Other Basic Examples

Here are some basic examples of DE’s taken from math and science. Except for example 1 we will not give solutions. We will do that and more with these DE’s as we go through the course.

Example 1. (From Calculus) dy Solve for y satisfying = 2x dx Solution. This problem is just asking for the anti-derivative of 2x: y(x) = x2 + c.

Notice that there are many solutions, parametrized by c. An expression like this, which parametrizes all the solutions is called the general solution.

Example 2. (Heat Diffusion) A body at temperature T sits in an environment of temperature TE. Newton’s law of cooling models the rate of change in temperature by T' = −k(T − TE), where k is a positive constant. Note, the minus sign guarantees that the temperature T is always heading towards the temperature of the environment TE.

Example 3. (Newton’s Law of Motion: Constant Gravity) Near the earth a body falls according to the law d2y = −g,dt 2 where y is the height of the body above the Earth and g is the acceleration due to gravity, 9.8 m/sec2.

Example 4. (Newton’s Law of Gravitation) Newton’s law of gravity says that the acceleration due to gravity of a body at distance r from the center of the Earth is d2r 2 = −GME/r ,dt 2

Other Basic Examples OCW 18.03SC

where ME is the mass of the Earth and G is the universal gravitational constant.

Example 5. (Simple Harmonic Oscillator: Hooke’s Law) Suppose a body of mass m is attached to a spring. Let x be the amount the spring is stretched from its unstretched equilibrium position. Hooke’s law combined with Newton’s law of motion says .. .. mx = −kx ⇔ mx + kx = 0, where k is the spring constant. The minus sign indicates that the force always points back towards equilibrium, as it does in the real world.

Example 6. (Damped Harmonic Oscillator) If we add a damping force proportional to velocity to the spring-mass system in example 5, we get .. . .. . mx = −kx − bx ⇔ mx + bx + kx = 0, . here −bx is the damping force and b is called the damping constant. Example 7. (Damped Harmonic Oscillator with an External Force) If we add a time varying external force F(t) to the system in example 6, we get .. . .. . mx = −kx − bx + F(t) ⇔ mx + bx + kx = F(t).