18.03 Recitation 2

Linear equations

1. (a) Using x for the independent variable and y for the dependent variable, what is standard linear form?

Is the equation $y' - (\tan x)y = 1$ linear? Is it separable?

In **(b)** and **(c)**, find the general solution of this equation by variation of parameters:

(b) Find a basic solution y_h of the associated homogeneous equation.

[18.01 reminder: $\int \tan x \, dx = \int \dfrac{\sin x}{\cos x} \, dx$ suggests the substitution $y = \cos x$.]

(c) Find $u(x)$ such that $y = uy_h$ is a solution. Write down the general solution of $y' - (\tan x)y = 1$.

(d) There is an alternative to variation of parameters which is sometimes fun; it's called the "method of integrating factors," and like so much else it was invented by Leonhard Euler. In it, you multiply your inhomogeneous linear equation through by some function (called an integrating factor) chosen by insight or calculation so that in the resulting equation the two terms on the left-hand side are the two terms in the formula for the derivative of a product.

In this example, try multiplying through by $\cos x$. Find the general solution this way.

(e) Solve
$$y' - xy = 1$$
by the method of integrating factors.