18.03 Recitation 1

Modeling: natural growth

A certain African government is trying to come up with a good policy regarding the hunting of oryx in a specific game preserve. They are using the following model: the oryx population has a positive natural growth rate of k years^{-1}, and there is assumed a constant harvesting rate of a oryxes/year.

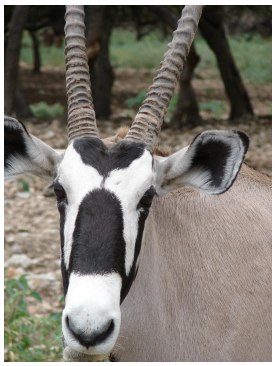

Figure 1: A picture of *Oryx gazella*, popularly known as a Gemsbok. An inhabitant of southern Africa.

1. Write down a differential equation modeling the oryx population. [First step: choose symbols and units.]

2. Discuss this model using the language of signals and systems.

3. Suppose $a = 0$: no hunters. What is the *doubling time* (in terms of k)? What is the relation between the population now and the population after k^{-1} years?

4. Find the general solution of the equation you found in **(1)**. Check that the proposed solution satisfies the ODE.

5. There is a constant solution. Find it. Does the way the solutions depend upon k and a make sense? That is, do the units come out right? Does the solution behave right when a is large or small? When k is large or small? Sketch the graphs of some solutions.

6. Notice that for initial values less than the equlibrium, the solutions stop having meaning in terms of the real-world situation they are modeling when they become negative. In these cases, predict the time t_e at which oryxes will be extirpated from this area. For example, suppose that $x(0) = x_0$ is less than the equilibrium population. For this initial condition what is t_e? Check units.

7. Would you recommend that the government base a policy on this idea, for any value of a?