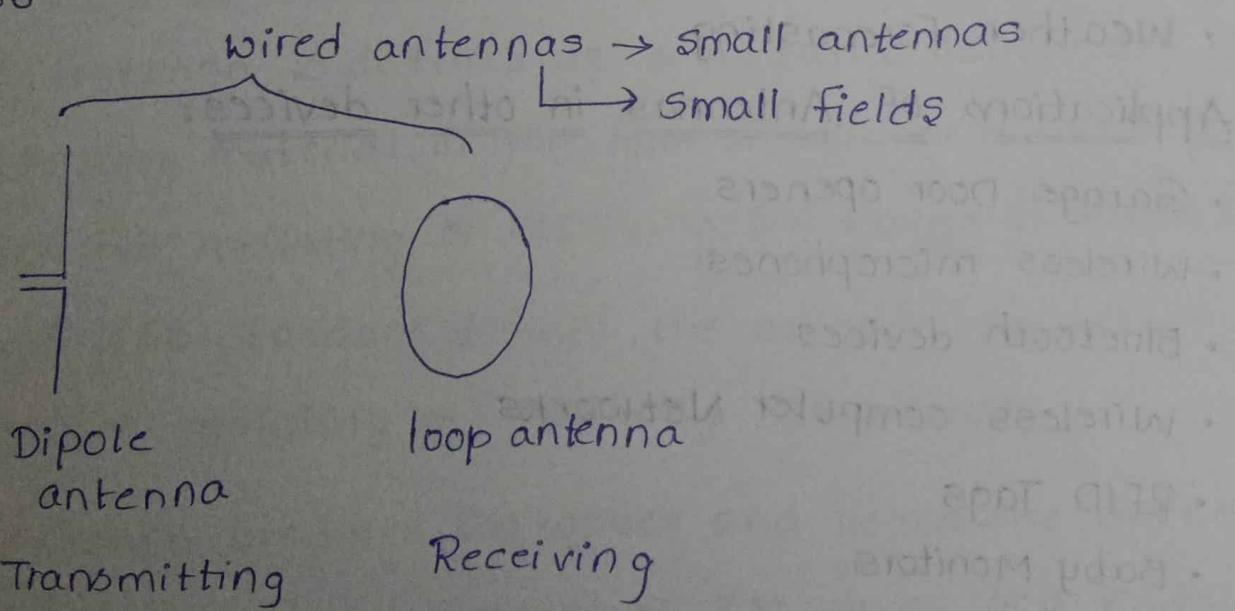


UNIT-I: ANTENNA FUNDAMENTALS.

Introduction

- Meaning of the word antenna is either of a pair of long, thin sensory appendages on the heads of insects or orthopods.
- Antenna in zoology is feeler i.e., part of insects with which they feel or organ of touch.
- Other names: Aerial or Radiator
- Antenna becomes an integral part of our day to day life. We find antennas everywhere at homes, workplaces, on cars, vehicles, aircrafts, ships and in mobiles.
- First antenna was assembled by Heinrich Hertz in 1886



short distance communication.

- further developed by Macconi for long distance communication in 1901

Symbol of antenna



Applications of antennas:

- Radio Broadcasting
- TV Broadcasting
- Two-way communication
- Communication Receivers
- Radar Systems
- Cell phones
- Satellite communications
- Weather Forecasting.

Applications of Antennas in other devices:

- Garage Door openers
- Wireless microphones
- Bluetooth devices
- Wireless computer Networks
- RFID Tags
- Baby Monitors

principle of Antenna:

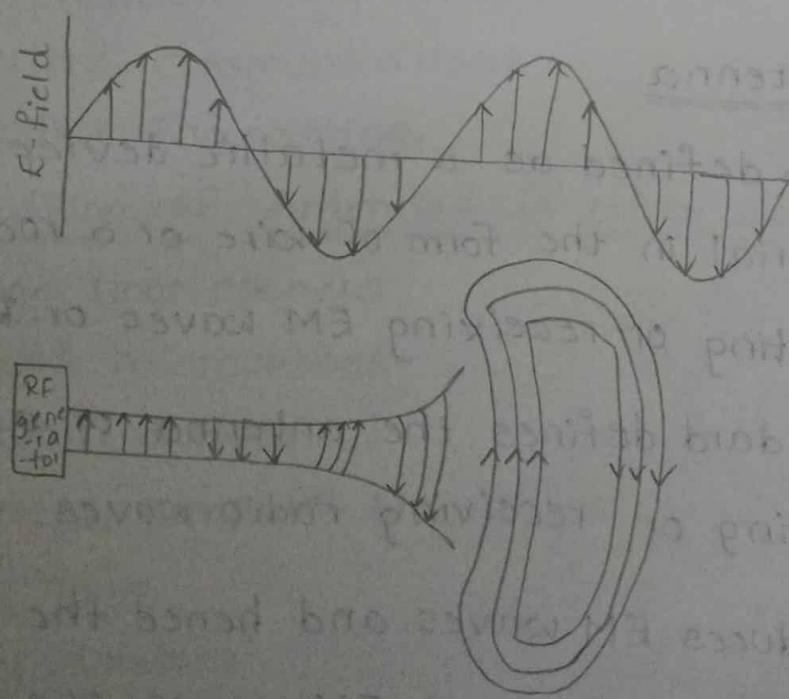
⇒ There are number of types of antennas, but all operate with the basic principles of Electromagnetics (Field theory).

- ⇒ Basically an antenna is a means of transporting signal from one end to other without any wire.
- ⇒ Antenna → Electromagnetic → E & H fields → Transmit & field
Receive EM energy → EM wave
- ⇒ when radio frequency (ac) signal is applied to an antenna, the E & H fields are produced and these fields constitute an EM wave.
- ⇒ Transmitting Antenna → RF i/P and EM o/P
- ⇒ Receiving Antenna → EM i/P and RF o/P

Definitions of Antenna

- ⇒ An antenna is defined as a metallic device or conducting material in the form of wire or a rod or a sheet for radiating or receiving EM waves or Radio waves
- ⇒ the IEEE standard defines the antenna or aerial as a means of radiating or receiving radio waves.
- ⇒ Antenna produces EM waves and hence the antenna can be defined as a source of EM waves or a Radiator of EM waves.
- ⇒ Antenna can be used to sense an EM wave and hence the antenna can be defined as a sensor of EM waves. (or) detector of EM waves.

- ⇒ Antenna can also be defined as a transducer which converts RF electrical current into an EM wave of same frequency as that of RF current and vice-versa.
- ⇒ Antenna can act as a coupling device between a generator or a transmitter and free-space / free space and a receiver.
- ⇒ Antenna is an impedance matching device between free-space and transmission line.
- ⇒ Antenna is the transitional structure between free-space and a guiding device or transmission line as shown.



Source → Transmission Antenna → Radiated
line free-space
wave

fig:- Antenna as a transition device

- 1) Switch ON the RF source.
- 2) Some charge on the Transmission line
- 3) Electric Flux lines \Rightarrow E-field & H-field
- 4) After every half cycle $I=0$, field = 0.
- 5) Electric flux lines can bent in antenna structure
- 6) Cannot extend upto infinity since Transmission line is Guide medium.

Radiation Mechanism & Sources of Radiation

- \Rightarrow The EM fields generated by the source, contained and guided within the transmission line and antenna and finally detached from the antenna to form a free-space wave. This mechanism is called radiation.
- \Rightarrow If a transmitting antenna is excited with RF ac signal, the initial motion is started by the balanced motion of charge in the antenna.
- \Rightarrow The resonant oscillations are produced by the supplied energy.
- \Rightarrow The E and H fields are generated due to sudden changes in charge.
- \Rightarrow When the charges around the antenna are set in motion first, the other charges are separated from the antenna and they are also set in motion.

- ⇒ The disturbance is spread from the antenna into free-space.
- ⇒ EM waves have no boundaries in free-space and the EM energy decreases as it propagates due to free-space conductivity (σ).

⇒ Antenna may radiate or may not be radiate

Conditions for no radiation

⇒ If the charge is stationary, the current is not created and hence there is no radiation.

⇒ If an electric charge is moving with a uniform velocity along the straight wire with infinite length, then also there is no radiation as shown in fig.

④

Static charge at rest

No radiation

Electric charge moving with uniform velocity

No radiation

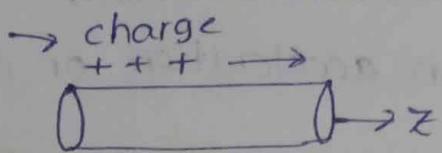
figure: conditions under which no radiation takes place

conditions for radiations:-

To explain the phenomenon of EM radiation, a relation between current and charge must be established. i.e., the fundamental relation of EM radiation (Basic radiation equation).

Relation between current and charge:

consider a straight wire and a pulse of electric charge moving with along the wire in z-direction.



→ Moving charges constitutes a current $I = \frac{Q}{t}$

$$V = \frac{l}{t} \quad t = \frac{l}{V}$$

$$\Rightarrow I = \frac{QV}{l}$$

$$I = \frac{QV}{l}$$

$$I_z = q_l V_z \text{ Amp}$$

q_l = charge/unit length

V_z = velocity in z-direction (m/sec)

I_z in time varying form

$$\frac{dI_z}{dt} = q_l \cdot \frac{dV_z}{dt}$$

$$\frac{dI_z}{dt} = q_l \cdot a_z$$

a_z → rate of change of velocity → acceleration (m/sec²)

→ The length of the wire ⇒ l

$$l \cdot \frac{dI_z}{dt} = l \cdot q_l \cdot a_z$$

$$\boxed{l \frac{dI_z}{dt} = q \cdot a_z}$$

Basic radiation equation

c.m/sec²

c/sec m/sec

(Am/sec) units

conditions for radiation:

- ⇒ $I \cdot \frac{dIz}{dt} = q \cdot az$
- ⇒ It simply states that ^{to} create radiation, there must be a time varying current ($\frac{dIz}{dt}$) or an acceleration or deceleration of charge (az)
- ⇒ current in time harmonic applications while charges in transients or pulses.
- ⇒ To create charge acceleration or deceleration, the wire must be curved, Bent, Discontinuous or terminated or Truncated.

Summary of radiations:-

- ⇒ If a charge is not moving, current is not created and there is no radiation.
- ⇒ If the charge is moving with a uniform velocity
- there is no radiation, if the wire is straight and infinite
 - There is radiation, if the wire is curved, Bent, discontinuous or terminated or truncated.
- ⇒ If the charge is oscillating in a time motion, it radiates even if the wire is straight.
- ⇒ Thus time changing current radiates and accelerated charge radiates.

Sources of Radiation:-

- ⇒ The following are the some of the basic sources of radiation
- single wire
 - two-wire
 - dipole

single wire Antenna:
 → conducting wires are materials whose prominent characteristics is in the motion of electric charges and creation of current flow.

→ Let us assume that an electric volume charge density represented by q_v (C/m^3) is distributed in a circular wire conductor of cross sectional area 'A' and volume V as shown in figure.

→ The total charge 'Q' within volume V is moving in z-direction with a uniform velocity v_z (m/sec).

volume current density

$$I_z = q_v v_z A/m^2$$

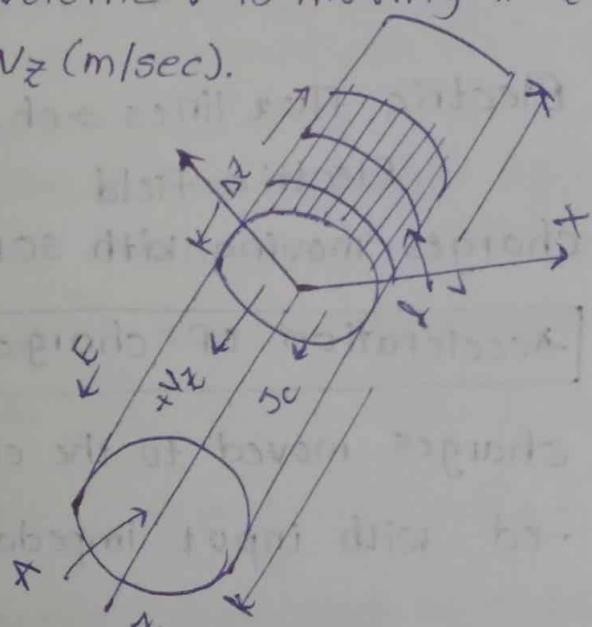
surface current density

$$I_s = q_s v_z A/m$$

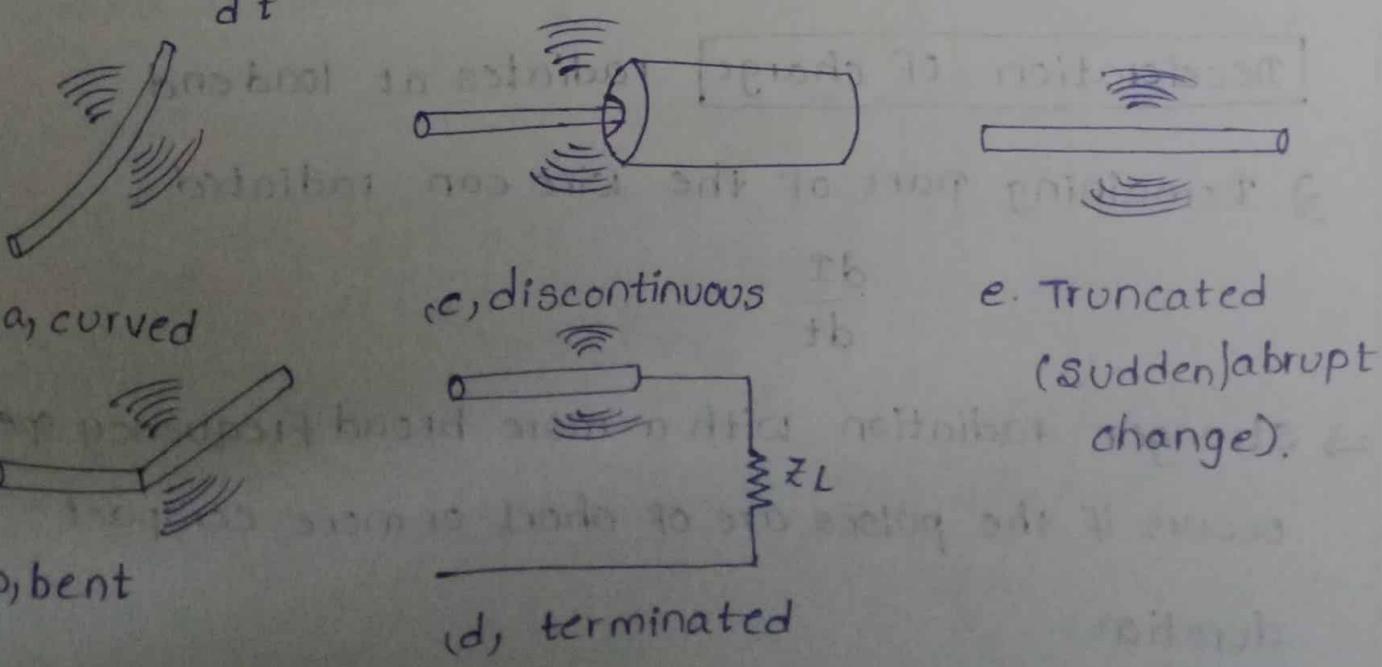
wire current

$$I_z = q_s v_z (A)$$

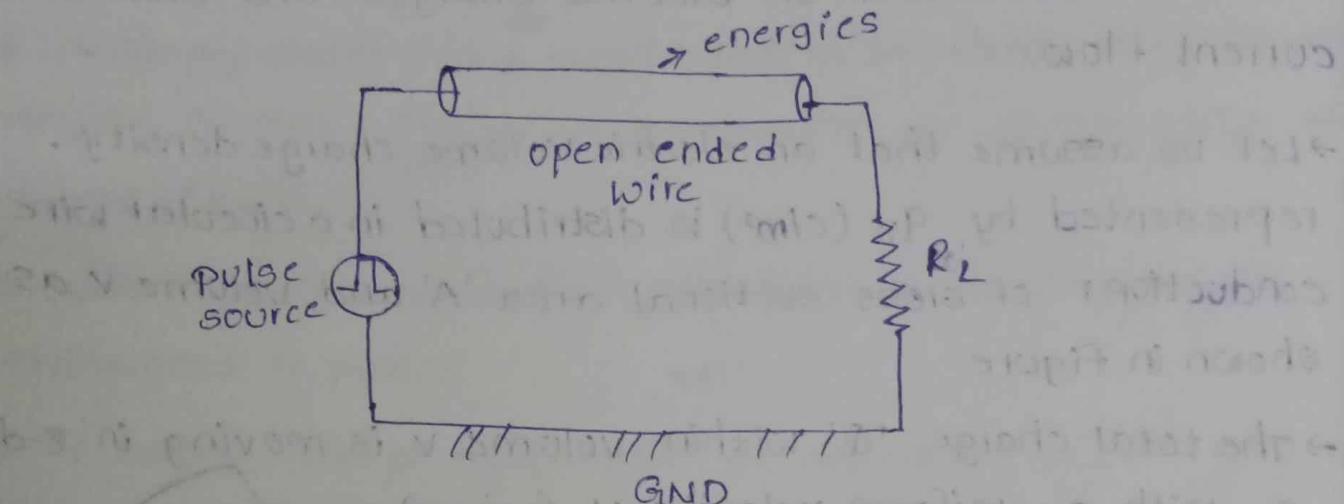
Basic Radiation Equation



$$I \cdot \frac{dI_z}{dt} = q \cdot a_z$$



open ended wire with termination for a Qualitative understanding of radiation mechanism.



i) Electric flux lines \rightarrow charge

Electric field

charges moving with some velocity

Acceleration of charge radiates at the source.

ii) charges moved to the end and the end is not terminated with input impedance (Z_0)

Reflection of charges

losses of energy for

Deceleration of charge radiates at load end

iii) Remaining Part of the wire can radiated

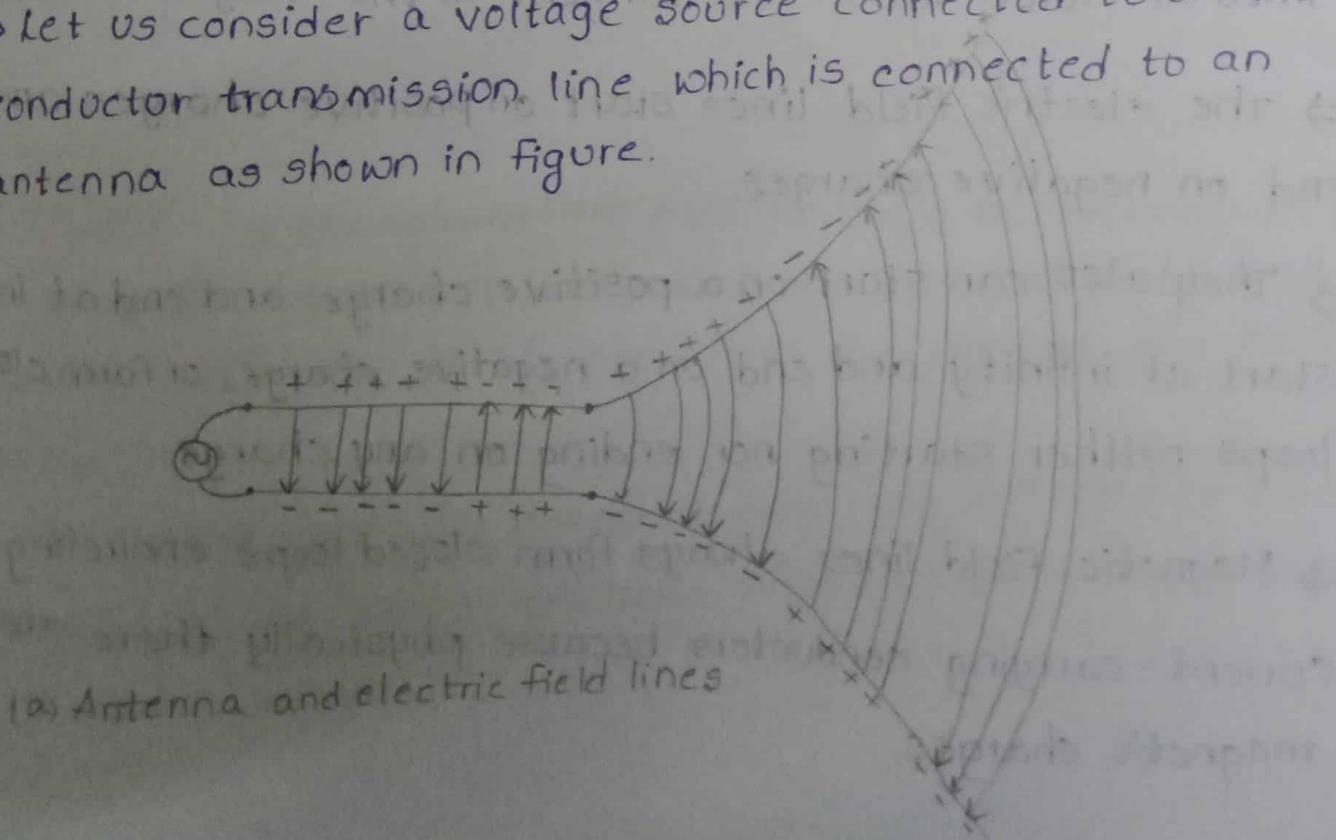
$$\text{balance} = \frac{dI}{dt}$$

\Rightarrow stronger radiation with a more broad frequency spectrum occurs if the pulses are of short or more compact duration.

- ⇒ continuous time-harmonic oscillating charge produces radiation of single frequency determined by frequency of oscillation.
- ⇒ The acceleration of the charge is accomplished by the external source in which forces set the charges in motion and the associated field radiation.
- ⇒ The deceleration of the charges at the end of the wire is accomplished by the internal forces, receives energy from the charge and its velocity reduced to zero at end of the wire (reflections).
- ⇒ Therefore charge acceleration due to external source, deceleration due to impedance mismatch (reflections) and smooth curves of the wire are mechanisms responsible for EM radiation.

Two-Wire Antenna:-

- ⇒ Let us consider a voltage source connected to a two-conductor transmission line which is connected to an antenna as shown in figure.

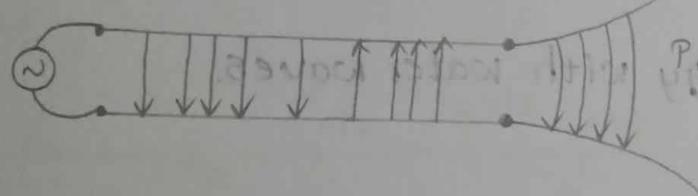


- ⇒ Applying a voltage across the two-conductor transmission line creates an electric field between the conductors.
- ⇒ The electric field has associated with it electric lines of force which are tangent to the electric field at each point and their strength is proportional to the electric field intensity.
- ⇒ The electric lines of force have a tendency to act on the free electrons (easily detachable from the atoms) associated with each conductor and force them to be displaced.
- ⇒ The movement of the charges creates a current that in turn creates a magnetic field intensity.
- ⇒ Associated with the magnetic field intensity are magnetic lines of force which are tangent to the magnetic field.
- ⇒ The electric field lines start on positive charges and end on negative charges.
- ⇒ They also can start on a positive charge and end at infinity, start at infinity and end on a negative charge, or form closed loops neither starting nor ending on any charge.
- ⇒ Magnetic field lines always form closed loops encircling current-carrying conductors because physically there are no magnetic charges.

⇒ The electric field lines drawn between the two conductors help to exhibit the distribution of charge.

⇒ If we assume that the voltage source is sinusoidal, we expect the electric field between the conductors to also be sinusoidal with a period equal to that of the applied source.

Antenna and free-space waves:-



b) Antenna and free-space waves

⇒ The free-space waves had also periodic but a constant phase. Point P_0 moves outwardly with the speed of light and travels a distance of $\lambda/2$ (to P_1) in the time of one-half of a period.

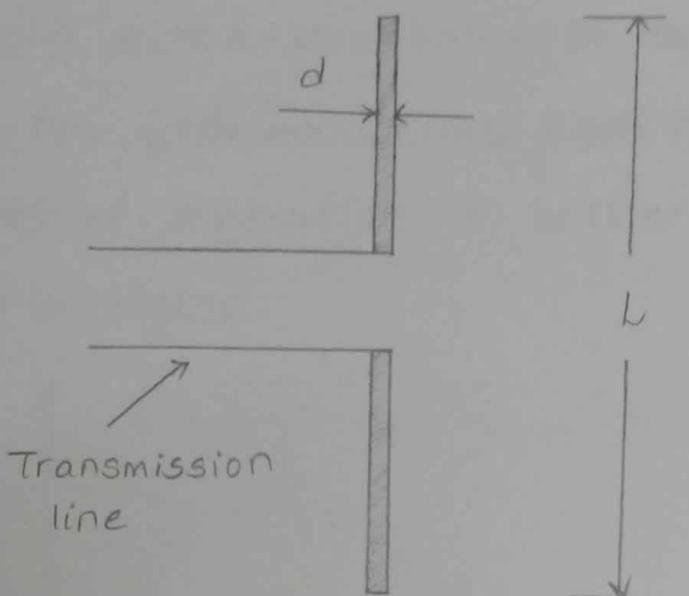
⇒ The guided waves are detached from the antenna to create the free-space waves as closed loops can be easily explained by water waves.

- ⇒ If the electric disturbance is of a continuous nature, electromagnetic waves exist continuously and follow in their travel behind the others.
- ⇒ When the electromagnetic waves are within the transmission line and antenna, their existence is associated with the presence of the charges inside the conductors.
- ⇒ However, when the waves are radiated, they form closed loops and there are no charges to sustain their existence.
- ⇒ This leads us to conclude that electric charges are required to excite the fields but are not needed to sustain them and may exist in their absence.
- ⇒ This is in direct analogy with water waves.

Dipole Antenna:

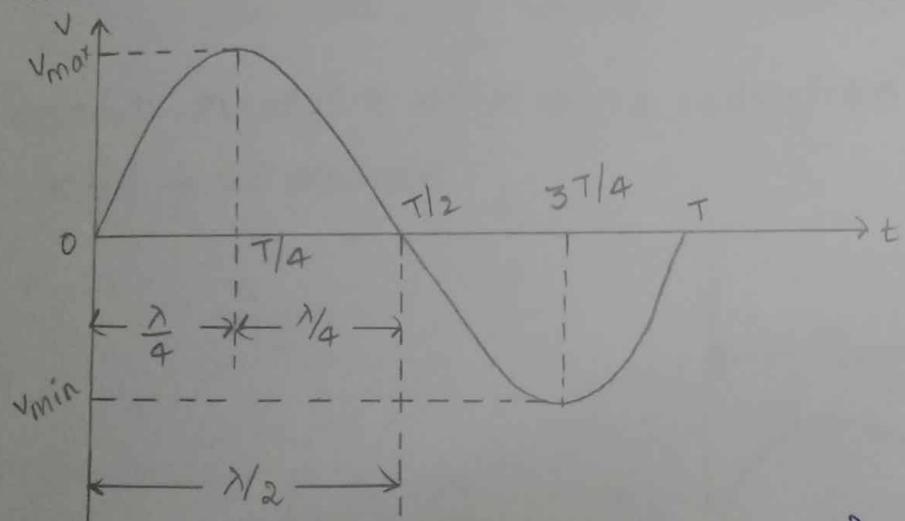
- ⇒ Dipole means two poles/charges/points and it has very small length wires.
- ⇒ Dipole: A pair of equal and opposite electric charges separated by a small distance.
- Dipole Antenna: An aerial half a wavelength long consisting of two rods connected to a transmission line at the centre.
- Short dipole/Hertzian dipole: A short linear conductor of finite length, even though it may be very short.
- Infinitesimal dipole : The dipole vanishingly short.

Dipole Antenna representation

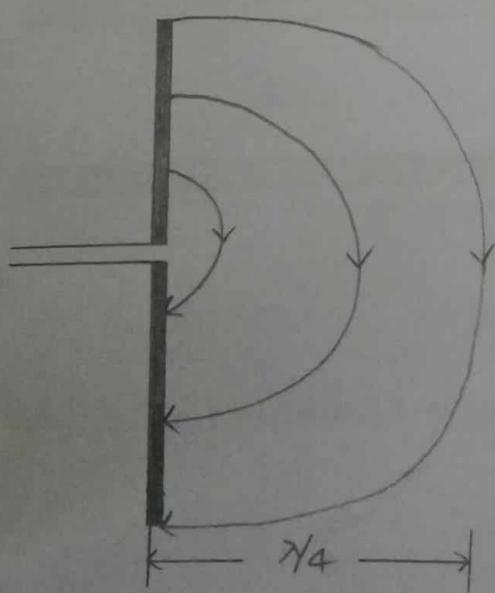


Radiation of Dipole antenna

consider a sinusoidal wave



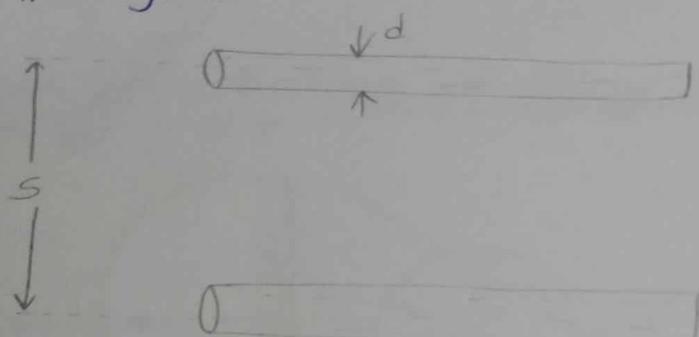
Dipole Antenna at $t = T/4$ (T =Period)



(a) $t = T/4$ (T =Period)

Current Distribution on a Thin Wire Antenna

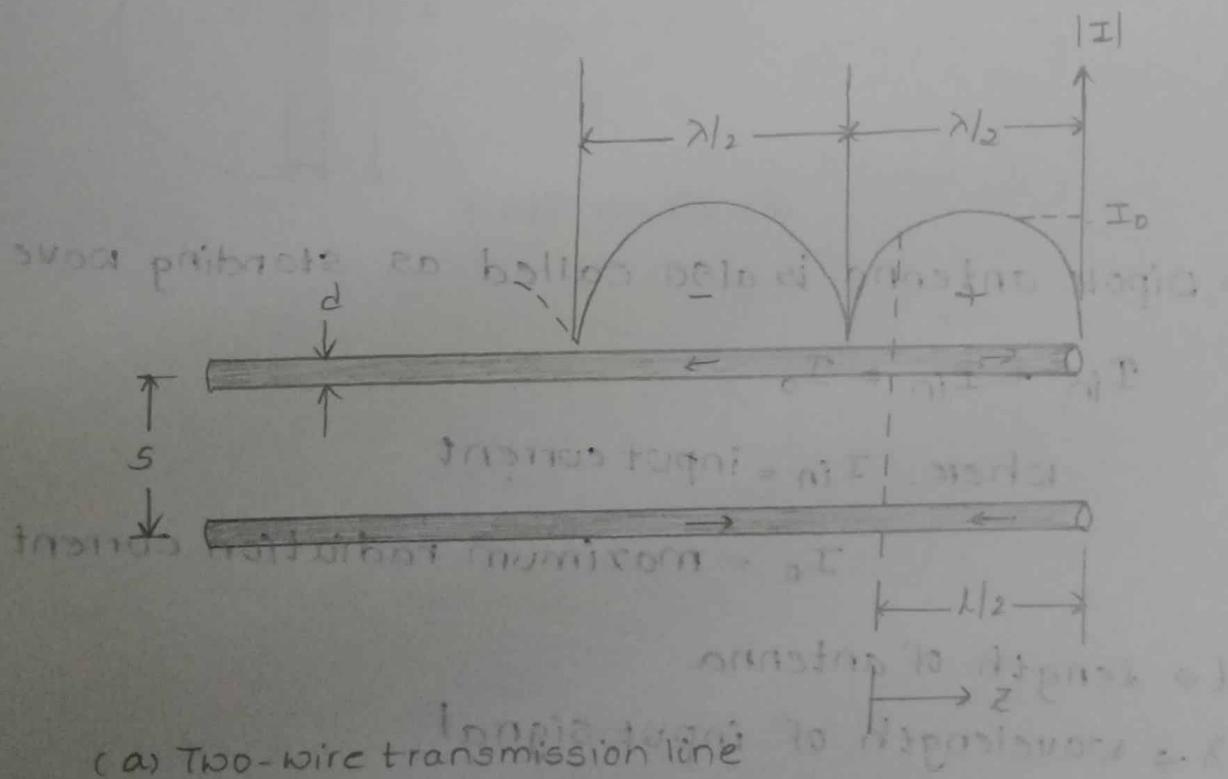
⇒ consider a two-wire balanced symmetrical lossless transmission line with conductors each of diameter 'd' and distance of separation 's' between two conductors as shown in figure.



(a) TWO WIRE LOSSLESS TRANSMISSION LINE

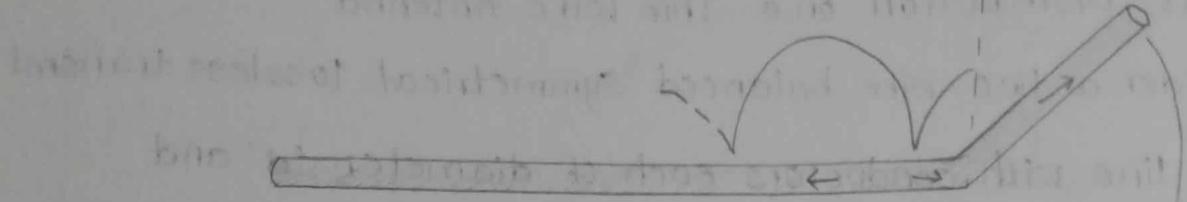
case ① straight wire ⇒ no radiation

wire ⇒ ac source



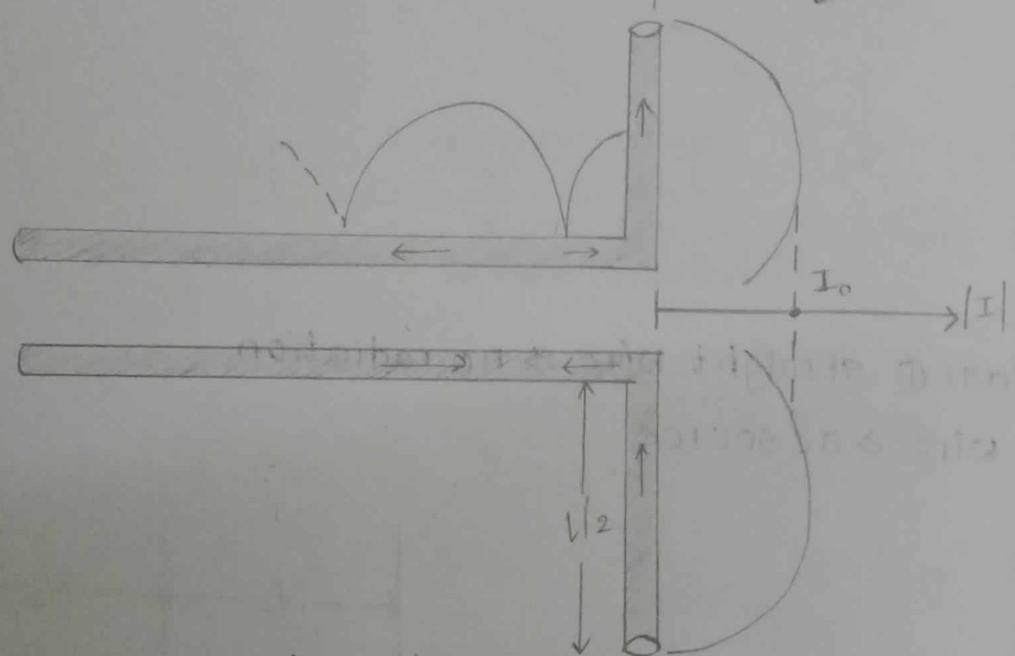
(a) TWO-WIRE TRANSMISSION LINE

bobbA sin abla ; 4.2
two conductors sin abla ; 6.1



in addition out resulting in net torque in upward direction

(b) Flared transmission line



(c) Linear dipole

⇒ Dipole antenna is also called as standing wave antenna

$$I_{in} + I_{in} = I_0$$

where, I_{in} = input current

I_0 = maximum radiation current

$l \rightarrow$ length of antenna

$\lambda \rightarrow$ wavelength of input signal

2 cases:-

i) $l < \lambda$; fields are Added

ii) $l > \lambda$; fields are Cancelled out

4 cases:-

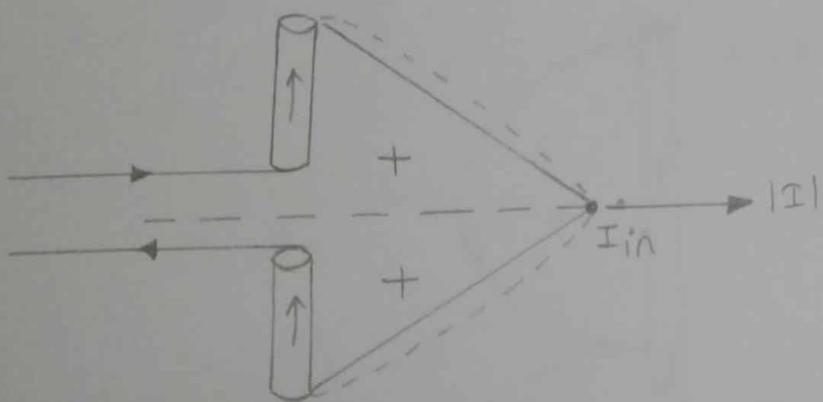
1) $L \ll \lambda \quad \frac{\lambda}{50} \leq l \leq \frac{\lambda}{10}$

2) $l = \frac{\lambda}{2} \Rightarrow$ half wavelength dipole antenna

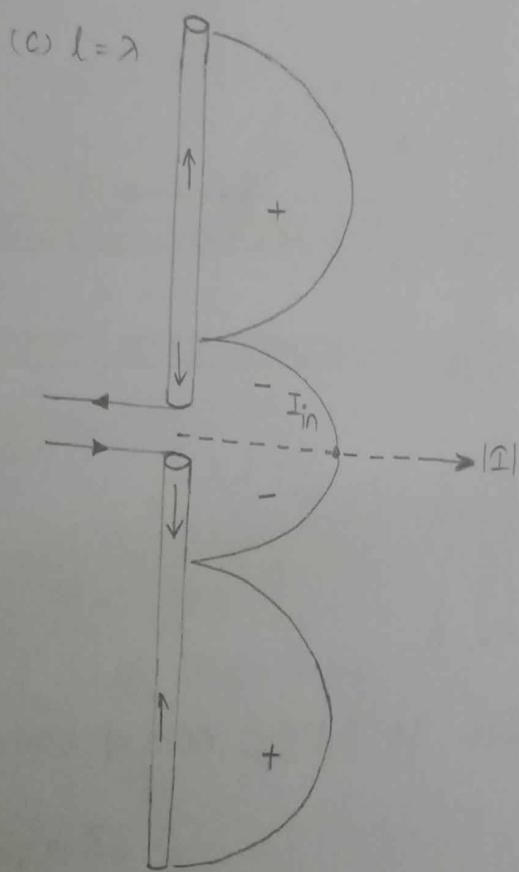
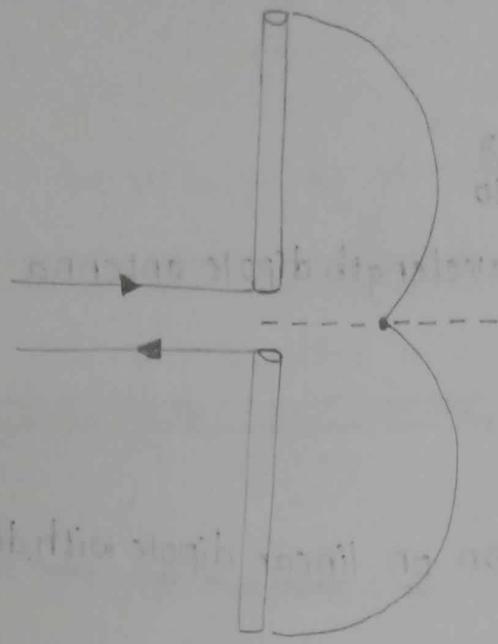
3) $l = \lambda$

4) $l = \frac{3\lambda}{2}$

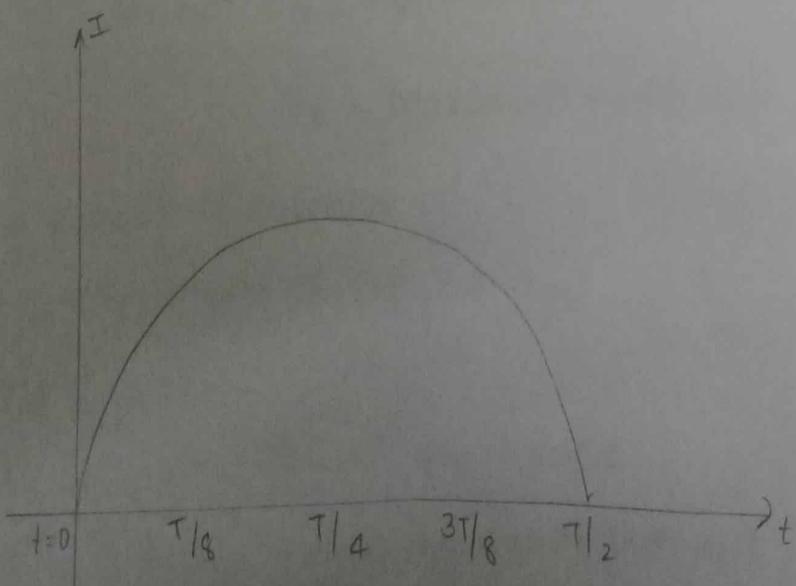
current distribution on linear dipole with different lengths



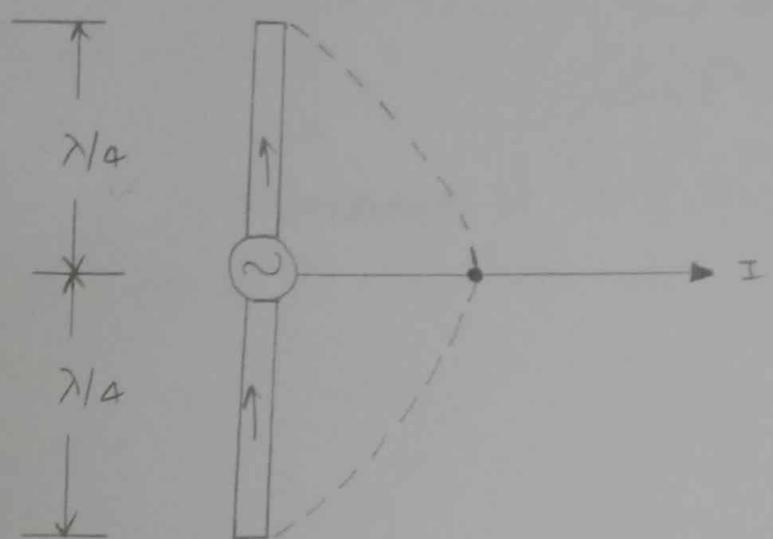
as $L \ll \lambda \quad \frac{\lambda}{50} \leq l \leq \frac{\lambda}{10}$



(d)



current distribution on $\lambda/2$ wire antenna for different times



(a) $t = 0$

Antenna parameters/specifications/characteristics

- To use an antenna
- To design an antenna
- To buy an antenna
- To judge the performance

⇒ Radiation pattern

- ⇒ Beam width
- ⇒ Polarization
- ⇒ Beam Area
- ⇒ Radiation intensity
- ⇒ Beam Efficiency
- ⇒ Directivity
- ⇒ Gain & Resolution
- ⇒ Antenna Apertures
- ⇒ Aperture efficiency
- ⇒ Effective height.

Radiation Pattern:-

- ⇒ The detachment of EM fields from the antenna into free-space is called radiation. Pattern means representation or graph.
- ⇒ Radiation Pattern is a graphical representation of EM waves or EM field in the free space.
- ⇒ The EM energy radiated by an antenna cannot be seen by human eyes. So how much EM energy is radiated or how the EM energy distributed in free-space can be studied with a pattern called radiation pattern.

⇒ An antenna radiation pattern is defined as a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates (ϕ, θ) .

⇒ Antenna radiation is independent of radius because practically no antenna can radiate equally in all directions.

Types of Radiation Patterns

The radiation patterns are different for different antennas and are affected by the location of the antenna w.r.t ground, polarization and length of the antenna.

There are two basic radiation patterns

- Field strength pattern
- Power radiation pattern

⇒ If the radiation of the antenna is expressed in terms of the field strength E (V/m), then the graphical representation is called Field strength pattern / Field radiation pattern.

⇒ If the radiation of the antenna is expressed in terms of the power per unit solid angle, then the graphical representation is called Power radiation pattern / Power pattern.

⇒ However, both are related to each other - a power pattern is proportional to the square of the field strength pattern and unless otherwise mentioned radiation pattern means field strength pattern.

Field radiation pattern:-

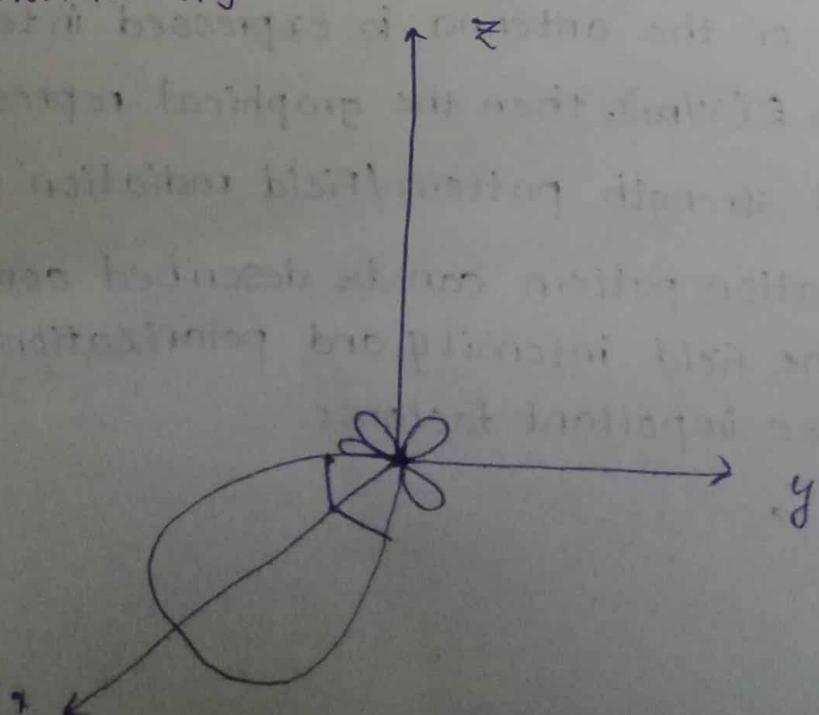
⇒ If the radiation of the antenna is expressed in terms of the field strength E (V/m), then the graphical representation is called field strength pattern / field radiation pattern.

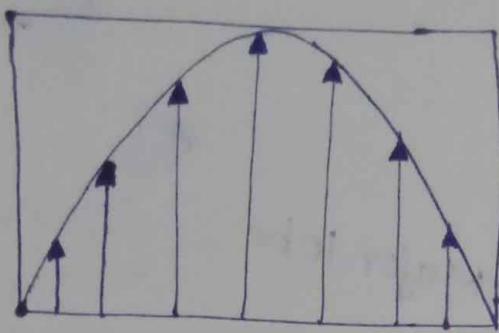
⇒ The field radiation pattern can be described completely with respect to the field intensity and polarization using the following three important factors:

$$(i) E_0(\theta, \phi) \rightarrow$$

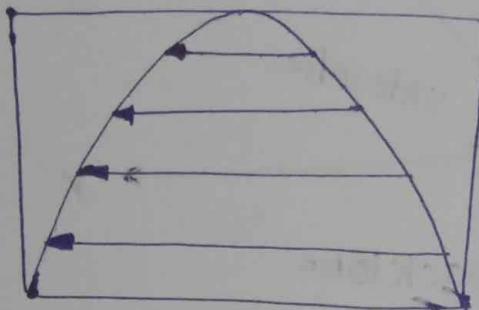
patterns in Principal planes / Principal patterns

- ⇒ principal plane: A plane that is perpendicular to the axis of the antenna and at which radiation parallel to the axis of the antenna.
- ⇒ For a linearly polarized antenna, performance is often described in terms of its principal planes.
- ⇒ There are two principal patterns
 - principal E-plane pattern
 - principal H-plane pattern
- ⇒ The E-plane is defined as the plane containing the electric field vector and maximum radiation.
- ⇒ The H-plane is defined as the plane containing the magnetic field vector and maximum radiation.
- ⇒ It is normal practice to orient most antennas in such a way that at least one of the principal plane patterns coincide with one of the geometric principal planes as shown in fig.





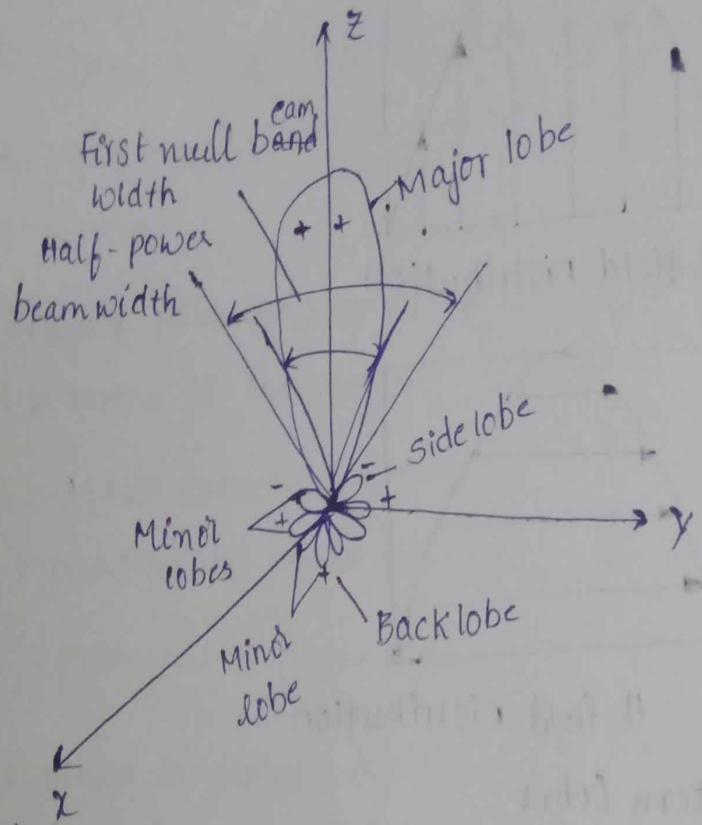
E-field Distribution



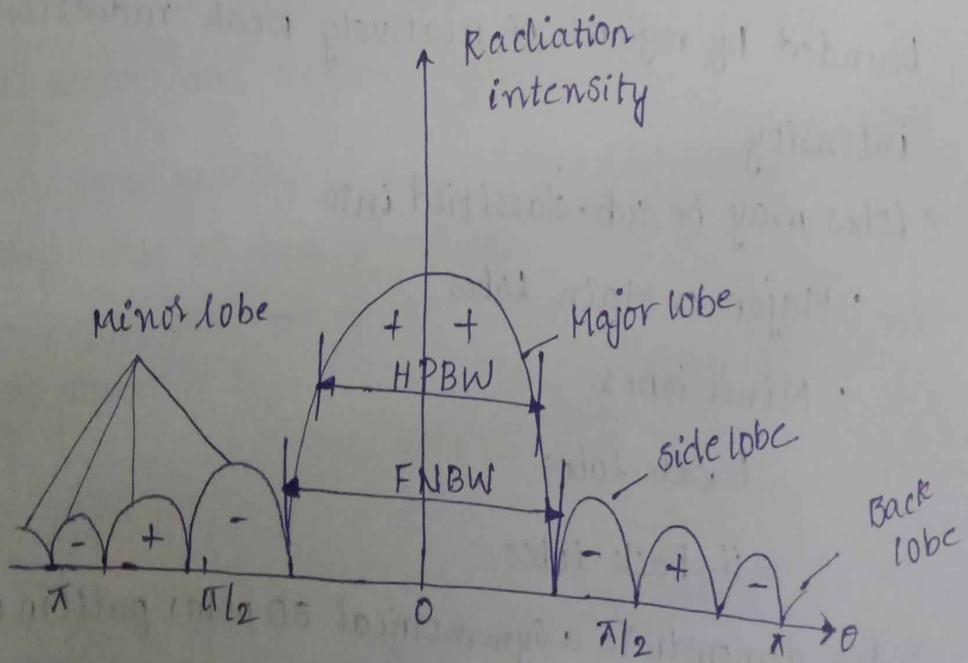
H-field Distribution

Radiation Pattern Lobes

- Various parts of a radiation are referred to as lobes
- A radiation lobe is a portion of the radiation pattern bounded by regions of relatively weak radiation intensity.
- Lobes may be sub-classified into
 - Major or Main lobes
 - Minor lobes
 - i. Side lobes
 - ii. Back lobes
- Fig. demonstrate a symmetrical 3D polar pattern with a no. of radiation lobes.



→ fig. shows a linear 2D pattern where the same pattern characteristics are indicated (power pattern)



→ A major lobe is also called main beam is defined as the radiation lobe containing the direction of maximum radiation

→ major lobe is pointing in the $\theta=0^\circ$ direction. In some antennas, there may exist

power radiated by the alternating current element / very small dipole/infinitermal small element:

$$P_d = \frac{d P_{rad}}{ds}$$

$$d P_{rad} = P_d \cdot ds$$

$$P_{rad} = \oint P_d \cdot s$$

According to pointing theorem

$$\bar{P} = \bar{E} \times \bar{H}$$

$$\bar{P} = \begin{vmatrix} \bar{a}_x & r\bar{a}_\theta & r\sin\phi \bar{a}_\phi \\ E_x & E_\theta & E_\phi \\ H_x & H_\theta & H_\phi \end{vmatrix}$$

$$= \begin{vmatrix} \bar{a}_x & r\bar{a}_\theta & r\sin\phi \bar{a}_\phi \\ E_x & E_\theta & 0 \\ 0 & 0 & H_\phi \end{vmatrix}$$

$$\bar{P} = \bar{a}_x [E_\theta H_\phi] - r\bar{a}_\theta [E_\phi H_\phi]$$

$$P_\theta = -rE_\theta H_\phi$$

$$P_\phi = E_\phi H_\phi$$

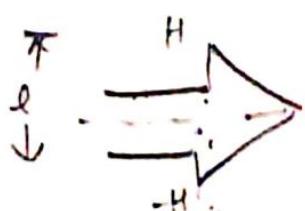
$$\text{Then, } P_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2 I_{rms}^2$$

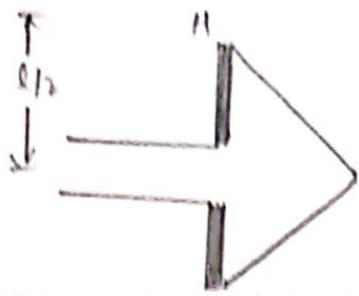
SHORT LINEAR ANTENNA'S:-

Assume an alternating current element is very very small antenna cannot exist practically.

practical antenna

- 1. dipole antenna
 - 2. monopole antenna
- These are short antenna





this antenna can also radiate since ground acts as a image antenna.

current element dipole

1) when the length and current of alternating current element & dipole are equal then,

$$\cdot (\text{Field})_{\text{dipole}} = \frac{1}{\lambda} (\text{Field})_{\text{ace}}$$

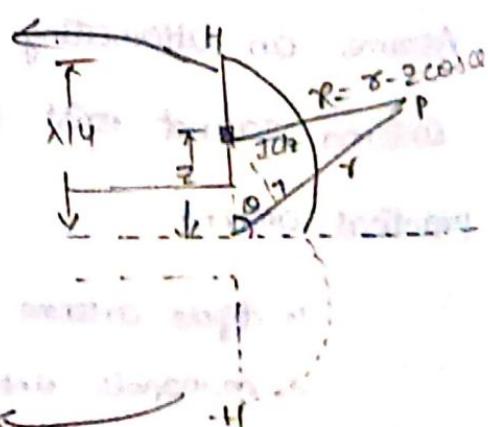
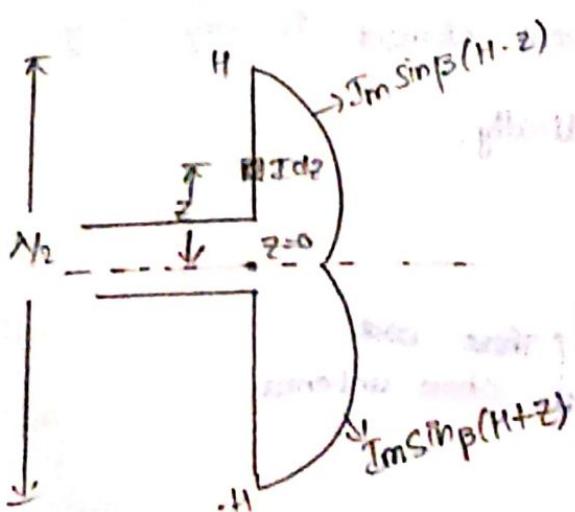
$$\cdot (P_{\text{rad}})_{\text{dipole}} = \frac{1}{4} (P_{\text{rad}})_{\text{ace}}$$

$$\cdot (R_{\text{rad}})_{\text{dipole}} = \frac{1}{4} (R_{\text{rad}})_{\text{ace}}$$

This valid up to $\lambda/4$ antennas.

18/7/16

Derivation for radiation resistance of quarter wave monopole & half wave dipole.



$$I = I_m \sin \beta (H-z) \quad , \text{for } z > 0$$

$$I = I_m \sin \beta (H+z) \quad , \text{for } z < 0$$

Procedure :-

1. Write the expressions for current distributions.

Obtain the expression for vector magnetic potential (A).

Obtain H from ' A '.

Obtain E from ' η_0 '.

Calculate the power radiated and average power.

Obtain the Radiation Resistance from power radiated.

Step - 1 :-

$$I = I_m \sin \beta (H-z) \quad \text{for } z > 0 \rightarrow ①$$

$$I = I_m \sin \beta (H+z) \quad \text{for } z < 0 \rightarrow ②$$

Step - 2 :-

$$dA_z = \frac{\mu}{4\pi} \cdot \frac{Idz}{R} \cdot e^{-JBR}$$

$$A_z = \int_{-H}^H \frac{\mu}{4\pi} \cdot \frac{Idz}{R} e^{-JBR}$$

$$= \frac{\mu}{4\pi R} \left[0 \int I \cdot e^{-J\beta(r-z\cos\theta)} dz + H \int I e^{-J\beta(r-z\cos\theta)} dz \right]$$

$$= \frac{\mu}{4\pi R} e^{-JBr} \left[0 \int I_m \sin \beta (H-z) e^{Jz \beta \cos \theta} dz + H \int I_m \sin \beta (H-z) e^{Jz \beta \cos \theta} dz \right]$$

$$A_z = \frac{\mu I_m}{4\pi R} e^{-JBr} \left[0 \int \sin \beta (H+z) e^{Jz \beta \cos \theta} dz + H \int \sin \beta (H-z) e^{Jz \beta \cos \theta} dz \right]$$

Now, for monopole $H = \lambda/4$ & $B = \frac{2\pi}{\lambda}$

$$\sin \beta(H+z) = \sin(\beta H + \beta z) = \sin\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{4} + \beta z\right)$$

$$= \sin\left(\frac{\pi}{2} + \beta z\right) = \cos \beta z$$

$$\sin \beta(H-z) = \cos \beta z$$

then,

$$\begin{aligned} A_z &= \frac{\mu_{\text{Im}}}{4\pi R} e^{-jBr} \left[\int_{-H}^0 \cos \beta z e^{jBz \cos \theta} dz + \int_0^H \cos \beta z e^{jBz \cos \theta} dz \right] \\ &= \frac{\mu_{\text{Im}}}{4\pi R} e^{-jBr} \left[\int_0^H \cos \beta z e^{jBz \cos \theta} dz + \int_0^H \cos \beta z e^{-jBz \cos \theta} dz \right] \\ &= \frac{\mu_{\text{Im}}}{4\pi R} e^{-jBr} \left[\int_0^H \cos \beta z (e^{jBz \cos \theta} + e^{-jBz \cos \theta}) dz \right] \end{aligned}$$

Then,

$$A_z = \frac{\mu_{\text{Im}}}{2\pi \beta R} e^{-jBr} \left[\frac{\cos \frac{\pi}{2} \cos \theta}{\sin^2 \theta} \right]$$

Now,

$$\vec{B} = \nabla \times \vec{A}$$

$$\mu \vec{H} = \nabla \times \vec{A}$$

$$\mu (H_r \bar{a}_r + H_\theta \bar{a}_\theta + H_\phi \bar{a}_\phi) = \nabla \times \vec{A}$$

$$\begin{aligned} \nabla \times \vec{A} &= \begin{vmatrix} \bar{a}_r & \tau \bar{a}_\theta & \tau \sin \theta \bar{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & \tau A_\theta & \tau \sin \theta A_\phi \end{vmatrix} = 0 \\ &\quad \frac{1}{\tau^2 \sin \theta} \end{aligned}$$

$$= \frac{1}{\tau^2 \sin \theta} \left[\tau \sin \theta \bar{a}_\phi \left(\frac{\partial}{\partial r} (\tau A_\theta) - \frac{\partial}{\partial \theta} A_r \right) \right]$$

$$= \frac{1}{\sigma} \left(\frac{\partial}{\partial \theta} (\gamma A_0) - \frac{\partial}{\partial \theta} A_\theta \right) \bar{A}_y$$

$$\therefore H_{Hf} = \frac{1}{\sigma} \left[\frac{\partial}{\partial \theta} (\gamma A_0) - \frac{\partial}{\partial \theta} (A_\theta) \right]$$

$$H_f = \frac{1}{\mu \sigma} \left[\frac{\partial}{\partial \theta} (\gamma A_0) \right]$$

$$= \frac{1}{\mu \sigma} \left[\frac{\partial}{\partial \theta} (-A_2 \sin \theta) \right]$$

$$= -\frac{1}{\mu \sigma} \left[\frac{\partial}{\partial \theta} \cdot \gamma \cdot \frac{\mu I_m e^{-jB\theta}}{2\pi\sigma\beta} \cdot \frac{\cos \frac{\pi}{\sigma} \cos \theta}{\sin^2 \theta} \sin \theta \right]$$

$$= -\frac{I_m}{2\pi\sigma\beta} \cdot \frac{\cos \frac{\pi}{\sigma} \cos \theta}{\sin^2 \theta} \frac{\partial}{\partial \theta} (e^{-jB\theta})$$

$$= -\frac{I_m}{2\pi\sigma\beta} \cdot \frac{\cos \frac{\pi}{\sigma} \cos \theta}{\sin^2 \theta} \cdot e^{-jB\theta} \cdot (-jB)$$

$$H_f = \frac{J I_m}{2\pi\sigma} e^{-jB\theta} \cdot \frac{\cos \frac{\pi}{\sigma} \cos \theta}{\sin^2 \theta}$$

Now,

$$\eta_0 = \frac{E_0}{H_f}$$

$$E_0 = \eta_0 H_f = 120\pi H_f$$

$$E_0 = \frac{60 J I_m}{\pi \sin^2 \theta} e^{-jB\theta} \cdot \cos \frac{\pi}{\sigma} \cos \theta$$

Now,

$$P_T = E_0 H_f$$

$$= \frac{60 J I_m}{\pi \sin^2 \theta} e^{-jB\theta} \cdot \cos \frac{\pi}{\sigma} \cos \theta \cdot \frac{J I_m}{2\pi\sigma} \cdot e^{-jB\theta} \cdot \frac{\cos \frac{\pi}{\sigma} \cos \theta}{\sin^2 \theta}$$

$$P_T = \frac{-60 J^2 m^2}{\pi^2 \sigma^2 \sin^4 \theta} (e^{-jB\theta} \cdot \cos \frac{\pi}{\sigma} \cos \theta)^2$$

$$P_T = \frac{-60 (I_m \sqrt{2})^2}{\pi^2 \sigma^2 \sin^4 \theta} (e^{-jB\theta} \cdot \cos \frac{\pi}{\sigma} \cos \theta)^2$$

$$P_T = \frac{-60}{\pi r^2 \sin^2 \theta} \cdot I_{rms}^2 (e^{-jBr} \cdot \cos \phi \cos \theta)^2$$

Now,

$$P_{radiated} = \oint P_T d\sigma$$

$$= \oint P_T \cdot r^2 \sin^2 \theta d\theta d\phi$$

$$P_{avg} = \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi} \frac{-60}{\pi r^2 \sin^2 \theta} I_{rms}^2 (e^{-jBr} \cdot \cos \phi \cos \theta)^2 r^2 \sin^2 \theta d\theta d\phi$$

$$= \frac{-60}{\pi} \cdot I_{rms}^2 \times 2\pi \times 0.609$$

$$= 60 I_{rms}^2 \times 0.609$$

$$P_{avg} = 36.54 I_{rms}^2 \rightarrow \text{monopole}$$

$$R_{rad} = 36.54 \rightarrow \text{monopole}$$

$$R_{rad} = 73.08 \rightarrow \text{dipole}$$

19/8/16

ANTENNA THEOREMS:

These antenna theorems are derived from network theorems.

1. Thevenin's theorem.
2. Superposition theorem.
3. Max. power transfer theorem.
4. Reciprocity theorem
5. Compensation theorem.

Need of Antenna theorem:-

To solve Antenna problems i.e; in case of antenna's the properties of transmitting antenna or receiving antenna

Unit - III

ANTENNA ARRAYS

- A group of antennas is called array of antennas or simply antenna arrays.
- Antenna array can be defined as the system of similar antennas directed to get the required high directivity and increased field strength (gain) in the desired direction.
- The total fields produced by the antennas array is the vector sum of the fields produced by the individual antennas in the array.
- The individual antenna in the array is called element of the array.
- The antenna array is said to be linear if the elements in the array are equally spaced along a straight line.
- The linear antenna array is said to be uniform linear array if all the elements are fed with a current of equal magnitude with progressive uniform phase shift along the line.
- Hence antenna array is a radiating system in which individual array elements contribute to obtain maximum field strength



Uniform linear array having no elements following the last one.

Following the elements NA is

Types of Arrays

1. Broad Side Array (BSA) bottom of antenna to ground

2. End Fire Array (EFA)

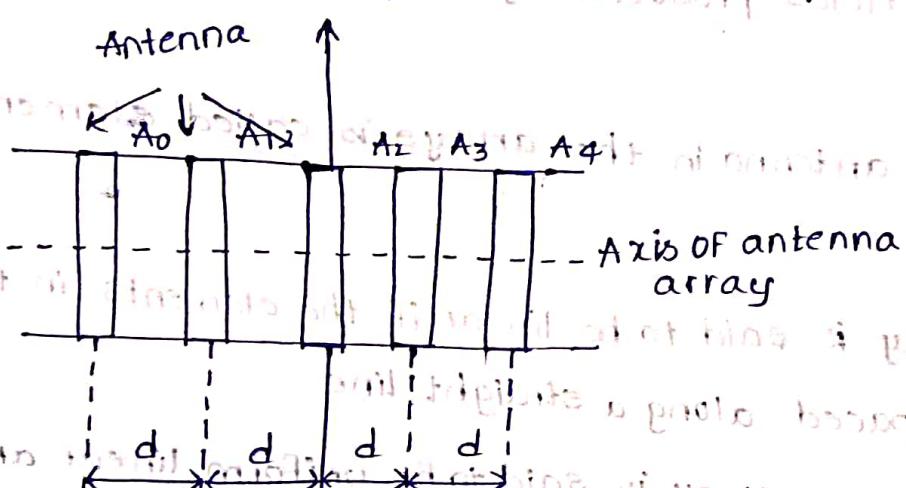
3. collinear Array (CA) horizontal line pattern

4. Parasitic Array (PA) add top of bottom surface

Broad side Array (BSA):-

maximum radiation

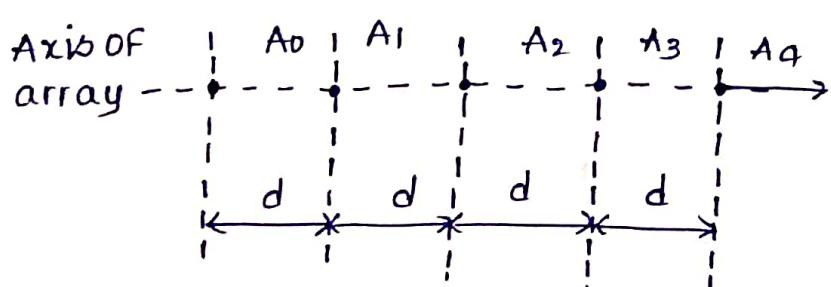
Antenna



a) TOP view

Broadside array of 4 vertical dipole antennas

antennas



b) front view.

→ Elements are placed in straight rod with equal spacing

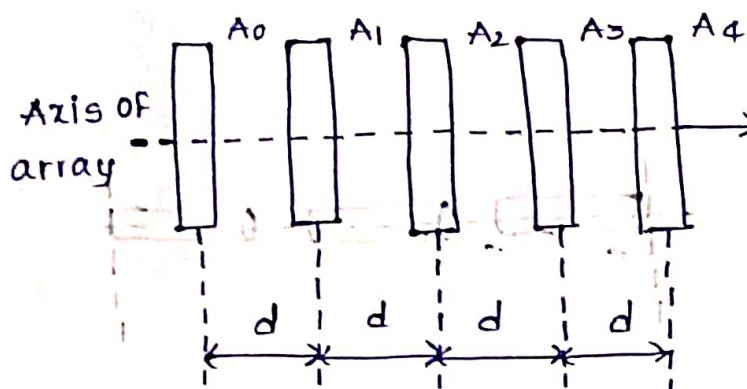
→ All elements are parallel

→ All elements \Rightarrow fed with I and equal magnitude & phase

→ Radiation pattern is Bidirectional

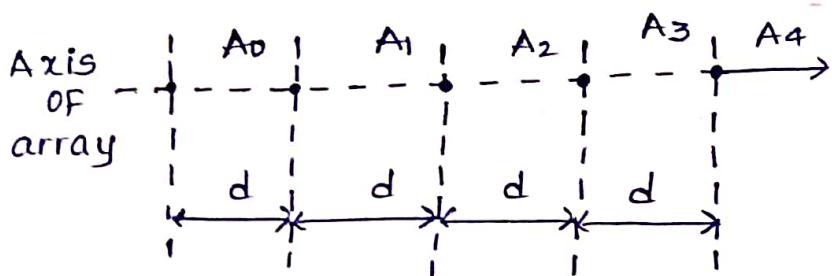
→

End fire Array (EFA) :-



Direction of maximum radiation
from left to right

(a) Top view



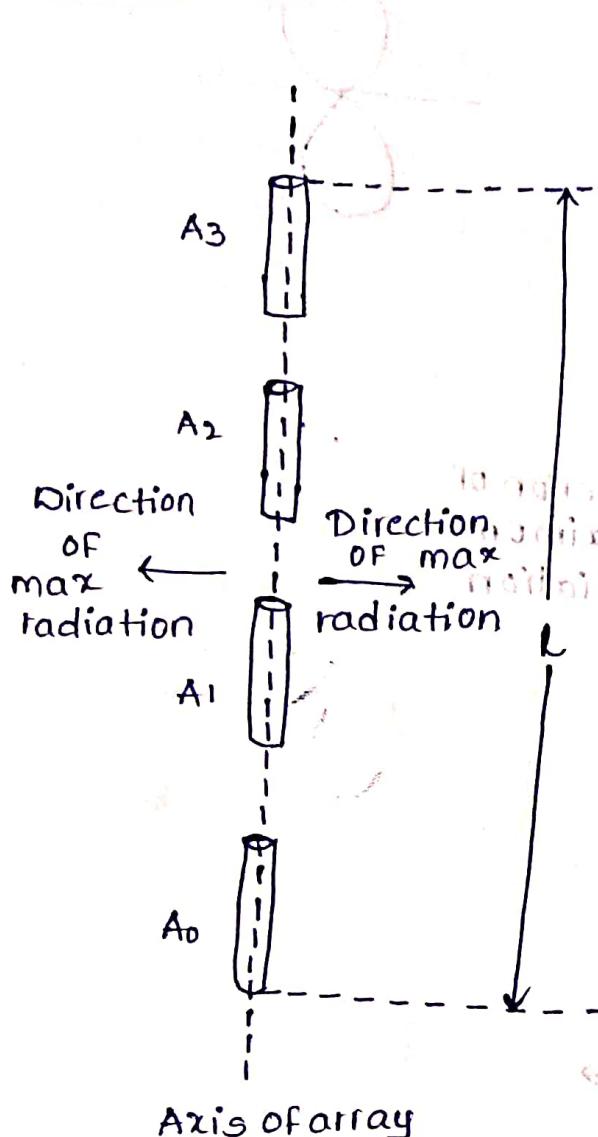
Pattern is unidirectional

(b) Front view

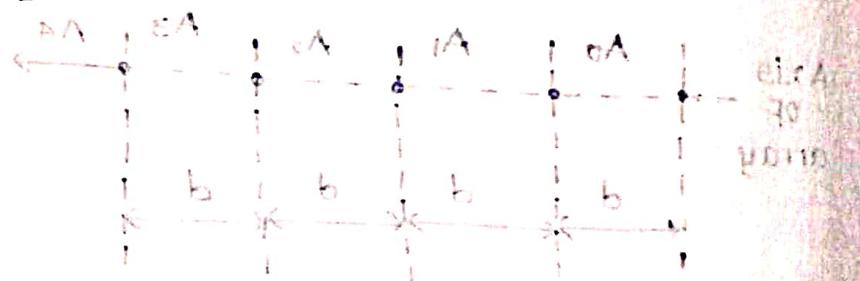
It is unidirectional pattern



collinear Array



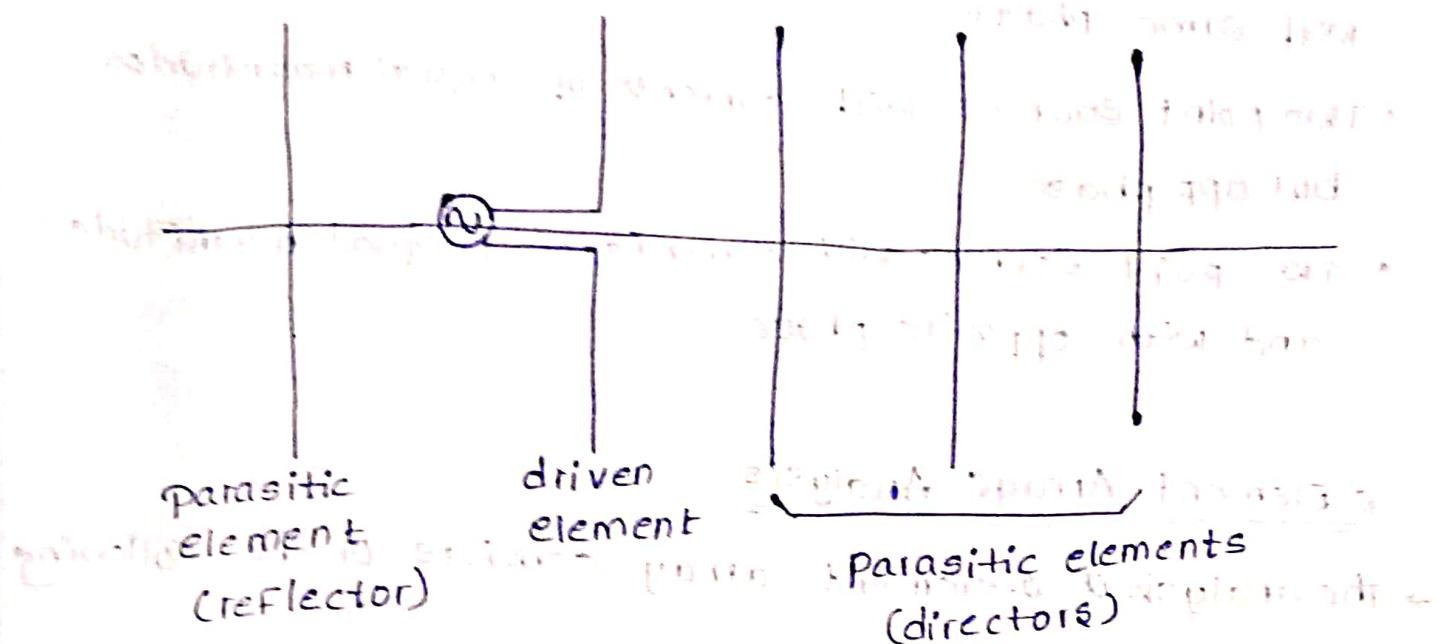
(a) Vertical



Divide into 5

Moving towards center of it

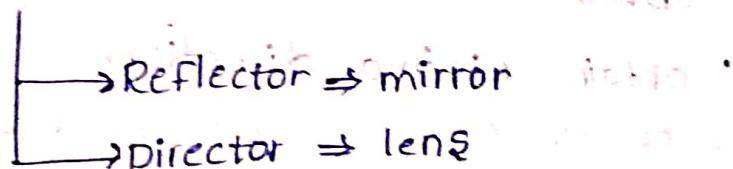
In parasitic array all the elements in the current loop are driven to produce the radiation pattern.



Elements without any current \Rightarrow parasitic elements.

Elements with current \Rightarrow driven element(s).

parasitic elements \Rightarrow current points.



Array of point sources / Element Arrays

- The array of point sources is nothing but the array of an isotropic radiators occupying zero volume.
- For large no. of point sources in the array, the analysis becomes complicated and time consuming.
- The simplest condition is array of two point sources and then extended to any no. of sources.
- Let us consider two point sources with a distance of separation 'd' between them. The following are different cases

of array of two point sources.

- Two point sources with currents of equal magnitudes and with same phase.
- Two point sources with currents of equal magnitudes but opp phase.
- Two point sources with currents of unequal magnitudes and with opposite phase.

2 Element Arrays Analysis

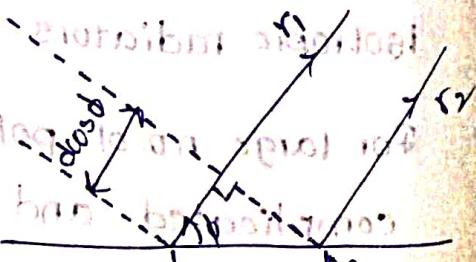
→ The analysis of 2-element array consists of the following steps.

- calculate phase shift (ψ)
- obtain Total field, (E_T)
- calculate Array Factor (AF)
- obtain maxima direction
- obtain minima direction
- obtain Half Power Point direction
- Draw the Radiation pattern

Two point sources with currents equal in Magnitude and phase.

→ Consider two point sources A_1 and A_2

separated by distance as shown in figure. consider that both the point have equal in magnitude and phase



→ Consider point P faraway from the array.

→ Let this distance between point P and the array.

and point sources A_1 and A_2 be r_1 and r_2 respectively. As these radial distances are extremely large as compared with the distance of separation between two point sources i.e., d , we can assume,

$$r_1 = r_2 = r$$

The radiation from the point source A_2 will reach earlier at point P than that from point source A_1 because of the path difference. The extra distance is travelled by the wave radiated from point source A_1 than that by the wave radiated from point source A_2 .

Hence path difference is given by,

$$\boxed{\text{path difference} = d \cos \phi}$$

The path difference can be expressed in terms of wavelength as,

$$\text{path difference} = \frac{d \cos \phi}{\lambda}$$

→ Hence, the phase angle ψ is given by

$$\text{phase angle} = \psi = 2\pi (\text{path difference})$$

$$= 2\pi \left(\frac{d \cos \phi}{\lambda} \right)$$

$$\boxed{\psi = \frac{2\pi}{\lambda} d \cos \phi, \text{ radians}}$$

(or)

$$\boxed{\psi = \beta d \cos \phi}$$

$$\therefore \text{where, } \beta = \frac{2\pi}{\lambda}$$

→ Let E_1 be the far field at a distant point P due to point source A_1 . Similarly let the far field at point P due to point source A_2 . Then the total field at point P addition

OF the two field components due to the point sources A_1 and A_2 . If $\psi = \beta d \cos\phi$ then the far field component at point to point source A_1 given by

$$E_1 = E_0 e^{-j\psi/2}$$

to write the far field component at point A_2 given by

→ Similarly, the far field component at point p due to the point source A_2 is given by

~~because both the point sources are identical hence their far field components will be same~~ $E_2 = E_0 e^{j\psi/2}$

→ Note that the amplitude of both the field components is E_0 as currents are same the point sources are identical.

⇒ The total field at point p is given by,

$$E_T = E_1 + E_2$$

$$= E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$= E_0 \left[e^{-j\psi/2} + e^{j\psi/2} \right]$$

$$= 2E_0 \left[\frac{e^{-j\psi/2} + e^{j\psi/2}}{2} \right]$$

$$\boxed{E_T = 2E_0 \cos\left(\frac{\psi}{2}\right)}$$

Sub ψ in equ①

$$E_T = 2E_0 \cos\left(\frac{\beta d \cos\phi}{2}\right)$$

Array factor is given by,

$$AF = \frac{|E_T|}{|E_{max}|}$$

$$= \frac{2E_0 \cos\left(\frac{\beta d \cos\phi}{2}\right)}{2E_0}$$

$$AF = \cos\left(\frac{\beta d \cos\phi}{2}\right)$$

→ The total field is max

$$\cos\left(\frac{\beta d \cos\phi}{2}\right) = \pm 1$$

→ Distance between two point source be $\lambda/2$

$$\cos\left[\frac{\beta(\lambda/2)\cos\phi}{2}\right] = \pm 1$$

$$\cos\left[\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos\phi\right] = \pm 1$$

$$\cos\left[\frac{\pi}{2} \cos\phi\right] = \pm 1$$

$$\text{i.e., } \frac{\pi}{2} \cos\phi_{\max} = \cos^{-1}(\pm 1) = \pm n\pi$$

→ If $n=0$, then

$$\frac{\pi}{2} \cos\phi_{\max} = 0$$

$$\cos\phi_{\max} = 0$$

$$\boxed{\phi_{\max} = 90^\circ \text{ (or) } 270^\circ}$$

Minima direction:-

Again from equ(4.4.9), total field strength is minimum when $\cos\left(\frac{\beta d \cos\phi}{2}\right)$ is minimum i.e, 0 as cosine of angle has minimum value 0. Hence the condition for minima is given by,

$$\therefore \cos\left(\frac{\beta d \cos\phi}{2}\right) = 0$$

Again assuming $d = \frac{\lambda}{2}$ and $\beta = \frac{2\pi}{\lambda}$, we can write

$$\cos\left(\frac{\pi}{2} \cos\phi_{\min}\right) = 0 \rightarrow 4.4.12$$

$$\therefore \frac{\pi}{2} \cos\phi_{\min} = \cos^{-1} 0 = \pm(2n+1)\frac{\pi}{2}, \text{ where } n=0, 1, 2, \dots$$

If $n=0$, then,

$$\frac{\pi}{2} \cos\phi_{\min} = \pm \frac{\pi}{2}$$

$$\text{i.e., } \cos\phi_{\min} = \pm 1$$

$$\text{i.e., } \boxed{\phi_{\min} = 0^\circ \text{ or } 180^\circ}$$

$\rightarrow 4.4.13$

Half power point directions

When the power is half, the voltage or current is $\frac{1}{\sqrt{2}}$ times the maximum value. Hence the condition for half power point is given by,

distance
b/w 2 elem
-cnts in led array \leftarrow $d = \frac{\lambda}{2}$, $\beta = \frac{2\pi}{\lambda}$

$$\cos\left(\frac{\beta d \cos\phi}{2}\right) = \pm \frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{\pi}{2} \cos\phi\right) = \pm \frac{1}{\sqrt{2}}$$

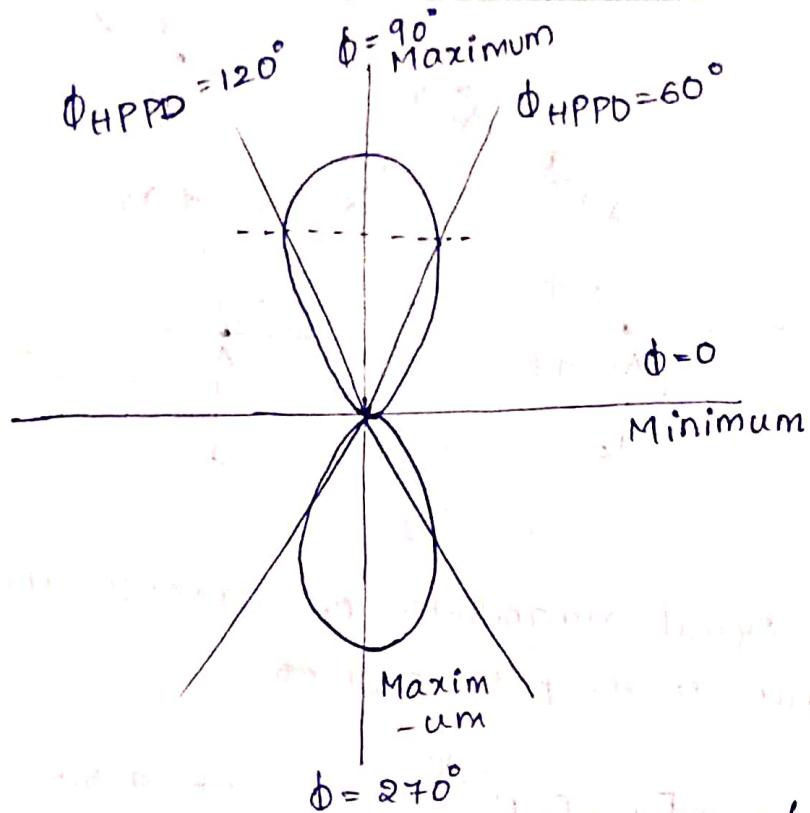
$$\text{i.e., } \frac{\pi}{2} \cos\phi = \cos^{-1}\left(\pm \frac{1}{\sqrt{2}}\right) = \pm(2n+1)\frac{\pi}{4}, \text{ where } n=0, 1, 2, \dots$$

If $n=0$, then

$$\frac{\pi}{2} \cos\phi_{HPPB} = \pm \frac{\pi}{4}$$

$$\text{i.e., } \cos\phi_{HPPB} = \pm \frac{1}{2}$$

$$\text{i.e., } \phi_{HPPB} = 60^\circ \text{ and } 120^\circ$$

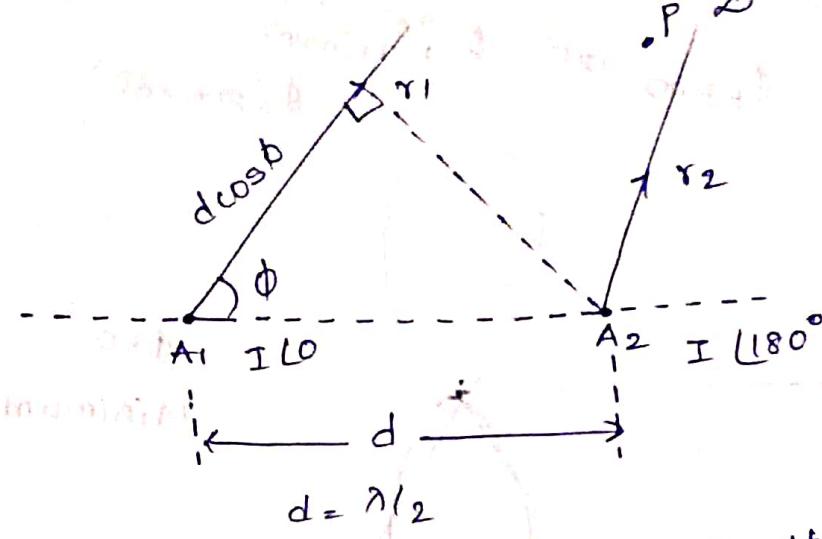


The field pattern obtained is bidirectional and it is a figure of eight (8). If this pattern is rotated by 360° about axis, it will represent three dimensional doughnut shaped

** Two point sources with currents Equal in magnitudes but opposite in phase.

Consider 2 point sources separated by distance d and supplied with currents equal magnitude but opposite in phase. consider Fig 4.4.1 from Section 4.4. All the directions are exactly same except the phase of the currents is opposite i.e., 180° . With the condition, the total field at far point P is given by,

$$E_T = (-E_1) + (E_2) \rightarrow (4.5.1)$$



Assuming equal magnitudes of currents, the fields at point P due to the point sources

$$E_1 = E_0 e^{-j\frac{\psi}{2}} \rightarrow 4.5.2$$

$$E_2 = E_0 e^{j\frac{\psi}{2}} \rightarrow 4.5.3$$

Substituting values of E_1 and E_2 in equation (4.5.1), we

get,

$$E_T = -E_0 \cdot e^{-j\frac{\psi}{2}} + E_0 \cdot e^{j\frac{\psi}{2}}$$

$$E_T = E_0 \left[e^{j\frac{\psi}{2}} - e^{-j\frac{\psi}{2}} \right]$$

Rearranging the terms in above equation, we get

$$E_T = (2j)(E_0) \left[\frac{e^{j\frac{\psi}{2}} - e^{-j\frac{\psi}{2}}}{2j} \right] \rightarrow 4.5.4$$

$$E_T = 2j E_0 \sin\left(\frac{\psi}{2}\right) \rightarrow 4.5.5$$

$$\psi = \beta d \cos \phi \rightarrow 4.5.6$$

$$E_T = j(2E_0) \sin\left(\frac{\beta d \cos \phi}{2}\right) \rightarrow 4.5.7$$

Maxima direction:-

from equation (4.5.7), the total field is maximum when $\sin\left(\frac{\beta d \cos\phi}{2}\right)$ is maximum i.e. as the max value of sine of angle is ± 1 . Hence condition for maxima is given by

$$\sin\left(\frac{\beta d \cos\phi}{2}\right) = \pm 1 \rightarrow 4.5.8$$

Let the spacing between two isotropic point sources be

equal to $\frac{\lambda}{2}$ i.e. $d = \frac{\lambda}{2}$. Substitute $d = \frac{\lambda}{2}$ and $\beta = \frac{2\pi}{\lambda}$

in equation (4.5.8), we get,

$$\sin\left(\frac{\pi}{2} \cos\phi\right) = \pm 1$$

$$\text{i.e., } \frac{\pi}{2} \cos\phi = \pm (2n+1) \frac{\pi}{2} \text{ where } n=0, 1, 2, \dots$$

i.e.

If $n=0$, then,

$$\frac{\pi}{2} \cos\phi_{\max} = \pm \frac{\pi}{2}$$

$$\text{i.e., } \cos\phi_{\max} = \pm 1$$

$$\text{i.e., } \phi_{\max} = 0^\circ \text{ and } 180^\circ \rightarrow 4.5.9$$

Minima direction:-

Again in equation (4.5.7), total field strength is minimum when $\sin\left(\frac{\beta d \cos\phi}{2}\right)$ is minimum i.e. 0.

Hence the condition for minima is

$$\sin\left(\frac{\beta d \cos\phi}{2}\right) = 0 \rightarrow 4.5.10$$

Assuming $d = \frac{\lambda}{2}$ and $\beta = \frac{2\pi}{\lambda}$ in equation (4.5.10) we get,

$$\sin\left(\frac{\pi}{2}\cos\phi\right) = 0$$

i.e., $\frac{\pi}{2}\cos\phi = \pm n\pi$, where $n=0, 1, 2, -1, -2, \dots$

If $n=0$, then

$$\frac{\pi}{2}\cos\phi_{\min} = 0$$

$$\text{i.e., } \cos\phi_{\min} = 0$$

$$\text{i.e., } \boxed{\phi_{\min} = +90^\circ \text{ or } -90^\circ} \rightarrow 4.5.11$$

Half power point Direction (HPPD)

When the power is half of maximum value, the voltage or current equals to $\frac{1}{\sqrt{2}}$ times the respective maximum value. Hence the condition for the half power point can be obtained from equation (4.5.7) as,

$$\sin\left(\frac{\beta d \cos\phi}{2}\right) = \pm \frac{1}{\sqrt{2}}, \text{ where } \rightarrow 4.5.12$$

Let $d = \frac{\lambda}{2}$ and $\beta = \frac{2\pi}{\lambda}$, we can write

$$\sin\left(\frac{\pi}{2}\cos\phi\right) = \pm \frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\phi = \pm (2n+1)\pi/4$$

(If $n=0$, $\frac{\pi}{2}\cos\phi_{\text{HPPD}} = \pm \frac{\pi}{4}$)

$$\phi_{\text{HPPD}} = 60^\circ \text{ or } 120^\circ$$

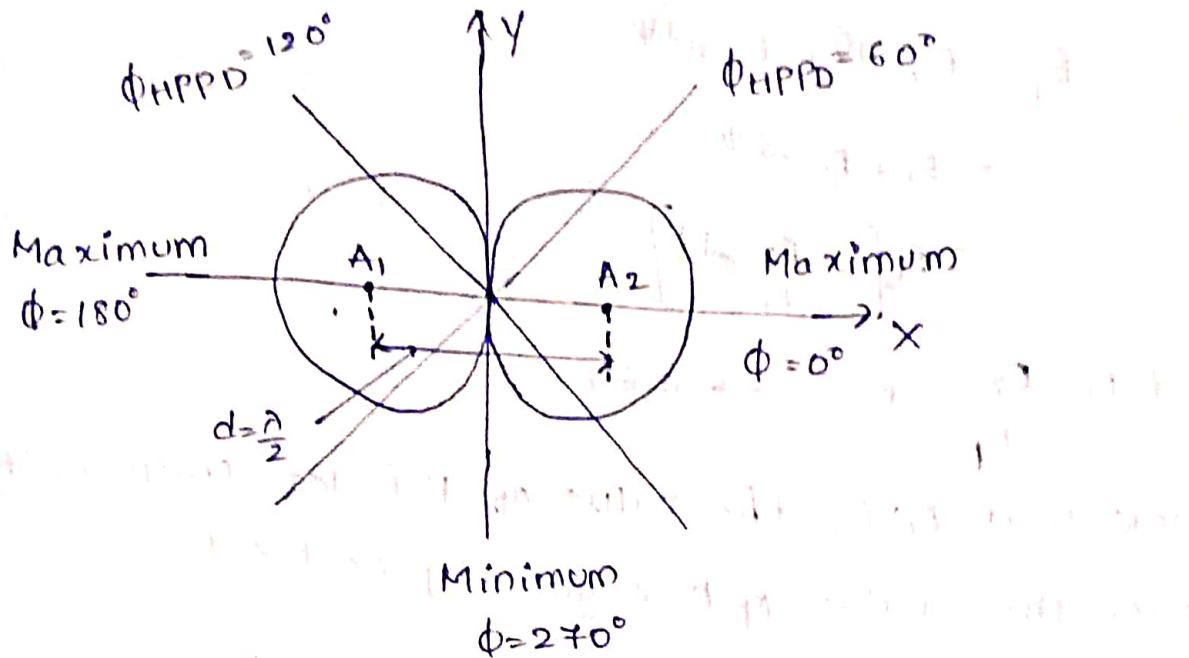


Fig 4.5.1: field pattern for two point sources with spacing $d = \frac{\lambda}{2}$ and fed with currents equal in magnitude but out of phase by 180° .

⇒ Two point sources with currents unequal in magnitudes and with any phase.

$\Psi \rightarrow \alpha$ factors

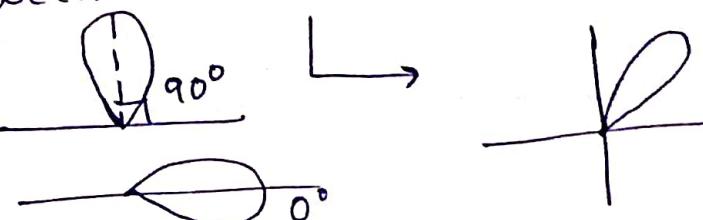
- i) due to path difference
- $\frac{2\pi}{\lambda} d \cos \phi = \beta d \cos \phi$
- ii) unequal phase

$$\Psi = \beta d \cos \phi + \alpha$$

α value lies in between 0° & 180° ($0^\circ < \alpha < 180^\circ$)

$\alpha = 0^\circ$, BSA

$\alpha = 180^\circ$, EFA



The total field $E_T = E_{11} + E_{22}$

$$\text{Reference} \leftarrow E_{11} = E_1 e^{j\psi} = E_1 e^{j0^\circ} = E_1$$

$$E_{22} = E_2 e^{j\psi}$$

$$E_T = E_1 e^{j\phi} + E_2 e^{j\psi}$$

$$= E_1 + E_2 e^{j\psi}$$

$$= E_1 \left[1 + \frac{E_2}{E_1} e^{j\psi} \right]$$

$$\text{Let, } \frac{E_2}{E_1} = K \rightarrow 4.6.2$$

Note that $E_1 > E_2$, the value of K is less than unity. Moreover the value of K is given by, $0 \leq K \leq 1$

$$\therefore E_T = E_1 \left[1 + K (\cos \psi + j \sin \psi) \right] \rightarrow 4.6.3$$

The magnitude of the resultant field at point P is given

by,

$$|E_T| = |E_1 [1 + K \cos \psi + j K \sin \psi]|$$

$$\therefore |E_T| = E_1 \sqrt{(1 + K \cos \psi)^2 + (K \sin \psi)^2} \rightarrow 4.6.4$$

The phase angle between two fields at the far point P is given by,

$$\theta = \tan^{-1} \frac{K \sin \psi}{1 + K \cos \psi} \rightarrow 4.6.5$$

N Element uniform linear Array :-

(Linear Arrays - All the elements in the array are equally spaced)

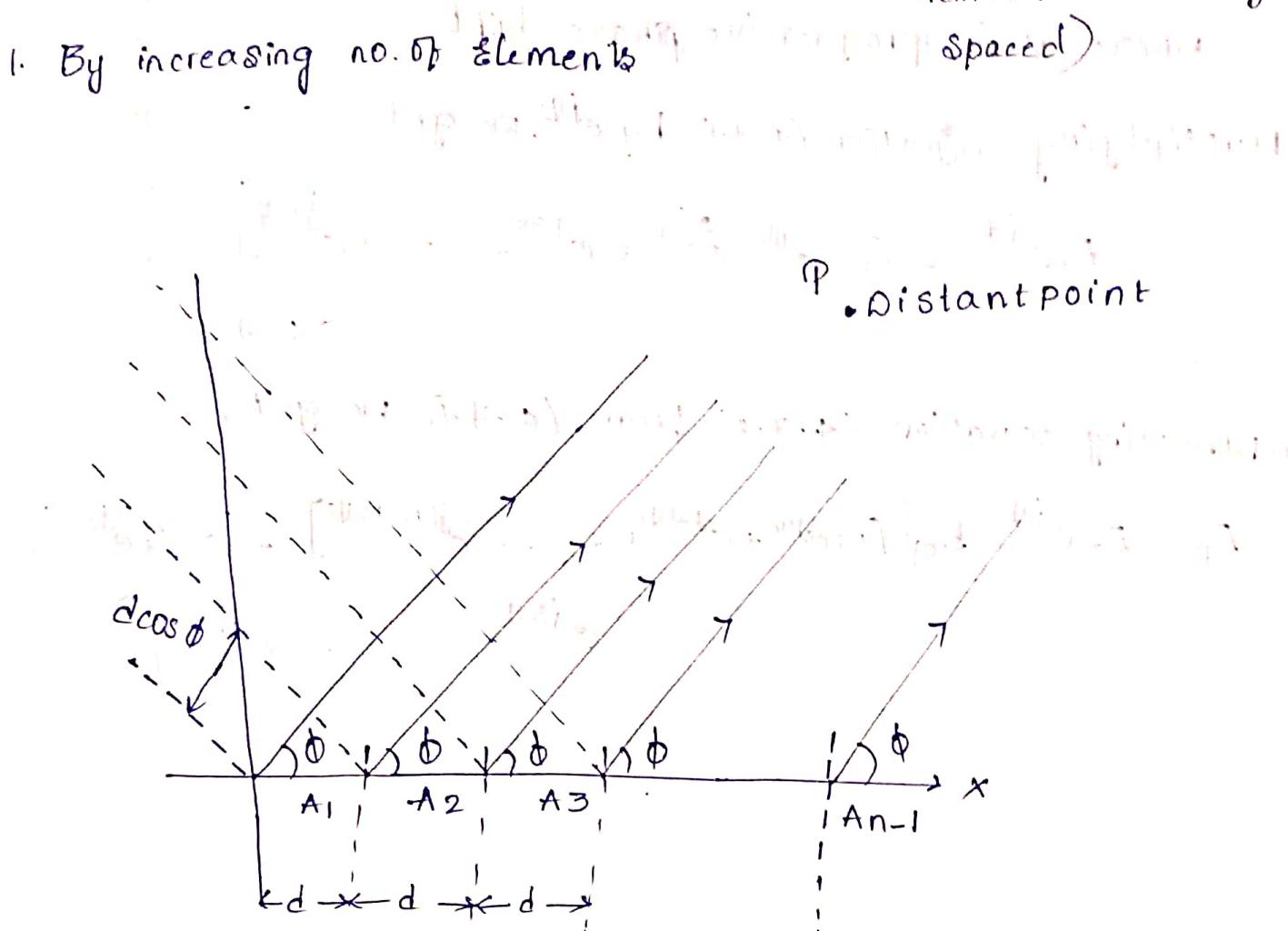


fig 4.7.1 uniform, linear array of n elements..

$$A_0 = I L_0$$

$$A_1 = I L \psi$$

$$A_2 = I L_2 \psi$$

$$\Psi = \beta d \cos \phi + \alpha \rightarrow \text{progressing phase shift}$$

The total resultant field at the distant point P is obtained by adding the fields due to dual sources.

Hence we can write,

$$E_T = E_0 e^{j\phi} + E_0 e^{j\psi} + E_0 e^{j2\psi} + \dots + E_0 e^{j(n-1)\psi}$$

$$E_T = E_0 [1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi}] \rightarrow 4.7.1$$

$$V = \beta d \cos \phi + \alpha$$

where α is progressive phase shift

Multiplying equation (4.7.1) by $e^{j\psi}$, we get

$$E_T e^{j\psi} = E_0 [e^{j\psi} + e^{2j\psi} + e^{3j\psi} + \dots + e^{jn\psi}]$$

→ 4.7.2

Subtracting equation (4.7.2) from (4.7.1), we get,

$$E_T - E_T e^{j\psi} = E_0 \{ [1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi}] - [e^{j\psi} + e^{j2\psi} + \dots + e^{jn\psi}] \}$$

$$\frac{E_T - E_T e^{j\psi}}{E_T} = \frac{[1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi}] - [e^{j\psi} + e^{j2\psi} + \dots + e^{jn\psi}]}{[1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi}]}$$

This ratio is growth constant, given by Eq. 4.7.3

$$ABT = e^{-\lambda T}$$

$$ABT < 100 \text{ m.s.}$$

ABT = 100

ABT = 100

ABT = 100

Finite band progressive wave

At a certain distance, diffraction losses become large so that amplitude of wave changes with position and frequency.

Diffraction loss is due to finite wavelength and finite aperture size.

$$ABT = \left(\frac{\lambda}{2d} \right)^2 \left(\frac{1}{\sin^2 \theta} + \frac{1}{\sin^2 (\theta + \phi)} \right)$$

The following are the Properties of N element BSA and EFA to draw radiation pattern.

→ obtain Total field (E_T)

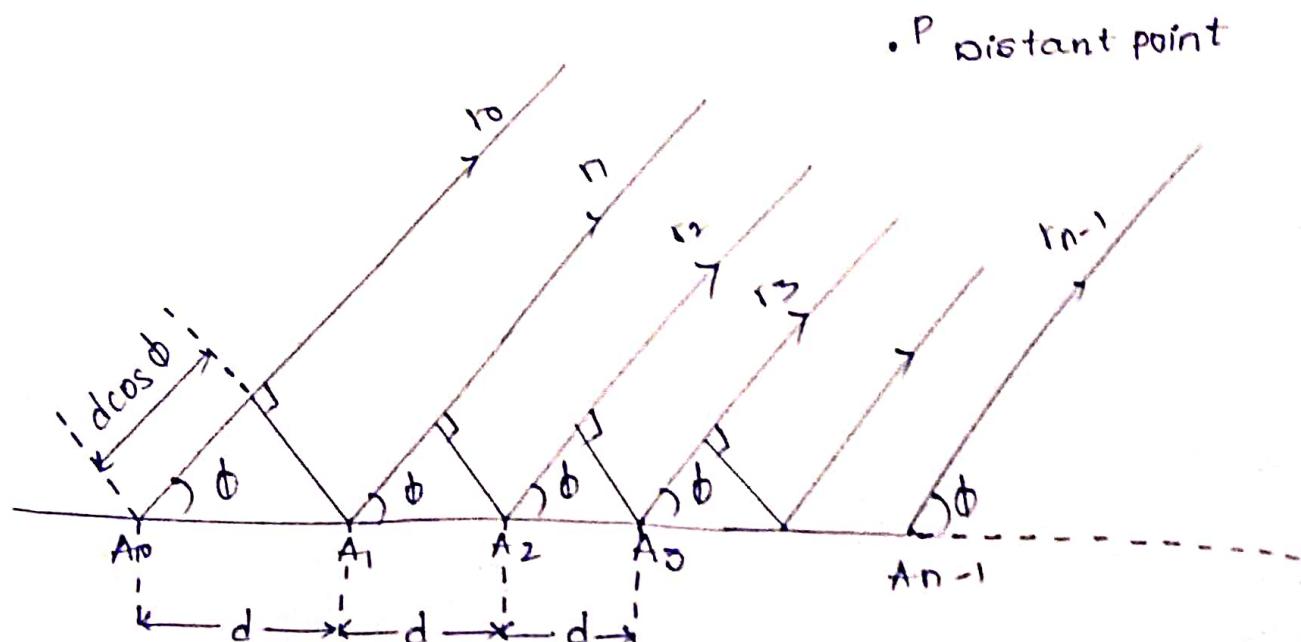
→ calculate Array Factor (AF)

→ properties or characteristics of BSA/EFA

- Major Lobe/ Maximum Radiation Direction
- Magnitude of major lobe
- Nulls/ No Radiation direction.
- Subsidary Maxima/Sidelobes
- Beamwidth of major lobe
- Directivity

→ Draw the Radiation Pattern.

Array of N Elements with equal Spacing & currents equal in magnitude and phase (N-Element Broadside Array)



$$A_0 = A_1 = A_2 = \dots = A_{n-1} = I \text{ } 10^{\circ}$$

$$r_0 \approx r_1 \approx r_2 \approx r_3 \dots \approx r_n$$

$$r_0 = r_1 + d \cos \phi$$

$d \cos \phi \rightarrow$ path difference

$$r_1 = r_0 - d \cos \phi$$

By

$$r_2 = r_1 - d \cos \phi$$

After one round trip, the path difference is

$d \cos \phi + d \cos \phi = 2d \cos \phi$

and the total path difference is

$2d \cos \phi \times n = 2nd \cos \phi$

and the path difference per round trip

$= 2d \cos \phi$

is the same as the distance between the two points.

After n round trips, the path difference will be $2nd \cos \phi$.

Opposite directions travel with nearly equal speeds.

Today's treatment:



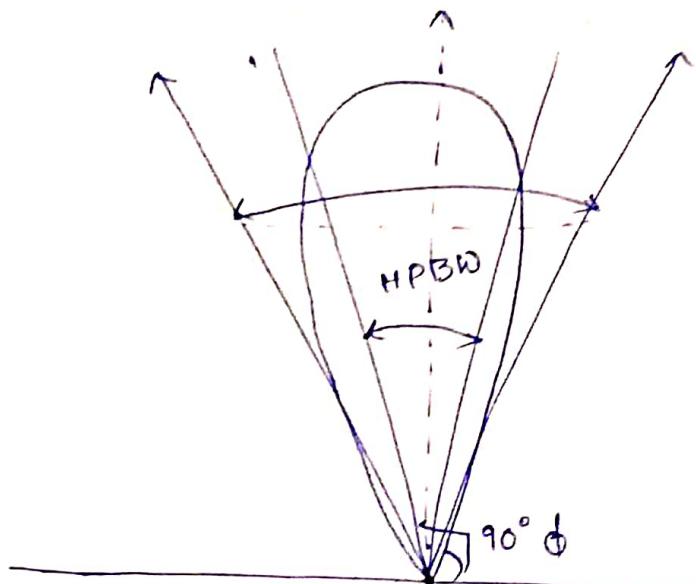
The angle between the path and the normal to the path is called the angle of incidence.

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Beamwidth of major lobe:

The beamwidth is defined as the angle between first nulls.
Alternatively beamwidth is the angle equal to twice the angle between first null and the major lobe maximum direction.



γ → angle in between first null & maximum radiation direction.

$$BWFN = 2 \times \gamma$$

$$\gamma = 90 - \phi$$

$$\gamma = 90 - \phi_{null}$$

$$\phi_{null} = 90 - \gamma$$

-Array of n Elements with Equal Spacing and currents Equal in Magnitude but with progressive phase shift- End fire Array

• consider n number of identical radiators supplied with equal current which are not in phase as shown in the fig 4.9.1
Assume that there is progressive phase lag of βd radians in each radiator.

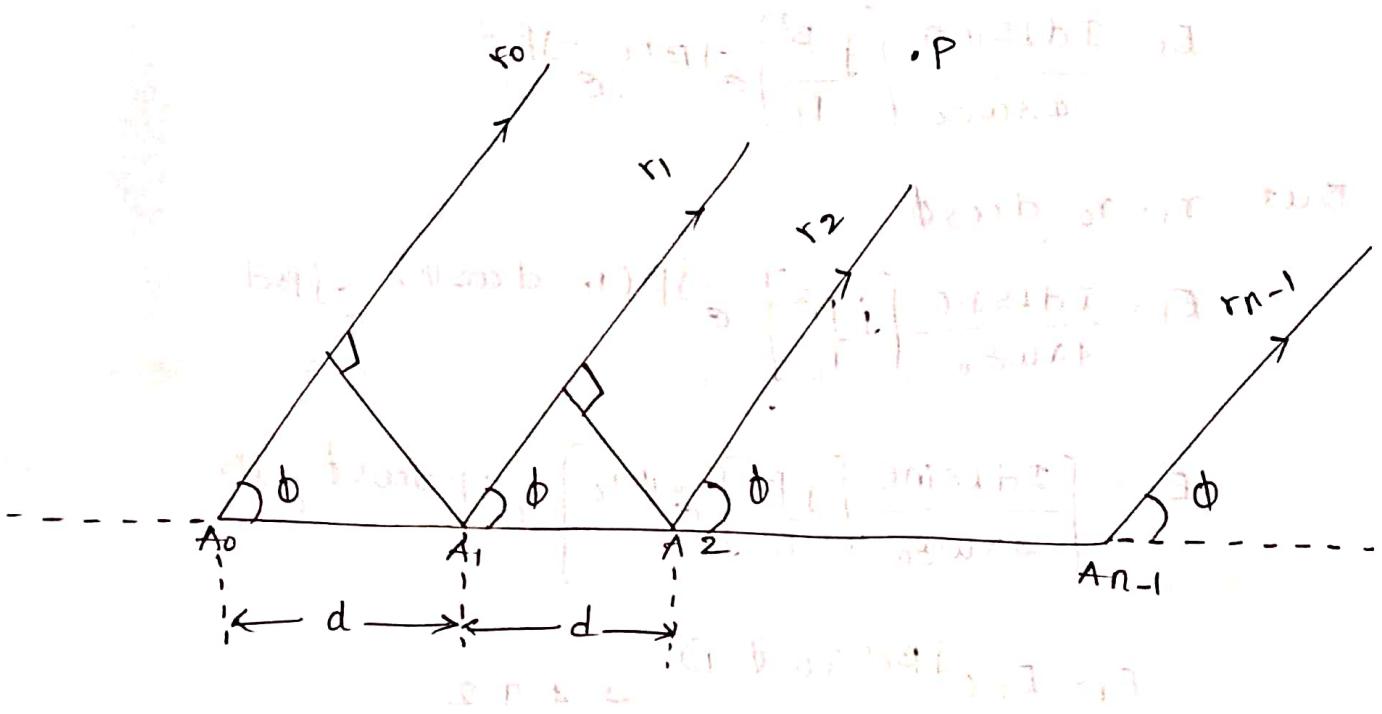


Fig. 4.9.1 End fire array

Consider that the current supplied to first element A_0 be I_0 . Then the current supplied to A_i is given by,

$$I_i = I_0 e^{-j\beta id}$$

Similarly the current supplied to A_2 is given by,

$$I_2 = I_0 e^{-j\beta d} = [I_0 e^{j\beta d}] e^{-j\beta d} = I_0 e^{-j2\beta d}$$

Thus the current supplied to the last element is given by,

$$I_{n-1} = I_0 e^{-j(n-1)\beta d}$$

The electric field produced at point P due to A_0 is

given by,

$$E_0 = \frac{IdL\sin\theta}{4\pi\omega\epsilon_0} \left[j \frac{\beta^2}{r_0} \right] e^{-j\beta r_0} \rightarrow 4.9.1$$

Then the electric field produced at point P, due to A_1 , is given by,

$$E_1 = \frac{IdL\sin\theta}{4\pi\omega\epsilon_0} \left[j \frac{\beta^2}{r_1} \right] e^{-j\beta r_1} e^{-j\beta d}$$

$$\text{But } r_1 = r_0 - d \cos\phi$$

$$E_1 = \frac{IdL\sin\theta}{4\pi\omega\epsilon_0} \left[j \frac{\beta^2}{r_0} \right] e^{-j\beta(r_0 - d \cos\phi)} e^{-j\beta d}$$

$$E_1 = \left[\frac{IdL\sin\theta}{4\pi\omega\epsilon_0} \left[j \frac{\beta^2}{r_0} \right] e^{-j\beta r_0} \right] e^{j\beta d \cos\phi} e^{-j\beta d}$$

$$E_1 = E_0 e^{j\beta d(\cos\phi - 1)} \rightarrow 4.9.2$$

$$\text{Let } \psi = \beta d(\cos\phi - 1) \rightarrow 4.9.3$$

$$\therefore E_1 = E_0 e^{j\psi}$$

The electric field produced at point P, due to A_2 is

given by,

$$E_2 = E_0 \cdot e^{j2\psi} \rightarrow 4.9.4$$

Similarly the electric field produced by A_{n-1} is given by,

$$E_{n-1} = E_0 e^{j(n-1)\psi} \rightarrow 4.9.5$$

Thus the resultant field at point P is given by,

$$E_T = E_0 + E_1 + E_2 + \dots + E_{n-1}$$

$$\therefore E_T = E_0 + E_0 e^{j\psi} + E_0 e^{j2\psi} + \dots + E_0 e^{j(n-1)\psi}$$

$$\therefore E_T = E_0 [1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi}] \rightarrow 4.9.6$$

using the concept of exponential series studied in previous section, we can write,

$$E_T = E_0 \frac{1 - e^{jn\psi}}{1 - e^{j\psi}}$$

$$\frac{E_T}{E_0} = \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \cdot e^{j \frac{(n-1)}{2}\psi} \rightarrow 4.9.7$$

considering only magnitude, we get,

$$\boxed{\left| \frac{E_T}{E_0} \right| = \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}}} \rightarrow 4.9.8$$

Binomial Array

- In linear arrays, there are n-isotropic sources of equal amplitudes are considered, but arrays of non-uniform amplitudes are also possible and binomial array is one of them.
- In this, the amplitudes of the radiating sources are arranged according to the coefficients of successive terms of binomial series $(a+b)^{n-1}$ and hence the name (where n is the no. of radiating sources).

$$(a+b)^{n-1} = a^{n-1} + \frac{(n-1)}{L_1} a^{n-2} b + \frac{(n-1)(n-2)}{L_2} a^{n-3} b^2 + \dots$$

B.P.A. \rightarrow
 L₁ L₂ L₃ \dots
 (n-1) (n-2) (n-3) \dots
 aⁿ⁻⁴ b³ \dots

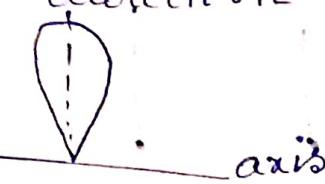
Where n = no. of radiating sources in the array.

- This was proposed by John Stone in 1929. He proposed that the secondary or side lobes in the linear array are to be eliminated totally if the radiating sources

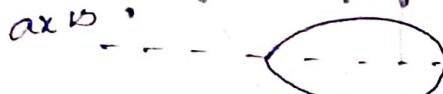
phased Arrays/Scanned Arrays

By using these Arrays we can find the maximum radiation pattern in any direction.

$$BSA \Rightarrow 90^\circ \text{ or } 270^\circ$$



$$EFA \Rightarrow 0^\circ$$



→ In BSA, the maximum radiation is perpendicular to the axis of the array.

→ In EFA, maximum radiation is along the axis of the array.

→ By adjusting the phase excitation b/w the elements, we can obtain the maximum radiation in any direction.

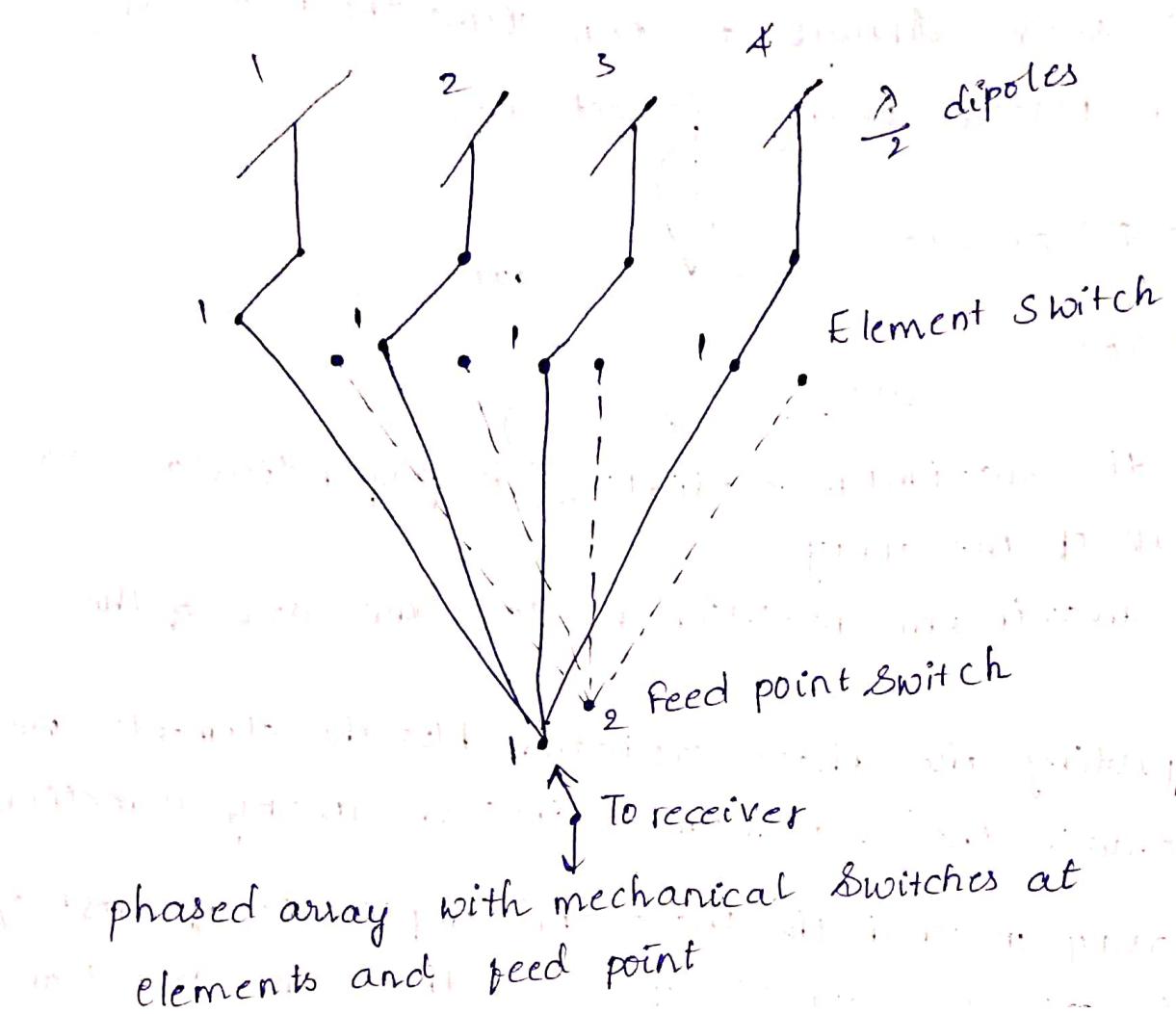
→ the array in which the phase and amplitude of most of the elements is variable, provided that the direction of maximum radiation and pattern shape along with the side-lobes is controlled is called phased or scanned array.

$$\psi = \beta d \cos\phi + \alpha = 0$$

$$\alpha = -\beta d \cos\phi_0$$

$$\beta = \frac{2\pi}{\lambda}, \quad d = \frac{\lambda}{2}$$

Basic Principle of Phased Arrays / Scanned Arrays



Yagi-Uda Antenna Design Calculations

To design Yagi-Uda antenna properly it is important to know the wavelength of the electromagnetic wave. The wavelength of the electromagnetic wave can be obtained by using relationship given as,

$$\lambda = \frac{c}{f}$$

where, c = velocity of light = 3×10^8 m/sec

f = frequency of electromagnetic wave

WKT, length of the dipole is given as,

$$L = \frac{\lambda}{2}$$

Putting value of λ in equ (6.5.2); we get

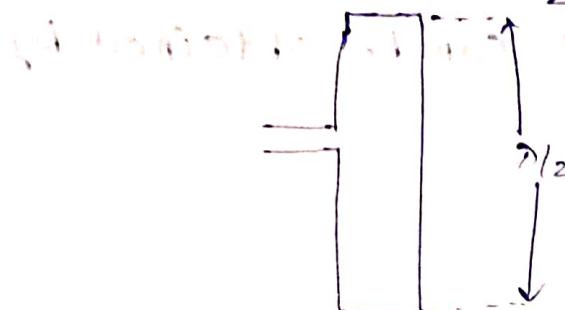
$$L = \frac{c/f}{2} = \frac{3 \times 10^8 / f \text{ (MHz)}}{2}$$
$$= \frac{150}{f} \text{ meter}$$

$$\boxed{L = \frac{150}{f \text{ (MHz)}} \text{ meter}}$$

Folded Dipole Antenna

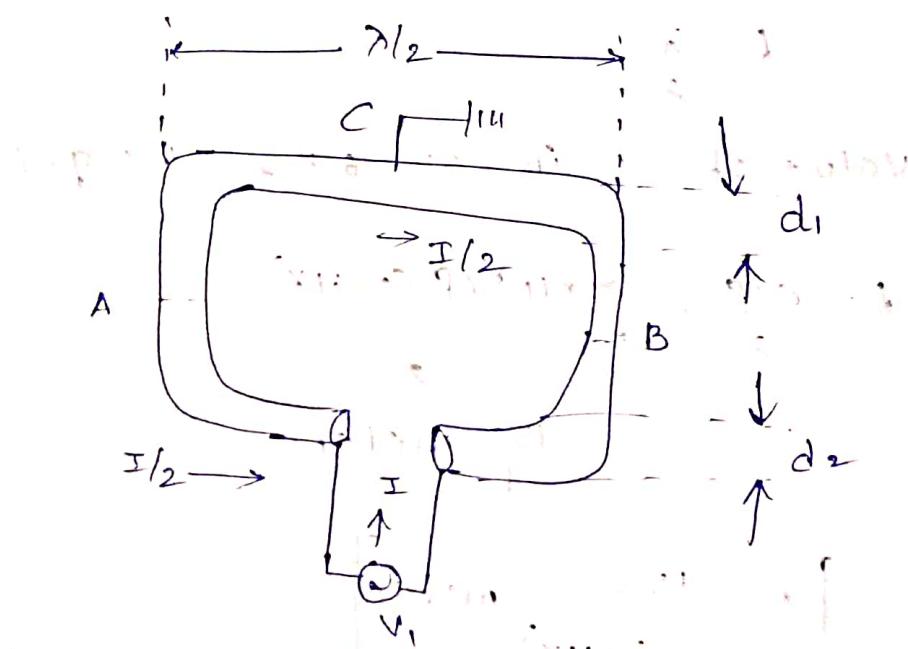
Straight dipole of $l = \frac{\lambda}{2} \Rightarrow \Omega$

Folded Dipole of $l = \frac{\lambda}{2} \Rightarrow \Omega/2$



bcoz of folded dipole the input impedance increases \uparrow

- folded dipole antenna is the important modification of the conventional half wave dipole in which the two half wave dipoles have been folded and joined together as shown in figure.



- one of the half wave dipoles is continuous while other is at the centre
- The folded dipole which is split at the centre is fed with balance transmission line.
- As a result the voltages at the ends of two dipoles are same.
- When the radiation fields are considered, the two dipoles are parallel.

for normal dipole $R_{\text{rad}} = 73 \Omega$

power radiated (P) = $I^2 R_{\text{rad}}$

$$R_{\text{rad}} = \frac{P}{I^2} = 73 \Omega$$

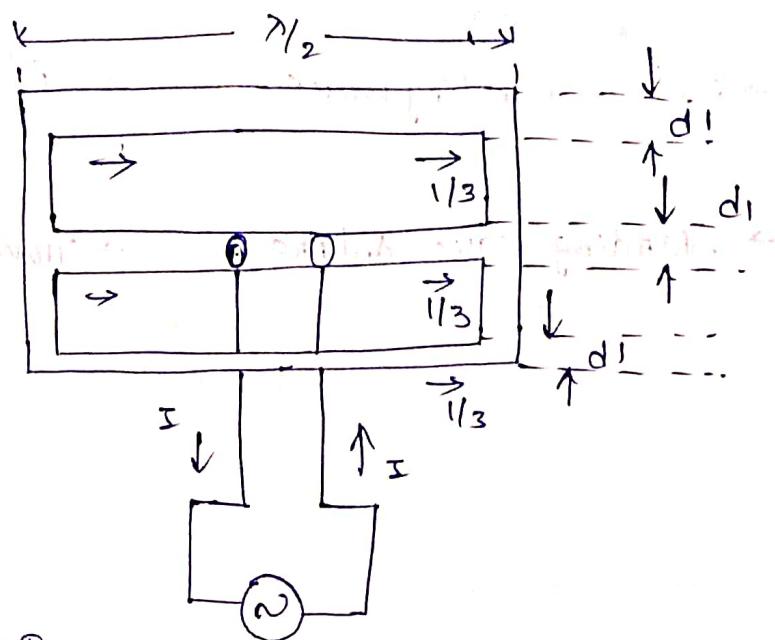
For Folded dipole (2-fold)

$$R_{\text{rad}} = \frac{P}{(I/2)^2} = 4 \cdot \frac{P}{I^2}$$

$$R_{\text{rad}} = 4 \times 73 \Omega = 292 \Omega.$$

If a T.L. of 300Ω is connected to a folded dipole of 2 folds then there will be

3-wire Folded Dipole Antenna or Triple.



for normal dipole $R_{\text{rad}} = 73 \Omega$

for 3-wire folded Dipole Antenna

$$R_{\text{rad}} = \frac{P}{(I/3)^2} = 9 \cdot \frac{P}{I^2}$$

$= 9 R_{\text{rad}}$ (normal dipole)

$$= 9 \times 73 \Omega$$

$$= 657 \Omega$$