18.031x Recitation 14

Euler's method

Euler's Method: Numerical method to compute $y(x)$ of $y' = f(x, y)$ with initial condition $y(x_0) = y_0$.

This is the only numerical method you are responsible for. (Memorize it.) The idea is to follow the direction field at a fixed step size $h > 0$. Start at $P_0 = (x_0, y_0)$ and the iterative formulas are given by

$$x_{n+1} = x_n + h; \quad y_{n+1} = y_n + h\, f(x_n, y_n)$$

Error Estimates: $E = y_{\text{approx}} - y_{\text{exact}}$

$$|E| \leq Ch$$

where C is on the order of $|f| + |\partial f/\partial x| + |\partial f/\partial y|$.

Remark. If $y'' < 0$ in a region (concave down), then $E > 0$. (The approximation is bigger than the actual answer.) Similarly, if $y'' > 0$, then $E < 0$. (The approximation is smaller than the actual answer.)

Tasks:

1. Use Euler's method to estimate the value at $x = 1.5$ of the solution of $y' = y^2 - x^2 = F(x, y)$ at $y(0) = -1$. Use $h = 0.5$. Recall the notation $x_0 = 0$, $y_0 = -1$, $x_{n+1} = x_n + h$, $y_{n+1} = y_n + m_n h$, $m_n = F(x_n, y_n)$. Make a table with columns $n, x_n, y_n, m_n, m_n h$. Draw the Euler polygon.

2. Is the estimate from **1.** likely to be too large or too small?