18.03 Recitation 8

Higher Order DEs

1. Which of the following equal $\text{Span}(e^{2t},\, e^{-2t},\, e^{i2t},\, e^{-i2t})$ (with complex coefficients)?

   - $\text{Span}(e^{2t},\, e^{-2t},\, \cos(2t),\, \sin(2t))$
   - $\text{Span}(e^{2t},\, -e^{-2t},\, 3\cos(2t),\, -\sin(2t))$
   - $\text{Span}(e^{2t} + e^{-2t},\, \cos(2t)/2,\, \sin(2t))$
   - $\text{Span}(e^{2t}, e^{-2t}, e^{2t} - e^{-2t},\, \cos(2t),\, \sin(2t))$
   - $\text{Span}(e^{2t}, e^{-2t}, e^{2t} - e^{-2t},\, \cos(2t - \pi/3),\, \sin(2t))$

2. If a vector space $V = \text{Span}(v_1,\, v_2,\, v_3)$, where $v_1,\, v_2,\, v_3$ are non-zero vectors, then which of the following can be the dimension of V?

   - 0
   - 1
   - 2
   - 2.5
   - 3
   - 4

3. If vectors $v_1,\, v_2,\, v_3$ belong to a vector space V, does it follow that $\text{Span}(v_1,\, v_2,\, v_3) = V$? $\text{Span}(v_1,\, v_2,\, v_3)$ is a subset of V?

4. Here's a third order linear equation: $\dfrac{d^3x}{dt^3} - \dfrac{dx}{dt} = 0$. A mystery!
   All question below relate to the DE above.

   (a) Find the characteristic roots of the DE above. There should be 3 of them since it is an order 3 equations.

   (b) How many solutions are there to the DE above that also satisfies $x(2) = 1;\ x'(2) = 1$?

   (c) Find a solution to the DE above that satisfies $x(0) =,\ x'(0) = 1$, and $x''(0) = 1$.

5. Find the simplest homogeneous constant-coefficient ODE having

$$\left(1 + 4t - t^2\right) e^{-2t} + 4e^{-t}\cos(3t)$$

   as one of its solutions.