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% MATLAB Demo (Least squares)
% File: lsdemo
%
% Find the least-squares parabola for the 5 data points:
%
%            t  |    y
%          ==============
%            0  |   -3
%            1  |    7.5
%            2  |    2.5
%            3  |   14.5
%            4  |   23.5
%
% Formulate problem in terms of finding the least-squares solution to Ax = b.
% Verify the results, and plot a graph of the data vs. the parabola.
% Compute the projection matrix for the column space of A.
%
%  Outline:
%  The matrix A has 5 rows; row k is [ 1   t_{k}   t_{k}^2 ].
%  The vector b has 5 components; component k  is y_{k}.
%  Use  ref([A b]) to show that Ax = b is NOT solvable.
%  Use 18.06 MATLAB command  lsq(A, b) to find the least-squares solution.
%  Verify: p = A*xbar, p+e = b, and e is orthogonal to p.
%  Give a plot of data points vs. the least-squares parabola.
%  Compute the projection matrix, or use 18.06 command  projmat(A)  .

>> diary lsdemo
>> b = [-3; 7.5; 2.5; 14.5; 23.5];
>> A = [1 1 1 1 1; 0 1 2 3 4; 0 1 4 9 16];
>> A = A'
A =
     1     0     0
     1     1     1
     1     2     4
     1     3     9
     1     4    16

>> Z = [A b]
Z =
    1.0000         0         0   -3.0000
    1.0000    1.0000    1.0000    7.5000
    1.0000    2.0000    4.0000    2.5000
    1.0000    3.0000    9.0000   14.5000
    1.0000    4.0000   16.0000   23.5000
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```
>> Z = ref(Z)
Z =
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
     0     0     0     0
%
%%% This reduced row echelon form [R d] shows that d cannot
%%% be expressed as a linear combination of the columns of R.
%%% Rx=d (and Ax=b) is not solvable.
%
>> help lsq
 LSQ    Least squares.
        [xbar,p,e] = LSQ(A,b) finds a least squares
        solution to the overdetermined system A*x ~= b.
        xbar = the solution to the normal equations.
        p = projection of b onto the column space.
        e = b - p.

>> [xbar, p, e] = lsq(A, b)
xbar =
    -1
     2
     1
p =
    -1
     2
     7
    14
    23
e =
   -2.0000
    5.5000
   -4.5000
    0.5000
    0.5000

%%% Check: A*xbar = p.
>> A * xbar
ans =
    -1
     2
     7
```

```
    14
    23

%%% Check: p+e = b.
>> p+e
ans =
   -3.0000
    7.5000
    2.5000
   14.5000
   23.5000

%%% Check: e is orthogonal to p.
>> p'*e
ans =
     0
%
%%% Plot the data points and parabola over the interval -1 <= t <= 5.
%
>> t = linspace(-1, 5, 21);
>> t = t';
>> xbar
xbar =
    -1
     2
     1
%
%%% Compute y(t) for each value of t in the interval [-1, 5].
%
>> y = xbar(1) + xbar(2)*t + xbar(3)*t.^2;
>> plot(t,y,'r-');
>> grid on; hold on
%
%%% Plot the 5 data points, and label the axes.
%
>> plot([0; 1; 2; 3; 4], b, 'g+');
>> xlabel('t');
>> ylabel('y(t)');
>> title('Least-squares parabola: y(t)');
>> print -dps parabola.ps

%
%%% Compute the projection matrix, P = A*inv(A'*A)*A'.
%%% Recall that A has 5 rows and 3 columns.
%%% A must have linearly independent columns in order for A'*A
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%%% to be invertible.
%
>> format rat
>> P = A*inv(A'*A)*A'
P =
    31/35          9/35         -3/35         -1/7           3/35
     9/35         13/35         12/35          6/35         -1/7
    -3/35         12/35         17/35         12/35         -3/35
    -1/7           6/35         12/35         13/35          9/35
     3/35         -1/7          -3/35          9/35         31/35

%%% Check: the columns of P span a 3-dimensional subspace of R^{5}.
%%% In particular, this subspace is the column space of the matrix A.
>> rank(P)
ans =
      3

>> help projmat
 PROJMAT Projection matrix onto the column space.
        P = projmat(A) is the symmetric, square matrix that
        projects any vector onto the column space of A.

>> projmat(A)
ans =
    31/35          9/35         -3/35         -1/7           3/35
     9/35         13/35         12/35          6/35         -1/7
    -3/35         12/35         17/35         12/35         -3/35
    -1/7           6/35         12/35         13/35          9/35
     3/35         -1/7          -3/35          9/35         31/35

%%% Check: The projection of b onto the column space of A is p.
>> projmat(A)*b
ans =
     -1
      2
      7
     14
     23
>> diary off
```