Introduction to algorithm and analysis
Assignment 10 solution

(1) Answer- c

Explanation:- When the total weight of the graph sum upto a (-ve) number then the graph is said to have a negetive weight cycle.

(2) Answer— b.

(3) Answer— b.

Explanation- There are 4 edges s-a, a-b, b-t and s-t of weights 1, 1, 1 and 4 respectively. The shortest path s to t is s-a, a-b, b-t. If we increase weight of every edges by 1, the shortest path changes to s-t.

(4) Answer— b.

(5) Answer— c

(6) Answer— c

Explanation- Time complexity of Bellman Ford algo is $\theta(VE)$ where V, E are numbers of vertices and edges respectively. For complete graph $V = n$, $E = \theta(n^2)$
So, overall time complexity is — $\theta(n^3)$

(7) Answer— b.

transitive closure of Graph is

$$t_{ij}^{(k)} = t_{ij}^{(k-1)} \vee \left(t_{ik}^{(k-1)} \wedge t_{kj}^{(k-1)} \right)$$

(8) Answer- a

By Applying Floyd warshall algo we get following matrix

$$D^{(0)} = \begin{bmatrix} 0 & 8 & \infty & 1 \\ \infty & 0 & 1 & \infty \\ 4 & \infty & 0 & \infty \\ \infty & 2 & 9 & 0 \end{bmatrix} \quad \text{Now, } D^{(1)} = \begin{bmatrix} 0 & 8 & \infty & 1 \\ \infty & 0 & 1 & \infty \\ 4 & 12 & 0 & 5 \\ \infty & 2 & 9 & 0 \end{bmatrix}$$

$D^{(2)} = \begin{bmatrix} 0 & 8 & 9 & 1 \\ \infty & 0 & 1 & \infty \\ 4 & 12 & 0 & 5 \\ \infty & 2 & 3 & 0 \end{bmatrix}$ $D^{(3)} = \begin{bmatrix} 0 & 8 & 9 & 1 \\ 5 & 0 & 1 & 6 \\ 4 & 12 & 0 & 5 \\ 7 & 2 & 3 & 0 \end{bmatrix}$

$D^{(4)} = \begin{bmatrix} 0 & 3 & 4 & 1 \\ 5 & 0 & 1 & 6 \\ 4 & 7 & 0 & 5 \\ 7 & 2 & 3 & 0 \end{bmatrix}$ which represents the shortest path between every pair of vertices.

(9) Answer – a

Explanation — Recurrence relation of Floyd-Washall algo. is

$$C_{ij}^{(k)} = \min_k \left\{ C_{ij}^{(k-1)}, C_{ik}^{(k-1)} + C_{kj}^{(k-1)} \right\}$$

(10) Answer – a

Explanation — Worst case time complexity of Johnson's algo. is - $O(|V|^2 \log|V| + |V||E|)$