Other Basic Examples

1. Other Basic Examples

Here are some basic examples of DE's taken from math and science. Except for example 1 we will not give solutions. We will do that and more with these DE's as we go through the course.

Example 1. (From Calculus)
Solve for y satisfying $\frac{dy}{dx} = 2x$

Solution. This problem is just asking for the anti-derivative of $2x$:

$$y(x) = x^2 + c.$$

Notice that there are many solutions, parametrized by c. An expression like this, which parametrizes all the solutions is called **the general solution**.

Example 2. (Heat Diffusion)
A body at temperature T sits in an environment of temperature T_E. Newton's law of cooling models the rate of change in temperature by

$$T' = -k(T - T_E),$$

where k is a positive constant. Note, the minus sign guarantees that the temperature T is always heading towards the temperature of the environment T_E.

Example 3. (Newton's Law of Motion: Constant Gravity)
Near the earth a body falls according to the law

$$\frac{d^2y}{dt^2} = -g,$$

where y is the height of the body above the Earth and g is the acceleration due to gravity, 9.8 m/sec^2.

Example 4. (Newton's Law of Gravitation)
Newton's law of gravity says that the acceleration due to gravity of a body at distance r from the center of the Earth is

$$\frac{d^2r}{dt^2} = -GM_E/r^2,$$

where M_E is the mass of the Earth and G is the universal gravitational constant.

Example 5. (Simple Harmonic Oscillator: Hooke's Law)
Suppose a body of mass m is attached to a spring. Let x be the amount the spring is stretched from its unstretched *equilibrium position*. Hooke's law combined with Newton's law of motion says

$$m\ddot{x} = -kx \quad \Leftrightarrow \quad m\ddot{x} + kx = 0,$$

where k is the **spring constant**. The minus sign indicates that the force always points back towards equilibrium, as it does in the real world.

Example 6. (Damped Harmonic Oscillator)
If we add a damping force proportional to velocity to the spring-mass system in example 5, we get

$$m\ddot{x} = -kx - b\dot{x} \quad \Leftrightarrow \quad m\ddot{x} + b\dot{x} + kx = 0,$$

here $-b\dot{x}$ is the damping force and b is called the **damping constant.**

Example 7. (Damped Harmonic Oscillator with an External Force)
If we add a time varying external force $F(t)$ to the system in example 6, we get

$$m\ddot{x} = -kx - b\dot{x} + F(t) \quad \Leftrightarrow \quad m\ddot{x} + b\dot{x} + kx = F(t).$$