Both the coils are pivoted on the spindle. The pressure coil A has no inductive resistance connected in series with the circuit, and the coil B has highly inductive coil connected in series with the circuit.



Single Phase Electrodynamometer
Type Power Factor Meter

The current in the coil A is in phase with the circuit while the current in the coil B lag by the voltage nearly equal to 90º. The connection of the moving coil is made through silver or gold ligaments which minimize the controlling torque of the moving system.

The meter has two deflecting torque one acting on the coil A, and the other is on coil B. The windings are so arranged that they are opposite in directions. The pointer is in equilibrium when the torques is equal

.Deflecting torque acting on the coil A is given as

$$T_A = KVIMcos\emptyset sin\theta$$

θ – Angular deflection from the plane of reference.
M_{max} – maximum value of mutual inductance between the coils.

The deflecting torque acting on coil B is expressed as

$$I_B = KVIM_{max}\,\mathrm{Cos}(90^\circ - \emptyset)\,Sin(90^\circ + \emptyset) \quad I_B = KVIM_{max}cos\emptyset sin\theta$$

The deflecting torque is acting on the clockwise direction.

The value of maximum mutual inductance is same between both the deflecting equations.

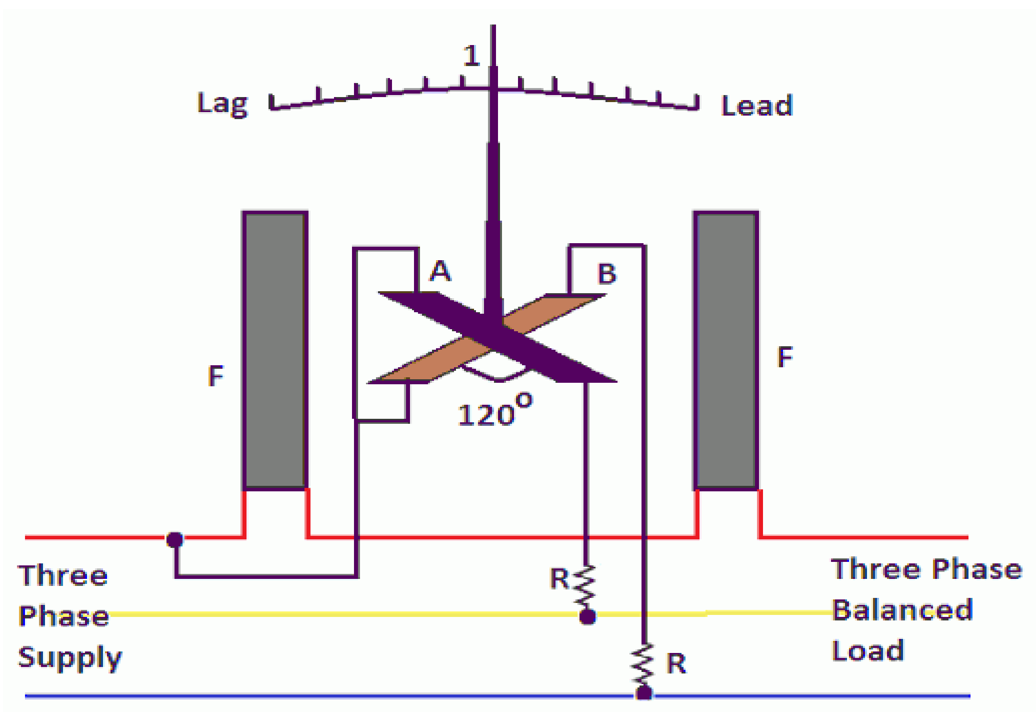
$$T_A = T_B$$

$$KVIMcos\emptyset sin\theta = KVIM_{max}cos\emptyset sin\theta$$

This torque acts on anti-clockwise direction. The above equation shows that the deflecting torque is equal to the phase angle of the circuit.

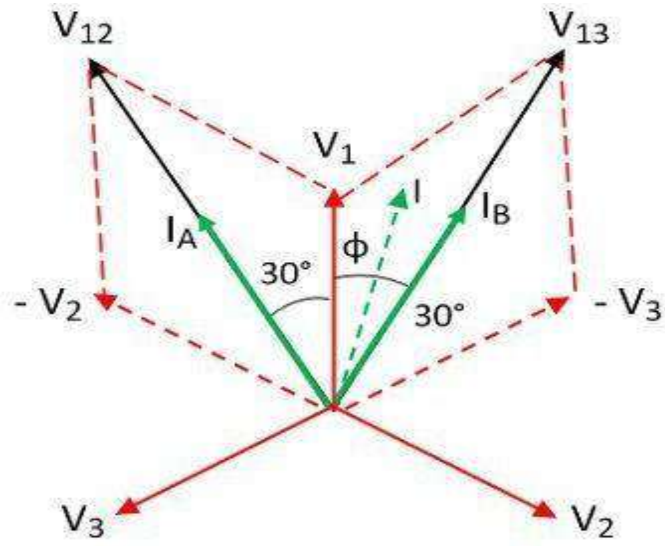
Three phase dynamo meter type power factor meter

The construction of the three phase meter is shown in the figure below. The electrodynamometer is only useful for the balanced load. The moving coil is placed at an angle of 120º. They are connected across different phases of the supply circuit. Both the coil has a series resistance.



The voltage across the coil A is V_{12} and the current across it I_{A1}. The circuit of the coil is resistive, and hence the current and voltage are in phase with each other. Similarly, the voltage V_{13} and the current I_{B1} is in phase with each other.

The phasor diagram of the three phase electrodynamic meter is shown in the figure below



Phasor Diagram of Three Phase Electrodynameter Type Power Factor Meter

Let Φ – phase angle of the circuit.
θ – angular deflection from the plane of reference.

$$T_A = KVI_{12}M_{max}\,\text{Cos}(30^\circ + \emptyset)\,Sin(60^\circ + \emptyset)$$

Torque acting on coil A is $T_A = \sqrt{3}KVI_{12}M_{max}\,\text{Cos}(30^\circ + \emptyset)\,Sin(60^\circ + \emptyset)$

$$T_B = KVI_{12}M_{max}\,\text{Cos}(30^\circ - \emptyset)\,Sin(120^\circ + \emptyset)$$

Torque acting on coil B is $T_B = KVI_{12}M_{max}\,\text{Cos}(30^\circ - \emptyset)\,Sin(120^\circ + \emptyset)$

The torque T_A and T_B are acting on the opposite directions.

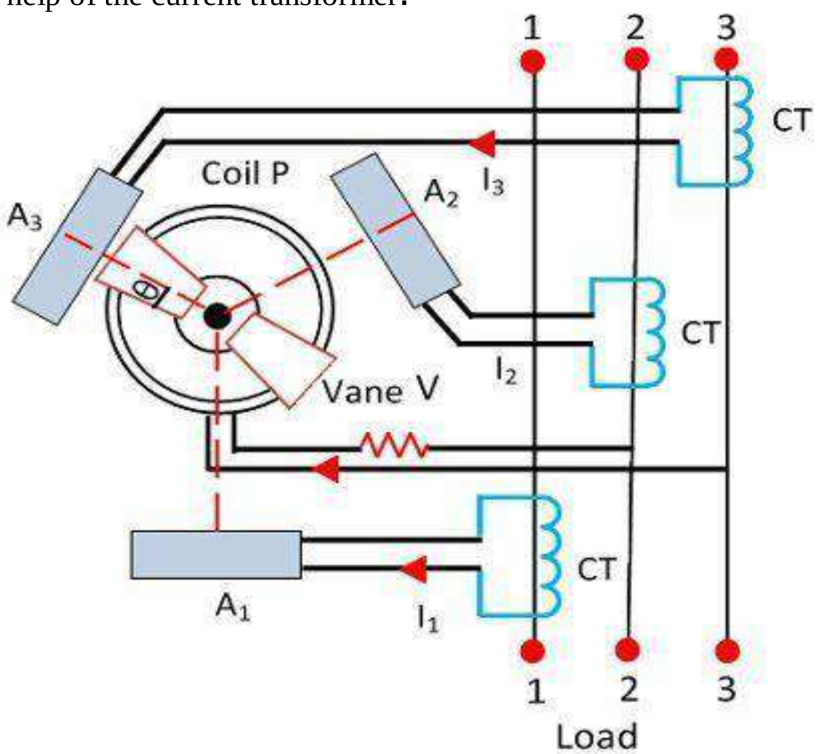
$$\text{Cos}(30^\circ - \emptyset)\,Sin(120^\circ + \emptyset) = \text{Cos}(30^\circ - \emptyset)\,Sin(120^\circ + \emptyset)$$

Thus the angular deflection of the coil is directly proportional to the phase angle of the circuit.

Moving Iron Power Factor Meter

The moving iron instrument is divided into two categories. They are the rotating magnetic field to some alternating fields.

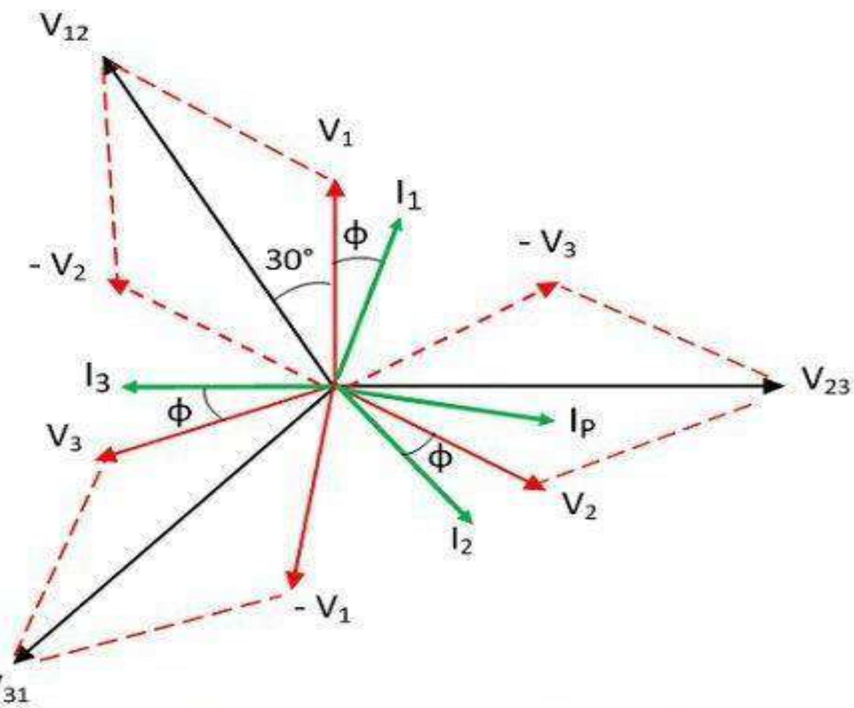
A. Rotating Field Power factor Meter – The following are the essential feature of the rotating magnetic field. The power factor meter has three fixed coils, and their axes are 120º displaced from each other. The axes are intersecting each other. The coils are connected to the three phase supply with the help of the current transformer.



Rotating Field Moving iron Power Factor Meter

The P is the fixed coil connected in series with the high resistance circuit across the phases 2 and 3. There is an iron cylinder across coil P. The two iron vanes are fixed to the cylinder. The spindles also carry damping vanes and pointer.

The phasor diagram of the power factor meter is shown in the figure.



Phasor Diagram for Moving Iron Type Power Factor Meter

$$[\mathrm{Cos}(90^\circ - \emptyset)Sin(90^\circ + \emptyset) + \mathrm{Cos}(330^\circ - \emptyset)Sin(210^\circ + \emptyset) + \mathrm{Cos}(210^\circ - \emptyset)Sin(330^\circ + \emptyset)] = 0$$

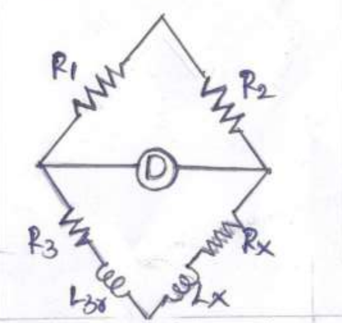
The coil P and the iron cylinders generate the alternating flux which interacts with the flux of the fixed coils. The interaction of the coil generates the moving system which determined the phase angle of the current. The vanes of the power factor meter are magnetized by the current of the moving coil which is in phase with the system line voltage.

Advantages of Moving Iron power Power Factor

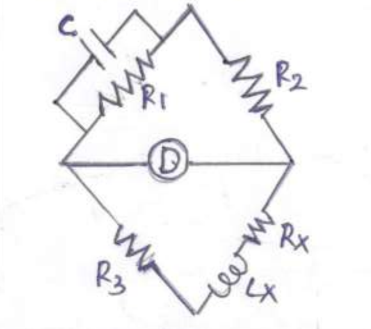
1. The meter requires large working force as compared to the electrodynamometer type meter.
2. The coils of the moving iron instruments are fixed permanently.
3. The range of the scale extends up to 360º.
4. The construction of the meter is robust and simple.
5. The moving iron instrument is cheap as compared to electrodynamics meter.

Measurement of Inductance:

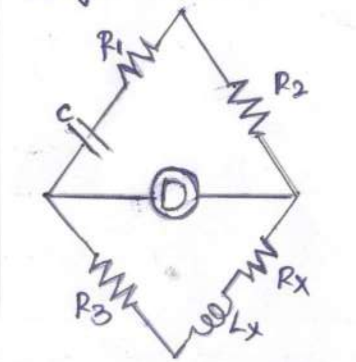
Maxwell Inductance:



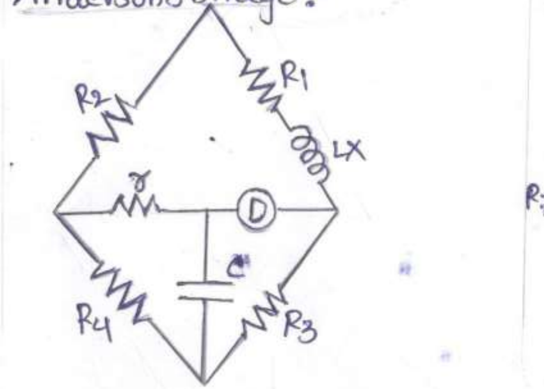
Maxwell Inductance & Capacitance:



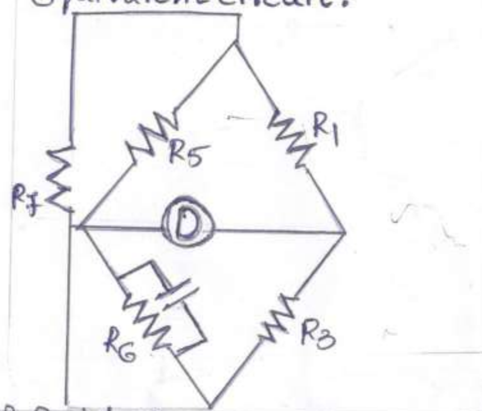
Hay's Bridge:



Anderson's Bridge:

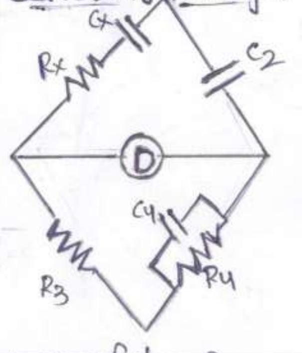


Equivalent circuit:

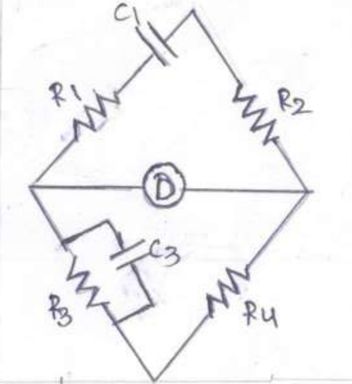


Measurement of Capacitance

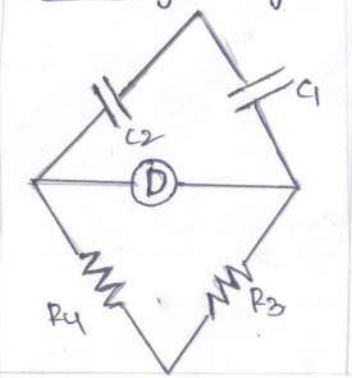
Schering Bridge:

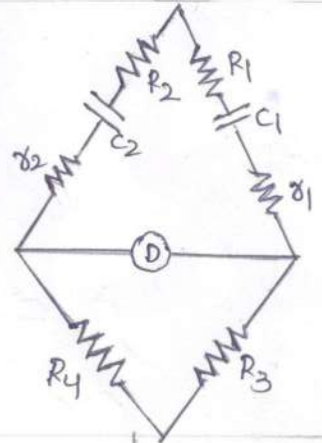


Weins Bridge:



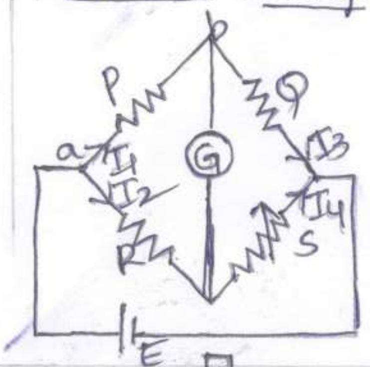
De-Sautys Bridge:



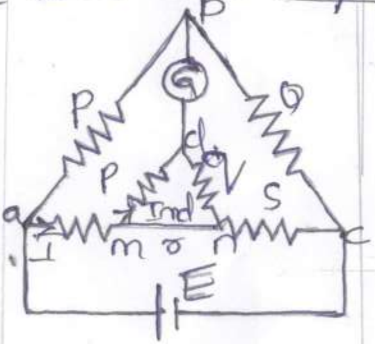


Measurement of Resistance

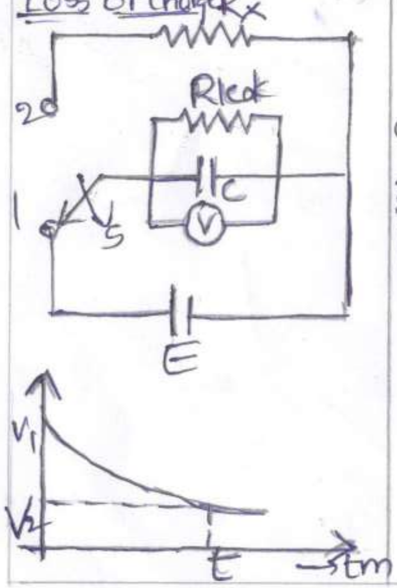
Wheatstone Bridge

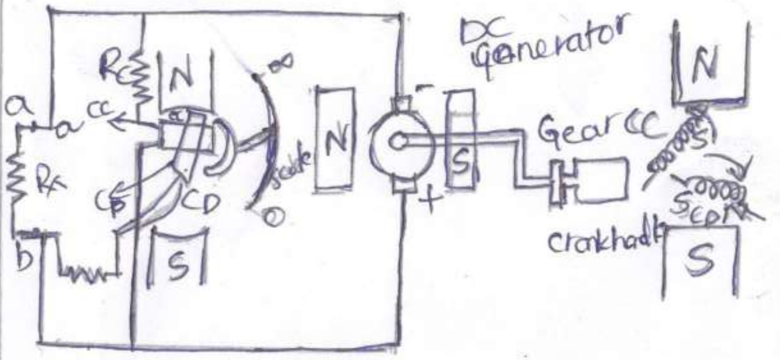


Kelvins double bridge



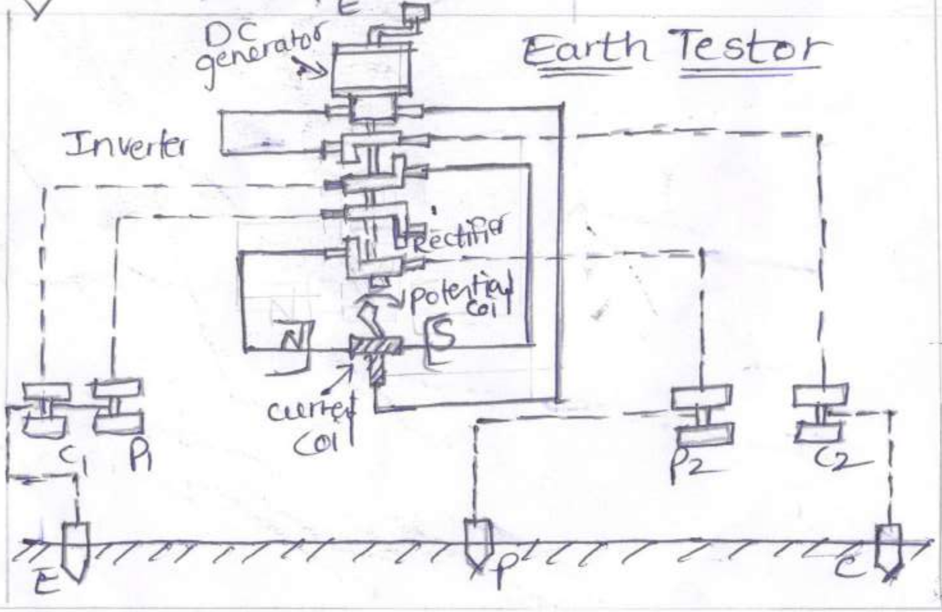
Loss of charge





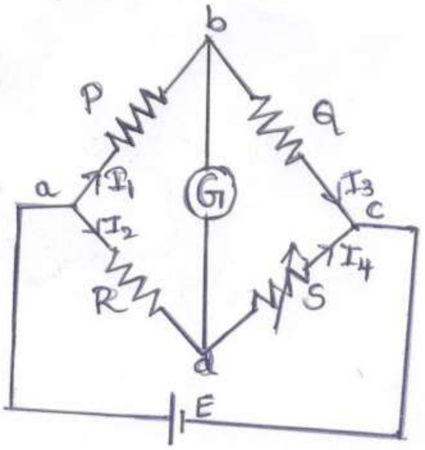
Megger

Earth Tester



UNIT - IV

Wheatstone bridge: (Measurement of medium Resistance)



S = Standard known Resistor

R = Unknown Resistor

Under balanced Conditions

$I_3 = I_1$ → Balance condition means galvanometer has to show zero deflection

$I_4 = I_2$

$V_{GS} = V_{ad}$

$\therefore I_1 P = I_2 R \rightarrow (1)$

$V_{bc} = V_{dc}$

$\therefore I_1 Q = I_2 S \rightarrow (2)$

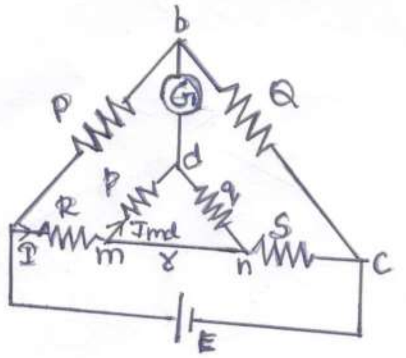
by dividing eqⁿ (1) by eqⁿ (2)

$$\frac{\cancel{I_1}P}{\cancel{I_1}Q} = \frac{\cancel{I_2}R}{\cancel{I_2}S}$$

$$\frac{P}{Q} = \frac{R}{S}$$

$$\boxed{R = \left(\frac{P}{Q}\right)S}$$

Kelvin's double bridge: (Measurement of low Resistance).

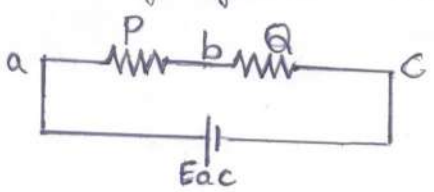


p and q Selected as $\frac{P}{Q} = \frac{p}{q} \rightarrow (1)$

under balanced condition current through galvanometer is zero

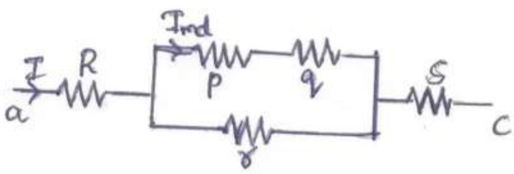
$$E_{ab} = E_{amd} \rightarrow (2)$$

$$E_{ab} = E_{ac} \times \frac{P}{P+Q} \rightarrow (3)$$



$$E_{ac} = IR + IS + I\left[(p+q)//r\right]$$

$$E_{ac} = IR + IS + I\left[\frac{(p+q)r}{p+q+r}\right] \rightarrow (4)$$



Substitute eqn (4) in eqn (2)

$$E_{ab} = I\left[R+S+\frac{(p+q)r}{p+q+r}\right] \times \frac{P}{P+Q} \rightarrow (5)$$

$$I_{md} = I\,\frac{r}{p+q+r}$$

$$E_{amd} = IR + I_{md}\,p$$

$$= IR + pI\left[\frac{r}{r+p+q}\right]$$

$$= I\left[R + \frac{pr}{r+p+q}\right] \rightarrow (6)$$

eqn (5) = eqn (6)

$$\frac{P}{P+Q}\left[R+S+\frac{(p+q)r}{p+q+r}\right] = R + \frac{pr}{p+q+r}$$

$$R+S+\frac{(p+q)r}{p+q+r} = \frac{P+Q}{P}\left[R+\frac{pr}{p+q+r}\right]$$

$$R+S+\frac{(p+q)r}{p+q+r} = \left[1+\frac{Q}{P}\right]\left[R+\frac{pr}{p+q+r}\right]$$

$$R+S+\frac{(p+q)r}{p+q+r} = R + \frac{pr}{p+q+r} + \frac{QR}{P} + \frac{Q}{P}\left[\frac{pr}{p+q+r}\right]$$

$$S = \frac{QR}{P} + \frac{Q}{P}\left(\frac{pr}{p+q+r}\right) + \frac{pr}{p+q+r} - \frac{(p+q)r}{p+q+r}$$

$$S = \frac{QR}{P} + \frac{Q}{P}\left[\frac{pr}{p+q+r}\right] - \frac{qr}{p+q+r}$$

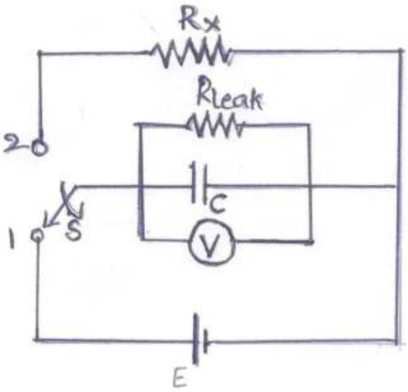
$$S = \frac{QR}{P} + \frac{pr}{p+q+r}\left[\cancel{\frac{Q}{P} - \frac{q}{p}}\right]^{\rightarrow 0} \rightarrow (7)$$

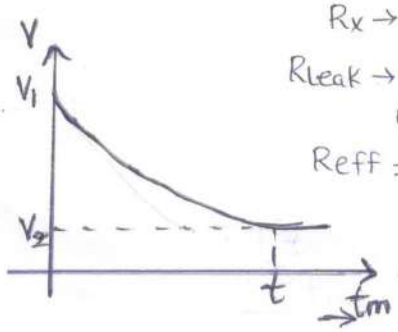
$$\frac{Q}{P} = \frac{q}{p} \text{ from } (1)$$

eqn (7) $\Rightarrow S = \frac{QR}{P}$

$$\boxed{R = \frac{PS}{Q}} \Rightarrow \boxed{PS = RQ}$$

Loss of charge method:





C → known Capacitor

R_x → unknown ~~Ca~~ Resistor

R_{leak} → Total leakage Resistance of Capacitor

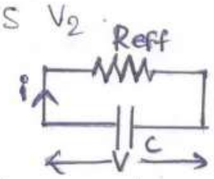
$R_{eff} = \cancel{\frac{R_{leak}}{R_x}} = \frac{R_{leak} + R_x}{R_{leak} \times R_x}$

→ when switch is at position 2, R_x and R_{leak} are in parallel

It is used to measure resistance of insulating material or leakage Resistance of Capacitor.

If switch is in position 1 charged to voltage V_1.

Switch is ~~swit plan~~ turned to position 2 and at time t voltage is V_2.



$$i = -\frac{dv}{dt} = -C\frac{dv}{dt} \rightarrow ①$$

$$i = \frac{v}{R_{eff}} \rightarrow ②$$

$$-C\frac{dv}{dt} = \frac{v}{R_{eff}}$$

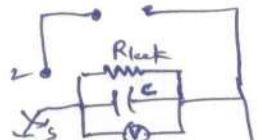
$$\frac{dv}{v} = \frac{-dt}{C R_{eff}}$$

Integrating on both sides

$$\int_{V_1}^{V_2} \frac{dv}{v} = \int_0^t \frac{-dt}{C R_{eff}}$$

$$(\ln v)_{V_1}^{V_2} = \frac{-t}{R_{eff}} \Rightarrow \ln V_2 - \ln V_1 = \frac{-t}{C R_{eff}} \Rightarrow \ln\left(\frac{V_2}{V_1}\right) = \frac{-t}{C R_{eff}}$$

$$\frac{V_2}{V_1} = e^{\left(\frac{-t}{C R_{eff}}\right)} \Rightarrow V_2 = V_1 e^{\left(\frac{-t}{C R_{eff}}\right)} \rightarrow ③$$



V_2, V_1, t and C are known parameters. only R_{eff} is unknown. To find R_{eff} →

→ Remove R_x and follow the same procedure

$$V_2 = V_1 e^{-(t_2/C R_{leak})} \quad [\because R_{eff} = R_{leak}]$$ → from this equation we know R_{leak} and we can find R_{eff}

V_2, V_1, t_2, C we know R_{leak} ~~fondamentals~~ find out

$$\frac{1}{R_{eff}} = \frac{1}{R_{leak}} + \frac{1}{R_x} \Rightarrow \boxed{\frac{1}{R_x} = \frac{1}{R_{eff}} - \frac{1}{R_{leak}}}$$

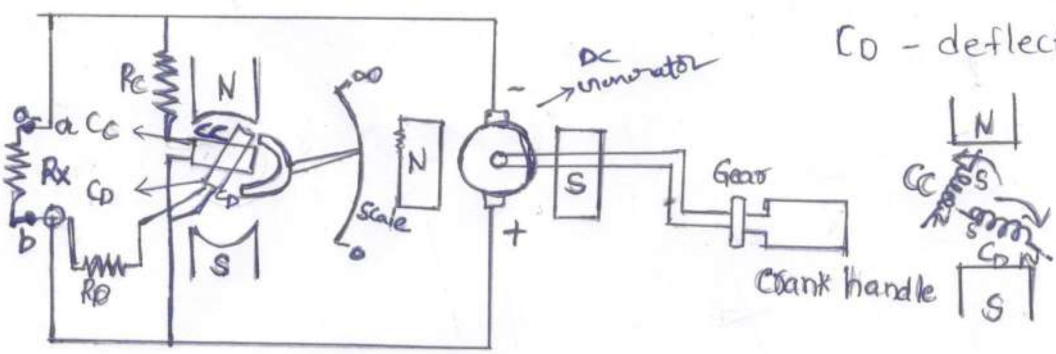
∴ unknown resistance R_x can be determined using expression

$$\boxed{V_2 = V_1 e^{-t_2/C R_{leak}}}$$

Mega ohmmeter (MEGGER):

CC - Control (or) voltage coil

CD - deflection (or) current coil



$C_C \to OC \to \infty$

$C_D \to SC \to 0$

→ Megger is used ~~for~~ to measure high Insulation Resistances.

→ Insulation Resistance of materials decreases with time due to electrical & Mechanical stresses, absorption of moisture, dust etc

→ Therefore Insulation resistance of materials is checked timely to avoid short circuits due to Insulation failure.

→ Because of leakage reactance reading will have some error Gaurd ring is used to compensate leakage resistance.

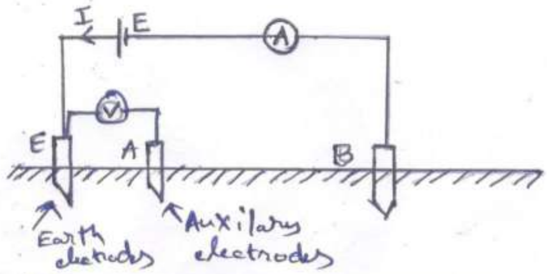
→ The Torque produced by both coils are opposite and align in a Position where torque produced by them is equal.

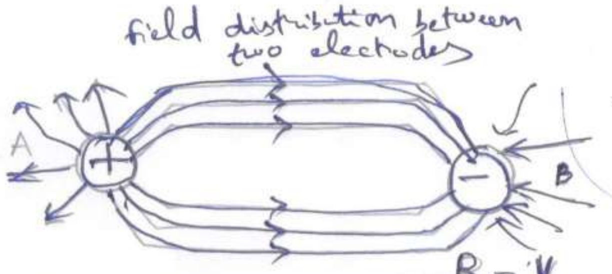
$T \propto$ ratio of current through coils ⇒ independent of voltage

→ when terminals a,b are short circuited which means deflection coil (CD) field dominates and it rotates in clockwise direction and shows "0" position of scale. which implies that $R_x = 0$.

→ when terminals a, b are open circuited which means control coil (CC) field dominates and it rotates in anticlockwise direction and shows "∞" position of scale. which implies that $R_x = \infty$

Earth Resistance

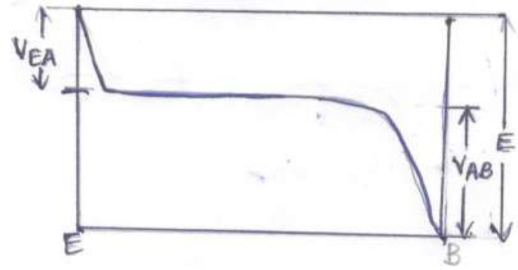




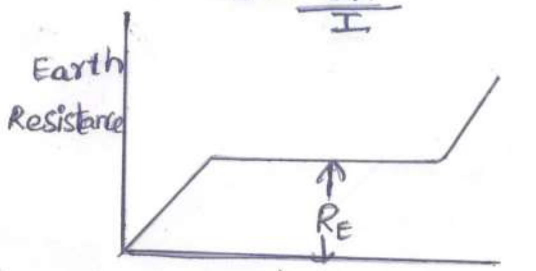
$R = \frac{V}{I}$

E- earth electrode

A - Auxilary electrode



$\therefore R_E = \frac{V_{EA}}{I}$



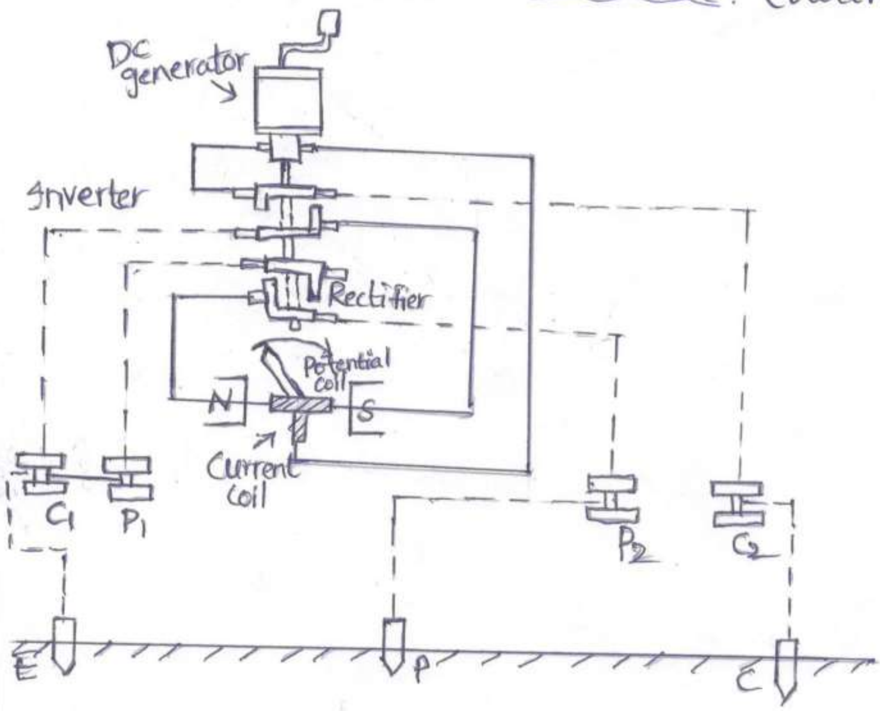
DC voltage is not used because of → electrolytic action → emf are induced → Errors

→ Earth resistance is not uniform and is varied nearer to the electrodes.

→ so while measuring earth resistance we need to take the reading where earth resistance is uniform.

→ we can do this by moving Auxilary electrode and take the reading of R_E.

Measurement of earth Resistance :- (Earth Tester)



Inverter converts DC — AC

Rectifier converts AC — DC

— line → DC

-- line → AC

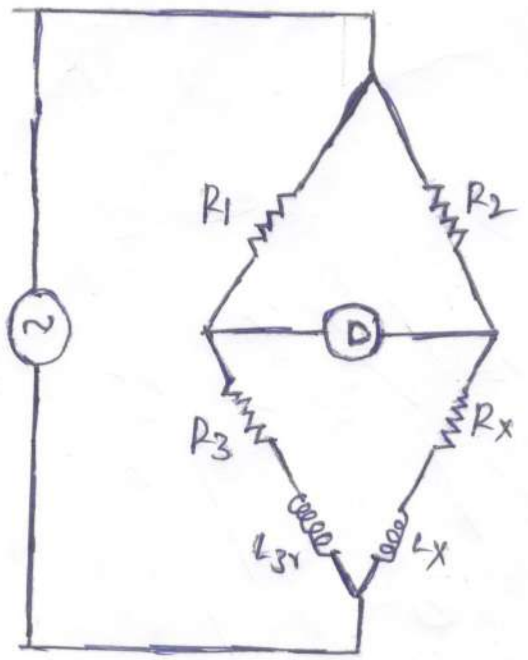
* It is basically a special type of megger having permanent magnet Dc generator with a inverter to supply AC to the electrodes and a rectifier to convert AC to DC to supply to ratio type ohm meter.

* Dc generator generates Dc and earth electrodes requires AC. So, invertor is used to supply AC to electrodes.

* For, measurement of resistance ratio type ohmmeter is used in megger. Ratio type ohmmeter needs DC. The AC current coming from electrodes is converted to DC by using rectifier.

MEASUREMENT OF INDUCTANCE

Maxwell Inductance Bridge



$Z_1 = R_1$

$Z_2 = R_2$

$Z_3 = (R_3 + r) + J\omega L_3$

$Z_4 = R_x + J\omega L_x$

$Z_1 Z_4 = Z_2 Z_3$

$R_1(R_x + J\omega L_x) = R_2(R_3 + r + J\omega L_3)$

$R_1 R_x + J R_1 L_x = R_2 R_3 + R_2 r + J R_2 L_3$

Equating real & imaginary parts

$R_1 R_x = R_2 (R_3 + r)$

$$\boxed{R_x = \frac{R_2 (R_3 + r)}{R_1}}$$

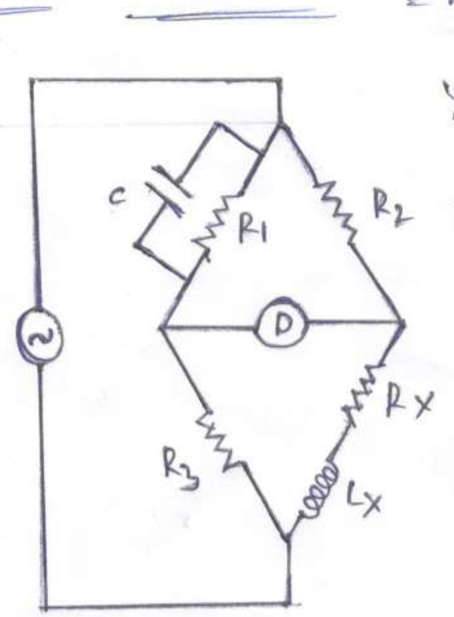
$R_1 L_x = R_2 L_3$

$L_x = \frac{R_2}{R_1} L_3$

$\cancel{\omega} L_x = \frac{R_2}{R_1} \cancel{\omega} L_3$

$$\boxed{L_x = \frac{R_2}{R_1} L_3}$$

Maxwell Inductance Capacitance Bridge.



$Y_1 = \frac{1}{R_1} + \frac{J}{X_c} \cdot \frac{1}{R_1} + J\omega C$

$Z_2 = R_2$

$Z_3 = R_3$

$Z_4 = R_x + J\omega L_x$

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_4 = \frac{Z_2 Z_3}{Z_1} = (Z_2 Z_3) \times Y_1 = R_2 R_3 \left(\frac{1}{R_1} + J\omega C\right)$$

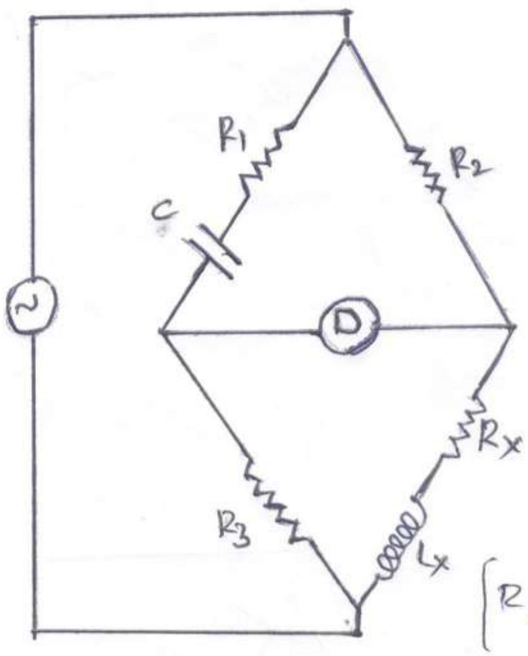
$$R_x + J\omega L_x = \frac{R_2 R_3}{R_1} + J R_2 R_3 \omega C$$

$$\boxed{R_x = \frac{R_2 R_3}{R_1}}$$

$$\cancel{\omega} L_x = \cancel{\omega} R_2 R_3 C$$

$$\boxed{L_x = R_2 R_3 C}$$

Hay's Bridge.



$$Y_1 = R_1 - \frac{J}{X_C} = R_1 - J\omega C$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_4 = R_x + J\omega L_x$$

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_x = \frac{Z_2 Z_3}{Z_1} = Z_2 Z_3 (Y_1)$$

$$\left[R_1 - J\left(\frac{1}{\omega C_1}\right)\right]\left[R_x + J\omega L_x\right] = R_2 R_3$$

$$R_1 R_x + J\omega R_1 L_x - \frac{J R_x}{\omega C_1} + \frac{L_x}{C_1} = R_2 R_3$$

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \quad \text{—} \ (1)$$

$$\omega R_1 L_x - \frac{R_x}{\omega C_1} = 0$$

$$L_x = R_x / \omega^2 C_1 R_1 \quad \text{—} \ (4)$$

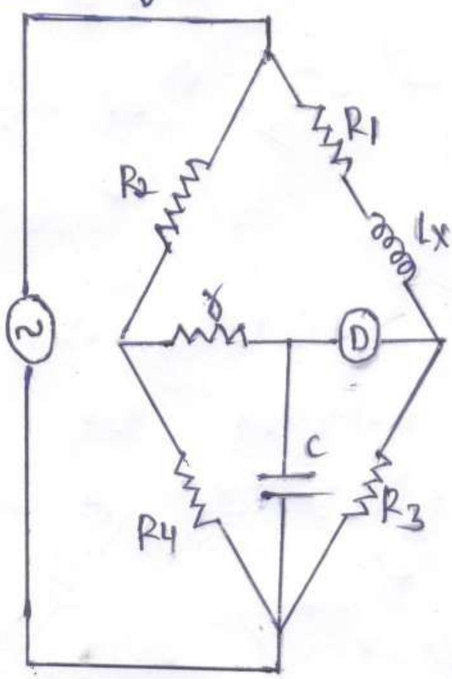
$$\boxed{L_x = \frac{R_2 R_3 C_1}{1+\omega^2 R_1^2 C_1^2}}$$

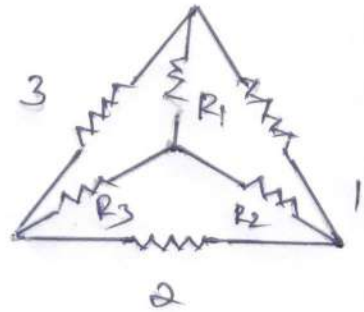
(4) in (1)

$$R_1 R_x + \frac{R_x}{\omega^2 C_1^2 R_1^2} = R_2 R_3$$

$$\boxed{R_x = \frac{R_2 R_3 \omega^2 R_1 C_1^2}{1+\omega^2 R_1^2 C_1^2}}$$

Andersson's Bridge.

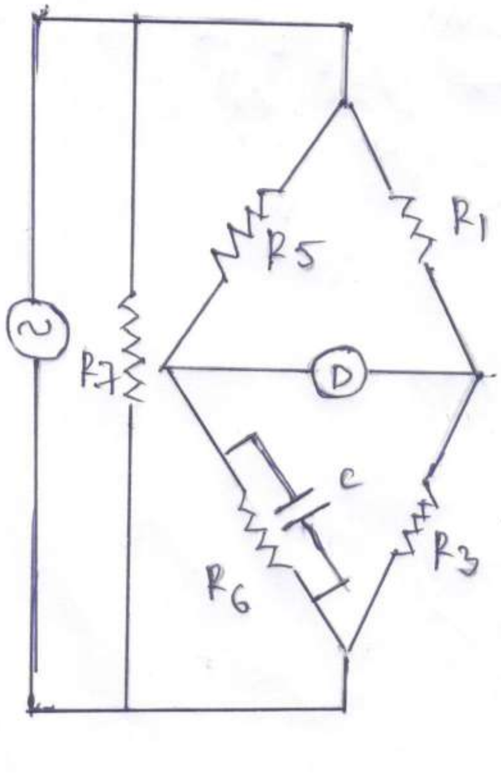




$$1 = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$2 = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$3 = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$



$$R_5 = \frac{R_2 r + r R_4 + R_4 R_2}{R_4}$$

$$R_6 = \frac{R_2 r + r R_4 + R_4 R_2}{R_2}$$

$$R_7 = \frac{R_2 r + r R_4 + R_4 R_2}{r}$$

$Z_1 = R_5$

$Z_2 = R_1 + j\omega_L$

$\frac{1}{Z_3} = Y_3 \Rightarrow \frac{1}{R_6} + j\omega_C$

$Z_4 = R_3$

$Z_1 Z_4 = Z_2 Z_3$

$R_5 R_3 = R_1 + j\omega L_x \; (Z_3)$

$R_1 + j\omega_L x = \frac{R_5 R_3}{Z_3}$

$R_1 + j\omega_L x = R_5 R_3 Y_3$

$R_1 + j\omega_L x = \frac{R_5 R_3}{R_6} + j R_5 R_3 \omega_c$

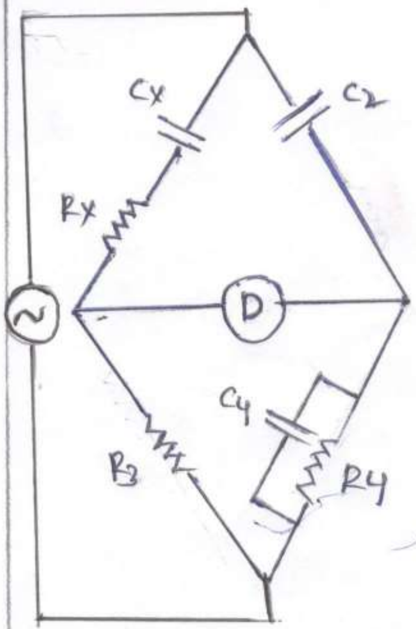
$\boxed{R_1 = \frac{R_5 R_3}{R_6}} \longrightarrow$ By Equating Real terms

$\cancel{\omega} L_x = R_5 R_3 \cancel{\omega} C$

$L_x = R_5 R_3 C$

MEASUREMENT OF ~~INDUCTANCE~~ CAPACITANCE

Schering Bridge



$Z_1 = R_x - \frac{J}{\omega C_x}$

$Z_2 = \frac{-J}{\omega C_2}$

$Z_3 = R_3$

$Y_4 = \frac{1}{R_4} + J\omega C_4$

$Z_1 Z_4 = Z_2 Z_3$

$Z_1 = \frac{Z_2 Z_3}{Z_4} = Z_2 Z_3 Y_4$

$R_x - \frac{J}{\omega C_x} = -\frac{J}{\omega C_2}(R_3)\left(\frac{1}{R_4} + J\omega C_4\right)$

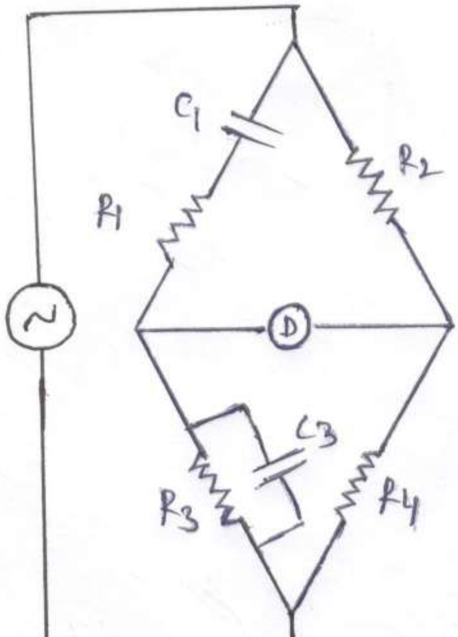
$= \frac{-JR_3}{\omega C_2 R_4} + \frac{\cancel{\omega} C_4 R_3}{\cancel{\omega} C_2}$

$\boxed{R_x = R_3 \frac{C_4}{C_2}}$

$\frac{1}{\cancel{\omega} C_x} = \frac{R_3}{\cancel{\omega} C_2 R_4}$

$\boxed{C_x = C_2 \frac{R_4}{R_3}}$

Weins Bridge



$Z_1 = R_1 - \frac{J}{\omega C_1}$

$Z_2 = R_2$

$Y_3 = \frac{1}{R_3} + J\omega C_3$

$Z_4 = R_4$

$Z_1 Z_4 = Z_2 Z_3$

$Z_2 = \frac{Z_1 Z_4}{Z_3} = Z_1 Z_4 Y_3$

$R_2 = \left(R_1 - \frac{J}{\omega C_1}\right) R_4 \left(\frac{1}{R_3} + J\omega C_3\right)$

$= \left(R_1 R_4 - \frac{JR_4}{\omega C_1}\right)\left(\frac{1}{R_3} + J\omega C_3\right)$

$R_2 = \frac{R_1 R_4}{R_3} + J\omega R_1 R_4 C_3 - \frac{JR_4}{\omega R_3 C_1} + \frac{\cancel{\omega} R_4 C_3}{\cancel{\omega} C_1}$

$$R_2 = \frac{R_1R_4}{R_3} + J\cancel{\omega R_1R_4} \quad \frac{R_4C_3}{C_1} + \omega R_1R_4C_3 - \frac{R_4}{\omega R_3C_1} = 0$$

$$\omega R_1R_4C_3 = \frac{R_4}{\omega R_3C_1}$$

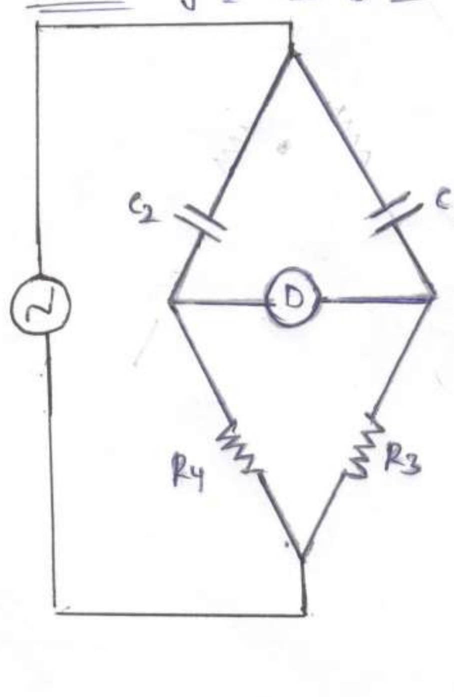
$$\omega^2 = \frac{\cancel{R_4}}{R_1\cancel{R_4}C_3R_3C_1}$$

$$\omega^2 = \frac{1}{R_1R_3C_1C_3}$$

$$\omega = \frac{1}{\sqrt{R_1R_3C_1C_3}}$$

$$\boxed{F = \frac{1}{2\pi\sqrt{R_1R_3C_1C_3}}}$$

Desautys Bridge



$$Z_1 = \frac{-J}{\omega C_2}$$

$$Z_2 = \frac{-J}{\omega C_1}$$

$$Z_3 = R_4$$

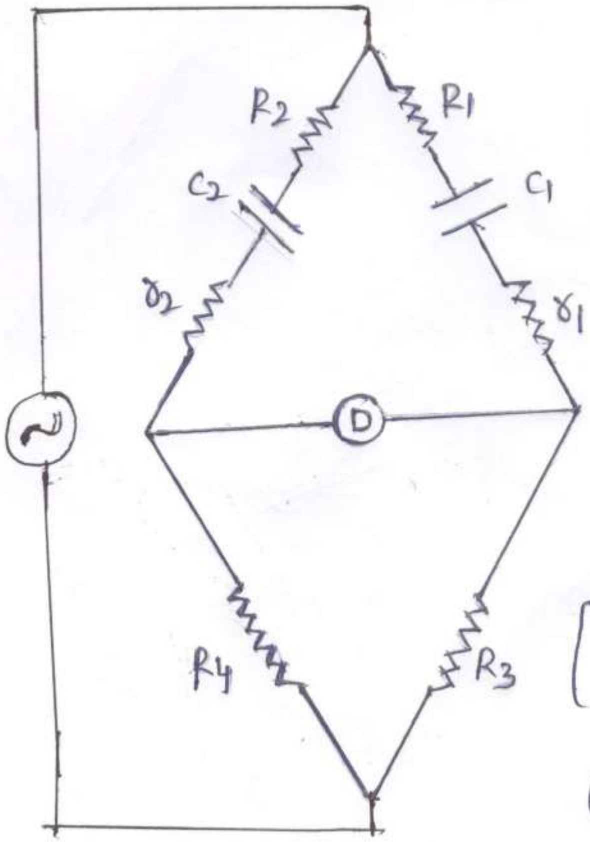
$$Z_4 = R_3$$

$$Z_1Z_4 = Z_2Z_3$$

$$\frac{-\cancel{J}R_3}{\cancel{\omega}C_2} = \frac{-\cancel{J}R_4}{\cancel{\omega}C_1}$$

$$\frac{C_1}{C_2} = \frac{R_4}{R_3}$$

Modified Desauty's Bridge



$$Z_1 = (R_2 + r_2) - \frac{J}{\omega C_2}$$

$$Z_2 = (R_1 + r_1) - \frac{J}{\omega C_1}$$

$$Z_3 = R_4$$

$$Z_4 = R_3$$

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left[(R_2 + r_2) - \frac{J}{\omega C_2}\right] R_3 = \left[(R_1 + r_1) - \frac{J}{\omega C_1}\right] R_4$$

$$(R_2 + r_2) R_3 - \frac{J R_3}{\omega C_2} = (R_1 + r_1) R_4 - \frac{J R_4}{\omega C_1}$$

$$(R_2 + r_2) R_3 = (R_1 + r_1) R_4$$

$$\frac{R_4}{R_3} = \frac{R_2 + r_2}{R_1 + r_1}$$

$$\frac{J R_3}{\omega C_2} = \frac{J R_4}{\omega C_1}$$

$$\frac{R_3}{C_2} = \frac{R_4}{C_1}$$

$$\frac{R_4}{R_3} = \frac{C_1}{C_2}.$$

$$\boxed{\therefore \frac{R_4}{R_3} = \frac{C_1}{C_2} = \frac{R_2 + r_2}{R_1 + r_1}}$$

1. An ac bridge was made up as follows: arm ab, a capacitor of 0.8 μF in parallel with 1 kΩ resistance, bc a resistance of 3 kΩ, arm cd an unknown capacitor C_x and R_x in series, arm da a capacitance of 0.4 μF. The supply at 1 kHz is connected across bd and a detector across ac. Determine the value of unknown capacitance C_x, unknown series resistance R_x and dissipation factor

Sol: Given

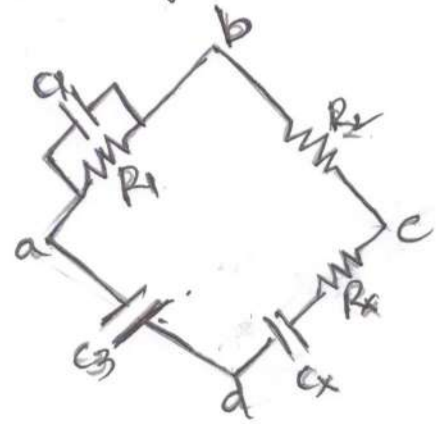
$R_1 = 1\,k\Omega$

$C_1 = 0.8\,\mu F$

$R_2 = 3\,k\Omega$

$C_3 = 0.4\,\mu F$

$f = 1\,KHz$



$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1} + J\omega C_1$$

$$Z_2 = R_2$$

$$Z_3 = \frac{-J}{\omega C_3}$$

$$Z_4 = R_x - \frac{J}{\omega C_x}$$

at balance condition, $Z_1 Z_4 = Z_2 Z_3$

$$Z_4 = \frac{Z_2 Z_3}{Z_1}$$

$$R_x - \frac{J}{\omega C_x} = R_2\left(\frac{-J}{\omega C_3}\right)\left(\frac{1}{R_1} + J\omega C_1\right)$$

$$R_x - \frac{J}{\omega C_x} = \frac{-JR_2}{\omega R_1 C_3} + \frac{R_2 \omega C_1}{\omega C_3}$$

$$R_x - \frac{J}{\omega C_x} = \frac{R_2 C_1}{C_3} - \frac{JR_2}{\omega R_1 C_3}$$

Comparing real & imaginary parts

$$\boxed{R_x = \frac{R_2 C_1}{C_3}}$$

$$\frac{1}{\omega C_x} = \frac{R_2}{\omega R_1 C_3}$$

$$\boxed{C_x = \frac{R_1 C_3}{R_2}}$$

$$C_x = \frac{1000 \times 0.4 \times 10^{-6}}{3000}$$

$$\boxed{C_x = 0.1333 \times 10^{-6} F}$$

$$R_x = \frac{3000 \times 0.8 \times 10^{-6}}{0.4 \times 10^{-6}}$$

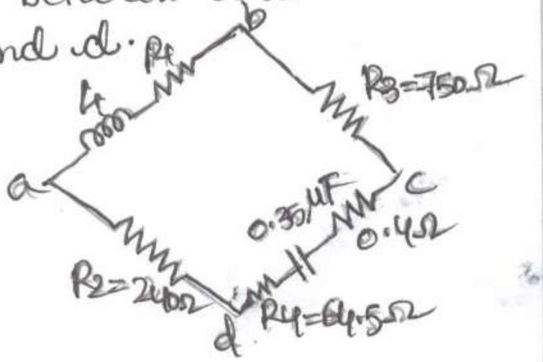
$$\boxed{R_x = 6000\,\Omega \text{ or } 6\,k\Omega}$$

Dissipation factor $D = \omega C_x R_x = 2\pi f C_x R_x = 2 \times 3.14 \times 1 \times 10^3 \times 0.133 \times 10^{-6} \times 6 \times 10^3$

$$\boxed{D = 5.011}$$

If Quality factor asked $= \frac{1}{D}$ or $\frac{X_C}{R}$

2). A bridge consists of arm ab, a choke coil having a resistance R_1 and inductance L_1. arm bc a non-inductive resistance R_3. when this bridge is fed from a source of 500Hz, balance is obtained under following conditions: $R_2 = 2410\Omega$, $R_3 = 750\Omega$ $C_4 = 0.35\mu F$, $R_4 = 64.5\Omega$. The series resistance of capacitance is $= 0.4\Omega$. calculate the resistance and inductance of the choke coil. The supply is connected between a and c and the detector is between b and d.



Sol:- $f = 500Hz$

$Z_1 = R_1 + J\omega L_1 = R_1 + J\,3140L_1$

$Z_2 = 2410\Omega$

$Z_3 = 750\Omega$

$Z_4 = 64.5 + 0.4 - JX_C$

$= 64.9 - \dfrac{J}{2\pi f \times 0.35 \times 10^{-6}} = 64.9 - \dfrac{J}{2 \times 3.14 \times 500 \times 0.35 \times 10^{-6}}$

$Z_4 = 64.9 - j909.918$

$Z_1 Z_4 = Z_2 Z_3$

$(R_1 + J\,3140L_1)(64.9 - J909.918) = 2410 \times 750$

$64.9R_1 - J909.918R_1 + J203786L_1 + 2857142.524 = 1807500$

$64.9R_1 + 2857142.524 = 1807500$ — ①

$-J909.918R_1 + J203786L_1 = 0$

$909.918R_1 - 203786L_1 = 0$ — ②

By Solving ① & ② we get

$R_1 = 141.1\Omega$

$L_1 = 0.63H$

3). The four arms of a bridge are:

arm ab : an imperfect capacitor C_1 with an Equivalent series resistor of r_1

arm bc : an non-inductive resistance R_3

arm cd : an non-inductive resistance R_4

arm da : an imperfect capacitor C_2 with an Equivalent Series resistance of r_2 Series with a resistance R_2

A Supply of 450HZ is given between terminals a and c detector is connected between b and d. At balance: $R_2 = 4.8$ ohms, $R_3 = 2000$ Ohms, $R_4 = 2850$ Ohms and $C_2 = 0.5\mu F$ and $r_2 = 0.4$ohms. Calculate the value of C_1 and r_1 and also the dissipating factor for this capacitor.

Sol:

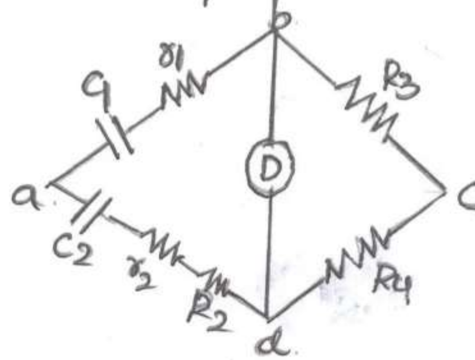
$R_2 = 4.8\Omega$

$R_3 = 2000\Omega$

$R_4 = 2850\Omega$

$C_2 = 0.5\mu F$

$r_2 = 0.4\Omega$



$$Z_1 Z_4 = Z_2 Z_3$$

$$\left(r_1 + \frac{1}{j\omega C_1}\right) R_4 = \left(r_2 + R_2 + \frac{1}{j\omega C_2}\right) R_3$$

Equating real Components

$$r_1 R_4 = r_2 R_3 + R_2 R_3$$

$$r_1 = (r_2 + R_2)\frac{R_3}{R_4}$$

$$r_1 = (0.4 + 0.8)\left(\frac{2000}{2850}\right)$$

$$\boxed{r_1 = 3.65\Omega}$$

Equating imaginary Components

$$\frac{R_4}{\cancel{j\omega} C_1} = \frac{R_3}{\cancel{j\omega} C_2}$$

$$C_1 = C_2 \frac{R_4}{R_3}$$

$$= (0.05 \times 10^{-6}) \times \frac{2850}{2000}$$

$$\boxed{C_1 = 0.712F}$$

Dissipating factor $D = \omega C_1 r_1$

$$= 2\pi \times 450 \times 0.712 \times 10^{-6} \times 3.65$$

$$= 0.00734$$

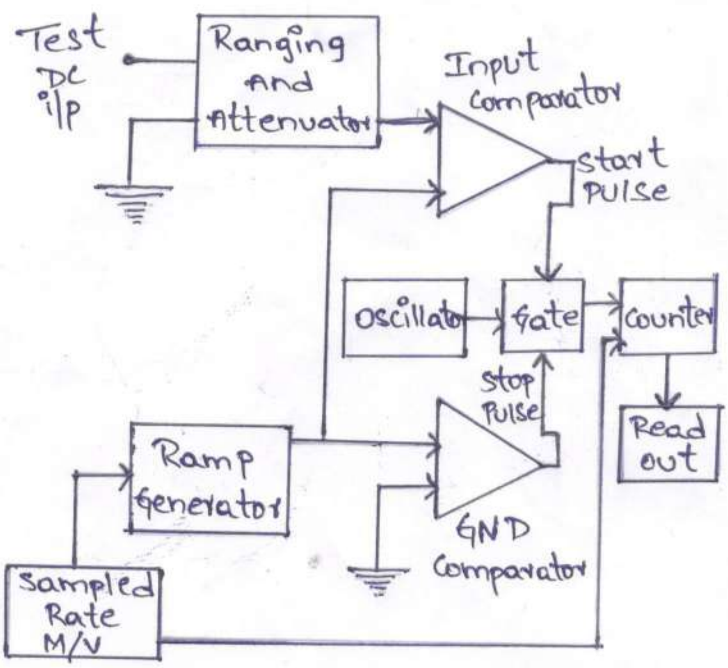
$$\boxed{D = 7.34 \times 10^{-3}}$$

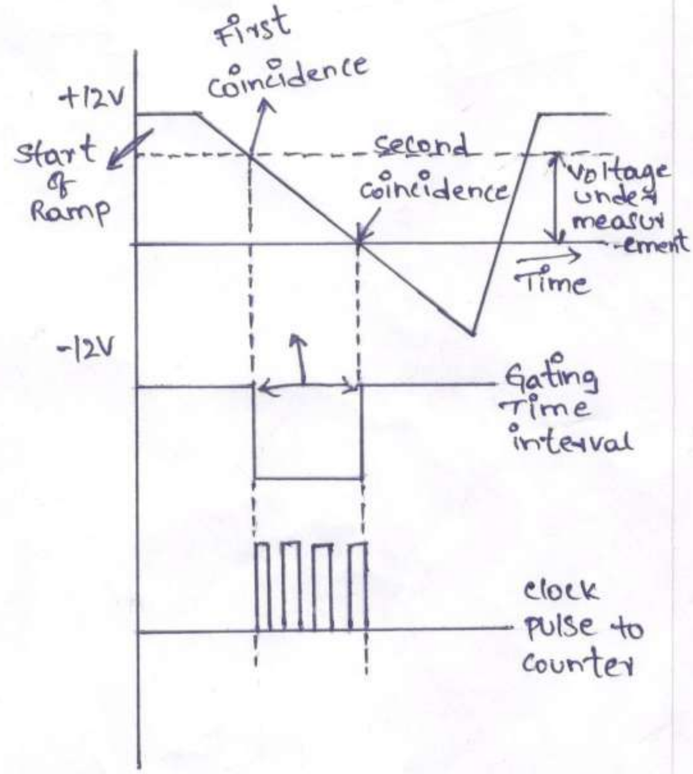
Unit - 5

Digital Voltmeters :-

- → Ramp type DVM
- → Integrating DVM
- → Successive approximation DVM
- → Continuous Balance DVM

⊛ Ramp Type DVM





Basic principle:- Based on the measurement of the time taken by a linear ramp to decrease from the level of the input voltage to zero voltage.

→ At First coincidence point ramp value is equal to voltage to be measured

→ Second Coincidence point is where ramp value is zero

→ At first coincidence point gate corresponding to counter is opened and at second coincidence point gate is closed

→ By measuring, this time interval we can measure the unknown voltage.

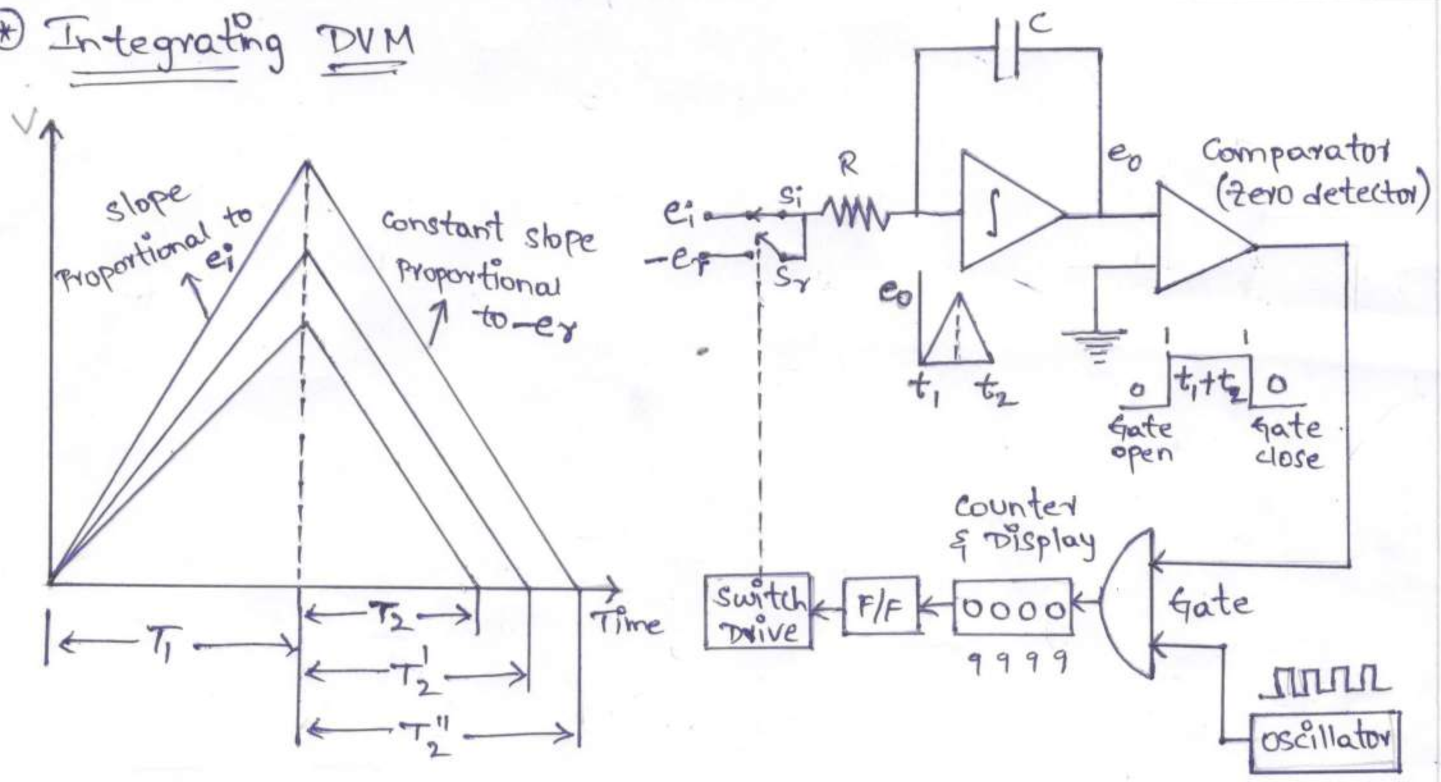
→ At the point when negative going ramp becomes equal to the input voltage, the comparator sends start pulse due to which gate opens.

→ Oscillator sends pulses to the counter and the counter starts counting the pulses when ramp is equal to ground voltage (0V) the ground comparator sends stop pulse to the gate and gate gets closed.

→ A definite no of pulses will be counted between start and stop pulses which is a measure of the input voltage and displayed in digital readout.

→ Sampled rate multivibrator determines the rate at which measurement cycles are initiated.

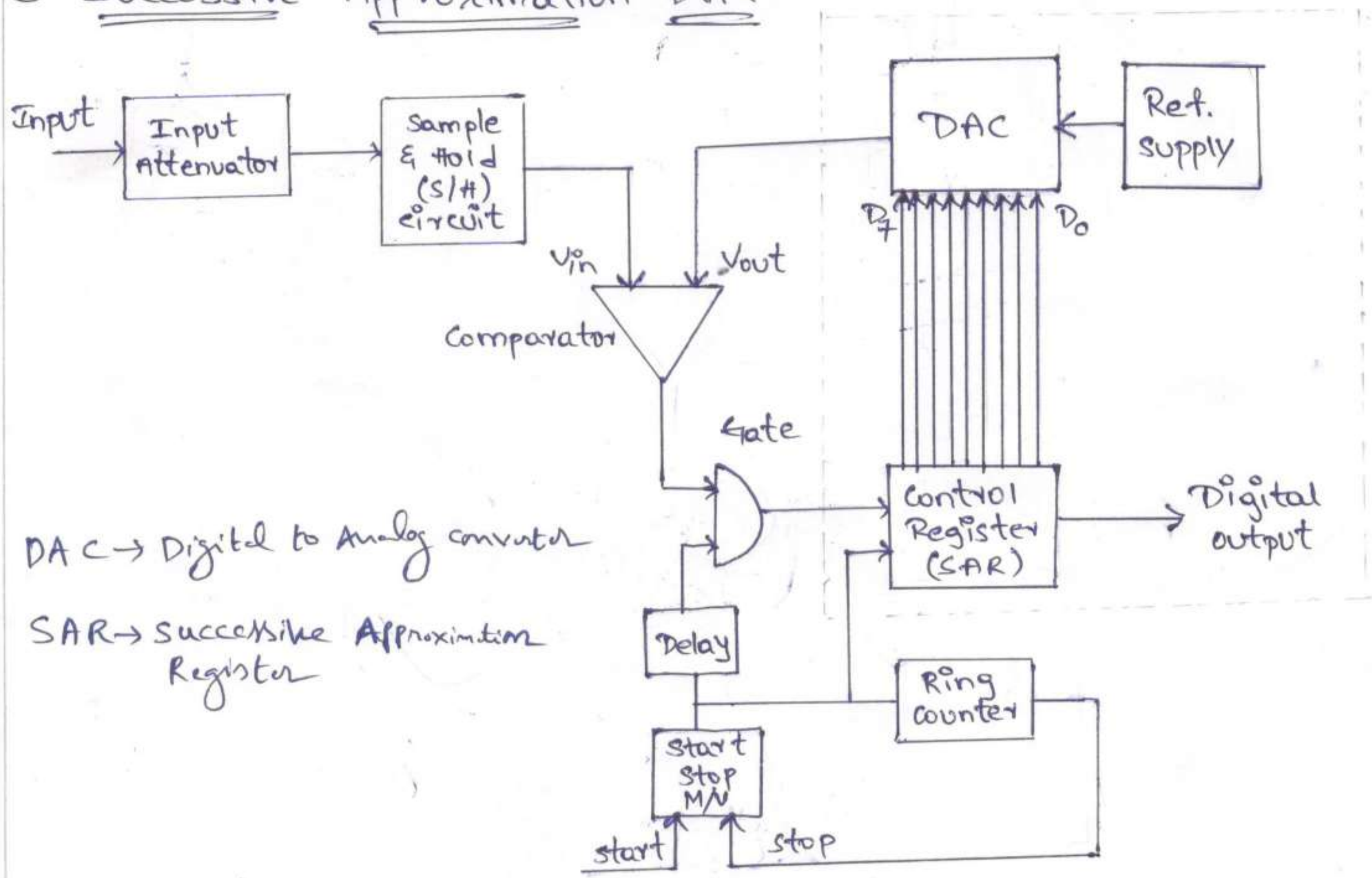
⊛ Integrating DVM



→ First oscillator sends a pulse in such a way that counter display and flip flop are set to logic "0" state and the switch drive sends a signal to S_1. Then S_1 is connected to i/p voltage e_i. In this condition the capacitor gets charged for time "T_1" and gives positive slope and this is applied to zero detector. It will detect zero point and gets activated and sends a signal to gate to open. As soon as gate opens pulses are sent and counter counts upto 9999. After this point counter is reset to "0000" and flip flop attains a state logic "1". This time switch drive closes switch "S_r". Now the capacitor starts discharging and it discharges upto a point where it touches

"0" position. In this position zero detector gets activated and sends to signal to gate and gate gets closed. The gate is opened for a total time period of (t_1+t_2). The counter displays a value which is corresponding to input voltage

Successive Approximation DVM



DAC → Digital to Analog convertor

SAR → Successive Approximation Register

V_{in}	opration	D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0		o/p
00110011 (51 < 128)	D_7 set (MSB)	1	0	0	0	0	0	0	0	$V_{in} < V_{out}$	D_7 Reset
00110011 (51 < 64)	D_6 set	0	1	0	0	0	0	0	0	$V_{in} < V_{out}$	D_6 Reset
00110011 (51 > 32)	D_5 set	0	0	1	0	0	0	0	0	$V_{in} > V_{out}$	D_5 set
00110011 (51 > 48)	D_4 set	0	0	1	1	0	0	0	0	$V_{in} > V_{out}$	D_4 set
00110011 (51 < 56)	D_3 set	0	0	1	1	1	0	0	0	$V_{in} < V_{out}$	D_3 Reset
00110011 (51 < 52)	D_2 set	0	0	1	1	0	1	0	0	$V_{in} < V_{out}$	D_2 Reset
00110011 (51 > 50)	D_1 set	0	0	1	1	0	0	1	0	$V_{in} > V_{out}$	D_1 set
00110011 (51 = 55)	D_0 set	0	0	1	1	0	0	1	1	$V_{in} = V_{out}$	D_0 set

It works on the principle of balancing the weights as in a simple balance

→ Initially a start pulse triggers the start/stop multivibrator.

→ The multivibrator provides clock input to ring counter. The clock pulse activates the ring counter which sets the MSB (most significant bit D_7) of SAR.

→ The SAR converts 10000000 into analog voltage by DAC (V_{out})

→ This V_{out} is compared with V_{in} and produces "0" or "1" depending on Magnitude.

→ If $V_{in} > V_{out}$ output is positive and D_7 is set to 1

→ If $V_{in} < V_{out}$ output is negative and D_7 is set to 0.

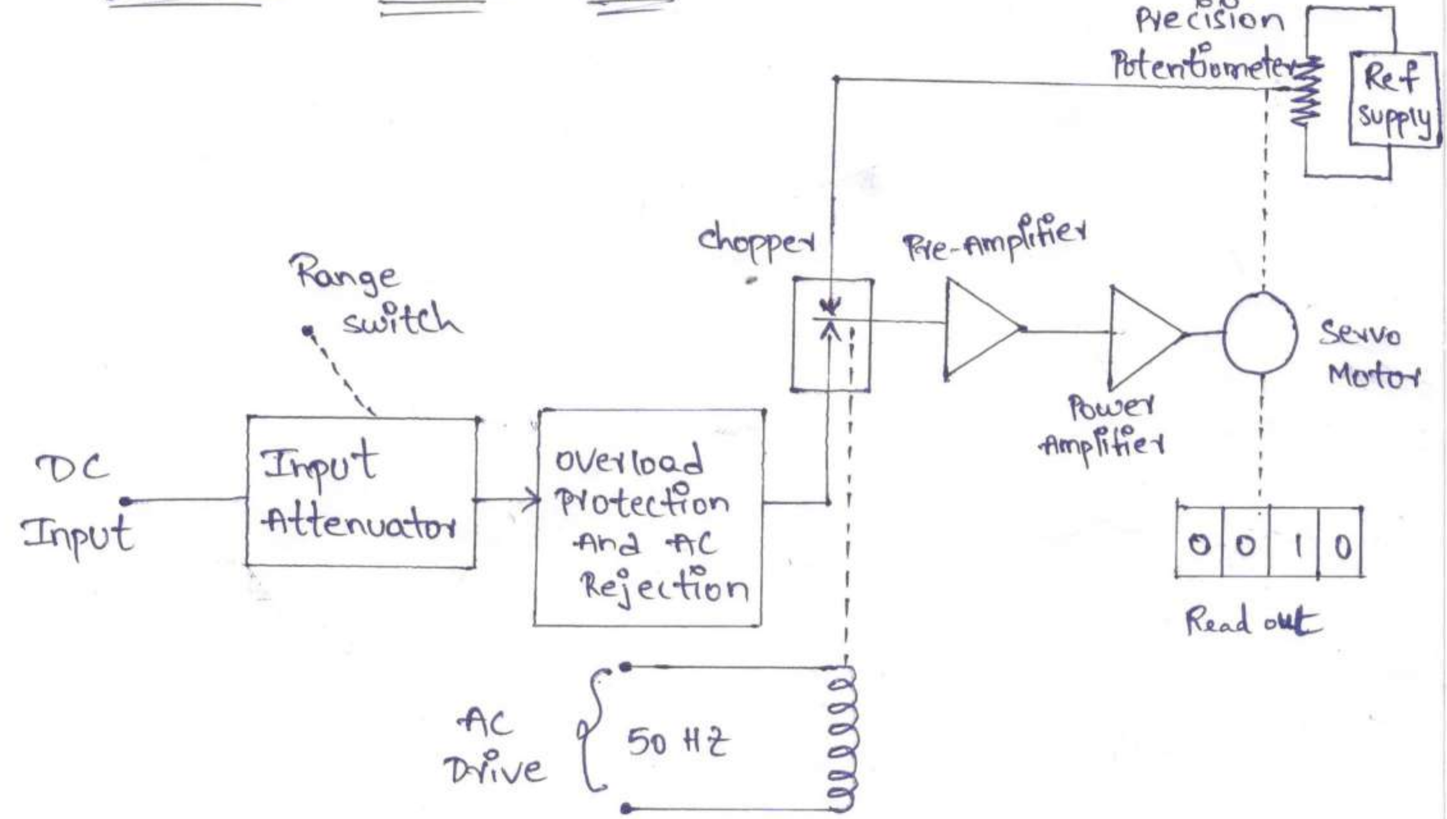
→ Similarly the bits beginning from D_7 to D_0 are set, reset and tested.

→ The measurement completes in 8 clock pulses.

<u>Advantages</u>:-
1) High speed measurement
2) This DVM takes "n" clock pulses to produce "n" bit result.
3) This voltmeter capable of producing 1000 readings per second.

<u>Disadvantages</u>:-
The control circuit is more complex in design and is costlier.

(*) Continuous Balance DVM :

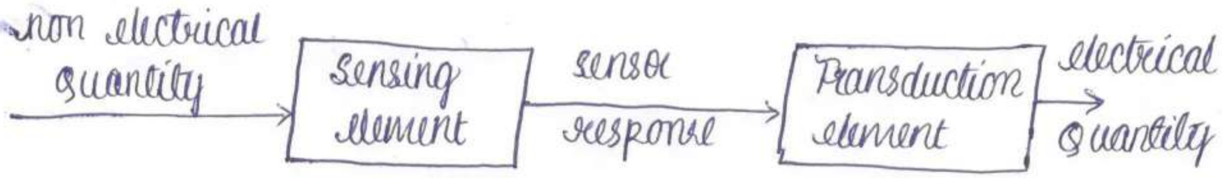


It mainly consists of a Mechanical chopper the chopper acts as a comparator and compares Voltage to be measured and reference voltage. After comparision the output is driven by ac drive at line frequency. A square wave signal is generated and given to pre amplifier. This signal is passed through power and amplifier and these pulses are given to ~~servo~~ servomotor. The servometer will drive the potentiometer arm in such a way that the difference between two voltages becomes zero. The corresponding reading proportional to voltage to be measured is displayed in the Read out

Input alternator is used to raise the voltage or reduce the voltages. overload protection and ac rejection is used remove the ripples which are generated in input alternator.

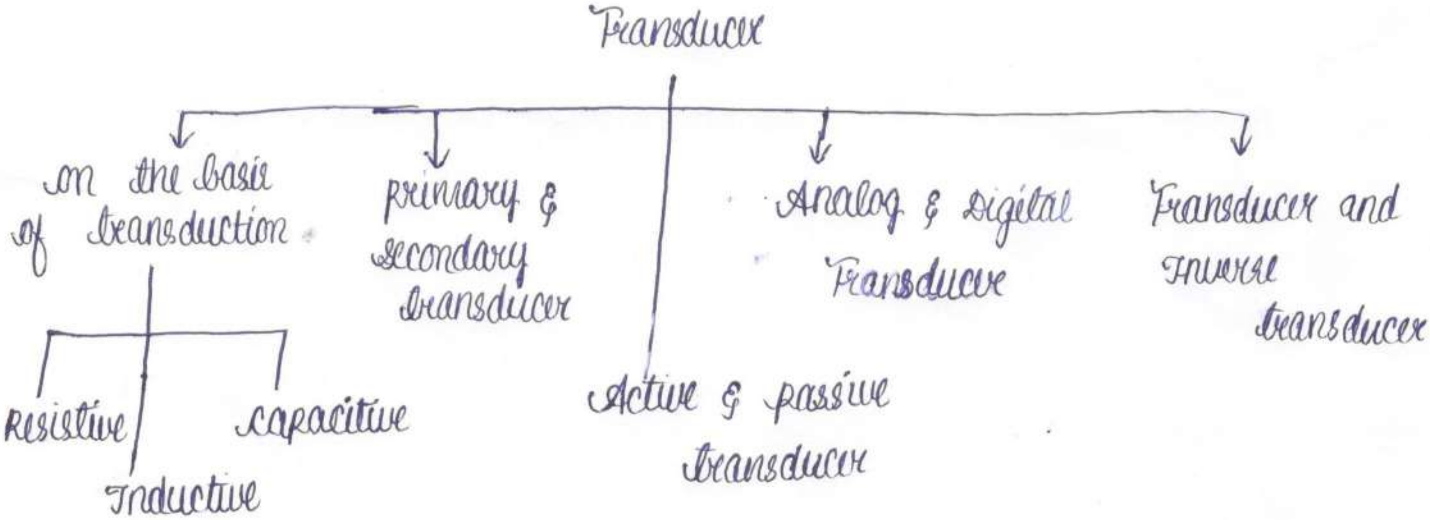
Transducer :-

The transducer changes the physical quantity into an electrical signal. It is an electronic device which has two main functions i.e, sensing and transduction. It senses the physical quantity and then converts it into electrical signals.



Classification :

The transducer is of many types, they are divided into



1. classification based on the principle of transduction :-

The transducer is divided by transduction medium. The transduction medium may be resistive, inductive or capacitive depends on the conversion process that how input transducer converts the i/p signal into resistive, inductance & capacitance resp.

2. primary & secondary transducer.

primary Transducer :- The transducer consists the mechanical as as the electrical devices. The mechanical devices of the transducer change the physical iput quantities into a mechanical signal. The mechanical device is known as primary transducer

Secondary transducer :- The secondary transducer converts the mechanical signal into an electrical signal. The magnitude of the output signal depends on the i/p mechanical signal.

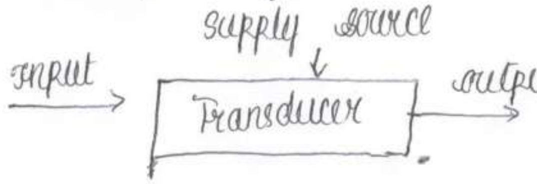
Example :- Suppose you need to measure pressure. In this case we use bourdon tube as primary transducer it senses the pressure and converts pressure into displacement of its free end. The displacement of free end moves core of LVDT which produces o/p voltage proportional to movement of core which is proportional to movement of core which is again proportional to pressure. So we are able to measure pressure. Here bourdon tube is primary transducer & LVDT is secondary transducer.

3. Passive and Active transducer.

Passive transducer - The transducer which requires the power from an external supply source is known as the passive transducer. They are also known as the external power transducer. The capacitive, resistive & inductive transducers are the example of the passive transducer.

Input → Transducer → Output

Active transducer :- The transducer which does not require the external power source is known as the active transducer. Such type of transducer develops their owns voltage or current, hence known as a self - generating transducer. The o/p signal is obtained from the physical input quantity. The piezoelectric crystal, photo-voltaic cell, tacho generator, thermocouples, photo voltaic cell are the examples of the active transducers.



4. Analog & Digital transducer

The transducer can also be classified by their O/P signals. The O/P signal of the transducer may be continuous or discrete.

Analog transducer -
These type of transducers which convert input signal into O/P in the form of a continuous function of time such as thermistor, LVDT, thermocouple etc.

~~Analog~~ Digital transducer :- These transducers convert an input quantity into a digital signal ~~into~~ or in the form of pulse. Here O/P is the form of square pulses & having two states (high & low). hence it is called a digital transducer.

5. Transducer & Inverse transducer :-

Transducer - The device which converts the non-electrical quantity into an electrical quantity is known as the transducer. ~~Here~~ ~~the~~ Example: position, Temperature, pressure etc are converted into electrical signals.

Inverse transducer :- The transducer which converts the electric quantity into a physical quantity such type of transducers is known as the inverse transducer. Example :- Piezo electric crystal. It converts electrical signal into a mechanical vibration.

→ Advantages of Electrical Transducer

Mostly Quantities to be Measured are non-electrical such as Temperature, pressure, displacement, humidity, fluid flow, speed etc. But these quantities cannot be measured directly. Hence such quantities are required to be sensed & changed into electrical quantities first & then measured. The function of converting non-electrical quantity into electrical one is accomplished by a device called the Electrical Transducer.

The production of these signals is based upon electrical effects which may be resistive, inductive, capacitive etc in nature. Electrical transducer suffer due to some draw-backs too, such as low reliability in comparsion to that of Mechanical transducers due to the ageing & comparative high cost of electrical transducers and associated signal conditioners. In some cases the accuracy & resolution are not as high compared to Mechanical transducers.

Some of the Advantages are :

→ Electrical Amplification & attenuation can be done easily with a Static Device.

→ The Effect to friction is Minimited.

→ The Electric or electronic system can be controlled with a very small electric power.

→ The electric power can be easily used, transmitted & process for the purpose of Measurement.

Factor to be Considered while Selecting Transducer

→ It should have high Input impedance & low output impedance, to avoid loading effect. It should have good resolution over its entire selected range.

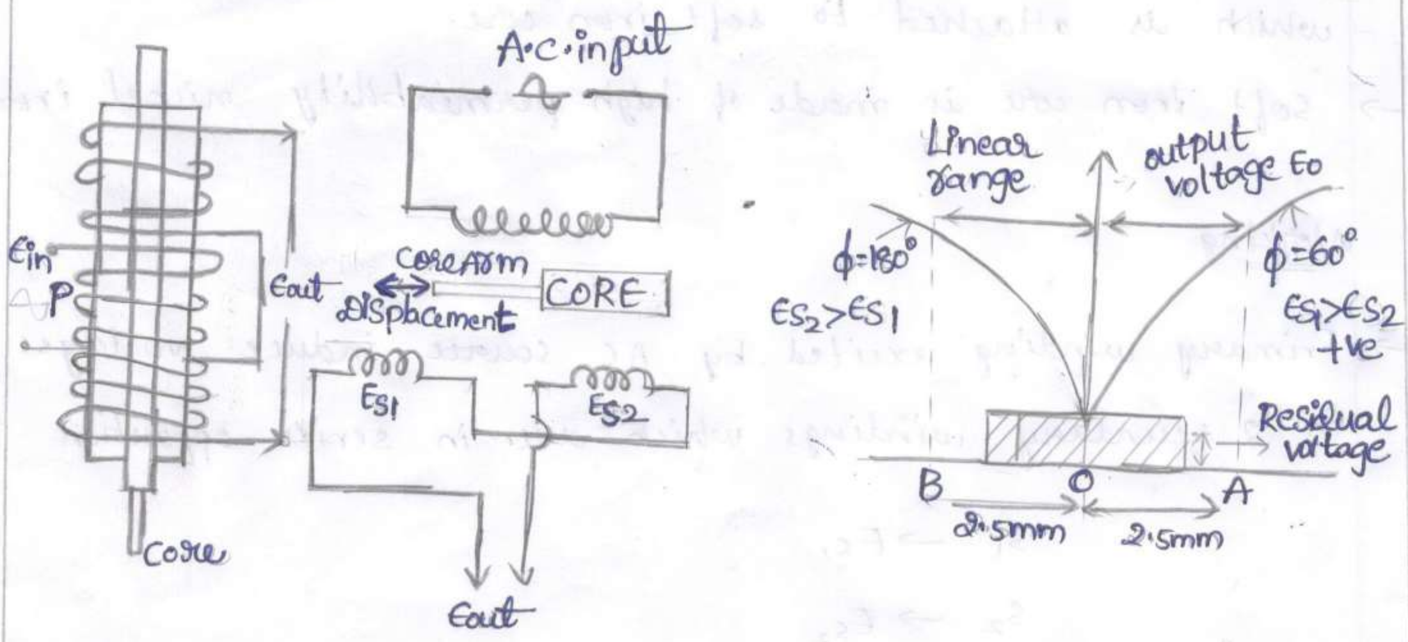
→ It must be highly sensitive to desired signal & insensitive to unwanted signal. preferable small in site.

- It should be able to work in corrosive environment.
- It should be able to withstand pressure, shock, vibrations etc.
- selected transducer must be free from errors.
- The Transducer circuit should have overload protection so that it will withstand overloads.

Requirements of a Good Transducers

- smaller in size & weight
- High Sensitivity
- Ability to withstand environmental conditions.
- Low cost.

LVDT (Linear Variable differential Transformer):-



⇒ It is most widely used inductive type transducer which is used to measure the displacement.

⇒ Displacement which is a non electrical quantity is measured in terms of voltage which is an electrical quantity.

⇒ It consists of 2 windings called primary and secondary windings. The output voltage is the difference between the voltages across secondary voltages.

⇒ LVDT converts the linear motion into electrical signals

⇒ The primary winding is P and secondary windings are S_1, S_2. The secondary windings are having equal no. of turns and are identically placed on either side of primary.

→ Displacement to be measured is applied to the arm which is attached to soft iron core.

→ Soft iron core is made of high permeability nickel iron

Working

→ Primary winding excited by AC source induce voltages in 2 secondary windings which are in series opposition.

$S_1 \rightarrow E_{S_1}$

$S_2 \rightarrow E_{S_2}$

output voltage $\boxed{E_0 = E_{S_1} - E_{S_2}}$

(i) when core is at null position $E_{S_1} = E_{S_2}$

$\therefore E_0 = 0$

(ii) when core is moved to left position

$E_{S_1} > E_{S_2}$

$E_0 = E_{S_1} - E_{S_2}$

$= \text{positive}$

(iii) when core is moved to right side

$E_{S_2} < E_{S_1}$

$E_0 = E_{S_1} - E_{S_2}$

$= \text{Negative}$

→ The amount of output voltage is measured to determine the displacement.

→ Output voltage of LVDT is a linear function of the core displacement within a limited range of motion 5mm from the null position.

Advantages:-

1) High range of displacement can be measured (1.25mm to 250mm)

2) Friction and electrical isolation → there is no physical contact between core and coils

3) Immunity from external effects like pressure, corrosive etc.

4) High input and high sensitivity

5) Ruggedness → can tolerate high shocks and vibrations.

6) LVDT shows low hysteresis effect.

7) Low power consumption.

Disadvantages:

1) Large displacements are difficult to measure

2) Sensitive to stray magnetic and electric fields

3) Dynamic response is slow

4) Temperature affects the performance.

Strain guage, guage factor:-

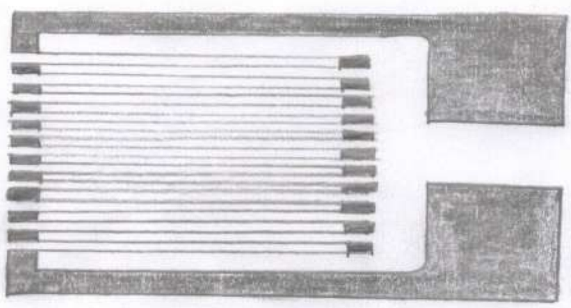
→ It is a device used to measure strain on an object.

→ The resistance of a strain guage varies with applied force and, it converts parameters such as force, pressure, fusion, weight etc into a change in resistance.

→ It is a passive and resistance transducer.

Strain Guage:

Schematic Diagram of Strain Guage

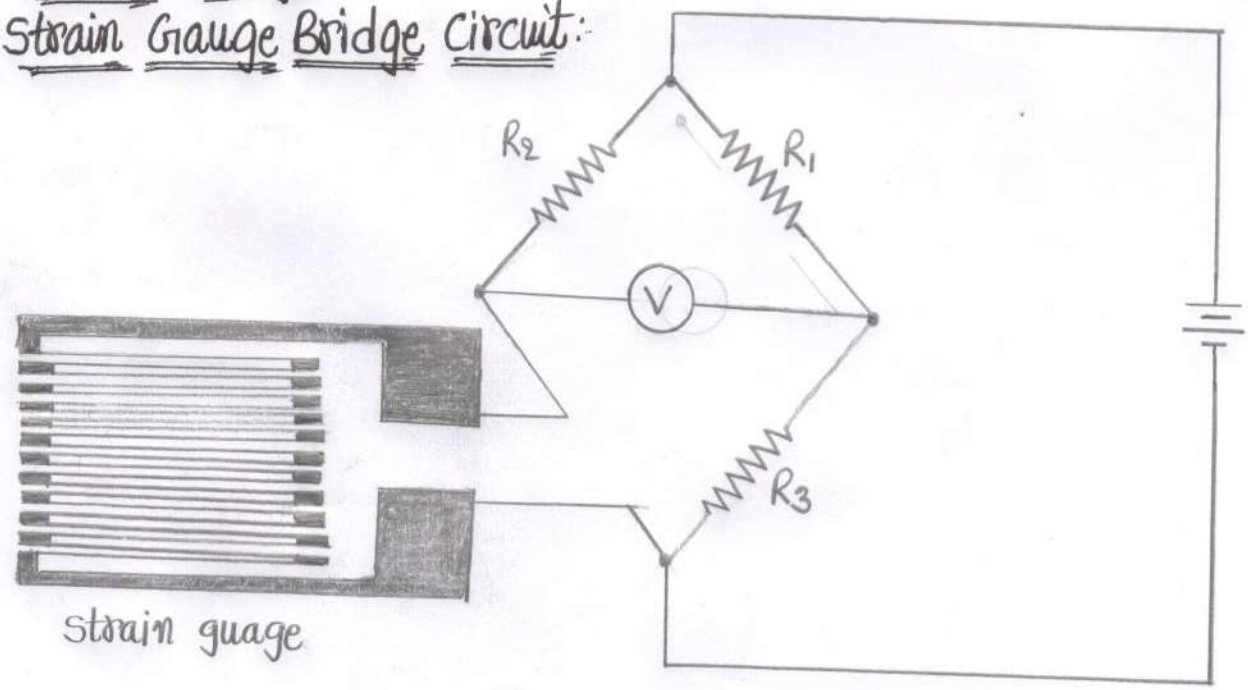


Construction details:-

- strain guage has a long thin metallic strip arrange in a zigzag pattern on a non-conducting materical called the carrier.
- It can enlarge the small amount of stress in the group of parallel lines and could be measured with high accuracy.
- It follows the principle of Piezo-resistance effect.

Strain Gauge:

Strain Gauge Bridge Circuit:-



strain guage

How does it work?

- In this circuit, R_1 and R_3 are the ratio arms equal to each other, and R_2 is the rheostat arm has a value equal to the strain guage resistance.
- When the guage is unstrained, the bridge is balanced, and voltmeter shows zero value.
- As there is a change in resistance of strain guage, the bridge gets unbalanced and producing an indication at the voltmeter.
- The output voltage from the bridge can be amplified further by a differential amplifier.

Principle:-

→ When an electrical conductor is stretched within the limits of its elasticity, it will became narrower and larger.

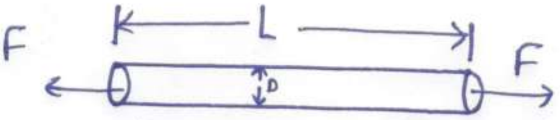
cross sectional area ↓ length of conductor ↑

which increases its electrical resistance.

$$R = \frac{\rho L}{A}$$

→ Similarly when a conductor is compressed, it will ~~droken~~ broken as shorter cross sectional area ↑ length ↓

which decreases its electrical resistance.



when a conductor is stretched then length changes from L to L+ΔL and diameter changes to D+ΔD

$R = \frac{\rho L}{A} \rightarrow$ ① here $R \propto L$, $R \propto \frac{1}{A}$ and $\rho = \frac{RA}{L}$

ΔL → change in length

ΔD → change in diameter

ΔR → change in resistance

ΔA → change in cross sectional area

Differentating ① with respect to strain S then

$$\frac{dR}{dS} = \frac{\rho}{A}\frac{\partial L}{\partial S} - \frac{\rho L}{A^2}\frac{\partial A}{\partial S} + L\frac{\partial \rho}{\partial S}$$

$$\frac{dR}{dS} = \frac{RA}{\frac{L}{A}}\frac{\partial L}{\partial S} - \frac{R\cancel{A}\cancel{L}}{\cancel{L}A^2}\frac{\partial A}{\partial S} + \frac{L}{A}\frac{\partial \rho}{\partial S}$$

$$\frac{1}{R}\frac{dR}{dS} = \frac{1}{L}\frac{\partial L}{\partial S} - \frac{1}{A}\frac{\partial A}{\partial S} + \frac{1}{\rho}\frac{\partial \rho}{\partial S} \rightarrow ②$$

Per unit change in resistance; per unit change in length; Per unit change in Area; Per unit change in resistivity

Here $A = \frac{\pi}{4}D^2 \Rightarrow \frac{\partial A}{\partial S} = 2\frac{\pi}{4}D\frac{\partial D}{\partial S} \Rightarrow \frac{1}{A}\frac{\partial A}{\partial S} = \frac{\frac{\pi}{2}D\frac{\partial D}{\partial S}}{\frac{\pi}{4}D^2} = \frac{2}{D}\frac{\partial D}{\partial S}$ — ③

substitute eq③ in ②

$$\frac{1}{R}\frac{dR}{dS} = \frac{1}{L}\frac{\partial L}{\partial S} - \frac{2}{D}\frac{\partial D}{\partial S} + \frac{1}{\rho}\frac{\partial \rho}{\partial S} \rightarrow ④$$

Poisson's ratio is the ratio of Lateral strain to longitudinal strain.

$$\vartheta = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{-\frac{\partial D}{D}}{\frac{dL}{L}}$$

$$\frac{\partial D}{D} = -\vartheta\frac{dL}{L} \rightarrow ⑤$$

substitute eq⑤ in ④

$$\frac{1}{R}\frac{dR}{dS} = \frac{1}{L}\frac{dL}{dS} + 2\vartheta\frac{dL}{dS} + \frac{1}{\rho}\frac{\partial \rho}{\partial S}$$

$$\boxed{\therefore \frac{\Delta R}{R} = \frac{\Delta L}{L} + 2\vartheta\frac{\Delta L}{L} + \frac{\Delta \rho}{\rho}}$$

Gauge factor:- Ratio of per unit change in resistance to per unit change in length.

$$\text{Gauge factor } GF = \frac{\Delta R/R}{\Delta L/L}$$

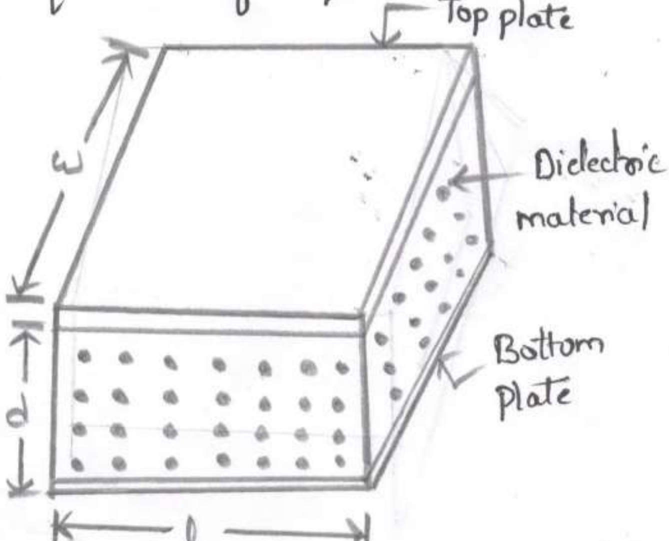
Capacitive Transducers

* Capacitive transducers Convert a non electrical Quantity into an electri -Cal Quantity by Means of changes in Capacitance.

Non Electrical Quantites → Displacement, force, pressure, flow, torque etc.

Electrical Quantites → Voltage, Current

The principal of operation of capacitive transducers is based on the Equation of Capacitance of a parallel plate capacitor.



Capacitance $C = \frac{\epsilon A}{d} = \frac{\epsilon_r \epsilon_0 A}{d}$

ϵ = permitivity of dielectric medium

ϵ_r = permitivity Relative.

ϵ_0 = permitivity of free space = 8.85×10^{-12} f/m

A = Overlapping area of plates

d = distance b/w two plates

Capacitance can be changed by Varying any one of the above parameters

* the Capacitive transducer work on the principle of change of Capacitance which may be caused by (i) Change in overlapping area, A
  ii) change in distance b/w the plates, d
  iii) change in dielectric constant, ϵ

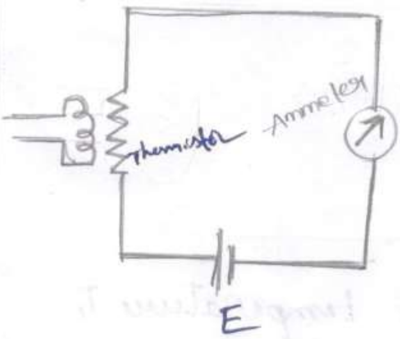
The Capacitance is measured with bridge circuits output impedence

$X_C = 1/2\pi f C$.

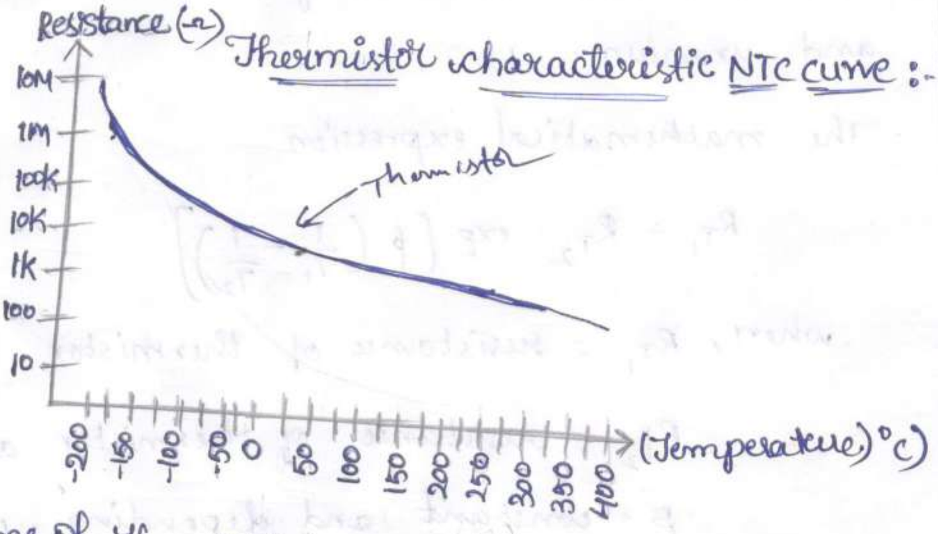
Advantages

* they require Extremly Small forces to operate them Very useful in Small Sytems
* they are highly sensitive.
* good frequency response.
* High input impedence, So less loading effect
* Good resolution.
* used to Measure both Linear & Angular displacement

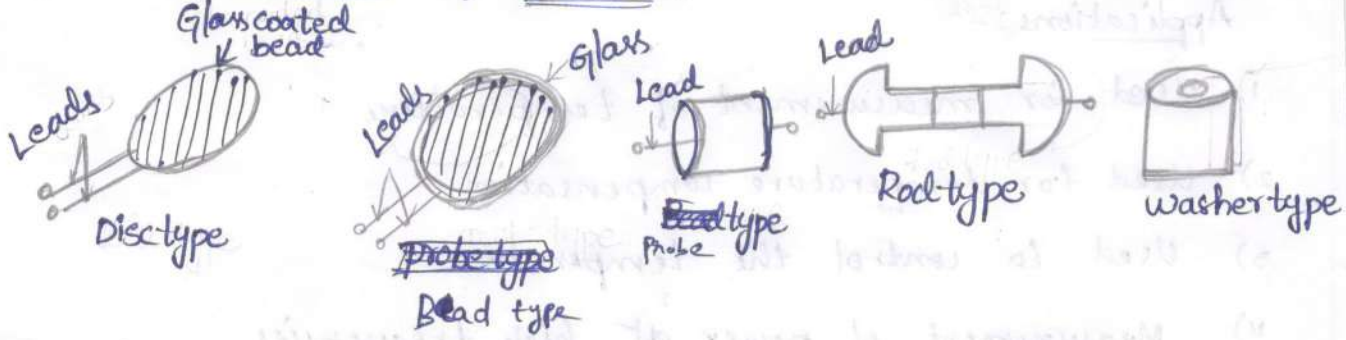
Thermistor (Thermal + Resistor) :-



Thermistor characteristic NTC curve :-



Different Constructions of Thermistor :-



Thermistor is a special type of resistor whose resistance changes with change in temperature.

In a conductor, if temperature rises, resistance decreases. and this is called negative temperature coefficient

Similarly, if temperature rises and resistance also increases, this is called positive temperature coefficient.

⇒ thermistor have a negative temperature coefficient of resistance

⇒ Generally 1°C rise in temperature results in 5% decrease in resistance.

⇒ Thermistors are highly sensitive but have a non linear characteristics of resistance versus temperature.

→ Materials used are manganese, nickel, cobalt, copper, iron, and uranium.

The mathematical expression

$$R_{T_1} = R_{T_2} \exp\left[\beta\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right]$$

where, R_{T_1} = resistance of thermistor at temperature T_1

R_{T_2} = resistance of thermistor at temperature T_2

β = constant and depending upon material of thermistor

Applications:-

1) Used for measurement of temperature
2) Used for temperature compensation
3) Used to control the temperature
4) Measurement of power at high frequencies

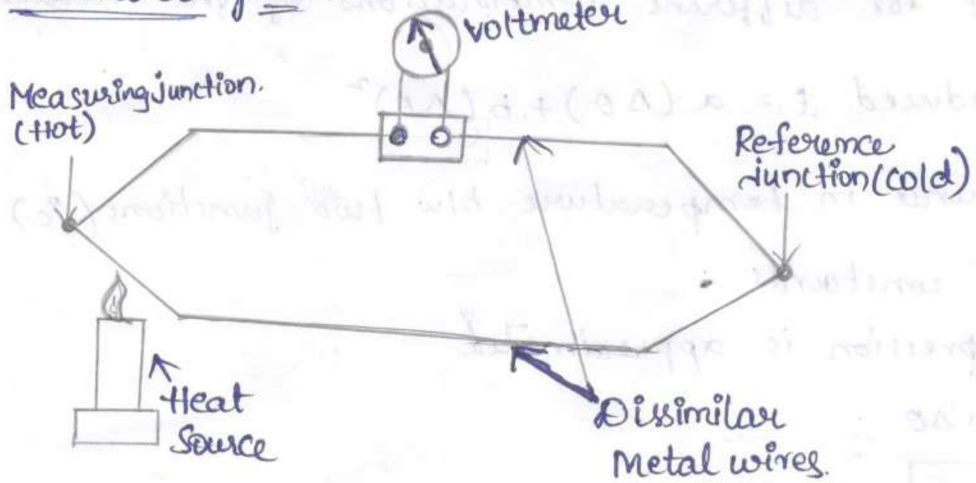
~~5) Used to control the temperature~~

5) Measurement of thermal conductivity
6) Providing time delay.

Advantages:-

1) They are compact, rugged and inexpensive
2) They have good stability, highly sensitive
3) Their response time is fast
4) ~~They~~ are not affected by stray magnetic and electric fields.

Thermo Couples :-



→ These are the devices which are used for the measurement of temperature variations. It is an active transducer

Principle:-

⇒ Thermocouple is composed of 2 metals joined together to form two junctions called hot junction and cold junction.

⇒ These junctions are also called measuring junction and reference junction.

⇒ The 2 junctions are placed at two different temperatures. So, due to change in temperature, e.m.f is induced in the thermocouple and output voltage can be obtained with the relationship between voltage and temperature.

⇒ The temperature to be measured (unknown) is placed at hot junction and known temperature is placed at cold junction.

Seebeck effect :- This effect states that when two different or unlike metals are joined together at two junctions, an electromotive force (emf) is generated at the 2 junctions.

⇒ It is different for different combinations of the metals.

The emf produced $E = a(\Delta\theta) + b(\Delta\theta)^2$

$\Delta\theta$ ⇒ difference in temperature b/w two junctions (°c)

a, b → constants

when the expression is approximated

$E = a\Delta\theta$

$$\boxed{\Delta\theta = \frac{E}{a}}$$

→ The reference junction is usually at 0°C

→ They can measure upto 1400°C

→ chromel + constantan

copper + constantan

chromel + Alumel are used.

Advantages

1. Follow the temperature changes with a small time-lag

2. Convenient for measuring the temperature at one particular point in a piece of apparatus

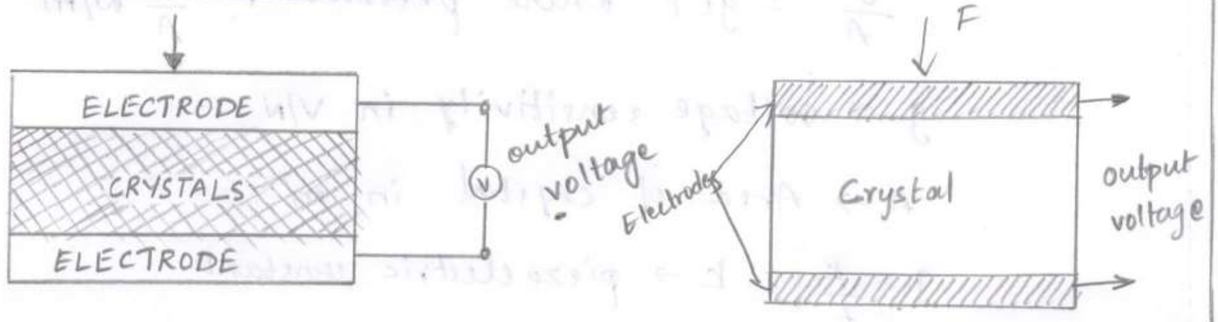
Disadvantages

1. Lower accuracy

2. It must be protected against contamination to ensure long life

3. Placed remote location from measuring devices.

Piezo electric Transducer:



→ the piezo electric transducer converts the physical quantities like pressure or mechanical stress into an electric voltage which is easily measured by analog and digital meters.

→ When mechanical force is applied to some material (crystal) along certain plane, they produce electric voltage.

→ If the varying voltage is applied to the crystal in proper axis it will change dimension of crystal by deforming it. This effect is known as piezoelectric effect.

Natural crystals → Quartz

Synthetic crystals → Rachelle salt, lithium sulphate etc.

$$E = \frac{gtF}{A} = gtP \quad \text{where pressure } P = \frac{F}{A} \ N/m^2$$

$g \rightarrow$ voltage sensitivity in V/N

$A \rightarrow$ Area of crystal in m^2

$g = \frac{k}{t}$ $\quad k \rightarrow$ piezo electric constant

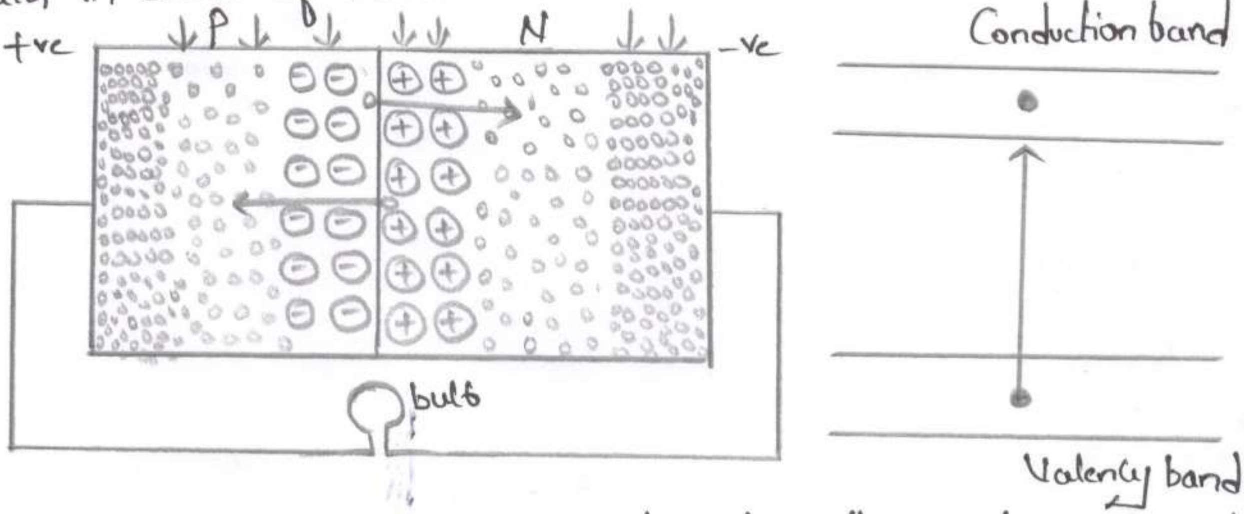
$t \rightarrow$ thickness of crystal

Applications:-

→ It is used in microphones, medical diagnostics, eletric lighter, inkjet printers etc.

Photovoltaic Cells.

* Convert light Energy to Electrical Energy.
* they are made up of Semi-Conductors materials such as silicon
* they are doping to form p type or N type semi-Conductors
* N type doping results in excess of electrons and p-type doping results in Excess of holes

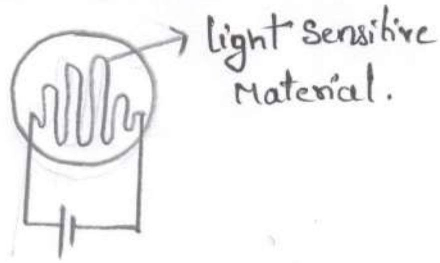


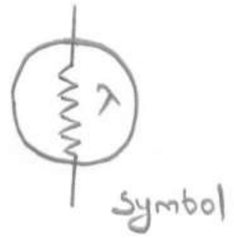
* Electrons diffuse across PN Junction from the N-type material Creating the change in the n-type material
* Holes diffuse across PN Junction from the p-type material Creating -ve charge in the p-type material
* when light is absorbed by semi-conductor Extra free electrons and holes are Created
* the electric field makes the Electron flow to the n-type and holes flow to the p-type material
* this is called photovoltaic effect

Photo Conductive Cell

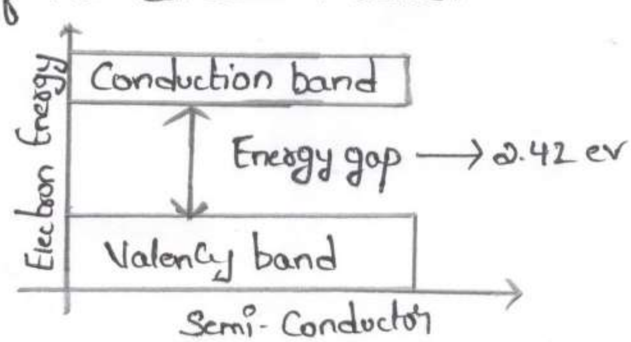
* photoconductive cell is a Semi-Conductor device whose resistance depends on the intensity of incident light
* Commonly used photo Conductive materials are

Cadmium Selenide (cdse) & Cadmium Sulphide (cds)





The Conductivity depends on Energy gap b/w top of the Valancy band and bottom of the Conduction band.

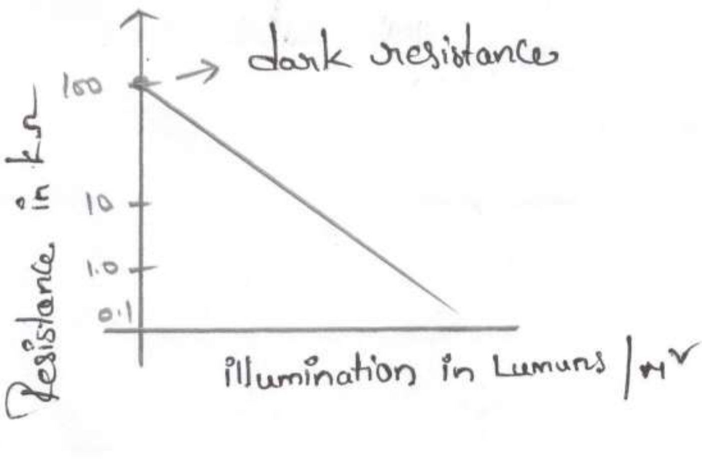


Change in resistance ∝ Light intensity

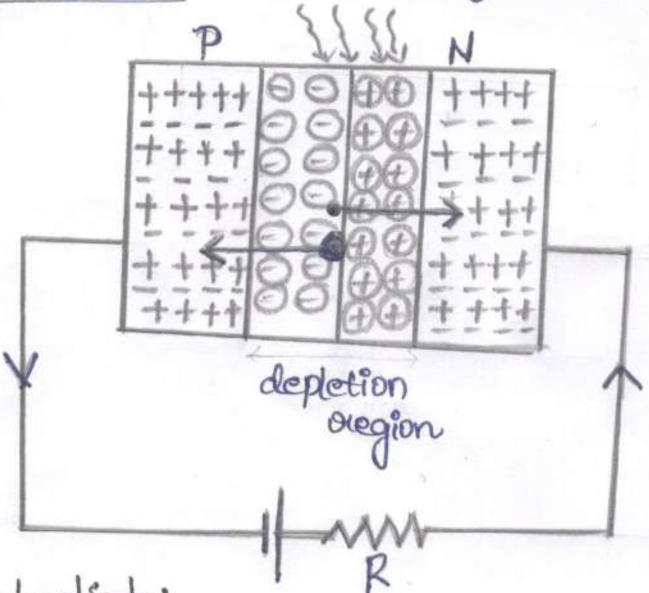
* when the cell is not illuminated its resistance is more than 100kΩ and is called as dark resistance.
* when light is incident on photo Conductive cells photons & absorbed by the Cadmium Sulphide and it releases free electrons from the Valency band this is due to increase in thermal Energy free electrons Jumps to the Conduction band and Causes electrical Current.

Applications

* light sensitive alarms
* Automatic street lights
* Detecting the location of air crafts / ships etc
* lighting control etc.



Photodiodes :- (light → Electricity)



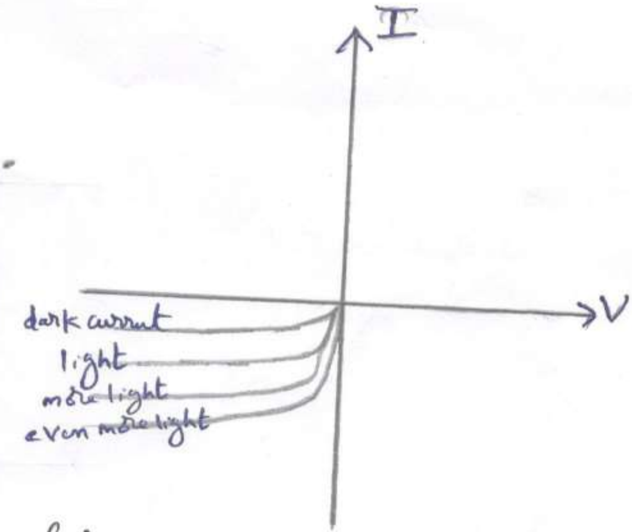
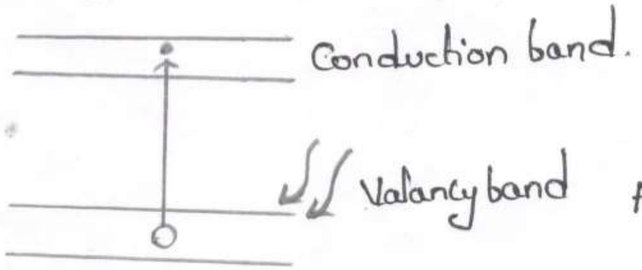
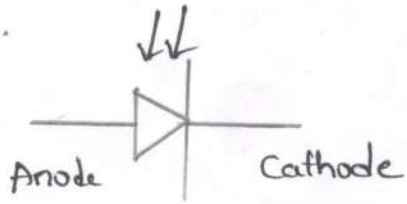


Photo diodes

* photodiodes detects the brightness of light
* Reverse biased PN Junction diode is used as a photodiode
* P Side Contains holes and N Side Contains electrons
* when we shine a light on photodiode electron hole pair is generated in depletion region and they move in opposite direction.





* Due to light flows.
* if more light is Shined then more electron hole pairs & generated in depletion region and Current
* Based on Current we can determine the brightness of light

Applications

* Smoke detectors
* Space applications
* pulse oximeters
* optical Communications
* used to Measure low light intensities etc.