Primer: Incorporating Legibility Into Optimal Control

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1 Introduction

The legibility of a trajectory is usually modeled using the ability of a Bayesian observer to efficiently infer the robot's actual goal. A trajectory is more legible if the intent can be inferred earlier. Unfortunately, this objective isn't quadratic and can not be easily decomposed into state-wise costs. Therefore, the legibility objective doesn't fit into the class of cost functions the standard optimal control methods are designed to solve. Our goal is to modify the legibility objective and the standard LQR formulation to be able to generate legible and dynamically feasible trajectories.

We compute a lower bound to the legibility score using Jensen's inequality; this lower bound can then be easily decomposed into state-wise costs. Our experiment results validate that both methods can generate legible and dynamically feasible robot motion.

1.1 Motivating example

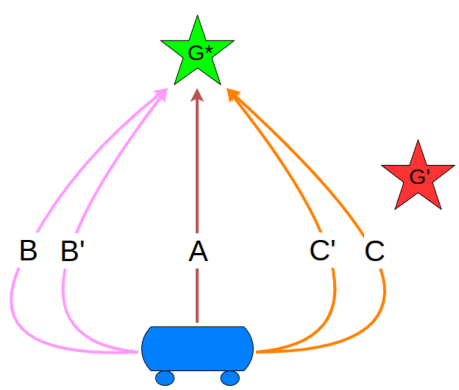

Figure 1: We see a motivating example on a dynamically constrained car that is trying to reach the green goal G^*.

Figure 1 shows a motivating scenario for coupling the optimal control trajectory optimization with the legibility optimization. Assume our blue car robot has dynamic constraints on its turning radius, has a heading on the x-axis, and is trying to reach the green goal G^*. It, therefore, needs to go in the +x axis and turn left, or go in the -x direction and turn right to reach the goal.

Pretend we start with an initial reference trajectory A and then run our favorite optimal control algorithm (OFOCA) to get the dynamically feasible trajectory C. Note that C can have a tight turning radius to minimize deviation from the reference trajectory. Afterward, we optimize for legibility (e.g., via gradient ascent) to get a more legibility path C'. C' is not necessarily dynamically feasible! Suppose we feed C' as a reference trajectory into OFOCA. In that case, we might easily get back C, resulting in no improvement in legibility.

Pretend we do the opposite. Suppose we start with a legible path and then apply OFOCA to get our dynamically feasible trajectory. If our legible path is anywhere on the right side of A (e.g., C', C), applying OFOCA will result in the same issue as above. However, if we are on the left side of A (e.g., B'), applying OFOCA would result in a dynamically feasible path B.

We hypothesize that by jointly optimizing legibility and dynamic feasibility, we can avoid the issue of legibility and dynamic feasibility causing trajectory changes in opposite directions. Instead, we can jointly optimize for a dynamically feasible and legible trajectory. In our example, given a reference trajectory A, there is a 50% chance that vanilla OFOCA picks C rather than B. With our techniques, we hope our joint optimization results in B, which has the same LQR cost but higher legibility than C.

2 Standard Legibility Definition

Mathematically, the legibility of a trajectory ξ given several goals G_i with target goal G^* is the following:

$$\text{Legibility}(\xi, G^*, G_i's) = \frac{\int_{t=0}^{T} P(G^*|\xi_{S\to\xi(t)})f(t)dt}{\int_{t=0}^{T} f(t)dt} \tag{1}$$

where $f(t)$ is a function to give more weight to earlier parts of a trajectory, e.g. $f(t) = T - t$ and $\int_{t=0}^{T}$ is the integral across a trajectory.

Let $Q = \xi(t)$, i.e. Q is an intermediate part of the path. Then to compute $P(G^*|\xi_{S\to Q})$, we notice $P(G|\xi_{S\to Q}) \propto P(\xi_{S\to Q}|G)*P(G)$. With a uniform prior over all goals G, this turns into:

$$P(\xi_{S\to Q}|G) = \frac{P(\xi_{S\to Q}) * \int_{\xi_{Q\to G}} P(\xi_{Q\to G})}{\int_{\xi_{S\to G}} P(\xi_{S\to G})}$$

Via some tricks, this gets reduced to the computationally feasible:

$$P(\xi_{S\to Q}|G) = \frac{\exp(-C(\xi_{S\to Q}) - C(\xi^*_{Q\to G}))}{\exp(-C(\xi^*_{S\to G}))}\frac{1}{Z} \tag{2}$$

where $C(\alpha)$ is the cost of a trajectory α (e.g. path length) and $Z = \sum_{G'} P(\xi_{S \to Q}|G')$. Intuitively, to increase legibility of a subpath through Q, we want to reduce $C(\xi_{S \to Q \to G*})$ (Equation 2 numerator) while increasing $C(\xi_{S \to Q \to G'})$ (via the Equation 2 proportionality). Previous work in legibility [1] generates legible trajectories by starting from the optimal minimum cost trajectory (i.e. a straight line to goal) and then doing functional gradient ascent on the legibility function.

3 Approach

The optimal control techniques such as LQR try to minimize an objective of the form:

$$J(x_{1:N}, u_{1:N-1}) = J_N(x_N) + \sum_{i=1}^{N-1} J(x_k, u_k)$$

where

$$J(x, u) = \frac{1}{2}(x - x_{ref})^T Q (x - x_{ref}) + \frac{1}{2}(u - u_{ref})^T R (u - u_{ref})$$

$$J_N(x) = \frac{1}{2}(x - x_{ref})^T Q_f (x - x_{ref})$$

Our main technical challenge is that the legibility L of a trajectory $x_{1:N}, u_{1:N-1}$ is not decomposable as a sum along x_i and u_i. Specifically, using Equations 1 and 2 and simplifying constants with $f(t) = 1/N$, we get $L(x_{1:N}, u_{1:N-1}) = \sum_i P(\xi_{S \to x_i}|G^*)$. Unfortunately $P(\xi_{S \to x_i}|G^*)$ is a function of the entire trajectory beforehand and contains exponentials, and therefore cannot be directly written as $x_i^T Q_i x_i + u_i^T R_i u_i$.

3.1 Log-legibility

We can get rid of the pesky exponential terms by utilizing Jensen's inequality and maximizing the lower bound of the log legibility. Mathematically:

$$\begin{aligned}
\text{argmax}_\xi L(\xi) &= \text{argmax}_\xi \ln L(\xi) \\
\text{argmax}_\xi \ln L(\xi) &= \text{argmax}_\xi \ln \sum_i P(\xi_{S \to x_i}|G^*) \\
\max_\xi \ln \sum_i P(\xi_{S \to x_i}|G^*) &\geq \sum_i \ln P(\xi_{S \to x_i}|G^*) \\
= \sum_i -C(\xi_{S \to x_i}) - C(\xi^*_{x_i \to G^*}) &+ C(\xi_{S \to G^*}) - \ln(Z)
\end{aligned} \tag{3}$$

We can rewrite each $C(A \to B) = (A - B)^T Q (A - B)$, which results in the final quadratic cost of

$$\begin{aligned}
L(x_i) = -(s - x_i)^T Q (s - x_i) &- (x_i - G^*)^T Q (x_i - G^*) \\
&+ (s - G^*)^T Q (s - G^*) - \ln(Z_{x_i})
\end{aligned} \tag{4}$$

Note that we assume here that $C(\xi_{S\to x_i})$ is a function of just x_i rather than the entire trajectory $\xi_{S\to x_i}$.

The new target stage cost $J(x_k, u_k)$ is defined as:

$$J(x_k, u_k) = -L(x_k) + \frac{1}{2}(u_k - u_{ref})^T R(u_k - u_{ref})$$

Since we want to maximize legibility, we include the negative legibility stage cost in the LQR stage cost, which is minimized. We additionally use the standard terminal cost to penalize deviance from the intended goal location.

4 Trajectories Generated by Legibility Controller

We run iLQR with the legibility formulation derived from maximizing the log-legibility.

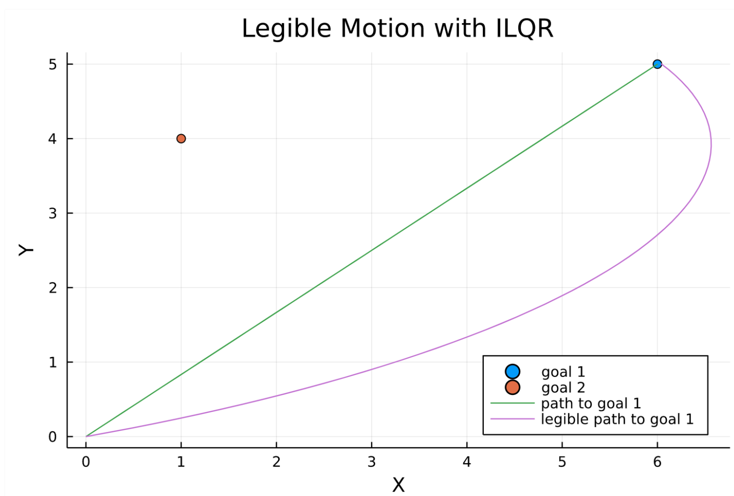

Figure 2: The iLQR controller is able to generate legible motion with the legibility-based stage cost over standard iLQR with goal-directed stage cost.

The original legibility formulation by Dragan et al. 2013 [2] frames legibility as identifying two goals that are about equidistant from the start and in the same direction. This formulation of LQR for legibility is able to generate legible motion using the same start to multiple goals setup.

We find that the amount of legibility in the planned robot path can be tuned as a function of time horizon. Higher planning time horizon would allow for the robot to take large deviations from the optimal straight-line trajectory, and still have enough time to make it to the goal in the end.

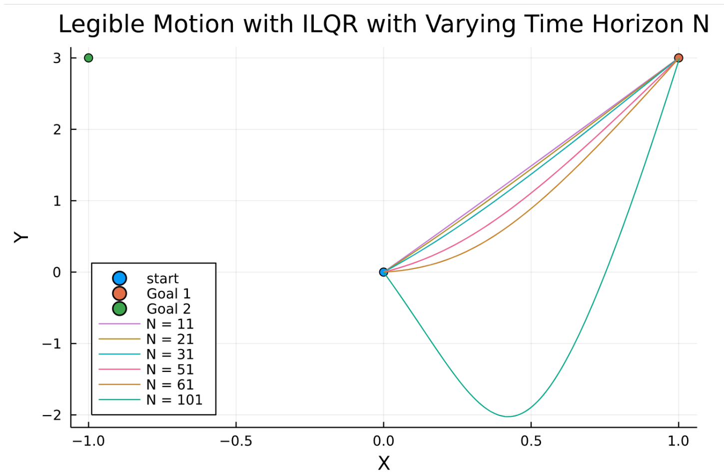

Figure 3: Legible path with increasing time horizon N.

There is limited benefit to this. When N is too large, the planned path becomes unstable.

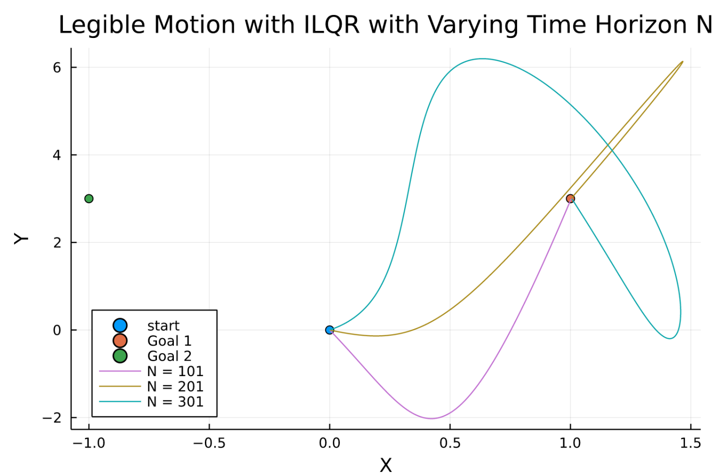

Figure 4: Legible path with increasing time horizon N fails when N is too large.

Additionally, we find that the amount of legibility in the planned robot path can be tuned as a function of a scale factor γ on non-intended goals in the normalization factor Z. Recall that the *stage legibility* $Legibility(x_k, u_k)$ is defined as: $L(x_k, u_k) = -C[\xi_{S\to x_k}] - C[\xi^*_{x_k\to G^*}]) + C[\xi_{S\to G^*}]) - \ln(Z_k)$ where $Z = \sum_G P(G|\xi_{S\to\xi(t)})$. We can modify Z so that there is a scale factor over non-intended goals

$$\hat{Z} = \sum_{G} P(G|\xi_{S\to\xi(t)}) \cdot (\gamma \mathbb{1}[G \neq G^*] + 1)$$

The amount of legibility in the planned robot path can be tuned as a function of a scale factor γ on non-intended goals in the normalization factor $\hat{Z}$. As γ increases, the amount the robot wants to avoid going towards non-intended goals increases, and thus willingness to deviate increases.

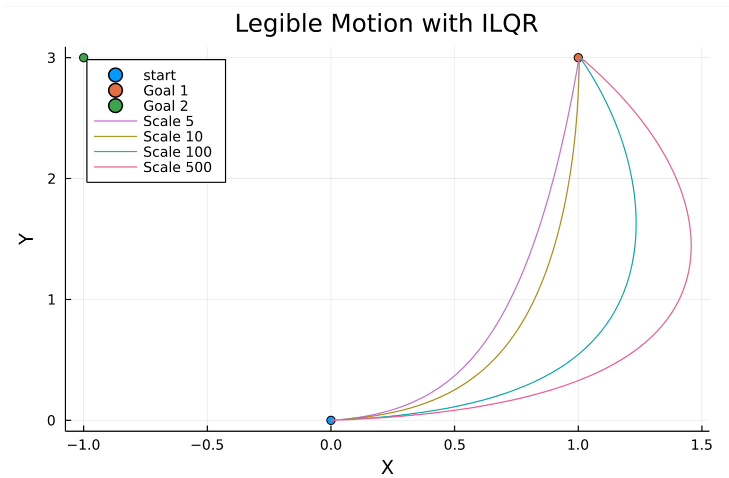

Figure 5: Legible path with increasing scale factor.

The LQR controller with the log-legibility objective works in cases with multiple goals. In the case where the intended goal is in between two non-intended goals, the best path for the robot to take is going straight to the middle goal because deviation to either left or right would indicate heading to those goals.

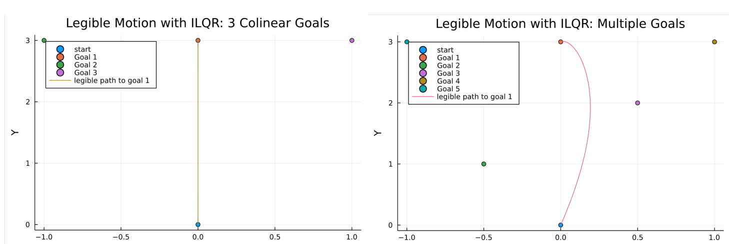

Figure 6: (Left) Legible motion with three goals. (Right) Legible motion with five goals.

In the 5 goal case, we consider a case where we might expect the motion to exhibit snaking behavior, moving away from the closest goal, then diverting away from a different goal once it becomes the next closest goal. However, we don't quite see this snaking motion.

4.1 Distance-Weighted Objective

We propose a weighted objective based on the closest non-intended goal. We again modify the normalization factor in the stage legibility, which is defined as $Legibility(x_k, u_k)$ is defined as:

$$L(x_k, u_k) = -C[\xi_{S \to x_k}] - C[\xi^*_{x_k \to G^*}]) + C[\xi_{S \to G^*}]) - \ln(Z_k)$$

where

$$Z_k = \sum_G P(G|\xi_{S \to \xi(t)})$$

. We modify Z using a scale factor multiplier, λ on the **closest** non-intended goal.

$$\tilde{Z}_k = \sum_G P(G|\xi_{S \to \xi(t)}) \cdot (\lambda \mathbb{1}[G \neq G^* \text{ AND } G = arg\, min_{G'}(x - G')^2] + 1)$$

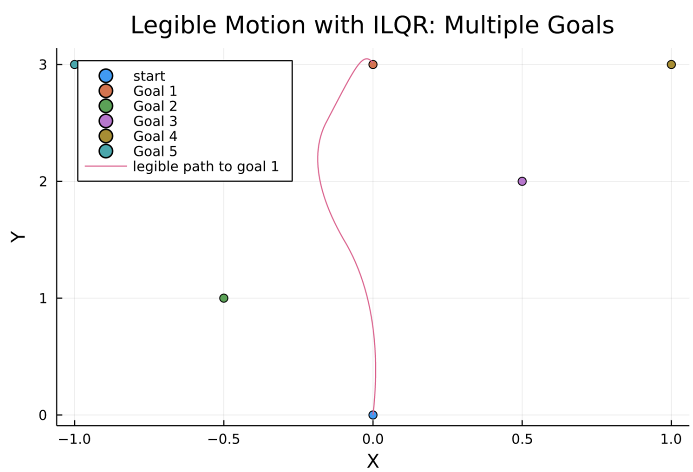

Figure 7: We generate snaking motion with a weighted objective based on the closest non-intended goal.

This modification generates snaking legible motion that diverges initially from the closest goal (green), and then once the path becomes close to the purple goal, diverges away from that goal in order to show that it is not headed to either of those two goals. This snakelike motion is debatably legible, but shows that this is another parameter that can be tuned in the legibility objective.

4.1.1 Failure Cases

We notice that when the non-intended goals are colinear to the start and intended goal (Figure 8), the iLQR controller with the log-legibility objective fails to avoid the non-intended goal lying on the straight-line path.

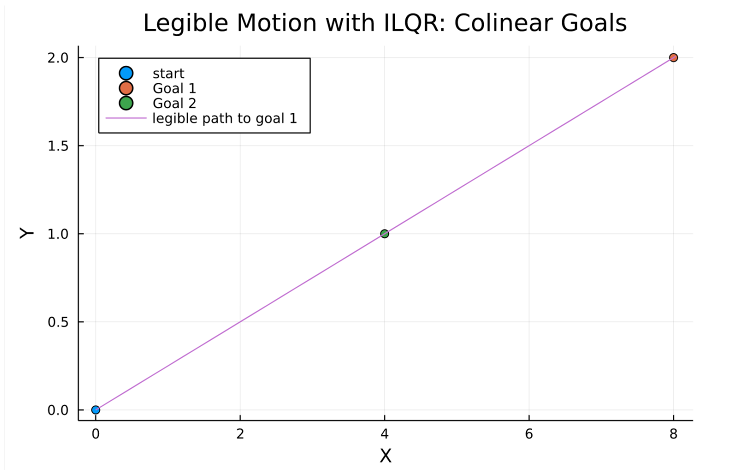

Figure 8: Legible path with colinear goals gives a straight line.

One solution to this is take the unintended goal that lines colinear to the intended goal and perturb the point, both uniformly or randomly, to create "proxy-goals." This solution gives us legible motion even with colinear goals.

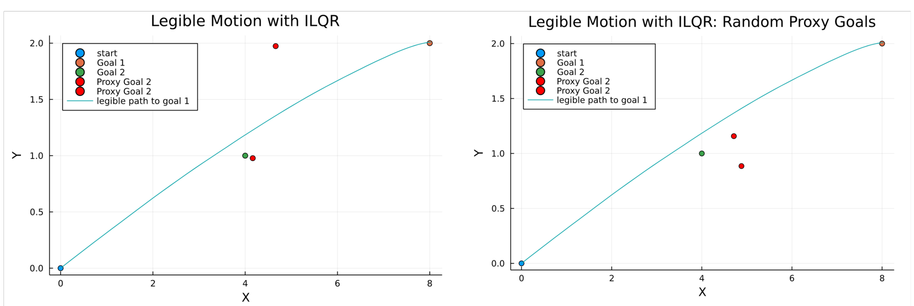

Figure 9: Legible path with colinear goals gives a legible path with proxy goals.

References

[1] Anca Dragan and Siddhartha Srinivasa. "Generating legible motion". In: (2013).

[2] Anca D. Dragan, Kenton C.T. Lee, and Siddhartha S. Srinivasa. "Legibility and predictability of robot motion". In: *2013 8th ACM/IEEE International Conference on Human-Robot Interaction (HRI)*. 2013, pp. 301–308. DOI: 10.1109/HRI.2013.6483603.