18.03 Recitation 3

Complex numbers

1. (a) Compute the product $(1 + \sqrt{3}i)(a + bi)$ (where a, b are real).

   (b) What are the polar coordinates of $1 + \sqrt{3}i$? Express $1 + \sqrt{3}i$ in the form $re^{i\theta}$, $r > 0$.

   (c) What are the polar coordinates of $(1 + \sqrt{3}i)(a + bi)$ in terms of the polar coordinates of $a + bi = re^{i\theta}$?

   (d) Describe geometrically what multiplying by $1 + \sqrt{3}i$ does.

   (e) Describe the sequence of powers of $1 + \sqrt{3}i$, positive and negative.

2. Explain why $|z^n| = |z|^n$ and $\arg(z^n) = n \arg(z)$ for n a positive integer.

3. Find an expression for $\sin(4t)$ in terms of powers of $\cos t$ and $\sin t$, using $(e^{it})^4 = e^{4it}$ and Euler's formula.