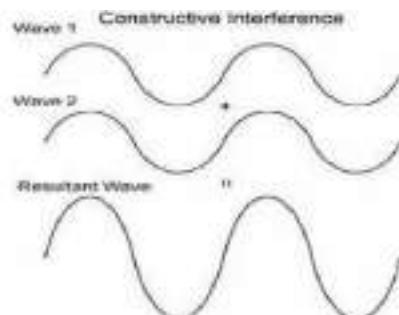
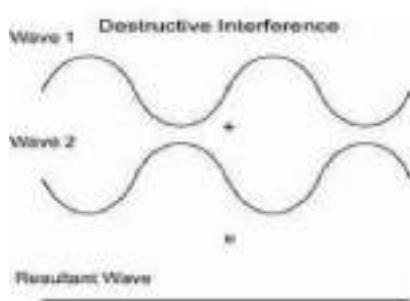


UNIT- I INTERFERENCE

Syllabus: INTERFERENCE: Introduction - Interference in thin films by reflection - Newton's rings.

1. Explain the principle of superposition of waves.

Principle of superposition of waves:-



Statement: when two or more waves travelling simultaneously in a medium, the resultant displacement at any point is the algebraic sum of the displacements due to individual waves.

If y_1 and y_2 are the instantaneous displacements of two waves, then the resultant displacement is given by

$$y = y_1 \pm y_2$$

'+' sign has to be taken when two both waves are in same phase and '-' sign when both waves are out of phase.

Coherence: Two waves are said to be coherent, if they have same frequency and constant phase difference.

2. Explain the phenomenon of interference.

Interference:

When two or more waves having same frequency and constant phase difference travelling in the same direction are superimposed with each other, then there is a modification of amplitude in the region of superposition. i.e. the energy is redistributed in the form of maxima and minima.

If a is the amplitude of the two waves, then the resultant amplitude for constructive interference is

$$\text{i.e. } y = a + a = 2a.$$

and for destructive interference is

$$\text{i.e. } y = a - a = 0$$

3. Write the conditions for obtaining interference Pattern.

- Two sources must be coherent.
- Two sources should be narrow and monochromatic.
- The separation between the two sources should be small.
- The distance between the sources and the screen should be large.
- The background should be dark.

4. With ray diagram discuss the theory of thin films in the condition for constructive and destructive interference in the case of reflected system.

INTERFERENCE IN THIN FILMS

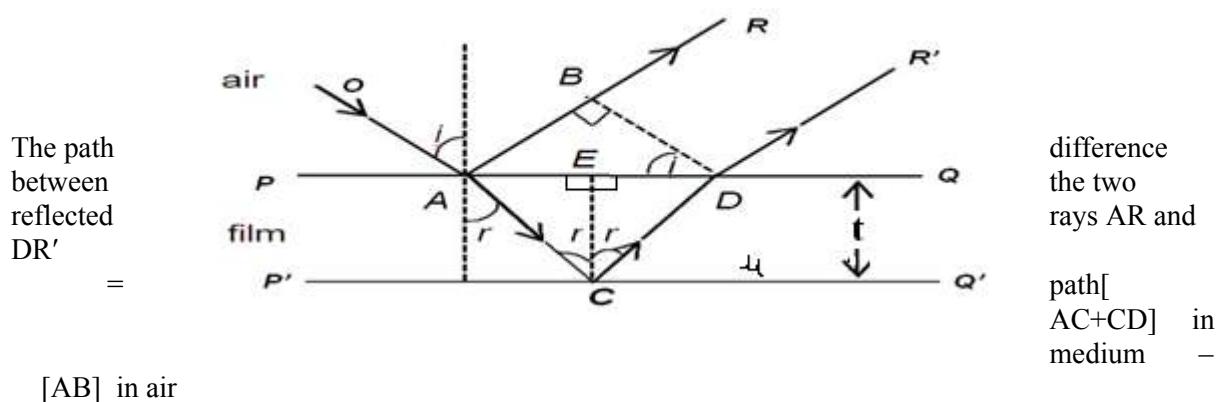
The colors of thin films, soap bubbles and oil slicks can be explained by using the phenomenon of interference on the basis of division of amplitude.

Interference in thin films will be discussed in two classes:

- Interference in uniform thin films.
- Interference in non-uniform thin films.

INTERFERENCE IN PLANE PARALLEL FILMS DUE TO REFLECTION (UNIFORM)

Consider two plane surfaces PQ and P'Q' are separated by a distance 't'. Let μ be the refractive index of the film between the surfaces. Let a ray of light OA be incident on the surface PQ at A, then some part of the light is reflected into air as AR ray, The remaining part is transmitted into the medium and is reflected at the lower surface P'Q' at C, meets the upper surface at D and emerges as DR' ray as shown in fig.



$$= \mu[AC+CD] - AB \quad \dots \dots \dots (1)$$

From ΔAEC , $\cos r = CE / AC$

$$= \frac{AC}{CE} - \frac{t}{(CE = t)}$$

$$\cos r = \frac{CO}{SR}$$

ΔAEC and ΔCED are similar, so
 $AC = CD$ and $AE = ED$

$$AC + CD = \frac{t}{\cos r} + \frac{t}{\cos r} = \frac{2t}{\cos r} \quad \dots \dots \dots (2)$$

From ΔABD , $\sin i = \frac{AB}{AD}$

$$AB = AD \sin i$$

We Know that $\mu = \frac{\sin i}{\sin r}$

$$\sin i = \mu \sin r$$

$$\therefore AB = AD \mu \sin r$$

From ΔAEC , $\tan r = \frac{AE}{CE}$

$$AE = CE \tan r = t \tan r$$

$$\therefore AD = AE + ED = 2t \tan r \quad (\text{AE} = ED)$$

$$\text{Hence } AB = 2t \tan r \times \mu \sin r$$

$$AB = \frac{2\mu t \sin^2 r}{\cos r} \quad \dots \dots \dots (3)$$

Substitute eqn (3) & (2) in eqn (1)

$$\delta = \frac{2\mu t}{\cos r} - \frac{2\mu t}{\cos r} \times \sin^2 r$$

\Rightarrow

$$\delta = \frac{2\mu t}{\cos r} (1 - \sin^2 r)$$

$$\delta = \frac{2\mu t}{\cos r} \times \cos^2 r$$

$$= 2\mu t \cos r$$

According to Stoke's law, **when the light is reflected by denser medium in to rarer medium,**

then the reflected ray suffers a phase change of π radians or undergoes a path change of $\frac{\lambda}{2}$

The path difference between the reflected rays AR and DR'

$$\delta = 2 \mu t \cos r - \frac{\lambda}{2}$$

Condition for maxima:

Hence the condition for maxima, i.e. for an air film to appear bright is

$$\delta = 2 \mu t \cos r - \frac{\lambda}{2} = n\lambda \Rightarrow 2 \mu t \cos r = (2n+1) \frac{\lambda}{2} \quad \text{where } n=0, 1, 2, \dots$$

Condition for minima:

Hence the condition for minima, i.e. for an air film to appear dark is

$$2 \mu t \cos r = (2n+1) \frac{\lambda}{2}$$

$$\mu t \cos r = (2n+1) \frac{\lambda}{2}$$

$$2 \mu t \cos r = (2n+2) \frac{\lambda}{2}$$

$$2 \mu t \cos r = (n+1) \lambda, \text{ where } n=0, 1, 2, 3, \dots$$

$$\text{or } 2 \mu t \cos r = n \lambda, \text{ where } n=0, 1, 2, 3, \dots$$

5. Explain the colours in a thin film when exposed to sunlight.

Colours in thin films:

- When white light is incident on a thin film, it gets splits up by reflection at the top and bottom surfaces of the film. These reflected rays interfere producing colors in thin films.
- Different points on the thin film satisfy the condition for the constructive interference i.e.,

$$2 \mu t \cos r = (2n+1) \frac{\lambda}{2}$$

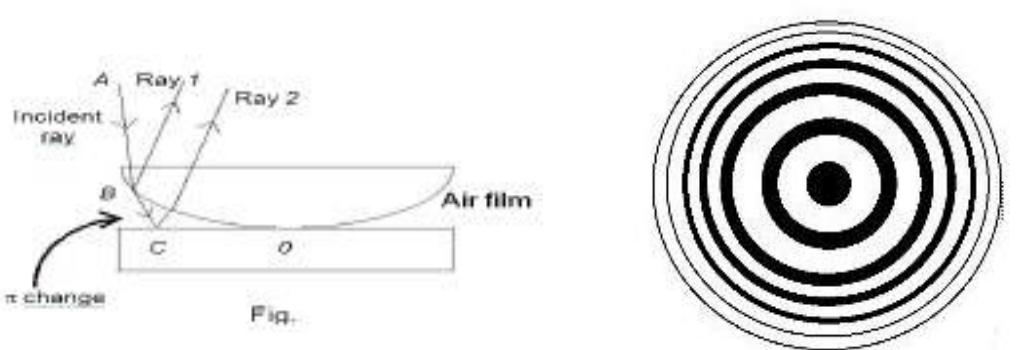
for different values of μ, λ and t when illuminated with white light and hence appear multicolored.

- In case of soap bubble, t is constant and varying values of μ, λ and r satisfy the above condition give multicolor.
- In case of oil slick, r is constant and varying values of μ, λ and t satisfy the above Condition gives multicolor.

6. Discuss the theory of Newton's rings with relevant diagram

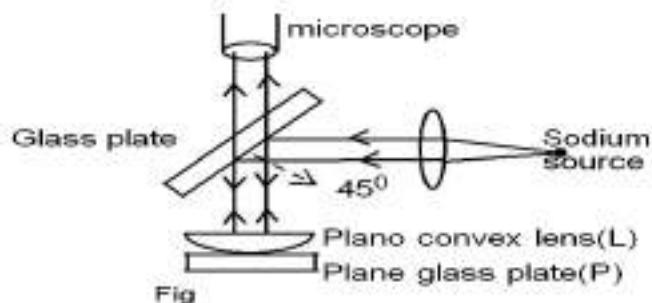
NEWTON'S RINGS: (INTERFERENCE IN NON-UNIFORM FILMS DUE TO REFLECTION)

When a Plano convex lens with its convex surface is placed on a plane glass plate, an air film of gradually increasing thickness is formed between them. The thickness of the film at the point of contact is zero. If a monochromatic light is allowed to fall normally and the film is viewed in reflected light, alternate dark and bright concentric circular rings are observed around the point of contact. This phenomenon was first described by Newton, so these rings are called Newton's rings.



EXPERIMENTAL ARRANGEMENT:-

The experimental arrangement consists of a Plano-convex lens 'L' of large radius of curvature and is placed on a plane glass plate 'P' as shown in fig. The light from monochromatic source is reflected normally on to the air film by means of glass plate G inclined at 45° . A part of the incident light is reflected by the curved surface of the lens 'L' (Ray 1) and remaining is transmitted. The transmitted light is reflected back from the plane surface of the glass plate 'P' (Ray 2) undergoes a path change of $\lambda/2$ (Stoke's principle). These two reflected rays are superimposed and produce an interference pattern in the form of bright and dark circular rings. These rings can be viewed in a microscope 'M' focused on the film.



The conditions for the bright and dark rings are governed by the following relations:

The condition for bright ring:

$$\delta = 2\mu t \cos r + \frac{\lambda}{2} = n \lambda$$

$$2\mu t \cos r = (2n - 1) \frac{\lambda}{2} \quad \text{where } n = 0, 1, 2, 3, \dots$$

For normal incidence, $\cos r = 1$ and for the air film $\mu = 1$

$$\therefore 2t = (2n - 1) \frac{\lambda}{2} \quad (\text{Bright fringes})$$

The condition for dark ring:

$$\delta = 2\mu t \cos r + \frac{\lambda}{2} = (2n + 1) \frac{\lambda}{2}$$

$$2\mu t \cos r = n \lambda \quad \text{Where } n=0,1,2,3, \dots$$

For normal incidence, $\cos r = 1$ and for the air film $\mu = 1$

$$\therefore 2t = n \lambda \quad (\text{Dark rings})$$

7. Derive the expressions for the diameters of n^{th} dark and bright rings by forming Newton's Rings.

Theory of Newton's rings:

In the reflected monochromatic light, Newton's rings are alternate bright and dark circles with a central dark spot. Let R be the radius of curvature of the lens. At Q , let the thickness of the film $PQ = t$ satisfies the condition for a dark ring to form by interference. Let it be an n^{th} dark ring with a radius, $SQ = r_n$.

Determination of radius of n^{th} dark ring

Consider an n^{th} dark ring with radius HE .

From the diagram
 $OE = R - t$, $HE = r_n$, $OH = R$

From the ΔOHE

$$OH^2 = HE^2 + OE^2$$

$$R^2 = r_n^2 + (R-t)^2$$

$$r_n^2 = R^2 - (R-t)^2$$

$$r_n^2 = R^2 - R^2 - t^2 + 2Rt$$

$$r_n^2 = 2Rt \quad (\text{as } 2Rt \gg t^2, \text{ } t^2 \text{ can be neglected})$$

$$r_n^2 = 2Rt$$

For dark rings, the condition is $2t = n\lambda$

$$r_n^2 = n\lambda R \text{ or}$$

$$r_n = \sqrt{n\lambda R}$$

The diameter of the dark ring is therefore given by

$$D_n = 2r_n = 2\sqrt{n\lambda R} \text{ or } D_n = \sqrt{4n\lambda R}$$

$$D_n \propto \sqrt{n}$$

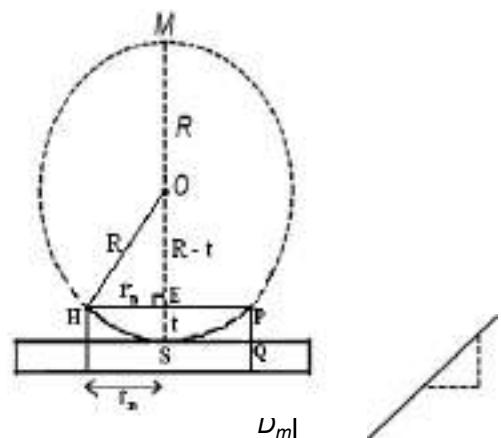
Thus, the radii (also diameters) of the dark rings are proportional to the square root of natural numbers.

Determination of radius of n^{th} bright ring

Let us suppose that a bright ring is located at the point Q . The radius of the n^{th} bright ring is given by $r_n^2 = 2Rt$.

For bright rings, the condition is

$$2t = (2n - \frac{\lambda}{2})$$



1)

$$\therefore r^2 = \frac{(2n-1)\lambda R}{2}$$

or $r = \sqrt{\frac{(2n-1)\lambda R}{2}}$

0 No. of ring, m

$$\therefore D_n = \sqrt{(2n-1)2\lambda R}$$

It is clear that **the radii (also diameters) of bright rings are proportional to the square root of the odd natural numbers.**

8. Why a Dark spot is observed at the center of the Newton's rings?

The thickness of the air film at the point of contact is zero i.e. $t = 0$, hence $\delta = \frac{\lambda}{2}$ which is the condition for destructive interference. So a dark spot is observed at the center of the Newton's rings.

9. Why Newton's rings are circular?

In Newton's rings arrangement, a thin air film is enclosed between a Plano-convex lens and a glass plate. The thickness of the air film at the point of contact is zero and gradually increases as we move outward. The locus of points where the air film has the same thickness then fall on a circle whose centre is the point of contact. Thus, the thickness of air film is constant at points on any circle having the point of lens-glass plate contact as the centre. The fringes are therefore circular.

10. Determine the wavelength of light source and Radius of curvature of given convex lens in Newton's Rings experiment.

The wave length of incident monochromatic light can be determined by forming Newton's rings and measuring the diameters of the dark rings using travelling microscope.

For m^{th} dark ring, $D_m^2 = 4m\lambda R$

Similarly, for n^{th} dark ring, $D_n^2 = 4n\lambda R$

$$D_m^2 - D_n^2 = 4m\lambda R - 4n\lambda R$$

$$D_m^2 - D_n^2 = 4(m-n)\lambda R$$

$$\lambda = \frac{D_m^2 - D_n^2}{4(m-n)\lambda} \quad \text{This is an expression for wavelength of incident light.}$$

$$R = \frac{D_m^2 - D_n^2}{4(m-n)\lambda} \quad \text{This is an expression for Radius of curvature of lens..}$$

PROBLEMS

1. A parallel beam of light of wavelength 5890 Å^0 is incident on a thin glass plate ($\mu = 1.5$) such that the angle of refraction into the plate is 60° . Calculate the smallest thickness of the glass plate which will appear dark in reflected light.

The condition is given by $2\mu t \cos r = m\lambda$. Taking $m = 1$, the smallest thickness of plate that causes destructive interference is

$$t = \frac{\lambda}{2\mu \cos r} = \frac{5890 \times 10^{-10}}{2 \times 1.5 \times \cos 60^\circ} = 0.39 \text{ } \mu\text{m}$$

2. A Newton's ring arrangement is used with a source emitting two wavelengths and $\lambda_2 = 4.5 \times 10^{-5} \text{ cm}$. It is found that n^{th} dark ring due to λ_1 coincides with $(n+1)_{th}$ dark ring for λ_2 . If the radius of curvature of the curved surface is 90 cm , find the diameter of n_{th} dark ring for λ_1 .

Solution : $r = \sqrt{m\lambda R}$, $m = 0, 1, 2, \dots$ for dark ring. Hence $r_n = \sqrt{n \times 6 \times 10^{-5} \times 90}$ and

$$r_{n+1} = \sqrt{(n+1) \times 4.5 \times 10^{-5} \times 90}$$

$$n \times 6 \times 10^{-5} \times 90 = (n+1)4.5 \times 10^{-5} \times 90$$

$$6n = 4.5(n+1) \quad \text{or} \quad 6n = 4.5n + 4.5 \quad \text{or} \quad 1.5n = 4.5 \quad \text{or} \quad n = 4.5/1.5 = 3$$

$$r_n = \sqrt{3 \times 6 \times 10^{-5} \times 90} = \sqrt{1620 \times 10^{-5}} = \sqrt{0.0162} = 0.127 \text{ cm}$$

$$\text{Diameter of } n_{th} \text{ ring} = 0.127 \times 2 = 0.254 \text{ cm.}$$

3. In a Newton's ring experiment, the diameter of the 4th and 12th dark rings are 0.400 cm and 0.700 cm, respectively. Find the diameter of the 20th dark ring.

Solution: We know that $D_m^2 - D_n^2 = 4(m-n)\lambda R$

$$\text{Here } m = 12, n = 4, \text{ and } (m-n) = 12-4 = 8$$

$$D_m = 0.700 \text{ cm} \quad \text{and} \quad D_n = 0.400 \text{ cm}$$

$$D_{12}^2 - D_4^2 = 4 \times 8 \times \lambda R$$

For 20th and 4th dark rings

$$D_{20}^2 - D_4^2 = 4 \times 16 \times \lambda R$$

2

Dividing (2) by (1)

$$\frac{D_{20}^2 - D_4^2}{D_{12}^2 - D_4^2} = \frac{4 \times 16 \times \lambda R}{4 \times 8 \times \lambda R} = 2$$

$$\text{or } \frac{D_{20}^2 - (0.4)^2}{(0.7)^2 - (0.4)^2} = 2 \quad \text{or} \quad \frac{D_{20}^2 - 0.16}{0.33} = 2$$

$$\text{or } D_{20}^2 - 0.16 = 2 \times 0.33 = 0.66$$

$$D_{20}^2 = 0.66 + 0.16 = 0.82 \quad \text{or } D_{20} = 0.906 \text{ cm.}$$

The diameter of the 20th dark ring = **0.906 cm**

4. In Newton ring's experiment the diameter of 10th ring changes from 1.40 cm to 1.27 cm when a liquid is introduced between the lens and the plate. calculate the refractive index of the liquid.

Solution: Given the diameter of 10th ring in air $D_{10} = 1.40 \text{ cm}$

the diameter of 10th ring in liquid $D'_{10} = 1.27 \text{ cm}$

the diameter of nth ring in air $D_n^2 = 4n\lambda R$

the diameter of nth ring in liquid $D'_n^2 = \frac{4n\lambda R}{\mu}$

From above two equations $\mu = \frac{D_n^2}{D'_n^2}$

For 10th ring $\mu = \frac{D_{10}^2}{D'_n^2}$

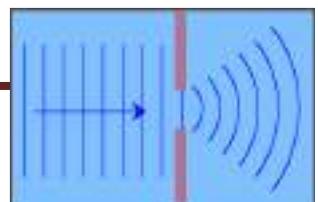
$$\mu = \frac{(1.40)^2}{(1.27)^2}$$

$$\mu = 1.2152$$

DIFFRACTION

Syllabus: Introduction - Fraunhofer diffraction - Fraunhofer diffraction at double slit (qualitative) - Diffraction grating - Grating spectrum - Resolving power of a grating - Rayleigh's criterion for resolving power.

I. What is Diffraction?



The phenomenon of bending of light waves around the edges of obstacle and spreading of light waves into the geometrical shadow of an obstacle placed in the path of light is called **diffraction**.

In general, diffraction of waves becomes noticeable only when the size of obstacle is comparable to the wavelength.

$$\lambda \approx d$$

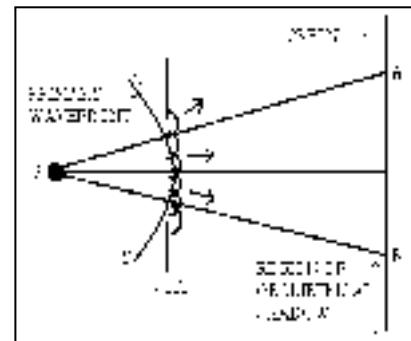
Examples:

- The spaced tracks on a CD or DVD act as a diffraction grating to form the familiar rainbow pattern.
- Sound produced inside one room reaches the other room after bending the edges of the doors.

2. Explain Huygen's wave theory.

HUYGEN'S WAVE THEORY

Let a primary wave front XY from point source S of monochromatic light reach the slit placed in its path. The light passing through the slit has to cause brightness inside AB on the screen and regions beyond A and B have to be dark i.e. regions of geometrical shadow. According to Huygens wave theory, every point on the primary wave front acts as a secondary source of disturbance and secondary wavelets are generated from these points. These secondary wavelets enter into the geometrical shadow region and interfere to produce diffraction pattern beyond A and B .

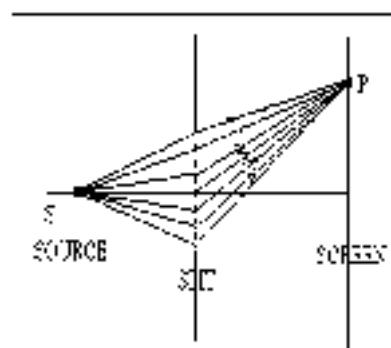


3. Write the differences between Fresnel and Fraunhofer diffraction.

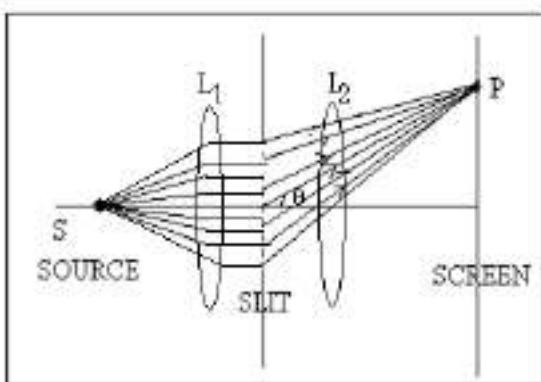
- 1) Fresnel diffraction 2) Fraunhofer diffraction

Fresnel diffraction:

1. In Fresnel diffraction, a point source or an illuminated narrow slit at finite distance is used.
2. The incident wave front is either spherical or cylindrical.
3. The source and screen are at finite distance from the aperture producing diffraction.
4. Lens is not used to focus the rays.
5. The centre of diffraction pattern may be bright or dark.
6. Mathematical investigations are complicated and approximate.



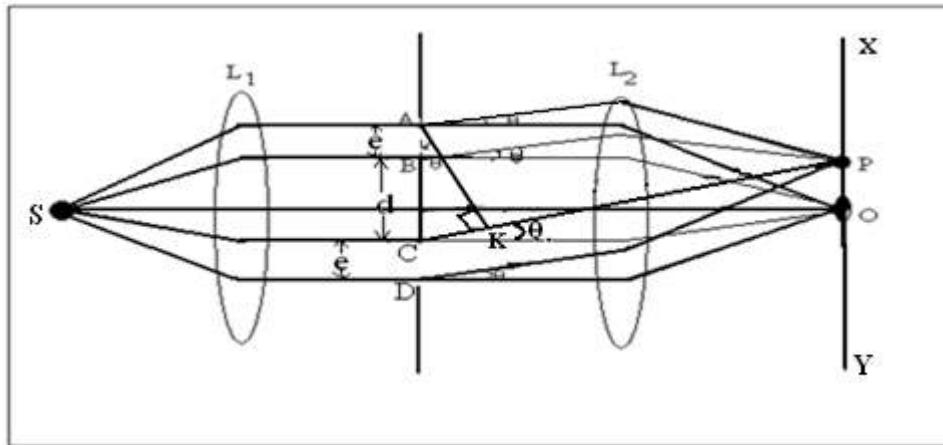
Fraunhofer diffraction:



1. In Fraunhofer diffraction, an extended source at infinite distance is used.
2. The incident wave front is plane wave front.
3. The source and screen are at infinite distance from the aperture producing diffraction.
4. Converging lens is used to focus the rays.
5. The centre of the diffraction pattern is always bright.
6. Mathematical investigations are easy and accurate.

4. Discuss in detail Fraunhofer diffraction at a double slit with suitable diagram and draw the intensity distribution curve.

Consider AB and CD be two parallel slits of equal width e and are separated by a distance d .



The distance between the corresponding points of the two slits is $(e+d)$. Let a parallel beam of monochromatic light of wavelength λ be incident normally upon the two slits. The diffracted light be focused by a convex lens L on the screen XY as shown in figure.

Explanation: By Huygens's principle, every point in the slits AB and CD sends out secondary wavelets in all directions. From the theory of diffraction at a single slit, the resultant amplitude due to the wavelets diffracted from each slit in direction θ is

$$R = \frac{A \sin \alpha}{\alpha}$$

Where A is constant and $\alpha = \frac{\pi e \sin \theta}{\lambda}$

let A and C be the points of the slits sending waves of amplitude $\frac{A \sin \alpha}{\alpha}$ in a direction θ .

∴ The resultant amplitude at a point P on the screen will be the result of interference between the waves of the same amplitude $\frac{A \sin \alpha}{\alpha}$ and having a phase difference ϕ

Let us draw AK perpendicular to CK. The path difference between the wavelets from A and C in the direction ' θ ' is $(e+d) \sin \theta$ i.e. $CK = (e+d) \sin \theta$

Hence the corresponding phase difference, $\phi = \frac{2\pi}{\lambda} \times \text{path diff.}$

$$\therefore \phi = \frac{2\pi}{\lambda} (e+d) \sin \theta.$$

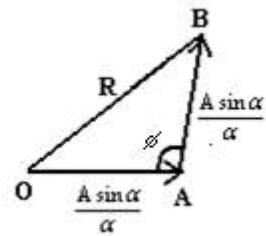
The resultant amplitude R at P can be determined by the vector amplitude diagram,

$$OB^2 = OA^2 + AB^2 + 2(OA)(AB) \cos \angle OAB$$

$$R^2 = \left(\frac{A \sin \alpha}{\alpha}\right)^2 + \left(\frac{A \sin \alpha}{\alpha}\right)^2 + 2\left(\frac{A \sin \alpha}{\alpha}\right)\left(\frac{A \sin \alpha}{\alpha}\right) \cos \phi$$

$$R^2 = \left(\frac{A \sin \alpha}{\alpha}\right)^2 (2 + 2 \cos \phi)$$

$$I = R^2 = 4 \frac{A^2 \sin^2 \alpha}{\alpha^2} \cos^2 \frac{\phi}{2}$$



Thus the resultant intensity at any point on the screen is given by the product of two factors i.e.

$$\left(\frac{A \sin \alpha}{\alpha}\right)^2$$

a) The diffraction term $\left(\frac{A \sin \alpha}{\alpha}\right)^2$ gives a central maximum in the direction $\theta = 0$ having alternately minima and secondary maxima of decreasing intensity on either side.

The minima are obtained in the directions given by

$$\sin \alpha = 0$$

$$\alpha = \pm n\pi$$

$$\frac{\pi e \sin \theta}{\lambda} = \pm n\pi$$

$$e \sin \theta = \pm n\lambda \quad \text{where } n=1,2,3,\dots$$

The position of secondary maxima appear at

$$\alpha = \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \pm \frac{7\pi}{2}, \dots$$

b) The interference term $\frac{\cos^2 \frac{\phi}{2}}{2}$ gives a set of equidistant dark and bright fringes.

The bright fringes are obtained in the directions given by

$$\cos^2 \frac{\phi}{2} = 1$$

$$\frac{\phi}{2} = \pm n\pi$$

$$\frac{\pi(e+d)\sin\theta}{\lambda} = \pm n\pi$$

$$(e+d)\sin\theta = \pm n\lambda \quad \text{Where } n=0, 1, 2, 3, \dots$$

The dark fringes are obtained in the directions given by

$$\cos^2 \frac{\phi}{2} = 0$$

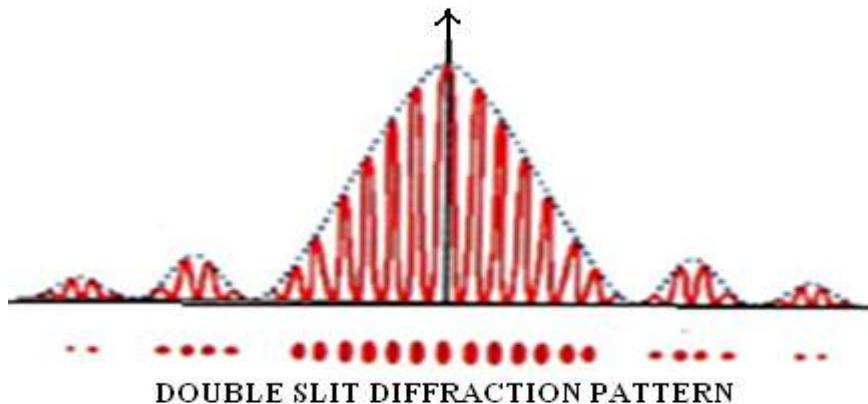
$$\frac{\phi}{2} = \pm(2n+1)\frac{\pi}{2}$$

$$\frac{\pi(e+d)\sin\theta}{\lambda} = \pm n\pi$$

$$(e+d)\sin\theta = \pm(2n+1)\frac{\lambda}{2} \quad \text{Where } n=0, 1, 2, 3, \dots$$

Therefore the intensity distribution of Fraunhofer diffraction due to double slit is the product of a

constant term 4, diffraction term $\left(\frac{A \sin \alpha}{\alpha}\right)^2$ and interference term $\cos^2 \frac{\phi}{2}$ as shown in figure

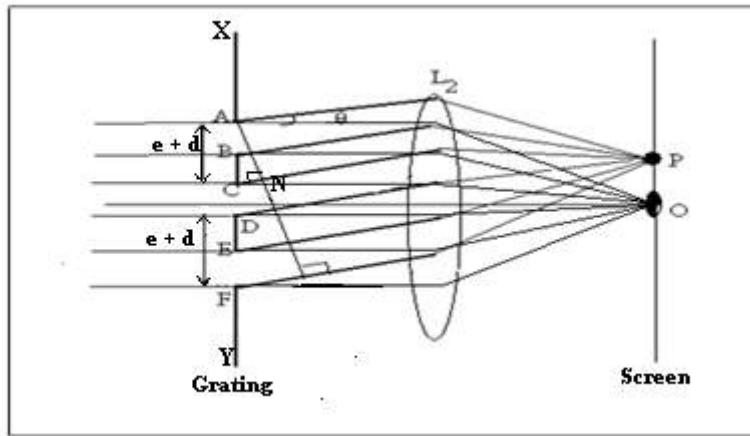


5. What is Diffraction Grating? Explain Fraunhofer diffraction at 'n' slits with necessary theory.

- A diffraction grating is an optically plane glass plate on which a large number of equidistant parallel lines are ruled.
- The ruling region becomes opaque and the region between the rulings is transparent. Therefore diffraction grating is nothing but closely placed multiple (N) slits.
- The combined width of a ruling and a slit is called grating element.
- A good quality of grating contains 15000 lines per inch.

Let XY represents the grating. The plane of the grating is perpendicular to the plane of paper. In grating AB, CD, EF etc. represents the slits, each of width 'e' and these slits are separated by equal opaque regions BC, DE, FG etc. each of width 'd'. (e + d) is called the grating element.

Let plane waves of monochromatic light of wave length ' λ ' incident normally on the grating. By Huygens –principle, every point on the plane wave front acts as a source of secondary wavelets. These secondary wavelets spread out in all directions.



The secondary wavelets travelling in the same direction of the incident light focus at point O on the screen. Now consider the secondary waves from each slit traveling in a direction θ are equivalent

to a single wave of amplitude $\frac{A \sin \alpha}{\alpha}$.

Let us consider the diffracted waves at the corresponding points A and C. AN is the normal drawn to CN.

The path difference between the waves on reaching the point P is CN.

$$\text{From } \Delta^{le} ACN \quad \sin \theta = \frac{CN}{AC}$$

$$CN = AC \sin \theta$$

$$CN = (e+d) \sin \theta$$

$$\frac{2\pi}{\lambda} (e+d) \sin \theta = 2\beta \text{ (say)}$$

And the corresponding phase difference is

Therefore the resultant amplitude at P in the direction of θ is

$$R = \frac{A \sin \alpha}{\alpha} \frac{\sin N\beta}{\sin \beta}$$

The Resultant intensity is

$$I = R^2 = \frac{A^2 \sin^2 \alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2 \beta}$$

$$\frac{A^2 \sin^2 \alpha}{\alpha^2}$$

The first factor gives a diffraction pattern due to a single slit, while the second factor

$$\frac{\sin^2 N\beta}{\sin^2 \beta}$$

gives the interference pattern due to N slits. The variation in intensity is due to second factor and is discussed below.

Case i: Principal Maxima

When $\sin \beta = 0$,

i.e $\beta = \pm n\pi$ where $n = 0, 1, 2, \dots$

$$\frac{\sin N\beta}{\sin \beta} = \frac{0}{0}$$

We have $\sin N\beta = 0$ and thus which is indeterminate

By applying L hospital rule

$$\text{Lt}_{\beta \rightarrow n\pi} \frac{\sin N\beta}{\sin \beta} = N$$

Then $\beta = \pm n\pi$

These maxima are most intense and are called principal maxima. These are obtained in the direction

$$\beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(e + d) \sin \theta = \pm n\pi$$

$$\underline{(e + d) \sin \theta = \pm n \lambda}$$

Where $n = 0, 1, 2, \dots$ we get zero order maximum, first order maximum....

Case 2: Condition for minima

When $\sin N\beta = 0$, but $\sin \beta \neq 0$

$$\frac{\sin N\beta}{\sin \beta} = 0, \text{ then } I = 0$$

These minima are obtained in the directions given by $\sin N\beta = 0$

$$N\beta = \pm m\pi$$

$$N \frac{\pi}{\lambda}(e + d) \sin \theta = m\pi$$

$$N(e+d) \sin \theta = m\lambda$$

Where m takes all the integral values except 0, N, 2N, ..., nN.

Case 3: Secondary maxima:

The positions are obtained by differentiating I w.r.t β and equating to 0

$$\frac{dI}{d\beta} = \frac{A^2 \sin^2 \alpha}{\alpha^2} \cdot 2 \left[\frac{\sin N\beta}{\sin \beta} \right] \frac{N \cos N\beta \sin \beta - \sin N\beta \cos \beta}{\sin^2 \beta} = 0$$

$$N \cos N\beta \sin \beta - \sin N\beta \cos \beta = 0$$

$$\tan N\beta = N \tan \beta$$

$$\frac{\sin^2 N\beta}{\sin^2 \beta} = \frac{N^2}{1 + (N^2 - 1) \sin^2 \beta}$$

$$I = \frac{A^2 \sin^2 \alpha}{\alpha^2} \frac{N^2}{1 + (N^2 - 1) \sin^2 \beta} \quad \text{In an actual grating, } N \text{ is very large. Hence these secondary maxima are not visible.}$$

- 6. Analyse the Grating Spectrum obtained when a plane grating is exposed to monochromatic and multi chromatic light.**

Grating Spectrum: Theory of formation of principal maxima

The position of the principal maxima is given by equation.

$$(e+d) \sin \theta_n = \pm n\lambda \quad (\text{where } n=0, 1, 2, 3, \dots)$$

This relation is called grating equation.

The angle of diffraction depends on the values of λ and n .

For a particular wavelength λ , the angle of diffraction θ is different for different orders of principal maxima.

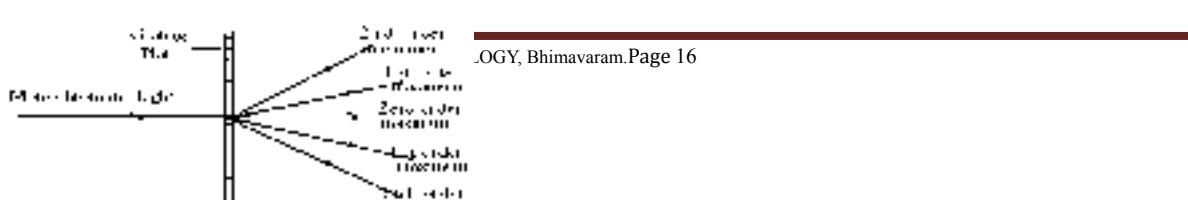
The principal maximum occurs at $\theta = 0$ irrespective of the wave length λ , which is known as

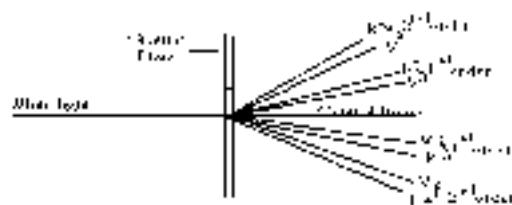
zeroth order.

The 1st order maxima obtained for $n=1$ then $(e+d) \sin \theta_1 = \lambda$

The 2nd order maxima obtained for $n=2$ then $(e+d) \sin \theta_2 = 2\lambda$ and so on.

Grating spectrum with the monochromatic and white light are shown below:





7. Derive an expression for maximum no of possible orders in a diffraction grating.

We know that the positions of the principal maxima are given by the equation

$$(e+d) \sin \theta = n\lambda \quad \text{where } n= 0, 1, 2, 3, \dots$$

$$n = \frac{(e+d) \sin \theta}{\lambda}$$

Where $(e+d)$ is the grating element.

n is the order of maxima.

λ is the wave length of the incident light.

The maximum angle of diffraction is 90° .

Hence maximum possible orders are given by

$$n \leq \frac{(e+d) \sin \theta}{\lambda} \leq \frac{e+d}{\lambda}$$

$$n \leq \frac{1}{N\lambda} \quad (\square N(e+d) = 1 \text{ inch} \Rightarrow (e+d) = \frac{1}{N} \text{ inch} = \frac{2.54}{N} \text{ cm.})$$

$$n_{\max} \leq \frac{1}{N\lambda}$$

10. Write the applications of Diffraction:

Diffraction is used

1. To measure the wavelength of spectral lines.
2. To determine the structure of crystals.
3. To find the velocity of the sound.
4. To know size and shape of the tumors etc., inside the human body can be assessed by Ultrasound scanning.

SOLVED PROBLEMS

1. Calculate the possible order of spectra with a plane transmission grating having 18000 lines per inch when light of wavelength $4500 \text{ } \text{\AA}$ is used.

Given : $d = 18000 \text{ lines/inch} = 7.09 \times 10^5 \text{ lines/m}$, $\lambda = 4500 \text{ } \text{\AA} = 4500 \times 10^{-10} \text{ m}$

Solution: Order of spectra, $n = \frac{d \sin \theta}{\lambda}$. The highest order occurs when $\theta = 90^\circ$

$$\therefore n = \frac{d}{\lambda} = \frac{1}{N\lambda} = \frac{1}{(7.09 \times 10^5 \text{ lines/m})(4500 \times 10^{-10} \text{ m})} = 3$$

2. In a plane transmission grating the angle of diffraction for the second order principal maximum is 30° for a light of wavelength $5 \times 10^{-5} \text{ cm}$. Calculate the number of lines/cm on the grating surface

Given: $\lambda = 5 \times 10^{-5} \text{ cm}$, $\theta = 30^\circ$, $m = 2$

Solution: The grating equation for normal incidence is $n\lambda = d \sin \theta$

$$\therefore d = \frac{n\lambda}{\sin \theta} = \frac{2 \times 5 \times 10^{-5}}{0.5} = 2 \times 10^{-4} \text{ cm.}$$

$$\text{Therefore number of lines/cm} = \frac{1}{d} = \frac{1}{2 \times 10^{-4}} = 5000.$$

The resolving power of a grating is $\frac{\lambda}{d\lambda} = nN$, where N is the total number of lines on a length of 2.5 cm.

$$\therefore N = \frac{\lambda}{n d\lambda} = \frac{5893 \times 10^{-8}}{2 \times 6 \times 10^{-8}} = 491$$

$$\text{Number of lines per cm} = \frac{491}{2.5} = 196.4$$

POLARIZATION

Introduction - Types of Polarization - Double refraction - Quarter wave plate and Half Wave plate.

1. What are the different types of polarized light?

Plane or Linearly polarized light

If electric component of light passing through the medium vibrates only along a single direction, Perpendicular to the direction of propagation, the wave is said to be plane polarized light or linearly polarized.

The electric vector can be resolved into two rectangular components E_x & E_y . Therefore electric Vector may be considered as superposition of two mutually perpendicular electric fields.

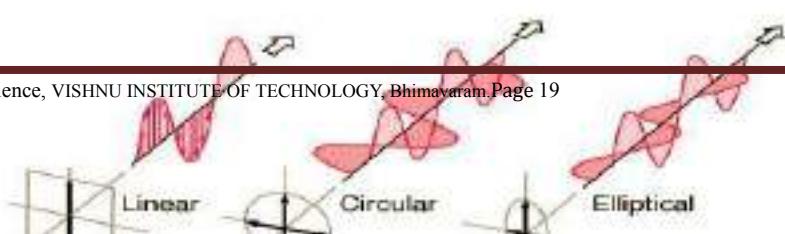


Circularly polarized light

If the two electric vectors E_x & E_y having same magnitude but vibrating in two mutually perpendicular planes at a phase difference of $\Pi/2$ radians superimpose, the magnitude of the resultant vector E remains constant about the direction of the propagation. Such light is called CIRCULARLY POLARIZED LIGHT.

Elliptically polarized light

If the two electric vectors E_x & E_y having unequal magnitudes vibrating in two mutually perpendicular planes superimpose at a phase difference of $\Pi/2$ radians, the magnitude of the resultant vector E changes with the time and the vector E sweeps a flattened helix in space. Such light is Called ELLIPTICALLY POLARIZED LIGHT.

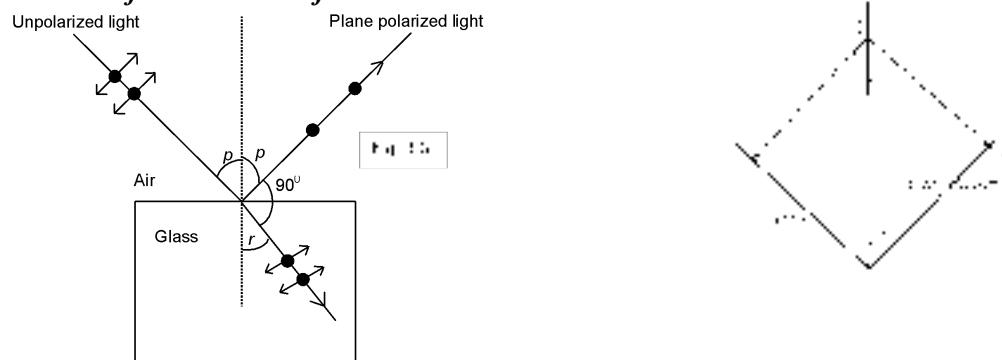


2. Explain Brewster's law and Malus Law.

Brewster's law:

When ordinary light is reflected from the surface of a transparent medium like glass, it becomes partially polarized. The degree of polarization varies with the angle of incidence.

Brewster proved that the tangent of the angle of polarization (p) is numerically equal to the refractive index of the medium.



$$\mu = \tan p \quad (\text{Brewster's law})$$

From Brewster's law,

$$\mu = \tan p = \frac{\sin p}{\cos p}.$$

From Snell's law, $\mu = \frac{\sin p}{\sin r}$.

From the above two equations, we get,
 $\sin r = \cos p = \sin (90^\circ - p)$

$$r = 90 - p \quad \text{or} \quad r + p = 90^\circ$$

Malus Law:

This law states that **the intensity of light transmitted from the analyzer is proportional to the square of the cosine of the angle between the planes of transmission of analyzer and polarizer.**

$$I = I_0 \cos^2 \theta$$

Where I_0 = the intensity of plane polarized light incident on the analyzer and θ is the angle

between the planes of transmission of polarizer and analyzer.

The intensity of transmitted light is maximum when the planes of polarizer and analyzer are parallel, i.e. when $\theta=0$, $I = I_o$ (max).

When the two planes are perpendicular, there is no transmitted light i.e.

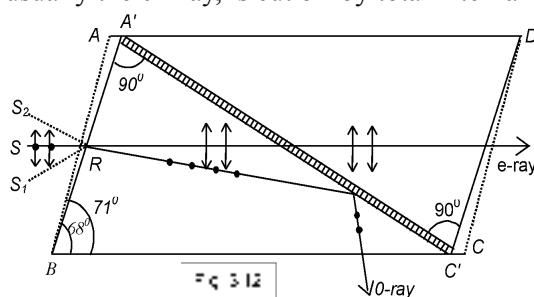
When $\theta=90^\circ$, $I = 0$.

3. Explain the construction and working Principle of Nicol Prism.

Nicol Prism is an optical device used for producing and analyzing plane polarized light.

Principle:

When light is passed through a doubly refracting crystal, it is split up into ordinary ray and extraordinary ray. Both these rays are plane polarized perpendicular to each other. One of these rays, usually the O - ray, is cut off by total internal reflection.



Construction:

Consider a calcite crystal whose length is three times as that of its width. The end faces of this crystal are ground in such a way that the angle in the principal section becomes 68° and 112° instead of 71° and 109° . The calcite crystal is cut into two pieces by a plane perpendicular to the principal section as shown in fig. The cut faces are ground and polished optically flat and then cemented together by Canada balsam which is a transparent substance. The refractive index of it is in between that of a O-ray and e-ray i.e. for sodium light $\mu_0 = 1.6584$,

$$\mu_{\text{canada balsam}} = 1.55 \text{ and } \mu_e = 1.4864.$$

Working:

When a beam of light enters into Nicol Prism, it is doubly refracted into o-ray and e-ray. Canada balsam acts as a rarer medium for an ordinary ray and denser medium for an extra ordinary ray. Therefore, the o-ray gets completely reflected by total internal reflection and the e-ray emerges out of the Nicol Prism with vibrations parallel to the Principal plane.

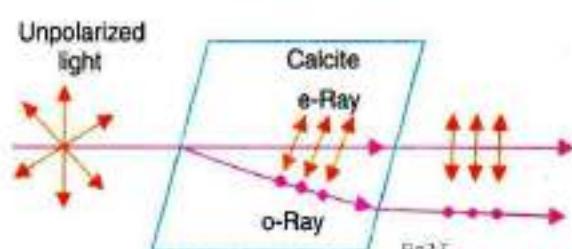
Uses:

Nicol Prisms are used as Polarizers and Analyzers.

4. Write a short note on Double refraction or birefringence.

Polarization by Double refraction:

When a beam of unpolarized light is allowed to fall on a calcite crystal or quartz crystal, it is split up into two refracted beams. The phenomenon is called double refraction or birefringence and such crystals are called doubly-refracting crystals.



The two refracted rays are plane polarized. The refracted ray which obeys the laws of refraction and having vibrations perpendicular to the principal section of the calcite crystal is known as the ordinary ray or the O - ray. The other refracted ray which does not obey the laws of refraction and having vibrations in the principal section is called the extraordinary ray or the e-ray.

5. Explain Quarter wave and Half Wave Plates.

A wave plate is a double refracting uniaxial crystal which introduces specific path difference between the o-ray and e-ray for a particular wavelength of light.

Consider a double refracting uniaxial crystal cut with its optic axis parallel to the refracting faces.

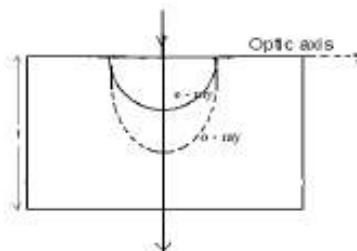
When a plane polarized light of wavelength λ is incident normally to the surface, the light splits up into ordinary and extraordinary rays. They travel in the same direction but with different velocities.

Quarter wave plate:

If the thickness of the crystal plate is such that it introduces a phase difference of $\pi/2$ radians or the path difference of $\lambda/4$, then it is called a Quarter wave plate.

The thickness 't' of the crystal to create a path difference $\lambda/4$ is given by

$$t(o \sim e) = \frac{\lambda}{4}$$



$$\therefore \text{Thickness of the quarter wave plate, } t = \frac{\lambda}{4(\mu_o - \mu_e)}$$

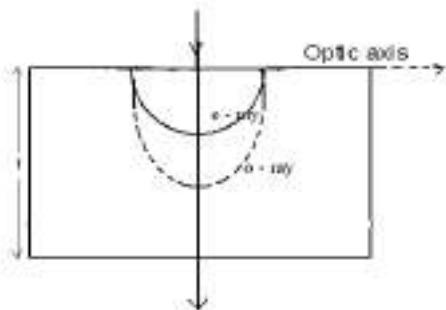
$$t = \frac{\lambda}{4(\mu_e - \mu_o)}$$

For a positive crystal (such as Quartz),

$$t = \frac{\lambda}{4(\mu_o - \mu_e)}$$

For a negative crystal (such as calcite),

A quarter wave plate is used to convert linearly polarized light into circularly polarized light. So it is used for the production and detection of circularly polarized light.



Half - Wave plate:

If the thickness of the crystal plate is such that it introduces a phase difference of radians or a path difference of

$$\frac{\lambda}{2}$$
, then it is called a half wave plate.

The thickness of the crystal to create a path difference of $\lambda/2$

$$t = \frac{\lambda}{2(\mu_o - \mu_e)}$$

is given by

$$t = \frac{\lambda}{2(\mu_e - \mu_0)}$$

For a positive crystal, like quartz,

$$t = \frac{\lambda}{2(\mu_o - \mu_e)}$$

For a negative crystal (such as calcite),

A half wave plate is used to convert the left handed circularly polarized in to right handed and vice versa.

SOLVED PROBLEMS

- 1. Calculate the thickness of a half wave plate of quartz for a wavelength of 5000A^0 . Given $\mu_e = 1.553$ and $\mu_0 = 1.544$.**

$$t = \frac{\lambda}{2(\mu_e - \mu_0)} = \frac{5000 \times 10^{-8}}{2(1.553 - 1.544)} = 2.78 \times 10^{-7} \text{ cm}$$

- 2. Calculate the thickness of the doubly refracting plate which can produce a path difference of $\frac{\lambda}{4}$ between the e-ray and o-ray**

Given: $\lambda = 5890 \text{ \AA}^0$, $\mu_0 = 1.53$, $\mu_e = 1.54$

$$\text{Solution: } t = \frac{\lambda}{4(\mu_e - \mu_0)} = \frac{5890 \text{ \AA}^0}{4(1.54 - 1.53)} = 1.45 \times 10^5 \text{ \AA}^0 = 14.5 \mu\text{m}$$

- 3. Plane polarized light passes through a calcite plate with its optic axis parallel to the faces. Calculate the least thickness of the plate for which the emergent light will be plane polarized.**

Given: $\mu_0 = 1.6584$, $\mu_e = 1.4864$ and $\lambda = 5000 \text{ \AA}^0$

Solution: When a plane polarized light is incident on a half wave plate, the emergent light will also be plane polarized.

$$t = \frac{\lambda}{2(\mu_o - \mu_e)} = \frac{5000 \times 10^{-10} \text{ m}}{2(1.6584 - 1.4864)} = 1.45 \mu\text{m}$$

UNIT II - LASERS & FIBER OPTICS**LASERS**

LASER is an acronym for Light Application by Stimulated Emission of Radiation.

- ❖ Albert Einstein in 1917 theoretically proved the process of stimulated emission.
- ❖ In 1954, C.H. Townes operated a microwave device MASER – "Microwave Amplification by Stimulated Emission of Radiation".
- ❖ In 1960, T.H. Maiman first achieved laser action at optical frequency using Ruby laser.
- ❖ In 1961, A. Javan developed first gas laser that is He-Ne laser.
- ❖ In 1964, C.K.N. Patel developed CO₂ laser.
- ❖ In 1962, Nathan and Hall developed GaAs semiconductor laser called diode laser.

Characteristics of laser:

The important characteristics of laser are:

1. High monochromaticity
2. High directionality
3. High intensity (brightness)
4. High coherence

1. Monochromaticity: Monochromaticity is represented in terms of line width or spectral width of the source $\Delta\nu$ which is the frequency spread of a spectral line. Frequency spread $\Delta\nu$ is related to wavelength spread $\Delta\lambda$ by

$$\Delta\lambda = \left(\frac{c}{\nu^2}\right) \Delta\nu$$

For white light source $\Delta\lambda = 300$ nm

For gas discharge lamp $\Delta\lambda = 0.01$ nm

For Laser light source $\Delta\lambda = 0.001$ nm

So laser light is highly monochromatic light.

2. Directionality: The degree of directionality is represented in terms of beam divergence.

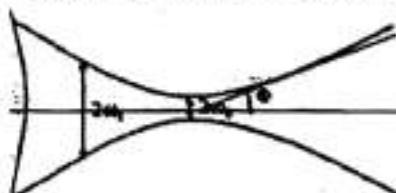
The beam divergence Φ is given in terms of the minimum spot size a_0 as $\Phi = (1.22\lambda/2a_0)$

For a typical small laser the beam divergence is about 1 milli radian. Laser beam increases in size about 1mm for every meter of beam travel for a beam divergence of 1 milli radian.

At d_1 and d_2 distances from the laser window, if the diameter of the spots are measured to be a_1 and a_2 respectively then the angle of divergence in degrees can be expressed as $\Phi = \frac{(a_2 - a_1)}{2(d_2 - d_1)}$.

For laser light divergence $\phi = 10^{-3}$ radians. Since the divergence of light is very low, we say that the laser light is highly directional.

3. Intensity: In laser many numbers of photons are in phase with each other, the amplitude of the resulting wave becomes ' nA ' and hence the intensity of laser is proportional to $(nA)^2$. Thus due to coherent addition of amplitude and negligible divergence the intensity increases enormously. 1mw He-Ne laser is highly intense than the sun.



4. Coherence: A predictable correlation of the amplitude and phase at any one point with other point is called coherence. There are two types of coherences.

- Temporal coherence
- Spatial coherence

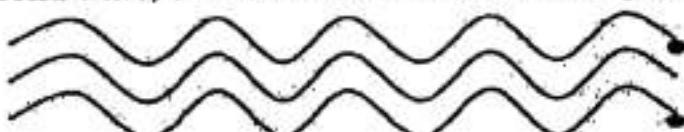
Temporal coherence (or longitudinal coherence):

The predictable correlation of amplitude and phase between any two points on a wave train is called temporal coherence.



Spatial coherence (or transverse coherence):

The predictable correlation of amplitude and phase at one point on the wave train w.r.t another point on a second wave, then the waves are said to be spatially coherent.



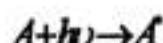
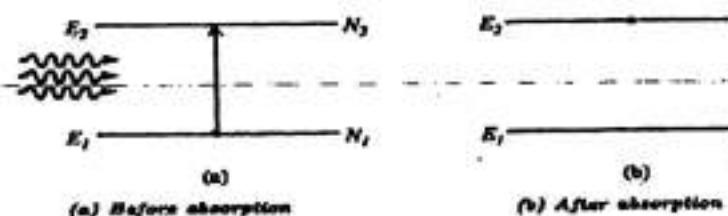
Laser light is highly coherent because two waves in laser beam maintain high degree of spatial and temporal coherences.

Interaction of radiation with matter:

In lasers the interaction between matter and light is of three different types.

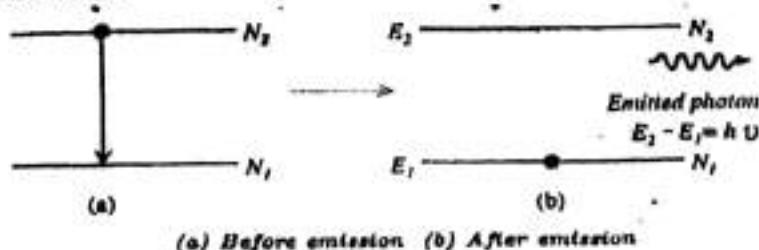
1. Stimulated absorption (or) induced absorption.
2. Spontaneous emission.
3. Stimulated emission

1. Stimulated absorption (or) induced absorption:



Initially let the atom is in ground state E_1 . When a photon of energy $h\nu = E_2 - E_1$ incident on this atom then the atom is excited to higher energy level (excited state) E_2 by absorbing incident energy. This is called stimulated absorption.

2. Spontaneous emission:

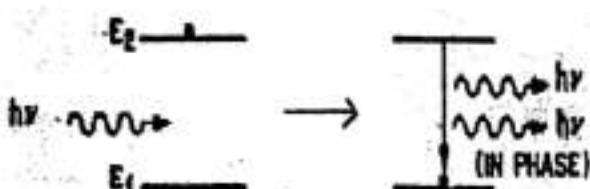


Initially the atom is in the excited state. This excited atom comes to ground state by emitting photon of Energy $\hbar\nu = E_2 - E_1$ on its own after the lifetime of excited state (i.e. 10^{-8} sec). This type of emission without any external agency is called spontaneous emission.

Different atoms of the medium emit photons at different times and in different directions. Hence there is no phase relationship among the emitted photons. So they are incoherent and intensity of light is very low.

Ex: Light from ordinary source (tube light, electric bulb, candle flame).

3. Stimulated emission:

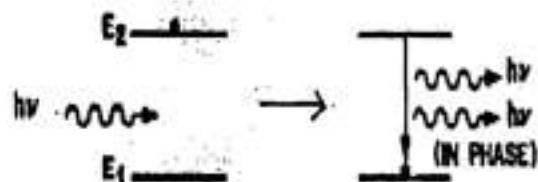
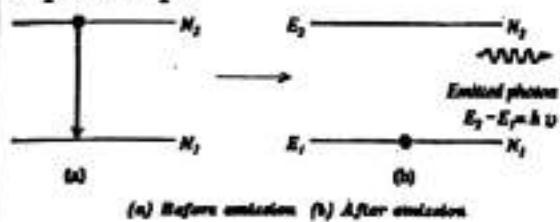


As shown in fig. a photon having energy $\hbar\nu = E_2 - E_1$ incident on an atom present in the excited state. The atom is stimulated to make transition to ground state and gives off a photon of energy $\hbar\nu = E_2 - E_1$. The emitted photon is in phase with the incident photon. The two photons travel in the same direction and they possess same energy and frequency. They are coherent and the intensity of light is very high. This type of forceful emission is called stimulated emission.

Ex: Light from LASER.

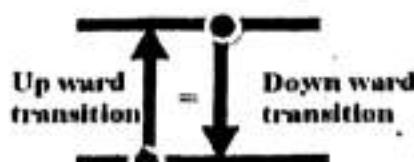
Differences between spontaneous and stimulated emission:

Spontaneous emission	Stimulated emission
1. Emission of light photon takes place immediately (10^{-8} sec) without any inducement during the transition of atoms from higher energy level to the lower energy level.	1. Emission of light photon takes place by the inducement of a photon having energy equal to the energy difference between transition energy levels $\hbar\nu = E_2 - E_1$.
2. Single photon is emitted.	2. Two photons with same energy are emitted.
3. The energy of emitted photon is equal to the energy difference of two energy levels.	3. The energy of the emitted photons is double the energy of stimulated photons.
4. This was postulated by Bohr.	4. This was postulated by Einstein.
5. Incoherent radiation	5. Coherent radiation
6. Less intensity	6. High intensity
7. Polychromatic radiation	7. Monochromatic radiation
8. Less directionality	8. High directionality
9. More angular spread during propagation	9. Less angular spread during propagation
10. Ex. Light from sodium or mercury vapor lamp.	10. Ex. Light from LASER

**Einstein coefficients:**

It establishes the relation between the three coefficients i.e. stimulated absorption, spontaneous emission and stimulated emission coefficients.

Let N_1 be the number of atoms per unit volume with energy E_1 and N_2 be the number of atoms per unit volume with energy E_2 and $p(v)$ be the density of photons. Let ' n ' be the number of photons per unit volume at frequency ' v ' such that $E_2 - E_1 = \hbar v$. The energy density of interacting photons $p(v) = nhv$. When the photons interact with ground level atoms, both upward (absorption) and downward (emission) transition occurs. At the equilibrium the upward transitions must be equal to downward transitions.



Relation between Einstein coefficients (B_{12} , A_{12} & B_{21}):

Upward transition (Stimulated absorption):

Stimulated absorption rate depends upon the number of atoms available in the lowest energy state as well as the energy density of photons.

Stimulated absorption rate $\alpha \rho(v)$

Stimulated absorption rate αN_1

Stimulated absorption rate = $B_{12} \rho(v) N_1$ (1)

Where B_{12} is the Einstein coefficient of stimulated absorption.

Downward transition (Emission):

Spontaneous emission: The atom in the excited state returns to ground state emitting a photon of energy (E) = $E_2 - E_1 = h\nu$, spontaneously known as spontaneous emission. The spontaneous emission rate depends up on the number of atoms present in the excited state.

Spontaneous emission rate αN_2

Spontaneous emission rate = $A_{21} N_2$ (2)

Where A_{21} is the Einstein coefficient of spontaneous emission.

Stimulated emission: The atom in the excited state can also returns to the ground state by applying external energy, thereby emitting two photons which are having same energy as that of incident photon. This process is called as stimulated emission. Stimulated emission rate depends upon the number of atoms available in the excited state as well as the energy density of incident photons.

Stimulated emission rate αN_2

Stimulated emission rate $\alpha \rho(v)$

Stimulated emission rate = $B_{21} \rho(v) N_2$ (3)

Where B_{21} is the Einstein coefficient of stimulated emission

At thermal equilibrium

Up ward transition = Down ward transition

\therefore Stimulated absorption = spontaneous emission + stimulated emission

$$B_{12} \rho(v) N_1 = A_{21} N_2 + B_{21} \rho(v) N_2$$

$$(B_{12} N_1 - B_{21} N_2) \rho(v) = A_{21} N_2$$

$$\rho(v) = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

$$\rho(v) = \frac{A_{21} N_2}{B_{21} N_2 \left(\frac{B_{12} N_1}{B_{21} N_2} - 1 \right)}$$

$$\rho(v) = \frac{A_{21}}{B_{21} \left(\frac{B_{12} N_1}{B_{21} N_2} - 1 \right)}$$

According to Boltzman distribution law

$$N_1 = N_0 e^{-E_1/kT}$$

$$N_2 = N_0 e^{-E_2/kT}$$

$$\frac{N_1}{N_2} = e^{E_2 - E_1/kT} = e^{\frac{h\nu}{kT}}$$

$$\therefore \rho(v) = \frac{A_{21}}{B_{21} \left(\frac{B_{12}}{B_{21}} e^{\frac{h\nu}{kT}} - 1 \right)} \dots \dots \dots (4)$$

According to the plank energy distribution law

$$\rho(v) = \frac{8\pi h v^3}{c^3 (e^{\frac{h\nu}{kT}} - 1)} \dots \dots \dots (5)$$

On comparing Eq (4) and (5)

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h v^3}{c^3} \dots \dots \dots (6)$$

$$\frac{B_{12}}{B_{21}} = 1 \Rightarrow B_{12} = B_{21} \dots \dots \dots (7)$$

The Eq (6) & (7) are called Einstein Relations

Conditions for light Amplifications:

At thermal equilibrium,

$$\frac{\text{Stimulated emission}}{\text{Spontaneous emission}} = \frac{B_{21} N_2 \rho(v)}{A_{21} N_2} = \frac{B_{21}}{A_{21}} \rho(v) \dots \dots \dots (8) \quad \text{And}$$

$$\frac{\text{Stimulated emission}}{\text{Stimulated absorption}} = \frac{B_{21} N_2 \rho(v)}{B_{12} N_1 \rho(v)} = \frac{N_2}{N_1} \dots \dots \dots (9)$$

From Eq(8) & (9) we concluded that for getting light amplification (getting Laser)

1. The radiation density $\rho(v)$ is to be made larger.
2. $N_2 > N_1$ (Population inversion)

Population inversion:

Let us consider two level energy system of energies E_1 and E_2 as shown in figure. Let N_1 and N_2 be the populations of energy levels E_1 and E_2 . The number of atoms present in an energy level is known as population of that energy level. At ordinary conditions, the population in the ground or lower state is always greater than the population in the excited or higher states.

The stage of making, population of higher energy level greater than the population of lower energy level is called population inversion.
According to Boltzmann's distribution the population of an energy level E at temperature T is given by

$$N_t = N_0 e^{\left(\frac{-E_t}{kT}\right)}$$

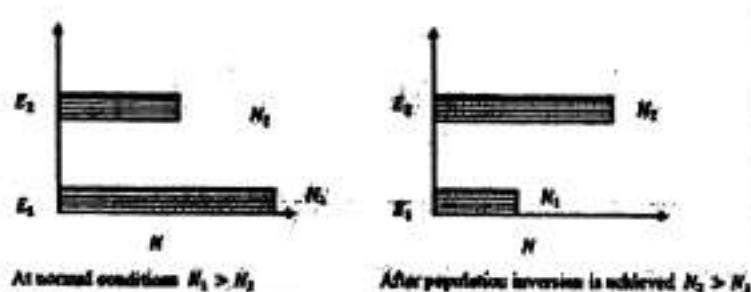
Where N_0 is the population of the lower level or ground state and K is the Boltzmann's constant.

From the above equation the population of energy levels E_1 & E_2 are given by

$$N_1 = N_0 e^{-E_1/kT}$$

$$N_2 = N_0 e^{-E_2/kT}$$

At ordinary conditions $N_1 > N_2$ i.e., the population in the lower state is always greater than the population in the higher states. The stage of making, population of higher energy level greater than the population of lower energy level ($N_2 > N_1$) is called population inversion or inverted population.



Conditions for population inversion are:

- The system should possess at least a pair of energy levels ($E_2 > E_1$), separated by an energy equal to the energy of a photon ($h\nu$).
- There should be a continuous supply of energy to the system such that the atoms must be raised continuously to the excited state.

Pumping mechanisms or Methods to produce population inversion:

The process of raising the particles from ground state to excited state to achieve population inversion is called pumping.

Methods of achieving population inversion:

Excitation of atom can be done by number of ways. The most commonly used excitation methods are

1. Optical pumping
2. Electrical discharge pumping
3. Chemical pumping
4. Injection current pumping

Optical pumping:

- Optical pumping is a process in which light is used to raise the atoms from a lower energy level to higher level to create population inversion.
- Optical pumping is used in solid state laser.
- The solid materials have very broad absorption band, so sufficient amount of energy is absorbed from the emission band of flash lamp to create population inversion.
- Xenon flash tubes are used for optical pumping.

Ex: Ruby laser, Nd:YAG Laser, Nd: Glass Laser

Electrical discharge pumping:

- In electric discharge pumping, atoms are excited into excited state by collisions with fast moving electrons in electric discharge tube.
- Electrical discharge pumping is used in gas lasers.
- Since gas lasers have very narrow absorption band, so optical pumping is not suitable for gas lasers.
- Ex: He-Ne laser, CO₂ laser, Argon-ion laser, etc

Chemical pumping:

- In this method the chemical energy released during the chemical process, that energy will excite the atoms to higher level and create population inversion.
- Whenever hydrogen reacts with fluorine, it liberates lot of heat energy. By utilizing this heat energy the atoms excites into higher states and create population inversion.
- Ex: HF and DF lasers.

Injection current pumping:

- This pumping mechanism is used in semiconductor lasers.
- In semiconductor lasers, by passing high currents across the junction, the population inversion will create.
- In semiconductors lasers the population inversion always creates among majority and minority charge carriers.
- Ex: InP and GaAs lasers

Pumping schemes or Levels of laser:

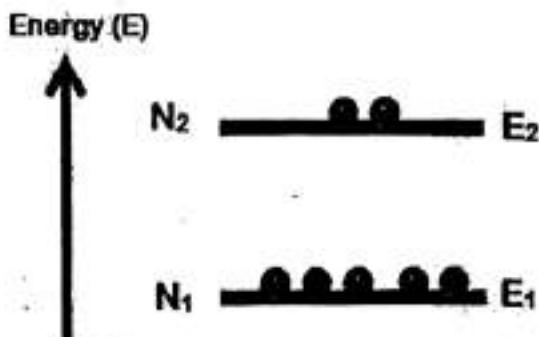
Two level pumping scheme:

Consider a system with two energy levels E_1 and E_2 in which the total number of atoms N are distributed. Let N_1 and N_2 be the number of atoms in these states.

$$N_1 = N_0 e^{-E_1/kT} \text{ and}$$

$$N_2 = N_0 e^{-E_2/kT}$$

$$\frac{N_1}{N_2} = e^{E_2 - E_1/kT} = e^{\frac{h\nu}{kT}}$$



If $h\nu > kT$ then $N_1 > N_2$

If $h\nu < kT$ then $N_1 = N_2$

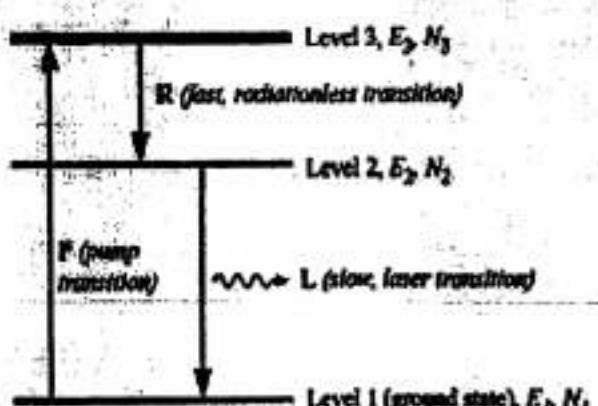
If $h\nu = kT$ then $N_1 = e.N_2$

That is the number of atoms in lower energy state is always greater than or equal to number of atoms in the higher energy state. Therefore in a two level system population inversion cannot be achieved and hence it is not suitable for the production of laser.

Three level pumping scheme:

In three level pumping scheme, three energy levels are involved in lasing action. They are the ground state energy (E_1), excited state energy (E_3) and metastable state energy (E_2) as shown in fig.

In the laser, initially all the lasing atoms are present in the ground state energy level, (E_1). Light photons of energy ($E_3 - E_1$) is made to incident on the ground state lasing atoms. The atoms in the ground state energy level will absorb the energy of photons and make transition to excited state energy (E_3). The atoms in the excited energy level (E_3), will remain for very short duration of the order of 10^{-8} sec and make non radiative transaction to metastable state E_2 . The atoms will remain for longer duration (10^{-3} sec) in metastable state. If photons are continuously supplied to ground to ground state atoms then more number of atoms makes transitions to excited state and then to metastable state fastly, so that more number of atoms will accumulate in metastable than in ground state. This is known as population inversion. Now a chance proton of transition from $(E_2 - E_1) = h\nu$ can trigger stimulated emission as shown in fig. The atoms present in E_2 energy level will make stimulated transition to E_1 energy level producing laser.



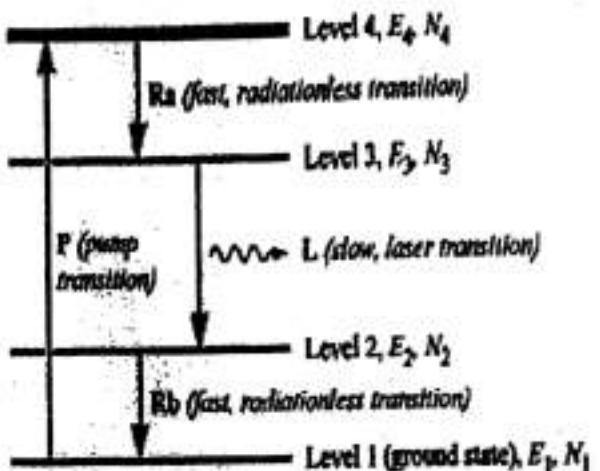
This scheme requires very high pumping power. Once the stimulated emission starts then quickly the metastable state becomes empty, hence population inversion ends. The system has to wait till the population inversion is to be re-established. Because of this, three level laser produces pulses of laser light.

Four level pumping scheme:

In four level pumping scheme, four energy levels are involved in lasing action. They are the ground state energy (E_1), excited state energy (E_4) and metastable state energy (E_3) and lower lasing level (E_2).as shown in fig.

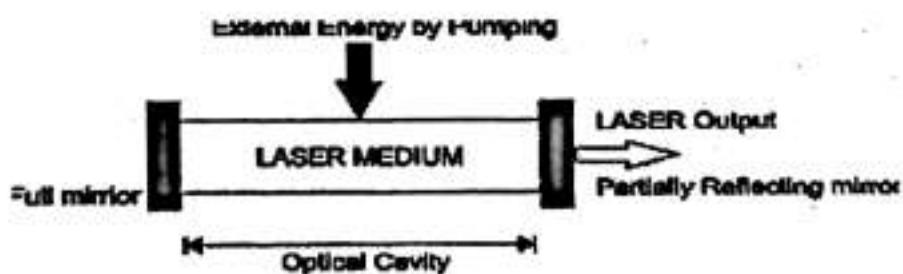
In the laser, initially all the lasing atoms are present in the ground state energy level, (E_1).light photons of energy $E_4 - E_1$ is made to incident on the ground state lasing atoms. The atoms in the ground state energy level will absorb the energy of photons and make transition to excited state energy (E_4).the atoms in the excited energy level (E_4), will remain for very short duration of the order of 10^{-9} sec and quickly drop down to the metastable state (E_3).after some time population inversion between the states E_3 and E_2 .A spontaneous emitted photon of energy $E_3-E_2 = h\nu$ can initiates stimulated emissions. This makes transition of atoms from E_3 to E_2 level. The atoms in the E_2 level will undergo non radiative transition to ground state E_1 level.

In this scheme the lower lasing level is nearly vacant, hence less pumping power is sufficient to get population inversion. The four level laser produces continuous wave (CW) output laser.

**Construction and components of laser:**

Any laser system consists of three important components.

1. Source of energy.
2. Active medium.
3. Optical resonant cavity.



Components of LASER

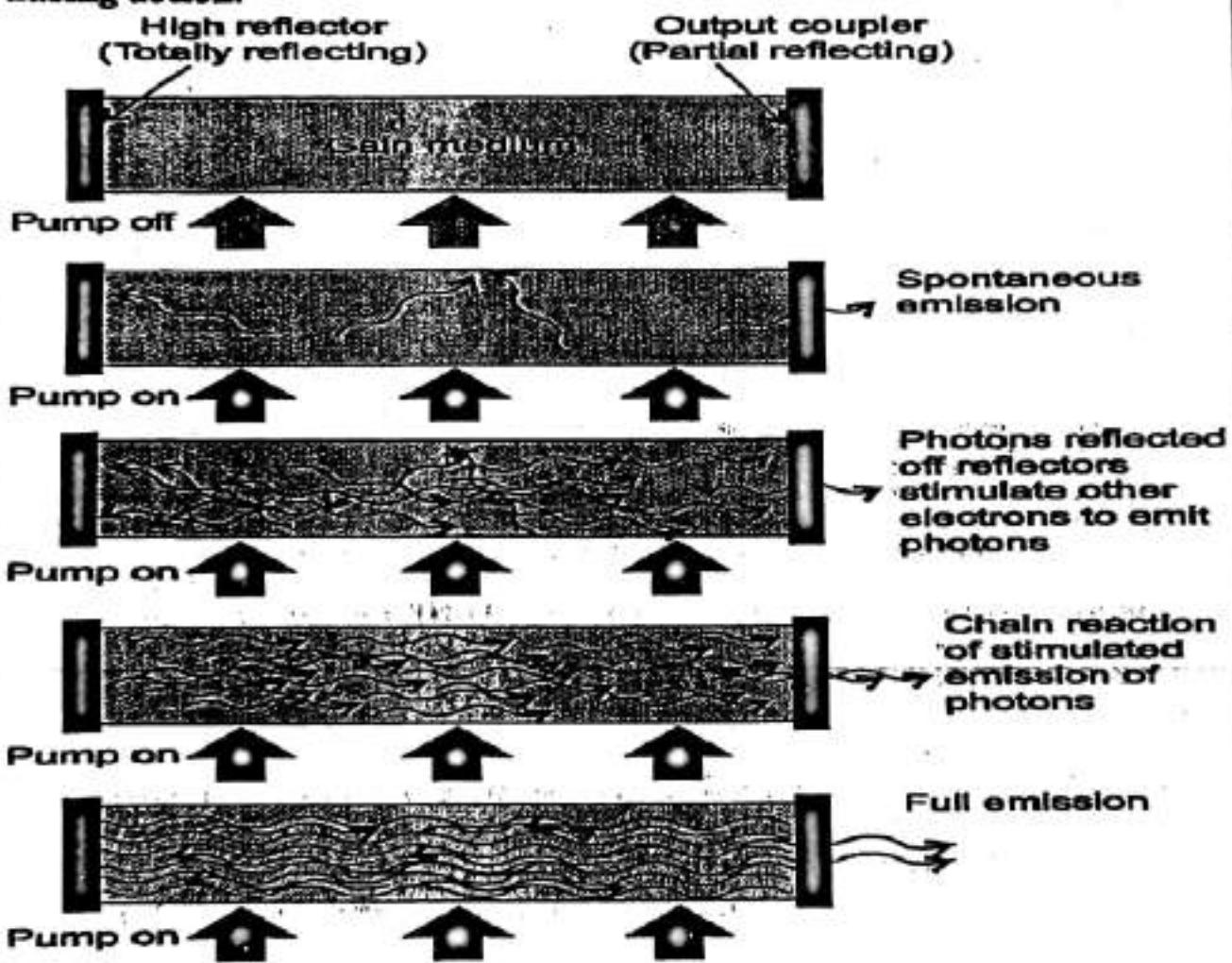
Source of energy: To get laser emission, first we must have population inversion in the system. The source of energy supplies sufficient amount of energy to the active medium by which the atoms (or) molecules in it can be excited to the higher energy level. As a result we get population inversion in an active medium.

Active Medium (or) Laser Medium: This is the medium where stimulated emission of radiation takes place. After receiving energy from the source, the atoms or molecules get

excited to higher energy levels. While transition to a lower energy level, the emitted photons start the stimulated emission process which result in laser emission.

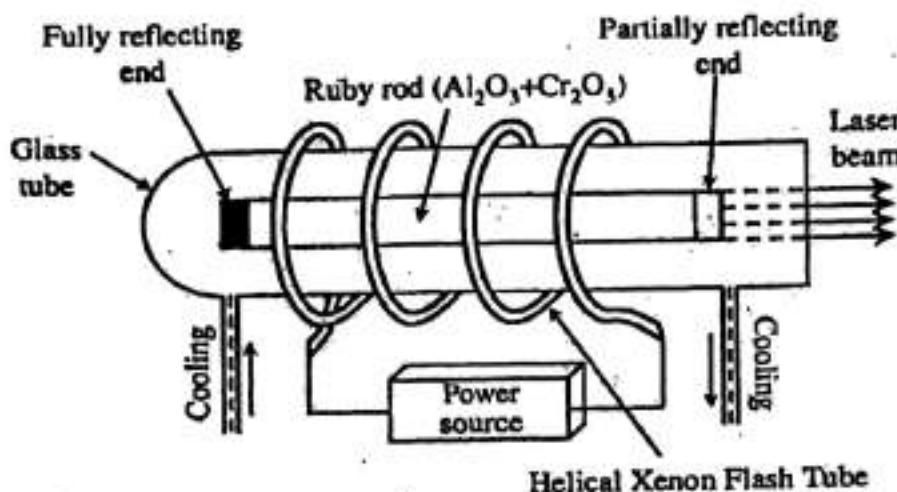
Optical cavity (or) Resonator: The active medium is enclosed between a fully reflecting mirror and a partially reflecting mirror. These mirrors constitute an optical cavity (or) resonator. The reflective portion of the mirrors reflects the incident radiation back into the active medium. These reflected radiations enhance the stimulated emission process with in the active medium. As a result we get high- intensity monochromatic and coherent laser light through the non-reflecting portion of the mirror.

Lasing action:



Ruby Laser:

Ruby laser is a three level solid state laser constructed by T.H. Maiman in the year 1960.



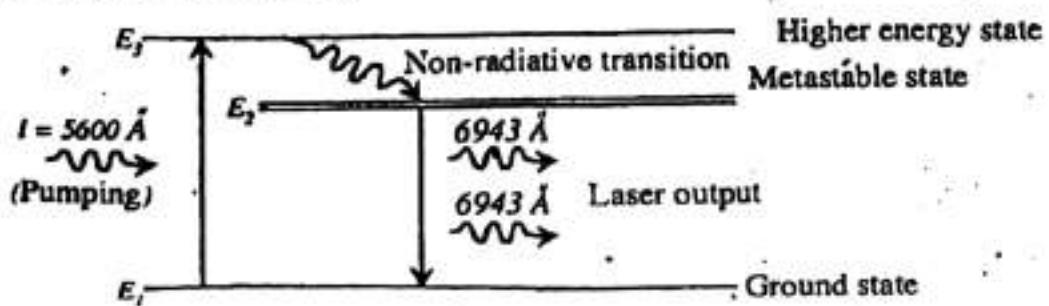
A schematic diagram of Ruby laser

Basically Ruby ($\text{Al}_2\text{O}_3\text{Cr}_2\text{O}_3$) is a Al_2O_3 crystal containing 0.05% of chromium atoms.

1. Active medium: Ruby rod.
2. Active centers: Chromium atoms
3. Pumping method : Optical pumping
4. Exciting source: Helical Xenon flash lamp
5. Cavity Resonator: The partially silvered face and fully silvered face of ruby rod
6. Power output: 10^4 - 10^5 Watts
7. Nature of output: Pulsed
8. Wavelength: 6943 \AA

Construction:

Ruby rod of 4 cm length and 5 mm diameter is kept inside the glass tube. The end faces are grounded and polished such that the end faces are exactly parallel to each other. One of the faces is silvered fully to get full reflection and other partially silvered to get partial reflection. The two silver faces acts as resonating cavity. The Ruby rod is surrounded by helical Xenon flash tube which acts as exciting source. It provides optical pumping for Cr^{3+} ions to move to the higher energy levels. Glass tube is provided with cold water circulation to prevent the damage due to heating from Xenon flash lamp.

Working (Energy level diagram):

Energy level diagram of Chromium ion

The energy levels of Cr^{3+} ions in the crystal lattice are shown in the fig. When xenon flash lamp is switched on, it produces very high intensity radiation. The Cr^{3+} ions in the ground state E_1 absorb sufficient energy of wavelength 5600 \AA and excited to higher energy state E_3 . The Cr^{3+} ions make spontaneous transitions $E_3 \rightarrow E_1$ and $E_3 \rightarrow E_2$ through non-radiative transitions. Since E_2 is metastable state and close to E_3 the transition $E_3 \rightarrow E_2$ is more dominant than the transition $E_3 \rightarrow E_1$. In the course of time the population in E_2 increases and therefore population inversion is achieved between E_2 and E_1 .

After completion of life time in E_2 one of the Cr^{3+} ions decay spontaneously to E_1 giving out a photon of wavelength 6943 \AA . The emitted photons moving parallel to the active system are reflected back by the silvered surfaces. These photons trigger stimulated emission for Cr^{3+} ions in E_2 , hence the rate of stimulated emission increases. When the intense beam is produced, it escapes from semi-silvered surface in the form of laser of wavelength 6943 \AA .

Applications of Ruby laser: Ruby laser is used in

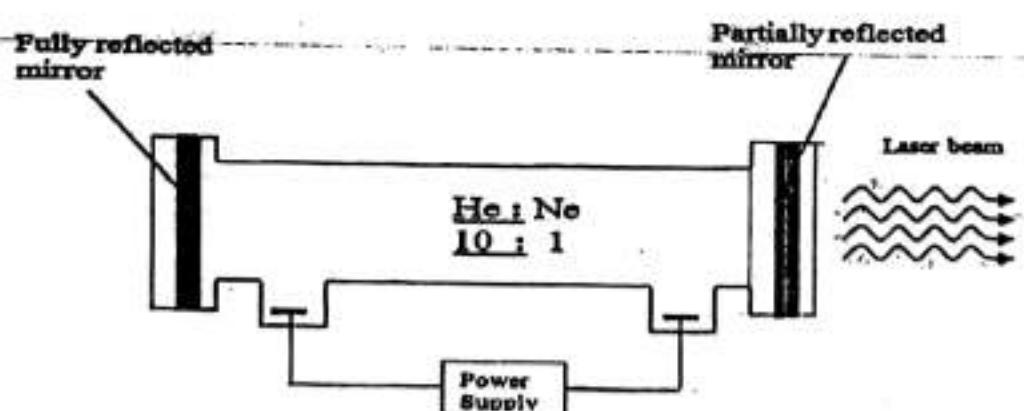
1. Pulsed holography
2. LIDAR
3. Remote sensing
4. Ophthalmology
5. Drilling small areas.

Drawbacks of Ruby laser:

1. The laser beam is not continuous and contains pluses.
2. The efficiency of laser beam is very poor.
3. The laser requires high pumping power to achieve population inversion.

He - Ne laser:

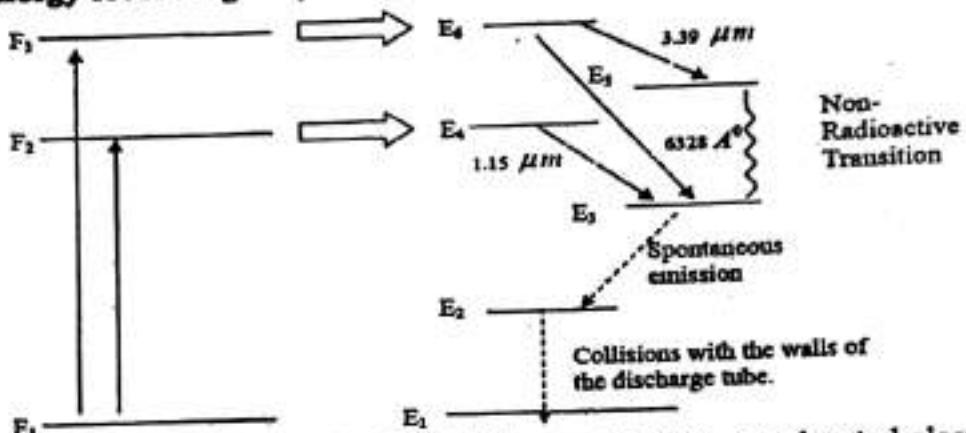
He-Ne laser was the first gas laser fabricated by Ali Javan and others in 1961.



1. Active medium: mixture of He and Ne in the ratio 10:1
2. Active centers: Ne atoms
3. Pumping method: Electrical pumping
4. Exciting Source: Electric discharge
5. Resonating Cavity: Partially & fully reflecting Mirrors
6. Power output: 0.5-50 mW
7. Nature of output: Continuous waveform
8. Wavelength of laser: 6328 \AA

Construction:

In He-Ne gas laser, He and Ne gases are taken in the ratio 10:1 in the gas discharge tube of length 80 cm and diameter 1 cm made up of quartz. Two reflecting mirrors are fixed on either ends of the discharge tube, in that, one is partially reflecting and the other is fully reflecting which serve as optical cavity or resonator. The out power of these lasers depends on the length of the discharge tube and pressure of the gas mixture. When the two windows are set at Brewster's angle, the output laser is linearly polarized.

Working (Energy level diagram):

When the electric discharge passed through the gas mixture, accelerated electrons collide with He atoms and excite them to higher levels F_2 and F_3 . These states are metastable states, so the He atoms stay longer time in these states. During this time He atoms collide with Ne atoms in the ground level E_1 and exchange energy through collisions. This results in the excitation of Ne atoms to the levels E_4 and E_6 and de-excitation of He atoms to ground state F_1 .

Due to the continuous excitation of Ne atoms, population inversion is achieved between the higher levels E_4 (E_6) and lower levels E_3 (E_5). The transitions $E_6 \rightarrow E_5$: $\lambda = 3.39 \mu m$ and $E_4 \rightarrow E_3$: $\lambda = 1.15 \mu m$ corresponds to IR region. The transition $E_6 \rightarrow E_5$ corresponds to visible red light $\lambda = 6328 \text{ Å}$. The Ne atoms present in the E_3 level are de-excited into E_2 level, by spontaneously emitting a photon of wavelength 6000 Å . When a narrow discharge tube is used, the Ne atoms present in the level E_2 collide with the walls of the tube and get de-excited to ground level E_1 . The excitation and de-excitation of He and Ne atoms is a continuous process and thus it gives continuous laser radiations.

Advantages:

1. He-Ne laser emits continuous laser radiation.
2. Due to the setting of end windows at Brewster's angle, the output laser is linearly polarized.
3. Gas lasers are more monochromatic and directional when compared with the solid state laser.

Applications: He-Ne laser is applied in the following fields,

1. Interferometry
2. Laser printing
3. Bar-code reading
4. In metrology in surveying, alignment etc.,
5. In three dimensional recording of objects called holography.

Applications of laser:

Due to high intensity, high mono-chromaticity and high directionality of lasers, they are widely used in various fields like

- | | | | |
|------------------|--------------|--------------|------------------------|
| 1. Communication | 2. Computers | 3. Chemistry | 4. Photography |
| 5. Industry | 6. Medicine | 7. Military | 8. Scientific research |

1. Communication:

- In case of optical communication, semiconductors laser diodes are used as optical sources.
- More channels can be sent simultaneously, signal cannot be tapped as the band width is large, more data can be sent.
- A laser is highly directional and less divergence, hence it has greater potential use in space crafts and submarines.

2. Computers:

- In LAN (local area network), data can be transferred from memory storage of one computer to other computer using laser for short time.
- Lasers are used in CD-ROMS during recording and reading the data.

3. Chemistry:

- Lasers are used in molecular structure identification.
- Lasers are also used to accelerate some chemical reactions.
- Using lasers, new chemical compound can be created by breaking bonds between atoms or molecules.

4. Photography:

- Lasers can be used to get 3-D lens less photography.
- Lasers are also used in the construction of holograms.

5. Industry:

- Lasers can be used to blast holes in diamonds and hard steel.
- Lasers are also used as a source of intense heat.
- Carbon dioxide laser is used for cutting drilling of metals and nonmetals.
- High power lasers are used to weld or melt any material.
- Lasers are also used to cut teeth in saws and test the quality of fabric.

6. Medicine:

- Pulsed neodymium laser is employed in the treatment of liver cancer.
- Argon and carbon dioxide lasers are used in the treatment of liver and lungs.
- Lasers used in the treatment of Glaucoma.

7. Military:

- Lasers can be used as a war weapon.
- High energy lasers are used to destroy the enemy air-crafts and missiles.
- Lasers can be used in the detection and ranging like RADAR.

8. Scientific research: Lasers are used

- in the field of 3D-photography.
- in Recording and reconstruction of hologram.
- to create plasma.
- to produce certain chemical reactions.
- in Raman spectroscopy to identify the structure of the molecule.

Questions

- What are the characteristics of laser beam? Explain.
- (a) With a neat diagram explain (i) absorption (ii) spontaneous emission (iii) stimulated emission
(b) Obtain the relation between Einstein's coefficients.
- What is population inversion and how can it be achieved.
- Describe the various methods to achieve population inversion in lasers.
- Explain the three level and four level laser systems. What are the advantages of four level laser system over three level laser system?
- With neat diagram, describe the construction and working of Ruby laser.
- Distinguish between spontaneous and stimulated emissions.
- Explain the construction and working of He-Ne laser with a neat diagram.

Problems

- Evaluate the wavelength of radiation given out by a laser with $(E_2 - E_1) = 3\text{eV}$.

(Hint: $[E_2 - E_1] = hC/\lambda$; Ans: 4410\AA)

- The He-Ne laser emits laser beam of wavelength 632.8 nm . Calculate the energy difference in eV between the two energy levels of the Neon atom.

(Hint: $[E_2 - E_1] = hC/\lambda$; Ans : 1.964 eV)

- Calculate the relative population in the laser transition levels in a ruby laser in thermal equilibrium ($T = 300\text{ K}$). The wavelength is 6943\AA .

(Hint: $N_1/N_2 = \exp(E_2 - E_1)/KT$; Ans: 1.064×10^{30})Objective Questions

- The spontaneous emission process depends upon
 - Properties of energy states E_1 and E_2
 - incident energy supplied to the atoms
 - both a and b
 - none
- The induced energy require for emission process
 - absorption
 - spontaneous emission
 - Stimulated emission
 - none
- The number of atoms in the lower energy state is _____ that of the higher.
 - more than
 - less than
 - equal to
 - none
- The life time of excited state is
 - 10^{-3} sec
 - 10^{-8} sec
 - 10^3 sec
 - none
- The life time of an atom in the metastable state is
 - 10^{-3} sec
 - 10^{-8} sec
 - 10^3 sec
 - None
- Laser stands for
 - light amplification by spontaneous emission of radiation
 - light modification by spontaneous emission of radiation
 - light amplification by stimulated emission of radiation
 - none
- The laser beam has characteristics of
 - perfect mono chromatic
 - high intensity
 - high coherence
 - all the above
- The mono chromatic light is
 - laser
 - sun light
 - high coherence
 - all the above
- The amount of time in which an atom stays in the excited state is
 - life-time
 - relaxation time
 - met stable time
 - none

10. The expression for number of atoms in any energy state at temperature T is

a. $N = N_0 e^{-\frac{E_1}{kT}}$

b. $N = N_0 e^{\frac{E_1}{kT}}$

c. $N_0 = N e^{-\frac{E_1}{kT}}$

d. none

11. The relation $A_{21}/B_{21} =$

a. $\frac{8\pi h\nu^3}{c^3}$

b. $\frac{8\pi h\nu^3}{c^3}$

c. $\frac{8\pi h\nu^3}{c^2}$

d. $\frac{8\nu^3}{c^2}$

12. The probable rate of transition for spontaneous emission of radiation is

a. $(P_{21})_{sp} = A_{21}$

b. $(P_{21})_{sp} = A_{12}$

c. $(P_{21})_{sp} = U(v)A_{21}$

d. $(P_{21})_{sp} = B_{21}$

13. If A_{21} is the Einstein coefficient for spontaneous emission then the spontaneous emission half time t_{sp} is

a. A_{21}

b. $\frac{1}{A_{21}}$

c. $\frac{1}{\sqrt{A_{21}}}$

d. $\frac{1}{A_{21}^2}$

14. If B is the Einstein coefficient for stimulated emission then the stimulated emission half time t_{sp} is

a. B_{21}

b. $\frac{1}{B_{21}^2}$

c. $\frac{1}{\sqrt{B_{21}}}$

d. $\frac{1}{B_{21}}$

15. The stimulated emission of radiation was forwarded by

a. Bohr

b. Sommerfeld

c. Einstein

d. Maimann

16. If N_1 and N_2 be the number of atoms in the lower and higher energy states E_1 and E_2 respectively then the condition for population inversion

a. $N_1 > N_2$

b. $N_2 > N_1$

c. $N_1 = N_2$

d. $N_1 \geq N_2$

17. The Ruby laser was invented by

a. Einstein

b. Bohr

c. Maiman

d. Ali Javan

18. Ruby laser is

a. Two level laser

b. Three level

c. Four level

d. None

19. The active atoms in the ruby laser

a. Al^{3+}

b. Cr^{3+}

c. Fe^{3+}

d. None

20. The emitted wave length of the laser beam in ruby laser is

a. 6328 Å

b. 6493 Å

c. 6943 Å

d. 5600 Å

21. The He-Ne laser is

a. Solid state laser

b. Continuous laser

c. Pulsed Laser

d. None

22. The ratio of Ne and He atoms in He-Ne laser is

a. 10:1

b. 1:2

c. 1:10

d. 1:1

23. The emitted wave length of the laser beam in He-Ne laser is

a. 6328 Å

b. 6493 Å

c. 6943 Å

d. 5600 Å

FIBER OPTICS

Introduction:

Fiber optics is a branch of physics which deals with the transmission and reception of light waves using optical fibers which acts as a guiding media. The transmission of light waves by optical fiber was first demonstrated by John Tyndall in 1870.

Advantages of optical fibers:

- Higher information carrying capacity.
- Light in weight and small in size.
- No possibility of internal noise and cross talk generation.
- No hazards of short circuits as in case of metals.
- Can be used safely in explosive environment.
- Low cost of cable per unit length compared to copper or G.I cables.
- No need of additional equipment to protect against grounding and voltage problems.
- Nominal installation cost.
- Using a pair of copper wires only 48 independent speech signals can be sent simultaneously whereas using an optical fiber 15000 independent speech signals can be sent simultaneously.

Optical Fiber:

Optical fiber is a long, thin transparent dielectric material made up of glass or plastic. It carries electromagnetic waves of optical frequencies (visible and infrared) from one end of the fiber to the other end by means multiple total internal reflections. Thus optical fiber works as guiding medium in optical communication systems.

Construction:

An optical fiber is a very thin, flexible transparent material made with plastic or glass. It has cylindrical shape consisting of three layers or sections

1) Core

2) Cladding

3) Sheath or outer jacket or buffer jacket

1) Core: Optical fiber consists of an inner cylindrical material made up of glass or plastic called core. Light is transmitted within the core which has refractive index

n_1 . It is a denser medium. Core is made up of silica (SiO_2).

2) Cladding: The core is surrounded by a cylindrical shell of glass or plastic called cladding. It has refractive index n_2 which is less than the refractive index of core i.e., ($n_1 > n_2$). It acts as a rarer medium. Typical refractive index values are $n_1 = 1.48$ and $n_2 = 1.46$. The core diameter is $\approx 50 \mu\text{m}$ and the thickness of cladding is ≈ 1 to 2 wavelengths of light propagate through the fiber. To lower the refractive index of cladding the silica is doped with phosphorous or bismuth material.

3) Sheath or outer or buffer Jacket: The cladding is surrounded by a polyurethane jacket called sheath. This layer protects the fiber from the surrounding atmosphere. It provides necessary toughness and tensile strength to the fiber.

Many fibers are grouped to form a cable. A cable may contain one to several hundred such fibers.



Parts of an Optical fiber

Working Principle of Optical Fiber:

Optical fiber works on the principle of total internal reflection. The light launched inside the core through its one end propagates to the other end due to total internal reflection at the core and cladding interface.

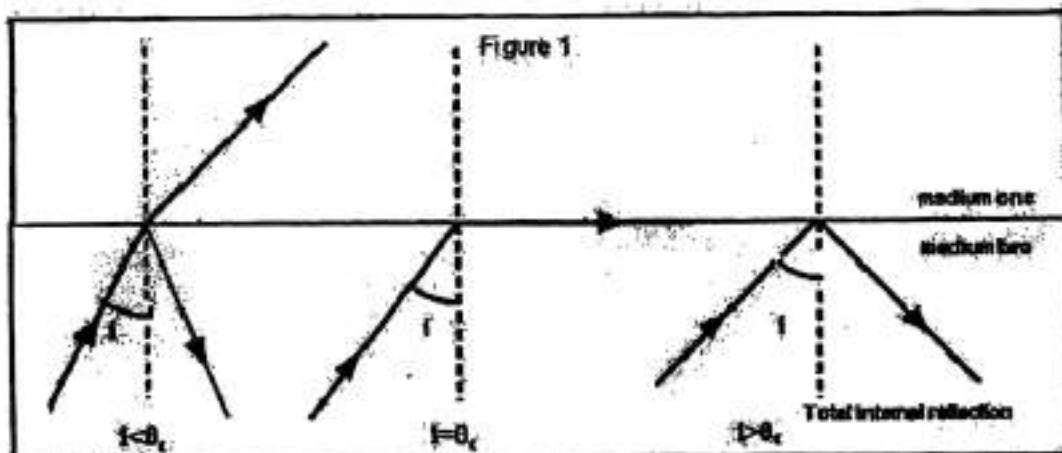
Total internal reflection:

Conditions for total internal reflection:

- 1) The light ray should move from denser to rarer medium.
- 2) The refractive index of core must be greater than cladding i.e. $n_1 > n_2$
- 3) The angle of incidence (i) must be greater than the critical angle (θ_c) i.e. $i > \theta_c$.
- 4) The critical angle $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

Explanation:

Let us consider a denser medium and a rarer medium of refractive indices n_1 and n_2 respectively and $n_1 > n_2$. Let a light ray move from denser to rare medium with ' i ' as the angle of incidence and ' r ' as angle of refraction. The refracted ray bends away from the normal as it travels from denser to rarer medium with increase of angle of incidence ' i' .



In this we get three cases

Case-1: When $i < \theta_c$, Then the light ray refracts into rarer medium as shown in figure.

Case-2: When $i = \theta_c$, then the light ray traverses along the two media as shown in figure.

For the two media, applying Snell's law

$$n_1 \sin i = n_2 \sin r$$

At critical angle $i = \theta_c$ and $r = 90^\circ$

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

$$n_1 \sin \theta_c = n_2$$

$$\sin \theta_c = \frac{n_2}{n_1}$$

For air $n_2 = 1$

$$\theta_c = \sin^{-1}\left(\frac{1}{n_1}\right)$$

Case-3: When $i > \theta_c$, then the light ray reflected back into the same medium as shown in figure. This phenomenon is called total internal reflection.

Acceptance angle, Acceptance and Numerical aperture:**Acceptance angle:**

Definition: The maximum angle of incidence at the core of an optical fiber so that the light can be guided through the fiber by total internal reflection is called as acceptance angle.

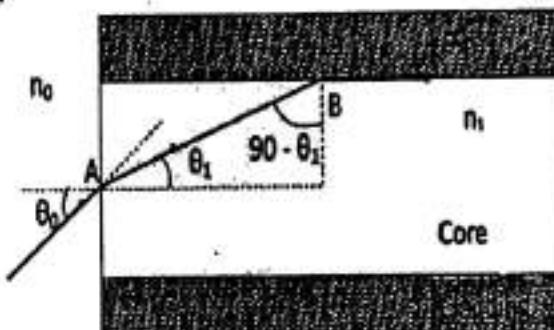
Let n_0 , n_1 and n_2 be the refractive indices of air, core and cladding media. Let a light ray OA is incident on the interface of air medium and core medium with an angle of incidence θ_0 , then the light ray refracts into the core medium with an angle of refraction θ_1 and the refracted ray AB is again incident on the interface of core and cladding with an angle of incident $(90^\circ - \theta_1)$.

If $(90^\circ - \theta_1)$ is equal to the critical angle of core and cladding media then the ray travels along the interface of core and cladding along the path BC. If the angle of incident at the interface of air and core $\theta_1 < \theta_c$, then $(90^\circ - \theta_1)$ will be greater than the critical angle. Therefore, the total internal reflection takes place.

Applying Snell's law $n_1 \sin \theta_1 = n_2 \sin \theta_2$ at point A

$$n_0 \sin \theta_0 = n_1 \sin \theta_1$$

$$\sin \theta_0 = \frac{n_1}{n_0} \sin \theta_1 \quad \dots \dots (1)$$



Applying Snell's law at point B

$$n_1 \sin(90^\circ - \theta_1) = n_2 \sin 90^\circ$$

$$n_1 \cos \theta_1 = n_2$$

$$\cos \theta_1 = \frac{n_2}{n_1}$$

$$\sin \theta_1 = \sqrt{1 - \cos^2 \theta_1}$$

$$\sin \theta_1 = \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\sin \theta_1 = \frac{\sqrt{n_1^2 - n_2^2}}{n_1} \quad \dots \dots (2)$$

Substituting eq.(2) in eq.(1), we get

$$\sin \theta_0 = \frac{n_1}{n_0} \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

$$\sin \theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

$$\theta_0 = \sin^{-1} \left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0} \right) \quad \dots \dots (3)$$

For air medium $n_0 = 1$

$$\text{Acceptance angle } \theta_0 = \sin^{-1} \left(\sqrt{n_1^2 - n_2^2} \right) \quad \dots \dots (4)$$

Acceptance cone: Rotating the acceptance angle about the fiber axis gives the acceptance cone of the fiber. Light launched at the fiber end within this acceptance cone alone will be accepted and propagated to the other end of the fiber by total internal reflection. Larger acceptance angles make launching easier.

Numerical aperture:

Definition: Numerical aperture is defined as the light gathering capacity of an optical fiber and it is directly proportional to the acceptance angle.

Numerically it is equal to the sin of the acceptance angle.

$$NA = \sin(\text{acceptance angle})$$

$$NA = \sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_2}$$

$$\text{Fractional change in refractive index } \Delta = \left(\frac{n_1 - n_2}{n_1} \right) \quad \dots\dots (5)$$

$$\Delta n_1 = (n_1 - n_2)$$

$$NA = \sqrt{(n_1 - n_2)(n_1 + n_2)}$$

$$NA = \sqrt{\Delta n_1 (n_1 + n_2)}$$

Since $n_1 = n_2$, So $(n_1 + n_2) = 2 n_1$

$$NA = \sqrt{\Delta n_1 (2n_1)}$$

$$NA = n_1 \sqrt{2 \Delta} \quad \dots\dots (6)$$

The above equation gives a relationship between numerical aperture and fractional change in relative refractive index.

Numerical aperture of the fiber depends only on refractive indices of the core and cladding materials and is not a function of fiber dimensions.

Classification of optical fibers:

Based on the refractive index of core medium, optical fibers are classified into two categories.

- i. Step index fiber
- ii. Graded index fiber

Based on the number of modes of transmission, optical fibers are classified into two categories

- i. Single mode fiber
- ii. Multi mode fiber

Based on the material used, optical fibers are may broadly classified into four categories

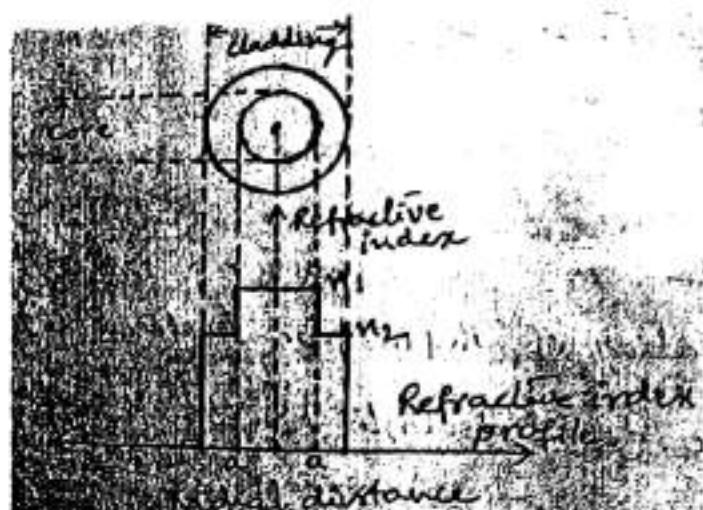
- i. All glass fibers
- ii. All plastic fibers
- iii. Glass core with plastic cladding fibers
- iv. Polymer clad silica fibers

Step index optical fibers:

In step index fibers the refractive index is uniform throughout the core medium (n_1). As we go radially in the fiber the refractive index undergoes a step change ($n_1 \rightarrow n_2$) at the core-cladding interface. The variation of refractive index with radial distance from the axis of the core is given by,

$$n(r) = n_1 \text{ for } r \leq a$$

$$n(r) = n_2 \text{ for } r > a$$

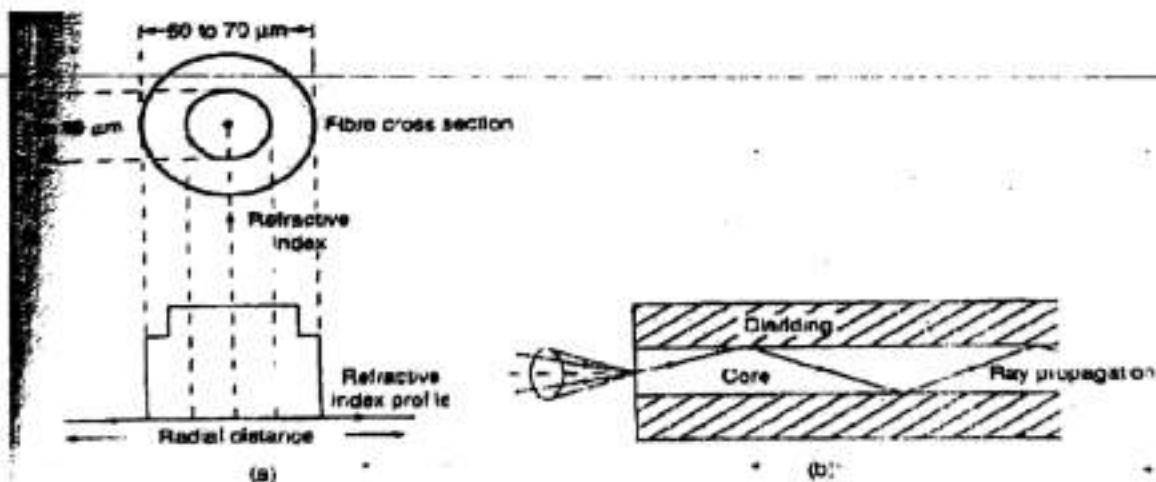


Based on the mode of propagation of light rays step index fibers are two types.

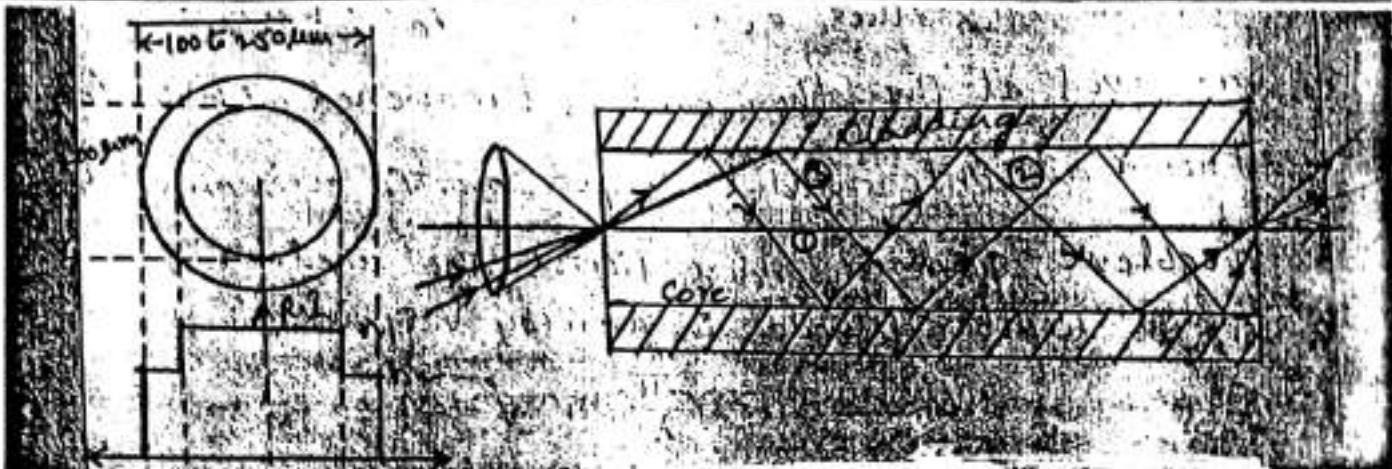
- a) Single mode step index fibers
- b) Multi mode step index fibers

Single mode step index fibers:

The core diameter of this fiber is about 8 to 10 μm and outer diameter of cladding is 60 to 70 μm . There is only one path for ray propagation. So, it is called single mode fiber. The cross sectional view, refractive index profile and ray propagation are shown in below fig. In this fiber, the transmission of light is by successive total internal reflections i.e. it is a reflective type fiber. The shape of propagation of the optical signal is in zigzag manner. Nearly 80% of the fibers manufactured today in the world are single mode fibers. They are mainly used in submarine cable system. Lasers are used as light sources in these fibers.

**Multi mode step index fibers:**

In this fiber the core and cladding diameters are much larger to have many paths for light propagation. The core diameter varies from 50 to 200 μm and the outer diameter of cladding varies from 100 to 250 μm . The cross-sectional view, refractive index profile and ray propagations are shown in below fig. Light propagation in this fiber is by multiple total internal reflections i.e., it is a reflective type fiber.



Generally the signal through the fiber is in digital form i.e. in the form of pulses representing 0s and 1s. From figure the ray 1 follows shortest path (i.e. travels along the axis of fiber) and the ray 2 follows longer path than ray 1. Hence the two rays reach the received end at different times. Therefore, the pulsed signal received at other end gets broadened. This is called intermodal dispersion. This difficulty is overcome in graded index fibers.

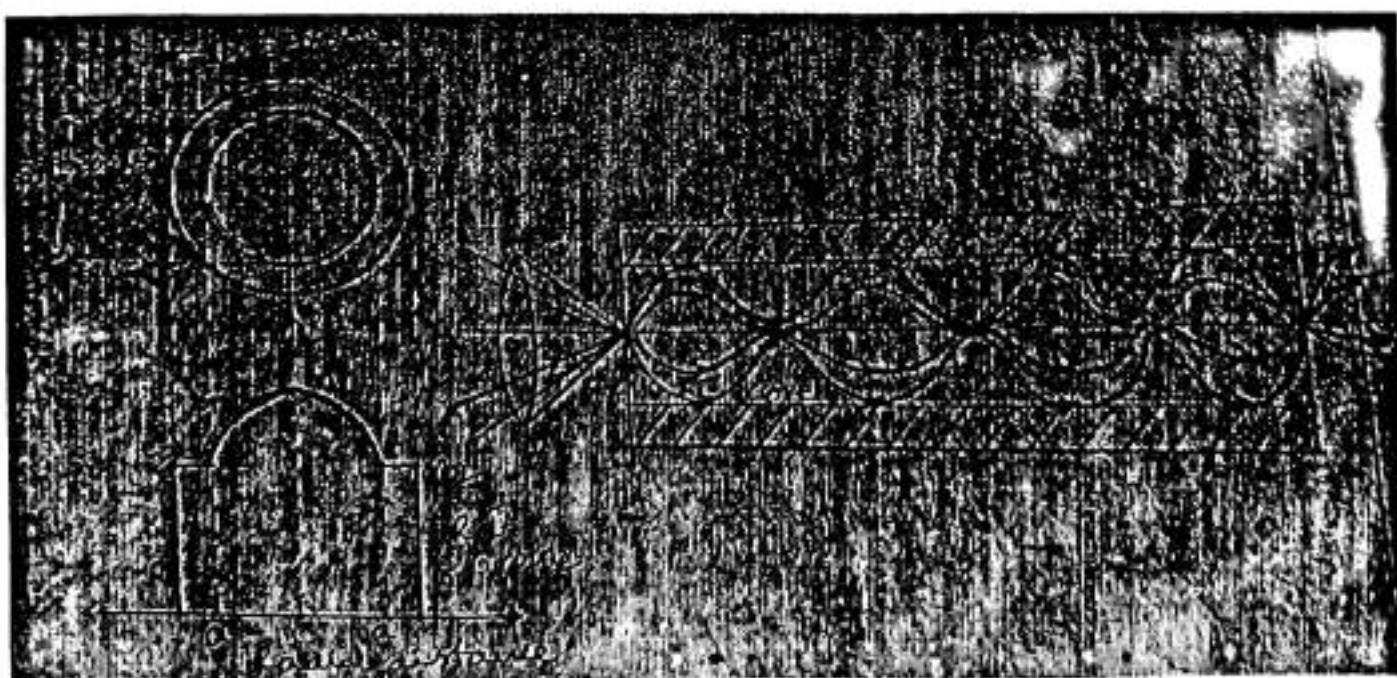
The number of modes is given by V-number = $\frac{2\pi}{\lambda} n_1 a \sqrt{2 \Delta}$

Number of modes of propagation $N = (V^2)/2$

Numerical aperture $NA = \sqrt{n_1^2 - n_2^2}$

Graded index optical fibers:

In graded index fibers the refractive index decreases continuously from center radially to the surface of the core. The refractive index is maximum at the center and minimum at the surface of core. This fiber can be single mode or multimode fiber. The refractive index profile is circularly symmetric. The propagation of light ray is not due to total internal reflection but by refraction.



The variation of refractive index with radial distance from the axis of core is given by,

$$n(r) = n_1 \sqrt{1 - 2\Delta \left(\frac{r}{a}\right)^2} \text{ for } r \leq a$$

$$n(r) = n_1 (1 - 2\Delta) = n_2 \text{ for } r > a$$

The cross sectional view, refractive index profile and ray propagation of multimode graded index fiber are shown in above fig. The diameter of core varies from 50 to 200 μm and outer diameter of cladding varies from 100 to 250 μm .

In graded index fiber, the light propagation is by refraction. Light rays travel at different speed in different paths of the fiber. Near the surface of the core, the refractive index is lower, so rays near the outer surface travel faster than the rays travel at the center. Because of this, all the rays arrive at the receiving end of the fiber approximately at the same time. So the intermodal dispersion is reduced. This fiber is costly.

Either laser or LED is used as light source. It is used in the telephone trunk between central offices.

The number of modes is given by V-number = $\frac{2\pi}{\lambda} a \sqrt{n_1^2 - n_2^2}$

Number of modes of propagation $N = (V^2)/4$

Numerical aperture $NA = n_1 \sqrt{2\Delta \left(1 - \left(\frac{r}{a}\right)^2\right)}$

Differences between step index fibers and graded index fibers:

Step index fiber	Graded index fiber
1. In step index fibers the refractive index of the core medium is uniform through and undergoes an abrupt change at the interface of core and cladding.	1. In graded index fibers, the refractive index of the core medium is varying in the parabolic manner such that the maximum refractive index is present at the center of the core.
2. The diameter of core is about 10 μm in case of single mode fiber and 50 to 200 μm in multi mode fiber.	2. The diameter of the core is about 50 μm .
3. The transmitted optical signal will cross the fiber axis during every reflection at the core cladding boundary.	3. The transmitted optical signal will never cross the fiber axis at any time.
4. The shape of propagation of the optical signal is in zigzag manner.	4. The shape of propagation of the optical signal appears in the helical or spiral manner
5. Attenuation is more for multi mode step index fibers but Attenuation is less in single mode step index fibers	5. Attenuation is very less in graded index fibers
6. Numerical aperture is more for multi mode step index fibers but it is less in single mode step index fibers	6. Numerical aperture is less in graded index fibers
7. Light source is laser	7. Light source is either laser or LED

Differences between single mode and multi mode fibers:

Single mode fiber	Multi mode fiber
1. In single mode optical fibers only one mode of propagation is possible	1. In multi mode optical fibers many number of modes of propagation are possible.
2. The diameter of core is about $10 \mu\text{m}$.	2. The diameter of core is 50 to $200 \mu\text{m}$.
3. The difference between the refractive indices of core and cladding is very small.	3. The difference between the refractive indices of core and cladding is also large compared to the single mode fibers.
4. There is no dispersion, so these are more suitable for communication.	4. Due to multi mode transmission, the dispersion is large, so these fibers are not used for communication purposes.
5. The process of launching of light into single mode fibers is very difficult.	5. The process of launching of light into single mode fibers is very easy.
6. Fabrication is very difficult and the fiber is costly.	6. Fabrication is very easy and the fiber is cheaper.

Applications of optical fibers:

1. Communication applications
 - i) In communication systems in exchange of information between different computers
 - ii) For exchange of information in cable televisions, space vehicles, submarines etc.
2. Sensing applications:
 - i) Displacement sensor
 - ii) Fluid level detector or Liquid level sensor
 - iii) Temperature and pressure sensor
 - iv) Chemical sensors
3. Medical applications
 - i) Optical fibers are used in medicine, in the fabrication of endoscopy for the visualization of internal parts of the human body, laparoscopic surgery and key hole surgery
 - ii) In ophthalmology, an optical fiber guided laser beam corrects the defects in vision.
 - iii) In cardiology for laser angioplasty
 - iv) The heart, respiratory system and pancreas can be investigated.
 - v) By measuring absorption of light by the blood the portion of hemoglobin in the blood can be estimated.
4. Military applications
 - i) An aircraft, a ship or a tank needs tons of copper wire for wiring of the communication equipment, control mechanisms, panel illumination etc. Use of optical fiber will reduce the weight and further maintain true communication.
 - ii) Fiber guided missiles are used in recent wars.

Questions

1. Explain briefly 'basic principle of an optical fiber' or explain the principle of total internal reflection in optical fibers.
2. Explain the terms numerical aperture and acceptance angle and derive expressions for the acceptance angle and numerical aperture in terms of fractional change in refractive index.
3. Write a note on the applications of an optical fiber.
4. Explain how the optical fibers are classified.
5. Describe different types of fibers based on refractive index profiles and number of modes.
6. Distinguish between step index fiber and graded index fiber.
7. Write the differences between single mode and multimode optical fiber.
8. With the help of suitable diagram explain the principle, construction and working of an optical fiber as a waveguide.

Problems

1. An optical fiber has a core material of refractive index of 1.55 and cladding material of refractive index of 1.50. The light is launched it in air. Calculate the its numerical aperture.
2. Calculate the angle of acceptance of a given optical fiber, if the refractive indices of the core and cladding are 1.563 and 1.498 respectively.
3. The numerical aperture of an optical fiber is 0.39. If the difference in the refractive indices of the material of its core and cladding is 0.05. Calculate refractive index of the core material.
4. Calculate fractional change in refractive for a given optical fiber if the refractive indices of the core and the cladding are 1.563 and 1.498 respectively.

Objective Questions

1. Light ray travels through the optical fiber by the principle of
 a) Reflection b) Transmission c) Total Internal Reflection d) None
2. The expression for Numerical Aperture is
 a) $\sqrt{n_1^2 - n_2^2}$ b) $\sqrt{n_1^2 + n_2^2}$ c) $\sqrt{(n_1 - n_2)}$ d) $\sqrt{(n_1^2 / n_2^2)}$
3. The refractive indices of step index fiber are
 a) Varies b) Constant c) Non-uniform . d) None
4. Numerical Aperture depends on
 a) Acceptance Angle b) Critical Angle . c) Refractive Angle d) None.
5. In step index fiber, the signal travels in ----- form
 a) Helical b) Parabolic c) Zig-zag d) None

6. In graded index fiber, the signal travels in ----- form
 a) Helical b) Parabolic c) Zig-zag d) None
7. If Δ is a fractional difference of refractive indices of fiber, then its numerical aperture is
 a) $n_2\sqrt{2\Delta}$ b) $n_1\sqrt{2\Delta}$ c) $n_1\sqrt{\Delta}$ d) $n_2\sqrt{\Delta}$
8. The attenuation in an optical fiber is a function of
 a) fiber material only b) wave length of light only
 c) length of the fiber only d) all the above
9. The refractive index of the core is
 a) greater than the cladding. b) less than the cladding.
 c) equal to cladding d) all the above.
10. To overcome the problem of inter model dispersion one has to use
 a) multimode fiber b) step index fiber c) graded index fiber d) plastic fiber.
11. When more than one mode is propagating, how is it dispersed?
 a) Dispersion b) Inter-modal dispersion c) Material dispersion d) Waveguide dispersion
12. Which of the following has more distortion?
 a) Single step-index fiber b) Graded index fiber
 c) Multimode step-index fiber d) Glass fiber
13. What is the principle of fiber optical communication?
 a) Frequency modulation b) Population inversion
 c) Total internal reflection d) Doppler Effect
14. The refractive index of a cladding of a fiber with refractive index 1.5 and numerical aperture 0.244 is
 a) 1.4 b) 1.325 c) 1.48 d) 1.656
15. Source of light in fiber optic system
 a) LED and laser diode b) LED c) laser diode d) photo diode

UNIT-III MAGNETIC AND DIELECTRIC PROPERTIES

MAGNETIC PROPERTIES

Magnetic permeability - Magnetization - Origin of magnetic moment - Classification of Magnetic materials - Dia, Para, Ferro, anti Ferro and Ferri-magnetism - Hysteresis curve.

1. **Magnetic field (H):** The region around the magnet up to which its influence is experienced by another magnet is called magnetic field.
2. **Magnetic flux (ϕ):** The total number of magnetic lines of forces passing normally through a surface is known as magnetic flux.

Units - Weber

3. **Magnetic flux density (B) or Magnetic induction:** The number of magnetic lines of forces passing through unit area of surface is known as magnetic field induction.

$$B = \frac{\phi}{A}$$

Units: $\frac{W}{m^2} = 1\text{Tesla}$

4. **Magnetic field intensity or intensity of magnetic field (H):** The force experienced by a unit North Pole placed at a point in the magnetic field is called magnetic intensity.

If 'F' is a force experienced by a north pole placed at a point in the magnetic field and 'm' is the pole strength of magnet then,

$$H = \frac{F}{m}$$

Units- Amp/meter

5. **Permeability:** (μ) The magnetic flux density 'B' is directly proportional to the magnetic field intensity 'H'

$$B \propto H$$

$$B = \mu H$$

Where μ is proportionality constant and is known as permeability of the medium

$$\mu = \frac{B}{H}$$

$$\mu = \frac{B}{H} \quad \frac{W/m^2}{A/m}$$

$$= \frac{B}{H} \quad \frac{w}{Am}$$

' μ ' is defined as the ratio of magnetic induction 'B' to magnetic field intensity 'H'

(or)

The measure of the degree of which the magnetic lines of forces can penetrate into the medium is called as the permeability of the medium.

The magnetic flux density in vacuum is $B \propto H$

$$B = \mu_o H$$

Where ' μ_o ' is permeability of free space.

$$\mu_o = 4\pi \times 10^{-7} H/m$$

6. Relative permeability: It is the ratio of permeability of medium to the permeability of free space or vacuum

$$\mu_r = \frac{\mu}{\mu_o} \Rightarrow \mu = \mu_r \mu_o$$

7. Intensity of Magnetization or Magnetization (I): The magnetic moment per unit volume of the material is called the intensity of magnetization. If M is the magnetic moment and V is the volume of the magnet then the intensity of magnetization is

$$I = \frac{M}{V}$$

8. Magnetic Susceptibility: The intensity of magnetization produced in a material is directly proportional to the magnetizing field H, that is

$$I \propto H \quad \text{or} \quad I = \chi H \quad \text{or} \quad \chi = \frac{I}{H}$$

It is defined as the ratio of intensity of magnetization to the magnetic field intensity. It has no units.

1. Derive the Relation between B, H, and I

When a magnetic material is placed in a uniform magnetic field of intensity H, two types of magnetic induction lines can pass through it.

- i) Due to magnetizing field, $B_o = \mu_o H$
- ii) Due to the magnetization of the material by induction, $B_I = \mu_o I$

The total flux density B will be given by

$$\begin{aligned} B &= B_o + B_I \\ B &= \mu_o H + \mu_o I \\ B &= \mu_o (H + I) \end{aligned}$$

2. Derive the Relation between μ_r and χ

We know that $B = \mu_o \mu_r H$ & $B = \mu_o (I + H)$

$$\begin{aligned} \mu_o \mu_r H &= \mu_o (I + H) \\ \mu_r = (H + I)/H &\Rightarrow \mu_r = 1 + \chi \end{aligned}$$

3. Explain the origin of magnetic moment in atoms.

Origin of magnetic moment:

The magnetic properties of solids originate from the motion of electrons. These electrons circulate about the nucleus in definite orbits and also spinning about their own axes. These moving electrons cause the magnetic moments and the atom behaves as tiny magnet. So, the magnetic moment in an atom is due to

1. Orbital magnetic moment of the electrons
2. Spin magnetic moment of the electrons
3. Spin magnetic moment of the nucleus

1. Orbital magnetic moment of electrons: consider an electron of charge ‘e’ and mass ‘m’ revolving around the nucleus in an orbit of radius ‘r’ with an angular velocity ω as shown in figure. The circumference of the orbit is $2\pi r$ and the area of the orbit is πr^2 .

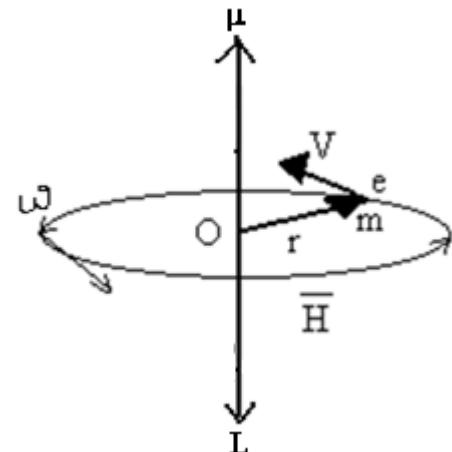
The linear velocity of the electron, $v = r\omega$

The time period of the orbit,

$$T = \frac{\text{Distance}}{\text{velocity}} = \frac{2\pi r}{v} = \frac{2\pi r}{r\omega} = \frac{2\pi}{\omega}$$

$$\text{The current in the orbit, } i = \frac{e}{T} = \frac{e\omega}{2\pi}$$

The magnetic moment of the electron = current \times area



$$\mu_o = i \times A$$

$$= \frac{e\omega}{2\pi} \pi r^2$$

$$\mu_o = \frac{e\omega r^2}{2}$$

We know that the angular momentum of the electron, $L = mvr = mr^2\omega$

$$\text{Therefore, } \mu_o = -\frac{e}{2m} \times L$$

the negative sign indicates that the magnetic moment and the angular momentum of the electron are oppositely directed.

According to the modern atomic theory, the angular momentum of the electron is

$L = \frac{m_l h}{2\pi}$; where m_l is magnetic orbital quantum number and it has $(2l+1)$ values and $m_l = -l, \dots, +l$ values; ‘h’ is Planck’s constant.

Substituting L, the orbital magnetic moment, $\mu_o = -\frac{e}{2m} \times \frac{m_l h}{2\pi}$

$$\mu_o = -\frac{e\hbar}{4\pi m} \times m_l$$

$$\mu_o = -\mu_B \cdot m_l$$

where $\mu_B = \frac{e\hbar}{4\pi m}$ is an atomic unit of magnetic moment called the Bohr magneton.

Here e is the charge of electron $= 1.602 \times 10^{-19} \text{ C}$,

m is the mass of the electron $= 9.109 \times 10^{-31} \text{ kg}$

\hbar is the Planck's constant $= 6.625 \times 10^{-34} \text{ J - sec}$.

Substituting these values we get, $\mu_B = 9.273 \times 10^{-24} \text{ ampere - meter}^2$. This represents the magnetic moment of elementary permanent magnetic dipole.

2. Spin magnetic moment of the electrons:

The magnetic moment associated with the spin motion of electron is called spin magnetic moment.

$$\mu_s = -\frac{e}{m} \mathbf{s} = -\frac{e}{m} (\pm \frac{1}{2} \hbar)$$

$$\mu_s = \pm \frac{e \hbar}{4\pi m} = \pm \mu_B$$

- The spin magnetic moment is mainly due to unpaired electrons.
- The spin magnetic moment is larger than orbital magnetic moment.
- $\frac{\mu_s}{s} = \frac{e}{m}$ is called spin gyromagnetic ratio.

3. Spin magnetic moment of the nucleus:

The magnetic moment produced by spin of the nucleus is known as nuclear magnetic moment

$$\mu_N = \frac{e \hbar}{4\pi m_p} \quad \text{Where } m_p \text{ is mass of proton.}$$

- μ_N is called the nuclear magneton which is very small and is neglected. Its value is $5.05 \times 10^{-27} \text{ amp-m}^2$.

The net magnetic moment of a atom is $\mu = \mu_o + \mu_s + \mu_N$

$\mu = \mu_o + \mu_s$ (μ_N is very small value, so it is neglected)

4. Explain How are magnetic materials are classified? Write the properties of magnetic materials.

Magnetic materials are classified on the basis of their behavior in external magnetic fields and they are

1. Dia magnetic materials.
2. Para magnetic materials.
3. Ferro magnetic materials.

4. Anti ferro magnetic materials.
5. Ferri magnetic materials.

DIAMAGNETIC MATERIALS:

Def: The substances which are weakly magnetized in opposite direction to the applied magnetic field.

1. Diamagnetic materials do not contain permanent dipoles.
2. In these materials, the net magnetic moment is zero because different orientation of different electron orbits in an atom
3. When an external field is applied, the induced magnetic moment is in opposite direction to the applied field, i.e., I and H are anti parallel. Hence diamagnetic materials are repelled by magnetic fields.
- 4.

$$I \propto -H \Rightarrow I = -\chi_{dia} H$$

$$\Rightarrow \chi_{dia} = -\frac{I}{H}$$

\therefore Diamagnetic susceptibility is negative

5. Relative permeability μ_r of diamagnetic materials is less than unity since

$$\mu_r = 1 + \chi_{dia} \Rightarrow \mu_r < 1 \text{ } (\because \chi_{dia} \text{ is negative})$$

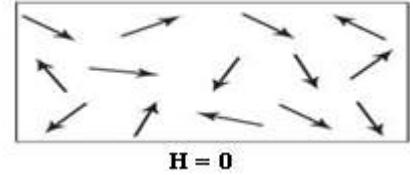
6. Diamagnetic susceptibility χ_{dia} is independent of temperature.

7. Examples of diamagnetic materials are hydrogen, air, water, majority of metals like gold, silver, bismuth.

PARAMAGNETIC MATERIALS

Def: The substances which are feebly magnetized in the same direction of the applied magnetic field.

1. Paramagnetic materials contain permanent dipoles.
2. Magnetic moments arising from orbital motion of electrons cause paramagnetism.
3. In the absence of external magnetic field, there is no net magnetization due to randomly oriented magnetic moments of the dipoles.
4. When an external magnetic field is applied, the magnetic moment of the dipoles tend to orient in the field direction resulting the small net magnetization I parallel to the field H .



$$\therefore I = \chi_{para} H \Rightarrow \chi_{para} = \frac{I}{H}$$

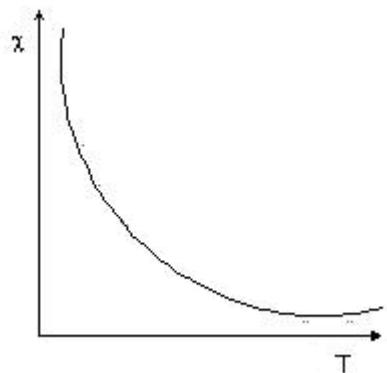
Paramagnetic susceptibility is small and positive .(is of the order of 10^{-6})

5. Relative permeability μ_r is slightly greater than one ($\mu_r > 1$) Since

$$\mu_r = 1 + \chi_{para} .$$

6. Paramagnetic susceptibility χ_{para} is strongly temperature dependent.

$$\chi_{para} \propto \frac{1}{T} \Rightarrow \chi_{para} = \frac{C}{T} \text{ Curie's law.}$$

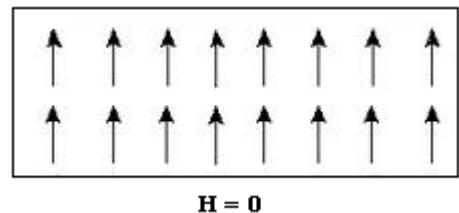


- 7.Examples of paramagnetic materials are aluminum, calcium, chromium, manganese, platinum, titanium etc.

FERROMAGNETIC MATERIALS

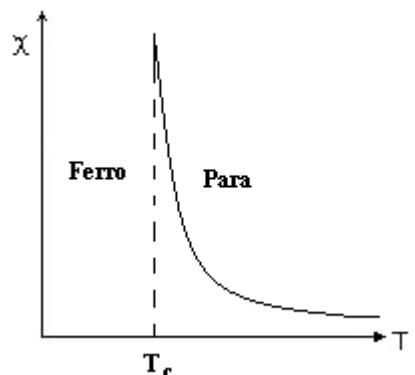
Def: The substances which are strongly magnetized in the same direction of the applied magnetic field.

1. Ferromagnetic materials contains large number of permanent dipoles which are aligned parallel in a particular direction
2. In these materials the net magnetic moment is non zero, even in the absence of external magnetic field. Ferromagnetic susceptibility is very large (χ_{ferro} is of the order of 10^6).



3. The materials exhibit Spontaneous magnetization.
5. Relative permeability μ_r is very much greater than unity and is of the order of 10^3 .
6. Every ferromagnetic material has a characteristic temperature T_c called Curie temperature.

$$\chi_{ferro} = \frac{C}{T - T_c} \text{ for } T > T_c \text{ ----- Curie Weiss law}$$



Below T_c , the material exhibits spontaneous magnetization and when the temperature is above T_c , the ferromagnetic material becomes paramagnetic.

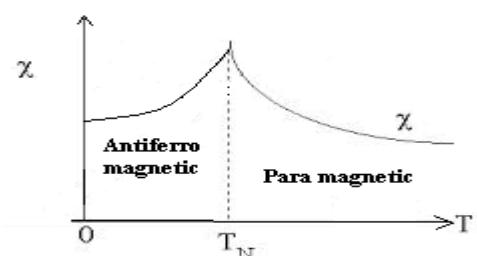
7. Ferromagnetic materials exhibit anisotropy i.e., they are high directional.
8. Hysteresis is an important characteristic of Ferromagnetic material.
9. Examples of ferromagnetic substances are iron, cobalt, nickel, Fe_2O_3 , etc.

7. Distinguish between Antiferro and Ferri Magnetic Materials

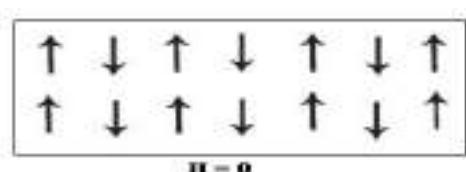
ANTIFERRO MAGNETIC MATERIALS

1. The substances which consists of antiparallel alignment of dipoles with equal magnitudes.
2. In the absence of external magnetic field, the net magnetization is zero.
3. Antiferromagnetic materials are crystalline materials having small positive value of susceptibility (χ) of the order of 10^{-3} to 10^{-5} .
4. The variation of susceptibility with temperature obeys modified Curie Weiss law

$$\chi_{af} = C/(T + T_N) \quad \text{for } T > T_N$$



5. Below Neel temperature the material is Antiferromagnetic and above T_N the material becomes Paramagnetic.
6. Most of the antiferromagnetic materials are ionic compounds.
7. Examples of antiferromagnetic materials are MnS , Cr_2O_3 , $NiCr$, NiO , $FeCl_2$ etc.

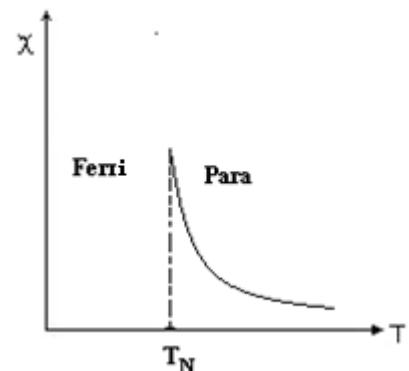
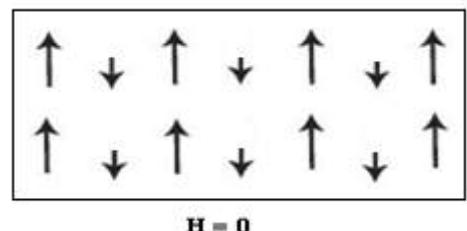


FERRI MAGNETIC MATERIALS

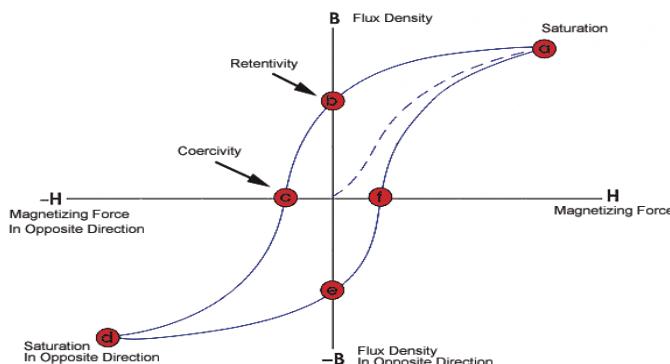
1. The substance in which magnitude of dipoles are unequal and oriented in opposite direction..
2. The net magnetic moment is non zero in the absence of external field.
3. The materials exhibit large magnetic susceptibility in the order of 10^3 to 10^5 .
4. $\chi_{\text{ferri}} = \frac{C}{T \pm T_N}$ for $T > T_N$

T_N is Neel Temperature and C is Curie temperature

5. The Ferrimagnetic materials behave like Ferromagnetic materials below the Neel temperature and are Paramagnetic above the Neel temperature.
6. Examples are complex Oxides as XFe_2O_4 , where X = Mn, Mg, Cu, Zn.,,



8. Explain Hysteresis or B-H curve for Ferromagnetic material placed in a magnetic field.



- A typical property of ferromagnetic material is hysteresis.
- Hysteresis may be defined as the lag of magnetization behind the variations of the magnetic field.
- It gives the relationship between the induced magnetic flux density(B) and the magnetizing field(H), often referred as the B-H loop or I-H loop.

From the hysteresis loop, the following properties of a magnetic material can be determined.

- i) **Retentivity:** It is the property of magnetic material in which the magnetic flux density remained even after the applied field is reduced to zero.
- ii) **Residual flux density (Br):** The magnetic flux density remained inside the material when the applied field is reduced from saturation to zero is called residual flux density.
- iii) **Coercivity:** It is the property of magnetic material in which the residual magnetic flux density becomes zero at certain value of reverse magnetic field applied to the material.
- iv) **Coercive field (Hc):** The amount of reverse magnetic field applied to the magnetic material to make the residual magnetic flux density zero is called coercive field.

9. Write the difference between Soft and Hard Magnetic Materials:

On the basis of Hysteresis, the magnetic materials can be divided into 2 classes:

- 1) Soft magnetic materials 2) Hard magnetic materials.

Properties of soft-magnetic materials:

- i) They are easily magnetized and demagnetized.
- ii) They have small hysteresis loss due to small hysteresis loop area.
- iii) These materials have large values of Permeability and susceptibility.
- iv) Coercivity and retentivity are small.

Properties of hard magnetic materials:

- i) They cannot easily magnetized and demagnetized.
- ii) These materials have large hysteresis loss due to large hysteresis loop area.
- iii) These materials have small values of permeability and susceptibility.
- iv) Coercivity and retentivity are large.

Solved problems:

1. find the relative permeability of a ferromagnetic material if a field of strength 220 amp/m produces a magnetization of 3300 amp/m in it.

Solution: Given field strength of ferromagnetic material , $H=220 \text{ amp/m}$

Magnetization $I = 3300 \text{ amp/m}$

$$\text{We know that } \mu_r = 1 + \frac{I}{H} = 1 + \frac{3300}{220} = 16$$

2. A magnetic material has a magnetization of 3300A/m and flux density of 0.0044 Wb/m² Calculate the magnetizing force and the relative permeability of the material.

Solution: Given magnetization, $M = 3300 \text{ A/m}$
Flux density, $B = 0.0044 \text{ Wb/m}^2$

$$\mathbf{B} = \mu_0 [\mathbf{H} + \mathbf{M}]$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$$

$$H = \frac{0.0044}{4 \times 3.14 \times 10^{-7}} - 3300$$

$$H = 203 \text{ A/m}$$

$$\mu_r = 1 + \frac{I}{H}$$

$$= 1 + \frac{3300}{200}$$

$$= 16.5 + 1$$

$$= 17.5$$

DIELECTRIC PROPERTIES

Introduction - Dielectric constant - Electronic, ionic and orientational polarization - internal fields - Clausius - Mossotti equation - Dielectric loss, Breakdown and Strength.

Definitions:

i) Dielectric constant ϵ_r

Dielectric constant is the ratio between the permittivity of the medium and the permittivity of the free space.

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

ii) Electric polarization

The process of producing electric dipoles which are oriented along the applied field direction is called polarization in dielectrics.

iii) Polarizability (α):

When the strength of the electric field E is increased the strength of the induced dipole μ also increases. Thus the induced dipole moment is proportional to the intensity of the electric field

$$\text{i.e. } \mu = \alpha E$$

where α , the constant of proportionality is called *polarizability*.

Polarizability is defined as induced dipole moment per unit electric field.

iv) Polarization vector \vec{P}

The dipole moment per unit volume of the dielectric material is called polarization vector. If \vec{P} is the average dipole moment per molecule and N is the number of molecules per unit volume the polarization vector

$$\vec{P} = N \vec{\mu}$$

The dipole moment per unit volume of the solid is the sum of all the individual dipole moments within that volume and is called the polarization of the solid.

v) **Electric flux density (or) Electric displacement D:** The electric flux density or electric displacement D at a point in a material is given by

$$D = \epsilon_0 \epsilon_r E$$

Where E is the electric field strength, ϵ_0 permittivity of free space (vacuum) and ϵ_r is the relative dielectric constant or relative permittivity of the material.

vi) Electric susceptibility χ

The polarization vector is proportional to the total electric flux density E and is in the same direction of E. Therefore the polarization vector can be written as

$$P = \epsilon_0 \chi_e E \quad \text{where } \chi_e = \epsilon_r - 1$$

Where χ_e is the electric susceptibility.

$$\therefore \chi_e = \frac{P}{\epsilon_0 E} = \frac{\epsilon_0 (\epsilon_r - 1) E}{\epsilon_0 E}$$

$$\chi_e = (\epsilon_r - 1)$$

1. Derive the Relation between three electric vectors \vec{E} , \vec{D} and \vec{P} .

The polarization measures the additional flux density arising from the presence of the material as compared to free space, it has the same units as D and is related to it as

$$D = \epsilon_0 \epsilon_r E$$

$$D = \epsilon_0 \epsilon_r E + \epsilon_0 E - \epsilon_0 E$$

$$D = \epsilon_0 E + \epsilon_0 E (\epsilon_r - 1)$$

$$D = \epsilon_0 E + P$$

Where,

$$P = \epsilon_0 (\epsilon_r - 1) E$$

Electric flux density D is similar to magnetic induction B in magnetism

2. Explain the different polarization mechanisms in dielectrics.

1. Electronic polarization
2. Ionic polarization
3. Orientational polarization

3. Define the electronic polarization and show that the electronic polarizability depends on the volume of the atom.

Electronic polarization: *The displacement of the positively charged nucleus and the (negatively) electrons of an atom in opposite directions, on application of an electric field, result in electronic polarization.*

The Induced dipole moment is proportional to the applied electric field strength and is given by

$$\mu \propto E$$

$$\mu = \alpha_e E$$

Where α_e is the electronic polarizability. This electronic polarizability is independent of temperature.

Calculation of electronic polarizability: α_e

Consider a classical model of an atom, the nucleus of charge Ze is surrounded by an electron cloud of charge $-Ze$ distributed in a sphere of radius R .

The charge density ρ is given by

$$\rho = \frac{-Ze}{\frac{4}{3}\pi R^3} = \frac{-3}{4} \left(\frac{Ze}{\pi R^3} \right)$$

When the external field of intensity E is applied, the nucleus and the electrons experience Lorentz forces of magnitude ZeE in opposite direction. Then coulomb force develops between them, which tends to oppose the displacement. When these forces are equal and opposite at equilibrium position. Let x be the displacement between the nucleus and the electron cloud.

Since nucleus is much heavier than the electron cloud it is assumed that only the electron cloud is displaced when the external field is applied.

$$\text{Lorentz force} = -ZeE$$

$$\text{Coulomb force} = Ze \times \frac{1}{4\pi\epsilon_0 x^2} \times \text{charge enclosed in the sphere of radius 'x'}$$

$$\text{The charge enclosed} = \frac{4}{3}\pi x^3 \rho$$

$$= \frac{4}{3}\pi x^3 \left[\frac{-3}{4} \left(\frac{Ze}{\pi R^3} \right) \right]$$

$$= \frac{-Zex^3}{R^3}$$

$$\text{Hence the Coulomb force is } \frac{Ze}{4\pi\epsilon_0 x^2} \left[\frac{-Zex^3}{R^3} \right] = -\frac{Z^2 e^2 x}{4\pi\epsilon_0 R^3}$$

$$\text{At equilibrium position, } F_L = F_C$$

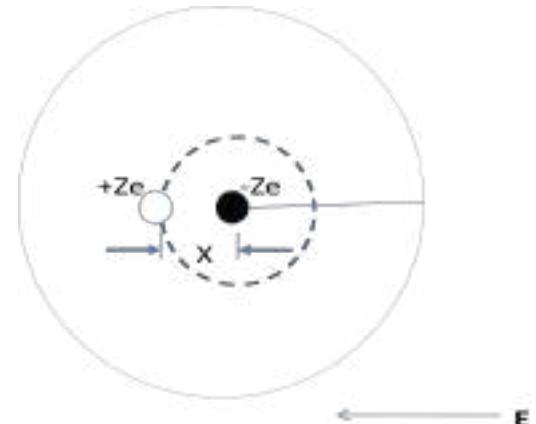
$$-ZeE = -\frac{Z^2 e^2 x}{4\pi\epsilon_0 R^3}$$

$$E = \frac{Zex}{4\pi\epsilon_0 R^3}$$

$$x = \frac{4\pi\epsilon_0 R^3 E}{Ze}$$

Thus the displacement of the electron cloud is proportional to the applied field.

\therefore Induced electric dipole moment is given by



$$\mu_e = Zex = \frac{Ze4\pi\epsilon_0 R^3 E}{Ze}$$

$$\mu_e = 4\pi\epsilon_0 R^3 E$$

$$\mu_e = \alpha_e E$$

Where $\underline{\alpha_e = 4\pi\epsilon_0 R^3}$ is called electronic polarizability.

The dipole moment per unit volume is called electronic polarization.

$$P_e = N\mu_e = N\alpha_e E$$

Where N is the number of atoms/ m³

$$\text{We know that } P_e = \epsilon_0 E (\epsilon_r - 1) = N\alpha_e E$$

$$\text{Hence } \alpha_e = \frac{\epsilon_0 (\epsilon_r - 1)}{N}$$

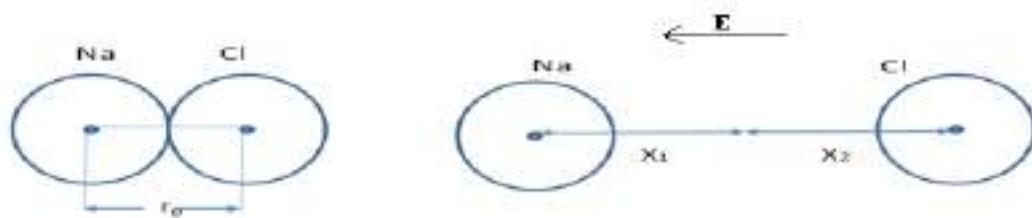
- 4. Explain Ionic Polarization and show that Ionic Polarizability varies inversely with the natural frequency of the ionic molecule.**

Ionic polarization:

The ionic polarization is due to the displacement of Cations and anions in opposite directions on the application of the electric field in an ionic solid.

Calculation of Ionic polarizability : α_i

Suppose an electric field is applied in the +ve X direction. The positive ions move to the right by X_1 and the negative ions move to the left by X_2 .



Assuming that each unit cell has one cation and one anion, the resultant dipole moment per unit cell due to ionic displacement is given by

$$\mu = e (x_1 + x_2)$$

If β_1 and β_2 are restoring force constants of cation and anion and 'F' is the force due to the applied field, then

$$F = \beta_1 x_1 = \beta_2 x_2$$

$$\text{Hence } x_1 = F / \beta_1$$

Restoring force constants depend upon the mass of the ion and angular frequency of the molecule in which ions are present.

Then

$$x_1 = eE / m\omega_0^2$$

$$x_2 = eE / M \omega_0^2$$

Where m is the mass of the anion and M is the mass of the cation.

$$\therefore x_1 + x_2 = eE / m\omega_0^2 (1/M + 1/m)$$

$$\text{and } \mu = e(x_1 + x_2) = e^2 E / \omega_0^2 (1/M + 1/m)$$

$$\alpha_i = \mu/E = e^2 / \omega_0^2 (1/M + 1/m)$$

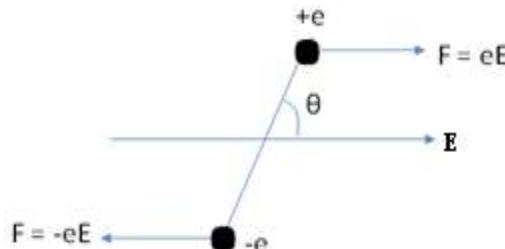
Thus the ionic polarizability α_i is inversely proportional to the square of the natural frequency of the ionic molecule.

5. Explain the Orientational Polarization and calculate the orientational polarizability.

Orientational polarization:

The Orientational polarization is a characteristic of Polar Dielectrics. When an electric field is applied on molecules which posses permanent dipole moment, they tend to align themselves in the direction of applied electric field. The polarization due to such alignment is called Orientational polarization.

Calculation of Orientational polarizability : α_o



The potential energy of the dipole $U = -\mu \cdot E = -\mu E \cos\theta$

According to Boltzmann statistics, the probability of finding a dipole within the solid angle 0 to π or $d\Omega$ is proportional to the Boltzmann factor

$$f(\theta) = e^{\frac{-U}{KT}} \Rightarrow f(\theta) = e^{\frac{\mu E \cos \theta}{KT}}$$

The average moment contribution per dipole lying within the solid angle can be determined by integrating from $= 0$ to π .

$$\bar{\mu} = \frac{\int_0^\pi \mu_x e^{\frac{\mu E \cos \theta}{KT}} d\Omega}{\int_0^\pi e^{\frac{\mu E \cos \theta}{KT}} d\Omega}$$

Where $\mu_x = \mu \cos \theta$, $d\Omega = 2\pi \sin \theta d\theta$,

$$\overline{\mu} = \frac{\mu \int_0^\pi \cos \theta e^{\frac{\mu E \cos \theta}{KT}} \times 2\pi \sin \theta d\theta}{\int_0^\pi e^{\frac{\mu E \cos \theta}{KT}} \times 2\pi \sin \theta d\theta}$$

Let $a = \frac{\mu E}{KT}$, $t = \cos \theta$, $dt = -\sin \theta d\theta$.

Deby

$$\overline{\mu} = \frac{\mu \int_{-1}^1 t e^{at} dt}{\int_{-1}^1 e^{at} dt}$$

By simplifying above integration, we get

$$\overline{\mu} = \frac{\mu^2 E}{3KT}$$

The Orientational polarizability, $\alpha_o = \frac{\overline{\mu}}{E} = \frac{\mu^2}{3KT}$

The total polarization

$$P = P_e + P_i + P_o$$

The total polarizability

$$\alpha = \alpha_e + \alpha_i + \alpha_o$$

$$\therefore \alpha = 4\pi\epsilon_o R^3 + \frac{e^2}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right) + \frac{\mu^2}{3KT}$$

The total polarization;

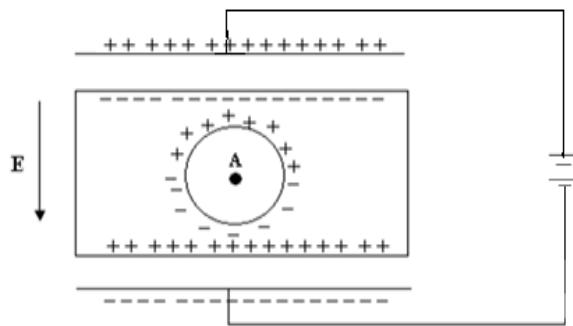
$$P = N\alpha E = NE \left\{ 4\pi\epsilon_o R^3 + \frac{e^2}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right) + \frac{\mu^2}{3KT} \right\}$$

This is known as Langevin-e equation

6. Derive an expression for an Internal Field or Local Field in Dielectrics.(Lorentz Method)

Def: The electric field at an atom site is called Internal field or local field.

Let a dielectric be placed between the plates of a parallel plate capacitor and let there be an imaginary spherical cavity around the atom A inside the dielectric as shown in fig.

Diagram

It is assumed that the radius of cavity is large compared to the radius of atom. The internal field at the atom site A can be considered as

$$E_i = E_1 + E_2 + E_3 + E_4$$

Where E_1 is electric field intensity at A due to the charge density on the plates.

E_2 is electric field intensity at A due to the charge density induced on the two sides of dielectric.

E_3 is electric field intensity at A due to the dipoles inside the cavity.

E_4 is electric field intensity at A due to the polarizations charges on the surface of the cavity.

Field E_1 : E_1 is electric field intensity at A due to the charge density on the plates without dielectric.

From the electric displacement we know that

$$D = \epsilon_0 E_1$$

$$E_1 = \frac{D}{\epsilon_0}$$

$$D = P + \epsilon_0 E$$

$$\therefore E_1 = \frac{P + \epsilon_0 E}{\epsilon_0} = E + \frac{P}{\epsilon_0}$$

Field E_2 : E_2 is electric field intensity at A due to the charge density induced on the two sides of dielectric.

$$D = \epsilon_0 E_2 + P$$

Here $D=0$ (D is not due to induced electric field)

$$E_2 = -\frac{P}{\epsilon_0}$$

Field E₃: E₃ is electric field intensity at A due to the dipoles inside the cavity. We are assuming a cubic structure of dielectric so E₃=0 because of symmetry.

Field E₄: E₄ is electric field intensity at A due to the polarization charges on the surface of the cavity and it was calculated by Lorentz as given below

If dA is the surface area of the sphere of radius r lying between θ and $\theta + d\theta$

Where θ is the direction with reference to the direction of applied field.

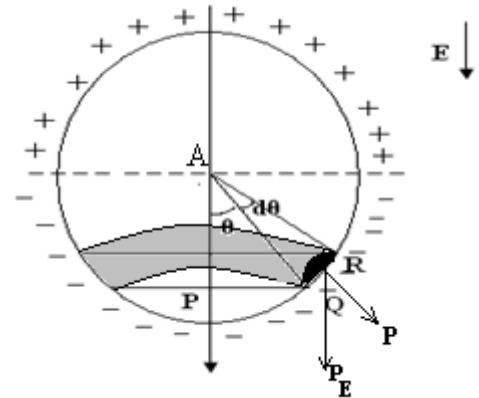
Then dA = circumference of the ring \times thickness

$$dA = 2\pi(PQ)(QR)$$

$$\text{From the triangle PQA } \sin \theta = \frac{PQ}{r} \quad PQ = r \sin \theta$$

$$(\text{from arc AQR}) \quad d\theta = \frac{QR}{r} \quad QR = rd\theta$$

$$\text{Hence } dA = 2\pi r \sin \theta r d\theta = 2\pi r^2 \sin \theta d\theta$$



The charge dq on the surface dA is equal to the normal component of the polarization multiplied by the surface area.

$$\therefore dq = P \cos \theta dA = P(2\pi r^2 \sin \theta \cos \theta d\theta)$$

The field due to this charge at A is denoted by dE₄ in the direction $\theta=0$ is

$$\begin{aligned} dE_4 &= \frac{dF}{q} = \frac{dq \cos \theta}{4\pi\epsilon_0 r^2} = \frac{P \cos \theta 2\pi r^2 \sin \theta d\theta \cos \theta}{4\pi\epsilon_0 r^2} \\ &= \frac{P}{2\epsilon_0} \cos^2 \theta \sin \theta d\theta \end{aligned}$$

Thus the total field E_4 due to the charges on the surface of the entire cavity is obtained by integrating

$$E_4 = \int dE_4 = \frac{P}{2\epsilon_0} \int_0^\pi \cos^2 \theta \sin \theta \, d\theta$$

$$= \frac{P}{2\epsilon_0} \int_0^\pi \cos^2 \theta \, d(-\cos \theta)$$

$$= \frac{-P}{2\epsilon_0} \left[\frac{\cos^3 \theta}{3} \right]_0^\pi$$

$$= \frac{P}{6\epsilon_0} [-1 - 1] = \frac{P}{3\epsilon_0}$$

$$E_i = E_1 + E_2 + E_3 + E_4$$

$$E_i = E + \frac{P}{\epsilon_0} - \frac{P}{\epsilon_0} + 0 + \frac{P}{3\epsilon_0}$$

$$E_i = E + \frac{P}{3\epsilon_0}$$

This is the expression for internal field or local field.

7. Deduce Clausius – Mosotti relation for dielectrics

Clausius –Mosotti equation relates the dielectric constant and the Polarizability inside the dielectric.

The dipole moment of an atom inside the dielectric is

$$\mu_i = \alpha E_i$$

If there are ‘N’ molecules or atoms per unit volume, then polarization, $P = N \propto_e E_i$

$$P = N\mu_i = N\alpha E_i$$

$$\Rightarrow P = N\alpha \left(E + \frac{P}{3\epsilon_0} \right)$$

$$\Rightarrow P = \frac{N\alpha}{3\epsilon_0} (3\epsilon_0 E + P)$$

$$\Rightarrow \frac{N\alpha}{3\epsilon_0} = \frac{P}{P+3\epsilon_0 E}$$

We know that $P = \epsilon_0 \chi_e E$

$$\Rightarrow P = \epsilon_0 (\epsilon_r - 1) E$$

$$\Rightarrow \frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_0 (\epsilon_r - 1) E}{\epsilon_0 (\epsilon_r - 1) E + 3\epsilon_0 E}$$

$$\Rightarrow \frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

The above relation is known as Clausius –Mosotti equation.

SOLVED PROBLEMS**1. Calculate the electronic polarizability of an argon atom.**

Given: $\epsilon_r = 1.0024$ and $N = 2.7 \times 10^{25}$ atoms/m³

Solution: Electronic polarization is given by $P_e = N \alpha_e E = \epsilon_0 (\epsilon_r - 1) E$

$$\begin{aligned}\therefore \alpha_e &= \frac{\epsilon_0 (\epsilon_r - 1)}{N} = \frac{8.85 \times 10^{-12} (0.0024)}{2.7 \times 10^{25}} \text{ F.m}^2 \\ &= \frac{8.85 \times 10^{-12} \times 2.4 \times 10^{-3}}{2.7 \times 10^{25}} = 7.86 \times 10^{-40} \text{ F.m}^2\end{aligned}$$

UNIT – IV**QUANTUM MECHANICS FOR ELECTRONIC TRANSPORT****QUANTUM MECHANICS**

Syllabus: Introduction - Schrödinger's time independent & time dependent wave equations - Physical signification of wave function - Particle in one dimensional potential box - operator version.

1. What is de Broglie hypothesis ?

As light exhibit dual nature, all fundamental moving matter particles like electron, proton, and neutron also exhibit dual nature.

Every moving fundamental particle is associated with a wave called de Broglie matter wave with a wave length given by $\lambda = \frac{h}{p} = \frac{h}{mv}$

Where $p = mv$ is the momentum of the particle.

2. Derive the expression for de Broglie wavelength in total energy (E).

$$\lambda = \frac{h}{mv}$$

The de Broglie wavelength λ is given by

This expression can be modified into different forms

If E is the K.E of the moving particle, then

$$E = \frac{1}{2} m v^2$$

$$mv = \sqrt{2mE}$$

$$\text{Hence } \lambda = \frac{h}{\sqrt{2mE}}$$

3. Derive the expression for de Broglie wavelength in terms of potential difference(V).

Consider a particle of mass 'm' and charge 'e' being accelerated by potential 'V' volts. If 'v' is the velocity acquired by the electron, then KE is

$$\frac{1}{2} m v^2 = eV \quad \text{or} \quad mv = \sqrt{2meV}$$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}}$$

For an electron :

$$\lambda = \frac{6.625 \times 10^{-34}}{(2 \times 9.1 \times 10^{-31} \times 1.632 \times 10^{-19} \times V)^{\frac{1}{2}}}$$

$$\lambda = \frac{12.26}{\sqrt{V}} \text{ Å}, \text{ where } V \text{ is the applied voltage in volts.}$$

4. Derive the expression for de Broglie wavelength of thermal neutrons.

The de Broglie wavelength associated with thermal neutrons at a temperature 'T' can be calculated using the laws of Kinetic theory of gases:

$$\frac{1}{2} m v^2 = \frac{3}{2} K_B T, \quad \text{where } v \text{ is the velocity of neutrons.}$$

$$\therefore m v = \sqrt{3 m K_B T}$$

$$\therefore \text{de Broglie wavelength } \lambda = \frac{h}{mv} = \frac{h}{\sqrt{3 m K_B T}}$$

At temperature T=300K (Room temperature), the de Broglie wavelength of neutrons is

$$\lambda = \frac{6.62 \times 10^{-34}}{(3 \times 1.67 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300)^{1/2}} = 0.146 \times 10^{-9} \text{ m}$$

5. What are matter waves and write the Properties of matter waves.

The waves which are associated with moving materialistic particle are known as matter waves.

$$\text{Since } \lambda = \frac{h}{p} = \frac{h}{mv}$$

- Lighter the particle, longer is the wavelength associated with it.
- Smaller the velocity of the particle, longer is the wavelength associated with it.
- For $v = 0$, $\lambda = \infty$. This means that matter wave is associated only with moving particles of matter.
- Matter waves are associated with all moving charged or uncharged particles.
- Matter waves are not electromagnetic waves since they travel faster than light.

6. Write about Heisenberg Uncertainty principle:

"It is impossible to determine both the exact position and exact momentum of a moving particle simultaneously".

If Δx denotes the uncertainty in the measurement of position and Δp denotes the uncertainty in the measurement of momentum, then $(\Delta x) \cdot (\Delta p) \geq \frac{h}{2}$

In terms of energy and time, the uncertainty principle is given by $(\Delta E) \cdot (\Delta t) \geq \frac{h}{2}$

7. Derive Schrodinger's time independent wave equation.

Let x, y, z be the coordinates of the particle and ψ is wave function which describes the wave displacement at any time t.

The classical differential equation of a wave motion is

$$\nabla^2 \Psi = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} \quad \dots\dots (1)$$

Where $\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$ is Laplacian Operator and 'v' is the wave velocity.

The solution of eqn (1) is given by

$$\Psi = \Psi_0 e^{-i\omega t} \quad \dots\dots (2)$$

Differentiating eqn (2) once, we get

$$\frac{\partial \Psi}{\partial t} = \Psi_0 e^{-i\omega t} (-i\omega)$$

Differentiating eqn (2) twice, we get

$$\frac{\partial^2 \Psi}{\partial t^2} = \Psi_0 e^{-i\omega t} (-i\omega)^2$$

$$\begin{aligned}\frac{\partial^2 \Psi}{\partial t^2} &= -(\omega)^2 \Psi \\ \frac{\partial^2 \Psi}{\partial t^2} &= -4\pi^2 v^2 \Psi \\ \frac{\partial^2 \Psi}{\partial t^2} &= -4\pi^2 \frac{V^2}{\lambda^2} \Psi\end{aligned}$$

$$\frac{\partial^2 \Psi}{\partial t^2}$$

Substituting the value of $\frac{\partial^2 \Psi}{\partial t^2}$ in eqn (1) we get

$$\nabla^2 \Psi = -\frac{4\pi^2 v^2}{\lambda^2} \Psi$$

$$\nabla^2 \Psi + \frac{4\pi^2}{\lambda^2} \Psi = 0$$

$$\text{From de Broglie's Principle, } \lambda = \frac{h}{mv}$$

$$\nabla^2 \Psi + \frac{4\pi^2 m^2 v^2}{h^2} \Psi = 0$$

If the particle has mass 'm' and velocity 'v', then

The Total energy = Kinetic Energy + Potential Energy

$$E = \frac{1}{2}mv^2 + V \quad \Rightarrow \quad E - V = \frac{1}{2}mv^2 \quad \Rightarrow \quad 2m(E - V) = m^2v^2$$

Then equation (2) becomes



Writing $\hbar = h/2\pi$ in the above eqn, we get



This equation is known as Schrodinger Time independent wave equation.

If the particle is free particle, then $V = 0$

$$\nabla^2 \Psi + \frac{8\pi^2 m E}{h^2} \Psi = 0$$

$$\nabla^2 \Psi + \frac{2mE}{\lambda^2} \Psi = 0$$

8. Derive Schrodinger's time dependent wave equation.

Schrodinger's time dependent wave equation:

$$\nabla^2 \Psi = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} \quad \text{----- (1)}$$

The classical differential equation of a wave motion is

The solution of eqn (1) is given by

$$\Psi = \Psi_0 e^{-i\omega t}$$

Differentiating eqn (2) w.r.t time 't' once, we get

$$\frac{\partial \Psi}{\partial t} = \Psi_0 e^{-i\omega t} (-i\omega)$$

$$\frac{\partial \Psi}{\partial t} = \Psi(-i\omega)$$

$$\frac{\partial \Psi}{\partial t} = \Psi(-i2\pi\nu)$$

$$\frac{\partial \Psi}{\partial t} = \Psi(-i2\pi) \frac{E}{h} (\sin ceE = h\vartheta)$$

$$\frac{-h}{2i\pi} \frac{\partial \Psi}{\partial t} = E\Psi$$

$$\frac{i\hbar}{2\pi} \frac{\partial \Psi}{\partial t} = E\Psi$$

$$i\hbar \frac{\partial \Psi}{\partial t} = E\Psi$$

The Schrödinger time independent wave Equation

$$\nabla^2 \Psi + \frac{2m(E - V)}{\hbar^2} \Psi = 0$$

$$\nabla^2 \Psi + \frac{2m(E\Psi - V\Psi)}{\hbar^2} = 0$$

Substituting E Ψ value in above equation

$$\nabla^2 \Psi + \frac{2m}{\hbar^2} (i\hbar \frac{\partial \Psi}{\partial t} - V\Psi) = 0$$

$$\nabla^2 \Psi = -\frac{2m}{\hbar^2} (i\hbar \frac{\partial \Psi}{\partial t} - V\Psi)$$

$$\frac{-\hbar^2}{2m} \nabla^2 \Psi + V\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$$(\frac{-\hbar^2}{2m} \nabla^2 + V)\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$$\hat{\mathbf{H}} \Psi = \frac{\hbar}{E} \Psi$$

Where \hat{E} is known as Energy Operator = $i\hbar \frac{\partial}{\partial t}$

$\hat{\mathbf{H}}$ is Hamiltonian Operator = $\frac{-\hbar^2}{2m} \nabla^2 + V$

9. What is the Physical significance of wave function ‘ Ψ ’ and mention its properties.

Physical significance:

According to Schrodinger, $\psi^* \psi = |\psi|^2$ is a measure of particle density.

According to Max Born, $\psi\psi^* = |\psi|^2$ gives the probability of finding the particle in the state ψ
i.e. $|\psi|^2$ is a measure of probability density.

∴ The Probability of finding the particle somewhere in space $\iiint |\Psi|^2 d\tau = 1$

Ψ Satisfying above requirement is said to be normalized.

Mathematical Properties of wave function (Ψ):

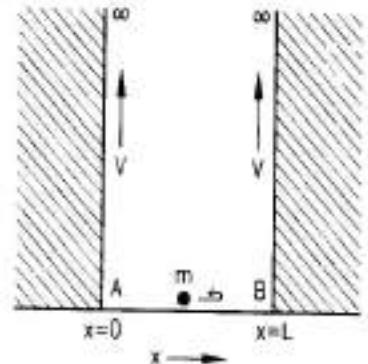
1. Ψ must be finite and single valued for all values of x, y and z.
2. Ψ must be continuous in all regions except where potential energy is infinite.
3. $\frac{\partial\Psi}{\partial x}$, $\frac{\partial\Psi}{\partial y}$ and $\frac{\partial\Psi}{\partial z}$ must be continuous.
4. Ψ and its first order derivative must vanish at boundaries.

10. Obtain the energy eigen values and eigen wave functions for a particle moving in one dimensional potential box.

Consider a particle moving inside a box along the x- direction. The particle is bouncing back and forth between the walls of the box. The box is supposed to have walls of infinite height at $x=0$ and $x=L$. The particle has a mass m and its position x at any instant is given by $0 < x < L$.

The potential energy $V(x)$ of the particle is

$$V(x) = 0, \quad 0 < x < L \quad \text{and} \quad V(x) = \infty, \quad x < 0 \text{ and } x > L$$



Since the potential energy outside the box is infinitely high, the probability of finding the particle outside must be zero.

Hence $|\Psi|^2 = 0$ in the region $0 > x > L \Rightarrow \Psi = 0$ at $x = 0$ and $x = L$

Inside the box the wave function is finite

$$\therefore |\Psi|^2 \neq 0, \text{ in the region } 0 < x < L; \quad U = 0$$

Within the box, the one - dimensional Schrodinger wave equation is

$$\frac{d^2\Psi}{dx^2} + \frac{2mE}{\hbar^2}\Psi = 0 \quad \frac{d^2\Psi}{dx^2} + k^2\Psi = 0$$

$$\text{where} \quad K^2 = \frac{2mE}{\hbar^2} \quad -- \quad (1)$$

The general solution of above equation is,

$$\Psi = A \sin Kx + B \cos Kx$$

To evaluate the constants A and B we must use the boundary conditions namely,

$\Psi = 0$ at $x=0$ and hence $B=0$

$\Psi = 0$ at $x=L$ and hence $0=A \sin KL$

$$\therefore \sin KL = 0 \quad \text{or} \quad KL = n\pi \quad \Rightarrow \quad K = n\pi/L \quad \dots\dots\dots (2)$$

where n is an integer

\therefore The wave function becomes $\Psi_n = A \sin \sin \left(\frac{n\pi x}{L} \right) \dots\dots\dots (3)$

$$\text{from (1) \& (2)} \quad \frac{2mE}{\hbar^2} = \frac{n^2 \pi^2}{L^2}$$

$\Rightarrow E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$ where E_n - Energy of the particle in the n^{th} energy state.

$$E_n = \frac{n^2 h^2}{8mL^2}$$

To find the value of A , we use the condition $\int_0^L |\Psi|^2 dx = 1$

Since the electron should exist within the box,

$$\int_0^L A^2 \sin^2 \left(\frac{n\pi x}{L} \right) dx = 1$$

$$\Rightarrow A^2 \int_0^L \sin^2 \left(\frac{n\pi x}{L} \right) dx = 1$$

$$\Rightarrow A^2 \int_0^L \left[\frac{1 - \cos \left(\frac{2n\pi x}{L} \right)}{2} \right] dx = 1$$

$$\Rightarrow \frac{A^2}{2} \int_0^L dx - \frac{A^2}{2} \int_0^L \cos \left(\frac{2n\pi x}{L} \right) dx = 1$$

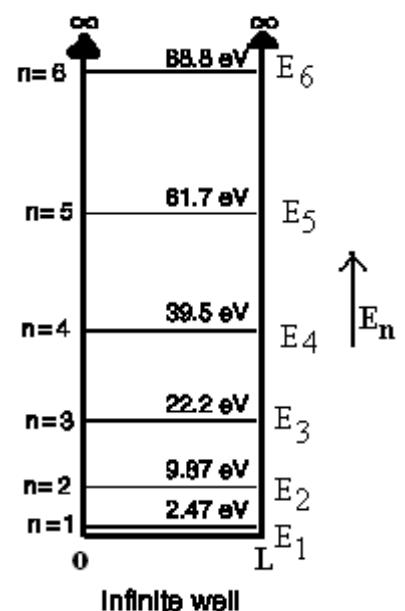
$$\Rightarrow \frac{A^2}{2} [x]_0^L = 1 \quad (\text{since } \int_0^L \cos \left(\frac{2n\pi x}{L} \right) dx = 0)$$

$$\Rightarrow \frac{A^2}{2} L = 1 \quad \Rightarrow \quad A^2 = \frac{2}{L} \quad \Rightarrow \quad A = \sqrt{\frac{2}{L}}$$

\therefore The normalized wave functions are given by $\Psi_n = \sqrt{\frac{2}{L}} \sin \sin \left(\frac{n\pi x}{L} \right)$

The normalized wave functions of a particle in a three dimensional box are given by

$$\Psi(x, y, z) = \sqrt{\frac{2}{a}} \sin \sin \left(\frac{n\pi x}{a} \right) \sqrt{\frac{2}{b}} \sin \sin \left(\frac{n\pi y}{b} \right) \sqrt{\frac{2}{c}} \sin \sin \left(\frac{n\pi z}{c} \right)$$



And the Energy eigen values are given by $E_n = \frac{h^2}{8m} \left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2} \right)$

Problems

1. Calculate the wavelength associated with an electron with energy 2000eV.

Solution: $E=2000\text{eV}=2000 \times 1.6 \times 10^{-19}\text{J}$

$$\begin{aligned}\lambda &= h/\sqrt{2mE} \\ &= 6.6 \times 10^{-34} / \sqrt{2 \times 9.1 \times 10^{-31} \times 2000 \times 1.6 \times 10^{-19}} \\ &= 0.0275 \times 10^{-9}\text{m} \\ &= 0.0275\text{nm}.\end{aligned}$$

2. An electron is confined in a one dimensional potential well having width of $3 \times 10^{-10}\text{m}$. Find the kinetic energy of the electron when it is in ground state

Solution:

$$a = 3 \times 10^{-10}\text{m}$$

$$n=1$$

$$\begin{aligned}E_n &= n^2 h^2 / 8ma^2 \\ &= \frac{1^2 \times (6.6 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (3 \times 10^{-10})^2} \\ &= 0.067 \times 10^{-17}\text{J} \\ E &= 0.067 \times 10^{-17} / 1.6 \times 10^{-19} \\ &= 4.2 \text{ eV}\end{aligned}$$

FREE ELECTRON THEORY

SYLLABUS: Classical free electron theory - electrical conductivity - Mean free path - Relaxation time and drift velocity - Quantum free electron theory - Fermi - Dirac (analytical) and its dependence on temperature - Fermi energy - density of states - derivations for current density.

1. Explain Classical free electron theory of metals.(Drude-Lorentz theory of metals) and derive expression for electrical conductivity in terms of mobility .

Drude and Lorentz proposed this theory in 1900.

According to this theory

1. Metals consist of free valence electrons. These electrons are free to move throughout the metal.
2. The free electrons move randomly in all directions and collide with positive ions or other free electrons.
3. So net current is zero in the absence of external field.
4. The free electrons move in a completely uniform potential field due to ions fixed in the lattice.
5. When an electric field is applied, the free electrons are accelerated in the direction opposite to the applied field.

Expression for electrical conductivity of metal:

When electric field is applied, then the Lorenz force acting on electrons $F_L = -eE$

$$\frac{mv_d}{\tau}$$

The frictional force on electron due to collision can be expressed as $F_F = \frac{mv_d}{\tau}$

From Newton's law, we know that

$$F = m \frac{dv_d}{dt}$$

$$\Rightarrow F = F_L + F_F$$

$$= m \frac{dv_d}{dt} = -eE - \frac{mv_d}{\tau}$$

For steady state condition $\frac{dv_d}{dt} = 0$

$$\frac{mv_d}{\tau} = -eE \Rightarrow v_d = \frac{-eE\tau}{m}$$

This is the steady state velocity of an electron which is known as drift velocity.

Drift velocity:

Definition: The average velocity acquired by an electron after steady state reached, in the presence of electric field is called drift velocity.

If 'n' is the no of electrons per unit volume,

The current density is $J = nev_d$

$$J = ne \times \frac{-e\tau}{m} E$$

$$J = \frac{ne^2\tau}{m} E$$

$$J = \sigma E \quad \text{which is known as ohm's law.}$$

Where $\sigma = \frac{ne^2\tau}{m}$ is the conductivity of electrons.

Mobility of the electron μ :

It is defined as the steady state drift velocity (v_d) per unit electric field.

$$\mu = \frac{|v_d|}{E}$$

$$\mu = \frac{\frac{e\tau}{m} E}{E} = \frac{e\tau}{m}$$

We know that $\sigma = \frac{ne^2\tau}{m}$

$$\sigma = ne \frac{e\tau}{m}$$

$$\sigma = ne\mu$$

This is the expression for electrical conductivity in terms of mobility of electrons.

The electrical conductivity σ depends on two factors, the charge density 'n' and their mobility μ . These quantities depend on temperature. The expression for resistivity of metal is given by

$$\rho = \frac{1}{\sigma} = \frac{1}{ne\mu} = \frac{m}{ne^2\tau_r}$$

Success of classical free electron theory:

1. It verifies ohm's law.
2. It explains the electrical and thermal conductivities of metals.
3. It derives Widemann-Franz law (The relation between electrical conductivity and thermal conductivity)
- 4.** It explains optical properties of metals.

Drawbacks of classical free electron theory:

1. The phenomenon such as photoelectric effect, Compton effect and black body radiation could not explain by this theory.
2. The theoretical electronic specific heat of metals is in disagreement with experimental values.
3. According to this theory, $\sigma \propto \frac{1}{\sqrt{T}}$, but experimentally it was found that $\sigma \propto \frac{1}{T}$

2. Define Mean free path (λ):

The average distance travelled by an electron between two successive collisions in the presence of applied field is known as mean free path. It can be expressed mathematically as

$$\lambda = c \Gamma$$

where c is RMS velocity of electrons
 Γ is mean collision time.

3. Explain about Relaxation time (τ_r):

Let us assume that the applied field is cutoff after the drift velocity of the electron has reached its steady value. Drift velocity after this instant is governed by

$$m \frac{dv_d}{dt} = -eE - \frac{mv_d}{T_r}$$

$$\text{If } E=0, m \frac{dv_d}{dt} = -\frac{mv_d}{\tau}$$

$$\frac{dv_d}{v_d} = \frac{1}{\tau} dt$$

Integrating the above eqn, we get

$$v_d(t) = v_d(0)e^{\frac{-t}{\tau}}$$

Where $v_d(0)$ is the drift velocity is at the instant the field E is cutoff i.e. $t=0$.

$$\text{If } \tau = t, \text{ then } v_d(t) = \frac{v_d(0)}{e}$$

Relaxation time: The time taken for the drift velocity to decay to $\frac{1}{e}$ of its initial value.

4. Explain Quantum theory of free electrons (Sommerfeld's theory of metals)

Quantum free electron theory was proposed by Sommerfeld in 1928

According to this theory

1. Metal has free electrons moving randomly in all directions. Here the electrons are treated as waves.
2. These free electrons are treated as non-interacting particles and confined in a three dimensional box of **constant internal potential**. Hence the energy of free electrons is quantized.
3. These free electrons occupy different energy levels subjected to Pauli's exclusion principle. Hence they possess different velocities.
4. These free electrons with different velocities in a metal can be shown in K-space representing a sphere known as Fermi sphere.

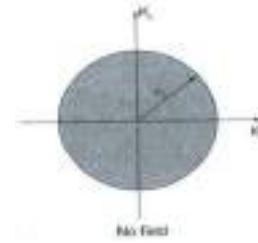
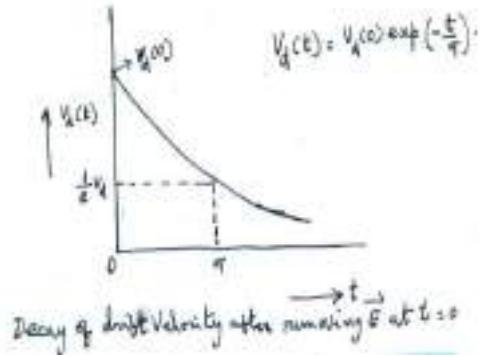
Expression for electrical conductivity of metal:

When electric field is applied on the free electrons along the positive X-direction, the Lorenz force exerted on the electron is $-eE$

$$F_L = -eE$$

The force is rate of change of momentum

$$F = \frac{dp}{dt}$$



$$-eE = \frac{dp}{dt}$$

From de Broglie's theory, $P = \frac{h}{\lambda} = \frac{h}{2\pi} \times \frac{2\pi}{\lambda} = \hbar K$

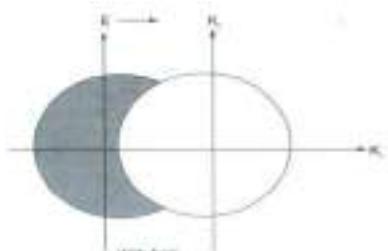
$$-eE = \hbar \frac{dK}{dt}$$

$$dK = \frac{-eE}{\hbar} dt$$

This means that origin of the K space moves through a distance dK in time dt on the application of external field. Because of collisions with imperfections, displacement of K space becomes steady.

Integrate above equation $\int_0^t dK = - \int_0^t \frac{eE}{\hbar} dt$

$$K(t) - K(0) = -\frac{eE}{\hbar} t$$



$$\Delta K = \frac{eE}{\hbar} \tau -$$

$$\frac{eE}{\hbar} \tau$$

$t = \tau$ is the mean collision

time

Since $P = mv = \hbar K$, the incremental velocity ΔV is given by

$$m\Delta v = \hbar \Delta K \Rightarrow \Delta v = \frac{\hbar}{m} \Delta K$$

$$\Delta v = \frac{\hbar}{m} \times \frac{-eE\tau}{\hbar}$$

$$\Delta v = \frac{-eE\tau}{m}$$

If Δv is increased velocity due to application of electric field, and 'n' is no of electrons per unit volume.

$$J = n(-e)\Delta v$$

$$J = -ne \times \frac{-eE\tau}{m} = \frac{ne^2\tau}{m} E$$

$$J = \sigma E \text{ (Ohm's law)}$$

where $\sigma = \frac{ne^2\tau}{m}$ is the electrical conductivity. $\sigma = \frac{ne^2\tau}{m}$

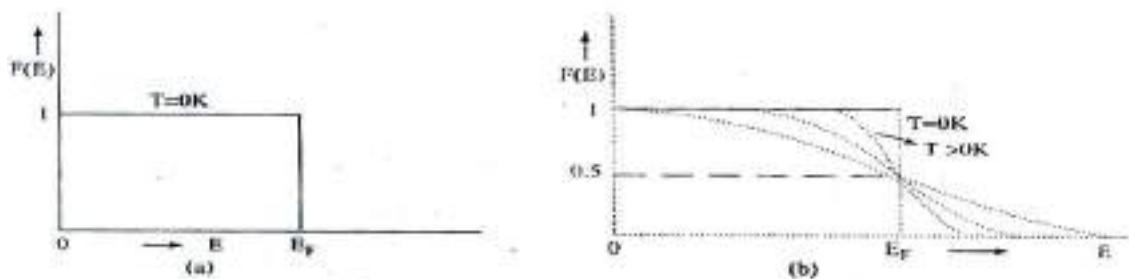
5. Explain Fermi-Dirac distribution and how it varies with temperature:

The probability of an electron occupancy in an energy level E is described by Fermi-Dirac statistics.

$$F(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}}$$

Where E_F is Fermi energy and E is the energy of the allowed state.

$F(E)$ is called Fermi-Dirac distribution function defined as the probability of an electron occupancy in an energy level E.



The Fermi distribution function at different temperatures.

- At $T=0K$, for $E < E_F$, then $F(E) = \frac{1}{1+e^{-\infty}} = \frac{1}{1+0} = 1$
This means that all energy states below E_F are filled.
- At $T=0K$, for $E > E_F$, then $F(E) = \frac{1}{1+e^{\infty}} = \frac{1}{1+\infty} = 0$
This means all the energy states above E_F are empty.

- At $T>0K$, for $E = E_F$, then $F(E) = \frac{1}{2}$

Fermi level: The energy state at which the probability of electron occupation is $\frac{1}{2}$ at any temperature above 0K or It is the level of maximum energy of the filled states at 0K.

Problems:

1. Find the relaxation time of conduction electrons in a metal of resistivity $1.54 \times 10^{-8} \text{ ohm-m}$. If the metal has $5.8 \times 10^{28} \text{ conduction electrons/m}^3$

Solution: Given

$$m = 9.1 \times 10^{-34} \text{ kg}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$n = 5.8 \times 10^{28} \text{ atoms per m}^3$$

$$\rho = 1.54 \times 10^{-8} \text{ ohm-m}$$

$$\rho = m/ne^2\tau$$

$$\tau = \frac{m}{ne^2\rho}$$

$$= 3.97 \times 10^{-14} \text{ s.}$$

BAND THEORY OF SOLIDS

Bloch theorem (qualitative) - Kronig - Penny model - Origin of energy band formation in solids - Classification of materials into conductors, semiconductors & insulators - Concepts of effective mass of electron - concept of hole.

1. Explain the Bloch theorem.

Bloch theorem states that if a potential is periodic with the periodicity of the lattice, i.e. $V(x+a) = V(x)$

then the solution of the Schrodinger's equation is given by

$$\psi(x) = u_k(x) e^{ikx}$$

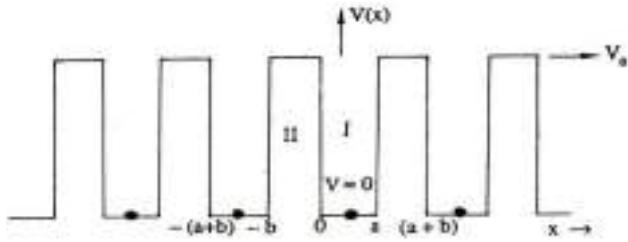
Where $u_k(x)$ is called Bloch function and has the periodicity of the lattice given by

$$u_k(x) = u_k(x + a).$$

The wave vector k represents the state of the electron motion with momentum $p = \hbar k$

2. Discuss The Kronig-Penny Model for motion of an electron in a Periodic Potential .

According to Kronig and Penny model, the electron moves in a periodic potential produced by positive ions in a crystal which is in the form of rectangular wells and barriers as shown in fig.



The potential wells are of zero potential energy of width 'a' (lattice periodicity). They are separated by rectangular barriers of width 'b' and height V_0 .

The wave functions ψ associated with the electron motion can be calculated by solving Schrodinger's equation for the two regions I and II. The time-independent Schrodinger's equations

$$\frac{d^2 \psi}{dx^2} + \frac{8\pi^2 m}{\hbar^2} E \psi = 0 \text{ for } 0 < x < a \text{ (region I)}$$

for the two regions are:

$$\frac{d^2 \psi}{dx^2} + \frac{8\pi^2 m}{\hbar^2} (E - V_0) \psi = 0 \text{ for } -b < x < 0 \text{ (region II)}$$

$$\text{Let } \alpha^2 = \frac{8\pi^2 m}{\hbar^2} E \text{ and } \beta^2 = \frac{8\pi^2 m}{\hbar^2} (E - V_0) .$$

$$\text{Thus } \frac{d^2 \psi}{dx^2} + \alpha^2 \psi = 0 \text{ (for region I)} \quad \text{and} \quad \frac{d^2 \psi}{dx^2} + \beta^2 \psi = 0 \text{ (for region II)}$$

The solutions of the above Schrodinger's equations can be found using Bloch function and are given by

$$\psi(x) = u_k(x) e^{ikx},$$

where $u_k(x)$ is periodic with the periodicity of the lattice, i.e., $u_k(x) = u_k(x + a)$.

Differentiating twice with respect to x and substituting in these following equations

$$\frac{d^2 \psi}{dx^2} + \alpha^2 \psi = 0, \text{ for } 0 < x < a \quad \text{and} \quad \frac{d^2 \psi}{dx^2} + \beta^2 \psi = 0, \text{ for } -b < x < a ,$$

We will arrive at two independent second order linear differential equations for the two regions. Applying the necessary boundary conditions, then we can arrive at the solutions of the wave equations.

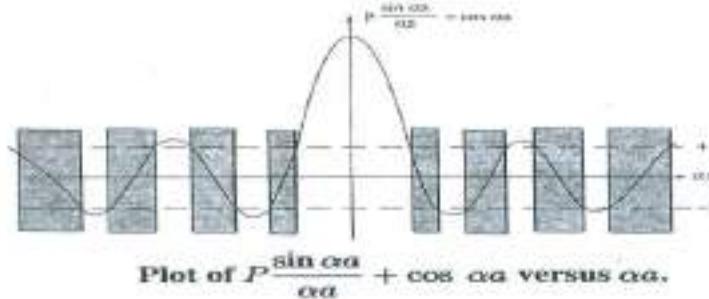
It is found that solutions of the wave equations are possible only for energies permitted by the equation

$$\frac{P \sin \alpha a}{\alpha a} + \cos \alpha a = \cos k a \quad \dots \dots \dots (1)$$

where $P = \frac{4\pi^2 m V_0 ab}{h^2}$ is the scattering power of the potential barrier.

Energy bands

There are only two variables α and k in above equation. The right hand side (RHS) is a cosine term. It can vary only between the limits +1 and -1. This condition of $\cos ka = \pm 1$ imposes a restriction on the left hand side (LHS). **This means that ' αa ' can take only certain range of values, i.e., values that make the LHS to vary between +1 and -1.** Fig. shows a plot of LHS of eq.1 as a function of ' αa '



Conclusions:

- (i) The energy spectrum of electrons in a crystal consists of alternate regions of allowed and forbidden energy bands.
- (ii) As the value of ' αa ' increases, the width of the allowed energy bands increases and the width of the forbidden energy bands decreases.

3. What is meant by the effective mass of an electron and explain the concept of hole?

Effective mass of an electron:

The variation of mass of an electron as function of its velocity is termed as effective mass of an electron.

The velocity of an electron in a one-dimensional lattice is described by its group velocity v_g . v_g is

$$v_g = \frac{d\omega}{dk} = \frac{2\pi}{h} \left(\frac{dE}{dk} \right) = \frac{1}{\hbar} \frac{dE}{dk} \quad \text{since } \omega = 2\pi v$$

The acceleration is

$$a = \frac{d v_g}{dt} = \frac{2\pi}{h} \frac{d}{dt} \left(\frac{d E}{dk} \right) = \left(\frac{2\pi}{h} \right) \frac{d^2 E}{dk^2} \frac{dk}{dt} \quad \dots\dots\dots (1)$$

where E is the energy of the electron behaving like a wave.

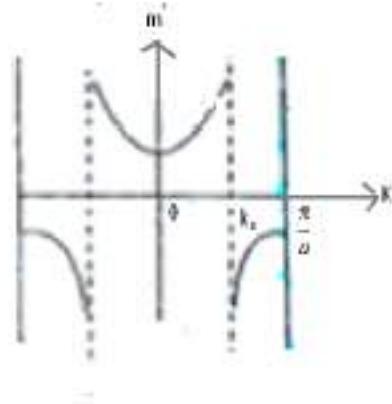
When an electric field \vec{E} is applied, a force ' $e\vec{E}$ ' acts on the electron. The momentum of electron is $p = \hbar k$

$$\Rightarrow \frac{dp}{dt} = \hbar \frac{dK}{dt} \Rightarrow F = \hbar \frac{dK}{dt} \Rightarrow \frac{dK}{dt} = \frac{F}{\hbar} \quad \dots\dots\dots (2)$$

From equations (1) & (2), we get

$$a = \frac{F}{m} \left(\frac{d^2 E}{dk^2} \right) \Rightarrow F = \left(\frac{\hbar^2}{\left(\frac{d^2 E}{dk^2} \right)} \right) a \Rightarrow F = m^* a$$

Where $m^* = \frac{\hbar^2}{\left(\frac{d^2 E}{dk^2} \right)}$ is the effective mass of an electron.



The effective mass m^* of an electron in a periodic potential as a function of k are shown in Fig.

Concept of hole:

At the top of the band ($k = \frac{\pi}{a}$) velocity decreases i.e., acceleration is negative, the lattice exerts a large retarding force on the electron i.e. the **electron has negative mass or behaves as a positively charged particle, a hole**.

4. Explain the origin of energy bands in solids and classifications of solids on the basis of band theory?

Origin of Energy bands in solids

The energy spectrum of electrons in a single isolated atom is discrete and sharp [Fig.(a)]

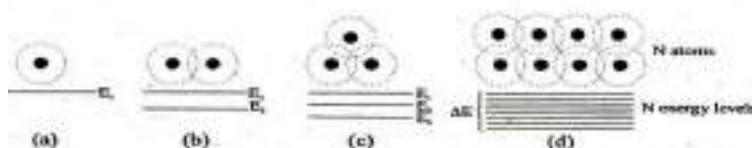
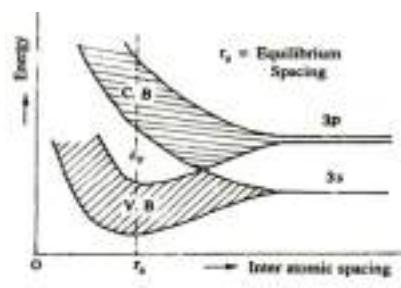


Figure shows the spreading of the energy levels, 3S and 3P bands of solid sodium.

- As the interatomic distance decreases, an energy level splits up into many closely spaced energy levels, thus forming an energy band.
- If N atoms are brought together, each of the energy levels splits into N closely spaced levels form an almost continuous band.
- The width of a given energy band depends on the interatomic separation r .**



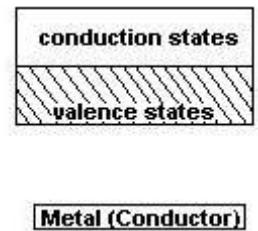
- The lower energy bands are completely filled by electrons and do not participate in electrical conduction.
- The outermost orbits consist of valence electrons and they form a band called valency band. The valence band may be partially or completely filled depending on the nature of the material.
- The next higher permitted band is the conduction band which is the lowest unfilled energy band. This band may be empty or partially filled. In this band, the electrons can move freely.
- The gap between these allowed energy bands is called forbidden energy gap or band gap.

5. Classify the solids into Conductors, Semiconductors and Insulators based on the band theory.

On the basis of energy gap, the solids are classified into three types namely conductors, semiconductors and insulators.

Conductors:

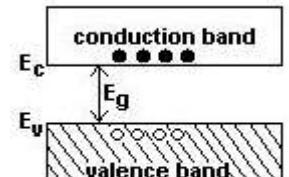
- In conductors, the valence band and conduction band overlap each other and there is no forbidden energy gap between them.
- Conductors possess high electrical conductivity due to plenty of free electrons in conduction band.
- Conductors have Positive temperature coefficient of resistance.
- Ex: Al, Cu, Fe etc.



Metal (Conductor)

Semiconductors:

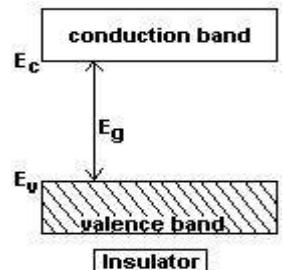
- In Semiconductors, the valence band and conduction band are separated by a small energy band gap ($E_g \sim 1\text{eV}$).
- At 0K, a semiconductor behaves as perfect insulator.
- At room temperature, the electrical conductivity of a semiconductor increases.
- Semiconductors have negative temperature coefficient of resistance.
- Ex: Ge and Si.



Semiconductor

Insulators:

- In insulators, the valence band and the conduction bands are separated by a large energy gap ($E_g \sim 7\text{eV}$).
- The valence band is completely filled and there are no electrons in the conduction band.
- The electrical conductivity of insulators is poor.
- Insulators have negative temperature coefficient of resistance.
- Ex: Diamond, wood, rubber etc.



Insulator

UNIT V SEMICONDUCTORS

Syllabus : Introduction - Intrinsic semiconductor and carrier concentration - Equation for conductivity - Extrinsic semiconductor and carrier concentration - Drift and diffusion - Einstein's equation - Hall Effect - direct & indirect band gap semiconductors - Electronic transport Mechanism for LEDs, Photo conductors and solar cells.

1. What are Semiconductors and different types of Semiconductors

Semiconductors are the materials whose electrical properties (such as energy gap, electrical conductivity etc.) are intermediate between that of conductors and insulators.

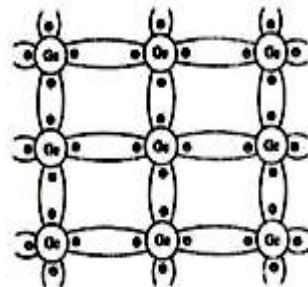
Depending upon purity, the semiconductors are classified in to two types:

- 1) Intrinsic semiconductors
- 2) Extrinsic semiconductors.

Intrinsic semiconductors:

- A semiconductor in its purest form is called intrinsic semiconductor.
- **In Intrinsic semiconductor, number of electrons and holes are equal.**
- Each semiconductor atom has 4 valence electrons, hence they are tetravalent.
Each atom forms four covalent bonds with four different semiconductor atoms.

EX: Si, Ge



Extrinsic semiconductor

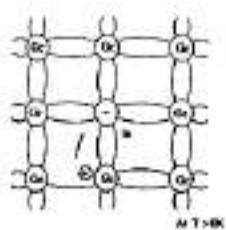
- Extrinsic semiconductors are impure semiconductors or doped semiconductors.
- When certain suitable impurities are added to pure semiconductors, then semiconducting material is known as **extrinsic semiconductor**.
- The process of adding impurities to a pure semiconductor is called **doping**.

Depending upon doping, extrinsic semiconductors are classified into two types

- i) **n-type semiconductors**
- ii) **P-type semiconductors**

n-type semiconductor:

- When pentavalent atoms such as As, Bi are added to a pure semiconductor, then **n-type semiconductor** is formed.
- Since these pentavalent impurity atoms donate electrons (negative charge), they are called **donors**
- Electrons are the majority charge carriers in n-type semiconductor.



P-type semiconductors

- When trivalent atoms such as B, In, are added to an intrinsic semiconductor, then p-type semiconductor is formed.
- Since these trivalent impurity atoms accept electrons (negative charge), they are called **acceptors**
- Holes are the majority charge carriers in p-type semiconductor.

2. Derive the Expression for the concentration of electrons in the conduction band of an intrinsic semiconductor.

The concentration of electrons in the conduction band is

$$n = \int_{E_c}^{\infty} Z(E) F(E) dE$$

We know that

$$Z(E) dE = \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E - E_c)^{1/2} dE$$

and

$$F(E) = \frac{1}{\exp\left(\frac{E - E_F}{K_B T}\right) + 1},$$

Where E_F is the Fermi energy which lies midway between E_c and E_v in intrinsic semiconductors.

$F(E)$ may be written as for all possible temperatures $E - E_F \gg KT$ Where,

$$F(E) = \exp\left(\frac{E_F - E}{K_B T}\right)$$

Therefore n becomes

$$n = \int_{E_c}^{\infty} \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E - E_c)^{1/2} \exp\left(\frac{E_F - E}{K_B T}\right) dE$$

Thus, the density of electrons in the conduction band of an intrinsic semiconductor is given by

$$n = 2 \left(\frac{2\pi m_e^* K_B T}{h^2} \right)^{3/2} \exp\left(\frac{E_F - E_c}{K_B T}\right)$$

3. Derive the expression for the concentration of holes in the Valence band of an intrinsic semiconductor:

The concentration of holes in the valence band is

$$p = \int_{-\infty}^{E_v} Z(E) [1 - F(E)] dE,$$

where $Z(E) dE$ is the density of states in the valence band in the energy range E and $E + dE$ and $[1 - F(E)]$ is the Fermi-Dirac distribution for holes in the valence band.

We know that

$$[1 - F(E)] \approx \exp\left(\frac{E - E_F}{K_B T}\right)$$

The density of energy states in the valence band in the energy interval E and $E+dE$ is given by

$$Z(E)dE = \frac{4\pi}{h^3} (2m_h^*)^{3/2} E^{1/2} dE$$

The concentration of holes in the valence band can be obtained by integrating the above equation from $-\infty$ to E_v . Thus

$$P = \int_{-\infty}^{E_v} \frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_v - E)^{1/2} \exp\left(\frac{E - E_F}{K_B T}\right) dE$$

The concentration of holes in the valence band of an intrinsic semiconductor is thus given by

$$p = 2 \left(\frac{2\pi m_h^* K_B T}{h^2} \right)^{3/2} \exp\left(\frac{E_v - E_F}{K_B T}\right)$$

4. Derive the expression for the Intrinsic Carrier Concentration

In an intrinsic semiconductor, the concentration of electrons n in the conduction band is equal to the concentration of holes p in the valence band. Hence

$n = p = n_i$, where n_i is called the intrinsic carrier concentration. Thus

$$n_i^2 = n p = 4 \left(\frac{2\pi K_B T}{h^2} \right)^3 (m_e^* m_h^*)^{3/2} \exp\left(-\frac{E_c - E_v}{K_B T}\right)$$

$$= 4 \left(\frac{2\pi K_B T}{h^2} \right)^3 (m_e^* m_h^*)^{3/2} \exp\left(\frac{-E_g}{K_B T}\right) ,$$

where $(E_c - E_v) = E_g$ is the forbidden energy gap.

$$\therefore n_i = 2 \left(\frac{2\pi K_B T}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} \exp\left(-\frac{E_g}{2 K_B T}\right)$$

$$\text{Thus } n_i \propto T^{3/2} \text{ and } n_i \propto \exp\left(\frac{-E_g}{2 K_B T}\right)$$

If $m_e^* = m_h^* = m$ (rest mass), then

$$n_i = 2 \left(\frac{2 \pi K_B m}{h^2} \right)^{3/2} T^{3/2} \exp \left(\frac{-E_g}{2 K_B T} \right)$$

5. Derive the expression for Intrinsic electrical conductivity σ_i

The intrinsic electrical conductivity σ_i is given by

$$\sigma_i = n e \mu_e + p e \mu_h ,$$

where μ_e and μ_h are the mobilities of electrons and holes, respectively.

For intrinsic semiconductors, $n = p = n_i$

$$\therefore \sigma_i = n_i e (\mu_e + \mu_h)$$

We know the expression for n_i . Thus

$$n_i = 2 \left(\frac{2 \pi K_B T}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} \exp \left(-\frac{E_g}{2 K_B T} \right)$$

Substituting for n_i in the equation for σ_i , we get

$$\sigma_i = 2e(\mu_e + \mu_h) \left(\frac{2 \pi k_B T}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} \exp \left(-\frac{E_g}{2 k_B T} \right)$$

6. Derive an expression for Carrier Concentration in n-type semiconductor

Density of electrons in the conduction band is given by

$$n = 2 \left(\frac{2 \pi m_e^* K_B T}{h^2} \right)^{3/2} \exp \left(\frac{E_F - E_c}{K_B T} \right)$$

Assuming that E_F lies more than a few $K_B T$ above the donor energy level E_d , the density of ionized donors is given by

$$N_d [1 - F(E_d)] \approx N_d \exp \left(\frac{E_d - E_F}{K_B T} \right)$$

$$\text{At very low temperature } n = N_d [1 - F(E_d)]$$

$$\text{or } E_F = \frac{(E_d + E_c)}{2} + \frac{K_B T}{2} \log \frac{N_d}{2 \left(\frac{2 \pi m_e^* K_B T}{h^2} \right)^{3/2}}$$

Hence, in n-type semiconductor, Fermi energy level at OK lies exactly midway between the donor energy level E_d and the bottom of the conduction band E_c .

7. Derive the expression for Concentration of electrons in the conduction band of an n-type semiconductor

The density of electrons in the conduction is given by

$$n = 2 \left(\frac{2\pi m_e^* K_B T}{h^2} \right)^{3/2} \exp \left(\frac{E_F - E_c}{K_B T} \right)$$

Substituting for E_F from in the exponential term above and rearranging, we get

$$n = \sqrt{2N_d} \left(\frac{2\pi m_e^* K_B T}{h^2} \right)^{3/4} \exp \left(\frac{E_d - E_c}{2K_B T} \right)$$

Thus $n \propto \sqrt{N_d}$ at moderately low temperatures. At higher temperatures, intrinsic behavior predominates and donor concentration becomes insignificant.

8. Derive an expression for Carrier concentration in p-type semiconductors

Density of holes in the valence band is given by

$$p = 2 \left(\frac{2\pi m_h^* K_B T}{h^2} \right)^{3/2} \exp \frac{E_v - E_F}{K_B T}$$

The density of ionized acceptors is given by

$$N_a F(E_a) = N_a \exp \left(\frac{E_F - E_a}{K_B T} \right)$$

Where N_a be the acceptor concentration and E_a is the acceptor energy level.

Since the density of holes in the valence band is equal to the density of ionized acceptors.

$$\text{or } E_F = \left(\frac{E_v + E_a}{2} \right) - \frac{k_B T}{2} \log \frac{N_a}{2 \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{3/2}}$$

$$\text{At } E_F = \frac{E_v + E_a}{2}$$

Hence, at OK, the Fermi level E_F in p-type semiconductors lies at the middle between the acceptor energy level E_a and the top of the valence band E_v .

9. Expression for Density of holes in the valence band of a p-type semiconductor

The density of holes in the valence band is given by

$$p = \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{3/2} \exp\left(\frac{E_v - E_F}{K_B T} \right)$$

Substituting for E_F in the exponential term above, we get

$$p = (2N_a)^{1/2} \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{3/4} \exp\left(\frac{E_v - E_a}{2 K_B T} \right)$$

Thus, $p \propto \sqrt{N_a}$ at moderately low temperatures. At higher temperatures, intrinsic behavior predominates and the contribution due to acceptor atoms becomes insignificant.

10. Explain Drift and Diffusion currents.

Drift Current:

“The total current due to holes and electrons in a semiconductor in the presence of applied electric field is called the drift current”.

According to Ohm's law, the net current density is given by

$$J_{drift} = (J_n)_{drift} + (J_p)_{drift} = ne\mu_n E + pe\mu_p E$$

The drift current density for holes is $(J_p) = p e \mu_h E_x$.

The drift current density for electrons is $(J_n) = n e \mu_e E_x$

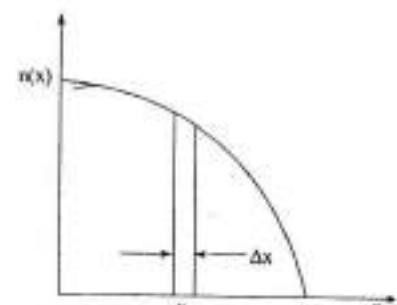
Diffusion current:

The motion of charge carriers (holes and electrons) from a region of higher concentration to a region of lower concentration without any electric field constitutes diffusion current.

The diffusion current is proportional to concentration gradient $\left(\frac{dn}{dx}\right)$.

According to Fick's law of diffusion,

$$\therefore \text{Diffusion electron flux} \propto \left(\frac{dn}{dx} \right) = D_n \frac{dn}{dx}$$



where D_n is the diffusion coefficient for electrons.

$$(J_n) = e D_n \frac{dn}{dx}$$

Hence diffusion current density for electrons is

Similarly for holes, the diffusion current density is

$$(J_p) = -e D_p \frac{dp}{dx}, \text{ where } D_p \text{ is the diffusion coefficient for holes.}$$

∴ The diffusion current density due both electrons and holes is given by

$$J_{diff} = (J_n)_{diff} + (J_p)_{diff} = eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}$$

Total current density in a semiconductor:

Hence, the total current flow in a semiconductor in one dimension is given by

$$J = J_{drift} + J_{diffusion}$$

$$J_s = J_n + J_p = \left[n e \mu_e E_x + e D_n \frac{dn}{dx} \right] + \left[p e \mu_h E_x - e D_p \frac{dp}{dx} \right]$$

11. Derive Einstein's relations? (OR) Derive the relation between Diffusion coefficient and mobility.

Einstein's relations:

The relation between mobility and diffusion coefficient of charge carriers in a semi conductors.

Under thermal equilibrium, the drift and diffusion currents due to an excess density of charge carriers are equal.

$$(\Delta n) e \mu_e E = e D_n \frac{d(\Delta n)}{dx} \quad \text{or} \quad F = (\Delta n) e E = \frac{e D_n}{\mu_e} \frac{d(\Delta n)}{dx},$$

Where $(\Delta n) e E$ is the force acting on the excess charge carriers Δn to restore equilibrium.

According to the kinetic theory of Gases, the force due to the pressure gradient is given by $F = K_B T \frac{d(\Delta n)}{dx}$.

$$K_B T \frac{d(\Delta n)}{dx}.$$

Since these two are analogues to each other, we can compare the corresponding equations. Thus

$$K_B T \frac{d(\Delta n)}{dx} = \frac{e D_n}{\mu_e} \frac{d(\Delta n)}{dx} \quad \text{or} \quad K_B T = \frac{e D_n}{\mu_e} \quad \text{or} \quad D_n = \frac{\mu_e K_B T}{e}$$

Similarly for holes,

$$D_p = \frac{\mu_h K_B T}{e}$$

$$\therefore \frac{D_n}{D_p} = \frac{\mu_e}{\mu_h}$$

The above relations are known as Einstein's relations.

12. State and explain Hall Effect and determine the Hall coefficient.

Hall Effect:

Statement: "When a current carrying semiconductor is placed in a transverse magnetic field, an electric field is produced in a direction normal to both the current and the magnetic field. This phenomenon is known as the Hall effect and the generated voltage is known as Hall Voltage."

Consider a n-type semiconductor in which a current I is flowing in positive X direction and magnetic field B along Y direction.

The Lorentz force is experienced by the electrons is given by

$$F_l = -Bev_d$$

Where v_d is the drift velocity of electrons.

Because of this force, the electrons will drift towards the upper surface. So the upper surface gets negative charge and the bottom surface becomes positive which establishes an electric field E_H . This electric field exerts an downward force.

Under
 $F_H = F_l$

$$v_d = \frac{E_H}{B}$$

If J is the

$$J = -nev_d \Rightarrow v_d = -\frac{J}{ne} \quad \dots\dots\dots (2)$$

$$\text{Equations (1)&(2)} \Rightarrow E_H = -\frac{JB}{ne}$$

The Hall effect is described in terms of Hall coefficient R_H and is given by

$$R_H = -\frac{1}{ne} = \frac{E_H}{JB} \quad \dots\dots\dots (3)$$

By substituting the values of E_H , J and B , the Hall coefficient R_H Can be found out.

Since $R_H = \frac{E_H}{JB}$, when $J = 1$ unit and $B = 1$ unit, then $R_H = E_H$

'Hall coefficient is defined as the Hall electric field per unit current density per unit magnetic induction.'

Determination of R_H

Let 'd' be the width of the sample across which Hall voltage V_H is measured.

$$E_H = \frac{V_H}{d}$$

If t is the thickness of the sample, then

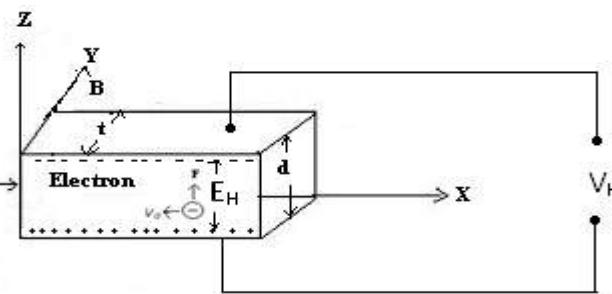


Fig: Hall Effect

$F_H = -eE_H$
equilibrium conditions,

$$\begin{aligned} &\Rightarrow -Bev_d = -eE_H \\ &\Rightarrow E_H = Bv_d \quad \Rightarrow \\ &\dots\dots\dots (1) \end{aligned}$$

current density, then

$$\text{Current density, } E_H = \frac{I}{A} = \frac{I}{dt}$$

Substitute E_H and J value in (3), we get

$$\therefore R_H = \frac{V_H t}{IB}$$

R_H is negative for electrons, and is positive for holes.

Applications of Hall effect:

Hall Effect is used to determine

- i) The sign of charge carriers
- ii) The charge density(n)
- iii) The mobility of charge carriers
- iv) Nature of the material(conductor or semiconductor)