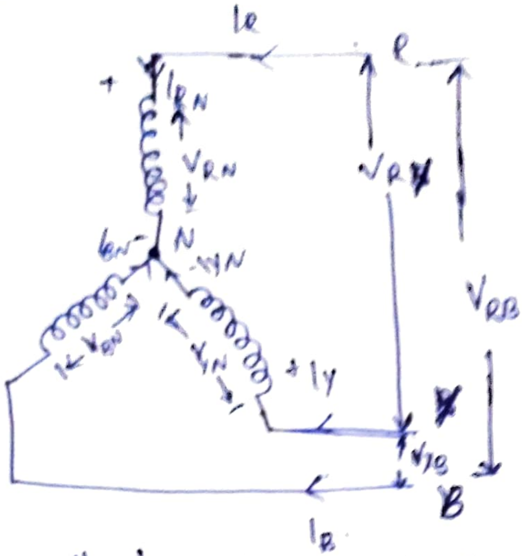
Star Connection:-

ECA-II



where,

V_{RN}, V_{YN}, V_{BN} are the phase voltages, I_{RN}, I_{YN}, I_{BN} are the Phase currents

V_{RY}, V_{BR}, V_{YB} are the line voltages I_R, I_Y, I_B are the line currents

$|I_{RN}| = |I_{YN}| = |I_{BN}| = I_{ph}$

$|V_{RN}| = |V_{YN}| = |V_{BN}| = V_{ph}$

* In star connection the line current = to the phase current

$\boxed{I_L = I_{ph}}$

$V_{RN} = V_{ph} \sin \omega t = V\angle 0°$

$V_{YN} = V_{ph} \sin(\omega t - 120°) = V\angle -120°$

$V_{BN} = V_{ph} \sin(\omega t + 120°) = V\angle 120°$ (or) $V_{ph}\angle -120°$

Applying KVL for (Y-Y-N-R-R).

$V_{YN} - V_{RN} + V_{RY} = 0$

$V_{RY} = V_{RN} - V_{YN}$

Similarly:-

$V_{YB} = V_{YN} - V_{BN}$

$V_{BR} = V_{BN} - V_{RN}$

Consider,

$V_{RY} = V_{RN} - V_{YN}$

$= V\angle 0 - V\angle -120°$ $[V\angle 0 = V(\cos 0 + j\sin 0)]$

$= V(\cos 0 + j\sin 0) - V[\cos(-120) + j\sin(-120°)]$

$= V(1) - V\left[-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right]$

$= V + \frac{V}{2} + j\frac{\sqrt{3}}{2}$

$= V\left[1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right]$

$= V\left[\frac{3}{2} + j\frac{\sqrt{3}}{2}\right]$

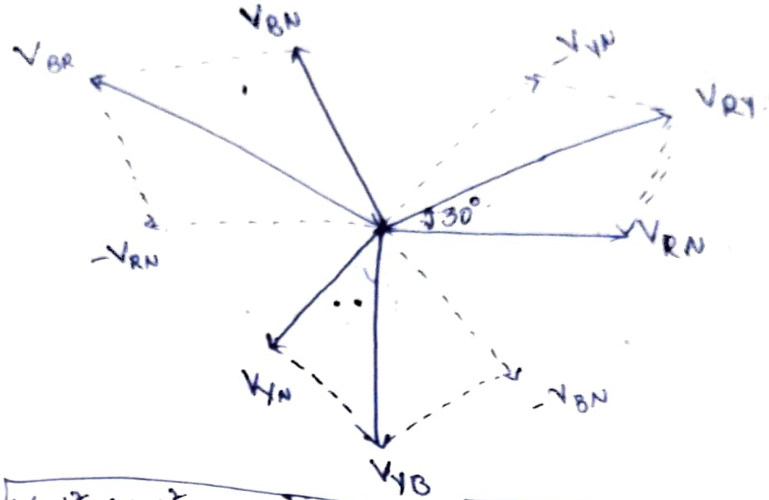
$= V_{ph}\, 1.732 \angle 30°$ $\boxed{V_L = \sqrt{3}\, V_{ph} \angle 30°}$

From the above equation it can conclude that, for a star connected connected 3-ϕ balanced system, the line voltage is $\sqrt{3}$ times of the phase voltage and the line voltage leads the phase voltage by 30°

$V_{RY} = V_{RN} - V_{YN}$

$V_{YB} =$

V_{BR}



$$|V_{RY}| = \sqrt{|V_{RN}|^2 + |V_{YN}|^2 + 2 V_{RN} V_{YN} \cos\theta} = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}^2 \cos\theta} = \sqrt{2V_{ph}^2 + 2V_{ph}^2 \cos 60°}$$

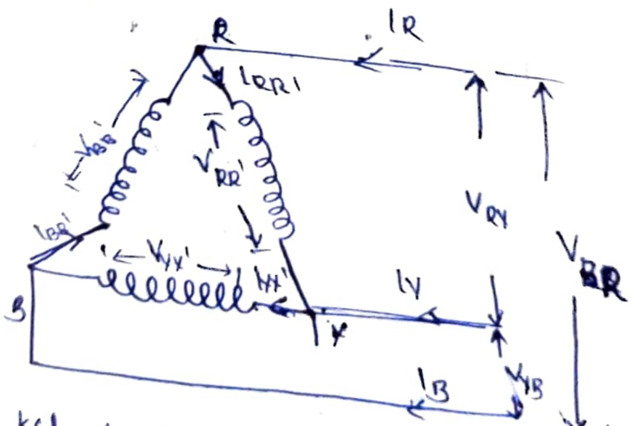
$$= \sqrt{2V_{ph}^2 + V_{ph}^2} = \sqrt{3}\, V_{ph}.$$

$$\boxed{\therefore V_L = \sqrt{3}\, V_{ph}}$$

$V_{RY} = \sqrt{3} V_{ph} \angle 30°$; $V_{YB} = \sqrt{3}\, V_{ph} \angle -120° + 30$; $V_{BR} = \sqrt{3}\, V_{ph} \angle 120° + 30$

$= \sqrt{3}\, V_{ph} \angle -90°$ $\qquad = \sqrt{3}\, V_{ph} \angle 150$

3-ϕ Delta connected system :-



From the diagram we can conclude that, $V_L = V_{ph}$ because they are in parallel connection.

Here,

$V_{RR'}$, $V_{YY'}$, $V_{BB'}$ are the phase voltages

V_{RY}, V_{YB}, V_{BR} are the line voltages

$I_{RR'}$, $I_{YY'}$, $I_{BB'}$ are the phase currents

I_R, I_Y, I_B are the line currents

Now - Applying KCL at Node R :-

$I_R + I_{BB'} = I_{RR'}$

$I_R = I_{RR'} - I_{BB'}$ — (2)

$I_Y = I_{YY'} - I_{RR'}$ — (3)

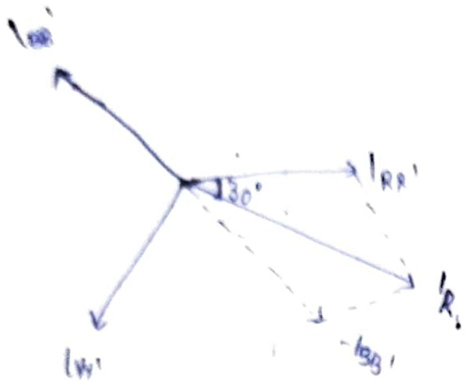
$I_B = I_{BB'} - I_{YY'}$. — (4)

$I_{RR'} = I_{ph} \sin \omega t = I_{ph} \angle 0°$

$I_{YY'} = I_{ph} \sin(\omega t - 120°) = I_{ph} \angle -120°$

$I_{BB'} = I_{ph} \sin(\omega t + 120°) = I_{ph} \angle 120°$.

Phasor diagram:-



From the phasor diagram we can say that the line currents lags the phase current $I_{RR'}$ by $30°$

$I_R = I_{ph} \angle -30°$.

$I_R = \sqrt{(I_{RR'})^2 + (I_{BB'})^2 + 2 I_{RR'} \cdot I_{BB'} \cos 60°}$

$= \sqrt{2 I_{ph}^2 + 2 I_{ph}^2 \cdot \frac{1}{2}}$

$= \sqrt{3}\, I_{ph}$

$\boxed{\therefore I_L = \sqrt{3}\, I_{ph}}$

$\therefore I_R = \sqrt{3} I_{ph} \angle -30°$

$I_Y = \sqrt{3} I_{ph} \angle -120° - 30°$

$= \sqrt{3} I_{ph} \angle -150°$

$I_B = \sqrt{3} I_{ph} \angle +120° - 30$

$= \sqrt{3}\, I_{ph} \angle 90°$.

In a 3-ϕ delta connected balanced system the line current is equal to $\sqrt{3}$ times of the phase current and the line current lags the phase current by $30°$.

Power in 3-ϕ star connected system:-

In a 3-ϕ system the active power (or) the true power is equal to the sum of the power of all the individual phases.

Active power, $P = 3 \times$ [Power in each phase] $= 3 \times V_{ph} I_{ph} \cos\phi$.

In star connection, $V_{ph} = \frac{V_L}{\sqrt{3}}$ $\Rightarrow$ $P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos\phi$

$I_L = I_{ph}$ $= \sqrt{3}\, V_L I_L \cos\phi$. W.

Reactive power, $Q = \sqrt{3} V_L I_L \sin\phi$ VAR.

Apparent power, $S = \sqrt{3} V_L \cdot I_L$ VA

Power in 3-ϕ delta connected system:-

Active power, $P = 3 \times$ (Power in each phase)

$= 3 \times V_{ph} I_{ph} \cos\phi$.

Reactive power, $Q = \sqrt{3} V_L I_L \sin\phi$ VAR

Apparent power, $S = \sqrt{3} \times V_L I_L$ VA

$\Rightarrow$ In delta, $V_{ph} = V_L$, $I_{ph} = \frac{I_L}{\sqrt{3}}$, $P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos\phi$ $\Rightarrow P = \sqrt{3} V_L I_L \cos\phi$ W

1(b) Sol:

Given, delta connected,

$Z_{ph} = (3+j15)\ \Omega$

Z_{ph} for all 3 phases are equal because of balanced load in delta.

$V_L = V_{ph} = 440V$

$I_L = \sqrt{3} I_{ph}$

Phase currents :-

$I_{ph(R)} = \frac{V_{ph}\angle 0°}{Z_{ph}} = \frac{440}{(3+j15)} = 28.76\angle -78.69°$

$I_{ph(Y)} = \frac{V_{ph}\angle -120°}{(3+j15)} = 28.76\angle -78.69 - 120°$

$= 28.76\angle -198.69°$

$I_{ph(B)} = \frac{V_{ph}\angle 120°}{(3+j15)} = 28.76\ \angle -78.69 + 120°$

$= 28.76\angle 41.31°$

Line currents :-

$I_L = \sqrt{3} I_{ph}\angle -30°$

$I_{RY} = \sqrt{3}\ 28.76\angle -78.69 - 30°$

$= 49.81\angle -108.69\ A$

$I_{YB} = 49.81\angle -78.69 - 150°$

$= 49.81\angle 228.69\ A$

$I_{BR} = 49.81\angle -78.69 + 90°$

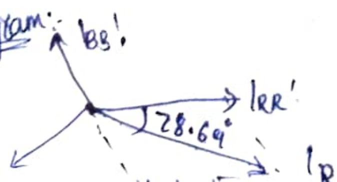
$= 49.81\angle 11.31\ A$.

Total Active power $= \sqrt{3} V_L I_L \cos\phi$

$= \sqrt{3} \times 440 \times 49.81 \cos(78.69)$

$\boxed{P = 7444.63W}$

Phasor diagram:-



2) b) Sol: Given data: Δ - connection

$V_L = V_{ph} = 230V$

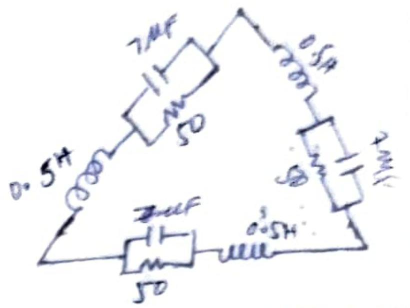
$L = 0.5H$, $C = 7\mu F$, $R = 50\ \Omega$.

$\omega = 400\ rad/sec$

$2\pi f = 400$

$f = \frac{400}{2\pi}$

$f = 63.66\ Hz$



$Z_{eq} = Z_L + Z_{C\|R}$

$Z_L = 2\pi f L = \omega L$

$= 400 \times 0.5$

$= j200\ \Omega$

$Z_{C\|R} = \frac{R(-jX_C)}{R - jX_C}$

$= \frac{RjX_C}{-R + jX_C} = \frac{50(357.14)j}{-50 + (357.14)j} = \frac{17857.14\angle 90°}{360.62\angle 97.96}$

$Z_{C\|R} = 49.51\angle -7.96$

$= 49.03 - j6.85$

$\therefore Z_{eq} = j200 + 49.03 - j6.85$

$= 49.03 + 193.15j$

$= 199.27\angle 75.75$

Phase currents :-

$I_{RR'} = \frac{V_{ph}\angle 0°}{Z_{ph}} = \frac{230\angle 0°}{199.27\angle 75.75} = 1.19\angle -75.75$

$I_{YY'} = \frac{V_{ph}\angle -120°}{Z_{ph}} = \frac{230\angle -120°}{199.27\angle 75.75} = 1.19\angle -195.75$

$I_{BB'} = \frac{V_{ph}\angle 120°}{Z_{ph}} = \frac{230\angle 120°}{199.27\angle 75.75} = 1.19\angle 44.25$

Line currents :-

$I_R = \sqrt{3} I_{RR'}\angle -30° = \sqrt{3} \times 1.19\ \angle -75.75 - 30$

$= 2.06\angle -105.75$

$I_Y = \sqrt{3} I_{YY'}\angle -30° = \sqrt{3} \times 1.19\ \angle -195.75 - 30°$

$= 2.06\angle 225.75$

$I_B = \sqrt{3} \times I_{BB'}\angle -30° = \sqrt{3} \times 1.19\angle 44.25 - 30$

$= 2.06\angle 14.25°$

Active power $= \sqrt{3} V_L I_L \cos\phi$

$= \sqrt{3} \times 230 \times 2.06 \times \cos(75.75) = 202\ W$.

3(a) $P_{delta} = 3\, P_{star}$

3(b)

Sol: $(20-j15) = Z$

$V_{RY} = 330\angle 0°$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ Δ connection, $I_L = \sqrt{3} I_{ph}$, $V_L = V_{ph}$

$I_{ph} = ?$, $I_L = ?$

$\therefore V_L = V_{ph} = 330$

∴ Phase currents:-

$I_{ph(R)} = \frac{V_{ph}}{Z_{ph}} = \frac{330\angle 0°}{20-j15}$

$I_{RR'} = 13.2\angle 36.86°$

$I_{YY'} = \frac{V_{ph}\angle -120°}{Z_{ph}} = \frac{330\angle -120°}{(20-j15)}$

$= 13.2\angle -83.13°$

$I_{BB'} = \frac{V_{ph}\angle 120°}{Z_{ph}} = \frac{330\angle 120°}{(20-j15)}$

$= 13.2\angle 156.86°$

Phase currents:-

$I_R = \sqrt{3} I_{RR'}\angle -30°$

$= \sqrt{3}\times 13.2\angle 36.86-30°$

$= 22.86\angle 6.86°$

$I_Y = \sqrt{3} I_{ph}\angle -150°$

$= \sqrt{3}\times 13.2\angle 36.86-150°$

$= 22.86\angle -113.14$

$I_B = \sqrt{3} I_{ph}\angle 90°$

$= \sqrt{3}\times 13.2\angle 36.86+90°$

$= 22.86\angle 126.86$

4(b)

Sol: Given data:

$L = 0.2\,H$

$C = 6\mu F$, $R = 90\,\Omega$

$V_L = V_{ph} = 400\,V$

$f = 50\,Hz$

$Z_1 = R + jX_L$

$Z_1 = X_L$

$= 2\pi f L$

$= 2\times \pi\times 50\times 0.2$

$= j62.83\,\Omega$

$Z_{eq} = Z_1 + Z_{C\|R}$

$Z_{C\|R} = \frac{R\,jX_C}{R+jX_C} = \frac{90\times 530.51j}{90+530.51j}$

$= \frac{47746.48\angle 90°}{538.09\angle 99.62}$

$= 88.73\angle -9.62°$

$= 87.48 - j14.82$

$\therefore Z_{eq} = j62.83 + 87.48 - j14.82$

$= 87.48 + j48.01$

$= 99.78\angle 28.75$

Phase currents:-

$I_{ph}(I_{RR'}) = \frac{V_{ph}\angle 0°}{Z_{ph}} = \frac{400\angle 0°}{99.78\angle 28.75} = 4\angle -28.75$

$I_{YY'} = \frac{V_{ph}\angle -120°}{Z_{ph}} = \frac{400\angle -120°}{99.78\angle 28.75} = 4\angle -148.75$

$I_{BB'} = \frac{V_{ph}\angle 120°}{Z_{ph}} = \frac{400\angle 120°}{99.78\angle 28.75} = 4\angle 91.25$

Line current's:-

$I_R = \sqrt{3} I_{RR'}\angle -30° = \sqrt{3}\times 4\angle -28.75-30°$

$= 6.92\angle -58.75°$

$I_Y = \sqrt{3} I_{YY'}\angle 30° = \sqrt{3}\times 4\angle -148.75-30° = 6.92\angle -178.75$

$I_B = \sqrt{3} I_{BB'}\angle -30° = \sqrt{3}\times 4\angle 91.25-30° = 6.92\angle 61.25°$

Active power, $P = \sqrt{3}\, V_L I_L \cos\phi$

$= \sqrt{3}\times 400\times 6.92\times \cos(28.75)$

$\boxed{P = 4203.30\,W}$

data:-
$L = 0.2\,H$
$C = 6\mu F,\ R = 90\Omega$
$V_{ph} = 400\,V$.
$f = 50Hz$.
$Z_1 = R_L$, $Z_2 = R + jX_L$
$X_L = 2\pi f L = 2\times\pi\times50\times0.2 = j62.83\Omega$.
$+ Z_C \| R$.

$\frac{R\,jX_C}{R - jX_C} = \frac{90\times530.51j}{-90+530.51j}$

$= \frac{47746.48\angle90^\circ}{538.09\angle99.62}$

$= 88.73\angle-9.62^\circ$.

$= 87.48 - j14.82$.

$j62.83 + 87.48 - j14.82$

$87.48 + j48.01$

$99.78\angle28.75$

$\frac{V_{ph}\angle0^\circ}{Z_{ph}} = \frac{400\angle0^\circ}{99.78\angle28.75} = 4\angle-28.75$

$\frac{V_{ph}\angle-120^\circ}{Z_{ph}} = \frac{400\angle-120^\circ}{99.78\angle28.75} = 4\angle-148.75$

$\frac{V_{ph}\angle120^\circ}{Z_{ph}} = \frac{400\angle120^\circ}{99.78\angle28.75} = 4\angle91.25$

$\sqrt3\times4\angle-28.75-30^\circ$

$= 6.92\angle-58.75^\circ$

$\sqrt3\times4\angle-148.75-30^\circ = 6.92\angle-178.75$

$\sqrt3\times4\angle91.25-30^\circ = 6.92\angle61.25$

$I_L\cos\phi$

$\times6.92\times\cos(28.75)$

$\boxed{3.30\,W}$

Given data:-
$Z_{ph} = (18+j12)\Omega$
$I_L = 22.5\angle35^\circ \Rightarrow I_L = \sqrt3 I_{ph}$
$I_{ph} = \frac{I_L}{\sqrt3} = \frac{22.5\angle35-30}{\sqrt3}$
$I_{ph} = 12.99\angle65^\circ$

$V_{ph} = I_{ph}\cdot Z_{ph}$
$= 12.99\angle65^\circ \times (18+j12)$
$= 281.01\angle33.69^\circ+65^\circ$
$= 281.01\angle98.69^\circ$

In delta, line $V_L = V_{ph} = 281.01\angle98.69^\circ$.

$I_{AB} = 12.99\angle65^\circ$.

Total Active powes, $P = \sqrt3\,V_L I_L\cos\phi$.

$= \sqrt3\times281.01\times22.5\cos(33.69)$

$\boxed{P = 9112.02\,W}$

Reactive powes, $Q = \sqrt3\,V_L I_L\sin\phi$

$= \sqrt3\times281.01\times22.5\times\sin(33.69)$

$\boxed{Q = 6074.66\ VAR}$

6b) Sol:- Given data:-
$R = 10\Omega,\ C = 10\mu F$.

We Assume that R&C are Connected in Parallel,

$Z = \frac{R\,jX_C}{-R+jX_C} = \frac{10(318.30)j}{-10+318.30j}$

$= \frac{3183.0\angle90^\circ}{-10+318.30j}$

$= 9.99\angle1.799^\circ$

In star connection,
$V_L = \sqrt3 V_{ph}$.
$V_L = 415\,V$.
$V_{ph} = \frac{V_L}{\sqrt3} = \frac{415}{\sqrt3} = 239.6\,V$.

Phase currents

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{239.6}{9.99\angle1.799} = 23.98\angle-1.799^\circ$

$I_{YN} = \frac{V_{ph}\angle-120^\circ}{Z_{ph}} = 23.98\angle-121.799^\circ$

$I_{BN} = \frac{V_{ph}\angle120^\circ}{Z_{ph}} = 23.98\angle118.201^\circ$

In star,
Line currents = phase currents.

∴ Real powes, $P = \sqrt3\,V_L I_L\cos\phi$

$= \sqrt3\times415\times23.98\cos(1.799)$

$\boxed{P = 17228.35\,W}$

∴ Reactive powes, $Q = \sqrt3\,V_L I_L\sin\phi$

$= \sqrt3\times415\times23.98\times\sin(1.799)$

$\boxed{Q = 541.12\ VAR}$

7)a) Sol:- Given data:-
$V_L = V_{ph} = 220\,V$ (Δ-connection)
$Z_{ph} = (15+j20)\,\Omega$

Phase currents:-

$I_{ph}(I_{RR'}) = \frac{V_{ph}}{Z_{ph}} = \frac{220}{(15+j20)} = 8.8\angle-53.13^\circ$

$(I_{YY'}) = \frac{V_{ph}}{Z_{ph}} = \frac{220\angle-120^\circ}{15+j20} = 8.8\angle-173.13^\circ$

$I_{BB'} = \frac{V_{ph}}{Z_{ph}} = \frac{220\angle120^\circ}{15+j20} = 8.8\angle66.87^\circ$

ii) Power in each phase $= V_{ph}\,I_{ph}\cos\phi$

$= 220\times8.8\times\cos(53.13)$

$\boxed{P_{in\,each\,ph} = 1161.60\,W}$

iii) Line currents:-

$I_R = \sqrt3\times8.8\angle-53.13-30^\circ = 15.24\angle-83.$

$I_Y = 15.24\angle-203.13^\circ$, $I_B = 15.24\angle36.87$

of line currents.

$= I_R + I_Y + I_B$

$= 15.24\angle -83.13 + 15.24\angle -203.13 + 15.24\angle 36.87$

$= \underline{0}$

8) a)

Sol:- Given data:-

$Z_{ph} = (8+j6)\,\Omega$

$V_L = 230$.

In star, $V_L = \sqrt{3} V_{ph}$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}}$

$V_{ph} = 132.79\,V$

a) $\Rightarrow I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{132.79}{8+j6}$

$= 13.27\angle -36.86$

b) power consumed by load,

$P_{in}/ph = V_{ph} I_{ph} \cos\phi$

$= \cancel{V_{ph}}\,132.79 \times 13.27 \times \cos(36.86)$

$\boxed{P/_{ph} = 1409.88\,W.}$

Total power $4229.64\,W$.

c) Power factor/load:-

$\cancel{\cos}\phi = \tan^{-1}\left(\frac{6}{8}\right)$

$\cancel{\cos}\phi = 36.86$

$\cos\phi = \cos(36.86)$

$\boxed{\cos\phi = 0.8}$

d) Reactive power $= \sqrt{3} V_L I_L \sin\phi$

$= \sqrt{3} \times 230 \times 13.27 \times 0.6$

$\boxed{Q = 3171.83\,VAR}$

9) b) $Z_{ph} = 20+j10\,\Omega$.

Sol:- $V_L = V_{ph} = 440V$.

~~line~~ Phase currents:-

$I_{RR'} = \frac{V_{ph}}{Z_{ph}} = \frac{440}{20+j10} = 19.67\angle -26.56$

$I_{YY'} = 19.67\angle -26.56-120° = 19.67\angle -146.56$

$I_{BB'} = 19.67\angle 120°-26.56 = 19.67\angle 93.44°$

Line currents:-

$I_R = \sqrt{3} \times I_{ph}\angle -30°$

$= \sqrt{3} \times 19.67\angle -26.56-30°$

$= 34.069\angle -56.56°$

$I_Y = \sqrt{3} \times I_{ph}\angle -150°$

$= \sqrt{3} \times 19.67\angle -26.56-150°$

$= 34.069\angle -176.56°$

$I_B = \sqrt{3} \times I_{ph}\angle 90°$

$= \sqrt{3} \times 19.67\angle -26.56+90°$

$= 34.069\angle 63.44°$

$\therefore$ Active power, P

$= \sqrt{3}\, V_L I_L \cos\phi$

$= \sqrt{3} \times 440 \times 34.069 \times \cos(26.56)$

$\boxed{P = 23223.98\,W}$

In Y- connection:

$\Rightarrow V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{V}{\sqrt{3}}$

$\Rightarrow I_L = I_{ph} = \frac{V_{RY}}{R}$

$P_Y = 3(I_{ph})^2 R$

$= \cancel{3}\cancel{\left(\frac{V_{RY}}{R}\right)^2}\cancel{\times R}\ 3 \times \frac{V_L}{\sqrt{3}} \times \frac{V}{\sqrt{3}} \times \frac{1}{R} R$

$= \cancel{3} \times \frac{V^2}{\cancel{3}R^{\cancel{2}}} \cdot \cancel{R} \qquad = \frac{\cancel{3}V^2}{\cancel{3}} R$

$= \frac{V^2}{R} \qquad = \frac{V^2}{R}$

$P_Y = \frac{V^2}{R}$ —(1)

Δ- connection:

$V_{RY} = V_L = V$

$I_L = \frac{I_{ph}}{\sqrt{3}} = \frac{V_{RY}}{R}$

$P_\Delta = 3(I_{ph})^2 \cdot R$

$= 3\left(\frac{V_{RY}}{R^2}\right) \cdot R$

$= 3 \cdot \frac{V^2}{R^2} \cdot R$

$P_\Delta = \frac{3V^2}{R}$ —(2)

from eq (1) & (2),

$\boxed{P_\Delta = 3 P_Y}$

(ii) Advantages of poly phase system over single phase system:-

- More power can be transmitted by the polyphase transmission system using the same amount of conducting material.
- polyphase motor has a more uniform torque than the single phase motors o/p torque is pulsating in nature.
- In the case of Induction motor polyphase induction motors are self-starting and more efficient but 1-ϕ induction motor is not self starting
- The o/p of 3-ϕ machine is always greater than the 1-ϕ machine of the same size.
- A polyphase transmission line requires less conductor material a 1-ϕ line for transmitting the amount of power at the same voltage.
- per the unit of output, the polyphase machine is very much cheaper.

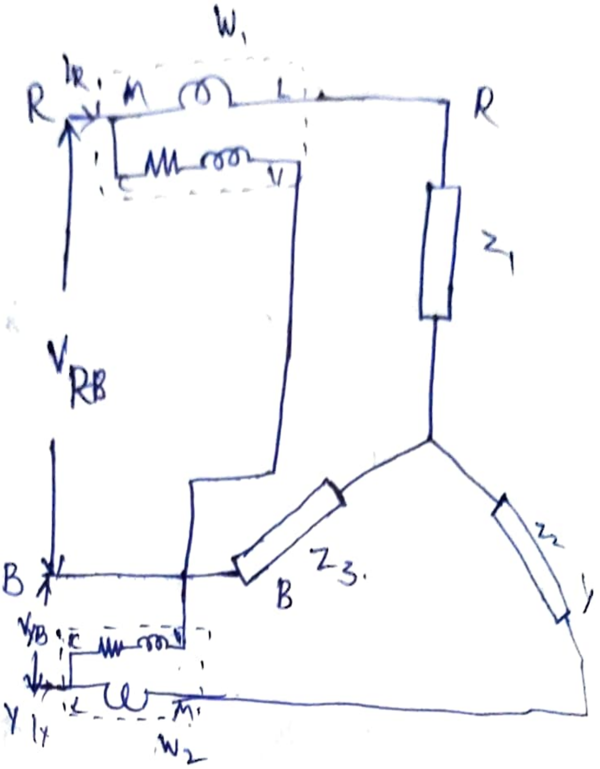
1) a) Show that,

UNIT - II

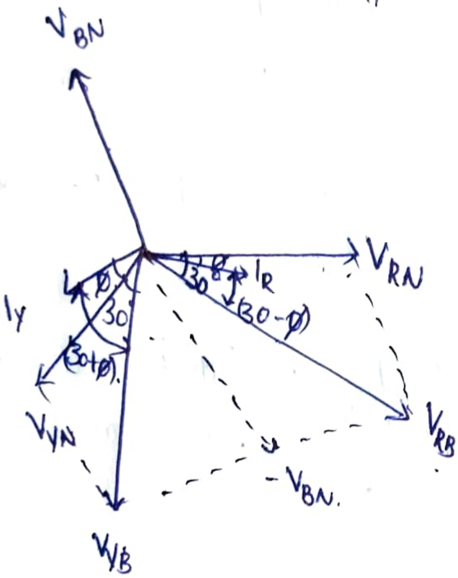
$W_1 + W_2$ = Total power by using two watt-meter method.

$V_{BR} = V_{BN} \cdot V_{RN} \Rightarrow V_{RB}$.

$V_{YB} = V_{YN} - V_{BN}$



phasor diagram:-



$W_1 = V_{RB} \cdot I_R \cos(\angle V_{RB} \& I_R)$

$W_1 = V_L \cdot I_L \cos(30-\phi)$

$W_2 = V_{YB} \cdot I_Y \cos(V_{YB} \& I_Y)$

$W_2 = V_L \cdot I_L \cos(30+\phi)$.

$\therefore W_T = W_1 + W_2$.

$= V_L I_L \cos(30-\phi) + V_L I_L \cos(30+\phi)$

$= V_L I_L [\cos(30-\phi) + \cos(30+\phi)]$

$= V_L I_L \, 2\cos\phi \cdot \cos 30$

$= V_L I_L \, \cancel{2} \cos\phi \cdot \frac{\sqrt{3}}{\cancel{2}}$

$$\boxed{W_T = \sqrt{3} \, V_L I_L \cos\phi}$$

$\therefore$ By using two watt-meter method, the sum of two watt meter readings is equal to total power

$V_{ph} = 200\ V$

$Z_1 = (7 + j5)\ \Omega = 8.6 \angle 35.5$

$Z_2 = (4 - j9)\ \Omega = 9.84 \angle -66.03$

$Z_3 = (8 - j6)\ \Omega = 10 \angle -36.86$

$I_{ph_1} = \frac{V_{ph}}{Z_1} = \frac{200}{8.6 \angle 35.5} = 23.25 \angle 35.5$

$I_{ph_2} = \frac{V_{ph}}{Z_2} = \frac{200 \angle -120°}{9.84 \angle -66.03} = 20.32 \angle -53.97°$

$I_{ph_3} = \frac{V_{ph}}{Z_3} = \frac{200 \angle -240°}{10 \angle -36.86} = 20 \angle -203.14°$

Complex power:

$S_1 = V_{ph_1} \times I_{ph_1} = 200 \times 23.25$
$= 4650\ VA$

$S_2 = V_{ph_2} \times I_{ph_2} = 200 \times 20.32$
$= 4064\ VA$

$S_3 = V_{ph} \times I_{ph_3} = 200 \times 20$
$= 4000\ VA$

$\therefore$ Total complex power,

$S_T = S_1 + S_2 + S_3$

$= 4650 + 4064 + 4000$

$\boxed{S_T = 12714\ VA}$

a)
Sol: Given:-

$V_L = 400$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\ V$

$Z_A = 10 \angle 0°$

$Z_B = 15 \angle 30°$

$Z_C = 10 \angle -30°$

$I_{ph} = \frac{230.94}{10} = 23.094\ A$

$I_{ph_2} = \frac{230.94 \angle -120°}{15 \angle 30°} = 15.396 \angle -150°$

$I_{ph_3} = \frac{230.94 \angle 120}{10 \angle -30°} = 23.94 \angle 150°$

$P_1 = V_{ph}\ I_{ph} \cos\phi$

$= 230.94 \times 23.094 \times \cos(0)$

$P_1 = 5333.32\ W$

$P_2 = V_{ph}\ I_{ph} \cos\phi$

$= 230.94 \times 15.396 \times \cos(30)$

$P_2 = 3079.198\ W$

$P_3 = V_{ph}\ I_{ph} \cos\phi$

$= 230.94 \times 23.094 \times \cos(-30)$

$= 4618.79\ W$

$\therefore$ Total power, P_T

$= P_1 + P_2 + P_3$

$\boxed{P_T = 13031.3\ W}$

4) b)
Sol:- Given data:-

$P_1 = 1560\ W$ | $V_L = 220\ V$

$P_2 = 2100\ W$

$P_T = P_1 + P_2 = 1560 + 2100$

$= 3660\ W$

(a) the per-phase average power is then

$P_p = \frac{1}{3} P_T = \frac{3660}{3} = 1220\ W$

(b) The total reactive power is,

$Q_T = \sqrt{3}(P_2 - P_1) = \sqrt{3}(2100 - 1560) = 935.3\ VAR$

Reactive power/Ph $= \frac{935.3}{3}$

$= 311.77\ VAR$

(c) The power angle is

$\theta = \tan^{-1}\left(\frac{Q_t}{P_1}\right)$

$= \tan^{-1}\left(\frac{935.3}{3660}\right)$

$\theta = 14.33^\circ$

Hence the power factor is,

$\cos\theta = 0.9689$ (lagging)

It is lagging pf because Q_t is +ve

$P_2 > P_1$

(d) The phase impedance is $Z_p = Z_p\angle\theta$

$Z_p = \frac{V_p}{I_p}$

We recall that for a delta-connected load,

$V_p = V_L = 220V$

$P_p = V_p I_p \cos\theta$

$\Rightarrow I_p = \frac{1220}{220 \times 0.9689} = 5.723 A$

$Z_p = \frac{220}{5.723} = 38.44\,\Omega$

$Z_p = 38.44\angle 14.33^\circ\,\Omega$

2(b). Effect of load power factor on watt meter readings in two-watt meter method :-

Case-i) :- If the P.f is unity then watt meter readings are equal ie, $W_1 = W_2$

we know that,

$\cos\phi = \cos\left[\tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right]$

$1 = \cos\left(\tan^{-1}\left(\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right)\right)$

$\tan(\cos^{-1}(1)) = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$

$\frac{0}{\sqrt{3}} = W_1 - W_2$

$\boxed{W_1 = W_2}$

Case-ii) :- If the P.f is 0 then both watt meter readings are equal and opposite sign.

$\tan(\cos^{-1}0) = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$

$\frac{1}{0} = \cancel{\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}}$

$W_1 + W_2 = 0$

$\boxed{W_1 = -W_2}$

case-iii) :- If P.f is 0.5 then

$\tan\left(\cos^{-1}\left(\frac{1}{2}\right)\right) = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$

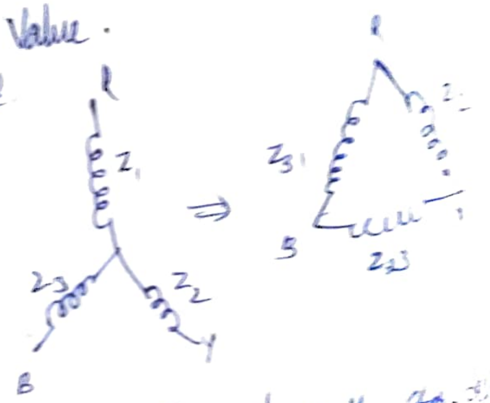
$\frac{\sqrt{3}}{\sqrt{3}} = \frac{W_1 - W_2}{W_1 + W_2}$

$\cancel{W_1} + W_2 = \cancel{W_1} - W_2$

$\boxed{W_2 = 0}$

∴ If P.f is 0.5 then one of the wattmeter reading shows zero value.

3(a)



The above figure shows the star-delta transformation

$$Z_{12} = \frac{Z_1Z_2+Z_2Z_3+Z_3Z_1}{Z_3}$$

$$Z_{23} = \frac{Z_1Z_2+Z_2Z_3+Z_3Z_1}{Z_1}$$

$$Z_{31} = \frac{Z_1Z_2+Z_2Z_3+Z_3Z_1}{Z_2}.$$

$I_{ph(1)} = \frac{V_{RY}}{Z_{12}}$; $I_{ph_2} = \frac{V_{YB}}{Z_{23}}$; $I_{ph_3} = \frac{V_{BR}}{Z_{31}}$

Line currents:-

$I_R = I_{RY} - I_{BR}$

$I_Y = I_{YB} - I_{RY}$

$I_B = I_{BR} - I_{YB}$

6th Proble b) Sol:- Given data:-

$V_{ab} = 220\angle 0^\circ$. $\Rightarrow Z_{AB} = -j5\Omega = 5\angle -90^\circ$

$V_{bc} = 220\angle 120^\circ$ $\quad Z_{BC} = j10\Omega = 10\angle 90^\circ$

$V_{ca} = 220\angle -120^\circ$. $\quad Z_{CA} = 10\Omega = 10\angle 0^\circ$

Phase currents:-

$$I_{AB} = \frac{V_{ab}}{Z_{AB}} = \frac{220}{5\angle -90^\circ} = 44\angle 90^\circ$$

$$I_{BC} = \frac{V_{bc}}{Z_{BC}} = \frac{220\angle 120^\circ}{10\angle 90^\circ} = 22\angle 30^\circ$$

$$I_{CA} = \frac{V_{ca}}{Z_{CA}} = \frac{220\angle -120^\circ}{10\angle 0^\circ} = 22\angle -120^\circ.$$

Line currents:-

$I_A = I_{AB} - I_{CA} = 44\angle 90^\circ - 22\angle -120^\circ$

$= 64\angle 80.10^\circ$

$I_B = I_{BC} - I_{AB} = 22\angle 30^\circ - 44\angle 90^\circ$

$= 38.105\angle -60^\circ$

$I_C = I_{CA} - I_{BC} = 22\angle -120^\circ - 22\angle 30^\circ$

$= 42.5\angle -135^\circ$

Power absorbed by AB phase, $P_{AB} = V_{ph} \times I_{ph} \cos\phi$

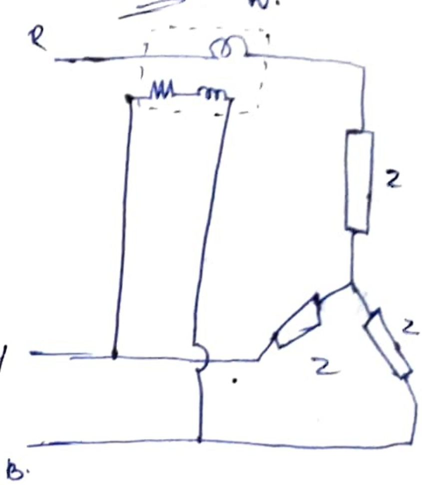
$P_{AB} = 220 \times 44 \times \cos(-90) = 0$

$P_{BC} = 220 \times 22 \times \cos(90) = 0$

$P_{CA} = 220 \times 22 \times \cos(0) = 4840\,W$.

Total real power absorbed $(P_T) = 4840\,W$.

5)a Ans:- Measurement of Reactive power by using single watt meter method:- W.



The power measured by the wattmeter is,

$W = V_{YB} \cdot I_R \cos(V_{YB} \& I_R)$

The angle b/w V_{YB} & I_R is $(90-\phi)$.

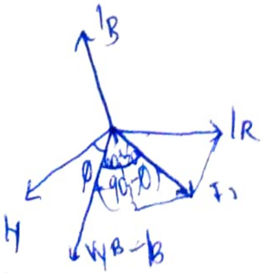
$W = V_{YB}\, I_R \cos(90-\phi)$

$W = V_{YB} \cdot I_R \sin\phi$.

$\boxed{W = V_L \cdot I_L \sin\phi.\ W}$

$\therefore$ The reactive power is,

$\boxed{Q = \sqrt{3}\, W}$ VAR



sol: Given data:-

because 4-wire system it is a λ-connection

$Z_1 = (5+j7)\,\Omega$

$Z_2 = (5+j7)\,\Omega$

$Z_3 = (8+j10)\,\Omega$

$V_L = 440V$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}}$

$= 254.03V.$

Phase currents:-

I_{phR} (I_{RY}) $= \frac{254.03}{5+j7} = 29.53\angle -54.46$

$I_{ph(Y)}$ (I_{YB}) $= \frac{254.03\angle -120^\circ}{5+j7} = 29.53\angle -174.46$

$I_{ph(B)}$ (I_{BR}) $= \frac{254.03\angle 120^\circ}{8+j10}$

$= 19.83\angle 68.65^\circ$

Line currents:-

$I_R = I_{RY} - I_{BR}$

$= 29.53\angle -54.46 - 19.83\angle 68.65$

$= 43.64\angle -76.82^\circ$

$I_Y = I_{YB} - I_{RY}$

$= 29.53\angle -174.46 - 29.53\angle -54.46$

$= 51.147\angle 155.54^\circ$

$I_B = I_{BR} - I_{YB}$

$= 19.83\angle 68.65 - 29.53\angle -174.46$

$= 42.36\angle 30.21^\circ$

$\therefore I_N = I_R + I_Y + I_B$

$= 1.84\times 10^{-3}\angle 65.10^\circ$

6) b)

sol:

$Z_1 = 20\angle -30^\circ$

$Z_2 = 40\angle 80^\circ$

$Z_3 = 10\angle 90^\circ$

$V_L = V_{ph} = 440V.$

$V_{RY} = 440\angle 0^\circ$

$V_{YB} = 440\angle -120^\circ$

$V_{BR} = 440\angle 120^\circ.$

Phase currents:-

$I_{RY} = \frac{V_{RY}}{Z_1} = \frac{440\angle 0^\circ}{20\angle -30^\circ} = 22\angle 30^\circ$

$I_{YB} = \frac{V_{YB}}{Z_2} = \frac{440\angle -120^\circ}{40\angle 80^\circ} = 11\angle -200^\circ$

$I_{BR} = \frac{V_{BR}}{Z_3} = \frac{440\angle 120^\circ}{10\angle 90^\circ} = 44\angle 30^\circ$

Line currents:-

$I_R = I_{RY} - I_{BR} = (22\angle 30^\circ) - (44\angle 30^\circ)$

$= 22\angle -150^\circ$

$I_Y = I_{YB} - I_{RY} = (11\angle -200^\circ) - 22\angle 30^\circ$

$= 30.26\angle -166.16$

$I_B = I_{BR} - I_{YB} = 44\angle 30^\circ - 11\angle -200$

$= 51.76\angle 20.63^\circ$

Power at each phase:-

$P_{RY} = V_{RY}\cdot I_{RY}\cdot\cos\phi$

$= 440\times 22\times\cos(-30^\circ)$

$= 8383.125\ W$

$P_{YB} = 440\times 11\times\cos(80^\circ) = 840.45$

$P_{BR} = 440\times 44\times\cos(90^\circ) = 0$

$\therefore$ Total power $= P_{RY} + P_{YB} + P_{BR}$

$= 8383.125 + 840.45 + 0$

$\boxed{P = 9223.575\ W}$

$Z_R = 8\angle 30^\circ$

$Z_Y = 10\angle 20^\circ$

$Z_B = 20\angle 0^\circ$

$V_L = 400V$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94$

$V_{ph} = 230.94\angle 30^\circ$
V_{RY}

$V_{YB} = 230.94\angle -90^\circ$

$V_{BR} = 230.94\angle 150^\circ$.

using star delta transformation:-

$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B}$

$= \frac{8\angle 30^\circ \cdot 10\angle 20^\circ + 10\angle 20^\circ \cdot 20\angle 0^\circ + 20 \cdot 8\angle 30^\circ}{20\angle 0^\circ}$

$= \frac{432.19\angle 29.02}{20}$

$= 21.6\angle 29.023^\circ$

$Z_{YB} = \frac{432.19\angle 29.02}{8\angle 30^\circ}$

$= 54.02\angle -0.98^\circ$

$Z_{BR} = \frac{432.19\angle 29.02^\circ}{10\angle 20^\circ}$

$= 43.219\angle 9.02^\circ$

phase currents:-

$I_{RY} = \frac{V_{RY}}{Z_{RY}} = \frac{230.94\angle 30^\circ}{21.6\angle 29.023}$

$= 10.69\angle 0.977^\circ$

$I_{YB} = \frac{V_{YB}}{Z_{YB}} = \frac{230.94\angle -90^\circ}{54.02\angle -0.98^\circ}$

$= 4.27\angle -89.02^\circ$

$I_{BR} = \frac{V_{BR}}{Z_{BR}} = \frac{230.94\angle 150^\circ}{43.219\angle 9.02^\circ}$

$= \cancel{5.343}\ 5.343\angle 140.98$

Line currents:-

$I_R = I_{RY} - I_{BR}$

$= 10.69\angle 0.977^\circ - 5.343\angle 140.98$

$= 15.17\angle -12.10^\circ$

$I_Y = I_{YB} - I_{RY}$

$= 4.27\angle -89.02 - 10.69\angle 0.977$

$= 11.51\angle -157.24^\circ$

$I_B = I_{BR} - I_{YB}$

$= 5.343\angle 140.98 - 4.27\angle -89.02$

$= 8.72\angle 118.95^\circ$

10)a.
Sol:- Applying kvl for loop-1:-

$100\angle 0^\circ + (1+j1)I_1 + (1+j2)(I_1 - I_2) - 100\angle 120^\circ = 0$

$(1+j1)I_1 + (1+j2)I_1 - (1+j2)I_2 + 173.2\angle 30^\circ = 0$

$I_1(1+j1+1+j2) - (1+j2)I_2 + 173.2\angle 30^\circ = 0$

$(2+j3)I_1 - (1+j2)I_2 = -173.2\angle 30^\circ$ — ①

Applying kvl for loop-2:-

$100\angle -120^\circ + (1+j2)(I_2 - I_1) + (3+j4)I_2 - 100\angle 240^\circ = 0$

$-(1+j2)I_1 + I_2(1+j2+3+j4) + 173.2\angle -90^\circ = 0$

$-(1+j2)I_1 + (4+j6)I_2 = -173.2\angle -90^\circ$ — ②

$$\begin{bmatrix} 2+j3 & -(1+j2) \\ -(1+j2) & (4+j6) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -173.2\angle 30^\circ \\ -173.2\angle -90^\circ \end{bmatrix}$$

$\Delta = (2+j3)(4+j6) - (1+j2)(1+j2)$

$\Delta = 26\angle 112.61 - 5\angle 126.86$

$\Delta = 21.189\angle 109.28^\circ$

$$\Delta_1 = \begin{vmatrix} -173.2\angle 30^\circ & -(1+j2) \\ -173.2\angle 90^\circ & (4+j6) \end{vmatrix}$$

$$\Delta_1 = -173.2\angle 30^\circ(4+j6) - (173.2\angle 90^\circ)(1+j2)$$

$$= -(80.38+j1246.37) - (346.4-j173.2)$$

$$= -80.38 - 346.4 + j1246.37 + j173.2$$

$$= -426.78 + j1419.57$$

$$= 1482.33\angle 106.73$$

$$\Delta_2 = \begin{vmatrix} 2+j3 & -173.2\angle 30^\circ \\ -(1+j2) & -173.2\angle -90^\circ \end{vmatrix}$$

$$= -(173.2\angle -90)(2+j3) - (173.2\angle 30)(1+j2)$$

$$= -(519.6-j346.4) - (-23.20+j386.59)$$

$$= -519.6+23.20-j346.4-j386.59$$

$$= -496.4 - j732.99$$

$$\Delta_2 = 885.26\angle -124.10$$

$$\therefore I_1 = \frac{\Delta_1}{\Delta} = \frac{1482.33\angle 106.73}{21.189\angle 109.28}$$

$$= 69.95\angle -2.55^\circ$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{885.26\angle -124.10}{21.189\angle 109.28}$$

$$= 41.77\angle 126.62^\circ$$

$$\therefore I_1 = I_A = 69.95A\angle -2.55^\circ$$

$$I_2 = I_C = 41.77A\angle 126.62^\circ$$

$$I_1 - I_2 = I_B = 69.95\angle -2.55^\circ - 41.77\angle 126.62$$

$$= 28.18^\circ A$$

8) b

Sol^n:

$V_L = 434$ V

3-φ $\quad I_m \rightarrow Z_{ph} = (1.25 + j2.17)$

$$V_{ph} = \frac{434}{\sqrt{3}} = 250.517 \text{ V}$$

$$I_{Im} = \frac{V_{ph}}{Z_{ph}} = \frac{250.517}{1.25+j2.17}$$

$$= 100\angle -60^\circ A$$

$I_{sm} = 120A$, at 0.87 leading.

$$\phi_{sm} = \cos^{-1}(0.87)$$

$$\phi_{sm} = 29.54^\circ$$

$$I_{sm} = 120\angle 29.54^\circ$$

$$I = I_{Im} + I_{sm}$$

$$= 100\angle -60^\circ + 120\angle 29.54$$

$$I = 156.82\angle -10.07 \text{ A}$$

Total power factor, $\cos\phi = \cos(\ldots)$

$$\cos\phi = 0.985 \text{ lagging.}$$

Input power, $P_{in} = \sqrt{3}V_L I_L \cos\phi$

$$P_{in} = \sqrt{3}\times 434 \times 156.82 \times 0.985$$

$$W_1 + W_2 = 116.11 \text{ KW} \quad -(1)$$

$$W_2 = 116.11\times 10^3 - W_1$$

But,

$$\phi = \tan^{-1}\left\{\frac{\sqrt{3}(W_1-W_2)}{W_1+W_2}\right\}$$

$$10.07 = \tan^{-1}\left(\frac{\sqrt{3}(W_1-W_2)}{W_1+W_2}\right)$$

$$0.178 = \frac{\sqrt{3}(W_1-W_2)}{W_1+W_2}$$

$$W_1+W_2 = \frac{\sqrt{3}}{0.178}(W_1-W_2)$$

$$W_1+W_2 = 9.73(W_1-W_2).$$

$\frac{116.11\times10^3}{9.73} = w_1 - w_2$

$11933.19 = w_1 - w_2$ — ②

By solving eq ① & ②

$w_1 + w_2 = 116.11\times10^3$

$w_1 - w_2 = 11933.19$

$2w_1 = 128043.19$

$w_1 = 64021.59$

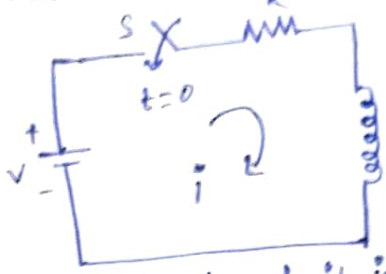
$\boxed{w_1 = 64.02\,KW}$

$w_2 = 116.11\times10^3 - 64.02\times10^3$

$\boxed{w_2 = 52.09\,KW}$

UNIT - 3

RL Series circuit with DC excitation:-



The inductor in the circuit is initially uncharged and it is in series with resistor R.

Apply KVL in the circuit,

$$-V + iR + L\frac{di}{dt} = 0$$

$$iR + L\frac{di}{dt} = V.$$

$$\frac{di}{dt} + \frac{iR}{L} = \frac{V}{L} \quad -①$$

The above eq is in the form of 1st order Differential equation and it is in the form of following standard first order differential equation.

$$\frac{dx}{dt} + Px = Q \quad -②$$

the sol. of 1st order we have two parts $<$ C.F, P.I

$C.F = I(C.F) = Ae^{-Pt}.$ by comparing with eq ①

$$P = \frac{R}{L}.$$

$\therefore$ C.F of eq ①, $i(C.F) = Ae^{-R/L\,t}.$

$$\tau = \frac{L}{R}.$$

$$i(C.F) = Ae^{-t/\tau}$$

$\therefore$ P.I, $x(P.I) = e^{-Pt}\int Q\,e^{Pt}\,dt.$

By comparing eq ① & ②, $Q = \frac{V}{L}.$

P.I of eq ①, $i(P.I) = e^{-R/L\,t}\int \frac{V}{L}e^{R/L\,t}\,dt$

$$= e^{-\frac{R}{L}t}\,\frac{V}{L}\left[\frac{e^{R/L\,t}}{\frac{R}{L}}\right]$$

$$= \left[e^{-R/L\,t}\cdot e^{R/L\,t}\right]\left[\frac{V}{L}\cdot\frac{L}{R}\right]$$

$$i(P.I) = V/R$$

The solution for eq (?)

$i(t) = Ae^{-R/L\, t} + \frac{V}{R}$

$i(t) = Ae^{-t/\tau} + \frac{V}{R}$

By substituting initial conditions

$t = 0,\ i(0) = 0$

$i(0) = Ae^{-0/\tau} + \frac{V}{R}$

$0 = A + \frac{V}{R}$

$\boxed{A = -\frac{V}{R}}$

$i(t) = \frac{V}{R}\left[1 - e^{-t/\tau}\right]$ at $[t \geq 0]$

at $t = \tau$ then, $i(t) = 0.632\frac{V}{R}$

$t = 2\tau$ then, $i = 0.865\frac{V}{R}$.

$t = 3\tau$ then $i = 0.95\frac{V}{R}$

$t = 4\tau$ then $i = 0.982\frac{V}{R}$.

consider, $i(t) = \frac{V}{R}\left[1 - e^{-t/\tau}\right]$.

$V_L(t) = \frac{L\, di(t)}{dt}$

$= L\cdot\frac{d}{dt}\left[\frac{V}{R}\left[1 - e^{-t/\tau}\right]\right]$

$\tau = L/R$

$= L\frac{d}{dt}\left[\frac{V}{R} - \frac{V}{R}e^{-R/L\cdot t}\right]$

$= L\left[0 - \frac{V}{R}e^{-R/L(t)} \cdot \frac{-R}{L}\right]$

$= \cancel{L}\left[Ve^{-R/L}\right]$ $[L = 1H]$

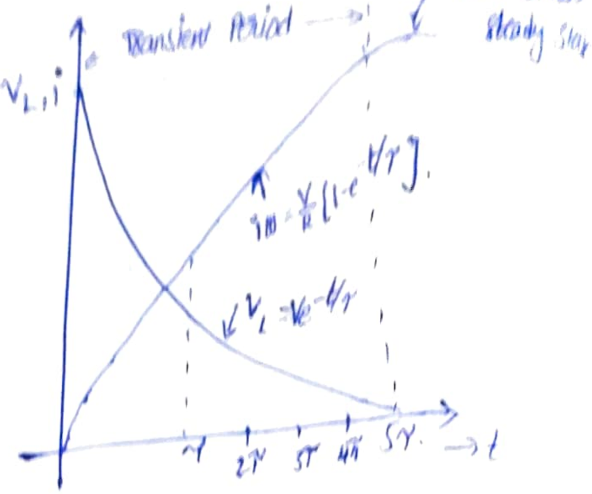
$= Ve^{-t/\tau}$.

$\boxed{V_L(t) = Ve^{-t/\tau}}$

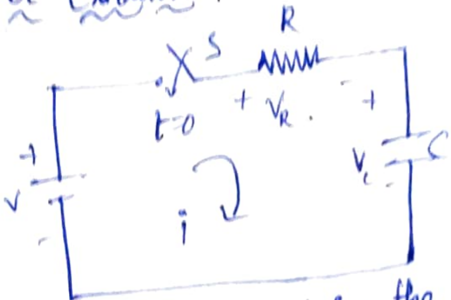
$V_R(t) = iR$

$= \frac{V}{R}\left[1 - e^{-t/\tau}\right] \times R$

$\boxed{V_R(t) = V\left[1 - e^{-t/\tau}\right]}$



Transient Response of RC series circuit with DC excitation :-



At $t=0$ we are closing the switch.

Apply KVL in above circuit.

$-V + V_R + V_c = 0$

$\cancel{V + iR + \frac{dV_c}{dt}}$

$V_R + V_c = V$.

$V_R = iR$.

$i = C\frac{dV_c}{dt}$;

$V_R = C\frac{dV_c}{dt} \times R$

$V = RC\frac{dV_c}{dt} + V_c = V$.

$$\frac{dv_c}{dt} + \frac{1}{RC} V_c = \frac{V}{RC} \quad (1)$$

this is in the form of

$$\frac{dx}{dt} + Px = Q \quad (2)$$

By comparing eq (1) with (2) we get,

$$P = \frac{1}{RC}, \quad Q = \frac{V}{RC}$$

the solution of 1st order differential eq is given below.

$$V_c(t) = CF + PI.$$

C.F $V_c(C.F) = Ae^{-Pt}$.

$$= Ae^{-\frac{1}{RC}t}$$

$$\boxed{V_c(C.F) = Ae^{-t/RC}.}$$

$\tau = RC$.

$$V_c(C.F) = Ae^{-t/\tau}.$$

P.I $V_c(P.I) = e^{-Pt}\int Q e^{Pt}\, dt$

$$= e^{-1/RC\, t}\int \frac{V}{RC} e^{\frac{t}{RC}} . dt$$

$$= e^{-1/RC\, t} . \frac{V}{RC}\left[\frac{e^{t/RC}}{\frac{1}{RC}}\right]$$

$$= e^{0} . \frac{V}{RC}\left[\frac{RC}{1}\right]$$

$$\boxed{V_c(P.I) = V}$$

$$\therefore V_c(t) = V_c(C.F) + V_c(P.I)$$

$$= Ae^{-t/\tau} + V.$$

we get the value of A by substituting initial condition,

at $t=0$.

$$0 = Ae^{0} + V$$

$$\boxed{A = -V}$$

$$V_c(t) = -Ve^{-t/\tau} + V$$

$$\boxed{V_c(t) = V[1 - e^{-t/\tau}]}$$

At $t=\tau$ then $V_c = 0.632\, V$

$t=2\tau$ then $V_c = 0.865\, V$

$t=3\tau$ then $V_c = 0.950\, V$

$t=4\tau$ then $V_c = 0.981\, V$

$t=5\tau \Rightarrow V_c = 0.993\, V$

$$i_c(t) = C\frac{dv_c}{dt}$$

$$= C\frac{d}{dt}[V\{1 - e^{-t/\tau}]$$

$$= C\frac{d}{dt}[V - Ve^{-t/\tau}]$$

$$= C[0 - V[\tfrac{-1}{RC}]e^{-t/RC}]$$

$$= C[\tfrac{V}{RC} e^{-t/\tau}]$$

$$\boxed{i_c(t) = \frac{V}{R} e^{-t/\tau}}$$

At $t=\tau$ then $i_c = 0.367\,\frac{V}{R}$

$t=2\tau \Rightarrow i_c = 0.135\,\frac{V}{R}$

$t=3\tau$ $i_c = 0.049\,\frac{V}{R}$

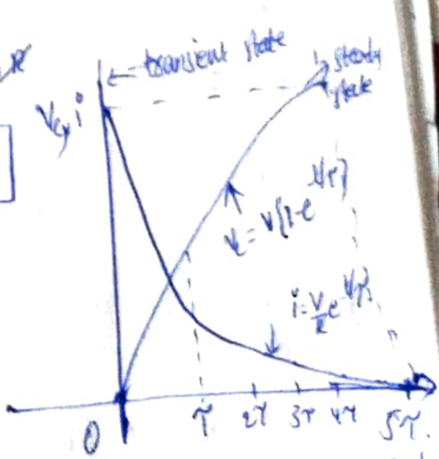
$t=4\tau$ $i_c = 0.018\,\frac{V}{R}$

$t=5\tau$ $i_c = 6.738\times10^{-3}\,\frac{V}{R}$ $\cong 0$.

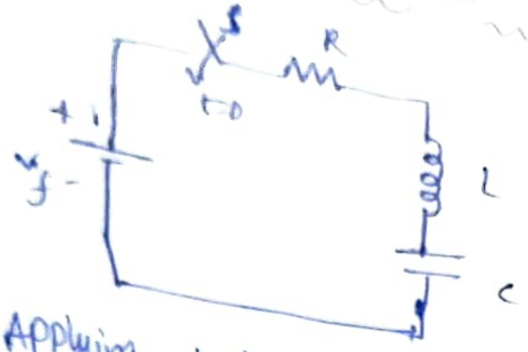
$$V_R(t) = i(R)$$

$$= \frac{V}{R}e^{-t/\tau} . R$$

$$\boxed{V_R(t) = Ve^{-t/\tau}}$$



RLC Series circuit with DC excitation:-



Applying KVL for the above circuit,

$$-V_f + V_R + V_L + V_c = 0$$

$$V_R + V_L + V_c = V_f$$

$$i(t) R + L\frac{di(t)}{dt} + V_c = V_f \quad \text{—(1)}$$

The current flowing through the capacitor is given by

$$i_c(t) = \frac{C\,dV_c(t)}{dt} = i(t)$$

Substitute in eq (1),

$$RC\frac{dv(t)}{dt} + L\frac{d}{dt}\left[\frac{C\,dV_c(t)}{dt}\right] + V(t) = V_f.$$

$$\frac{LC\,d^2V_c(t)}{dt^2} + RC\frac{dV_c t}{dt} + V_c(t) = V_f.$$

Divide LC with above equation on both sides.

$$\frac{d^2V_c(t)}{dt^2} + \frac{R}{L}\frac{dV_c(t)}{dt} + \frac{V_c}{LC}(t) = \frac{V_f}{LC}$$

Let $\frac{d}{dt} = D$, $\frac{d^2}{dt^2} = D^2$

$$D^2V_c(t) + \frac{R}{L}DV_c(t) + \frac{V_c}{LC}(t) = \frac{V_f}{LC}$$

$$\left(D^2 + D\frac{R}{L} + \frac{1}{LC}\right)V_c(t) = \frac{V_f}{LC} \quad (2)$$

The characteristic of eq (2) is given by

$$D^2 + D\frac{R}{L} + \frac{1}{LC} = 0$$

Case-i): The roots are real and different

$$V_c(t) = V_f + Ae^{D_1t} + Be^{D_2t}.$$

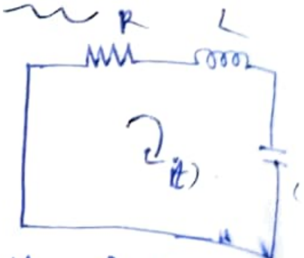
Case-ii): The roots are real and equal

$$V_c(t) = V_f + (A+Bt)e^{D_1t}$$

Case-iii): The roots are complex

$$V_c(t) = V_f + (A\cos\omega_d t + B\sin\omega_d t)e^{-\alpha t}$$

Source free circuit:-



Apply KVL for the circuit

$$i(t)R + L\frac{di}{dt}(t) + \frac{1}{C}\int i\,dt = 0$$

Differentiate with r.to 't'

$$\frac{di(t)}{dt}R + L\frac{di^2(t)}{dt^2} + \frac{i}{C} = 0$$

$$L\frac{di^2(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{i}{C} = 0$$

$$\frac{di^2(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{t} + \frac{i}{LC} = 0$$

let, $\frac{d}{dt} = D$, $\frac{d^2}{dt^2} = D^2$

$$D^2 i(t) + \frac{R}{L} D i(t) + \frac{i(t)}{LC} = 0$$

$$\left(D^2 + \frac{R}{L} + \frac{1}{LC}\right) i(t) = 0 \quad -(2)$$

the characteristic equation of eq (2) is given by $D^2 + \frac{R}{L} D + \frac{1}{LC} = 0$.

case i): the roots are real and unequal

$$i(t) = Ae^{D_1 t} + Be^{D_2 t}$$

case ii): the roots are real & equal

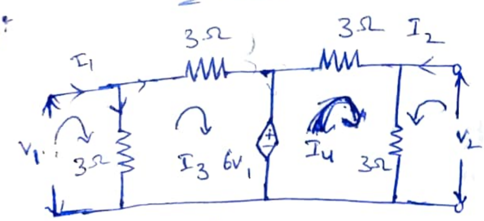
$$i(t) = (At+B)e^{D_1 t}$$

case iii): the roots are complex.

$$i(t) = (A\cos \omega dt + B\sin \omega dt)e^{-\alpha t}.$$

UNIT-4.

Q):



Applying KVL for loop-1.

$$3(I_1 - I_3) = V_1$$

$$3I_1 - 3I_3 = V_1 \quad -(1)$$

Apply KVL for loop-2:-

$$3I_3 + 6V_1 + 3(I_3 - I_1) = 0$$

$$-3I_1 + 6I_3 + 6V_1 = 0$$

$$I_3 = \frac{-6V_1 + 3I_1}{6}$$

$$I_3 = -V_1 + \frac{1}{2}I_1 \quad -(2)$$

Substitute eq (2) in eq(1),

$$3I_1 - 3\left(-V_1 + \frac{1}{2}I_1\right) = V_1$$

$$3I_1 + 3V_1 - \frac{3}{2}I_1 = V_1$$

$$3I_1 - \frac{3I_1}{2} = V_1 - 3V_1$$

$$\frac{3}{2}I_1 = -2V_1$$

$$\frac{3}{2}I_1 + 2V_1 = 0 \quad -(1)$$

Applying KVL for loop-3:-

$$3I_4 + 3(I_4 - I_2) - 6V_1 = 0$$

$$6I_4 - 3I_2 - 6V_1 = 0.$$

ABCD equal
$V_1 = AV_2 - BI_2$
$I_1 = CV_2 - DI_2$

Applying KVL for loop-4:

$$3(I_2 - I_4) = V_2$$

$$3I_2 - 3I_4 = V_2$$

$$I_4 = \frac{V_2 - 3I_2}{3}$$

$$I_4 = \frac{1}{3}V_2 - I_2$$

Substitute I_4 value in above equation

$$6\left(\frac{1}{3}V_2 - I_2\right) - 3I_2 - 6V_1 = 0$$

$$2V_2 - 6I_2 - 3I_2 - 6V_1 = 0.$$

$$-9I_2 - 6V_1 = -2V_2 \Rightarrow V_1 = \frac{-2V_2 + 9I_2}{-6}$$

$$\cancel{-}(9I_2 + 6V_1) = \cancel{-}2V_2 \qquad V_1 = \frac{1}{3}V_2 - \frac{3}{2}I_2$$

$$\frac{9}{2}I_2 + 3V_1 = V_2$$

$$V_1 = \frac{V_2 - \frac{9}{2}I_2}{3}$$

$$V_1 = \frac{1}{3}V_2 - \frac{3}{2}I_2 \quad -(2)$$

Substitute eq (2) in eq (1)

$$\frac{3}{2}I_1 + 2\left(\frac{1}{3}V_2 - \frac{3}{2}I_2\right) = 0$$

$$\frac{3}{2}I_1 + \frac{2}{3}V_2 - 3I_2 = 0$$

$I_1 = \frac{1}{3}V_1 + \frac{1}{3}I_1$

$I_1 = -\frac{1}{3}V_1 \times \frac{3}{3} + \frac{8}{3}I_1 \times \frac{3}{3}$

$I_1 = -\frac{4}{4}V_2 + 2I_1$

$A = \frac{1}{3}, \; B = \frac{3}{8}$

$C = -\frac{4}{4}, \; D = -2$

3(ii)

Sol: Given, $Z_{11} = Z_{22} = 6\,\Omega$

$Z_{12} = Z_{21} = 4\,\Omega$

The z-parameter equations are,

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

$V_1 = 6I_1 + 4I_2$ — ①

$V_2 = 4I_1 + 6I_2$ — ⑤.

h-param

$V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

$I_2 = \frac{V_2 - 4I_1}{6}$

$I_2 = \frac{1}{6}V_2 - \frac{2}{3}I_1$

$I_2 = -\frac{2}{3}I_1 + \frac{1}{6}V_2$ — ⑥

Substitute eq ⑥ in eq ①

$V_1 = 6I_1 + 4\left(-\frac{2}{3}I_1 + \frac{1}{6}V_2\right)$

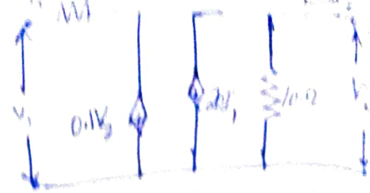
$V_1 = 6I_1 - \frac{8}{3}I_1 + \frac{2}{3}V_2$

$V_1 = \frac{10}{3}I_1 + \frac{2}{3}V_2$

h-parameters: $\begin{bmatrix} 3.33\,\Omega & 0.666 \\ -0.666 & 0.166 \end{bmatrix}$

3(b)

Sol:



The above figure shows the equivalent of h-parameters.

H-Parameter equation

Apply KVL for loop 1:

$V_1 = 4I_1 + 0.1V_2$ — ①.

$I_2 = 20I_1 + \frac{1}{10}V_2$

$I_2 = 20I_1 + 0.1V_2$ — ②

Y-Parameters

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

Consider eq ①,

$I_1 = \frac{V_1 + 0.1V_2}{4}$

$I_1 = \frac{1}{4}V_1 + \frac{0.1}{4}V_2$ — ③

Substitute eq ③ in eq ②.

$I_2 = 20\left(\frac{V_1}{4} + \frac{0.1}{4}V_2\right) + 0.1V_2$

$I_2 = 5V_1 + 0.5V_2 + 0.1V_2$

$I_2 = 5V_1 + 0.6V_2$ — ④

By comparing eq ③ & ④ with Y-Parameters we get,

$Y_{11} = \frac{1}{4}\mho; \; Y_{12} = \frac{0.1}{4}\mho$

$Y_{21} = 5\mho; \; Y_{22} = 0.6\mho$

4(a) H-Parameters in terms of T-parameters.

Sol:

T-Parameters

$V_1 = AV_2 - BI_2$ — ①

$I_1 = CV_2 - DI_2$ — ②

H-param

$V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

$I_2 = \frac{CV_2 - I_1}{D} \Rightarrow I_2 = \frac{C}{D}V_2 - \frac{1}{D}I_1$ — ③

$AV_2 \; B\left[\frac{A}{?}V_2 - \frac{1}{D}I_1\right]$

$AV_2 \; \frac{BC}{D}V_2 + \frac{B}{D}I_1$

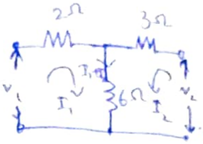
$\frac{AD-BC}{D}V_2 + \frac{B}{D}I_1$ —(4)

comparing eq (A) & (3) with h-parameters

$h_{11} = \frac{B}{D}$; $h_{12} = \frac{A}{D}$

$h_{21} = \frac{-1}{D}$ $h_{22} = \frac{C}{D}$

h parameters



$V_1 = 8I_1 + 6I_2$ —(3)

$V_2 = 6I_1 + 9I_2$

$I_2 = \frac{V_2 - 6I_1}{9}$

$I_2 = \frac{1}{9}V_2 - \frac{2}{3}I_1$ —(4)

$V_1 = h_{11}I_1 + h_{12}V_2$ —(1)

$I_2 = h_{21}I_1 + h_{22}V_2$ —(2)

substitute eq (4) in eq (3) we get,

$V_1 = 8I_1 + 6\left[\frac{1}{9}V_2 - \frac{2}{3}I_1\right]$

$V_1 = 8I_1 + \frac{2}{3}V_2 - 4I_1$

$V_1 = 4I_1 + \frac{2}{3}V_2$ —(5)

comparing eq (5) & (4) with (1) & (2)

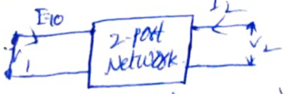
The h-parameters are,

$h_{11} = 4$; $h_{12} = \frac{2}{3}$

$h_{21} = -\frac{2}{3}$ $h_{22} = \frac{1}{9}$

$$\begin{bmatrix} 4 & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{9} \end{bmatrix}$$

Condition for symmetry:- ABCD



$V_1 = AV_2 - B(I_2)$ —(1)

$I_1 = CV_2 - D(I_2)$ —(2)

o/p is open circuited i.e, $I_2 = 0$.

$I_1 = CV_2$.

$\frac{I_1}{V_2} = C \Rightarrow \frac{V_2}{I_1} = \frac{1}{C}$. $V_2 = \frac{I_1}{C}$.

substitute $I_2 = 0$ in (1)

$CV_2 = DI_2$.

$\frac{V_2}{I_2} = \frac{D}{C}$ —(3)

$V_1 = AV_2$

$V_1 = A\left(\frac{I_1}{C}\right)$

$\frac{V_1}{I_1} = \frac{A}{C}$

$V_1 C = AI_1$

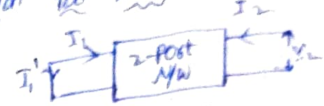
$\frac{V_1}{I_1} = \frac{A}{C}$

According to definition of symmetry,

$\frac{V_1}{I_1} = \frac{V_2}{I_2} \Rightarrow \frac{A}{C} = \frac{D}{C}$

$\boxed{A = D}$

Reciprocity condition for ABCD:-



$V_1 = 0$ $\frac{V_2}{I_1'} = ?$

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$-I_1' = CV_2 - DI_2$

$AV_2 = BI_2$

$\frac{V_2}{I_2} = \frac{B}{A}$

$I_2 = \frac{A}{B}V_2$

$-I_1' = CV_2 - D\left(\frac{A}{B}V_2\right)$

$-I_1' = \left(\frac{BC-AD}{B}\right)V_2$

$\frac{I_1'}{V_2} = \frac{AD-BC}{B}$ —(4)

If o/p port is short circuited $V_2 = 0, I_1 = I_1$

$I_2 = -I_2'$

$\frac{I_2'}{V_2}$

$I_1 = -DI_2$

$\frac{-I_1}{D} = I_2$.

$V_1 = A(0) - BI_2$

$V_1 = -BI_2$

$V_1 = -B(-I_2')$

$V_1 = BI_2'$

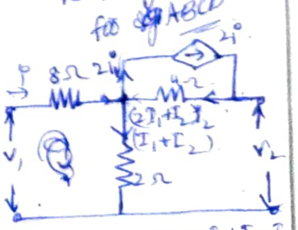
$\boxed{\frac{1}{B} = \frac{I_2'}{V_1}}$

$\frac{AD-BC}{B} = \frac{1}{B}$

$\boxed{AD - BC = 1}$

Reciprocity condition for ABCD

5b) Sol: Applying kvl for loop-1:-



$8I_1 + 2(I_1 + I_2) = V_1$

$10I_1 + 2I_2 = V_1$ —(1).

Apply kvl for loop-2:-

$2(I_1+I_2) + 4(2I_1+I_2) = 0$

$2I_1 + 2I_2 + 8I_1 + 4I_2 = 0$.

$10I_1+6I_2 \quad V_2$

Z-Parameters are $\begin{bmatrix} 10 & 2 \\ 10 & 6 \end{bmatrix}$

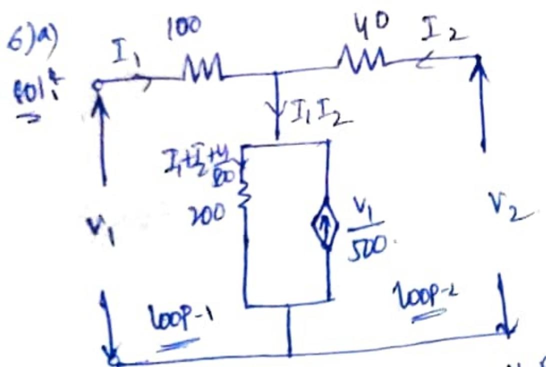
Y-Parameters are:

$$Y = \frac{1}{40}\begin{bmatrix} 6 & -2 \\ -10 & 10 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} \frac{6}{40} & \frac{2}{40} \\ \frac{-10}{40} & \frac{10}{40} \end{bmatrix}$$

$$Y = \begin{bmatrix} 0.15 & -0.05 \\ -0.25 & 0.25 \end{bmatrix}$$

6)a) sol:



Y-Parameters.

Applying kvl for loop-1:-

$$100I_1 + 200\left(I_1+I_2+\frac{V_1}{500}\right) = V_1$$

$$300I_1+200I_2+\frac{2}{5}V_1 = V_1$$

$$300I_1+200I_2 = V_1-\frac{2}{5}V_1$$

$$300I_1+200I_2 = \frac{3}{5}V_1$$

$$\boxed{\cancel{V_1 = 500I_1 + \frac{5}{3}(200)I_2} \quad \cancel{V_1 = 500I_1 + 333.33I_2}}$$

$$I_1 = \frac{\frac{3}{5}V_1 - 200I_2}{300}$$

$$I_1 = \frac{3}{5}\times\frac{1}{300}V_1 - \frac{200}{300}I_2$$

$$I_1 = \frac{1}{500}V_1 - \frac{2}{3}I_2 \quad \text{—(1)}$$

Applying kvl for loop-2:-

$$40I_2 + 200\left(I_1+I_2+\frac{V_1}{500}\right) = V_2$$

$$240I_2+200I_1+\frac{2}{5}V_1 = V_2$$

$$I_2 = \frac{V_2-\frac{2}{5}V_1-200I_1}{240}$$

$$I_2 = \frac{1}{240}V_2 - \frac{1}{600}V_1 - \frac{5}{6}I_1 \quad \text{—(2)}$$

substitute eq (2) in eq (1).

$$I_1 = \frac{1}{500}V_1 - \frac{2}{3}\left[\frac{1}{240}V_2 - \frac{1}{600}V_1 - \frac{5}{6}I_1\right]$$

$$I_1 = \frac{1}{500}V_1 - \frac{1}{360}V_2 + \frac{1}{900}V_1 + \frac{5}{9}I_1$$

$$I_1 - \frac{5}{9}I_1 = V_1\left(\frac{1}{500}+\frac{1}{900}\right) - \frac{1}{360}V_2$$

$$\frac{4}{9}I_1 = V_1\left(\frac{7}{2250}\right) - \frac{1}{360}V_2$$

$$I_1 = \frac{7}{2250}\times\frac{9}{4}V_1 - \frac{1}{360}\times\frac{9}{4}V_2$$

$$I_1 = \frac{7}{1000}V_1 - \frac{1}{160}V_2 \quad \text{—(3)}$$

substitute eq (3) in eq (2).

$$I_2 = \frac{1}{240}V_2 - \frac{1}{600}V_1 - \frac{5}{6}\left(\frac{7}{1000}V_1 - \frac{1}{160}V_2\right)$$

$$I_2 = \frac{1}{240}V_2 - \frac{1}{600}V_1 - \frac{7}{1200}V_1 + \frac{1}{192}V_2$$

$$I_2 = V_1\left(\frac{-1}{600}-\frac{7}{1200}\right) + V_2\left(\frac{1}{240}+\frac{1}{192}\right)$$

$$I_2 = \frac{-3}{400}V_1 + \frac{3}{320}V_2$$

Y-Parameters are,

$$Y = \begin{bmatrix} \frac{7}{1000} & \frac{-1}{160} \\ \frac{-3}{400} & \frac{3}{320} \end{bmatrix}$$

... of Y parameters

Y parameters

$I_1 = Y_{11}V_1 + Y_{12}V_2$ (1)

$I_2 = Y_{21}V_1 + Y_{22}V_2$ (2)

Consider

$V_1 = \frac{I_2 - Y_{22}V_2}{Y_{21}}$

$V_1 = \frac{1}{Y_{21}}I_2 - \frac{Y_{22}}{Y_{21}}V_2$ (3)

Substitute V_1 in eq (1)

$I_1 = Y_{11}\left[\frac{1}{Y_{21}}I_2 - \frac{Y_{22}}{Y_{21}}V_2\right] + Y_{12}V_2$

$I_1 = \frac{Y_{11}}{Y_{21}}I_2 - \frac{Y_{11}Y_{22}}{Y_{21}}V_2 + Y_{12}V_2$

$I_1 = \frac{Y_{11}}{Y_{21}}I_2 + V_2\left[\frac{-Y_{11}Y_{22} + Y_{12}Y_{21}}{Y_{21}}\right]$

$I_1 = \frac{Y_{11}}{Y_{21}}I_2 + V_2\left[\frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}\right]$.

ABCD Parameters are:-

$A = \frac{+Y_{22}}{Y_{21}}$; $B = \frac{-1}{Y_{21}}$

$C = \frac{-\Delta}{Y_{21}}$; $D = \frac{-Y_{11}}{Y_{21}}$ //.

Z parameters in terms of ABCD:-

ABCD:-

$V_1 = AV_2 - BI_2$ —(1)

$I_1 = CV_2 - DI_2$ —(2).

$I_2 = \cancel{\frac{CV_2 - I_1}{D}}$

Z:-

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

$V_2 = \frac{I_1 + DI_2}{C}$

$V_2 = \frac{1}{C}I_1 + \frac{D}{C}I_2$ —(3)

$V_1 = A\left(\frac{1}{C}I_1 + \frac{D}{C}I_2\right) - BI_2$

$V_1 = \frac{A}{C}I_1 + \frac{AD}{C}I_2 - BI_2$

$V_1 = \frac{A}{C}I_1 + I_2\frac{AD - BC}{C}$ —(4)

ABCD in terms of Y parameters

$V_1 = AV_2 - BI_2$; $I_1 = CV_2 - DI_2$

$I_2 = \frac{AV_2 - V_1}{B}$

$I_2 = \frac{A}{B}V_2 - \frac{1}{B}V_1$

Substitute I_2 in eq(2)

$I_1 = CV_2 - D\left(\frac{A}{B}V_2 - \frac{1}{B}V_1\right)$

$I_1 = CV_2 - \frac{AD}{B}V_2 + \frac{D}{B}V_1$

$I_1 = \frac{BC - AD}{B}V_2 + \frac{D}{B}V_1$

$Y_{11} =$ [illegible] ; $Y_{12} = \frac{\Delta}{B}$

$Y_{21} = \frac{-1}{B}$; $Y_{22} = \frac{A}{B}$

3)(a) $Z_{11} = 13\,\Omega$, $Z_{22} = 25\,\Omega$, $Z_{12} = Z_{21} = 10\,\Omega$

$V_1 = 13I_1 + 10I_2$

$V_2 = 10I_1 + 25I_2$

ABCD

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$I_1 = \frac{V_2 - 25I_2}{10}$

$I_1 = \frac{1}{10}V_2 - \frac{25}{10}I_2$ —(1)

Substitute I_1 value in above eq

$V_1 = 13\left(\frac{1}{10}V_2 - \frac{25}{10}I_2\right) + 10I_2$

$V_1 = \frac{13}{10}V_2 - \frac{325}{10}I_2 + 10I_2$

$V_1 = I_2\left(\frac{-325}{10} + 10\right) + \frac{13}{10}V_2$

$V_1 = \frac{-45}{2}I_2 + \frac{13}{10}V_2$

$A = \frac{13}{10}$, $B = \frac{45}{2}$, $C = \frac{1}{10}$, $D = \frac{25}{10}$

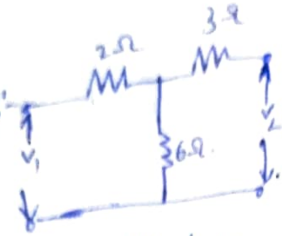
Sol:

9a: Z_{11} 9a = 8-b Repeated

9b)

Sol:

Applying kvl for loop 1:



$2I_1 + 6(I_1+I_2) = V_1$

$8I_1 + 6I_2 = V_1$ —(1)

$8I_1 + 6(\frac{1}{9}V_2 - \frac{2}{3}I_1) = V_1$

$8I_1 + \frac{2}{3}V_2 - 4I_1 = V_1$

$V_1 = \frac{2}{3}V_2 + 4I_1$

Applying kvl for loop 2:

$9I_2 + 6I_1 = V_2$

~~$I_2 = \frac{V_2 - 6I_1}{9}$~~

$I_1 = \frac{V_2 - 9I_2}{6}$

$I_1 = \frac{1}{6}V_2 - \frac{3}{2}I_2$ —(1)

~~$I_2 = \frac{1}{9}V_2 - \frac{2}{3}I_1$~~

ABCD:

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$8(\frac{1}{6}V_2 - \frac{3}{2}I_2) + 6I_2 = V_1$

$\frac{4}{3}V_2 - 12I_2 + 6I_2 = V_1$

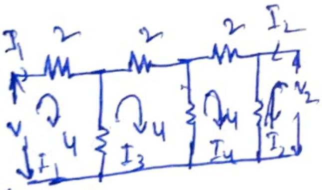
$V_1 = -6I_2 + \frac{4}{3}V_2$ —(2)

$\therefore A = \frac{4}{3}, B = 6$

$C = \frac{1}{6}, D = \frac{3}{2}$

10) a) Sol: 7(a)

10) b) Sol:



Applying kvl for loop-1:

$2I_1 + 4(I_1 - I_3) = V_1$

$6I_1 - 4I_3 = V_1$

Applying kvl for loop-2:

$2I_3 + 4(I_3 - I_1) + 4(I_3 - I_4) = 0$

$10I_3 - 4I_1 - 4I_4 = 0$ (2)

Applying kvl for loop-3:

$2I_4 + 4(I_4 + I_2) + 4(I_4 - I_3) = 0$

$10I_4 + 4I_2 - 4I_3 = 0$ —(4)

Applying kvl for loop 4:

$4(I_2 + I_4) = V_2$

$4I_2 + 4I_4 = V_2$

$I_4 = \frac{V_2 - 4I_2}{4}$

$I_4 = \frac{1}{4}V_2 - I_2$

Consider eq (1),

$-4I_3 = V_1 - 6I_1$

$I_3 = \frac{-1}{4}V_1 + \frac{3}{2}I_1$ —(6)

Substitute I_4, I_3 value in eq (4)

$10(\frac{-1}{4}V_1 + 3/2 I_1) - 4I_1 - 4(\frac{1}{4}V_2 - I_2)$

$-\frac{10}{4}V_1 + 15I_1 - 4I_1 - V_2 + 4I_2 = 0$

$11I_1 - \frac{5}{2}V_1 + 4I_2 = V_2$

$\Rightarrow 10(\frac{1}{4}V_2 - I_2) + 4I_2 - 4(\frac{-1}{4}V_1 + \frac{3}{2}I_1) = 0$

$\frac{5}{2}V_2 - 10I_2 + 4I_2 + V_1 - 6I_1 = 0$

$-6I_1 - 6I_2 + \frac{5}{2}V_2 = V_1$

11) Ans: Symmetry for Y-parameters:

$I_1 = Y_{11}V_1 + Y_{12}V_2 \Rightarrow I_1 = 0$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

$I_1 = 0$

$\frac{V_2}{I_2}$

$-Y_{11}V_1 = Y_{12}V_2$

$V_1 = \frac{-Y_{12}}{Y_{11}}V_2$

$I_2 = Y_{21}(\frac{-Y_{21}}{Y_{11}}V_2) + Y_{22}V_2$

$I_2 = \frac{-Y_{12} \cdot Y_{21}}{Y_{11}}V_2 + Y_{22}V_2$

$I_2 = \frac{-Y_{12}Y_{21}V_2 + Y_{11}Y_{22}V_2}{Y_{11}}$

$I_2 = \frac{\Delta Y}{Y_{11}} \Rightarrow \frac{V_2}{I_2} = \frac{Y_{11}}{\Delta Y}$

If port is open circuited $I_2 = 0$

$$Y_{21}V_1 = -Y_{22}V_2$$

$$V_2 = -\frac{Y_{21}}{Y_{22}} V_1$$

Substitute V_2 in eq ①

$$I_1 = Y_{11}V_1 + Y_{12}\left(\frac{-Y_{21}}{Y_{22}} V_1\right)$$

$$I_1 = Y_{11}V_1 + \frac{-Y_{12}Y_{21}}{Y_{22}} V_1$$

$$I_1 = \left(\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{22}}\right) V_1$$

$$\frac{I_1}{V_1} = \frac{\Delta}{Y_{22}}$$

$$\frac{V_1}{I_1} = \frac{Y_{22}}{\Delta}$$

By the Definition of symmetry,

$$\frac{V_1}{I_1} = \frac{V_2}{I_2}$$

$$\frac{Y_{11}}{\Delta} = \frac{Y_{22}}{\Delta}$$

$$\boxed{Y_{11} = Y_{22}}$$

Reciprocity condition for Y parameters:-

If I/P port is short circuited, then,

$V_1 = 0 \quad I_1 = -I_1' , \quad V_2 = V_2 \Rightarrow \frac{V_2}{I_1'}$

$$-I_1' = Y_{12} V_2$$

$$\frac{-I_1'}{V_2} = Y_{12} \Rightarrow \frac{V_2}{I_1'} = -\frac{1}{Y_{12}}$$

If o/p port is short circuited, then,

$V_2 = 0 \quad , I_2 = -I_2' \quad V_1 = V_1 \Rightarrow \frac{V_1}{I_2'}$

$$-I_2' = Y_{21} V_1$$

$$\frac{-I_2'}{V_1} = Y_{21}$$

$$\frac{V_1}{I_2'} = -\frac{1}{Y_{21}}$$

By the definition,

$$\frac{V_1}{I_2'} = \frac{V_2}{I_1'} \Rightarrow -Y_{12} = -Y_{21}$$

$$\boxed{Y_{21} = Y_{12}}$$

(1) Z Parameters in terms of h-parameters

Sol: h parameters

z parameters

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$V_2 = \frac{I_2 - h_{21}I_1}{h_{22}}$$

$$V_2 = \frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1 \quad \text{—①}$$

$$V_1 = h_{11}I_1 + h_{12}\left(\frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1\right)$$

$$V_1 = h_{11}I_1 + \frac{h_{12}}{h_{22}} I_2 - \frac{h_{12}h_{21}}{h_{22}} I_1$$

$$V_1 = \frac{h_{12}}{h_{22}} I_2 + I_1\left(\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right)$$

$$V_1 = \frac{\Delta}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

$\therefore \quad Z_{11} = \frac{\Delta}{h_{22}} \qquad Z_{21} = \frac{-h_{21}}{h_{22}}$

$\quad Z_{12} = \frac{h_{12}}{h_{22}} \qquad Z_{22} = \frac{1}{h_{22}}$

(5).

Applying kvl for loop1:-

$$4I_1 + 3(I_1 - I_3) = V_1$$

$$7I_1 - 3I_3 = V_1$$

$$I_3 = \frac{V_1 - 7I_1}{-3}$$

Applying kvl for loop 2:

$$3(I_3 - I_1) + V_2 + 5(I_3 + I_2) = 0$$

$$3I_3 - 3I_1 + V_2 + 5I_3 + 5I_2 = 0$$

$$8I_3 - 3I_1 + 5I_2 = -V_2$$

$$-8I_3 + 3I_1 - 5I_2 = V_2 \quad \text{—④}$$

Applying kvl for loop3:-

$$5(I_2 + I_3) = V_2$$

$$5I_2 + 5I_3 = V_2 \quad \text{—③}$$

$$5I_2 + 5I_3 = -8I_3 + 3I_1 - 5I_2$$

$$10I_2 + 13I_3 = 3I_1$$

$$I_3 = \frac{3I_1 - 10I_2}{13} \quad \text{—⑤}$$

$$\frac{V_1 - 7I_1}{3} \times \frac{3I_1 - 10I_2}{13}$$

$$13V_1 - 91I_1 = -9I_1 + 30I_2$$

$$13V_1 = -9I_1 + 91I_1 + 30I_2$$

$$13V_1 = 82I_1 + 30I_2$$

$$V_1 = \frac{82}{13}I_1 + \frac{30}{13}I_2$$

$$\boxed{V_1 = 6.30 I_1 + 2.307 I_2}$$

Substitute eq (5) in eq (3),

$$5I_2 + 5\left[\frac{3I_1 - 10I_2}{13}\right] = V_2$$

$$5I_2 + \frac{15}{13}I_1 - \frac{50}{13}I_2 = V_2$$

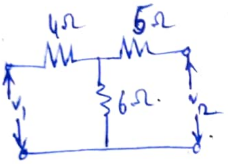
$$\frac{15}{13}I_1 + I_2\left(5 - \frac{50}{13}\right) = V_2$$

$$\boxed{1.153 I_1 + 1.153 I_2 = V_2}$$

$\therefore Z_{11} = 0.485\,\Omega$, $Z_{12} = 2.307\,\Omega$

$Z_{21} = 1.153\,\Omega$, $Z_{22} = 1.153\,\Omega$.

14) Sol:



Z-Parameters

$$V_1 = 10I_1 + 6I_2$$

$$V_2 = 6I_1 + 11I_2$$

$$I_2 = \frac{V_2 - 6I_1}{11}$$

$$\boxed{I_2 = -\frac{6}{11}I_1 + \frac{1}{11}V_2}$$

h-Parameters

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$V_1 = 10I_1 + 6\left(\frac{V_2 - 6I_1}{11}\right).$$

$$V_1 = 10I_1 + \frac{6}{11}V_2 - \frac{36}{11}I_1$$

$$V_1 = 10I_1 - \frac{36}{11}I_1 + \frac{6}{11}V_2$$

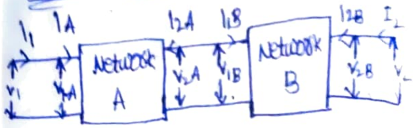
$$V_1 = \frac{74}{11}I_1 + \frac{6}{11}V_2$$

$\therefore$ h-parameters:-

$h_{11} = \frac{74}{11}$; $h_{12} = \frac{6}{11}$

$h_{21} = -\frac{6}{11}$; $h_{22} = \frac{1}{11}$

Cascade connection:-



from above network; $V_1 = V_1A$

$V_2A = V_1B$

$V_2B = V_2$

$I_1 = I_1A$

$I_2A = I_1B$ (or) $I_1B = -I_2A$

$I_2B = I_2$

ABCD parameters are highly useful in characterizing the cascade

Unit-5 - Two Port Network.

Two port Network:

A pair of terminals through which a current may enter or leave a network is known as port.

Two terminal devices or elements (such as resistor, capacitors, and inductors) results in one-port network.

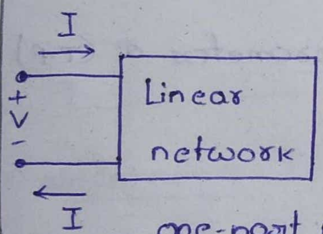
Most of the circuits we have dealt with so far are two-terminal or one-port circuits.

A two-port network is an electrical network with two separate ports for input and output.

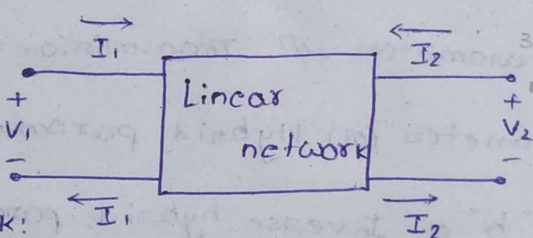
It has two terminal pairs acting as access points. The current entering one terminal of a pair leaves the other terminal in the pair.

Port → A pair of terminals which are accesible for external connection.

1 port → 2 variables
2 port → 4 variables
3 port → 8 variables
n port → 2^n variables.



one-port network:



Two port network:

Reasons why to study two port network:

1. Such networks are useful in communication, control system, power systems and electronics.
2. Knowing the parameters of a two-port network enables us to treat it as a "black box" when embedded within a larger network.

From the network, we can observe that there are 4 variables that is I_1, I_2, V_1 and V_2, which two are independent.

The various term that relate these voltages and currents are called parameters.

$\left.\begin{matrix} V_1 \\ I_1 \end{matrix}\right\}$ dependent $\left.\begin{matrix} V_2 \\ I_2 \end{matrix}\right\}$ Independent variables ⟶ ①

$\left.\begin{matrix} I_1 \\ I_2 \end{matrix}\right\}$ dependent $\left.\begin{matrix} V_1 \\ V_2 \end{matrix}\right\}$ Independent variables $\longrightarrow$ ②

$\left.\begin{matrix} V_1 \\ V_2 \end{matrix}\right\}$ dependent $\left.\begin{matrix} I_1 \\ I_2 \end{matrix}\right\}$ Independent variables $\longrightarrow$ ③.

$\left.\begin{matrix} V_2 \\ I_2 \end{matrix}\right\}$ dependent $\left.\begin{matrix} V_1 \\ I_1 \end{matrix}\right\}$ Indepent $\longrightarrow$ ④

$\left.\begin{matrix} V_1 \\ I_2 \end{matrix}\right\}$ dependent $\left.\begin{matrix} V_2 \\ I_1 \end{matrix}\right\}$ Independent $\longrightarrow$ ⑤

$\left.\begin{matrix} I_1 \\ V_2 \end{matrix}\right\}$ dependent $\left.\begin{matrix} V_1 \\ I_2 \end{matrix}\right\}$ Independent variables $\longrightarrow$ ⑥

∴ six set of parameters.

→ "Z" parameters

→ "Y" parameters

→ "ABCD" parameters or Transmission line parameters or (T.P)

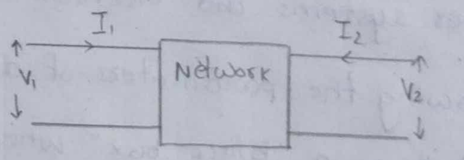
→ "h" - parameters (or) Hybrid parameters.

→ Inverse "h" or Inverse hybrid parameters.

1) Z parameters:

$\left.\begin{matrix} V_1 \\ V_2 \end{matrix}\right\}$ Dependent $\left.\begin{matrix} I_1 \\ I_2 \end{matrix}\right\}$ Independent variables

$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow$ ①



$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow$ ②

O.C ⇒ I = 0

S.C ⇒ V = 0

$Z_{11} = \frac{V_1}{I_1}$; $Z_{12} = \frac{V_1}{I_2}$; $Z_{21} = \frac{V_2}{I_1}$; $Z_{22} = \frac{V_2}{I_2}$

Z parameters are also called "open circuit characteristics"

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

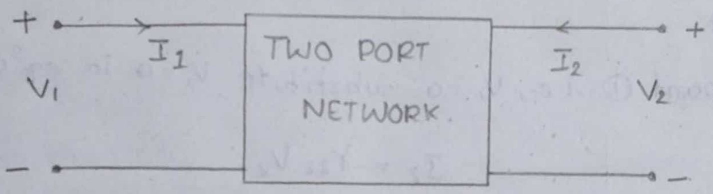
To find Z_{11}, Z_{21} & Z_{12}, Z_{22} we need to open circuit the port 1 & port 2.

Z_{11} → input driving point impedance.

Z_{12} → Transfer impedence from port 1 to port 2.

Before starting actual analysis of two port network, it is necessary to assume following:

1. The voltages and current in the actual network inside box are not available for the measurements. In other words, the measurements can be made on a box consisting network using only port variables.
2. The network inside the box is assumed to consist only the linear elements. Also the network may consist dependent sources but independent sources are not allowed.
3. If the network consists energy storing elements such as inductor and capacitor, then the intial conditionon them is assumed to be zero.



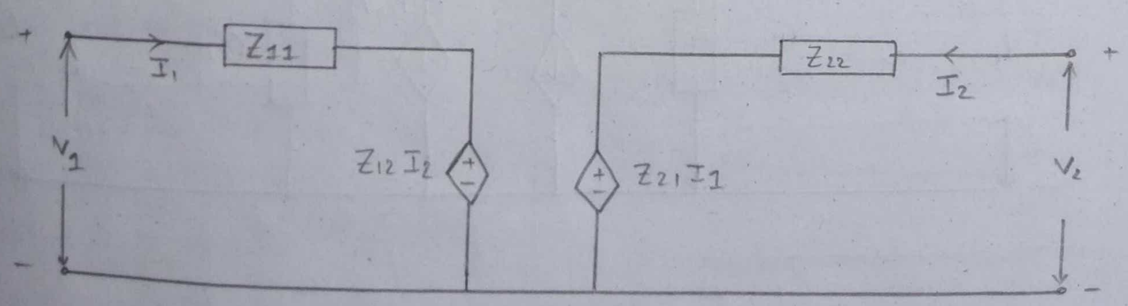
Two port network with standard positive port Voltages and currents.

Z parameters:

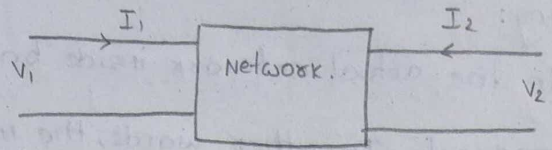
$V_1 = Z_{11} I_1 + Z_{12} I_2$; $V_2 = Z_{21} I_1 + Z_{22} I_2$.

$I_2 = 0$	$I_1 = 0$
$Z_{11} = V_1/I_1$	$Z_{12} = V_1/I_2$
$Z_{21} = V_2/I_1$	$Z_{22} = V_2/I_2$

Equivalent circuit of Z-parameters eqn's:



2) Y-parameters (or) admittance parameters (or) short ckt parameters



I_1, I_2 acts as dependent ; V_1, V_2 acts as independent variables.

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \longrightarrow ①$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \longrightarrow ②$$

Short ckt the port ② i.e., $V_2 = 0$. substitute $V_2 = 0$ in eqⁿ ① & ②

$I_1 = Y_{11} V_1$

$Y_{11} \Rightarrow I_1 / V_1$

input driving point admittance

$I_2 = Y_{21} V_1$

$Y_{21} = I_2 / V_1$

Transfer ~~impedence,~~ admittance

Short ckt the port ① i.e., $V_1 = 0$. substitute $V_1 = 0$ in eqⁿ ① & ②

$I_1 = Y_{12} V_2$

$Y_{12} = I_1 / V_2$

Transfer admittance

$I_2 = Y_{22} V_2$

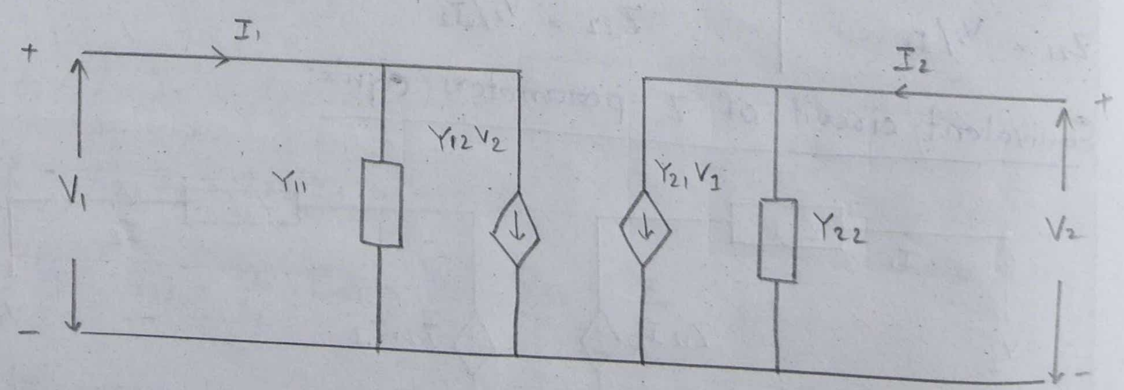
$Y_{22} = I_2 / V_2$

output driving point admittance.

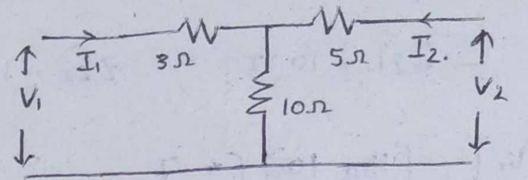
In Matrix form;

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Equivalent circuit of y-parameters:



Eg ①: Find the Z parameters for the network shown below.



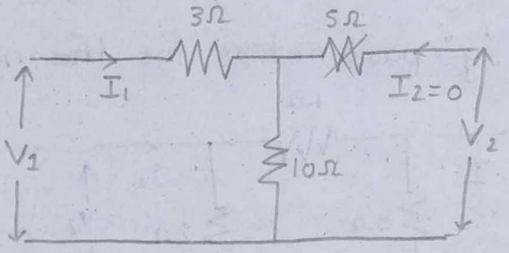
Sol: **Method ①:**

Z parameters eq'ns are

$$V_1 = Z_{11} I_1 + Z_{12} I_2 .$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 .$$

Port ② is open circuited i.e., put $I_2 = 0$:-



$Z_{11} = \frac{V_1}{I_1}$; $Z_{21} = \frac{V_2}{I_1}$.

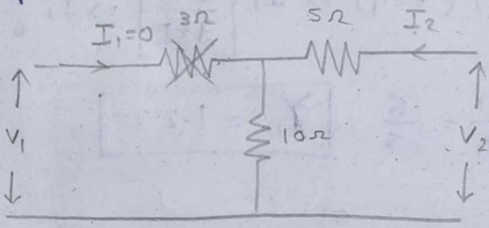
$V_1 = I_1 R_{eq} \Rightarrow I_1 [3+10] \Rightarrow 13 I_1$

$$\boxed{\therefore Z_{11} = \frac{V_1}{I_1} = 13\Omega}$$

$V_2 = I_1 \times 10$

$$\boxed{Z_{21} = \frac{V_2}{I_1} = 10\Omega}$$

port ① is open circuited i.e., put $I_1 = 0$:-



$Z_{22} = \frac{V_2}{I_2}$; $V_{12} = \frac{V_1}{I_2}$

$V_2 = I_2 R_{eq} \Rightarrow I_2 (10+5) \Rightarrow 15 I_2$

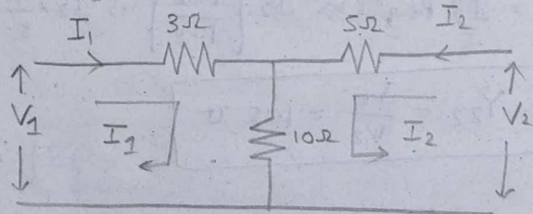
$$\boxed{\therefore Z_{22} = \frac{V_2}{I_2} = 15\Omega}$$

$V_1 = 10 I_2$

$$\boxed{\therefore Z_{12} = \frac{V_1}{I_2} = 10\Omega}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 13\Omega & 10\Omega \\ 10\Omega & 15\Omega \end{bmatrix}}_{\text{Z parameters}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Method ②:



Apply kvl to loop ①

$3I_1 + 10[I_1 + I_2] = V_1$

$V_1 = 13 I_1 + 10 I_2 . \longrightarrow ①$

Apply kvl to loop ②

$5 I_2 + 10 [I_2 + I_1] = V_2$

$V_2 = 10 I_1 + 15 I_2 \longrightarrow ②$

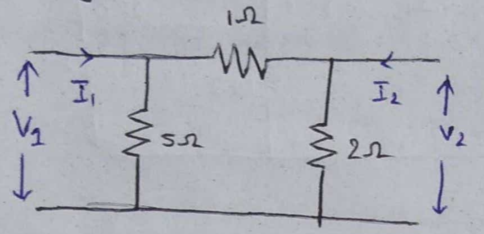
Compare the eq^n ① & ② with Z parameters equations.

$\therefore Z_{11} = 13\,\Omega \qquad Z_{12} = 10\,\Omega$

$Z_{21} = 10\,\Omega \qquad Z_{22} = 15\,\Omega.$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 13\Omega & 10\Omega \\ 10\Omega & 15\Omega \end{bmatrix}}_{\text{Z-parameters}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Eg ② Find "y" parameters for the network shown below.

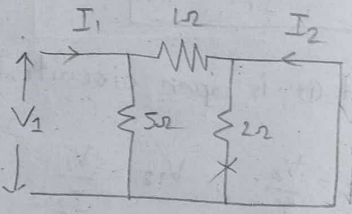


Sol:

Method ①: $I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

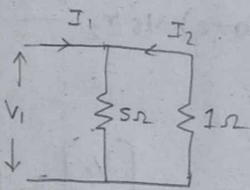
Take $V_2 = 0$ i.e., short circuit the port ②



$V_1 = I_1 (R_{eq}) \Rightarrow I_1 \left[\frac{1\times 5}{1+5}\right] \Rightarrow I_1 \frac{5}{6}$

$\therefore \frac{I_1}{V_1} = \frac{6}{5} \qquad \boxed{Y_{11} = 1.2\,\mho}$

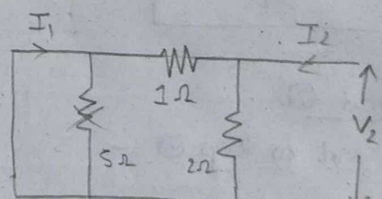
~~$I_2 = -I \times \frac{5}{}$~~



$V_1 = -1 I_2.$

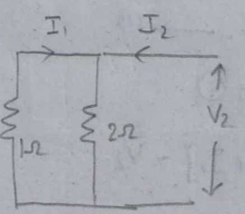
$$\boxed{Y_{21} = \frac{I_2}{V_1} = -1\,\mho}$$

Take $V_1 = 0$ i.e., short circuit the port ①.



$V_2 = I_2 R_{eq} \Rightarrow I_2 \left[\frac{1\times 2}{1+2}\right] \Rightarrow I_2 \times \frac{2}{3}$

$$\boxed{\therefore Y_{22} = \frac{I_2}{V_2} = 1.5\,\mho}$$

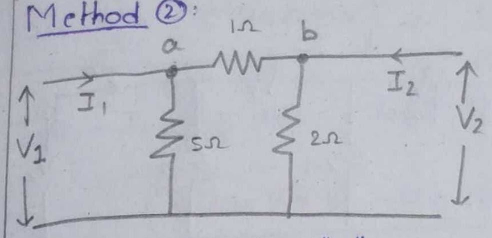


$V_2 = -1 I_1$

$$\boxed{\therefore Y_{12} = \frac{I_1}{V_2} = -1\,\mho}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1.2\mho & -1\mho \\ -1\mho & 1.5\mho \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Method ②:



Consider a,b are 2 nodes.

Apply KCL to node "a"; $I_1 = \frac{V_1}{5} + \frac{V_1 - V_2}{1} \Rightarrow V_1\left[\frac{1}{5}+1\right] + V_2(-1)$

$$\boxed{\therefore I_1 = 1.2\,V_1 + V_2(-1) \longrightarrow ①}$$

Apply KCL to node "b"; $I_2 = \frac{V_2}{2} + \frac{V_2 - V_1}{1} \Rightarrow V_1(-1) + V_2\left[1+\frac{1}{2}\right]$

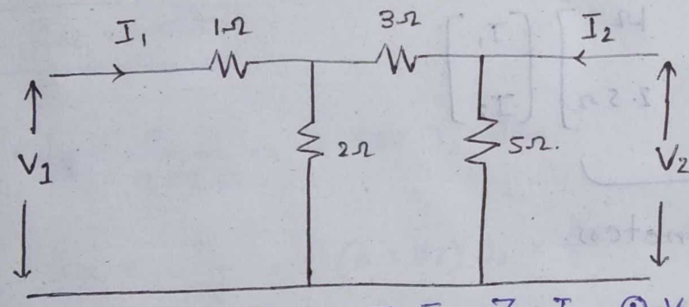
$$\boxed{\therefore I_2 = V_1(-1) + V_2(1.5) \longrightarrow ②}$$

Compare the Y parameter equation with equations ① & ②.

$Y_{11} = 1.2\,℧$; $Y_{12} = -1\,℧$; $Y_{21} = -1\,℧$; $Y_{22} = 1.5\,℧$.

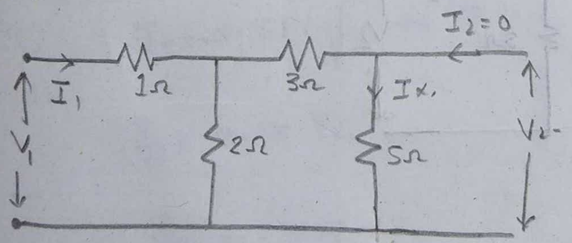
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1.2℧ & -1℧ \\ -1℧ & 1.5℧ \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Eg: ③ Find the Z-parameters for the circuit shown in figure.



Sol: The equations are ① $V_1 = Z_{11}I_1 + Z_{12}I_2$; ② $V_2 = Z_{21}I_1 + Z_{22}I_2$.

① open CKT the port ② i.e., $I_2 = 0$.



$V_1 = I_1 R_{eq}$

$V_1 = I_1\left[\left[(3+5)//2\right]+1\right]$

$V_1 = I_1\, 2.6$

$Z_{11} = \frac{V_1}{I_1}$; $Z_{21} = \frac{V_2}{I_1}$

$$\boxed{Z_{11} = \frac{V_1}{I_1} = 2.6}$$

$$\boxed{\therefore Z_{11} = 2.6\,\Omega}$$

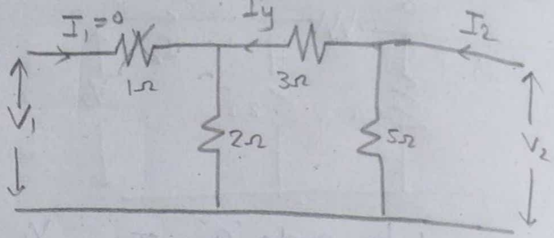
$\therefore V_2 = 5\,I_x$

I_x can be find out by using current division rule.

$I_x = I_1 \times \frac{2}{2+3+5} = 0.2\,I_1$

$V_2 = 5\times(0.2I_1) \Rightarrow 1\,I_1$ $\quad\therefore \frac{V_2}{I_1} = Z_{21} = 1\Omega$ $\quad\boxed{\therefore Z_{21} = 1\Omega}$

② open circuit port ① i.e., $I_1 = 0$.



$Z_{22} = \frac{V_2}{I_2}$; $Z_{12} = \frac{V_1}{I_2}$.

$V_2 = I_2 \, Req$

$Req = (2+3) // 5 \Rightarrow 2.5\Omega$.

$\therefore \frac{V_2}{I_2} = 2.5\Omega$ $\boxed{Z_{22} = 2.5\Omega}$

$V_1 = I_y 2$

I_y can be found out by taking current division rule.

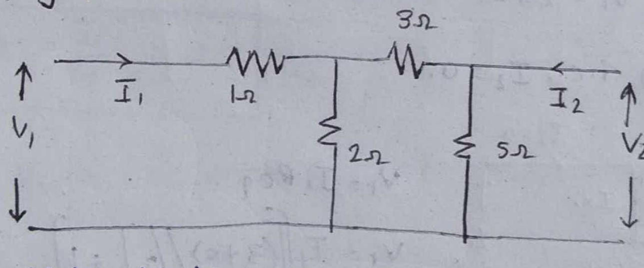
$I_y = I_2 \times \frac{5}{5+2+3} = 0.5 I_2$.

$\therefore V_1 = 2(0.5 I_2) = 1 I_2$

$\therefore \frac{V_1}{I_2} = 1\Omega$ $\boxed{\therefore Z_{12} = 1\Omega}$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 2.6\Omega & 1\Omega \\ 1\Omega & 2.5\Omega \end{bmatrix}}_{Z\text{-parameters.}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Eg: ④ Find y parameters.

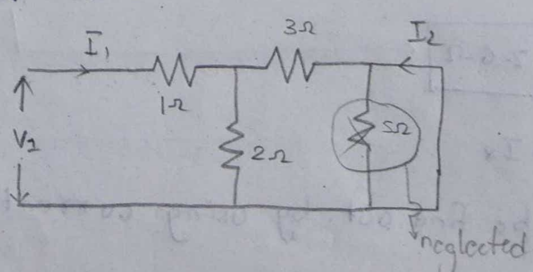


Sol: $I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$.

step ①: short ckt the port; $V_2 = 0$.

$Y_{11} = \frac{I_1}{V_1}$; $Y_{21} = \frac{I_2}{V_1}$



$V_1 = I_1 \, Req$

$V_1 = I_1 \left[[2 // 3] + 1 \right]$

$\frac{V_1}{I_1} = 2.2\Omega$ $\therefore \frac{I_1}{V_1} = \frac{1}{2.2} = 0.45℧$

$$\boxed{Y_{11} = \frac{I_1}{V_1} = 0.45\,mho.} \Rightarrow I_1 = 0.45\,V_1$$

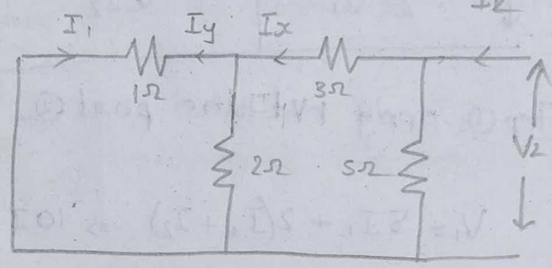
$I_2 = -I_1 \frac{2}{2+3} \Rightarrow -I_1(0.4)$ ⟶ substitute above I_1 value in this eqn

$I_2 = -(0.45V_1)(0.4) \Rightarrow -0.18\,V_1$

$\therefore \frac{I_2}{V_1} = -0.18.$

$$\boxed{Y_{21} = -0.18\,\mho}$$

Step ②: short circuit the port ①; $V_1 = 0$.



$Y_{12} = \frac{I_1}{V_2}$; $Y_{22} = \frac{I_2}{V_2}$.

$I_2 = V_2/R_{eq} \Rightarrow V_2 / \left[((1//2)+3)//5\right]$

$I_2 = V_2/(3.66//5) \Rightarrow V_2/2.11$

$[3 + (2//1) \Rightarrow 3.667\Omega]$

$\therefore \frac{I_2}{V_2} = \frac{1}{2.11} \Rightarrow 0.47\,\mho$

$$\boxed{\therefore Z_{22} = 0.47\,\mho}$$

$I_x = +I_2 \times \frac{5}{5+3.66} \Rightarrow 0.577\,I_2$ Amps

$\therefore I_y = I_x \times \frac{2}{1+2} \Rightarrow (0.572)\,I_2 \times \frac{2}{3} \Rightarrow 0.381\,I_2$ Amps.

$$\boxed{\therefore I_1 = -I_y = -0.381\,I_2}$$

$I_1 = -0.381\,[0.47\,V_2]$

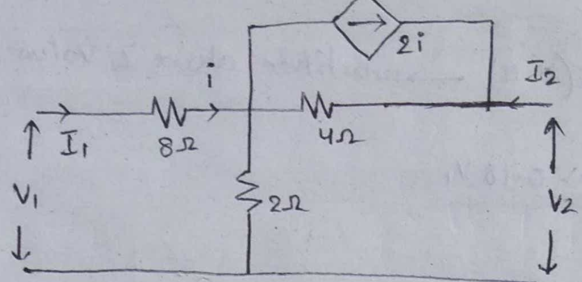
$I_1 = -0.18\,V_2$

$\therefore \frac{I_1}{V_2} = -0.18$

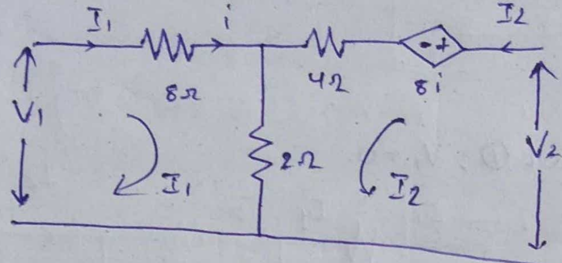
$$\boxed{Y_{12} = -0.18\,\mho}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0.45\,\mho & -0.18\,\mho \\ -0.18\,\mho & 0.47\,\mho \end{bmatrix}}_{\text{Y-parameters.}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Eg:⑤ find Z-parameters for the network shown below.



Sol:



$$\begin{bmatrix} V_S = CS \times 4\Omega \\ V_S = 2i \times 4 = 8i \end{bmatrix}$$

Step ①: apply KVL to the port ②, Port ①:

$$V_1 = 8I_1 + 2(I_1 + I_2) \Rightarrow 10I_1 + 2I_2 \longrightarrow ①$$

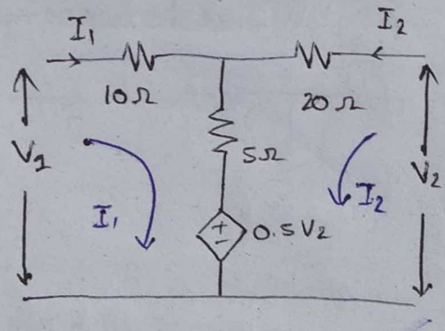
$$V_2 = 8i + 4I_2 + 2(I_2 + I_1) \Rightarrow 8I_1 + 4I_2 + 2I_2 + 2I_1$$

$$V_2 = 10I_1 + 6I_2 \longrightarrow ②$$

Compare the eqn's ① & ② with Z-parameters equations.

$$Z = \begin{bmatrix} 10\Omega & 2\Omega \\ 10\Omega & 6\Omega \end{bmatrix}$$

Eg:⑥ find Z parameters.



$$\left.\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned}\right\} \text{Z parameter equations.}$$

Apply KVL to both loops;

$$V_1 = 10I_1 + 5(I_1 + I_2) + 0.5V_2 \Rightarrow 15I_1 + 5I_2 + 0.5V_2 \longrightarrow ①$$

$$V_2 = 20I_2 + 5(I_2 + I_1) + 0.5V_2 \Rightarrow 5I_1 + 25I_2 + 0.5V_2$$

$$V_2 - 0.5V_2 = 5I_1 + 25I_2$$

$$0.5V_2 = 5I_1 + 25I_2 \longrightarrow ②$$

$$V_2 = 10I_1 + 50I_2 \longrightarrow ③$$

Substitue eqⁿ ② in eqⁿ ①.

$$V_1 = 15 I_1 + 5 I_2 + 0.5 V_2 \Rightarrow 15 I_1 + 5 I_2 + [5 I_1 + 25 I_2]$$

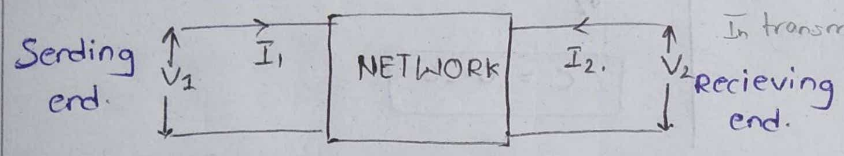
$$V_1 = 20 I_1 + 30 I_2 \longrightarrow ④$$

Compare eqⁿ ③ & ④ with Z parameter equation.

$$\therefore Z_{11} = 20\Omega,\ Z_{12} = 30\Omega,\ Z_{21} = 10\Omega,\ Z_{22} = 50\Omega.$$

$$Z = \begin{bmatrix} 20\Omega & 30\Omega \\ 10\Omega & 50\Omega \end{bmatrix}$$

③ ABCD parameters (or) Transmission line (or) General parameters:



In transmission line current is in opposite direction of I_2.

$$\left.\begin{aligned} V_1 &= AV_2 - BI_2 \rightarrow ① \\ I_1 &= CV_2 - DI_2 \rightarrow ② \end{aligned}\right\} \text{General (or) chain parameters.}$$

① Take $I_2 = 0$ i.e., open circuit the port ②.

$V_1 = AV_2$	$I_1 = CV_2$
$A = \frac{V_1}{V_2}$	$C = \frac{I_1}{V_2}$
↓	↓
reverse voltage gain (ratio)	Transfer admittance (℧).

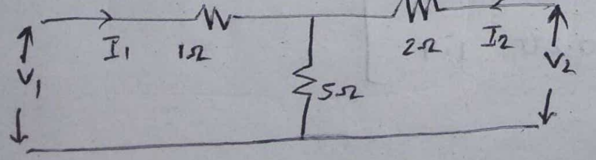
② Take $V_2 = 0$ i.e., short circuit the port ②.

$V_1 = -BI_2$	$I_1 = -DI_2$
$B = -\frac{V_1}{I_2}$	$D = -\frac{I_1}{I_2}$
↓	↓
Transfer impedence	reverse current gain (ratio)

$\therefore \frac{1}{D}$ = forward current gain.

$\therefore \frac{1}{A}$ = forward voltage gain.

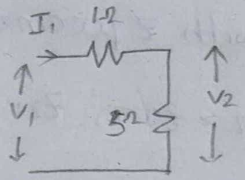
Eg: ⑦ Find the general CKT parameters for network shown below



$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2.$

① open CKT the port ② i.e., $I_2 = 0$.



$V_1 = I_1 \, Req$

$\frac{V_1}{I_1} = Req = (1+5) = 6\Omega$

$\frac{V_1}{\frac{V_2}{5}} = 6\Omega$

$\frac{V_1}{V_2} = \frac{6}{5} = 01.2 = A$

$\boxed{\therefore A = 1.2}$

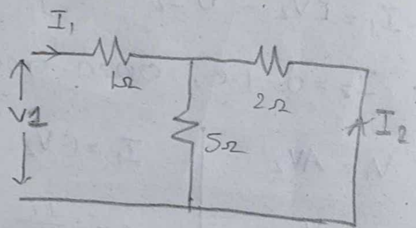
$\left[\begin{array}{l} \therefore V_2 = 5 I_1 \\ \therefore I_1 = \frac{V_2}{5} \end{array}\right]$

$\frac{I_1}{V_2} = \frac{1}{5} = 0.2 \mho = C$

$\boxed{\therefore C = 0.2\mho}$

② short CKT the port ② i.e., $V_2 = 0$.

$D = -\frac{I_1}{I_2}$ $\qquad B = -\frac{V_1}{I_2}$



$I_2 = -I_1\left(\frac{5}{5+2}\right) = -I_1/1.4$

$\therefore \frac{I_2}{I_1} = \frac{-1}{1.4}$

$\frac{I_1}{I_2} = -1.4 = D$ $\quad \boxed{\therefore D = -1.4}$

$V_1 = I_1 (Req) \Rightarrow I_1 \left([2 \| 5] + 1\right) \Rightarrow I_1 (2.4)$

$V_1 = I_1 (2.4)$

$V_1 = (I_2 \times -1.4)(2.4)$

$\frac{V_1}{I_2} = -3.36\Omega$

$-\frac{V_1}{I_2} = -(-3.36\Omega) = 3.36\Omega$

$\boxed{\therefore B = 3.36\Omega}$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1.2 & 3.36\Omega \\ 0.2\mho & 1.4 \end{bmatrix}$$

For inverse transmission line parameters:-

$$V_2 = A'V_1 - B'I_1$$

$$I_2 = C'V_1 - D'I_1$$

(4) Hybrid parameters or H-parameters:

These are widely used in modelling of electronic component i.e., particularly transistors.

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

(1) short circuit the port (2) i.e., $V_2 = 0$.

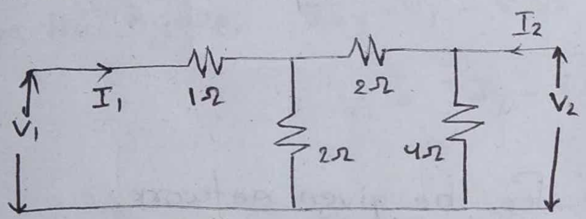
$h_{11} = \frac{V_1}{I_1}$ (Input impedence); $h_{21} = \frac{I_1}{I_2}$ [Current gain]

(2) open circuit the port (1) i.e., $I_1 = 0$.

$h_{22} = \frac{I_2}{V_2}$ $\qquad$ $h_{12} = \frac{V_1}{V_2}$ [voltage gain]

(output admittance)

Eg:(8) Find the "h" parameters for the given circuit.

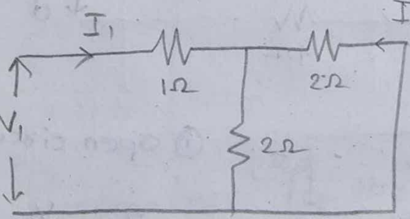


$V_1 = h_{11} I_1 + h_{12} [V_2]$

$I_2 = h_{21} I_1 + h_{22} V_2$

(1) short circuit the port (2) $V_2 = 0$.

$h_{11} = \frac{V_1}{I_1}$ | $h_{21} = \frac{I_2}{I_1}$



$V_1 = I_1 (R_{eq}) \Rightarrow I_1 [1 + 2 // 2]$

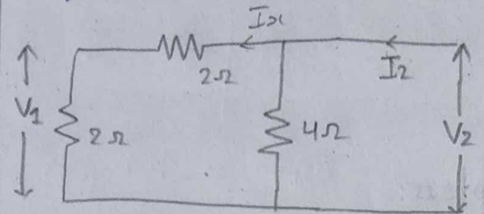
$\therefore \frac{V_1}{I_1} = 2\Omega$ $\quad \boxed{h_{11} = 2\Omega}$

$-I_2 = I_1 \left(\frac{2}{2+2}\right) = I_1 \frac{2}{4}$

$\frac{I_2}{I_1} = -\frac{1}{2} = -0.5$

$\boxed{h_{21} = -0.5}$

② open circuit the port ①, i.e, $I_1 = 0$. ($I_1 = 0$)



$$h_{12} = \frac{V_1}{V_2} \qquad h_{22} = \frac{I_2}{V_2}$$

$$V_2 = I_2 (R_{eq}) \Rightarrow I_2 \left((2+2)//4\right) \Rightarrow 2 I_2.$$

$$\frac{V_2}{I_2} = 2 \Rightarrow \frac{I_2}{V_2} = \frac{1}{2} = 0.5\,℧ \qquad [\therefore I_2 = 0.5 V_2]$$

$$\boxed{\therefore h_{22} = 0.5\,℧}$$

$$V_1 = 2 I_x \Rightarrow 2\left[I_2 \times \frac{4}{4+4}\right] \Rightarrow 2\left[I_2 \times \frac{1}{2}\right]$$

$$V_1 = I_2$$

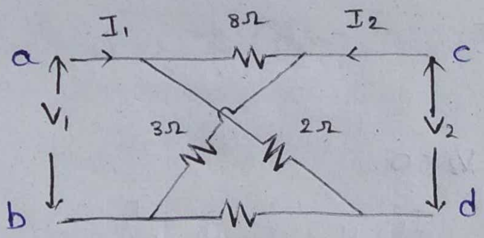
$$V_1 = 0.5 V_2$$

$$\frac{V_1}{V_2} = 0.5$$

$$\boxed{h_{12} = 0.5}$$

$$h = \begin{bmatrix} 2 & +0.5 \\ -0.5 & 0.5 \end{bmatrix}$$

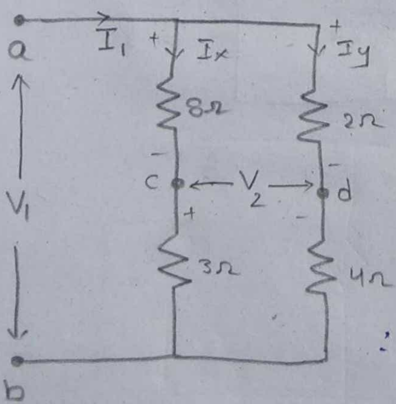
Eg: ⑨ Find out the Z-parameters for the given network.



(Lattice network)

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Sol:



① open circuit the port ② i.e., $I_2 = 0$.

$$Z_{11} = \frac{V_1}{I_1} \; ; \; Z_{21} = \frac{V_2}{I_1}$$

$$V_1 = I_1 (R_{eq}) \Rightarrow I_1 \left[(8+3)//(2+4)\right] \Rightarrow I_1 \left(\tfrac{11}{6}\right)$$

$$\therefore \frac{V_1}{I_1} = 3.88\,\Omega \qquad \boxed{\therefore Z_{11} = 3.88\,\Omega}$$

$$\therefore V_2 = 2(I_y) - 8(I_x)$$

from the fig; $2 I_y - V_2 - 8 I_x = 0$.

$$V_2 = 2I_y - 8I_x \Rightarrow 2\left[I_1\left(\frac{11}{6+11}\right)\right] - 8\left[I_1\left(\frac{6}{6+11}\right)\right]$$

$$\therefore V_2 \Rightarrow 2I_1\left[\frac{11}{17}\right] - 8I_1\left[\frac{6}{17}\right]$$

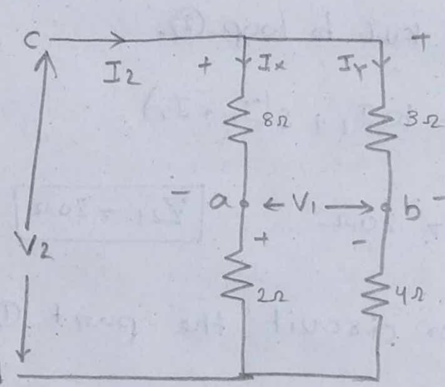
$$V_2 = I_1\left[\frac{22}{17}\right] - I_1\left[\frac{48}{17}\right] \Rightarrow I_1\left[\frac{22-48}{17}\right] \Rightarrow I_1\left[\frac{-26}{17}\right]$$

$$\therefore \frac{V_2}{I_1} = -1.52\,\Omega$$

$$\boxed{\therefore Z_{21} = -1.52\,\Omega}$$

(2) open circuit the port (1), i.e., $I_1 = 0$.

$$Z_{12} = \frac{V_1}{I_2} \quad \Big| \quad Z_{22} = \frac{V_2}{I_2}$$



$$\therefore V_2 = I_2\,(R_{eq}) \Rightarrow I_2\,(10 /\!/ 7)$$

$$\frac{V_2}{I_2} = \frac{10\times 7}{10+7} \Rightarrow \frac{70}{17} \Rightarrow 4.11\,\Omega$$

$$\boxed{\therefore Z_{22} = 4.11\,\Omega}$$

from the figure, $3I_y - V_1 - 8I_x = 0$

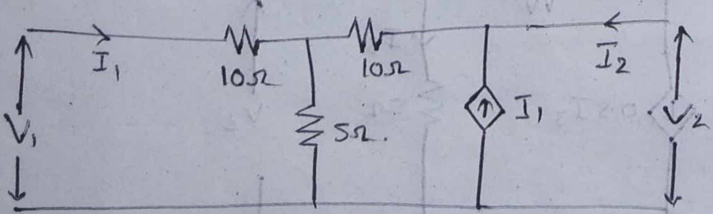
$$V_1 = 3I_y - 8I_x \Rightarrow 3\left[I_2\,\frac{10}{10+7}\right] - 8\left[I_2\,\frac{7}{10+7}\right]$$

$$\therefore \frac{V_1}{I_2} = \frac{30}{17} - \frac{56}{17} \Rightarrow \frac{-26}{17} = -1.52\,\Omega$$

$$\boxed{\therefore Z_{12} = -1.52\,\Omega}$$

$$Z \Rightarrow \begin{bmatrix} 3.88\,\Omega & -1.52\,\Omega \\ -1.52\,\Omega & 4.11\,\Omega \end{bmatrix}$$

Eg: (8) Determine the open circuit parameters for the given network.

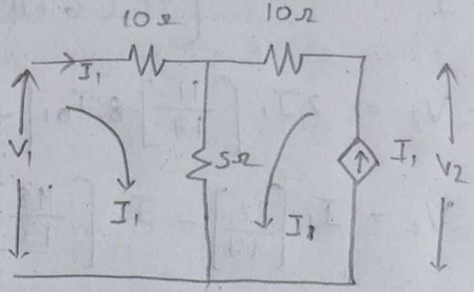


$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

① open circuit the port ② i.e., $I_2 = 0$.

$$Z_{11} = \frac{V_1}{I_1} \quad \Bigg| \quad Z_{21} = \frac{V_2}{I_1}$$



Apply KVL to loop ①

$$V_1 = 10 I_1 + 5 (I_1 + I_1)$$

$$V_1 = 10 I_1 + 10 I_1$$

$$\frac{V_1}{I_1} = 20\Omega \qquad \boxed{\therefore Z_{11} = 20\Omega}$$

Apply KVL to loop ②.

$$V_2 = 10 I_1 + 5(I_1 + I_1)$$

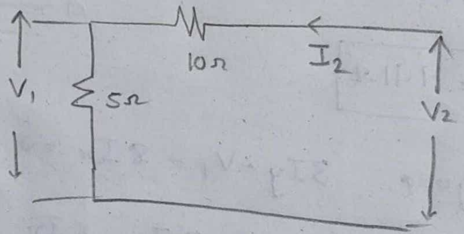
$$\frac{V_2}{I_1} = 20\Omega \qquad \therefore \boxed{Z_{21} = 20\Omega}$$

② open circuit the port ① i.e., $I_1 = 0$.

$$V_2 = I_2 (R_{eq})$$

$$\frac{V_2}{I_2} = 10 + 5 = 15\Omega$$

$$\boxed{\therefore Z_{22} = 15\Omega}$$



$$Z_{12} = \frac{V_1}{I_2}$$

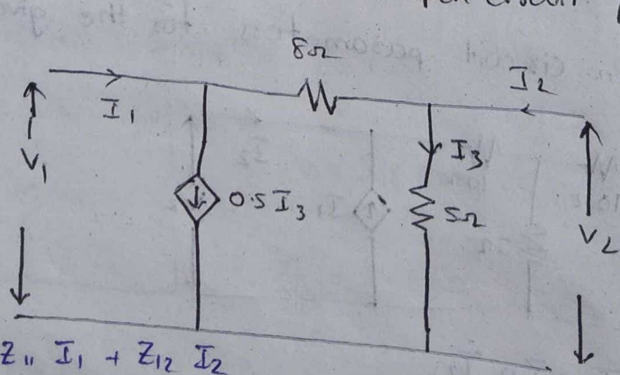
$$Z_{22} = \frac{V_2}{I_2}$$

$$V_1 = 5 I_2$$

$$\frac{V_1}{I_2} = 5\Omega \qquad \boxed{\therefore Z_{12} = 5\Omega}$$

$$Z = \begin{bmatrix} 20\Omega & 5\Omega \\ 20\Omega & 15\Omega \end{bmatrix}$$

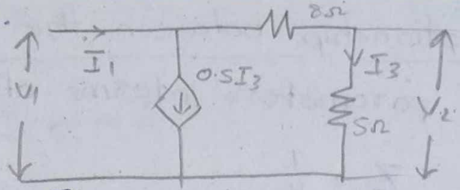
Eg ⑪: A network shown below obtain open circuit parameters.



$$(V_1) = Z_{11} I_1 + Z_{12} I_2$$

$$(V_2) = Z_{21} I_1 + Z_{22} I_2 .$$

① Open CKT the port ② i.e., $I_2 = 0$.



$Z_{11} = V_1/I_1$ | $Z_{21} = V_2/I_1$

$$\left[\begin{array}{l} I_1 = 0.5I_3 + I_3 \\ I_1 = 1.5I_3 \\ I_3 = \frac{I_1}{1.5} \end{array}\right]$$

$V_1 = 8I_3 + 5I_3$

$V_1 = 13I_3 \Rightarrow 13\left(\frac{I_1}{1.5}\right)$

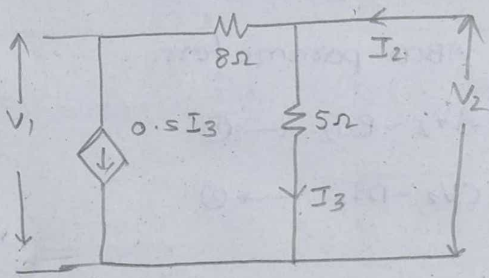
$\frac{V_1}{I_1} = 8.66\,\Omega$ $\boxed{Z_{11} = 8.66\,\Omega}$

$V_2 = 5I_3 \Rightarrow 5\left(\frac{I_1}{1.5}\right)$

$\frac{V_2}{I_1} = 3.33\,\Omega$ $\boxed{Z_{21} = 3.33\,\Omega.}$

② Open CKT the port ① i.e., $I_1 = 0$.

$Z_{12} = \frac{V_1}{I_2}$ | $Z_{22} = \frac{V_2}{I_2}$



$I_2 = I_3 + 0.5I_3$

$I_2 = I_3(1.5)$

$I_3 = \frac{I_2}{1.5}$

$V_2 = 5I_3 = 5\left(\frac{I_2}{1.5}\right) \Rightarrow \frac{V_2}{I_2} = \frac{5}{1.5} = 3.33$ $\boxed{\therefore Z_{22} = 3.33\,\Omega}$

$\boxed{\therefore V_2 = I_2 \times 3.33}$

$V_2 = 8(0.5I_3) + V_1$

$V_2 = 8\left(0.5 \times \frac{I_2}{1.5}\right) + V_1$

$3.33I_2 - 2.66I_2 = V_1$

$V_1 = 0.66I_2$

$\frac{V_1}{I_2} = 0.66\,\Omega$ $\boxed{\therefore Z_{12} = 0.66\,\Omega}$

$$Z = \begin{bmatrix} 8.66\,\Omega & 0.66\,\Omega \\ 3.33\,\Omega & 3.33\,\Omega. \end{bmatrix}$$

Relationship between the parameters:

1) a) Z parameters interms of Y parameters.

$$Z = \frac{1}{y}$$

$$[z] = [y]^{-1}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1} \Rightarrow \frac{1}{\Delta Y}\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

Where $\Delta Y = Y_{11}Y_{21} - Y_{12}Y_{21}$

$$\boxed{Z_{11} = \frac{Y_{22}}{\Delta Y}; \quad Z_{12} = \frac{-Y_{12}}{\Delta Y}; \quad Z_{21} = \frac{-Y_{21}}{\Delta Y}; \quad Z_{22} = \frac{Y_{11}}{\Delta Y}.}$$

b) Z parameters interms of ABCD parameters:-

Eqn's of ABCD parameters.

$$V_1 = AV_2 - BI_2 \longrightarrow ①$$

$$I_1 = CV_2 - DI_2 \longrightarrow ②$$

Z parameter equations.

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2.$$

from eqn ②

$$CV_2 = I_1 + DI_2.$$

$$V_2 = \frac{1}{C}I_1 + \frac{D}{C}I_2 \longrightarrow ③$$

Substitute eqn ③ in eqn ①.

$$V_1 = A\left[\frac{1}{C}I_1 + \frac{D}{C}I_2\right] - BI_2 \Rightarrow I_1\frac{A}{C} + I_2\left[\frac{AD-BC}{C}\right]I_2$$

$$V_1 = I_1\left[\frac{A}{C}\right] + I_2\left[\frac{AD-BC}{C}\right] \longrightarrow ④$$

Compare the eqn ③ & ④ with Z parameter equations.

$$\boxed{\begin{array}{ll} Z_{11} = \frac{A}{C} & Z_{12} = \frac{AD-BC}{C} \\ Z_{21} = \frac{1}{C} & Z_{22} = \frac{D}{C} \end{array}}$$

c) Parameters (z) interms of h-parameters.

$$V_1 = h_{11}I_1 + h_{12}V_2 \longrightarrow ①$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \longrightarrow ②$$

$$\left.\begin{array}{l} V_1 = Z_{11}I_1 + Z_{12}I_2 \\ V_2 = Z_{21}I_1 + Z_{22}I_2 \end{array}\right\} \text{Z parameter equations.}$$

from eqn ②

$$h_{22}V_2 = -h_{21}I_1 + I_2$$

$$\therefore V_2 = \frac{-h_{21}}{h_{22}}I_1 + \frac{1}{h_{22}}I_2. \longrightarrow ③$$

Substitute eqn ③ in ①.

$$V_1 = h_{11}I_1 + h_{12}\left[\frac{-h_{21}}{h_{22}}I_1 + \frac{1}{h_{22}}I_2\right]$$

$$V_1 = h_{11} I_1 + h_{12} \times \frac{-h_{21}}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_1 = \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right] I_1 + \left[\frac{h_{12}}{h_{22}}\right] I_2 \longrightarrow ④$$

Compare eqⁿ ③ & ④ with Z-parameter equations.

$$\boxed{Z_{11} = \frac{\Delta h}{h_{22}} \; ; \; Z_{12} = \frac{1}{h_{22}} \; ; \; Z_{21} = \frac{-h_{21}}{h_{22}} \; ; \; Z_{22} = \frac{h_{12}}{h_{22}}}$$

2) a) Y-parameters interms of Z-parameters:

$$[Y] = [Z]^{-1}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} \Rightarrow \frac{1}{\Delta Z} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$$\boxed{Y_{11} = \frac{Z_{22}}{\Delta Z} \; ; \; Y_{12} = -\frac{Z_{12}}{\Delta Z} \; ; \; Y_{21} = -\frac{Z_{21}}{\Delta Z} \; ; \; Y_{22} = \frac{Z_{11}}{\Delta Z}}$$

Where; $\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$

b) Y-parameters interms of ABCD parameters.

Equations of ABCD parameters

$$V_1 = A V_2 - B I_2 \longrightarrow ①$$

$$I_1 = C V_2 - D I_2 \longrightarrow ②$$

Y-parameter eqⁿ's

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

from eqⁿ ①

$$B I_2 = -V_1 + A V_2$$

$$I_2 = \frac{-1}{B} V_1 + \frac{A}{B} V_2 \longrightarrow ③$$

substitute eqⁿ ③ in eqⁿ ②.

$$I_1 \Rightarrow C V_2 - D\left[\frac{-1}{B} V_1 + \frac{A}{B} V_2\right] \Rightarrow C V_2 + \frac{D}{B} V_1 - \frac{AD}{B} V_2$$

$$\therefore I_1 = \frac{D}{B} V_1 + \frac{BC - AD}{B} V_2 \longrightarrow ④$$

Compare eqⁿ ③ & ④ with Y-parameter equations.

$$\boxed{\begin{array}{ll} Y_{11} = \frac{D}{B} & Y_{12} = \frac{BC-AD}{B} \\ Y_{21} = \frac{-1}{B} & Y_{22} = \frac{A}{B} \end{array}}$$

c) Y-parameters interms of h-parameters:

h-parameters eqn's

$$V_1 = h_{11} I_1 + h_{12} V_2 \longrightarrow ①$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow ②$$

Y-parameter eqn's

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

from eqn ①.

$$h_{11} I_1 = V_1 - h_{12} V_2$$

$$I_1 = \frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2 \longrightarrow ③$$

substitute eqn ③ in eqn ②.

$$I_2 = h_{21} \left[\frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2 \right] + h_{22} V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 - \frac{h_{21} h_{12}}{h_{11}} V_2 + h_{22} V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{11}} V_2 \longrightarrow ④$$

compare eqn ③ & ④ with Y-parameter equations.

$$\boxed{Y_{11} = \frac{1}{h_{11}} \; ; \; Y_{12} = \frac{-h_{12}}{h_{11}} \; ; \; Y_{21} = \frac{h_{21}}{h_{11}} \; ; \; Y_{22} = \frac{\Delta h}{h_{11}}}$$

3) a) ABCD parameters interms of Z-paramers:

Z parameter eqn's

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow ①$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow ②$$

ABCD parameter eqn's

$$V_1 = A V_2 - B I_2$$

$$I_1 = C V_2 - D I_2$$

from eqn ②.

$$Z_{21} I_1 = V_2 - Z_{22} I_2$$

$$I_1 = \left(\frac{1}{Z_{21}}\right) V_2 - \left(\frac{Z_{22}}{Z_{21}}\right) I_2 . \longrightarrow ③.$$

substitute eqn ③ in eqn ①.

$$V_1 = Z_{11} \left[\frac{V_2}{Z_{21}} - \frac{Z_{22}}{Z_{21}} I_2 \right] + Z_{12} I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{Z_{11} Z_{22}}{Z_{21}} . I_2 + Z_{12} I_2 \Rightarrow \frac{Z_{11}}{Z_{21}} V_2 - \left[\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}} \right] I_2 \longrightarrow ④$$

Compare the eqn ③ & ④ with ABCD parameters.

$$\boxed{A = \frac{Z_{11}}{Z_{21}} \; ; \; B = \frac{\Delta Z}{Z_{21}} \; ; \; C = \frac{1}{Z_{21}} \; ; \; D = \frac{Z_{22}}{Z_{21}}}$$

b) ABCD parameters interms of Y-parameters:

Y-parameter eq^n's

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \longrightarrow ①$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \longrightarrow ②$$

ABCD-parameters.

$$V_1 = AV_2 - BI_2.$$

$$I_1 = CV_2 - DI_2.$$

from eq^n ②:-

$$Y_{21}V_1 = -Y_{22}V_2 + I_2$$

$$V_1 = \left[-\frac{Y_{22}}{Y_{21}}\right]V_2 - \left[-\frac{1}{Y_{21}}\right]I_2 \longrightarrow ③$$

substitute eq^n ③ in eq^n ①.

$$I_1 = Y_{11}\left[\frac{-Y_{22}}{Y_{21}}V_2 + \frac{I_2}{Y_{21}}\right] + Y_{12}V_2 \Rightarrow \frac{-Y_{11}Y_{22}}{Y_{21}}V_2 + \frac{I_2 Y_{11}}{Y_{21}} + Y_{12}V_2$$

$$\therefore I_1 = \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}V_2 - \left[\frac{-Y_{11}}{Y_{21}}\right]I_2 \longrightarrow ④$$

Compare eq^n ③ & ④ with ABCD eq^n's

$$\boxed{A = \frac{-Y_{22}}{Y_{21}} \; ; \; B = -\frac{1}{Y_{21}} \; ; \; C = \frac{-\Delta Y}{Y_{21}} \; ; \; D = \frac{-Y_{11}}{Y_{21}}}$$

c) ABCD parameters interms of H-parameters.

H-parameter equations

$$V_1 = h_{11}I_1 + h_{12}V_2 \longrightarrow ①$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \longrightarrow ②$$

ABCD eq^n's

$$V_1 = AV_2 - BI_2.$$

$$I_1 = CV_2 - DI_2.$$

from eq^n ②

$$h_{21}I_1 = I_2 - h_{22}V_2 \Rightarrow I_1 = \left[\frac{-h_{22}}{h_{21}}\right]V_2 - \left[\frac{-1}{h_{21}}\right]I_2 \longrightarrow ③$$

substitute eq^n ③ in eq^n ①.

$$V_1 = h_{11}\left[\frac{-h_{22}}{h_{21}}V_2 + \frac{I_2}{h_{21}}\right] + h_{12}V_2 \Rightarrow \left[\frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}\right]V_2 + \left[\frac{h_{11}}{h_{21}}\right]I_2$$

$$\therefore V_1 = \left[\frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}\right]V_2 - \left[-\frac{h_{11}}{h_{21}}\right]I_2 \longrightarrow ④$$

Compare eq^n ③ & ④ with ABCD parameter equations.

$$\boxed{A = \frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}} \quad B = \frac{-h_{11}}{h_{21}} \quad C = \frac{-h_{22}}{h_{21}} \quad D = \frac{-1}{h_{21}}}$$

4. a) h-parameters interms of Z-parameters.

WKT; h-parameter eqn's

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

Z-parameter equation,

$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow ①$

$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow ②$

From eqn ②.

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$Z_{22} I_2 = V_2 - Z_{21} I_1$

$I_2 = \frac{V_2}{Z_{22}} - \frac{Z_{21}}{Z_{22}} I_1 \Rightarrow \left[-\frac{Z_{21}}{Z_{22}}\right] I_1 + \left[\frac{1}{Z_{22}}\right] V_2 \longrightarrow ③$

substitute eqn ③ in eqn ①.

$V_1 = Z_{11} I_1 + Z_{12} \left[-\frac{Z_{21}}{Z_{22}} I_1 + \frac{V_2}{Z_{22}}\right]$

$= Z_{11} I_1 - \frac{Z_{12} Z_{21}}{Z_{22}} I_1 + \frac{Z_{12}}{Z_{22}} V_2$

$V_1 = \left[\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{22}}\right] I_1 + \frac{Z_{12}}{Z_{22}} V_2 \longrightarrow ④$

From eqn ③ & ④. compare with the eq's of h-parameters.

$h_{11} = \frac{\Delta Z}{Z_{22}}$ $h_{12} = \frac{Z_{12}}{Z_{22}}$

$h_{21} = \frac{-Z_{21}}{Z_{22}}$ $h_{22} = \frac{1}{Z_{22}}$.

b) h-parameters interms of Y-parameters:

WKT, Y-parameter eqn's

$I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow ①$

$I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow ②$

H-parameter eqn's

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

From eqn ①

$Y_{11} V_1 = I_1 - Y_{12} V_2$

$V_1 = \frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2 \longrightarrow ③$

substitute eqn ③ in eqn ②.

$$I_2 = Y_{21}\left[\frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2\right] + Y_{22} V_2.$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + V_2 \left[\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}}\right] \longrightarrow (4).$$

from (3) & (4).

$$h_{11} = \frac{1}{Y_{11}} \qquad h_{12} = -\frac{Y_{12}}{Y_{11}}$$

$$h_{21} = \frac{Y_{21}}{Y_{11}} \qquad h_{22} = \frac{\Delta Y}{Y_{11}}$$

c) H-interms of ABCD parameters:

ABCD parameters — H-parameters.

$$V_1 = AV_2 - BI_2 \longrightarrow (1) \qquad V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_1 = CV_2 - DI_2 \longrightarrow (2). \qquad I_2 = h_{21} I_1 + h_{22} V_2.$$

from eq^n (2).

$$I_1 = CV_2 - DI_2$$

$$-DI_2 = I_1 - CV_2.$$

$$I_2 = \frac{1}{-D} I_1 + \frac{C}{D} V_2 \longrightarrow (3).$$

subshitute eq^n (3) in (1).

$$V_1 = AV_2 - B\left[-\frac{1}{D} I_1 + \frac{C}{D} V_2\right]$$

$$V_1 = AV_2 + \frac{B}{D} I_1 - \frac{BC}{D} V_2.$$

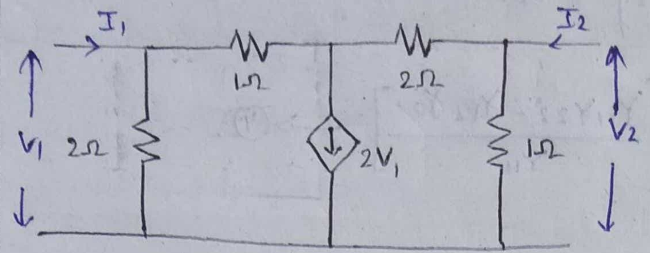
$$V_1 = \frac{B}{D} I_1 - \frac{AD - BC}{D} V_2. \longrightarrow (4)$$

Compare eq^n (3) & (4) with H-parameter equations.

$$H_{11} = \frac{B}{D} \qquad h_{12} = \frac{AD - BC}{D}$$

$$h_{21} = \frac{-1}{D} \qquad h_{22} = \frac{C}{D}.$$

Eg: (12) obtain ABCD parameters for the network shown below.



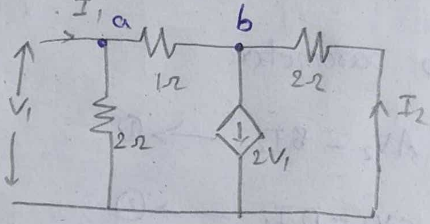
Let us slove Y-parameters for given network.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2.$$

(1) S.C port (2), i.e, $V_2 = 0$.

$$Y_{11} = \frac{I_1}{V_1} \quad \Big| \quad Y_{21} = \frac{I_2}{V_1}$$



Apply KCL at node (a)

$$I_1 = \frac{V_1}{2} + \frac{V_1 - V_x}{1}$$

$$I_1 = V_1\left(\frac{1}{2}+1\right) - V_x = V_1(1.5) - V_x. \longrightarrow (1)$$

Apply KCL at node (b)

$$2V_1 + \frac{V_x - V_1}{1} - I_2 = 0.$$

$$2V_1 + \frac{V_x - V_1}{1} - \frac{(-V_x)}{2} = 0.$$

$$2V_1 - V_1 + V_x\left[1 + \frac{1}{2}\right] = 0.$$

$$V_1 + 01.5V_x = 0.$$

$$\boxed{V_x = \frac{-V_1}{01.5}}$$ substitute in eq (1)

$$I_1 = V_1(1.5) - V_x \Rightarrow V_1 \times 1.5 - \left(\frac{-V_1}{01.5}\right)$$

$$I_1 \Rightarrow V_1\left[1.5 + \frac{1}{01.5}\right]$$

$$I_1 \Rightarrow V_1\,(+2.16)$$

$$\boxed{Y_{11} = \frac{I_1}{V_1} = +2.16\,\mho} \Rightarrow \boxed{\therefore Y_{11} = 2.16\,\mho}$$

$$\cancel{\therefore 2I_2 = -2V_1}$$

$$\cancel{\therefore \frac{I_2}{V_1} = -\frac{2}{2} = -1\,\mho}$$

$$\cancel{\boxed{Y_{21} = -1\,\mho}}$$

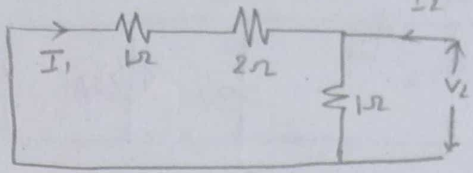
(or) $I_2 = \frac{-V_x}{2} = +\frac{V_1}{01.5 \times 2}$

$$\therefore \frac{I_2}{V_1} = +\frac{1}{3} \Rightarrow 0.33\,\mho$$

$$\boxed{Y_{21} = 0.33\,\mho}$$

(2) S.C the port (1) i.e., $V_1 = 0$.

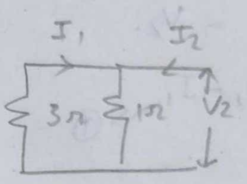
$Y_{22} = \frac{I_2}{V_2}$ | $Y_{12} = \frac{I_1}{V_2}$



$V_2 = I_2 (R_{eq}) \Rightarrow I_2 \left(3 // 1\right)$

$V_2 \Rightarrow I_2 \frac{3}{4}$

$\frac{I_2}{V_2} = \frac{4}{3} = 1.33 ℧$ $\boxed{Y_{22} = 1.33 ℧}$



$-3I_1 = V_2$

$\frac{I_1}{V_2} = -\frac{1}{3} \Rightarrow -0.33 ℧$

$\boxed{Y_{12} = -0.33 ℧}$

Now ABCD parameters are

$A = \frac{-Y_{22}}{Y_{21}} \Rightarrow \frac{-1.33}{+0.33} = -4.033$

$B = -\frac{1}{Y_{21}} = \frac{-1}{+0.33} = -3.03\Omega \Rightarrow -3.03\Omega$

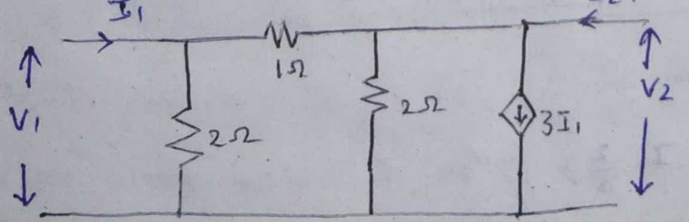
$C = \frac{-\Delta Y}{Y_{21}} \Rightarrow \frac{Y_{12} Y_{21} - Y_{11} Y_{22}}{Y_{21}} \Rightarrow \frac{(-0.33 \times 0.33) - (2.15 \times 1.33)}{+0.33}$

$\Rightarrow -9.0354$ mho's $\Rightarrow -9.0354 ℧$.

$D = -\frac{Y_{11}}{Y_{21}} \Rightarrow -\frac{2.15}{+0.33} = 6.54$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -40.33 & -3.03\Omega \\ -9.035 \text{ mho} & 6.54 \end{bmatrix}$$

Eg: (13) obtain H parameters for the given network.

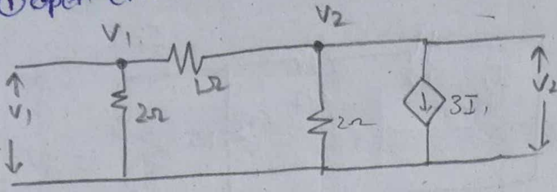


Sol: Let us find out Z-parameters for given network.

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$.

① open circuit the port ② i.e., $I_2 = 0$.



$Z_{11} = \frac{V_1}{I_1}$

$Z_{21} = \frac{V_2}{I_1}$

Apply kcl at node ①

$I_1 = \frac{V_1}{2} + \frac{V_1 - V_2}{1}$

$I_1 = V_1\left(\frac{1}{2}+1\right) - V_2$

$I_1 = 1.5V_1 - V_2$.

$V_2 = 1.5V_1 - I_1 \rightarrow ①$

Apply kcl at ②.

$\frac{V_2 - V_1}{1} + \frac{V_2}{2} + 3I_1 = 0$.

$1.5V_2 - V_1 + 3I_1 = 0$.

substitute eq^n ① in above eq^n

$1.5(1.5V_1 - I_1) - V_1 + 3I_1 = 0$.

$2.25V_1 - 1.5I_1 - V_1 + 3I_1 = 0$.

$1.25V_1 = -1.5I_1$

$\frac{I_1}{V_1} = -0.833℧$.

$\boxed{\therefore Y_{11} = -0.833℧}$

$\boxed{\frac{V_1}{I_1} = -1.2\Omega}$ $\boxed{Z_{11} = -1.2\Omega}$

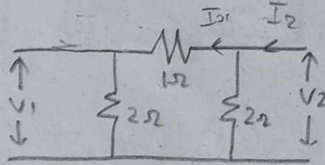
$\therefore V_1 = -1.2I_1$

$V_2 = 1.5V_1 - I_1 \Rightarrow V_2 = 1.5(-1.2I_1) - I_1$

$\frac{V_2}{I_1} = -1.8 - 1 \Rightarrow -2.8\Omega$

$\boxed{Z_{21} = -2.8\Omega}$

② o-c the port ① i.e., $I_2 = 0$.



$Z_{22} = \frac{V_2}{I_2}$

$Z_{12} = \frac{V_1}{I_2}$

$V_2 = I_2 R_{eq}$

$\frac{V_2}{I_2} = 3//2 \Rightarrow \frac{6}{5} = 1.2\Omega$

$\boxed{\therefore Z_{22} = 1.2\Omega}$

$I_x = I_2\left(\frac{2}{3+2}\right) = I_2 \frac{2}{5}$.

$\therefore V_1 = 2I_x \Rightarrow 2 \times \frac{2}{5} \times I_2$

$\frac{V_1}{I_2} = \frac{4}{5} \Rightarrow 0.8\Omega$

$\boxed{\therefore Z_{12} = 0.8\Omega}$

$$H_{11} = \frac{\Delta Z}{Z_{22}} = \frac{(-1.2)(1.2) - (-2.8)(0.8)}{1.2} \Rightarrow 0.667\Omega.$$

$$h_{12} = \frac{Z_{12}}{Z_{22}} = \frac{0.8}{1.2} = 0.667$$

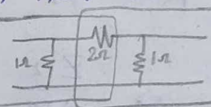
$$h_{21} = \frac{-Z_{21}}{Z_{22}} \Rightarrow \frac{-(-2.8)}{1.2} = 2.33$$

$$h_{22} = \frac{1}{Z_{22}} = 0.833\mho.$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 0.667\Omega & 0.667 \\ 2.33 & 0.833\mho \end{bmatrix}$$

Repsrocity (ci) and symmetry condition in Two-port network parameters:-

- A network is termed to be reciprocal, if the ratio of the response variable to the excitation variable remains same even if the positions of response and excitation are interchanged.
- A symmetrical network is defined as a network which can be divided into two halves with each half as a mirror image of other. (or)



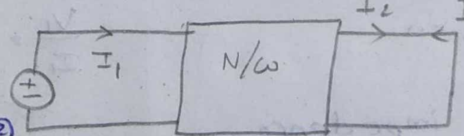
If the impedence measured at one port is equal to the impedence measured at the other port with remaining port is open circuited then network is said to be symmetrical.
(or) when the input port impedence is equal to output port impedence then it is called symmetrical. $(Z_{11} = Z_{22})$

①a. condition of reciprocity for Z-parameters:

The equation's of Z-parameters are.

$V_1 = Z_{11}I_1 + Z_{12}I_2 \rightarrow ①$

$V_2 = Z_{21}I_1 + Z_{22}I_2 \rightarrow ②$



$V_1 = V_1$; $V_2 = 0$; $I_2 = -I_2'$ [∴ from the fig]

substitute above values in eq^n ① & ②

$V_1 = Z_{11}I_1 - Z_{12}I_2'$

$0 = Z_{21}I_1 - Z_{22}I_2'$

$I_1 = \frac{Z_{22}I_2'}{Z_{21}} \longrightarrow ③$

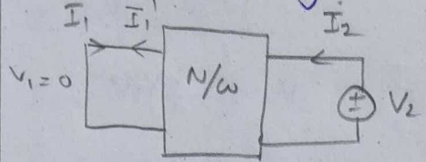
Substitute I_1 value.

$V_1 = Z_{11}\left[\frac{Z_{22}}{Z_{21}}I_2' - Z_{12}I_2'\right]$

$$V_1 = I_2'\left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}\right]$$

$$\boxed{\therefore \frac{I_2'}{V_1} = \frac{Z_{21}}{\Delta Z}} \longrightarrow (4)$$

Now interchange response and excitation positions.



From the fig: $V_2 = V_2$, $V_1 = 0$, $I_1 = -I_1'$

Substitute the values in eq^n (1) & (2).

$$0 = -Z_{11}I_1' + Z_{12}I_2.$$

$$I_2 = \frac{Z_{11}I_1'}{Z_{12}}$$

$$V_2 = -Z_{21}I_1' + Z_{22}I_2$$

$$V_2 = -Z_{21}I_1' + Z_{22}\left[\frac{Z_{11}I_1'}{Z_{12}}\right]$$

$$V_2 = \left[\frac{-Z_{12}Z_{21} + Z_{11}Z_{22}}{Z_{12}}\right]I_1'$$

$$\boxed{\therefore \frac{I_1'}{V_2} = \frac{Z_{12}}{\Delta Z}} \longrightarrow (5)$$

For condition of reciprocity.

$$\frac{I_1'}{V_2} = \frac{I_2'}{V_1} \Rightarrow \frac{Z_{12}}{\cancel{\Delta Z}} = \frac{Z_{21}}{\cancel{\Delta Z}} \Rightarrow \boxed{Z_{12} = Z_{21}}$$

(1) b, :. symmetry condition for Z-parameters:

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \longrightarrow (1)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2. \longrightarrow (2).$$

(1) i/p port impedence.

$$\left.\frac{V_1}{I_1}\right|_{I_2 = 0}$$

from eq^n (1) $V_1 = Z_{11}I_1$

$$\frac{V_1}{I_1} = Z_{11}. \longrightarrow (3)$$

(2) O/P port impedence.

$$\left.\frac{V_2}{I_2}\right|_{I_1 = 0}$$

From eq^n (2) $V_2 = Z_{22}I_2$

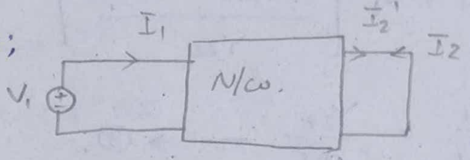
$$\frac{V_2}{I_2} = Z_{22} \longrightarrow (4)$$

from eq^n (3) & (4).

$$\frac{V_1}{I_1} = \frac{V_2}{I_2} \Rightarrow \boxed{Z_{11} = Z_{22}}$$

② a, Reciprocity condition for Y-parameters:

The equations of Y parameters;



$I_1 = Y_{11}V_1 + Y_{12}V_2 \longrightarrow$ ①

$I_2 = Y_{21}V_1 + Y_{22}V_2 \longrightarrow$ ②

From the fig; $V_2 = 0$, $I_2 = -I_2'$, $V_1 = V_1$

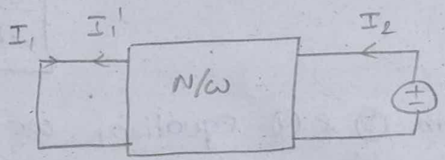
Substitute above values in eqn's ② only.

$$-I_2' = Y_{21}V_1 + 0.$$

$$\boxed{\frac{I_2'}{V_1} = -Y_{21}} \Rightarrow ③.$$

Now interchange the response and excitation positions.

From the fig: $V_1 = 0$, $I_1 = -I_1'$,
$V_2 = V_2$.



substitute above values in eqn ①.

$$[-I_1'] = 0 + Y_{12}V_2$$

$$\boxed{\frac{I_1'}{V_2} = -Y_{12}} \longrightarrow ④.$$

From ③ & ④ reciprocity condition.

$$\frac{I_2'}{V_1} = \frac{I_1'}{V_2} \Rightarrow \cancel{-}Y_{21} = \cancel{-}Y_{12}$$

Condition for reciprocity $\therefore$ $\boxed{Y_{21} = Y_{12}}$

② b, symmetry condition For Y parameters:

Y eqn's ; $I_1 = Y_{11}V_1 + Y_{12}V_2 \longrightarrow$ ①

$I_2 = Y_{21}V_1 + Y_{22}V_2 \longrightarrow$ ②.

① i/p port impedence.

$$\left.\frac{V_1}{I_1}\right| I_2 = 0.$$

substitute $I_2 = 0$ in eq ②.

$$0 = Y_{21}V_1 + Y_{22}V_2.$$

$$\therefore V_2 = \frac{-Y_{21}}{Y_{22}}V_1$$

substitute V_2 in eqn ① $I_1 = Y_{11}V_1 + Y_{12}\left[-\frac{Y_{21}}{Y_{22}}V_1\right]$

$$I_1 = V_1\left[\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{22}}\right]$$

$$\frac{V_1}{I_1} = \frac{Y_{22}}{\Delta Y} \longrightarrow (3).$$

(2) Now, o/p port Impedence $\left.\frac{V_2}{I_2}\right|_{I_1=0}$.

substitute $I_1=0$ in eq^n (1).

$$0 = Y_{11}V_1 + Y_{12}V_2$$

$$\therefore V_1 = -\frac{Y_{12}}{Y_{11}}V_2$$

$$I_2 = Y_{21}\left[-\frac{Y_{12}}{Y_{11}}V_2\right] + Y_{22}V_2$$

$$I_2 = \left[\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}}\right]V_2$$

$$\boxed{\frac{V_2}{I_2} = \frac{Y_{11}}{\Delta Y}} \longrightarrow (4)$$

From (3) & (4) equations we get symmetry condition

$$\frac{V_1}{I_1} = \frac{V_2}{I_2} \Rightarrow \frac{Y_{22}}{\Delta Y} = \frac{Y_{11}}{\Delta Y}$$

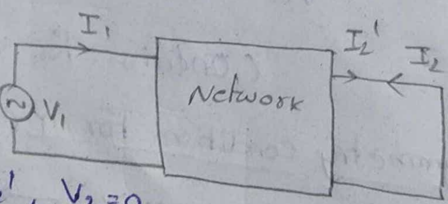
$$\boxed{Y_{22} = Y_{11}} \longrightarrow \text{symmetry condition.}$$

(3) a(ABCD parameters) reciprocity Conditions for ABCD parameters:

The equations of ABCD parameters;

$$V_1 = AV_2 - BI_2 . \longrightarrow (1)$$

$$I_1 = CV_2 - DI_2 . \longrightarrow (2).$$



From the fig: $V_1 = V_1$, $I_2 = -I_2'$, $V_2 = 0$.

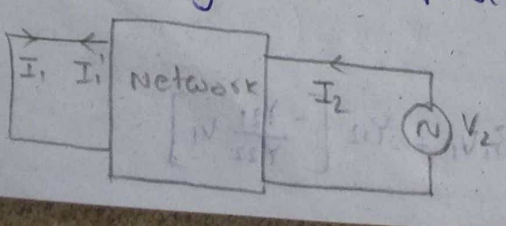
Substitute the above values in the eq^n (1) & (2).

$$V_1 = A(0) - B(-I_2')$$

$$V_1 = BI_2'$$

$$\boxed{\frac{I_2'}{V_1} = \frac{1}{B}} \longrightarrow (3)$$

Now interchange the response and excitation positions.



From the fig: $V_2 = V_2$, $V_1 = 0$,

$I_1' = -I_1'$

Substitute the above values in eq^n ②

$$-I_1' = CV_2 - DI_2 \longrightarrow ④$$

substitute the above values in eq^n ①.

$$0 = AV_2 - BI_2$$

$$\therefore I_2 = \frac{AV_2}{B}$$

substitute the value of I_2 in eq^n ④.

$$-I_1' = CV_2 - \left[\frac{AV_2}{B} \times D\right]$$

$$-I_1' = CV_2 - \frac{AD\,V_2}{B}$$

$$-I_1' = \left[\frac{BC - AD}{B}\right] V_2$$

$$\boxed{\frac{I_1'}{V_2} = \frac{AD - BC}{B}} \longrightarrow ⑤.$$

equate the equations ③ & ⑤ to get reciprocity condition.

$$\frac{I_2'}{V_1} = \frac{I_1'}{V_2}$$

$$\frac{1}{B} = \frac{AD - BC}{B}$$

$\boxed{\therefore AD - BC = 1}$ Condition for reciprocity.

③ b, symmetry condition for ABCD parameters:

ABCD parameter equations are; $V_1 = AV_2 - BI_2 \longrightarrow ①$

$I_1 = CV_2 - DI_2 \longrightarrow ②$

① i/p port impedence.

$$\left.\frac{V_1}{I_1}\right|_{I_2 = 0.}$$

from eq^n ①

$$V_1 = AV_2 - 0$$

$$\therefore V_2 = \frac{V_1}{A}$$

from eq^n ②

$$I_1 = C\left(\frac{V_1}{A}\right) - 0.$$

$$\boxed{\therefore \frac{V_1}{I_1} = \frac{A}{C}} \longrightarrow ③.$$

② o/p port impedence.

$$\left.\frac{V_2}{I_2}\right|_{I_1 = 0.}$$

from eq^n ②.

$0 = CV_2 - DI_2.$

$CV_2 = DI_2$

$$\boxed{\frac{V_2}{I_2} = \frac{D}{C}} \longrightarrow ④.$$

∴ from ③ & ④ equations.

$$\frac{V_1}{I_1} = \frac{V_2}{I_2}$$

$$\frac{A}{C} = \frac{D}{C}$$

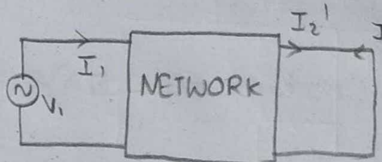
$\boxed{\therefore A = D}$ is the condition for symmetricity network

④ a, Reciprocity condition for h-parameters:

The equations of h parameters;

$V_1 = h_{11} I_1 + h_{12} V_2 \longrightarrow ①$

$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow ②.$



from fig; $V_2 = 0,\ V_1 = V_1,\ I_2 = -I_2'$

Substitute the above values in eq^n ① & ②.

$V_1 = h_{11} I_1 + 0.$

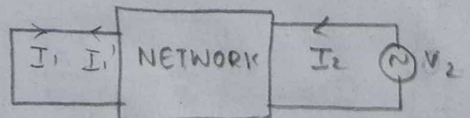
$V_1 = h_{11}\left[\frac{-I_2'}{h_{21}}\right]$

$-I_2' = h_{21} I_1$

$I_1 = \left[\frac{-I_2'}{h_{21}}\right]$

$$\boxed{\therefore \frac{I_2'}{V_1} = -\frac{h_{21}}{h_{11}}} \longrightarrow ③.$$

Now replace the response and excitation positions.



from fig: $V_1 = 0,\ I_1 = -I_1',\ V_2 = V_2$

Substitute the above values in eq^n ①.

$0 = h_{11}(-I_1') + h_{12} V_2$

$h_{11} I_1' = h_{12} V_2$

$$\boxed{\therefore \frac{I_1'}{V_2} = \frac{h_{12}}{h_{11}}} \longrightarrow ④$$

for reciprocity condition equate ③ & ④ equations.

$$\frac{I_2'}{V_1} = \frac{I_1'}{V_2}$$

$$\frac{-h_{21}}{h_{11}} = \frac{h_{12}}{h_{11}}$$

$\boxed{\therefore h_{12} = -h_{21}}$ is the condition of reciprocity.

④ b. symmetry condition for h-parameters:

The equations of h-parameters; $V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow ①$

$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow ②$.

① i/p port impedence.

$$\frac{V_1}{I_1}\Big|_{I_2=0}$$

from eq^n ①.

$V_1 = h_{11} I_1 + h_{12} V_2$

$V_1 = h_{11} I_1 + h_{12}\left[\frac{-h_{21} I_1}{h_{22}}\right]$

from eq^n ②

$0 = h_{21} I_1 + h_{22} V_2$

$V_2 = \frac{-h_{21} I_1}{h_{22}}$

$\therefore V_1 = \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right] I_1$

$$\boxed{\frac{V_1}{I_1} = \frac{\Delta h}{h_{22}}} \longrightarrow ③$$

② o/p port impedence.

$$\frac{V_2}{I_2}\Big|_{I_1=0}.$$

from eq^n ②.

$I_2 = h_{21}(0) + h_{22} V_2$

$I_2 = h_{22} V_2$

$$\boxed{\frac{V_2}{I_2} = \frac{1}{h_{22}}} \longrightarrow ④.$$

from eq^n's ③ & ④.

$$\frac{V_1}{I_1} = \frac{V_2}{I_2}$$

$$\frac{1}{h_{22}} = \frac{\Delta h}{h_{22}}.$$

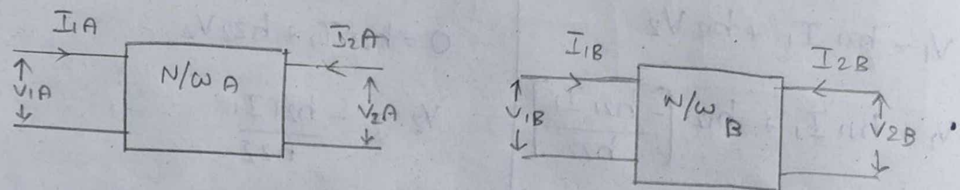
$\boxed{\therefore \Delta h = 1}$ is the condition for symmetry.

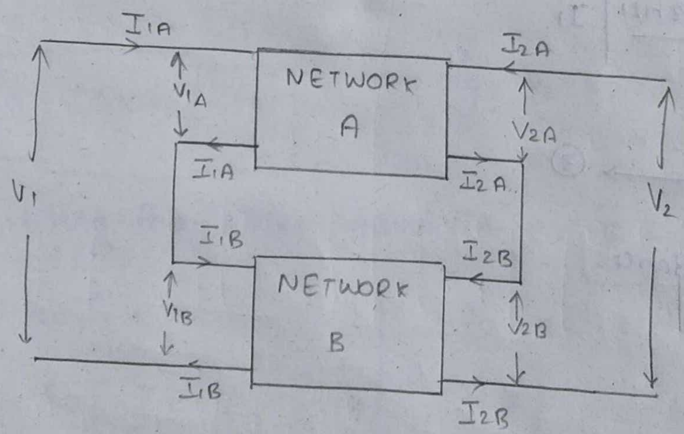
	Reciprocity condition	Symmetry condition.
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
ABCD	$AD - BC = 1$	$A = D$
h	$h_{12} = -h_{21}$	$\Delta h = 1$

Inconnection of Two-port Networks:

1. Series connection
2. parallel connection.
3. Cascade connection.

Series connection:—





From the above network:

$\therefore I_1 = I_{1A} = I_{1B}$

$I_2 = I_{2A} = I_{2B}$

$\therefore V_1 = V_{1A} + V_{1B}$

$V_2 = V_{2A} + V_{2B}$

The open circuit or Z-parameters are highly preferable in characterizing the series connected two port network.

for n/w (A) Z parameter equations are:-

$$V_{1A} = Z_{11A} I_{1A} + Z_{12A} I_{2A}$$

$$V_{2A} = Z_{21A} I_{1A} + Z_{22A} I_{2A}.$$

For n/w (B) Z parameter equation are.

$$V_{1B} = Z_{11B} I_{1B} + Z_{12B} I_{2B}$$

$$V_{2B} = Z_{21B} I_{1B} + Z_{22B} I_{2B}.$$

$$\therefore V_1 = V_{1A} + V_{1B} \Rightarrow Z_{11A} I_1 + Z_{12A} I_2 + Z_{11B} I_1 + Z_{12B} I_2$$

$$\therefore V_1 = I_1 (Z_{11A} + Z_{11B}) + I_2 [Z_{12A} + Z_{12B}]$$

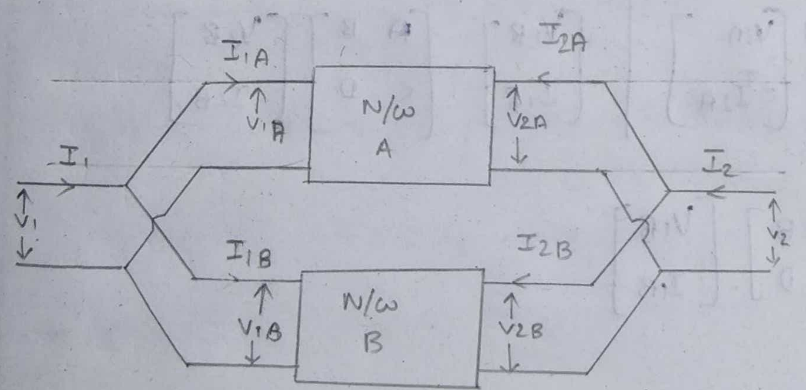
$$\therefore V_2 = V_{2A} + V_{2B} \Rightarrow Z_{21A} I_1 + Z_{22A} I_2 + Z_{21B} I_1 + Z_{22B} I_2$$

$$\therefore V_2 = I_1 (Z_{21A} + Z_{21B}) + I_2 (Z_{22A} + Z_{22B})$$

overall Z-parameters;

$$\underbrace{\begin{bmatrix} Z_{11A} & Z_{12A} \\ Z_{21A} & Z_{22A} \end{bmatrix}}_{\text{for N/w (A)}} + \underbrace{\begin{bmatrix} Z_{11B} & Z_{12B} \\ Z_{21B} & Z_{22B} \end{bmatrix}}_{\text{For N/w (B)}} = \underbrace{\begin{bmatrix} Z_{11A} + Z_{11B}. & Z_{12A} + Z_{12B} \\ Z_{21A} + Z_{21B} & Z_{22A} + Z_{22B} \end{bmatrix}}_{\text{for overall network.}}$$

Parallel connection:



$$V_1 = V_{1A} = V_{1B}$$

$$V_2 = V_{2A} = V_{2B}$$

$$I_1 = I_{1A} + I_{1B}$$

$$I_2 = I_{2A} + I_{2B}.$$

The Y parameters are highly useful to characterize parallel connected two port network.

For network (A) Y parameter eqn's.

$$I_{1A} = Y_{11A} V_{1A} + Y_{12A} V_{2A}$$

$$I_{2A} = Y_{21A} V_{1A} + Y_{22A} V_{2A}$$

For network (B) Y-parameter eqn's

$$I_{1B} = Y_{11B} V_{1B} + Y_{12B} V_{2B}$$

$$I_{2B} = Y_{21B} V_{1B} + Y_{22B} V_{2B}.$$

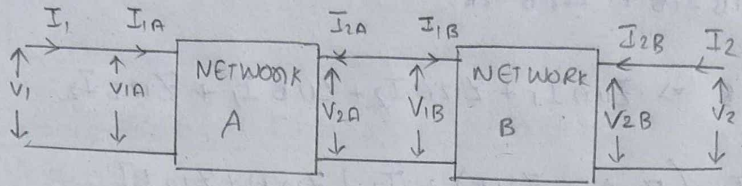
$$I_1 = I_{1A} + I_{1B} \Rightarrow Y_{11A} V_1 + Y_{12A} V_2 + Y_{11B} V_1 + Y_{12B} V_2$$

$$\therefore I_1 \Rightarrow V_1 [Y_{11A} + Y_{11B}] + V_2 [Y_{12A} + Y_{12B}]$$

similarly, $I_2 = V_1\left[Y_{21A} + Y_{21B}\right] + V_2\left[Y_{22A} + Y_{22B}\right]$

overall Y - parameters $\begin{bmatrix} Y_{11A} + Y_{11B} & Y_{12A} + Y_{12B} \\ Y_{21A} + Y_{21B} & Y_{22A} + Y_{22B} \end{bmatrix}$

- Cascade connection:



From above network; $V_1 = V_{1A}$ | $I_1 = I_{1A}$

$V_{2A} = V_{1B}$ | $I_{2A} = -I_{1B}$ (or) $I_{1B} = -I_{2A}$

$V_{2B} = V_2$ | $I_{2B} = I_2$

ABCD parameters are highly useful in characterizing the cascade connected two port network.

For Network (A), ABCD eq^n's

$V_{1A} = A V_{2A} - B I_{2A}$

$I_{1A} = C V_{2A} - D I_{2A}$

$$\begin{bmatrix} V_{1A} \\ I_{1A} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{2A} \\ -I_{2A} \end{bmatrix}$$

For n/w - B, ABCD eq^n's.

$V_{1B} = A V_{2B} - B I_{2B}$

$I_{1B} = C V_{2B} - D I_{2B}$.

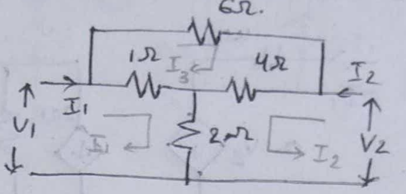
$$\begin{bmatrix} V_{1B} \\ I_{1B} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{2B} \\ -I_{2B} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{1B} \\ I_{1B} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{2B} \\ -I_{2B} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Eg: ① obtain H-parameters for the given network.



Sol: Apply kvl to loop ①.

$V_1 = (I_1 - I_3)1 + 2[I_1 + I_2]$

$V_1 \Rightarrow 3I_1 + 2I_2 - I_3 \longrightarrow ①.$

Apply kvl to loop ②:-

$V_2 = 4[I_2 + I_3] + 2[I_2 + I_1]$

$V_2 = 2I_1 + 6I_2 + 4I_3 \longrightarrow ②.$

Apply kvl to loop ③:

$6I_3 + 4[I_3 + I_2] + 1[I_3 - I_1] = 0.$

$-I_1 + 4I_2 + 10I_3 + 1I_3 = 0.$

$11I_3 = I_1 - 4I_2$

$I_3 = 0.09I_1 - 0.36I_2.$

substitute I_3 in eq^n ① & ②.

$V_1 = 3I_1 + 2I_2 - [0.09I_1 - 0.36I_2]$

$V_1 = 2.91I_1 + 2.36I_2 \longrightarrow ③$

$V_2 = 2I_1 + 6I_2 + 4[0.09I_1 - 0.36I_2]$

$V_2 = 2.36I_1 + 4.56I_2 \longrightarrow ④.$

from ③ & ④

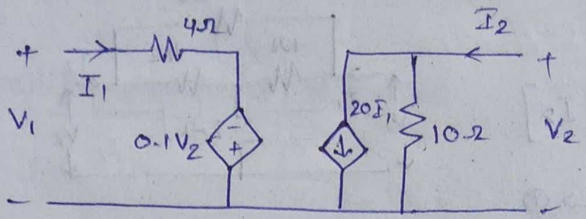
$$\underbrace{\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2.91\Omega & 2.36\Omega \\ 2.36\Omega & 4.56\Omega \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}_{\text{Z-parameter equations,}}$$

H parameters equations;

$h_{11} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}} = \frac{2.91 \times 4.56 - 2.36^2}{4.56} \Rightarrow 1.68\Omega.$

$h_{12} = \frac{Z_{12}}{Z_{22}} = \frac{2.36}{4.56} \Rightarrow 0.517$; $h_{21} = \frac{-Z_{21}}{Z_{22}} \Rightarrow \frac{-2.36}{4.56} \Rightarrow -0.517$; $h_{22} = \frac{1}{Z_{22}} = \frac{1}{4.56} \Rightarrow 0.21\mho$

Eg ① Determine the Y- parameters for the ckt shown in fig.



Sol: WKT, h parameter equations are,

$V_1 = h_{11}I_1 + h_{12}V_2 \longrightarrow ①$ $\quad I_2 = h_{21}I_1 + h_{22}V_2 \longrightarrow ②$

Given circuit is an equivalent circuit of hybrid parameters.

Apply KVL to 1st loop.

$V_1 = 4I_1 - 0.1V_2 \longrightarrow ③$

Apply KCL to 2nd half

$I_2 = \frac{V_2}{10} + 20I_1$.

$I_2 = 20I_1 + 0.1V_2 \longrightarrow ④$

Compare ③ & ④ with ① & ②.

$$h \text{ parameters} = \begin{bmatrix} 4\Omega & -0.1 \\ 20 & 0.1℧ \end{bmatrix}$$

Now Y-parameters in terms of H parameters.

$Y_{11} = \frac{1}{h_{11}} = \frac{1}{4} = 0.25℧$.

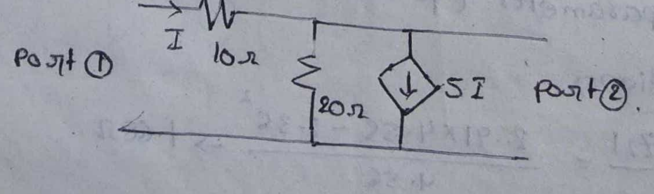
$Y_{12} = \frac{-h_{12}}{h_{11}} = \frac{0.1}{4} \Rightarrow 0.025℧$.

$Y_{21} = \frac{h_{21}}{h_{11}} = \frac{20}{4} \Rightarrow 5℧$.

$Y_{22} = \frac{\Delta h}{h_{11}} \Rightarrow \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}} \Rightarrow \frac{4\times0.1 + 0.1\times20}{4} \Rightarrow 0.6℧$.

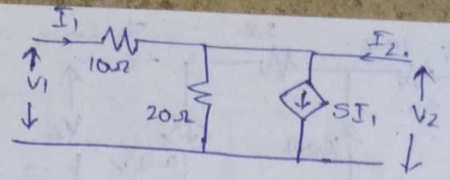
$$Y\text{-parameters are} = \begin{bmatrix} 0.25℧ & 0.025℧ \\ 5℧ & 0.6℧ \end{bmatrix}$$

Eg:
2) Determine the impedence parameters for the the network shown in fig.



Z parameter eq^n's. $V_1 = Z_{11} I_1 + Z_{12} I_2$

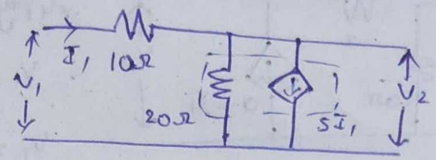
$V_2 = Z_{21} I_1 + Z_{22} I_2$



open circuit the port ② i.e., $I_2 = 0$.

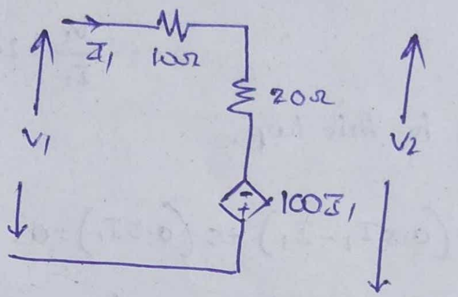
$Z_{11} = \frac{V_1}{I_1}$, $Z_{21} = \frac{V_2}{I_1}$

Apply KVL to loop.



$V_1 = 10I_1 + 20I_1 - 100I_1$

$V_1 = -70I_1$



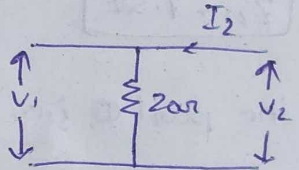
$\therefore \frac{V_1}{I_1} = \boxed{Z_{11} = -70\Omega}$

$V_2 = 20I_1 - 100I_1$

$\therefore \frac{V_2}{I_1} = \boxed{Z_{21} = -80\Omega}$

open circuit the port ① i.e., $I_1 = 0$.

$Z_{12} = \frac{V_1}{I_2}$, $Z_{22} = \frac{V_2}{I_2}$.



$\therefore V_2 = 20I_2$

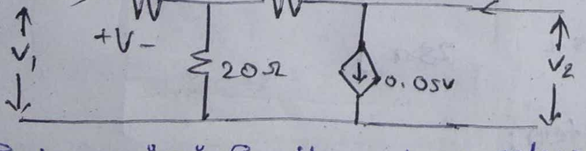
$\frac{V_2}{I_2} = 20\Omega \Rightarrow \boxed{Z_{22} = 20\Omega}$

$\therefore V_1 = 20I_2$

$\frac{V_1}{I_2} = 20\Omega \Rightarrow \boxed{Z_{12} = 20\Omega}$

Z parameter equations are $\begin{bmatrix} -70\Omega & 20\Omega \\ -80\Omega & 20\Omega \end{bmatrix}$

③ Determine H-parameters for the network shown in figure.



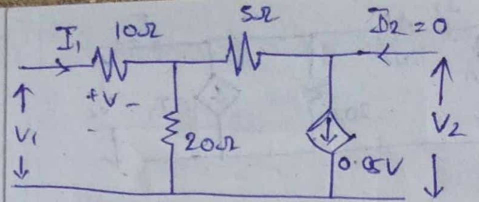
Let us find out "Z" for the above network

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$.

① open ckt port ② $I_2 = 0$.

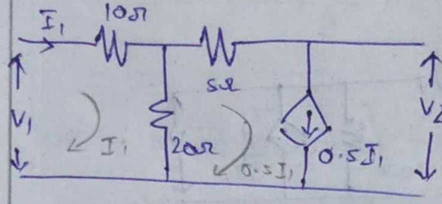
$Z_{11} = \frac{V_1}{I_1}$, $Z_{21} = \frac{V_2}{I_1}$



$V = 10 I_1$

$0.05V = 0.05(10 I_1)$

$= 0.5 I_1$



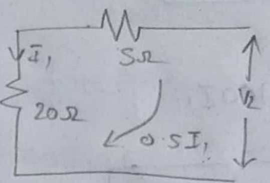
Apply KVL to loop ①.

$V_1 = 10 I_1 + 20(I_1 - 0.5 I_1)$

$V_1 = 10 I_1 + 20 I_1 - [10 I_1]$

$\frac{V_1}{I_1} = 20\Omega$ $\boxed{\therefore Z_{11} = 20\Omega}$

Apply KVL to this loop.



$V_2 + 20(0.5 I_1 - I_1) + 5(0.5 I_1) = 0.$

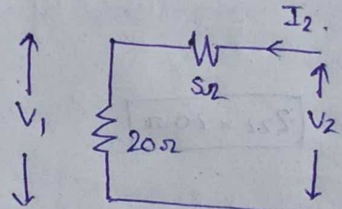
$V_2 = -10 I_1 + 20 I_1 - 2.5 I_1$

$V_2 = 7.5 I_1$

$\frac{V_2}{I_1} = 7.5\Omega$ $\boxed{Z_{21} = 7.5\Omega}$

② open circuit the port ① i.e, $I_1 = 0$.

$Z_{22} = \frac{V_2}{I_2}$; $Z_{12} = \frac{V_1}{I_2}$.



$\therefore V_2 = 5 I_2 + 20 I_2$

$\therefore \frac{V_2}{I_2} = \boxed{Z_{22} = 25\Omega}$

$\therefore V_1 = 20 I_2$

$\therefore \frac{V_1}{I_2} = 20\Omega$ $\boxed{\therefore Z_{12} = 20\Omega}$

Z parameters $= \begin{bmatrix} 20\Omega & 20\Omega \\ 7.5\Omega & 25\Omega \end{bmatrix}$

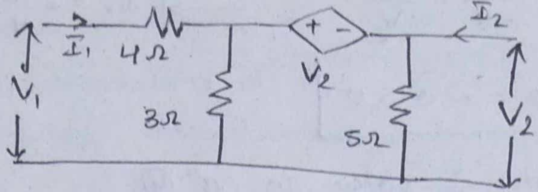
h interms of Z parameters;

$h_{11} = \frac{\Delta Z}{Z_{22}} = 14\Omega$ $h_{21} = -\frac{Z_{21}}{Z_{22}} = -0.3$

$h_{12} = \frac{Z_{12}}{Z_{22}} = 0.8$ $h_{22} = \frac{1}{Z_{22}} \Rightarrow 0.04 \mho$

④ Find Y & Z parameters for given network.

Y & Z parameters.

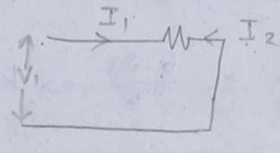


Sol: let us

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

① short circuit the port ② i.e., $V_2 = 0$.



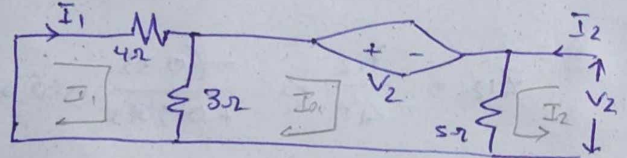
$V_1 = 4I_1$ $\qquad$ $V_1 = 4(-I_2)$

$\frac{I_1}{V_1} = \frac{1}{4} = Y_{11}$ $\qquad$ $\frac{I_2}{V_1} = -\frac{1}{4} = Y_{21}$

$\boxed{\therefore Y_{11} = 0.25\,\mho}$ $\qquad$ $\boxed{\therefore Y_{21} = -0.25\,\mho}$

② S.C the port ① i.e., $V_1 = 0$.

$Y_{12} = \frac{I_1}{V_2}$ $\qquad$ $Y_{22} = \frac{I_2}{V_2}$.



Apply KVL to loop ①

$4I_1 + 3(I_1 - I_x) = 0.$

$7I_1 - 3I_x = 0$

$I_x = \frac{7}{3}I_1 \rightarrow ①$

Apply KVL to loop ②

$V_2 = 5(I_2 + I_x)$

$V_2 = 5I_2 + 5I_x. \rightarrow ②$

Apply KVL to loop ③

$3(I_x - I_1) + V_2 + 5(I_x + I_2) = 0.$

$8I_x - 3I_1 + 5I_2 + V_2 = 0. \rightarrow ③$

$8I_x = 3I_1 - V_2 - 5I_2.$

$I_x = \frac{3}{8}I_1 - \frac{5}{8}I_2 - \frac{V_2}{8}$

substitute eqⁿ ① in eqⁿ ③.

$8\left(\frac{7}{3}\right)I_1 = 3I_1 - V_2 - 5I_2$

$\frac{56}{3}I_1 - 3I_1 + 5I_2 = -V_2.$

$18.66I_1 - 3I_1 + 5I_2 = -V_2.$

$I_1(15.66) + 5I_2 = -V_2 \rightarrow ④$

substitute eqⁿ ① in eqⁿ ②.

$V_2 = 5I_2 + 5\left(\frac{7}{3}I_1\right)$

$V_2 = 11.66I_1 + 5I_2. \rightarrow ⑤$

subtract ⑤ − ④

$[V_2 = 11.66I_1 + 5I_2] - [-V_2 = 15.66I_1 + 5I_2]$

$2V_2 = -4I_1$

$$\frac{2}{-4} = \frac{I_1}{V_2} \longrightarrow \left[I_1 = \frac{2V_2}{-4} \right]$$

$$\boxed{\therefore Y_{12} = -0.5\mho}$$

Substitute I_1 value in eqn ④

$$15.66\left(\frac{2V_2}{-4}\right) + 5I_2 = -V_2$$

$$-7.83V_2 + 5I_2 = -V_2$$

$$5I_2 = -V_2 + 7.83V_2$$

$$\frac{I_2}{V_2} = \frac{6.83}{5} \Rightarrow 1.366\mho$$

$$\boxed{\therefore Y_{22} = 1.366\mho}$$

Y parameters are = $\begin{bmatrix} 0.25\mho & -0.5\mho \\ -0.25\mho & 1.366\mho \end{bmatrix}$

$\therefore$ Z parameters interms of Y parameters are

$$Z_{11} = \frac{Y_{22}}{\Delta Y} \Rightarrow \frac{1.366}{+0.2168s} \Rightarrow +6.32\Omega \qquad Z_{12} = -\frac{Y_{12}}{\Delta Y} \Rightarrow \frac{-(0.5)}{+0.2168s} \Rightarrow 2.314\Omega$$

$$Z_{21} = \frac{-Y_{21}}{\Delta Y} \Rightarrow \frac{-(-0.25)}{+0.2168s} \Rightarrow 1.157\Omega \qquad Z_{22} = \frac{Y_{11}}{\Delta Y} \Rightarrow \frac{0.25}{+0.2168s} \Rightarrow +1.157\Omega$$

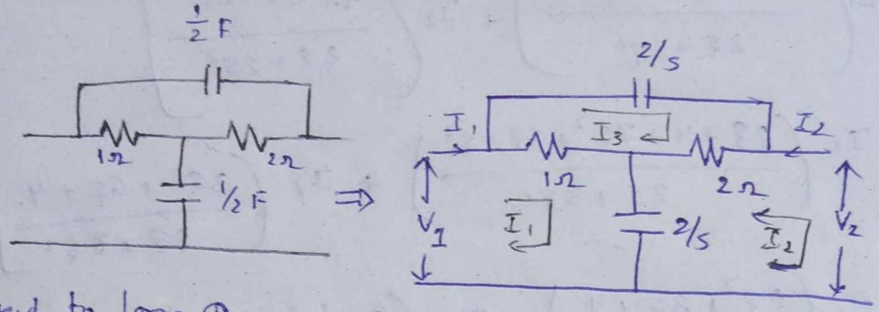
$\therefore$ Where $\Delta Y \Rightarrow \boxed{Y_{11}Y_{22} - Y_{12}Y_{21}}$

$$\Delta Y \Rightarrow 0.25 \times 1.366 - 0.5 \times 0.25$$

$$\Rightarrow +0.2168s$$

Z parameters are = $\begin{bmatrix} 6.32\Omega & 2.314\Omega \\ 1.157\Omega & 1.157\Omega \end{bmatrix}$

Eg: Find the open circuit $\left[\therefore \left(\frac{1}{2}F\right) \Rightarrow \frac{1}{Cs} \Rightarrow \frac{1}{\frac{1}{2}s} = \frac{2}{s}.\right]$



Sol: Apply KVL to loop ①.

$$1(I_1 - I_3) + \frac{2}{s}\left(I_1 + I_2\right] = V_1$$

$$I_1\left(1+\frac{2}{s}\right) + \frac{2}{s} I_2 - I_3 = V_1$$

$$V_1 = I_1\left(\frac{s+2}{s}\right) + I_2\left(\frac{2}{s}\right) - I_3 \longrightarrow ①.$$

Apply KVL to loop ②:

$$2(I_2 + I_3) + \frac{2}{s}(I_2 + I_1) = V_2$$

$$V_2 = \frac{2}{s} I_1 + I_2\left(2 + \frac{2}{s}\right) + 2I_3$$

$$V_2 = \frac{2}{s} I_1 + I_2\left(\frac{2s+2}{s}\right) + 2I_3 \longrightarrow ②.$$

Apply KVL to loop ③:

$$\frac{2}{s} I_3 + \left(1\left[I_3 - I_1\right]\right) + 2(I_3 + I_2) = 0.$$

$$-I_1 + 2I_2 + I_3\left(\frac{2}{s} + 1 + 2\right) = 0.$$

$$-I_1 + 2I_2 + I_3\left(\frac{2+3s}{s}\right) = 0.$$

$$I_3\left(\frac{2+3s}{s}\right) = I_1 - 2I_2.$$

$$I_3 = I_1\left(\frac{s}{2+3s}\right) - I_2\left(\frac{2s}{2+3s}\right) \longrightarrow ③.$$

sub eqⁿ ③ in ① & ②:—

$$V_1 = I_1\left(\frac{s+2}{s}\right) + I_2\left(\frac{2}{s}\right) - I_1\left(\frac{s}{2+3s}\right) + I_2\left(\frac{2s}{2+3s}\right)$$

$$= I_1\left(\frac{s+2}{s} - \frac{s}{2+3s}\right) + I_2\left(\frac{2}{s} + \frac{2s}{2+3s}\right).$$

$$= I_1\left(\frac{(s+2)(2+3s)-s^2}{2s+3s^2}\right) + I_2\left(\frac{4+6s+2s^2}{2s+3s^2}\right)$$

$$= I_1\left(\frac{2s+4+3s^2+6s-s^2}{2s+3s^2}\right) + I_2\left(\frac{2s^2+6s+4}{2s+3s^2}\right)$$

$$V_1 = I_1\left(\frac{2s^2+8s+4}{2s+3s^2}\right) + I_2\left(\frac{2s^2+6s+4}{2s+3s^2}\right) \longrightarrow (4)$$

from eq^n (2)

$$V_2 = \frac{2}{s} I_1 + I_2\left(\frac{2s+2}{s}\right) + 2\left[I_1\left(\frac{s}{2+3s}\right) - I_2\left(\frac{2s}{2+3s}\right)\right]$$

$$V_2 \Rightarrow \frac{2}{s} I_1 + I_2\left(\frac{2s+2}{s}\right) + I_1\left(\frac{2s}{2+3s}\right) - I_2\left(\frac{4s}{2+3s}\right)$$

$$V_2 = I_1\left[\frac{2}{s} + \frac{2s}{2+3s}\right] + I_2\left[\frac{2s+2}{s} - \frac{4s}{2+3s}\right]$$

$$V_2 = I_1\left[\frac{4+6s+2s^2}{3s^2+2s}\right] + I_2\left[\frac{4s+6s^2+4+6s-4s^2}{3s^2+2s}\right]$$

$$V_2 = I_1\left[\frac{2s^2+6s+4}{3s^2+2s}\right] + I_2\left[\frac{2s^2+10s+4}{3s^2+2s}\right] \longrightarrow (5).$$

Compare eq^n (4) & (5) with Z-parameter equations.

$$V_1 = Z_{11} I_1 + Z_{12} I_2 ; \quad V_2 = Z_{21} I_1 + Z_{22} I_2 .$$

$$Z_{11} = \frac{2s^2+8s+4}{3s^2+2s}, \quad Z_{12} = \frac{2s^2+6s+4}{3s^2+2s}$$

$$Z_{21} = \frac{2s^2+6s+4}{3s^2+2s}, \quad Z_{22} = \frac{2s^2+10s+4}{3s^2+2s}.$$