18.03 Recitation 10

ERF and complex replacement

ERF: If $p(r) \neq 0$ then $x_p = A\dfrac{e^{rt}}{p(r)}$ is a solution of $p(D)x = Ae^{rt}$.

1. (a) Find a sinusoidal solution of $\ddot{x} + 2\dot{x} + 2x = \cos(2t)$. What is the general solution? Let $x(t)$ be *any* solution. Describe what it looks like for t sufficiently large.

(b) Find a solution of $\ddot{x} + 2\dot{x} + 2x = e^{-t}\cos(2t)$ using the method of complex replacement.

2. Find the sinusoidal solution of

$$\ddot{x} + \omega_n^2 x = a\cos(\omega t) \qquad \text{and of} \qquad \ddot{x} + \omega_n^2 x = b\sin(\omega t)$$

(Careful about when the two frequencies coincide!)

3. Find a particular solution of the equation

$$\ddot{x} + 4x = 2\cos t + 3\cos(2t) + 4\sin(3t)$$

4. (a) Find the complex gain of the following system given that the input is $\cos t$ and the response is x.

$$(D^4 + 2D^2 + 5)x = (D^2 + D)\cos(t)$$

(b) Find the complex gain of the following system as a function of a (where it is defined) given that the input is $\cos at$ and the response is x.

$$(D^4 + 2D^2 + 5)x = (D^2 + D)\cos(at)$$

(c) What is the magnitude of the complex gain when a is close to 0?

(d) What is the magnitude of the complex gain when a is large?

(e) Is this system stable?