

18.03 Recitation 4

Complex exponential

1.

(a) Suppose $z^n = 1$. What must $|z|$ be? What are the possible values of $\arg(z)$?

(b) Do the examples $n = 4$ and $n = 6$: that is, find the fourth and the sixth roots of 1.

(c) Find the cube roots of i .

2. Trajectories of $e^{(a+bi)t}$ can vary a lot, depending upon the value of the complex number $a+bi$. The “Complex Exponential” Mathlet shows this clearly. Invoke this java applet if you can. You can use it to gain insight into the following questions.

(a) Sketch the trajectory of the complex-valued function $e^{(-1+2\pi i)t}$, and the graphs of its real and imaginary parts.

For each of the following shapes, decide on all the values of $a + bi$ for which the trajectory of $e^{(a+bi)t}$ has this shape.

(b) A circle centered at 0, traversed counterclockwise. What circles are possible?

(c) A circle centered at 0, traversed clockwise.

(d) A ray (straight half line) heading away from the origin.

(e) A curve heading to zero as $t \rightarrow \infty$.

3. Since you differentiate a vector-valued function coordinatewise, you should also integrate a vector-valued function coordinatewise.

(a) What is $\int e^{rt} dt$? (Here r is a constant, perhaps complex.)

(b) Integrals like $\int e^{2t} \cos(3t) dt$ are memorably annoying to compute. Do this integral by writing $e^{2t} \cos(3t)$ as the real part of a complex exponential and invoking (a).