18.03 Recitation 7

Damped harmonic oscillators

1. **(a)** An important use of damping is to bring a system into equilibrium. In many mechanical systems, vibrations are a noisy nuisance or even dangerous. For example if your airplane wing starts to vibrate, then you want it to settle down promptly. The spring-mass-dashpot

$$m\ddot{x} + b\dot{x} + kx = 0$$

   is our basic way of modeling vibrations. We will study how the damping constant b affects the rate at which vibrations settle down in the mathlet Damped Vibration.

   Use the sliders to set the values of $m = 1/2$, $b = 1$ and $k = 1/2$ and drag the point to change the initial conditions to $x(0) = 1$, $\dot{x}(0) = -1$.

   Use the crosshairs by mousing over the graph to see x and t values; report to two decimals the time it takes before the value settles to equilibrium. For this purpose we will define "reaching equilibrium" to mean when the system reaches the range $\pm.005$.

   (Look at the system at scale $x = 1$ to get the overall picture and then zoom in to scale .01 to get a more accurate view of the behavior near $x = 0$.)

   Try this for $b = 0.6, 0.8, 1.0, 1.2$, and 1.4. Note that the value $b = 1.0$ is the case of critical damping.

   (b)

   Which value of b brings the system to equilibrium fastest? For this purpose we will define "reaching equilibrium" to mean when the system reaches the range $\pm.005$. Report this "best" value of b for the initial conditions $(x(0), \dot{x}(0)) = (1, -1)$ above and also for $(x(0), \dot{x}(0)) = (0, -1)$ and $(x(0), \dot{x}(0)) = (1, 0)$.

   By what percentage do the best values of b differ for the given initial conditions?

2. A series of problems related to oscillations of a boat in the water can be found interspersed between a series of videos that introduce the problem, context, and model.