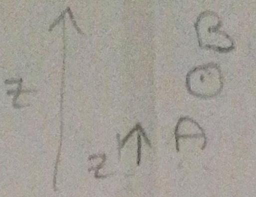
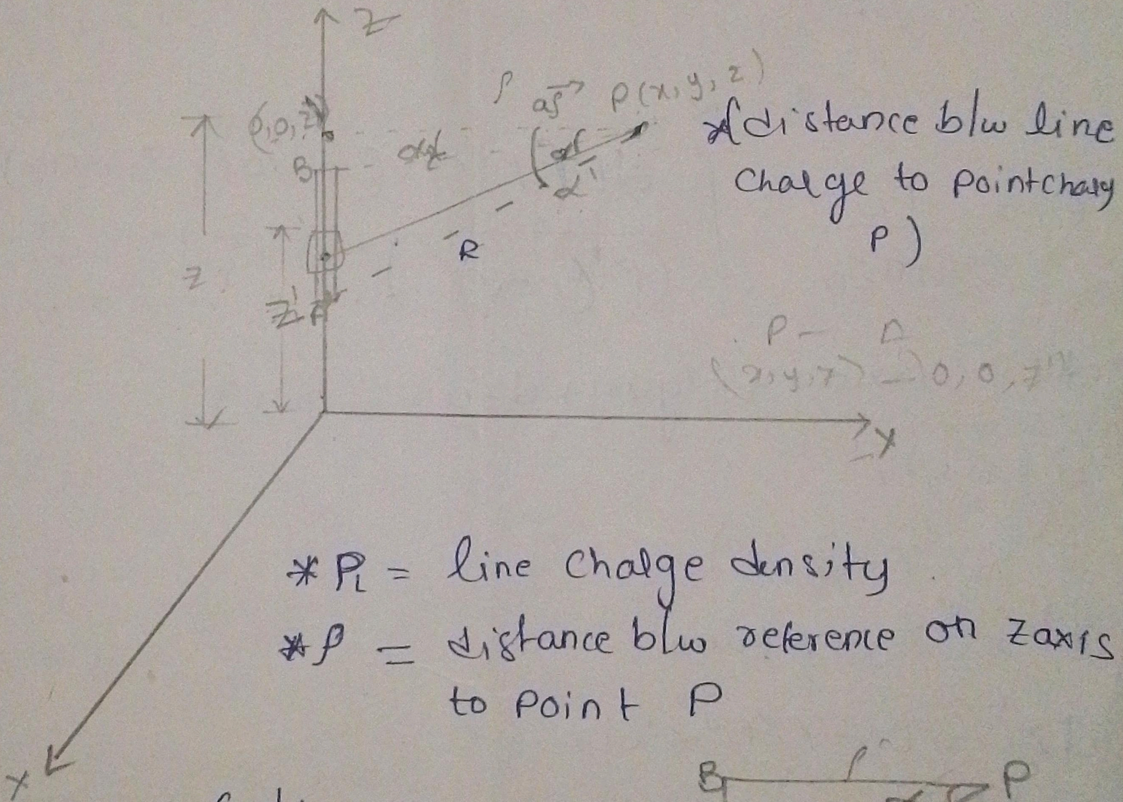
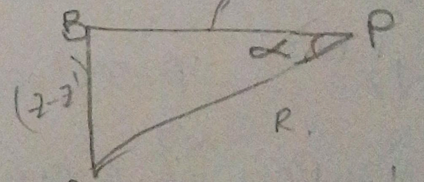


* Electric field intensity due to a line charge

Let Consider a line charge of finate length along the Z-axis



* P_L = line charge density
* ρ = distance b/w reference on Z axis to point P



$$E = \frac{P_L}{4\pi\varepsilon_0}\int_L \frac{dL}{R^3}\vec{R}$$

$\vec{R} = (x,y,z) - (0,0,z')$

$\vec{R} = x a_x + y a_y + (z-z')a_z$

$\vec{\rho} = (x,y,z) - (0,0,z)$

$\vec{\rho} = x a_x + y a_y$

$\sin\alpha = \frac{z-z'}{R}$

$\boxed{\cos\alpha = \frac{\rho}{R}}$

$\tan\alpha = \frac{z-z'}{\rho}$

$\rho' = R\cos\alpha$

$$\vec{R} = \rho a_\rho + (z - z')a_z$$

$$= R(\cos\alpha\, a_\rho) + R\sin\alpha\, a_z$$

$$= R(\cos\alpha\, a_\rho + \sin\alpha\, a_z)$$

$$|R| = \sqrt{\rho^2 + (z-z')^2}$$

$$= \sqrt{\rho^2 + (\rho\tan\alpha)^2}$$

$$= \sqrt{\rho^2(1+\tan^2\alpha)}$$

$$= \rho\sec\alpha$$

$$E = \frac{\rho_L}{4\pi\varepsilon_0}\int \frac{dL\,[\rho\sec\alpha(\cos\alpha\, a_\rho + \sin\alpha\, a_z)]}{(\rho^3\sec\alpha)^3}$$

$$dL = dx\,a_x + dy\,a_y + dz\,a_z$$

$$dL = dz'$$

$$z' = z - (z - z')$$

$$z' = z - \rho\tan\alpha$$

$$dz' = 0 - \rho\sec^2\alpha\, d\alpha$$

$$dz' = -\rho\sec^2\alpha\, d\alpha$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2} \frac{(-\rho\cancel{\sec^2\alpha})}{\rho^3\sec^3\alpha}\, d\alpha\, \rho\cancel{\sec\alpha}\,[\cos\alpha\, a_\rho + \sin\alpha\, a_z]$$

$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0\rho}\int_{\alpha_1}^{\alpha_2}(\cos\alpha\, a_\rho + \sin\alpha\, a_z)\, d\alpha$$

$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0\rho}\left[(\sin\alpha\, a_\rho]_{\alpha_1}^{\alpha_2} + (-\cos\alpha\, a_z)_{\alpha_1}^{\alpha_2}\right]$$

$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0\rho}\left[(\sin\alpha_2 - \sin\alpha_1)a_\rho - (\cos\alpha_2 - \cos\alpha_1)a_z\right]$$

<u>case-(i)</u> : If the line charge is infinate

'A' is at $(0,0,-\infty)$

'B' is at $(0,0,\infty)$

$\alpha_1 = +\pi/2\ (90^\circ)$ (clockwise)

$\alpha_2 = -\frac{\pi}{2}$ Anticlockwise

$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0\rho}\left[(\sin\alpha_2 - \sin\alpha_1)a_\rho - (\cos\alpha_2 - \cos\alpha_1)a_z\right]$$

$$= \frac{-\rho_L}{4\pi\varepsilon_0\rho}\left[\left(\sin\left(-\frac{\pi}{2}\right) - \sin\left[\frac{\pi}{2}\right]\right]a_\rho - \left(\cos\left[\pi/2\right] - \cos\left(\pi/2\right)a_z\right)\right]$$

$$= \frac{-\rho_L}{4\pi\varepsilon_0\rho}\left[-1 + (-1)a_\rho - 0\right]$$

$$= \frac{-\rho_L}{4\pi\varepsilon_0\rho}(-2)\,a_\rho$$

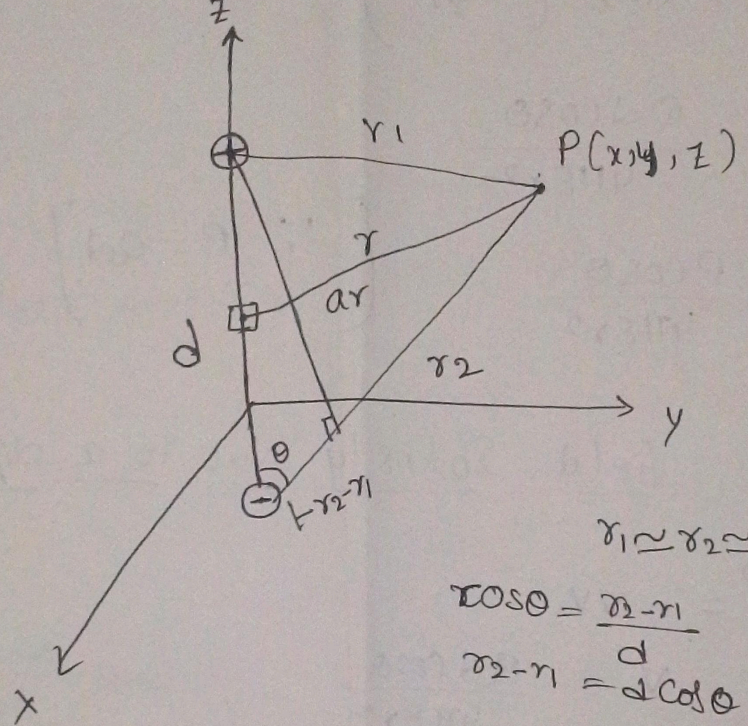
$$= \frac{+\rho_L}{2\pi\varepsilon_0\rho}a_\rho$$

e a pair of
opposite polarity

ge and istance,

Electric potential due to a Dipole:

Let consider a electric ~~dipole~~ dipole along z with distance 'd'.



$r_1 \simeq r_2 \simeq = r$

$\cos\theta = \dfrac{r_2 - r_1}{d}$

$r_2 - r_1 = d\cos\theta$

$$V = -\int \vec{E}\, dl$$

$$\vec{E} = \frac{Q}{4\pi\epsilon r^2} . a_r$$

$$V = -\int_{r_2}^{r_1} \frac{Q}{4\pi\epsilon r^2} . a_r\, dr\, a_r$$

$dl = dr\, a_r$

limits r_2 to r_1

$$= -\int_{r_2}^{r_1} \frac{Q}{4\pi\epsilon r^2} dr$$

$$= \frac{Q}{4\pi\epsilon}\left[\frac{1}{r}\right]_{r_2}^{r_1} = \frac{Q}{4\pi\epsilon}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

$$= \frac{Q}{4\pi\varepsilon}\left(\frac{d\cos\theta}{r^2}\right)$$

$$V = \frac{Qd\cos\theta}{4\pi\varepsilon r^2}$$

$$V = \frac{P\cos\theta}{4\pi\varepsilon r^2} \qquad \left[\because P = Qd\right]$$

Electric field Intensity due to a dipole:

$$\vec{E} = -\nabla V$$

where $V = \dfrac{Qd\cos\theta}{4\pi\varepsilon r^2}$

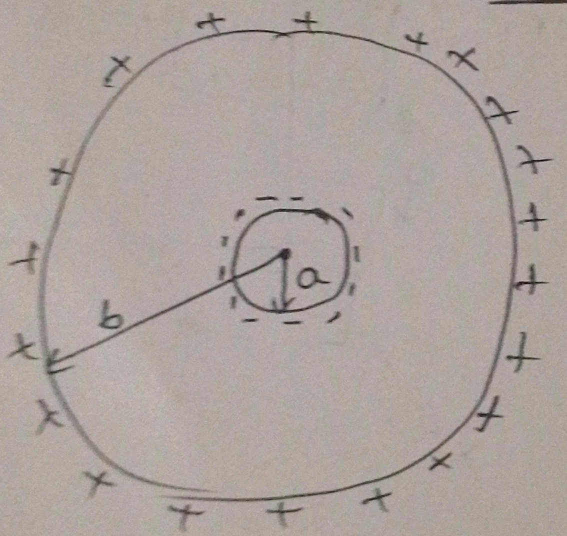
$$\nabla = \frac{\partial}{\partial r}a_r + \frac{1}{r}\frac{\partial}{\partial\theta}a_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}a_\phi$$

$$\vec{E} = -\left[\frac{\partial}{\partial r}\frac{Qd\cos\theta}{4\pi\varepsilon r^2}a_r + \frac{1}{r}\frac{\partial}{\partial\theta}\frac{Qd\cos\theta}{4\pi\varepsilon r^2}a_\theta\right]$$

$$= -\left(\frac{Qd\cos\theta}{{}_2\cancel{4}\pi\varepsilon}\left[\frac{-\cancel{2}}{r^3}\right]a_r + \frac{1}{r}\frac{Qd}{4\pi\varepsilon r^2}\left[-\sin\theta\right]a_\theta\right)$$

$$\boxed{\vec{E} = \frac{Qd\cos\theta}{2\pi\varepsilon r^3}a_r + \frac{Qd\sin\theta}{4\pi\varepsilon r^3}a_\theta}$$

② Spherical plate Capacitor:



$$C = \frac{Q}{V}$$

$$V = -\int \vec{E} \cdot d\vec{l}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon r^2} a_r \quad ; \quad d\vec{l} = dr\,a_r$$

$$V = \int \frac{Q}{4\pi\varepsilon r^2} a_r (dr\,a_r)$$

$$= \frac{Q}{4\pi\varepsilon} \int_a^b \frac{1}{r^2} dr$$

$$= \frac{Q}{4\pi\varepsilon} \left[-\frac{1}{r}\right]_a^b$$

$$= \frac{Q}{4\pi\varepsilon} \left[-\frac{1}{b} - \left(-\frac{1}{a}\right)\right]$$

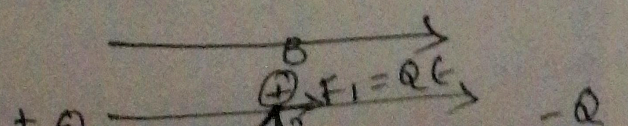
$$V = \frac{Q}{4\pi\varepsilon} \left[\frac{1}{a} - \frac{1}{b}\right]$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon}\left[\frac{1}{a} - \frac{1}{b}\right]}$$

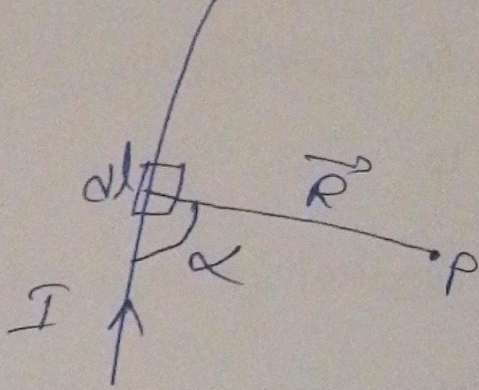
$$\boxed{C = \frac{4\pi\varepsilon}{\left(\frac{1}{a} - \frac{1}{b}\right)}} \text{ farads}$$

Torque acting on electric dipole;

Let us consider an electric dipole in a external electric field



* Biot - Savart's law :



$$dH \propto I\,dl$$

$$\propto \sin\alpha$$

$$\propto \frac{1}{R^2}$$

$$dH \propto \frac{I\,dl \sin\alpha}{R^2}$$

$$dH = k\frac{I\,dl \sin\alpha}{R^2} \qquad \left[\because k = \frac{1}{4\pi}\right]$$

$$\boxed{dH = \frac{I\,dl \sin\alpha}{4\pi R^2}} \; A/m$$

$$\boxed{dH = \frac{I\,dl \times a_R}{4\pi R^2}} \; A/m \qquad \left[\begin{aligned} dL \times A &= |dL|\,|A| \sin\alpha \\ &= dL \sin\alpha \end{aligned}\right]$$

$$\boxed{dH = \frac{I\,dl \times \vec{R}}{4\pi R^3}} \; A/m \qquad \left[\vec{a_R} = \frac{\vec{R}}{|\vec{R}|}\right]$$

$$\boxed{H = \int_{L} \frac{I\, dL \times \vec{R}}{4\pi R^3}} \; A/m$$

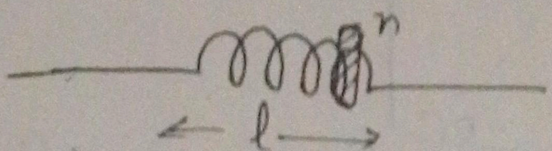
$$\boxed{H = \int_{S} \frac{K\, dS \times \vec{R}}{4\pi R^3}} \; A/m$$

$$\boxed{H = \int_{V} \frac{J\, dv \times \vec{R}}{4\pi R^3}} \; A/m$$

intensity due to a straigh

* Magnetic field Intensity ~~due~~ to Solenoide

Let Consider a Solenoide with "N" no of turns and Length "l" meters

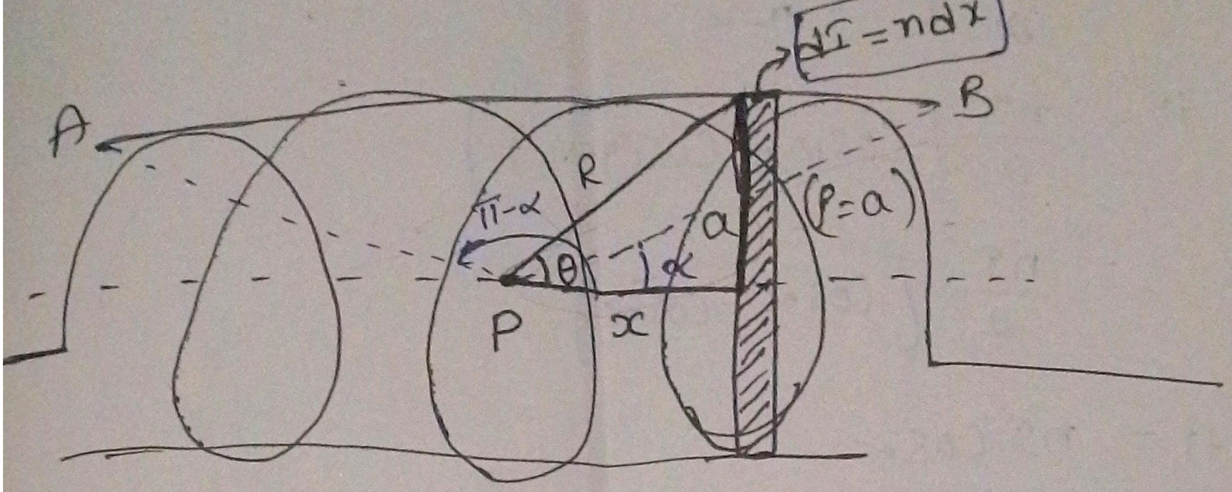


$n = \frac{N}{l}$ (Turns density)

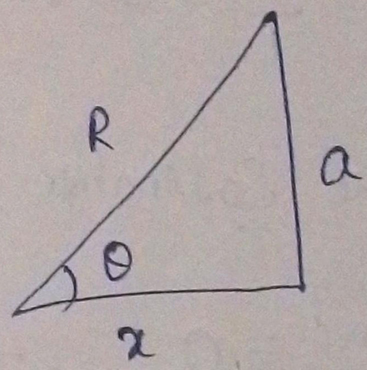
A Solenoide is compressing of multiple no of Single turned conductors

$$H = \frac{I\rho^2}{2(\rho^2+h^2)^{3/2}} a_z$$

$$H = \frac{I\rho^2}{2R^3} a_z \left[\because \rho^2+h^2 = R^2 \quad (R^2)^{3/2} = R^3 \right]$$



$$dH = \frac{I a^2}{2R^3} n\,dx$$



$$dH = \frac{I a^2}{2 a^3 \operatorname{cosec}^3\theta} n\left(-a \operatorname{cosec}^2\theta \, d\theta\right)$$

$$\tan\theta = \frac{a}{x}$$

$$x = a\cot\theta$$

$$\boxed{dx = -a\operatorname{cosec}^2\theta \, d\theta}$$

$$= \frac{I}{2\operatorname{cosec}\theta} - n\,d\theta$$

$$= -\frac{nI}{2\operatorname{cosec}\theta} d\theta$$

$$\sin\theta = \frac{a}{R}$$

$$R = a\operatorname{cosec}\theta$$

$$= -\frac{nI}{2} \sin\theta \, d\theta$$

$$H = \int_{\pi-\alpha}^{\alpha} -\frac{nI}{2} \sin\theta \, d\theta$$

$$= \frac{nI}{2}\left[\cos\theta\right]_{\pi-\alpha}^{\alpha}$$

$$H = \frac{nI}{2}\left[\cos\theta\right]_{\pi-\alpha}^{\alpha}$$

$$H = \frac{nI}{2}\left[\cos\alpha - \cos(\pi-\alpha)\right]$$

$$= \frac{nI}{2}\left[\cos\alpha + \cos\alpha\right]$$

$$H = nI\cos\alpha$$

$$H = \frac{N}{l} I\cos\alpha$$

*If the Solinoide is of nifinate length

$\alpha = 0$, then $\cos\alpha = 1$

$$H = \frac{NI}{l} \text{ (or) } nI$$

$\lambda_{21} = M I_2$

$M I_2 = N_1 \phi_{21}$

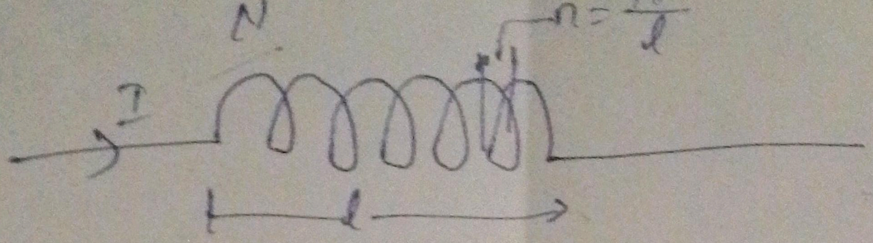
$$\boxed{M_{21} = \frac{N_1 \phi_{21}}{I_2}}$$ At coil-1 wrt coil 2

$L_{11} = \frac{N_1 \phi_{11}}{I_1}$; $L_{22} = \frac{N_2 \phi_{22}}{I_2}$

-> Self Inductance of a Solenoide:

Let us consider a solenoide of length 'l' mts having turns 'N' & turns density 'n'.

where, $\boxed{n = \frac{N}{l}}$



$$L = \frac{N\phi}{I} \;;$$

$$\phi = \int_s B \cdot ds = B \cdot A$$

$$B = \mu \cdot H$$

$$B = \mu \cdot nI = \mu \frac{NI}{l}$$

$$\phi = \frac{\mu N I A}{l}$$

$$L = \frac{N}{\cancel{I}}\left(\frac{\mu N \cancel{I} A}{l}\right)$$

$$\boxed{L = \frac{\mu N^2 A}{l}}$$ Henry

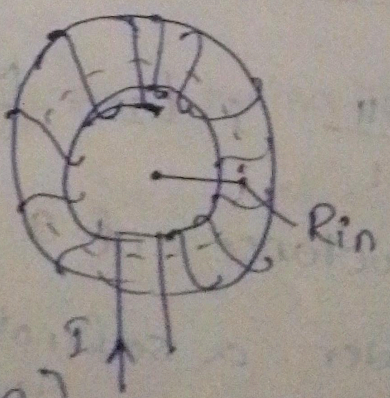
→ <u>Self Inductance of a Thoroide</u>

$$L = \frac{N\phi}{I}$$

$$\phi = \int B \cdot dS = BA$$

$$B = \mu \cdot H$$

$$B = \mu \cdot nI = \frac{\mu NI}{2\pi R_{in}} \qquad [l = 2\pi R_{in}]$$



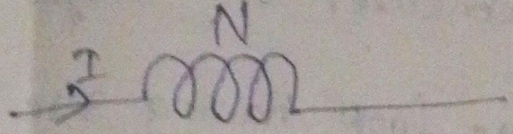
$$\phi = \frac{\mu N I A}{2\pi R_{in}}$$

$$L = \frac{N}{\cancel{I}}\left(\frac{\mu N \cancel{I} A}{2\pi R_{in}}\right)$$

$$\boxed{L = \frac{\mu N^2 A}{2\pi R_{in}}}$$ Henry

→ Energy stored

Let us consider an inductor 'L' ~~to~~ 'N' turns & carrying Cur
'I'



$e = L\frac{di}{dt}$

$P = vi = ei$

$dw = ei\,dt$

$\left[\begin{array}{l} P = \frac{dw}{dt} \\ dw = P\,dt \end{array}\right]$

$dw = L\frac{di}{\cancel{dt}}\, i\, \cancel{dt}$

$dw = Li\,di$

$$\boxed{w = \frac{1}{2}Li^2}$$

→ Energy density :

$$\text{Energy density} = \frac{\text{Energy}}{\text{Unit volume}}$$

$w_D = \frac{w_E}{V}$

$L = \frac{\mu N^2 A}{l}$

$= \frac{\frac{1}{2}LI^2}{Al}$

$= \frac{\frac{1}{2}\left(\frac{\mu N^2 \cancel{A}}{l}\right)I^2}{Al} = \frac{\mu N^2 I^2}{2l^2}.$

$w_D = \left(\frac{\mu^2 N^2 I^2}{l^2}\right)\cdot\frac{1}{2\mu}$

$$B = \mu \cdot H = \mu \cdot \frac{NI}{l}$$

$$w_D = \frac{B^2}{2\mu}$$

$$\boxed{w_D = \frac{1}{2} \mu H^2}$$