18.03 Recitation 24, Fall 2014

Direction fields, Integral Curves, Isoclines, Separatrices

1. The following questions concern the differential equation

$$\frac{dy}{dx} = x - 2y.$$

You may want to use the Isoclines Mathlet to work through these problems.

(a) Draw a big axis system and plot some isoclines, especially the nullcline. On each isocline, draw short line segments of the appropriate slope. Then use this information to plot a few solutions. Do this by hand first. Later you might want to refer to the `Isoclines` Mathlet.

(b) One of the integral curves seems to be a straight line. Is this true? What straight line is it? (i.e. for what m and b is $y = mx + b$ a solution?)

(c) In general – for the general differential equation $y' = F(x, y)$ – if a straight line is an integral curve, how is it related to the isoclines of the equation? What happens in our example?

(d) Where are the critical points of the solutions of $y' = x - 2y$? How many critical points can a single solution have? For what values of y_0 does the solution y with $y(0) = y_0$ have a critical point? When there is one, is it a minimum or a maximum? You can see an answer to this from your picture. Can you also use the second derivative test to be sure? It looks as though none of the solutions has a point of inflection (where the second derivative vanishes). Is this true?

2. The following questions concern the differential equation $y' = y^2 - x$. (This is also on the `Isoclines` Mathlet, and was discussed in lecture.)

   (a) As in the previous problem, make a big picture of some isoclines and use them to sketch the direction field, and then sketch a few solutions.

   (b) A "separatrix" is a solution such that solutions above it have a fate (as x increases) entirely different from solutions below it. The equation $y' = y^2 - x$ exhibits a separatrix. Sketch it and describe the differing behaviors of solutions above it and below it. (You may need the applet to get a precise idea where the separatrix is.)